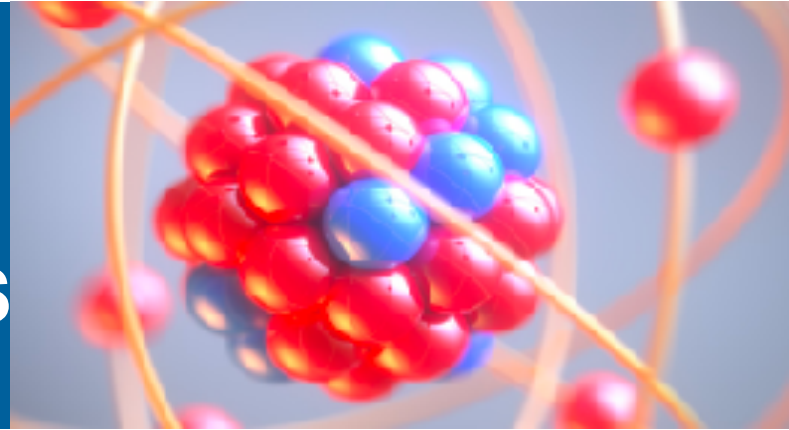


# NUCLEI AND NUCLEONIC MATTER WITH NEURAL- NETWORK QUANTUM STATES



ALESSANDRO LOVATO

Workshop on Progress in Ab Initio Nuclear Theory

TRIUMF, Vancouver, BC

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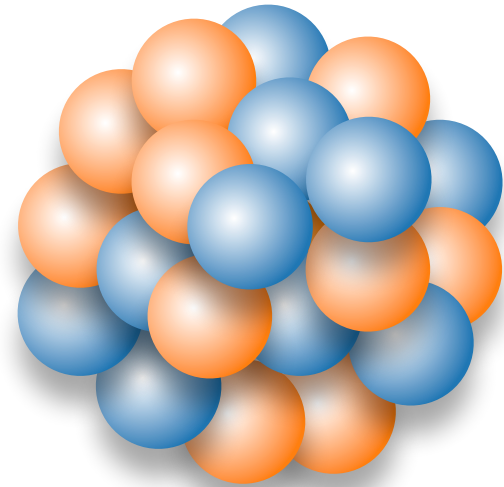
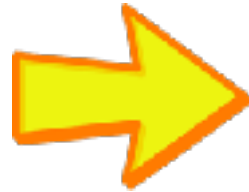
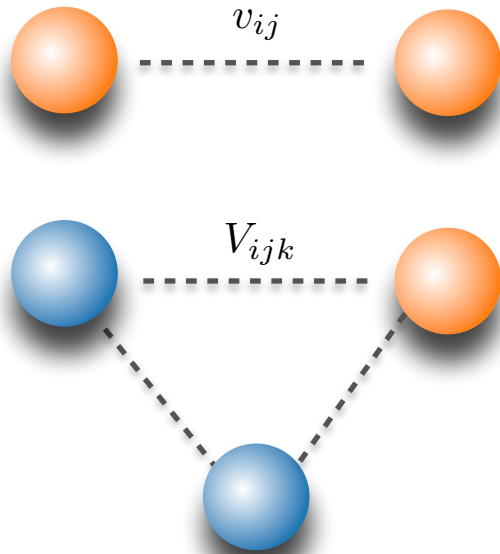


A. Di Donna, F. Pederiva

# THE QUANTUM MANY-BODY PROBLEM

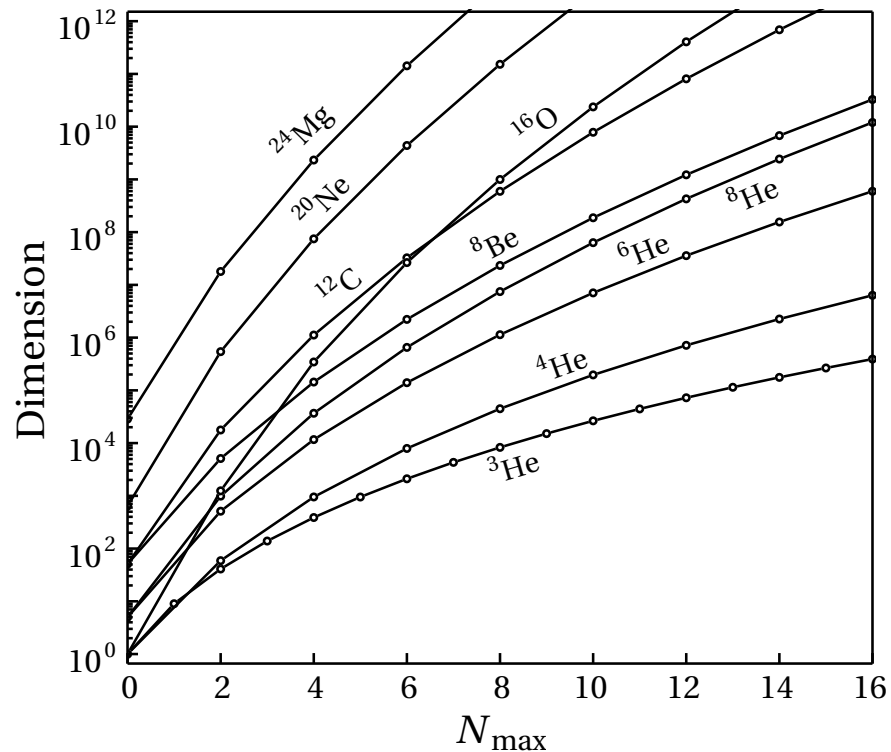
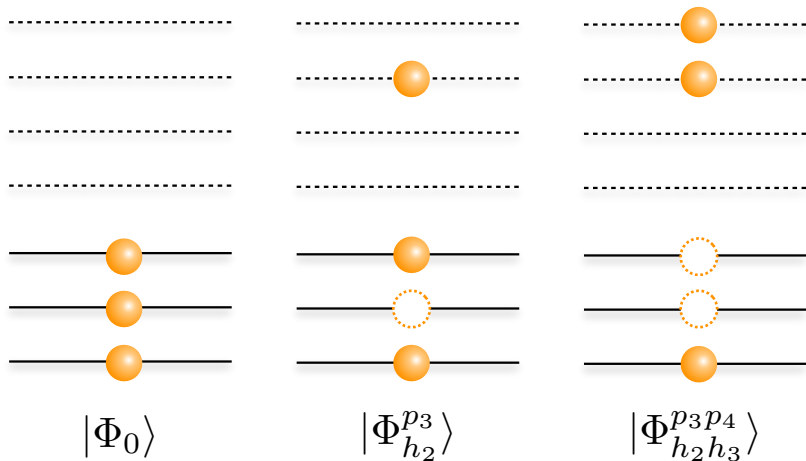
$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i < j} v_{ij} + \sum_{i < j < k} V_{ijk}$$

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$



# CONFIGURATION-INTERACTION METHODS

$$\Psi_0(x_1, \dots, x_A) = \sum_n c_n \Phi_n(x_1, \dots, x_A)$$





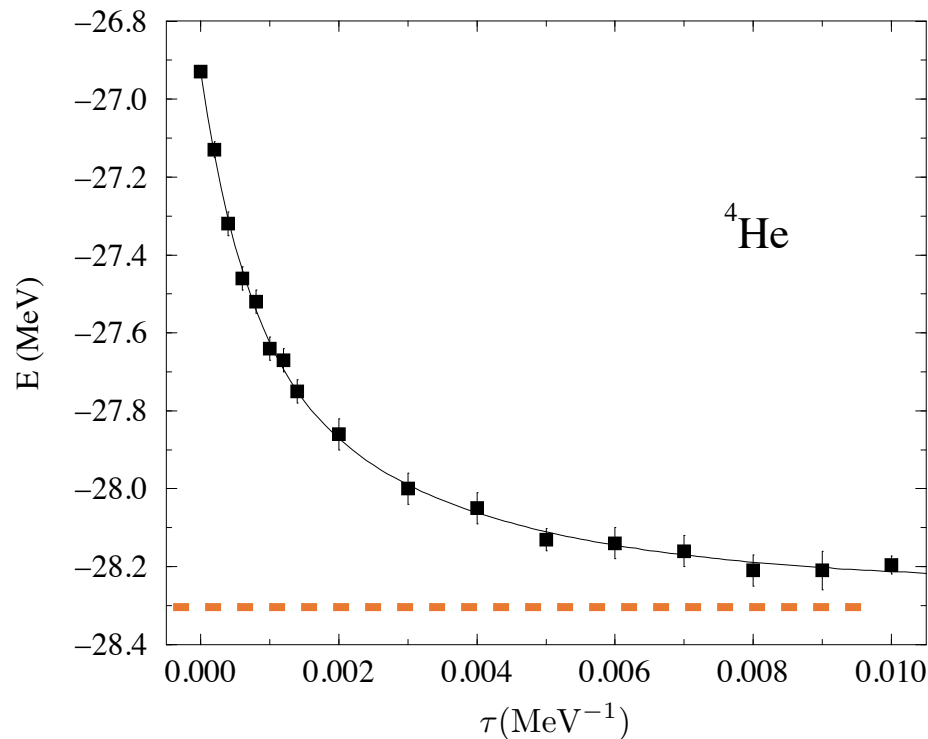
# GREEN'S FUNCTION MONTE CARLO

The GFMC projects out the lowest-energy state using an imaginary-time propagation

$$|\Psi_V\rangle = \sum_n c_n |\Psi_n\rangle \quad ; \quad H_n |\Psi_n\rangle = E_n |\Psi_n\rangle$$

$$\lim_{\tau \rightarrow \infty} e^{-(H-E_0)\tau} |\Psi_V\rangle =$$

$$= \sum_n c_n e^{-(E_n-E_0)\tau} |\Psi_n\rangle = c_0 |\Psi_0\rangle$$

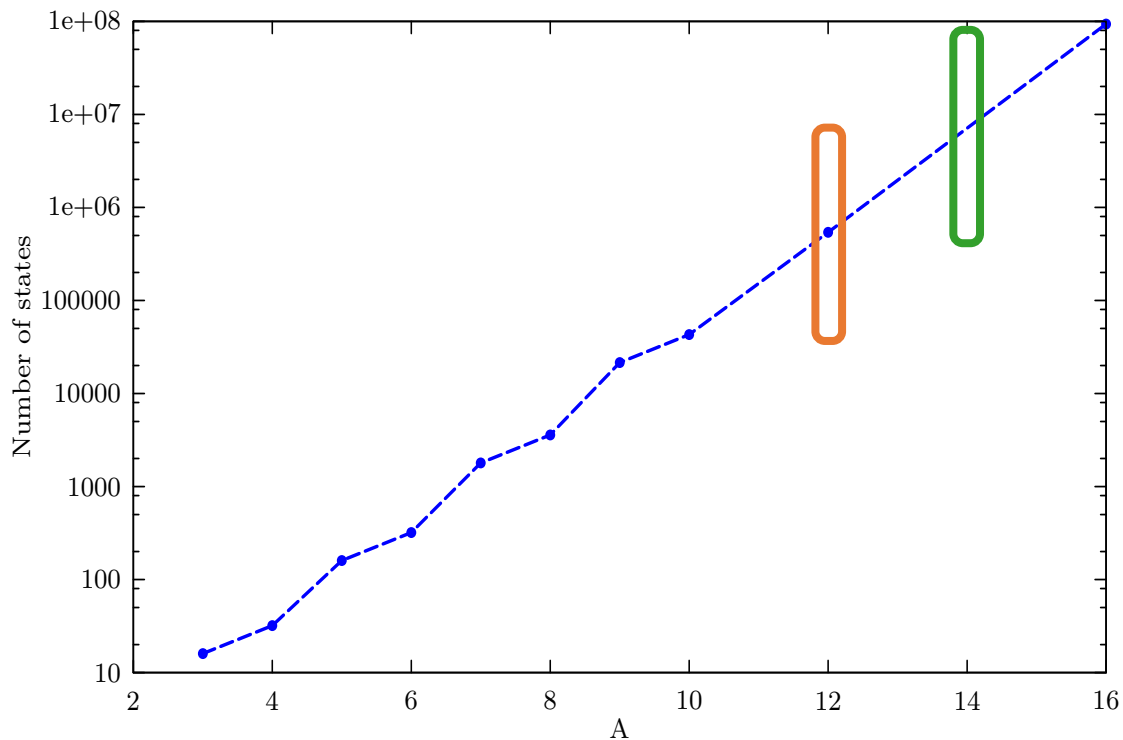


*J. Carlson Phys. Rev. C* **36**, 2026 (1987)

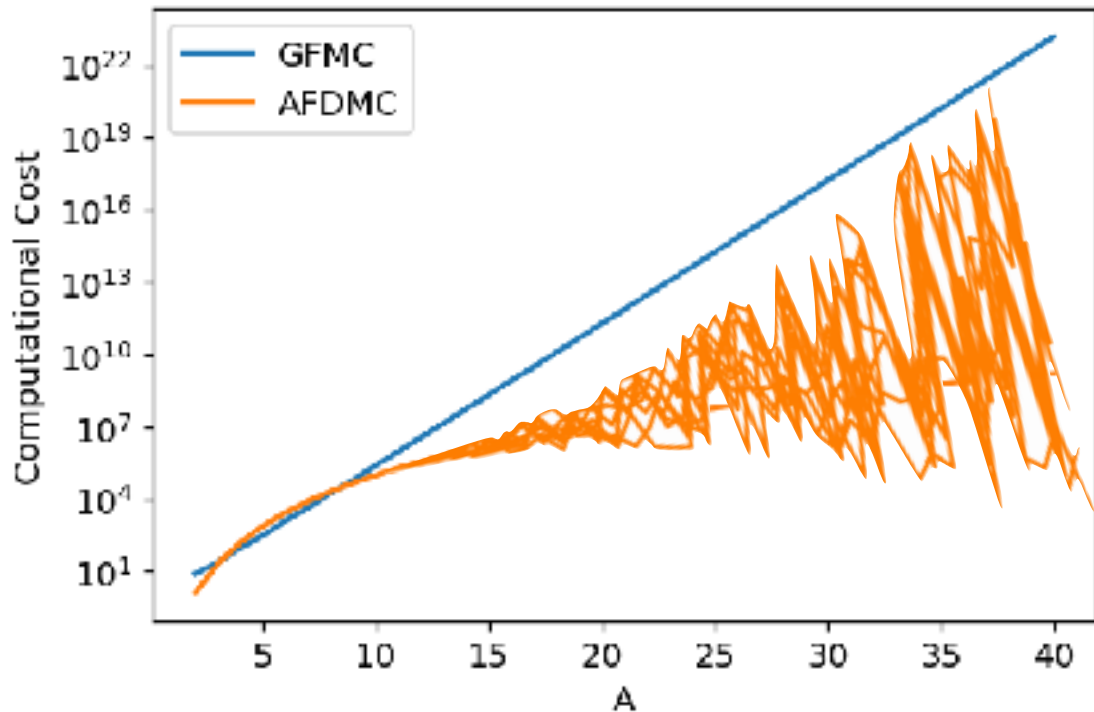
*B. Pudliner et al., PRC* **56**, 1720 (1997)

# CONVENTIONAL QUANTUM MONTE CARLO

$$\langle \mathbf{R} | \Psi(\tau) \rangle = \begin{pmatrix} a_{\uparrow\uparrow\uparrow}(\mathbf{R}) \\ a_{\downarrow\uparrow\uparrow}(\mathbf{R}) \\ a_{\uparrow\downarrow\uparrow}(\mathbf{R}) \\ a_{\downarrow\downarrow\uparrow}(\mathbf{R}) \\ a_{\uparrow\uparrow\downarrow}(\mathbf{R}) \\ a_{\downarrow\uparrow\downarrow}(\mathbf{R}) \\ a_{\uparrow\downarrow\downarrow}(\mathbf{R}) \\ a_{\downarrow\downarrow\downarrow}(\mathbf{R}) \end{pmatrix}$$



# HOW TO TACKLE (EVEN) LARGER NUCLEI?



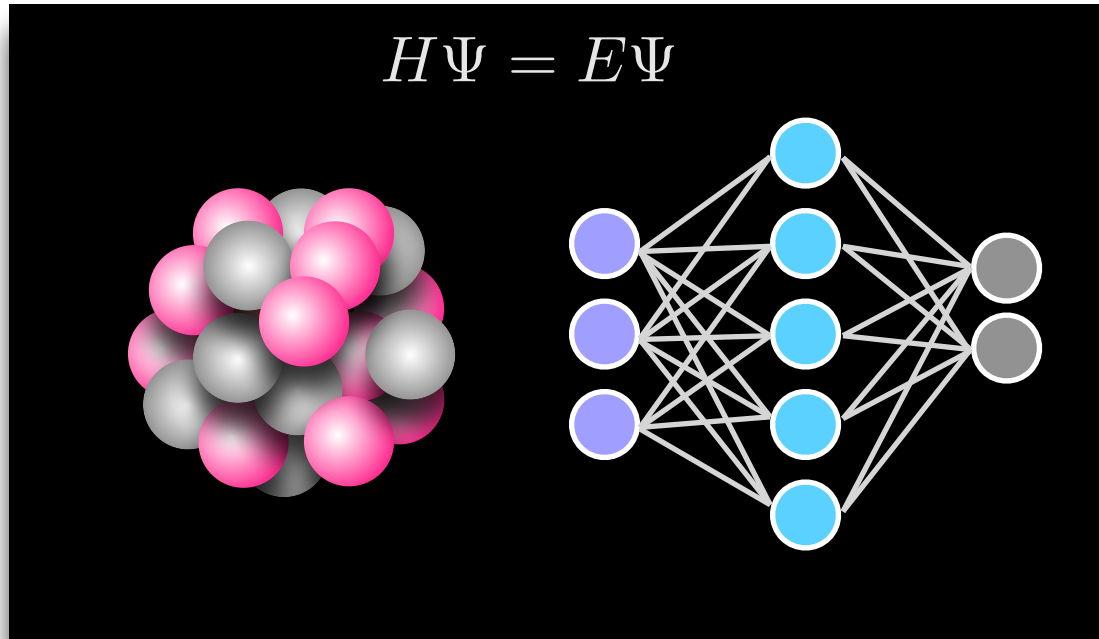
GFMC

$$|\Psi_T\rangle \sim \prod_{i<j} F_{ij} |\Phi\rangle$$

AFDMC

$$|\Psi_T\rangle \sim (1 + \sum_{i<j} F_{ij}) |\Phi\rangle$$

# NEURAL NETWORK QUANTUM STATES



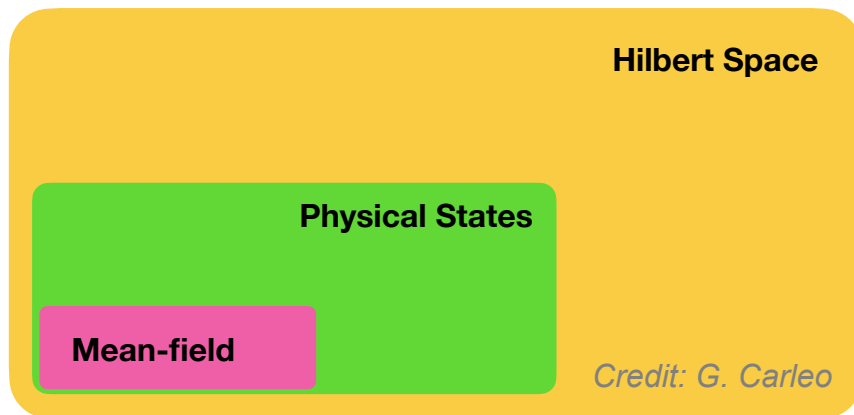
# NEURAL-NETWORK QUANTUM STATES

$$H_{TIF} = -h \sum_i \sigma_i^x - \sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z$$

Finding the exact ground-state is, in principle, **exponentially hard**

$$|\Psi\rangle = c_{\uparrow\uparrow\uparrow\dots} |\uparrow\uparrow\uparrow\dots\rangle + c_{\downarrow\uparrow\uparrow\dots} |\downarrow\uparrow\uparrow\dots\rangle + \dots + c_{\downarrow\downarrow\downarrow\dots} |\downarrow\downarrow\downarrow\dots\rangle$$

Quantum states of physical interest have distinctive features and intrinsic structures

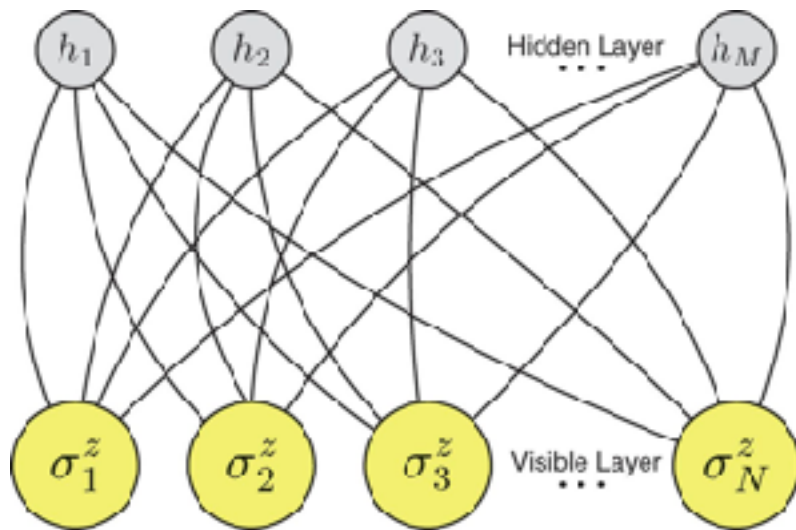


# NEURAL-NETWORK QUANTUM STATES

$$\left\{ \begin{array}{l} c_{\uparrow\uparrow\uparrow\dots} \equiv \langle \uparrow\uparrow\uparrow \dots | \Psi \rangle \equiv \Psi(\uparrow\uparrow\uparrow \dots) \\ c_{\downarrow\uparrow\uparrow\dots} \equiv \langle \downarrow\uparrow\uparrow \dots | \Psi \rangle \equiv \Psi(\downarrow\uparrow\uparrow \dots) \\ \\ c_{\downarrow\downarrow\downarrow\dots} \equiv \langle \downarrow\downarrow\downarrow \dots | \Psi \rangle \equiv \Psi(\downarrow\downarrow\downarrow \dots) \end{array} \right. \longleftrightarrow c_S \equiv \langle S | \Psi \rangle \equiv \Psi(S)$$

**Idea:** use neural networks to represent the quantum many-body wave function

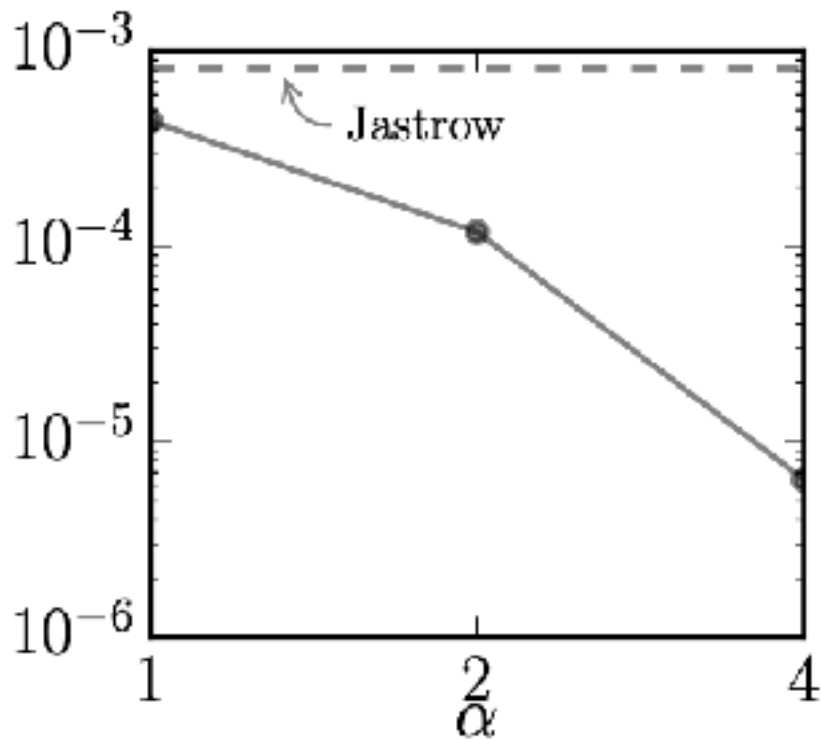
$$\hat{\Psi}(S) = \sum_{h_i=0,1} e^{\sum_j a_j \sigma_j^z + \sum_i b_i h_i + \sum_{ij} W_{ij} h_i \sigma_j}$$



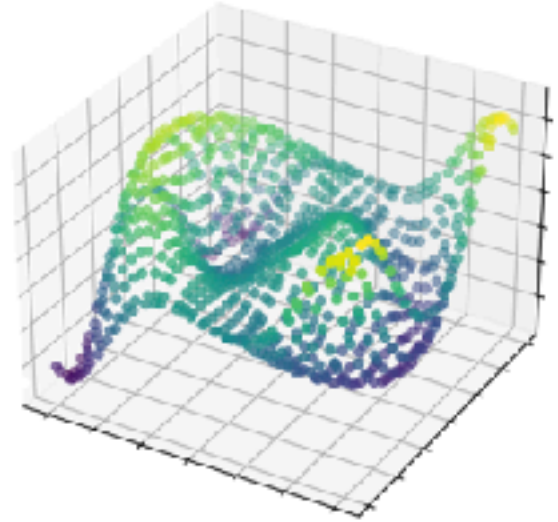
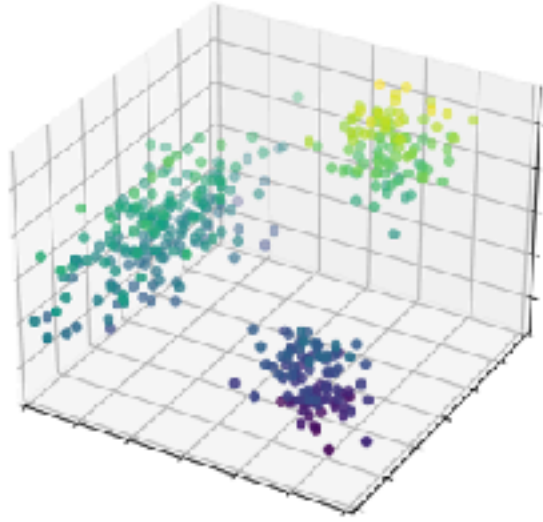
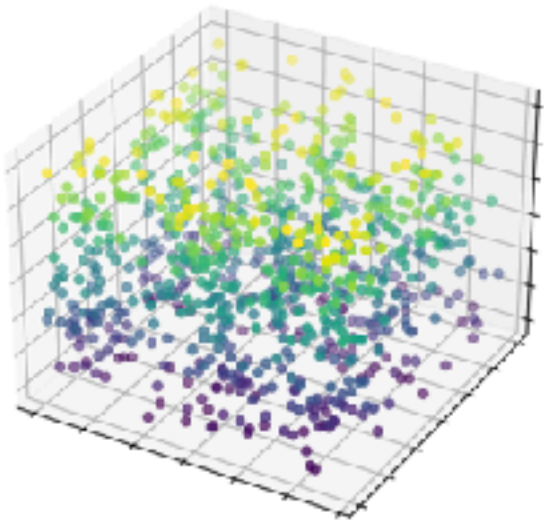
G. Carleo et al. *Science* **355**, 602 (2017)

# NEURAL-NETWORK QUANTUM STATES

G. Carleo and M. Troyer proved that Restricted Boltzmann machines outperform traditional Jastrow



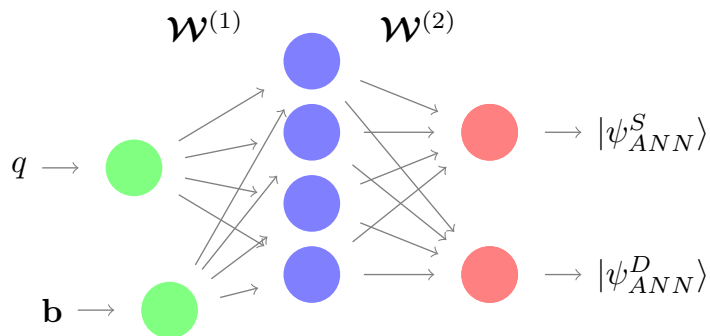
# NEURAL-NETWORK QUANTUM STATES





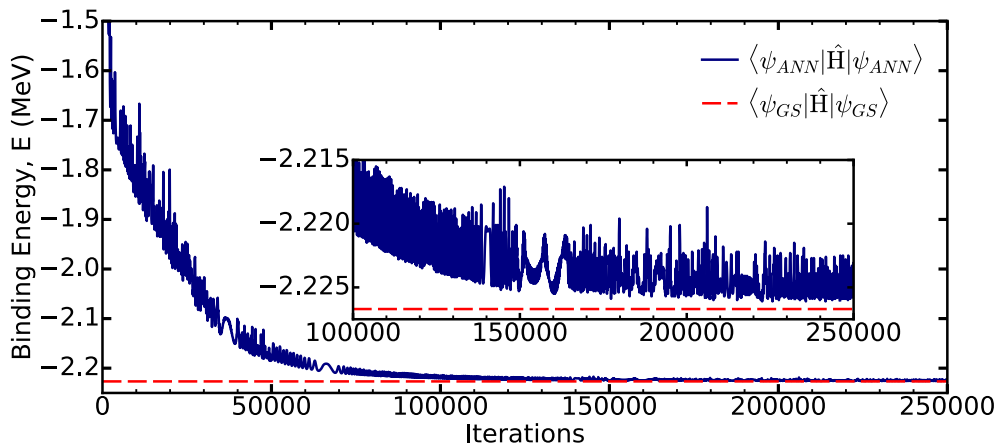
# MACHINE-LEARNING THE DEUTERON

ANNs solve the deuteron using the  
N3LO Entem-Machleidt chiral-EFT  
NN potential



The parameters of the ANN are optimized minimizing the variational energy using RMSprop

$$E^{\mathcal{W}} = \frac{\langle \Psi_{ANN}^{\mathcal{W}} | \hat{H} | \Psi_{ANN}^{\mathcal{W}} \rangle}{\langle \Psi_{ANN}^{\mathcal{W}} | \Psi_{ANN}^{\mathcal{W}} \rangle}$$

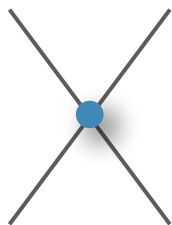


# PIONLESS EFT HAMILTONIAN

We take as input a LO pionless-EFT Hamiltonian

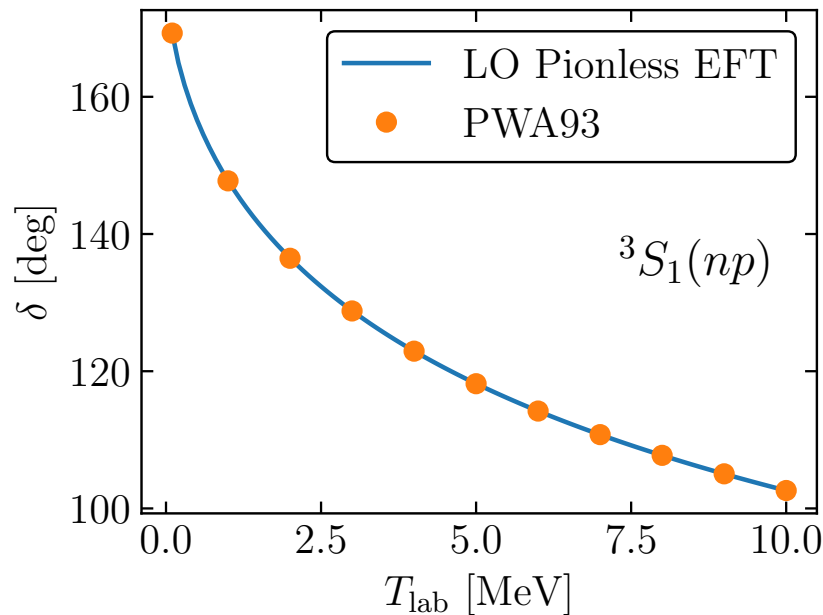
$$H_{LO} = - \sum_i \frac{\vec{\nabla}_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

- NN potential fit to s-wave np scattering lengths and effective ranges



$$v_{ij}^{\text{CI}} = \sum_{p=1}^4 v^p(r_{ij}) O_{ij}^p,$$

$$O_{ij}^{p=1,4} = (1, \tau_{ij}, \sigma_{ij}, \sigma_{ij}\tau_{ij})$$

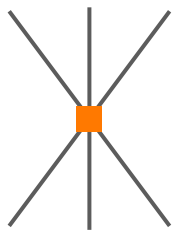


# PIONLESS EFT HAMILTONIAN

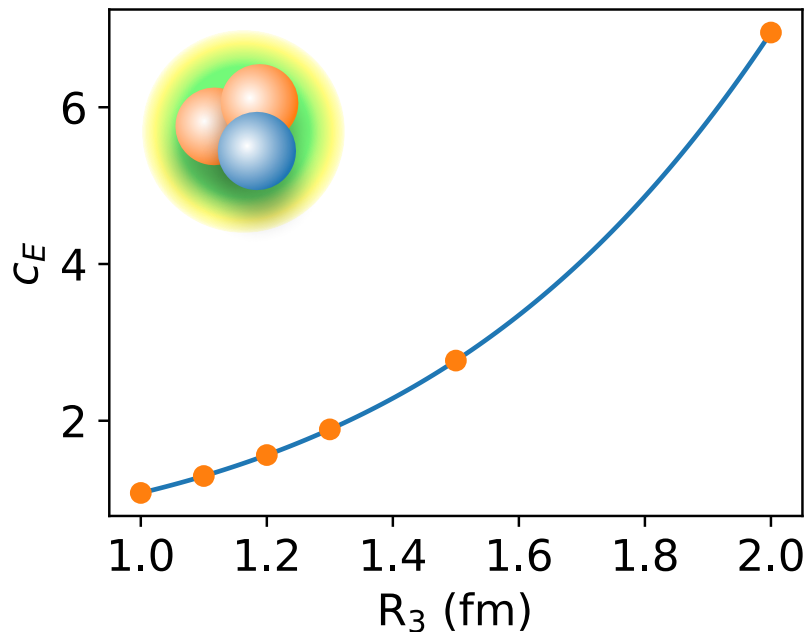
We take as input a LO pionless-EFT Hamiltonian

$$H_{LO} = -\sum_i \frac{\vec{\nabla}_i^2}{2m_N} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}$$

- 3NF adjusted to reproduce the energy of  ${}^3\text{H}$ .



$$V_{ijk} \propto c_E \sum_{\text{cyc}} e^{-(r_{ij}^2 + r_{jk}^2)/R_3^2}$$



# NEURAL SLATER-JASTROW ANSATZ

Product of mean-field state modulated by a flexible correlator factor

$$\Psi_{SJ}(X) = e^{J(X)}\Phi(X)$$

# NEURAL SLATER-JASTROW ANSATZ

Product of mean-field state modulated by a flexible correlator factor

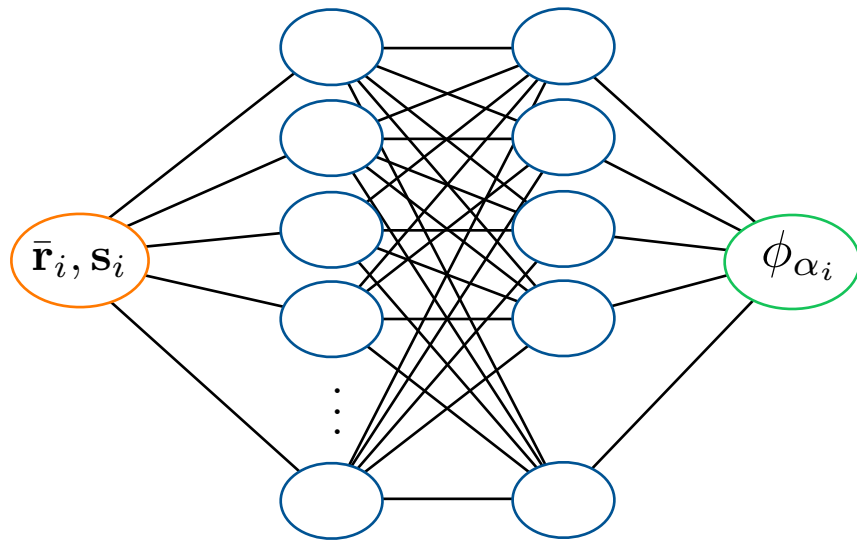
$$\Psi_{SJ}(X) = e^{J(X)}\Phi(X)$$

Mean-field: Slater determinant of single-particle orbitals

$$\det \begin{bmatrix} \phi_1(\mathbf{x}_1) & \phi_1(\mathbf{x}_2) & \cdots & \phi_1(\mathbf{x}_N) \\ \phi_2(\mathbf{x}_1) & \phi_2(\mathbf{x}_2) & \cdots & \phi_2(\mathbf{x}_N) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_N(\mathbf{x}_1) & \phi_N(\mathbf{x}_2) & \cdots & \phi_N(\mathbf{x}_N) \end{bmatrix}$$

Each orbital is a FFNN that takes as input

$$\bar{\mathbf{r}}_i = \mathbf{r}_i - \mathbf{R}_{CM}$$



# NEURAL SLATER-JASTROW ANSATZ

“Manually” imposing permutation-invariance scales factorially with A

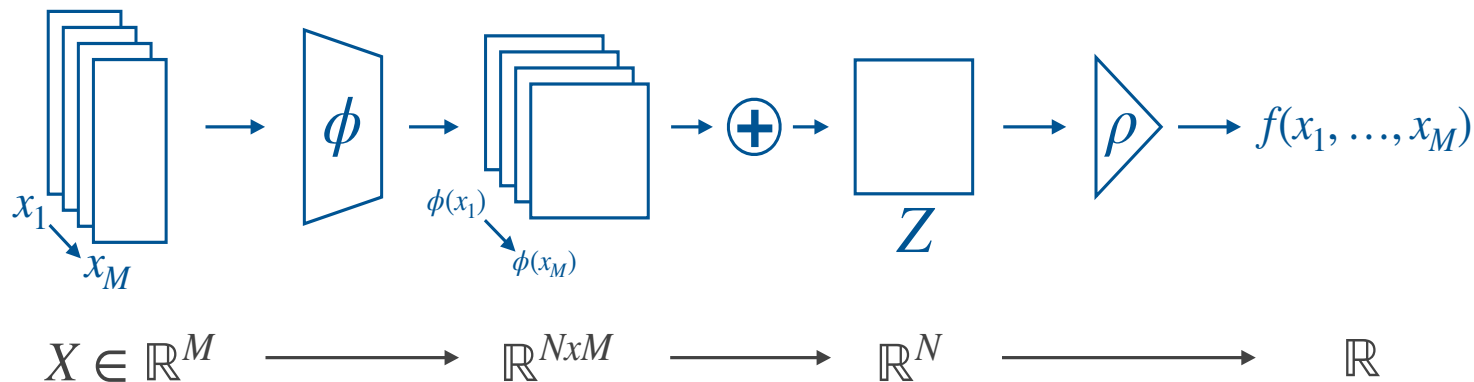
$$J(X) = j(x_1, x_2, x_3) + j(x_1, x_3, x_2) + j(x_2, x_1, x_3) + j(x_2, x_3, x_1) + j(x_3, x_1, x_2) + j(x_3, x_2, x_1)$$

# NEURAL SLATER-JASTROW ANSATZ

“Manually” imposing permutation-invariance scales factorially with A

$$J(X) = j(x_1, x_2, x_3) + j(x_1, x_3, x_2) + j(x_2, x_1, x_3) + j(x_2, x_3, x_1) + j(x_3, x_1, x_2) + j(x_3, x_2, x_1)$$

**Solution:** “deep-sets”  $\longrightarrow J(X) = \rho_F \left[ \sum_i \vec{\phi}_{\mathcal{F}}(\bar{\mathbf{r}}_i, \mathbf{s}_i) \right]$



# SAMPLING COORDINATES AND SPIN

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} = \frac{\sum_S \int dR |\Psi_V(R, S)|^2 \frac{\langle RS | H | \Psi_V \rangle}{\langle RS | \Psi_V \rangle}}{\sum_S \int dR |\Psi_V(R, S)|^2}.$$

Markov Chain Monte Carlo algorithm to sample the Hilbert space

$$P_{\text{acc}} = \min \left( 1, \frac{|\Psi_V(R', S')|^2}{|\Psi_V(R, S)|^2} \right)$$

Observables estimated by averaging over the sampled configurations

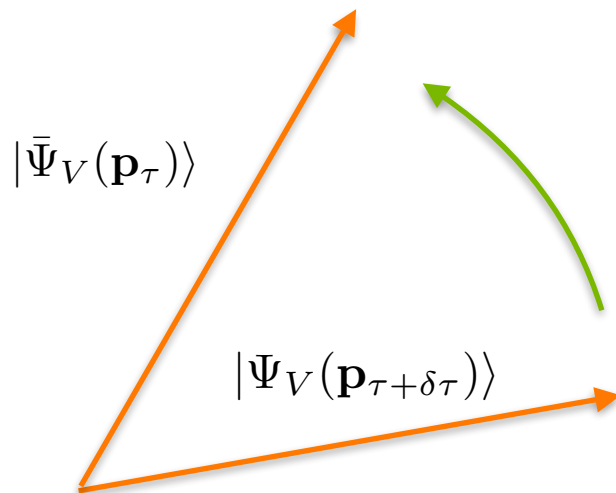
$$\frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \simeq \frac{1}{N} \sum_{R, S} \frac{\langle R, S | H | \Psi_V \rangle}{\langle R, S | \Psi_V \rangle}$$



# WAVE FUNCTION OPTIMIZATION

ANN trained by performing an imaginary-time evolution in the variational manifold

$$\left\{ \begin{array}{l} |\bar{\Psi}_V(\mathbf{p}_\tau)\rangle \equiv (1 - H\delta\tau)|\Psi_V(\mathbf{p}_\tau)\rangle \\ \mathbf{p}_{\tau+\delta\tau} = \arg \max_{\mathbf{p} \in R^d} \left( |\langle \bar{\Psi}_V(\mathbf{p}_\tau) | \Psi_V(\mathbf{p}_{\tau+\delta\tau}) \rangle|^2 \right) \end{array} \right.$$



The parameters are updated as

$$\mathbf{p}_{\tau+\delta\tau} = \mathbf{p}_\tau - \delta\tau S^{-1} \mathbf{g}_\tau$$

# WAVE FUNCTION OPTIMIZATION

ANN trained by performing an imaginary-time evolution in the variational manifold

*See Mehdi Drissi's  
talk on Friday*

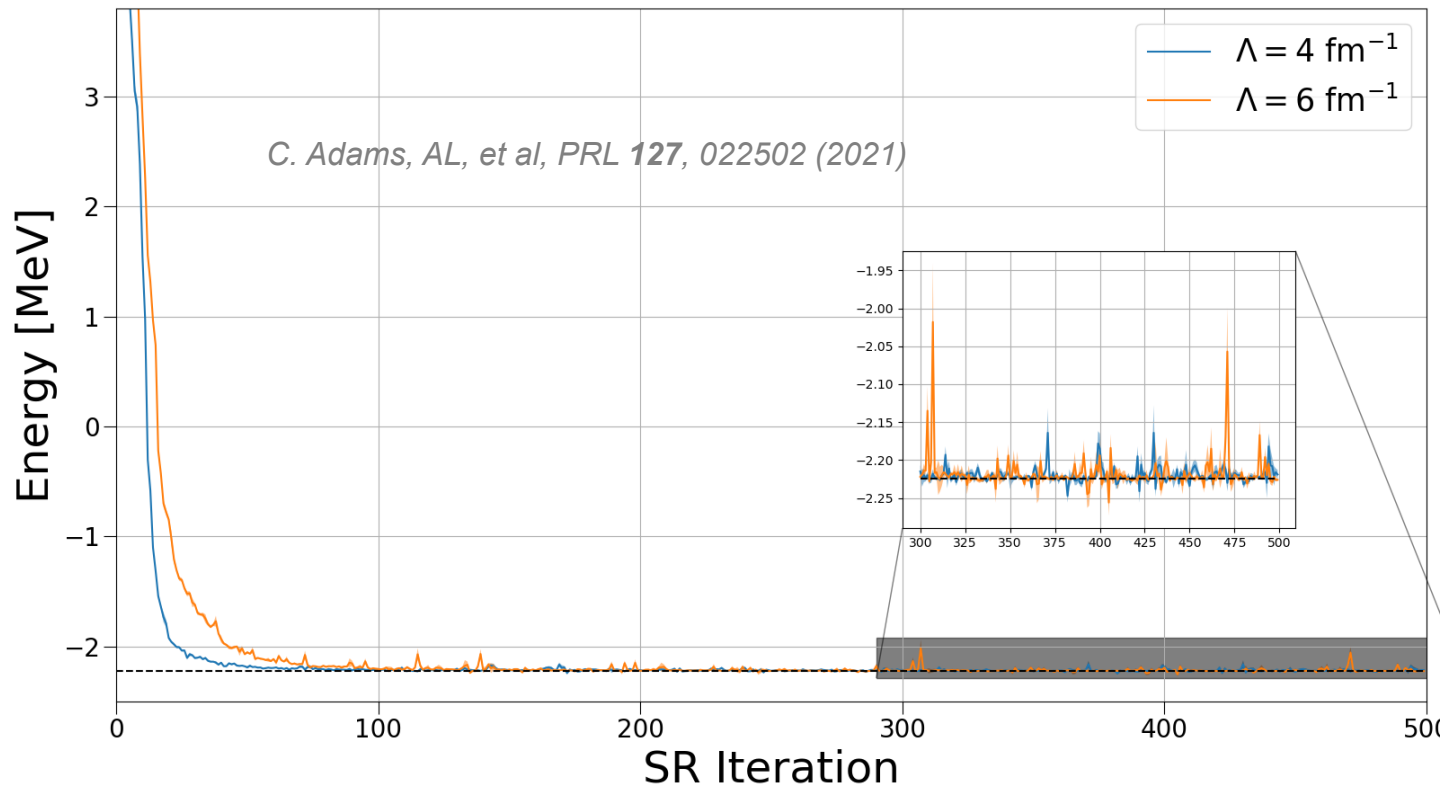
The parameters are updated as

$$\mathbf{p}_{\tau+\delta\tau} = \mathbf{p}_{\tau} - \delta\tau S^{-1} \mathbf{g}_{\tau}$$

*J. Stokes, et al., Quantum 4, 269 (2020).*

*S. Sorella, Phys. Rev. B 64, 024512 (2001)*

# STOCHASTIC RECONFIGURATION



# COMPARISON WITH GFMC

- The ANN Slater Jastrow ansatz outperforms conventional Jastrow correlations

	$\Lambda$	VMC-ANN	VMC-JS	GFMC	GFMC <sub>c</sub>
${}^2\text{H}$	$4 \text{ fm}^{-1}$	-2.224(1)	-2.223(1)	-2.224(1)	-
	$6 \text{ fm}^{-1}$	-2.224(4)	-2.220(1)	-2.225(1)	-
${}^3\text{H}$	$4 \text{ fm}^{-1}$	-8.26(1)	-7.80(1)	-8.38(2)	-7.82(1)
	$6 \text{ fm}^{-1}$	-8.27(1)	-7.74(1)	-8.38(2)	-7.81(1)
${}^4\text{He}$	$4 \text{ fm}^{-1}$	-23.30(2)	-22.54(1)	-23.62(3)	-22.77(2)
	$6 \text{ fm}^{-1}$	-24.47(3)	-23.44(2)	-25.06(3)	-24.10(2)

- Differences with the GFMC due to deficiencies in the Slater-Jastrow ansatz

$$\Psi_{SJ}(X) = e^{J(X)}\Phi(X)$$

# HIDDEN NUCLEONS ANSATZ

$$\Phi(X) = \det \begin{bmatrix} \phi_1(x_1) & \phi_1(x_2) & \phi_1(x_3) & \phi_1(x_4) \\ \phi_2(x_1) & \phi_2(x_2) & \phi_2(x_3) & \phi_2(x_4) \\ \phi_3(x_1) & \phi_3(x_2) & \phi_3(x_3) & \phi_3(x_4) \\ \phi_4(x_1) & \phi_4(x_2) & \phi_4(x_3) & \phi_4(x_4) \end{bmatrix}$$

# HIDDEN NUCLEONS ANSATZ

$$\Psi_{\text{HN}}(X) = \det \begin{bmatrix} \phi_1(x_1) & \phi_1(x_2) & \phi_1(x_3) & \phi_1(x_4) & \phi_1(y_1) & \phi_1(y_2) \\ \phi_2(x_1) & \phi_2(x_2) & \phi_2(x_3) & \phi_2(x_4) & \phi_2(y_1) & \phi_1(y_2) \\ \phi_3(x_1) & \phi_3(x_2) & \phi_3(x_3) & \phi_3(x_4) & \phi_3(y_1) & \phi_1(y_2) \\ \phi_4(x_1) & \phi_4(x_2) & \phi_4(x_3) & \phi_4(x_4) & \phi_4(y_1) & \phi_1(y_2) \\ \chi_1(x_1) & \chi_1(x_2) & \chi_1(x_3) & \chi_1(x_4) & \chi_1(y_1) & \chi_2(y_2) \\ \chi_2(x_1) & \chi_2(x_2) & \chi_2(x_3) & \chi_2(x_4) & \chi_2(y_1) & \chi_2(y_2) \end{bmatrix}$$

Visible orbitals on visible coordinates

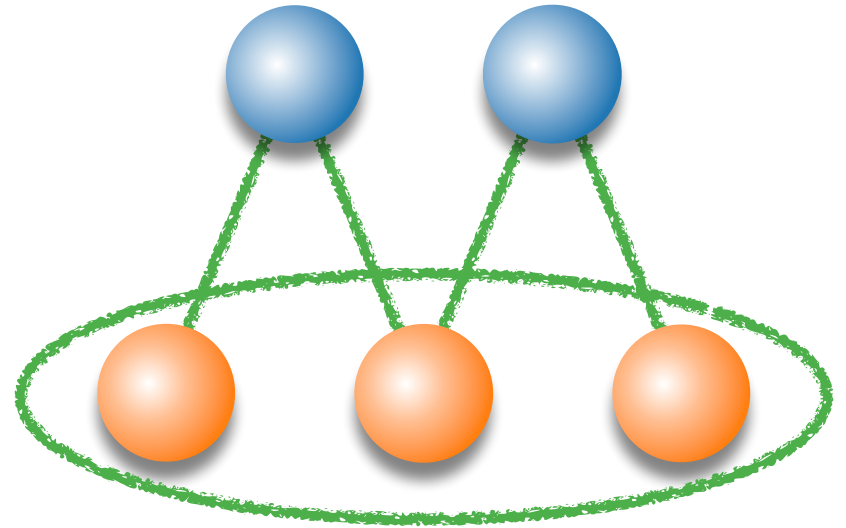
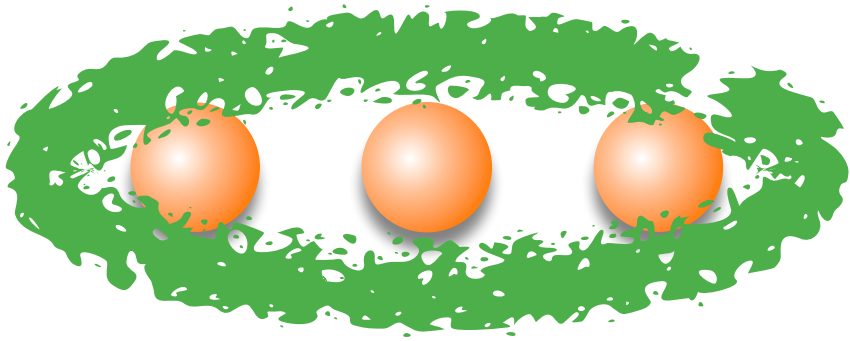
Visible orbitals on hidden coordinates

Hidden orbitals on visible coordinates

Hidden orbitals on hidden coordinates

*J. R. Moreno, et al., PNAS 119, 2122059119(2022)*

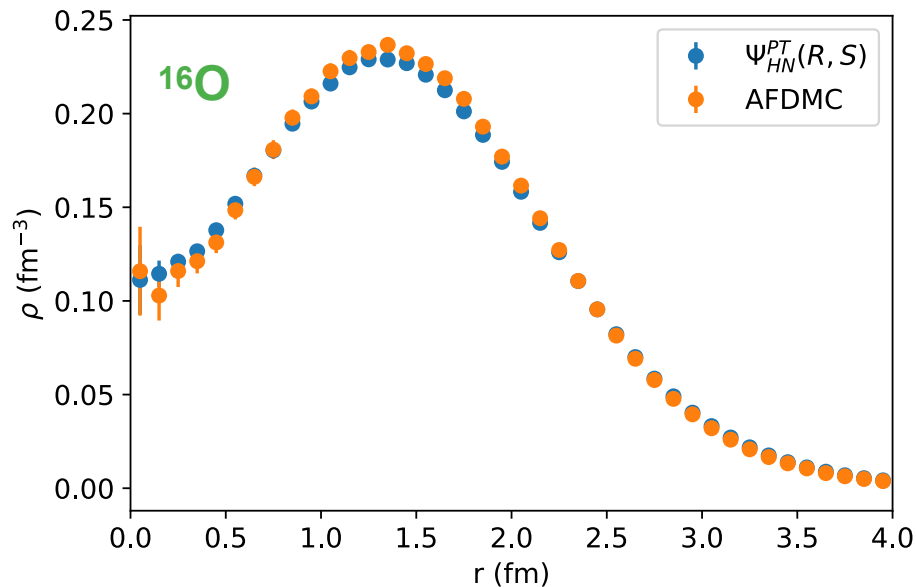
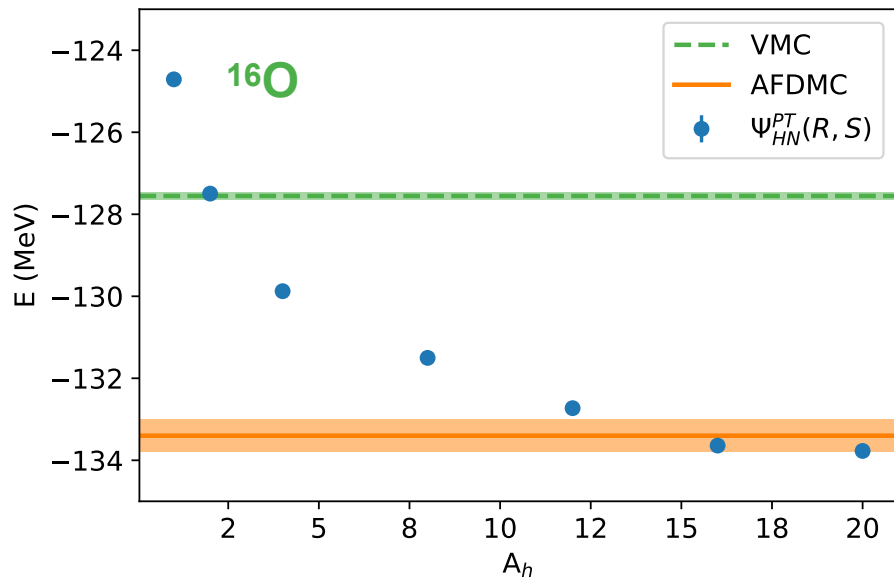
# HIDDEN NUCLEONS ANSATZ



# HIDDEN NUCLEONS ANSATZ

We extend the reach of neural quantum states to  $^{16}\text{O}$

In addition to its ground-state energy, we evaluate the point-nucleon density of  $^{16}\text{O}$  with  $A_h=16$



AL, et al., Phys. Rev. Res. 4 (2022) 4, 043178

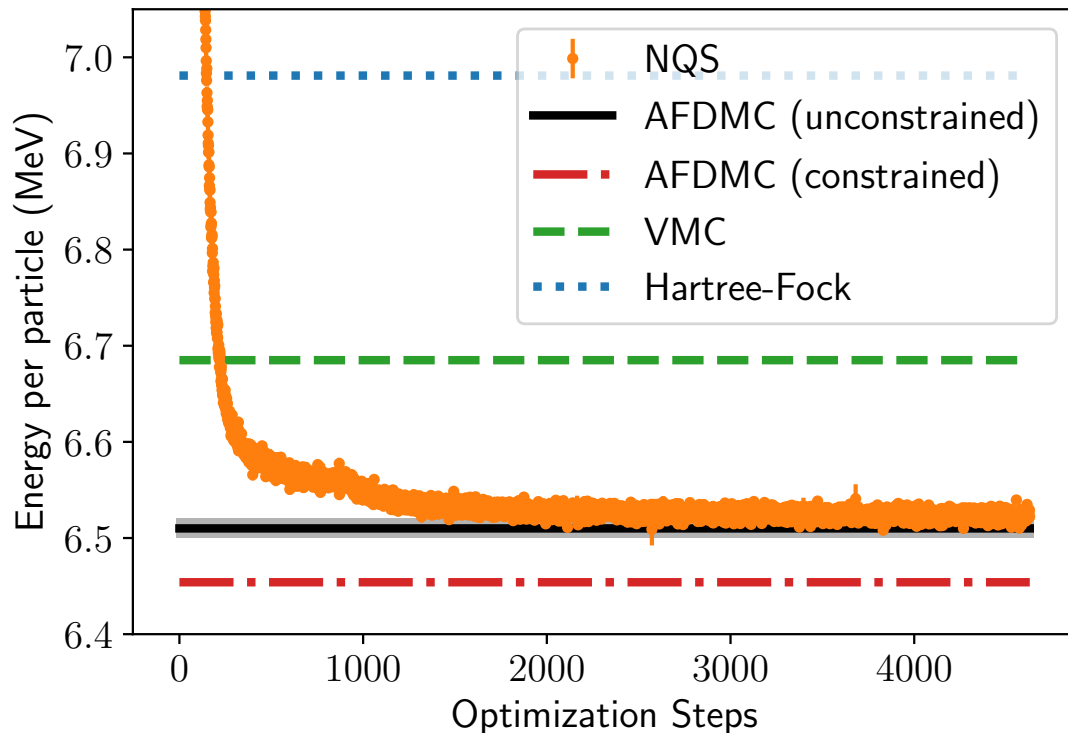


# DILUTE NEUTRON MATTER

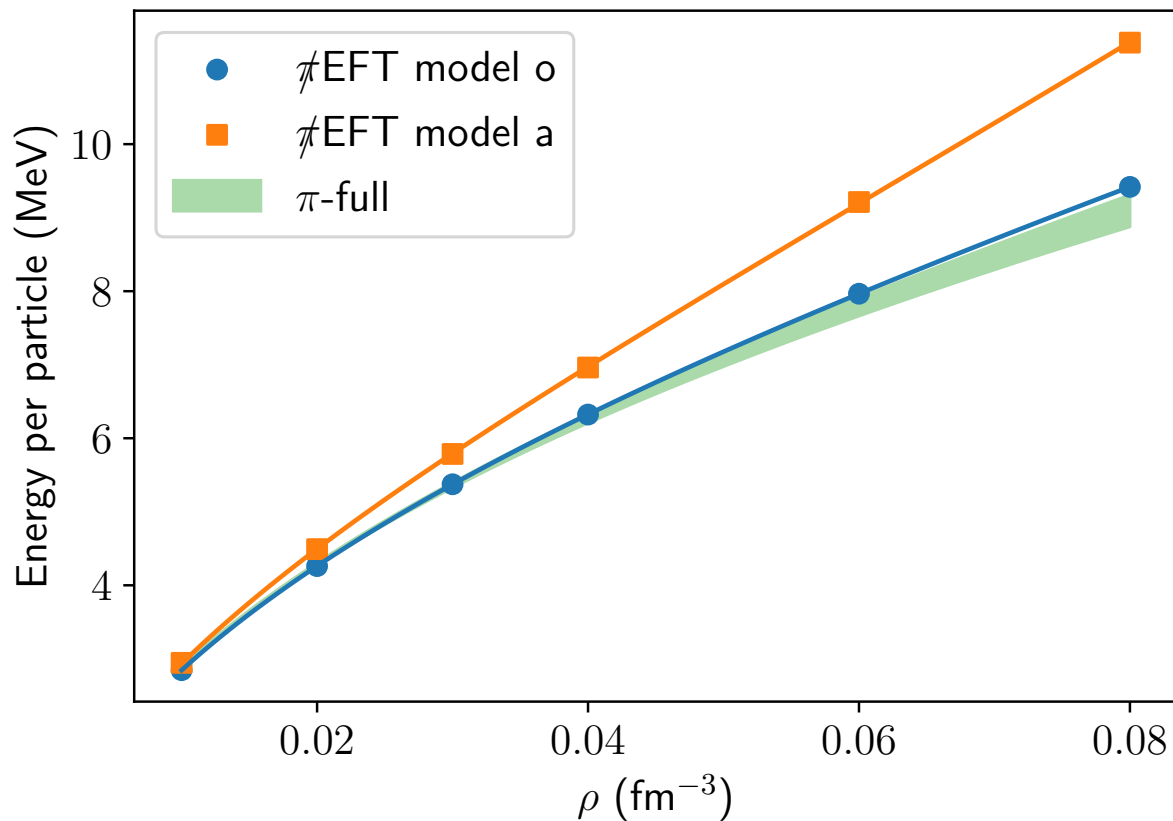
Periodic hidden-nucleons ansatz to model low-density neutron matter

- **NQS**: 100 GPU hours
- **AFDMC**: 1.2 million CPU hours

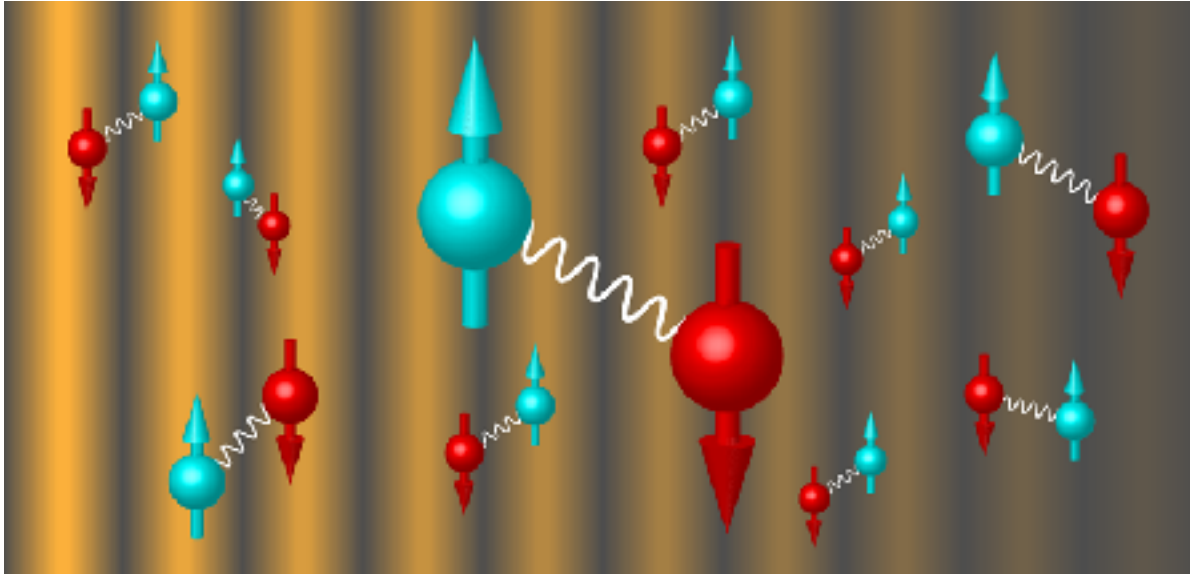
14 Neutrons @  $\rho=0.04 \text{ fm}^{-3}$



# DILUTE NEUTRON MATTER

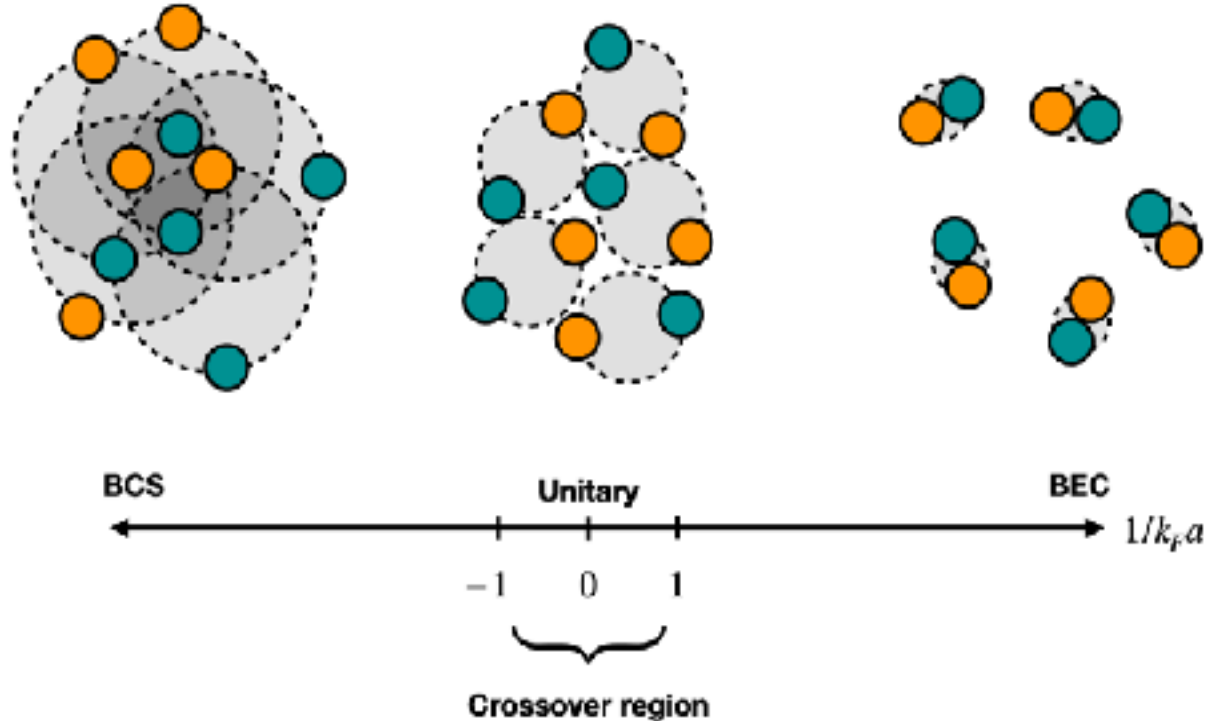


# CONDENSED-MATTER DETOUR



# COLD FERMION GASES

Periodic-NQS to solve the two-components Fermi gas in the BCS- BEC crossover region



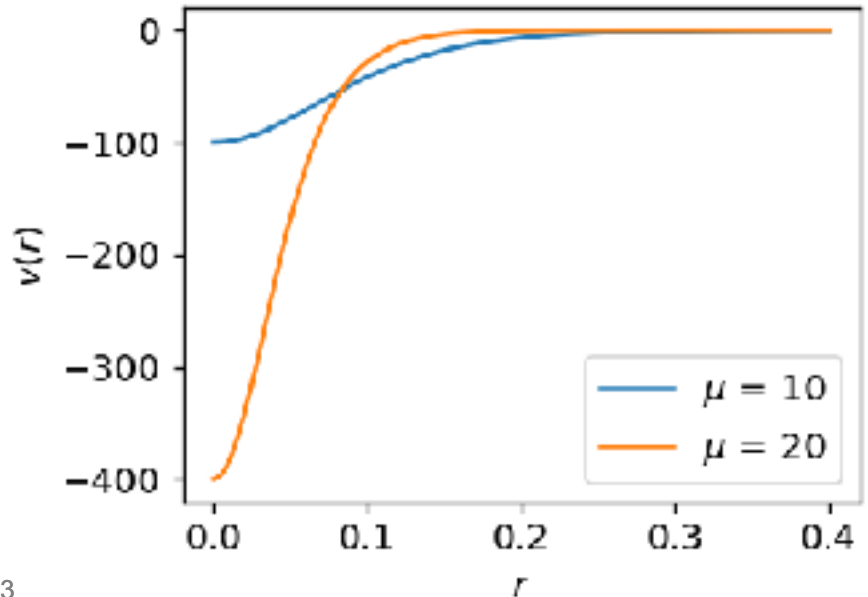
# COLD FERMI GASES

We model the 3D unpolarized gas of fermions with the Hamiltonian

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij}.$$

- Modified Pöschl-Teller potential between opposite-spin particles

$$v_{ij} = (\delta_{s_i, s_j} - 1) v_0 \frac{2\hbar^2}{m} \frac{\mu^2}{\cosh^2(\mu r_{ij})}$$



# COLD FERMION GASES

We introduce a Pfaffian-Jastrow ansatz

$$\Phi_{PJ}(X) = \text{pf} \begin{bmatrix} 0 & \phi(x_1, x_2) & \cdots & \phi(x_1, x_N) \\ \phi(x_2, x_1) & 0 & \cdots & \phi(x_2, x_N) \\ \vdots & \vdots & \ddots & \vdots \\ \phi(x_N, x_1) & \phi(x_N, x_2) & \cdots & 0 \end{bmatrix}$$

In order for the above matrix to be skew-symmetric, the neural pairing orbitals are taken to be

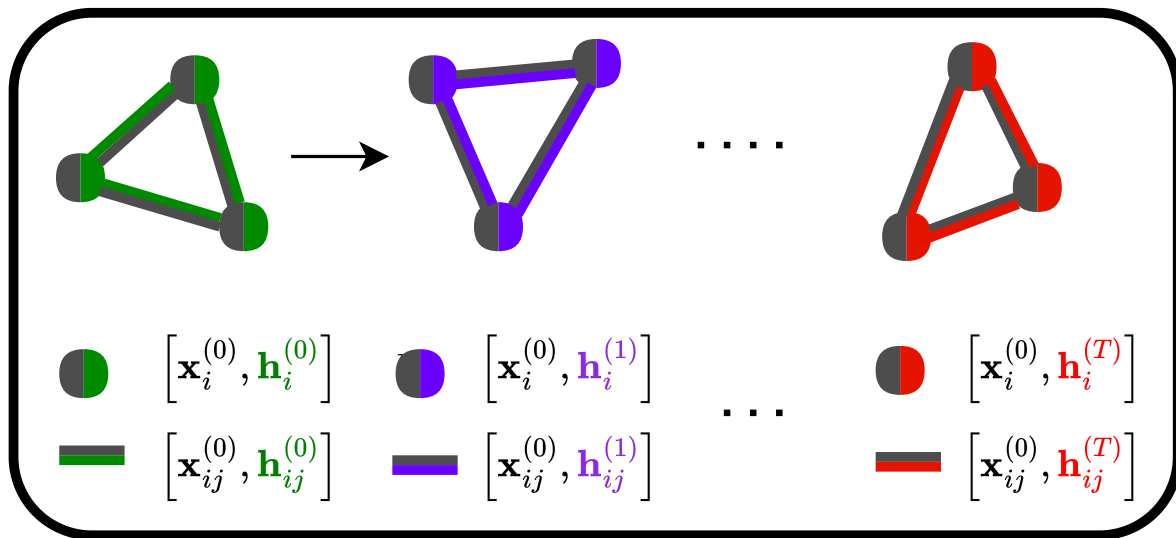
$$\phi(x_i, x_j) = \eta(x_i, x_j) - \eta(x_j, x_i)$$

Example:

$$\text{pf} \begin{bmatrix} 0 & \phi_{12} & \phi_{13} & \phi_{14} \\ -\phi_{12} & 0 & \phi_{23} & \phi_{24} \\ -\phi_{13} & -\phi_{23} & 0 & \phi_{34} \\ -\phi_{14} & -\phi_{24} & -\phi_{34} & 0 \end{bmatrix} = \phi_{12}\phi_{34} - \phi_{13}\phi_{24} + \phi_{14}\phi_{23}$$

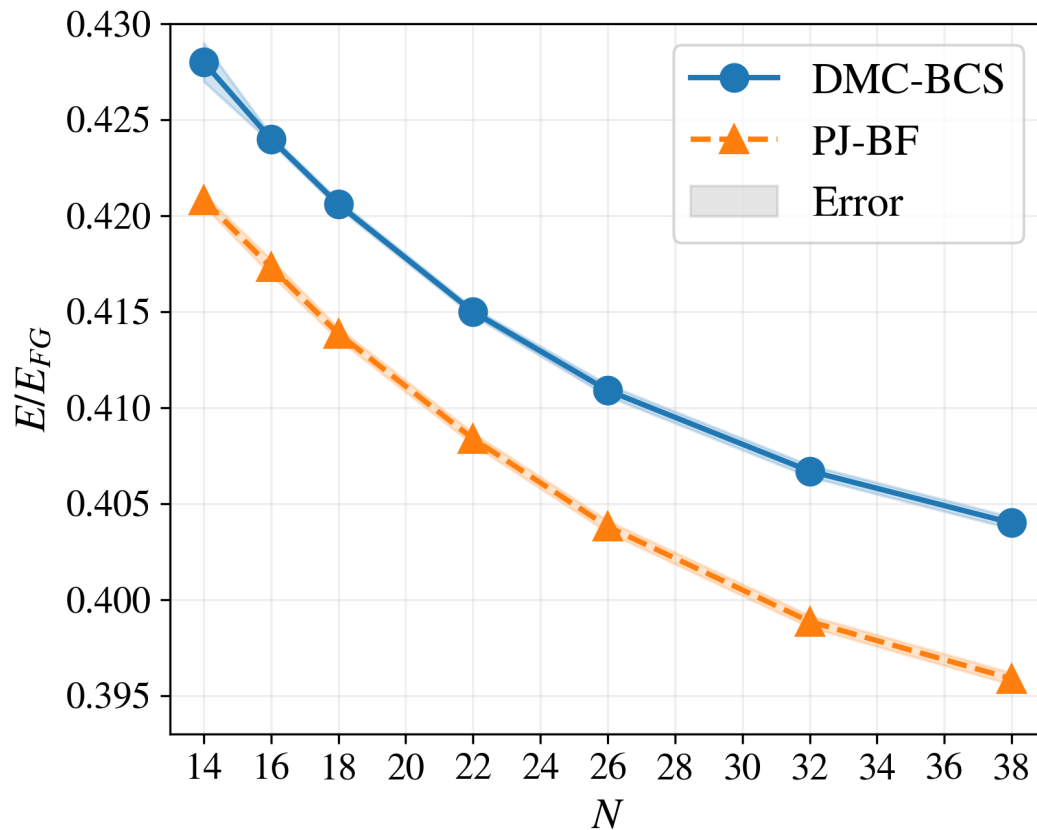
# HOMOGENEOUS ELECTRON GAS

The nodal structure is improved with neural back-flow transformations  $\mathbf{x}_i \longrightarrow \phi(\mathbf{x}_i; \mathbf{x}_{j \neq i})$



Translation-invariant (and periodic) by construction:  $\longrightarrow \mathbf{x}_i^{(0)} = \mathbf{e}, \quad \mathbf{x}_{ij}^{(0)} = [\mathbf{r}_{ij}, \|\mathbf{r}_{ij}\|, s_i \cdot s_j].$

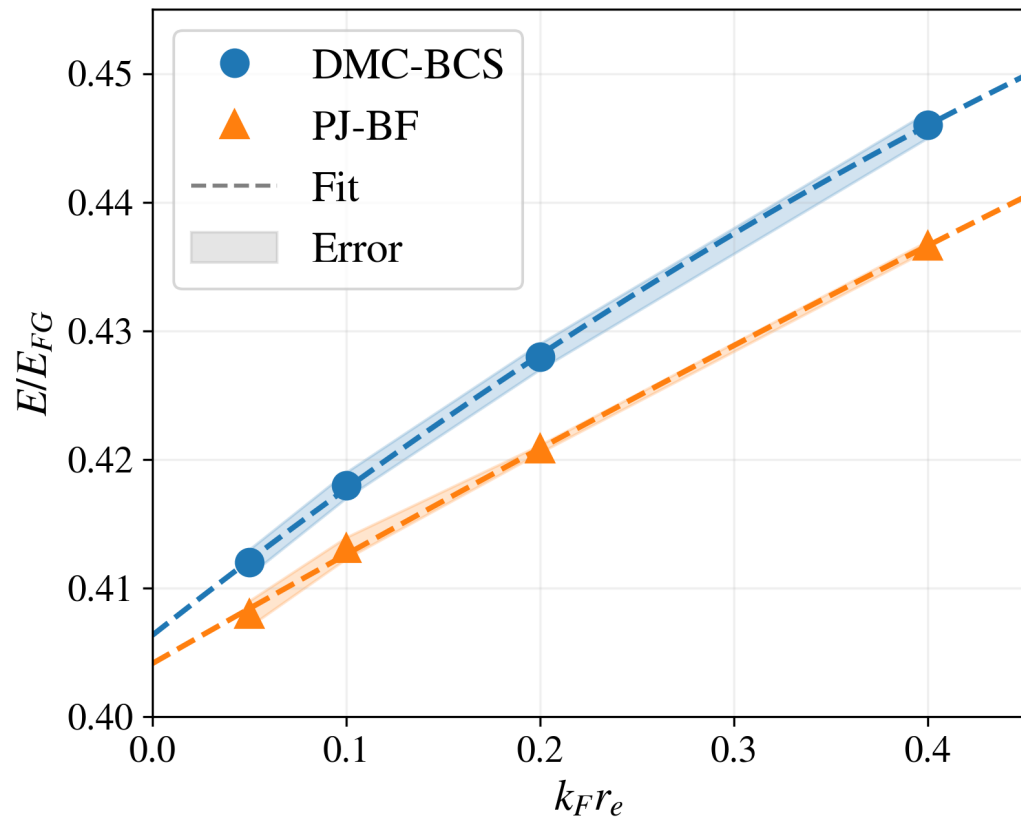
# COLD FERMI GASES



$$\left( \frac{E}{E_{FG}} \right)_{\text{exp}} = \xi = 0.376(5)$$

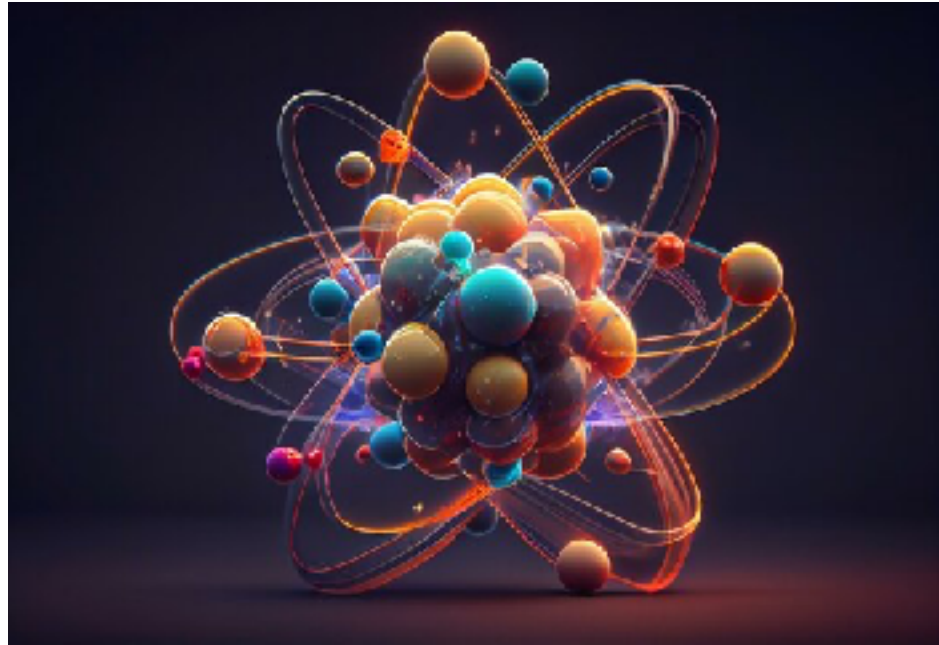


# COLD FERMI GASES

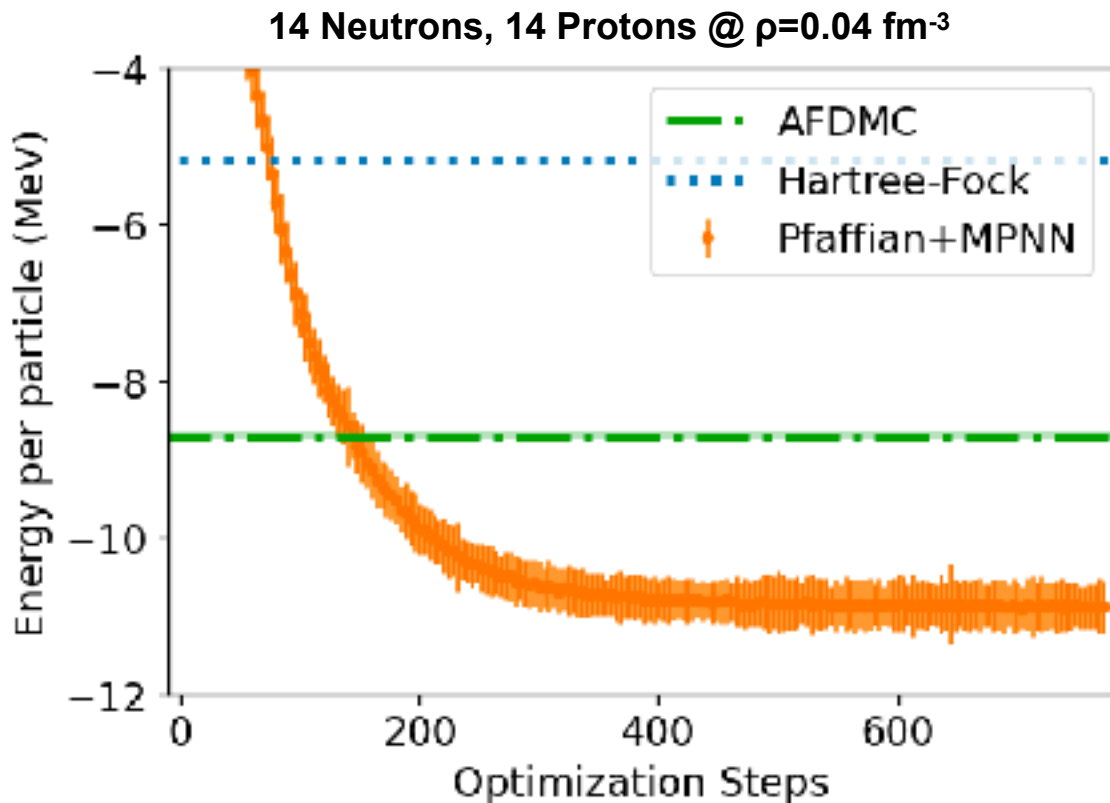


$$\left( \frac{E}{E_{FG}} \right)_{\text{exp}} = \xi = 0.376(5)$$

# BACK TO NUCLEAR PHYSICS

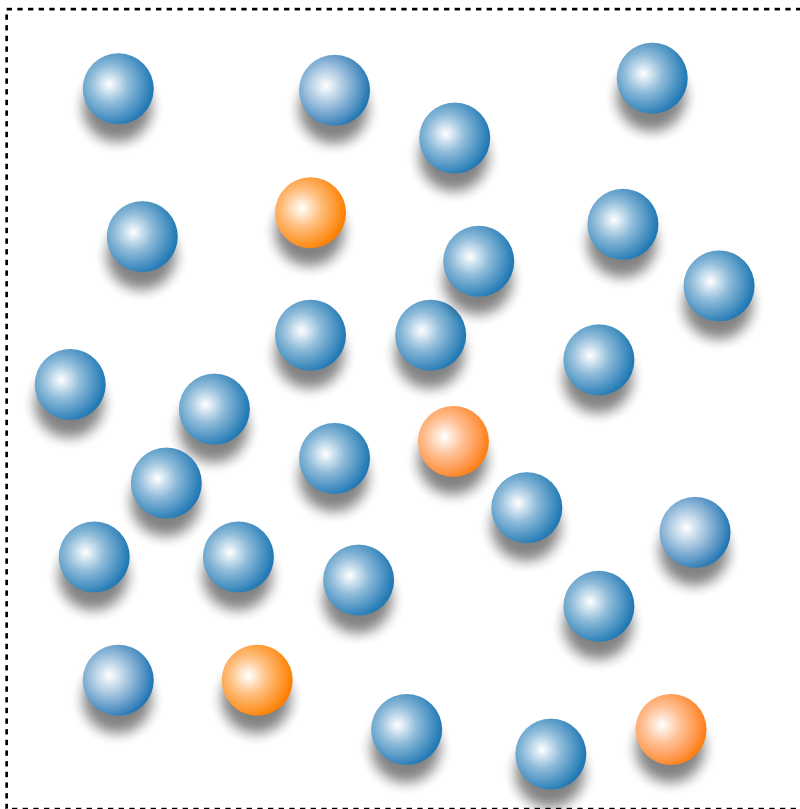


# DILUTE NUCLEONIC MATTER WITH MPNN



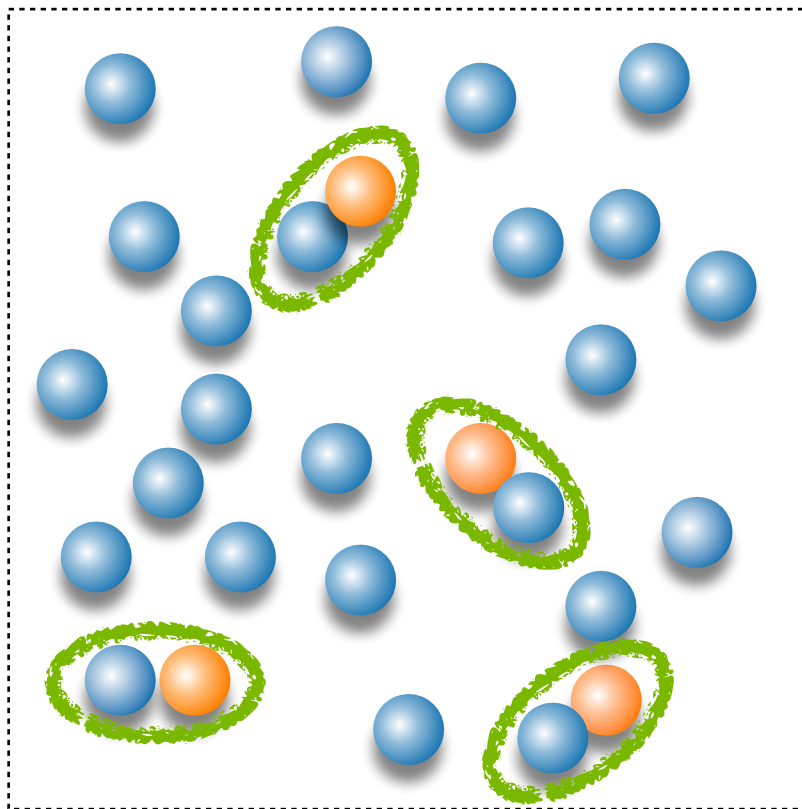
# DILUTE NUCLEONIC MATTER WITH MPNN

Liquid phase



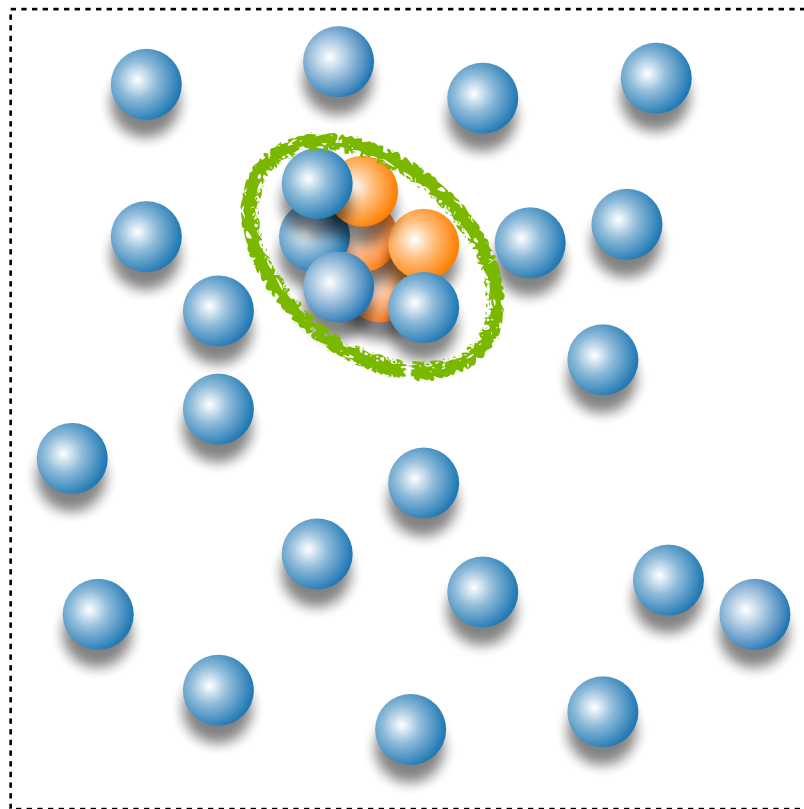
# DILUTE NUCLEONIC MATTER WITH MPNN

$^2\text{H}$  clusters



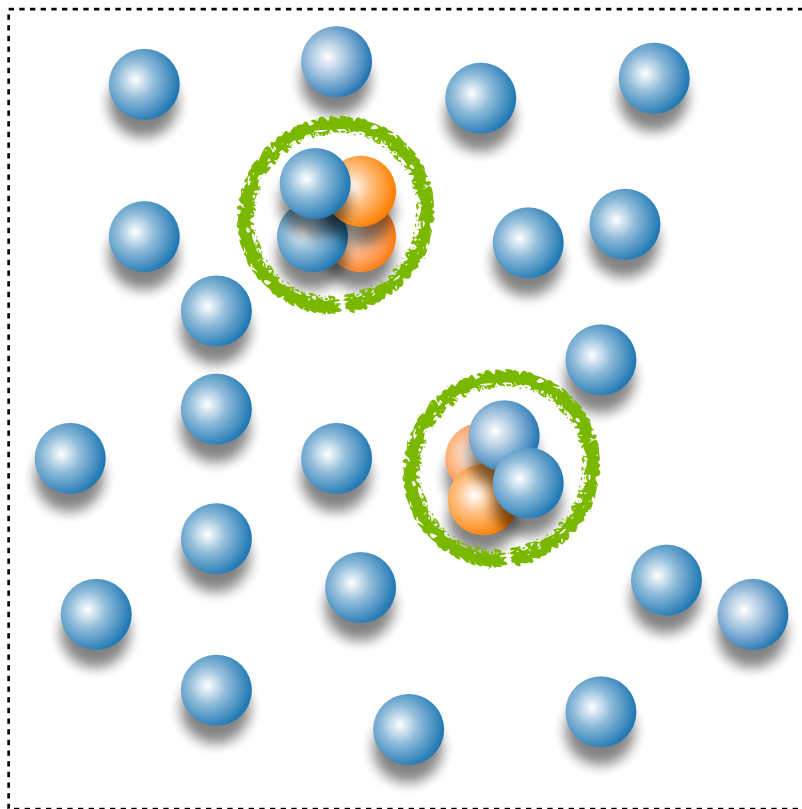
# DILUTE NUCLEONIC MATTER WITH MPNN

$^8\text{Be}$  clusters



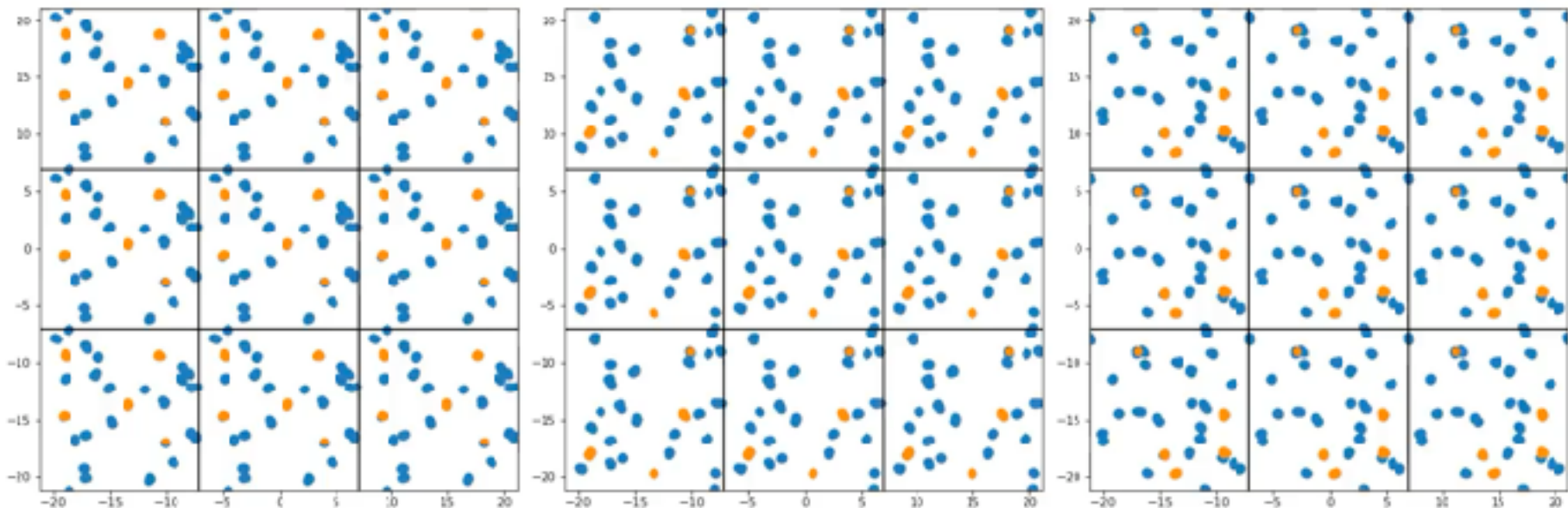
# DILUTE NUCLEONIC MATTER WITH MPNN

$^4\text{He}$  clusters



# DILUTE NUCLEONIC MATTER WITH MPNN

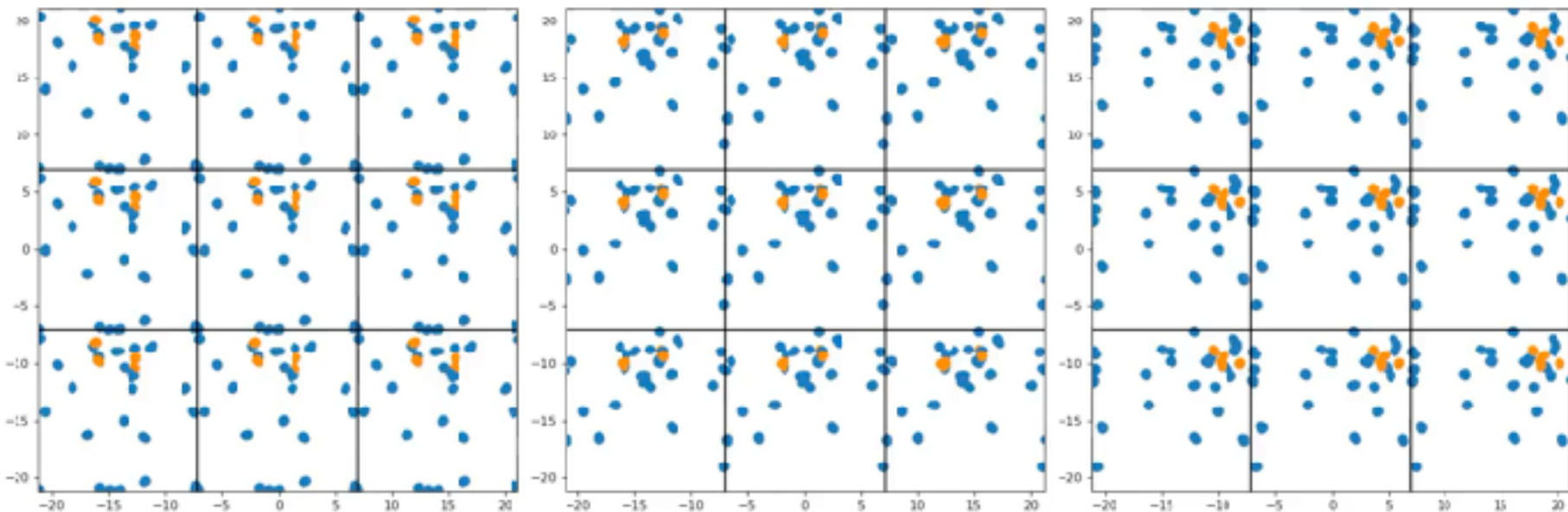
24 Neutrons, 4 Protons @  $\rho=0.01 \text{ fm}^{-3}$



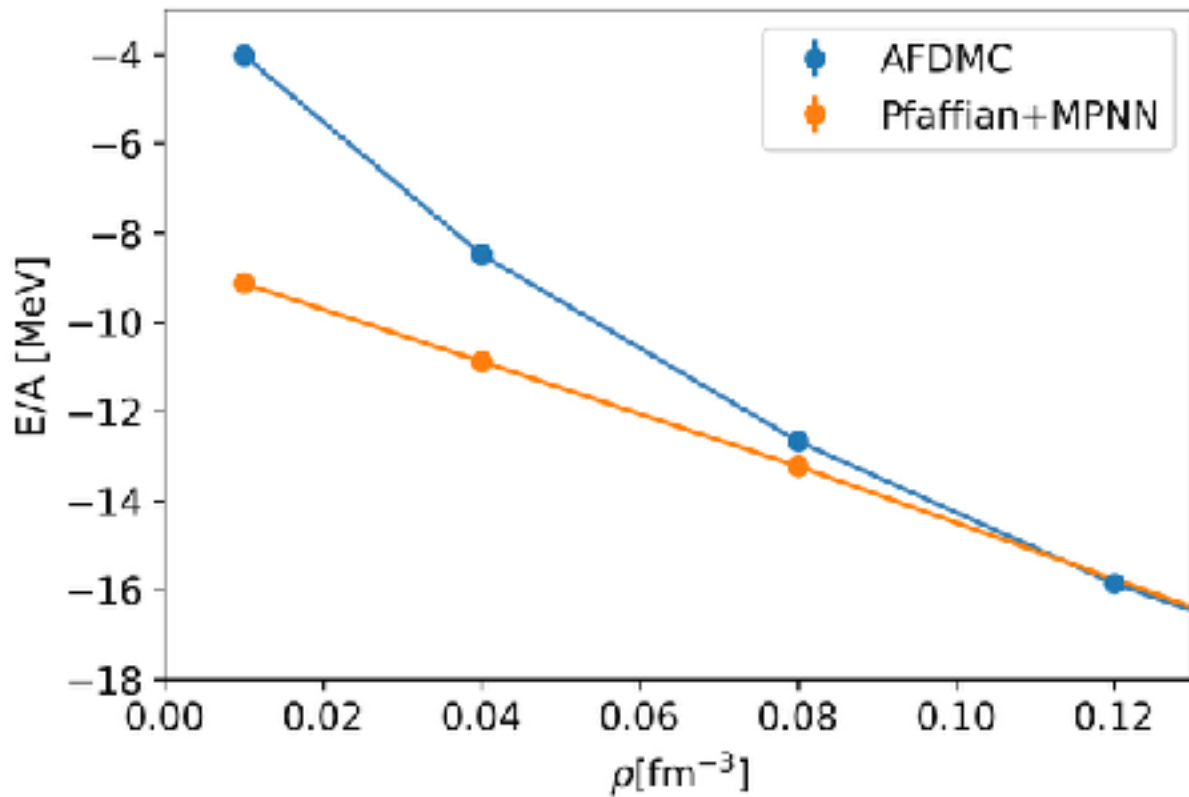


# DILUTE NUCLEONIC MATTER WITH MPNN

24 Neutrons, 4 Protons @  $\rho=0.01 \text{ fm}^{-3}$



# DILUTE NUCLEONIC MATTER WITH MPNN



# ESSENTIAL ELEMENTS OF NUCLEAR BINDING

Physics Letters B 793 (2019) 134802



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## Essential elements for nuclear binding

Ding-Nan Lu<sup>a</sup>, Ning Li<sup>a</sup>, Serdar Elhatisari<sup>b,c</sup>, Dean Lee<sup>a,\*</sup>, Evgeny Epelbaum<sup>d</sup>,  
Ulf-G. Meißner<sup>b,e,f</sup>

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<sup>b</sup> Helmholtz Institute for Nuclear Physics and Helmholtz Institute for Technical Physics, Helmholtz-Bau 10, D-52715 Bonn, Germany

<sup>c</sup> Institute of Engineering, Karlsruhe Institute of Technology, Karlsruhe, 76131, Germany

<sup>d</sup> State Institute for Nuclear and Subatomic Physics, University of Wrocław, ul. Wyspiańskiego 27, 50-033 Wrocław, Germany

<sup>e</sup> Institute for Advanced Study, Institute for Research in Physics, Paris Lodron Universität Salzburg, 5020 Salzburg, Austria

<sup>f</sup> Helmholtz Institute for Nuclear Physics, Helmholtz-Bau 10, D-52715 Bonn, Germany

<sup>\*</sup> E-mail: dean.lee@msu.edu



PHYSICAL REVIEW C **103**, 054003 (2021)

## Two- and three-nucleon contact interactions and ground-state energies of light- and medium-mass nuclei

R. Schiavilla<sup>1,2</sup>, L. Girlanda<sup>3,4</sup>, A. Ghosh<sup>5,6</sup>, A. Kievsky<sup>7</sup>, A. Lovato<sup>8,9</sup>, L. E. Marcucci<sup>5,\*</sup>, M. Piarulli<sup>9</sup> and M. Viviani<sup>9</sup>

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<sup>3</sup>Department of Mathematics and Physics, University of Salerno, 84084 Lacco, Italy

<sup>4</sup>INFN-LNS, 75100 Lecce, Italy

<sup>5</sup>Department of Physics, University of Pisa, 56127 Pisa, Italy

<sup>6</sup>INFN-Pisa, 56127 Pisa, Italy

<sup>7</sup>Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

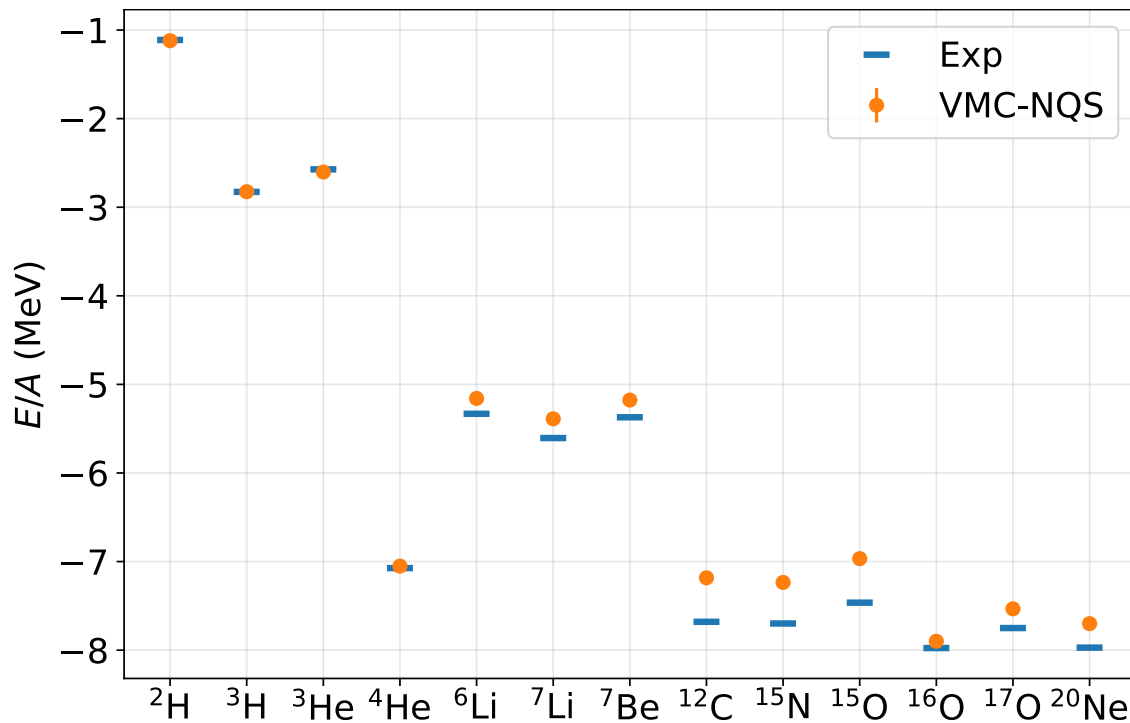
<sup>8</sup>INFN TIFPA Trento Institute of Fundamental Physics and Applications, 38122 Trento, Italy

<sup>9</sup>Department of Physics, Washington University in St. Louis, St. Louis, Missouri 63130, USA

(Received 3 February 2021; accepted 6 May 2021; published 24 May 2021)

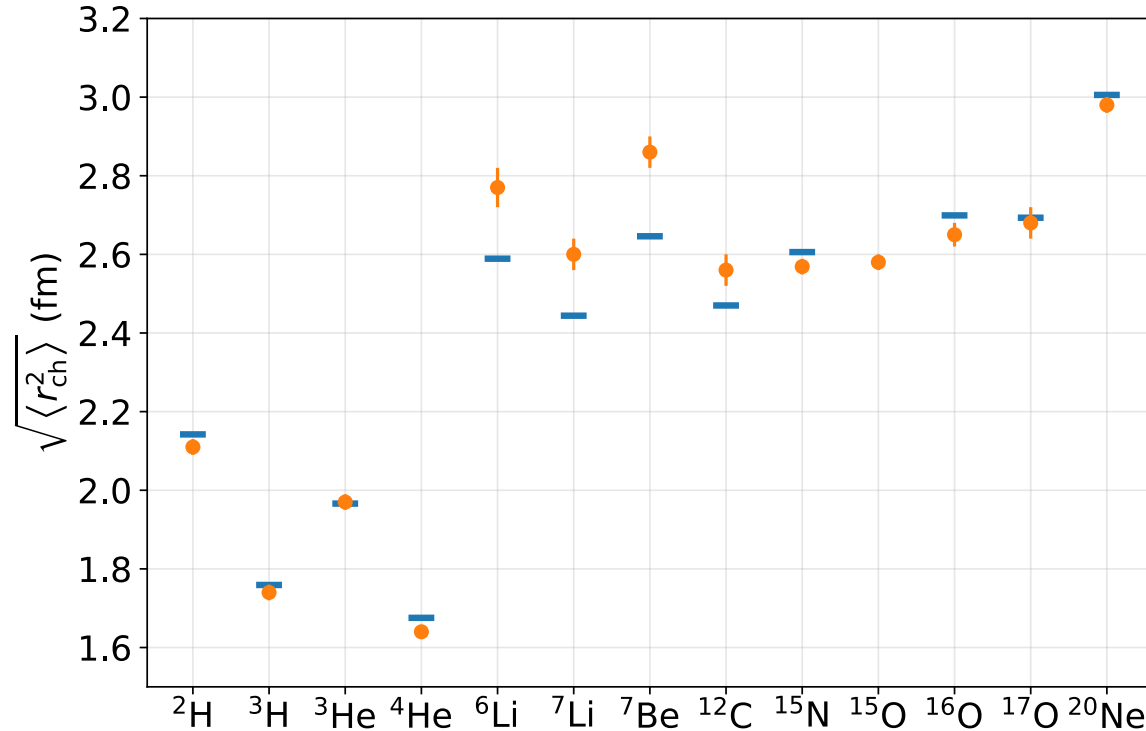
# ESSENTIAL ELEMENTS OF NUCLEAR BINDING

A simple pionless-EFT Hamiltonian reproduces well the spectrum of different nuclei



# ESSENTIAL ELEMENTS OF NUCLEAR BINDING

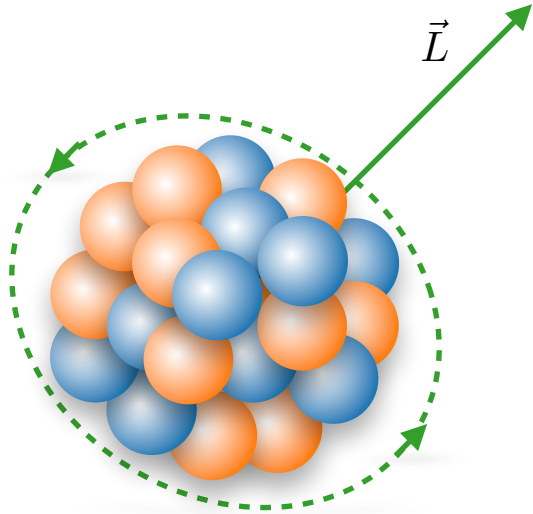
Deviations from experiments are more pronounced for charge radii



# MAGNETIC MOMENTS

The ground-state is generate in  $L_z$

$$|\Psi_{HN}; L, S\rangle = \sum_{L_z S_z} c_{L_z S_z}^{L, S} |L, L_z; S, S_z\rangle.$$



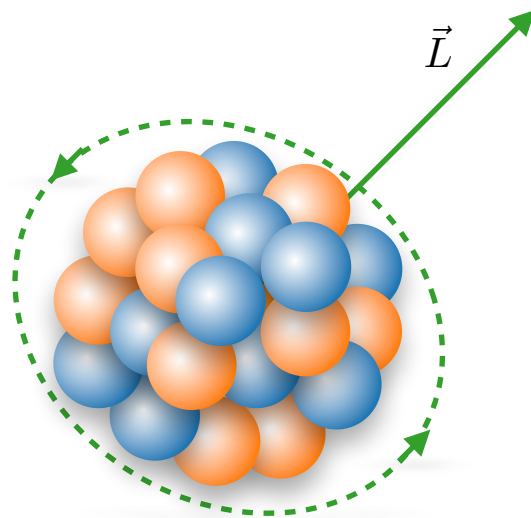
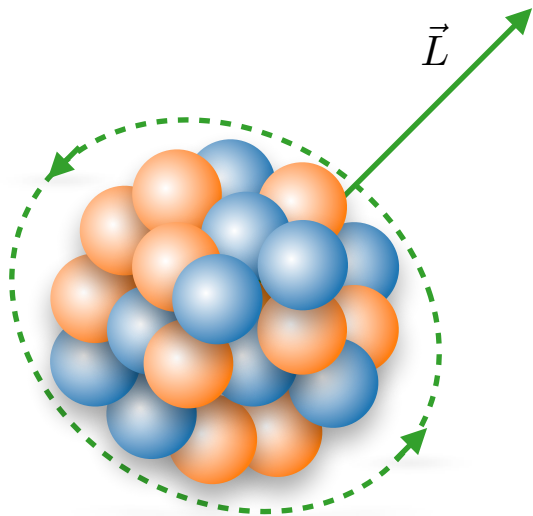
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$$|\Psi_{HN}; L, S\rangle = \sum_{L_z S_z} c_{L_z S_z}^{L, S} |L, L_z; S, S_z\rangle.$$

Remove this degeneracy by

$$H \rightarrow H - B_z L_z$$



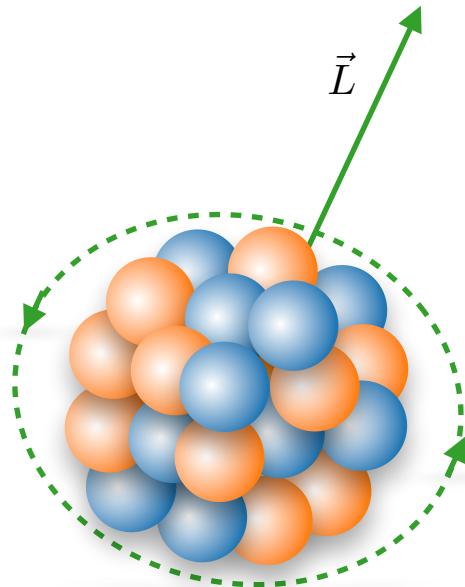
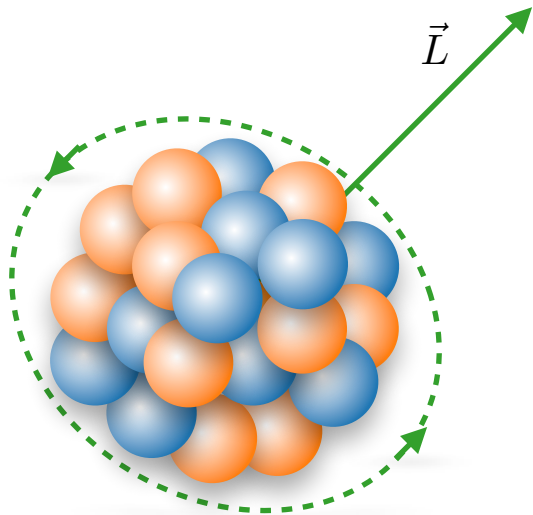
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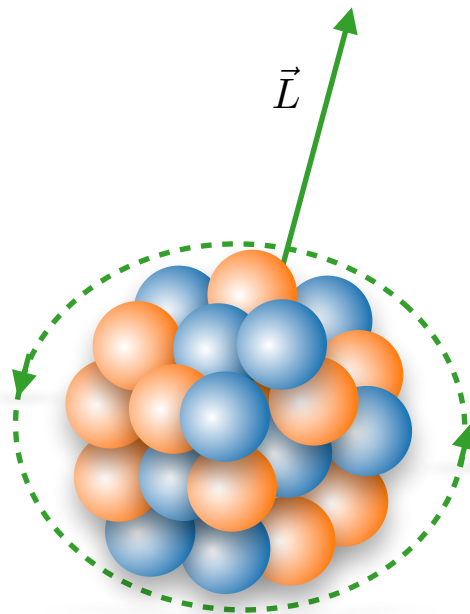
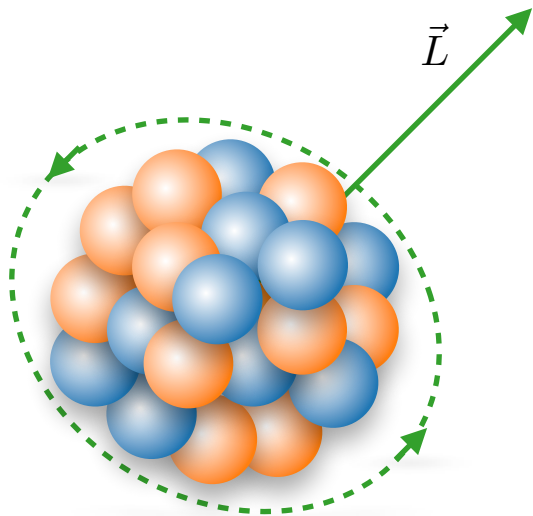
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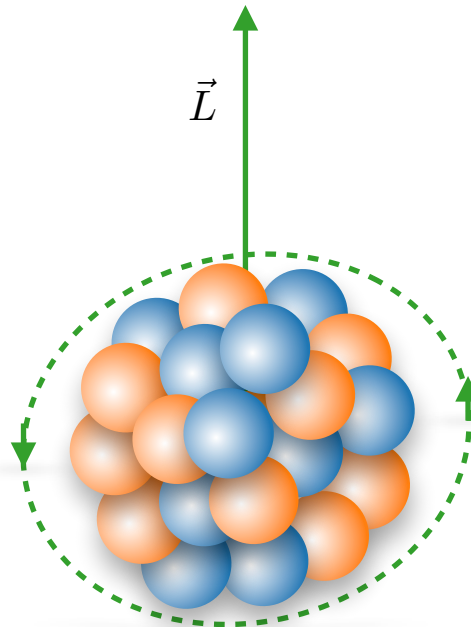
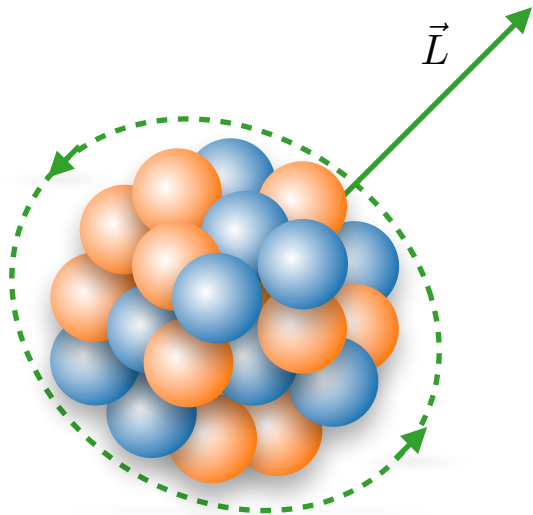
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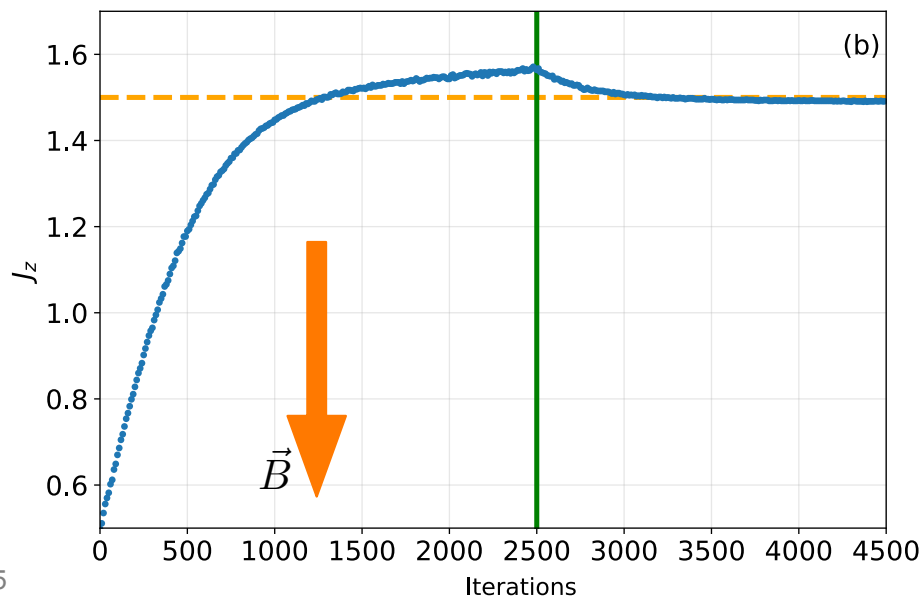
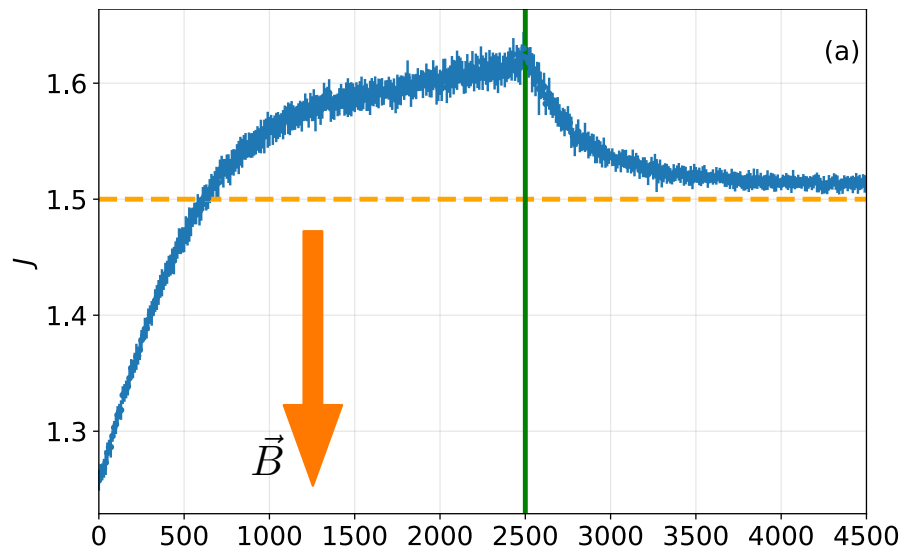
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Remove this degeneracy by

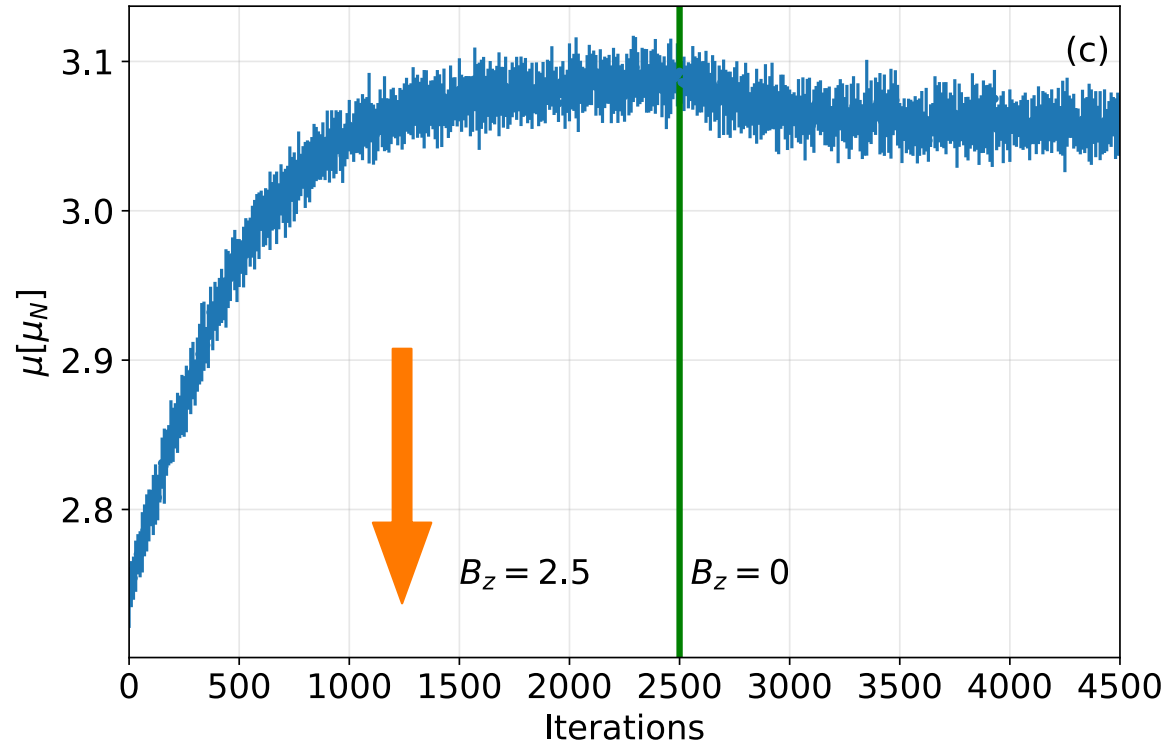
$$H \rightarrow H - B_z L_z$$



# MAGNETIC MOMENTS

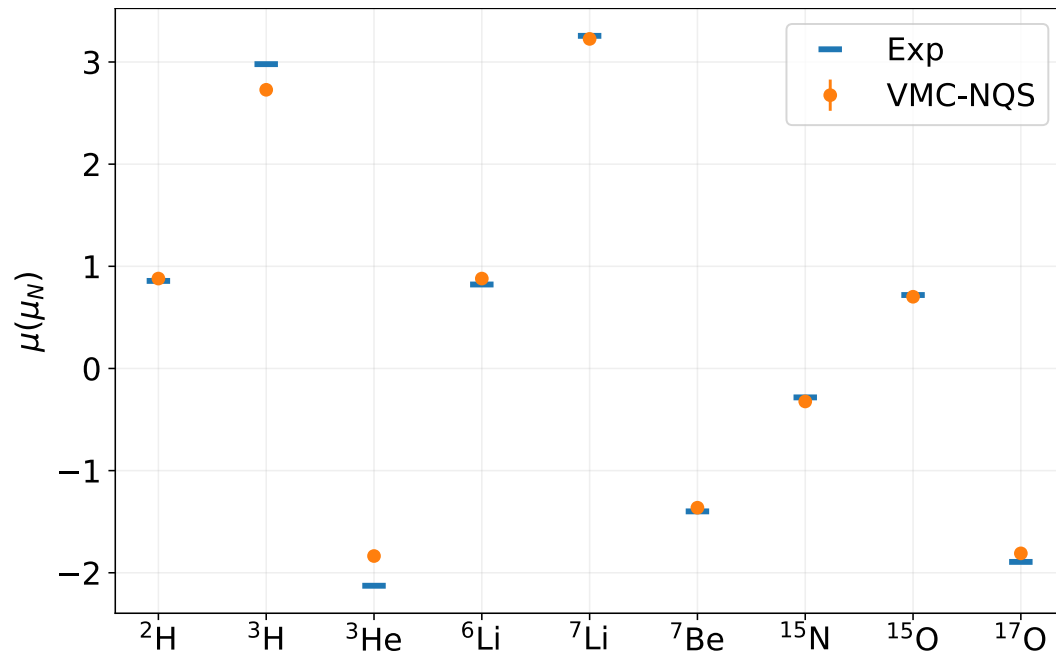


# MAGNETIC MOMENTS



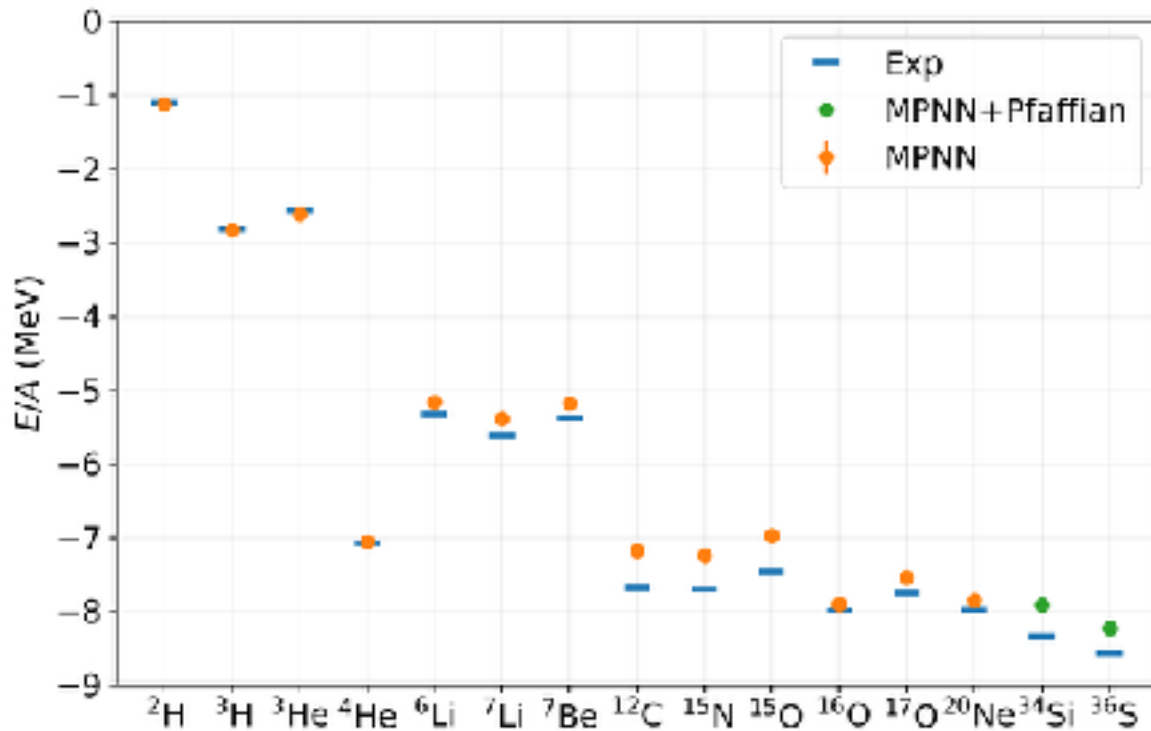
# MAGNETIC MOMENTS WITH MPNN

In addition to energies and single-particle densities, we compute electroweak properties



# ESSENTIAL ELEMENTS OF NUCLEAR BINDING

A simple pionless-EFT Hamiltonian reproduces well the spectrum of different nuclei



# HIGH-PERFORMANCE COMPUTING

The variational Monte Carlo with neural network quantum state code is by design scalable to leadership-class hybrid CPU/GPU computers

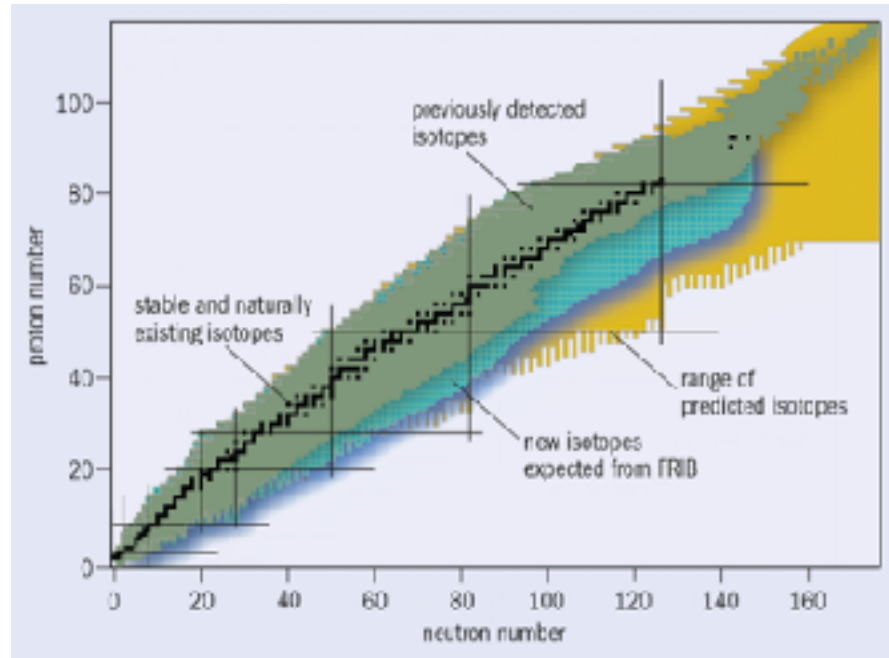


# PERSPECTIVES

- **Larger nuclei** are within reach on leadership-class machines

➔ Optimized version of the code reaches  $^{76}\text{Ge}$  on Polaris @ANL

➔ Work in progress to port it to Intel Ponte Vecchio ( $^{100}\text{Sn}$  and beyond on Aurora @ANL)





# PERSPECTIVES

- **Real-time dynamics** is the prototypical exponentially-hard problem in many-body theory

→ Relevant for: fission, fusion lepton- and hadron-nucleus scattering

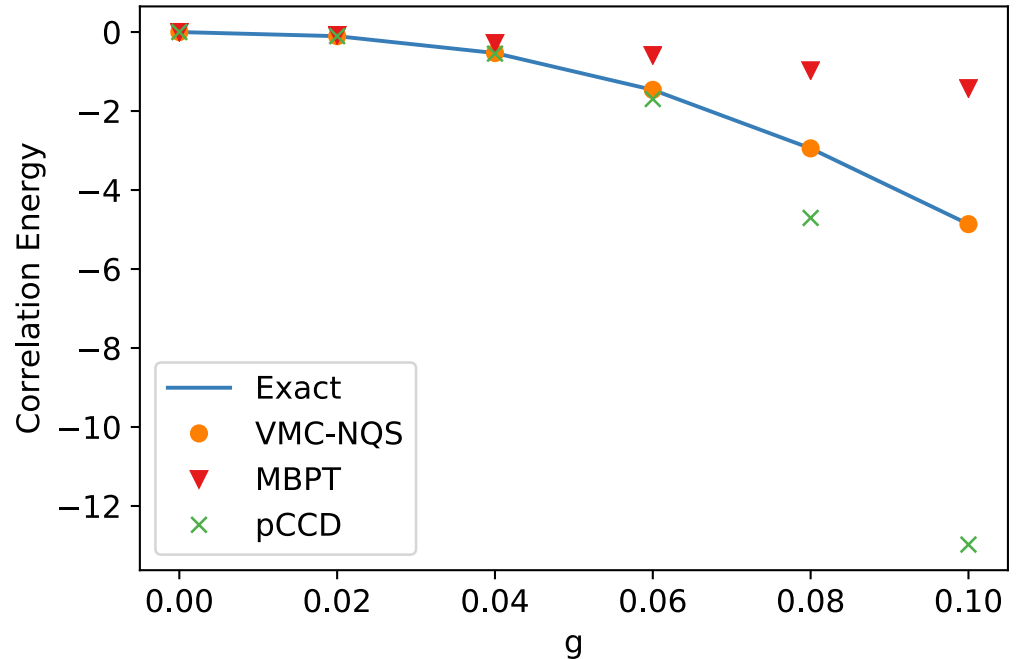
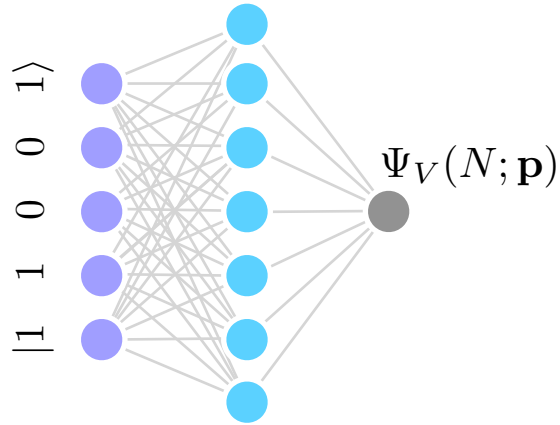
→ Learn from TDHF calculations: collaboration with K. Godbey, W. Nazarewicz, and N. Rocco.

$$\mathcal{D} (|\Psi(\mathbf{p}_{t+\delta t})\rangle, e^{-iHt}|\Psi(\mathbf{p}_t)\rangle)^2 = \arccos \left( \sqrt{\frac{\langle \Psi(\mathbf{p}_{t+\delta t}) | e^{-iHt} | \Psi(\mathbf{p}_t) \rangle \langle \Psi(\mathbf{p}_t) | e^{iHt} | \Psi(\mathbf{p}_{t+\delta t}) \rangle}{\langle \Psi(\mathbf{p}_{t+\delta t}) | \Psi(\mathbf{p}_{t+\delta t}) \rangle \langle \Psi(\mathbf{p}_t) | \Psi(\mathbf{p}_t) \rangle}} \right)^2$$

→ Alternative approach based on integral transform, with N. Barnea, E. Parnes, and N. Rocco

# SOME PERSPECTIVES:

- **Occupation number formalism** allows to naturally satisfy Pauli's exclusion principle



A solid green vertical bar is located on the left side of the slide.

**THANK YOU**