



JOHANNES GUTENBERG  
UNIVERSITÄT MAINZ



# Electric dipole polarizability in medium-mass nuclei

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In collaboration with:

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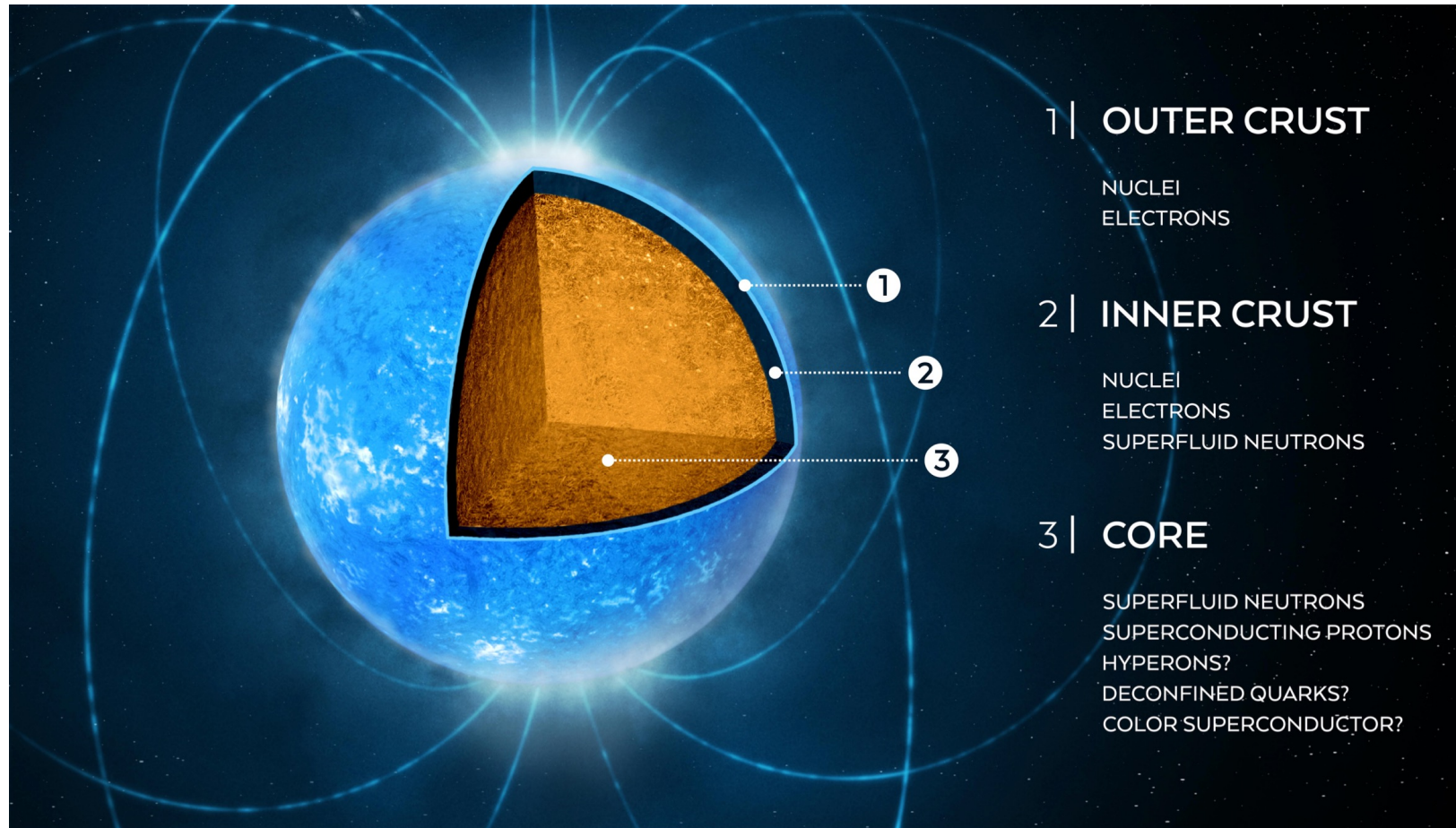
Gaute Hagen (ORNL)

Gustav R. Jansen (ORNL)

Thomas Papenbrock

(ORNL/UTK)

# Extreme matter in neutron stars



A. Watts et al, RMP 88, 021001 (2016).

# Neutron matter equation of state (EOS)

Infinite **nuclear matter EOS**:

$$\mathcal{E}(\rho, \alpha) = \mathcal{E}_{\text{SNM}}(\rho) + \alpha^2 \mathcal{S}(\rho) + \mathcal{O}(\alpha^4)$$

$$\rho = (\rho_n + \rho_p)$$

$$\alpha = (\rho_n - \rho_p) / \rho$$

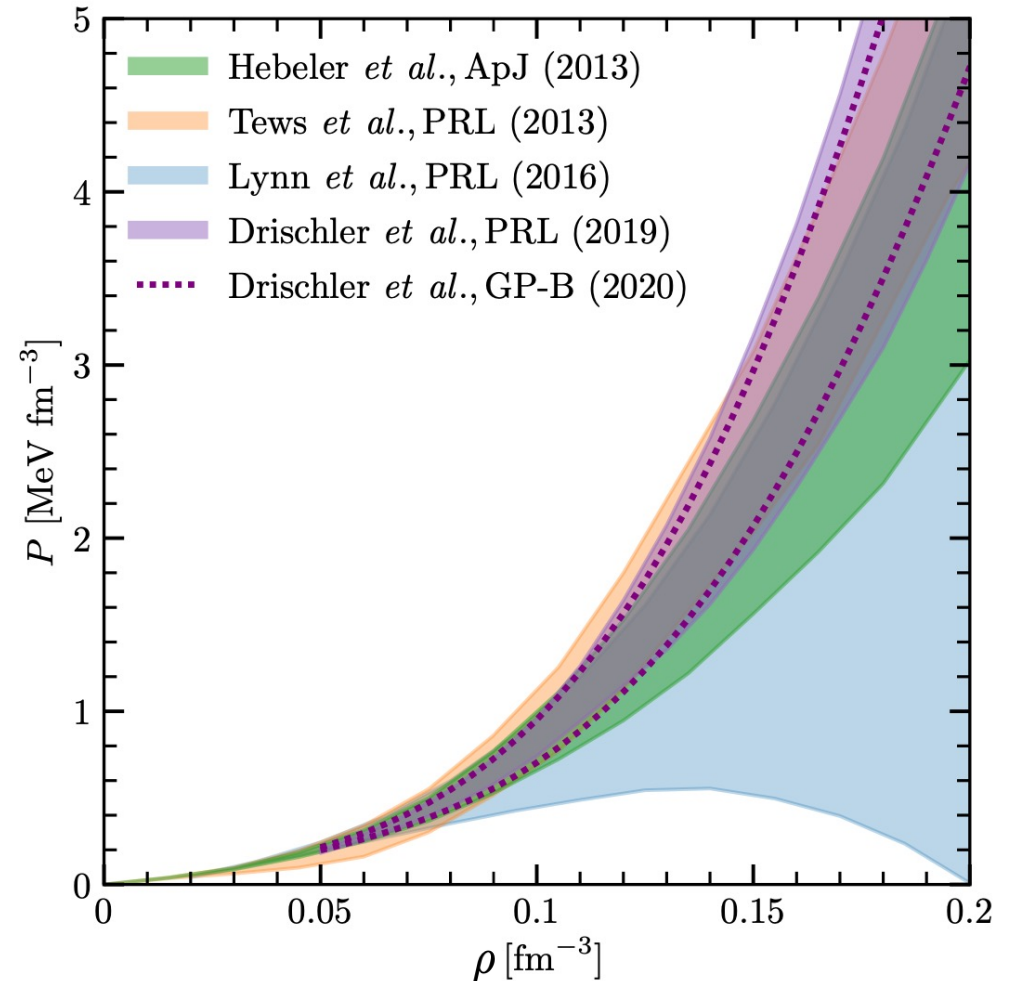
where the **symmetry energy** is:

$$\mathcal{S}(\rho) = J + L \frac{(\rho - \rho_0)}{3\rho_0} + \dots$$

**symmetry energy**  
at saturation density

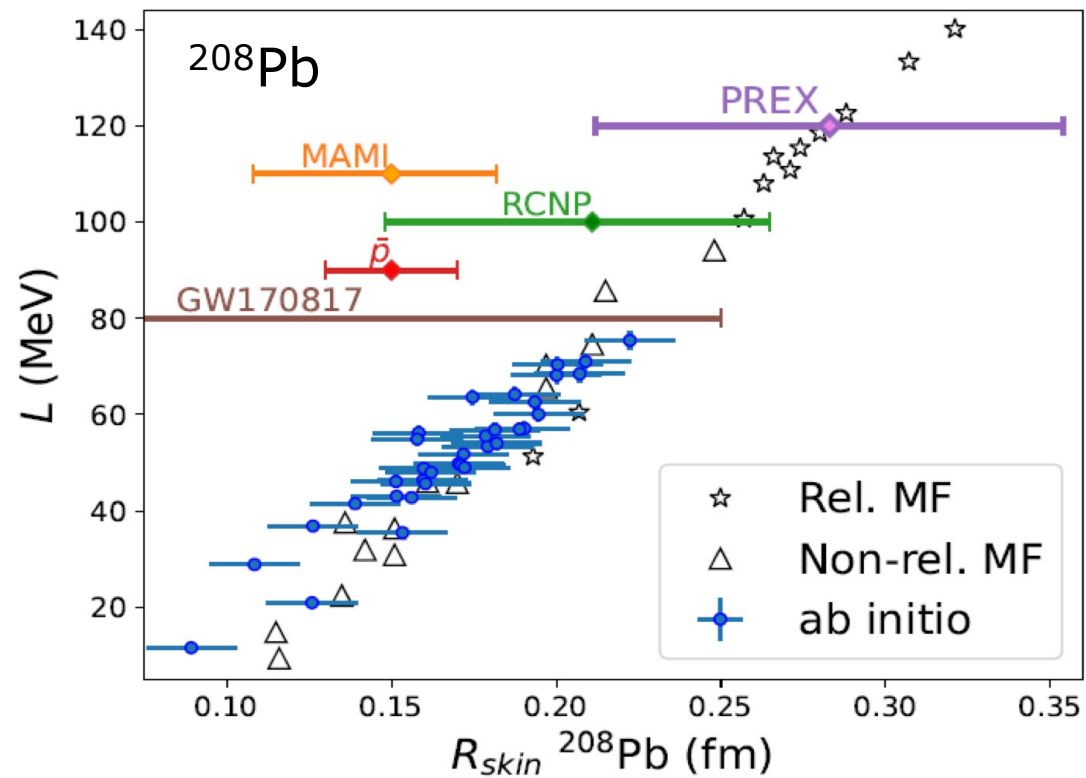
**slope parameter**,  
related to **pressure of  
pure neutron matter**  
at saturation density

S. Huth et al, PRC 103, 025803 (2021).



# How to constrain $L$ ?

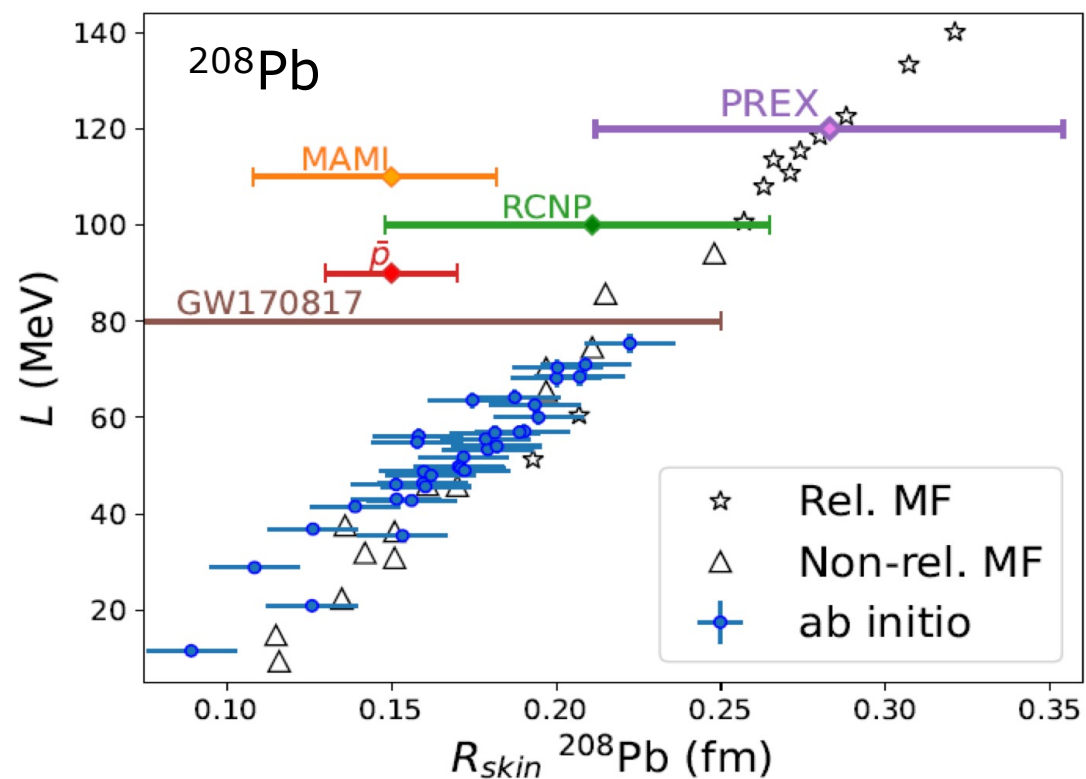
## Neutron-skin thickness



B. Hu et al, Nat. Phys. **18**, 1196–1200 (2022).

# How to constrain L?

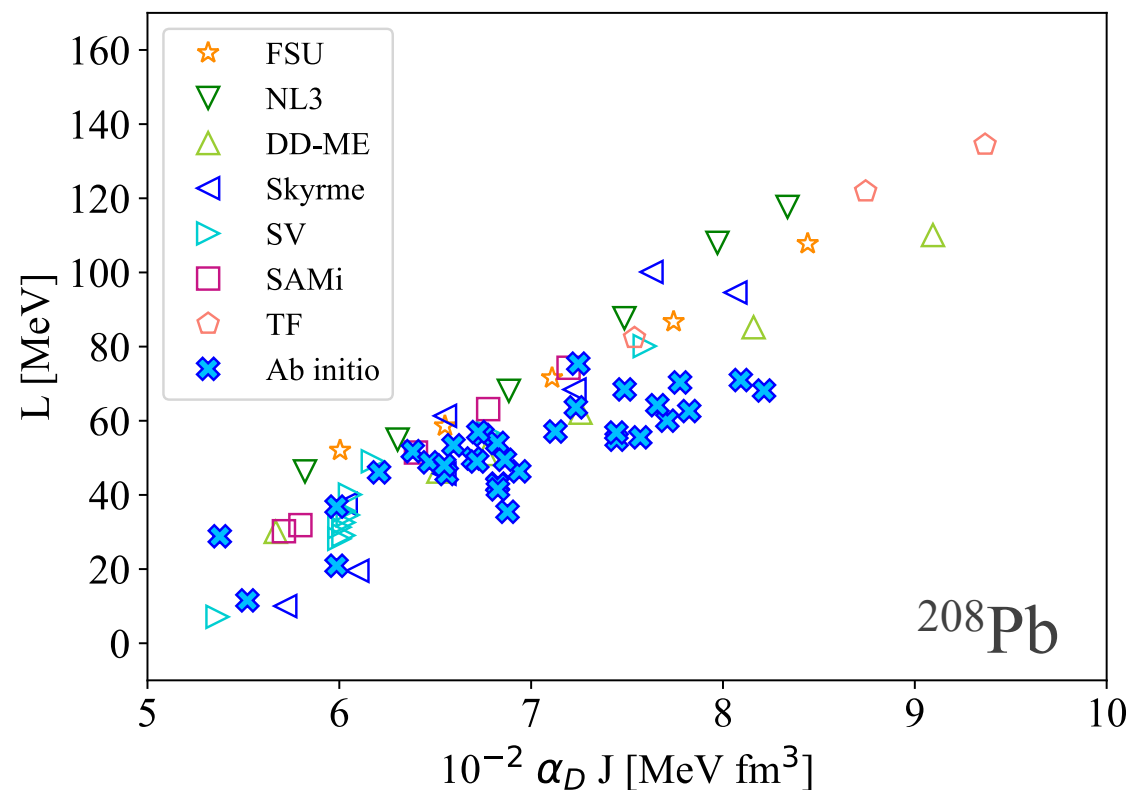
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Electric dipole polarizability

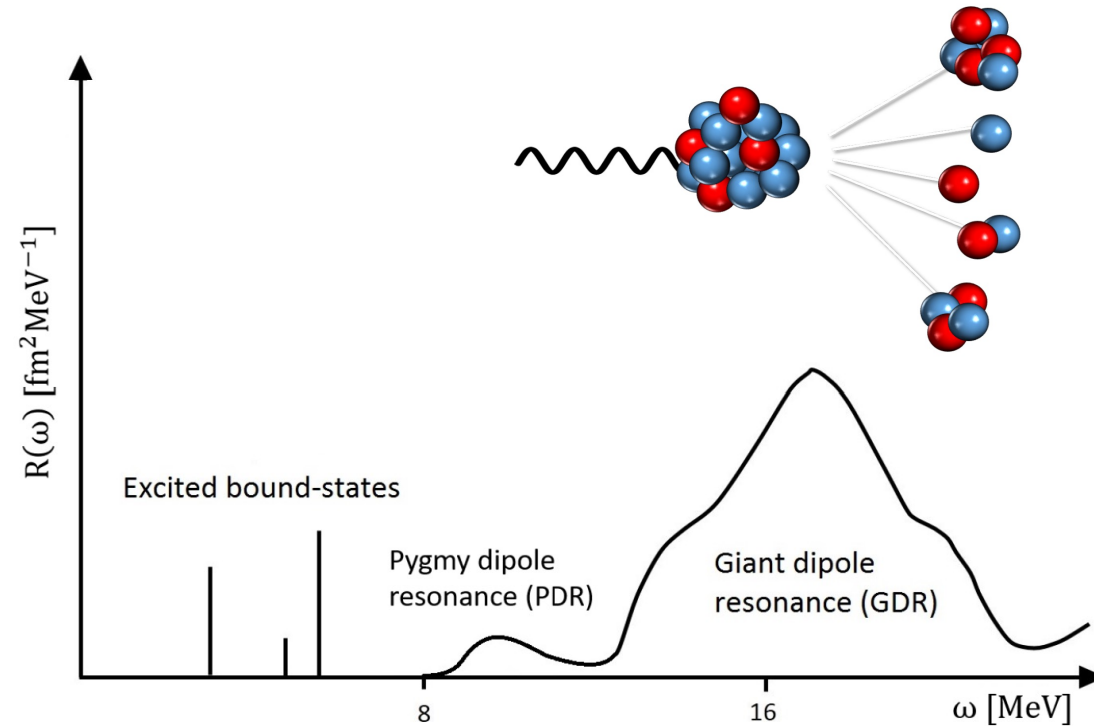
$$\alpha_D = 2\alpha \int d\omega \frac{R(\omega)}{\omega}$$



Data from X. Roca-Maza et al, PRC **88**, 024316 (2013),  
B. Hu et al, Nat. Phys. **18**, 1196-1200 (2022).

# Nuclear response functions

$$R(\omega) = \sum_f |\langle \Psi_f | \Theta | \Psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$



# Coupled-cluster theory

- Starting point: **Hartree-Fock** reference state on the HO basis  $|\Phi_0\rangle$
- Add correlations via:

$$e^{-T} H e^T |\Phi_0\rangle = \bar{H} |\Phi_0\rangle = E_0 |\Phi_0\rangle$$

Similarity-transformed  
Hamiltonian  
(non-Hermitian)

with

$$T = \sum t_i^a a_a^\dagger a_i + \sum t_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i + \sum t_{ijk}^{abc} a_a^\dagger a_b^\dagger a_c^\dagger a_k a_j a_i + \dots$$

→ coefficients from  
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equations

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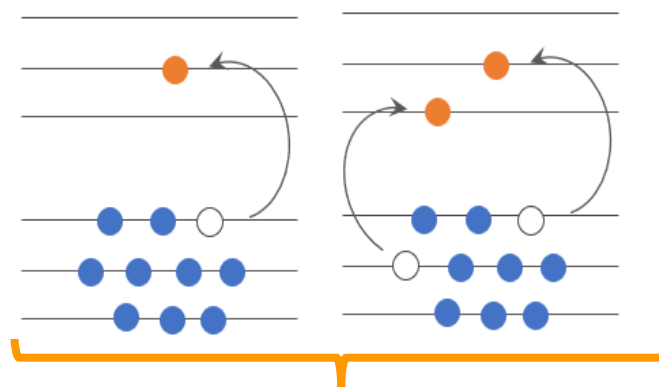
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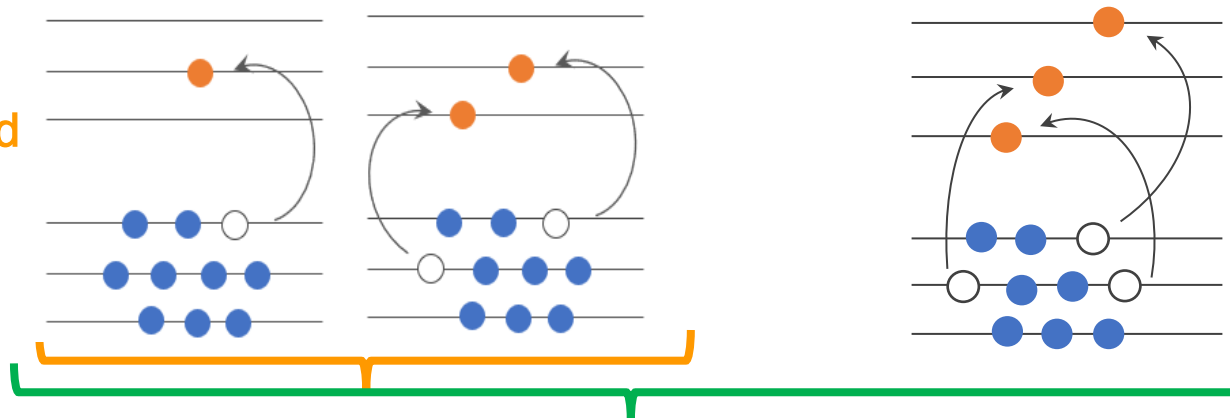
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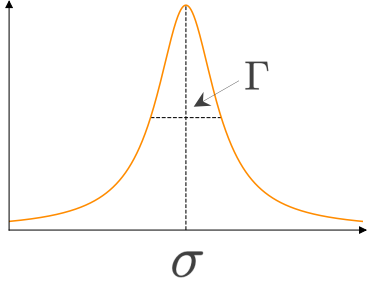
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+ triples  
(CCSDT-1)

# From bound to dipole-excited states

$$R(\omega) = \sum_f |\langle \Psi_f | \Theta | \Psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$

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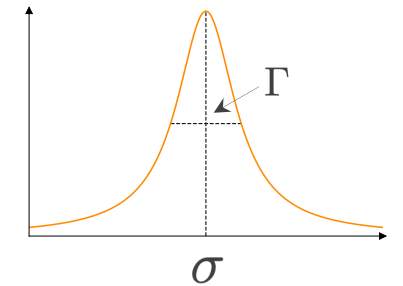
$$R(\omega) = \sum_f |\langle \Psi_f | \Theta | \Psi_0 \rangle|^2 \delta(E_f - E_0 - \omega)$$
$$L(\sigma, \Gamma) = \frac{\Gamma}{\pi} \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2} = \frac{\Gamma}{\pi} \langle \Psi_L | \Psi_R \rangle$$


Lorentz Integral Transform (LIT)

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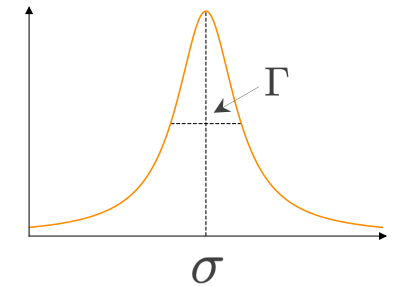
$$(\bar{H} - E_0 - \sigma - i\Gamma) |\Psi_R\rangle = \bar{\Theta} |\Phi_0\rangle$$

CC equation of motion  
with a source

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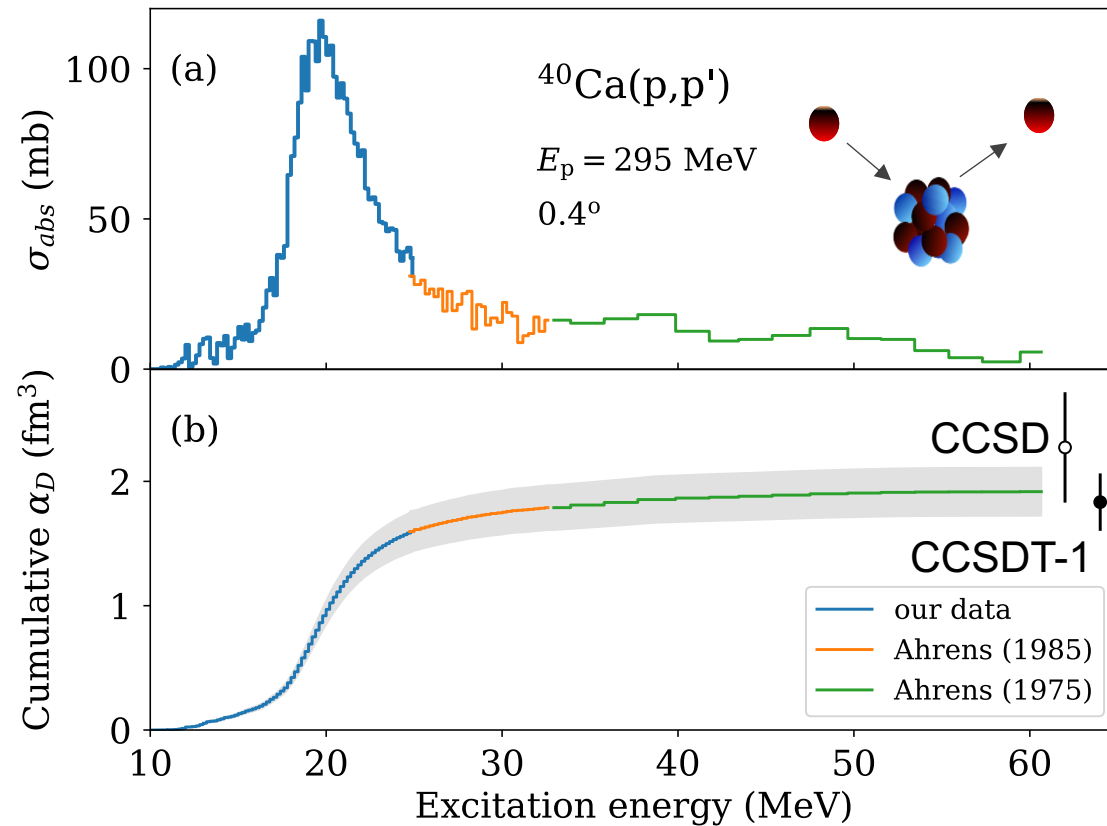
CC equation of motion  
with a source

LIT-CC ansatz for closed-shell nuclei:

$$|\Psi_R\rangle = \mathcal{R} |\Phi_0\rangle \quad \mathcal{R} = r_0 + \sum_i r_i^a a_a^\dagger a_i + \sum_{ij} r_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i + \dots$$

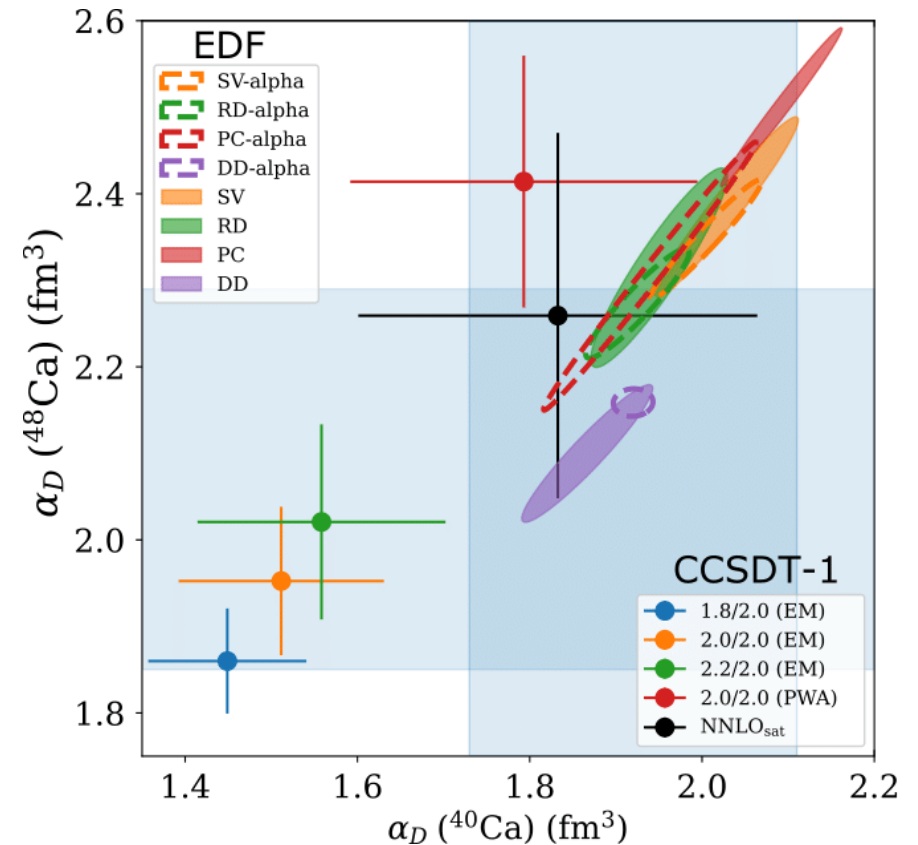
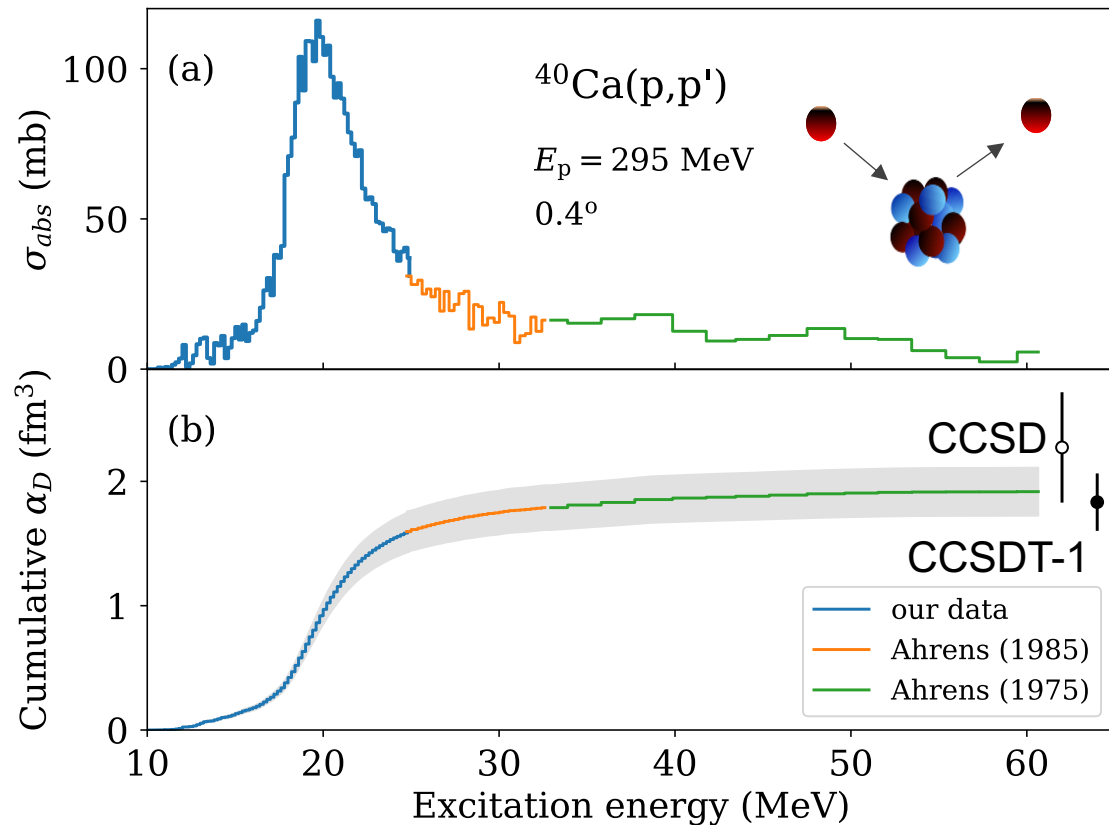
# The case of $^{40}\text{Ca}$

R. Fearick, P. von Neumann-Cosel, S. Bacca, FB et al, Phys. Rev. Research 5, L022044 (2023).



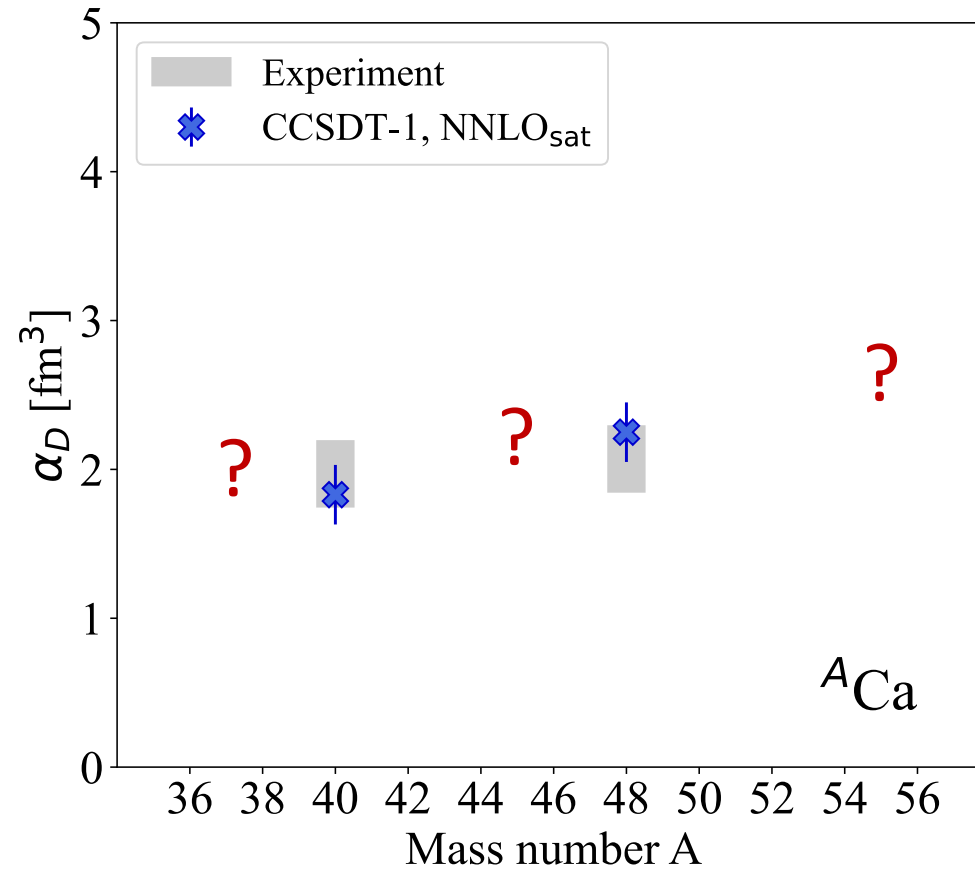
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R. Fearick, P. von Neumann-Cosel, S. Bacca, FB et al, Phys. Rev. Research 5, L022044 (2023).



Constraints on **slope parameter** from chEFT interactions of this work:  $L = 41 - 49$  MeV  
 Global analysis of all Ca+Pb skin data [J. Lattimer, Particles (2023)]:  $L = 40 \pm 8$  MeV

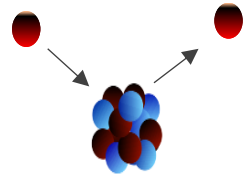
# Happy ending for $^{40,48}\text{Ca}$ ... but what's next?



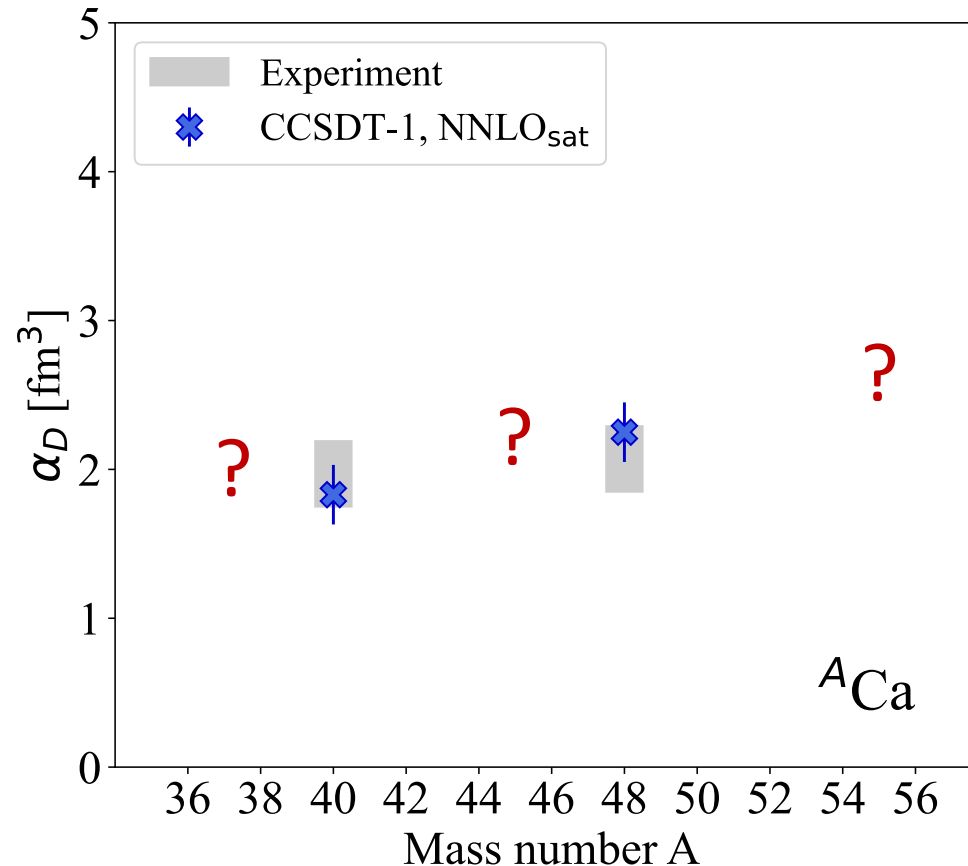


# Happy ending for $^{40,48}\text{Ca}$ ... but what's next?

Ongoing **inelastic proton scattering experiments** to measure  $\alpha_D$

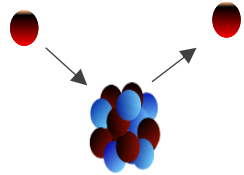


at iThemba Labs, South Africa and RNCN, Japan of **nuclei near closed shells**, e.g.  $^{42}\text{Ca}$ ,  $^{58}\text{Ni}$ ...

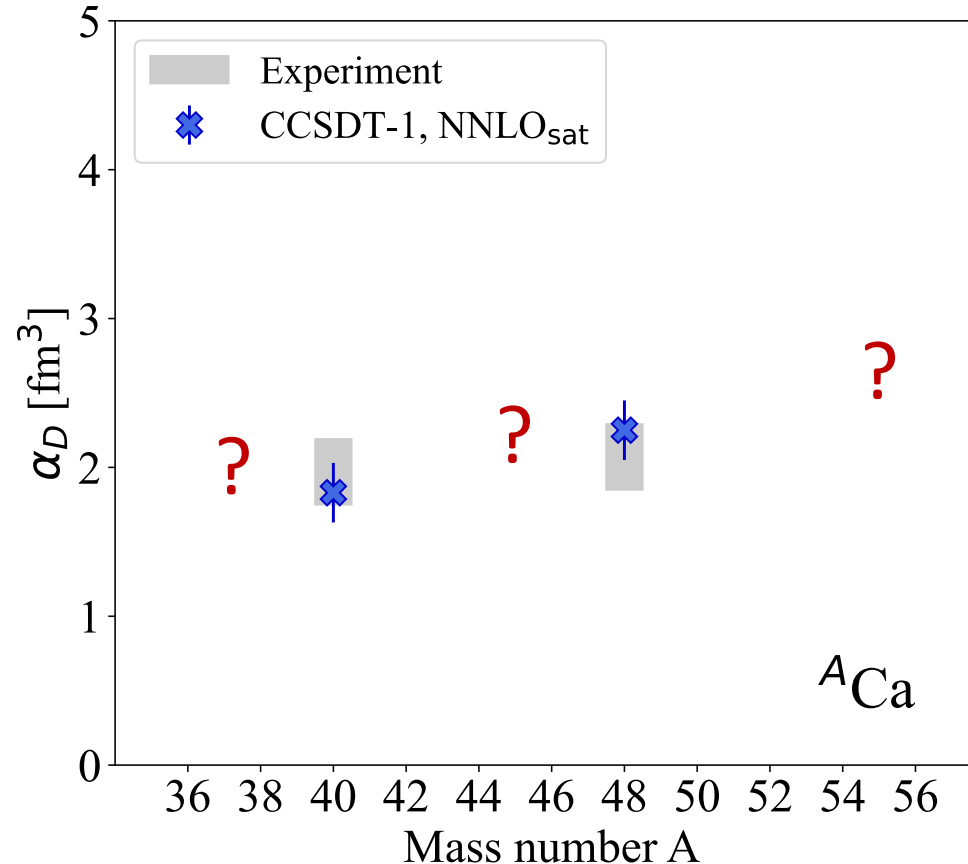


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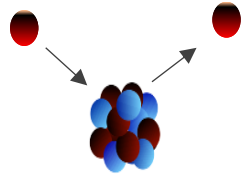
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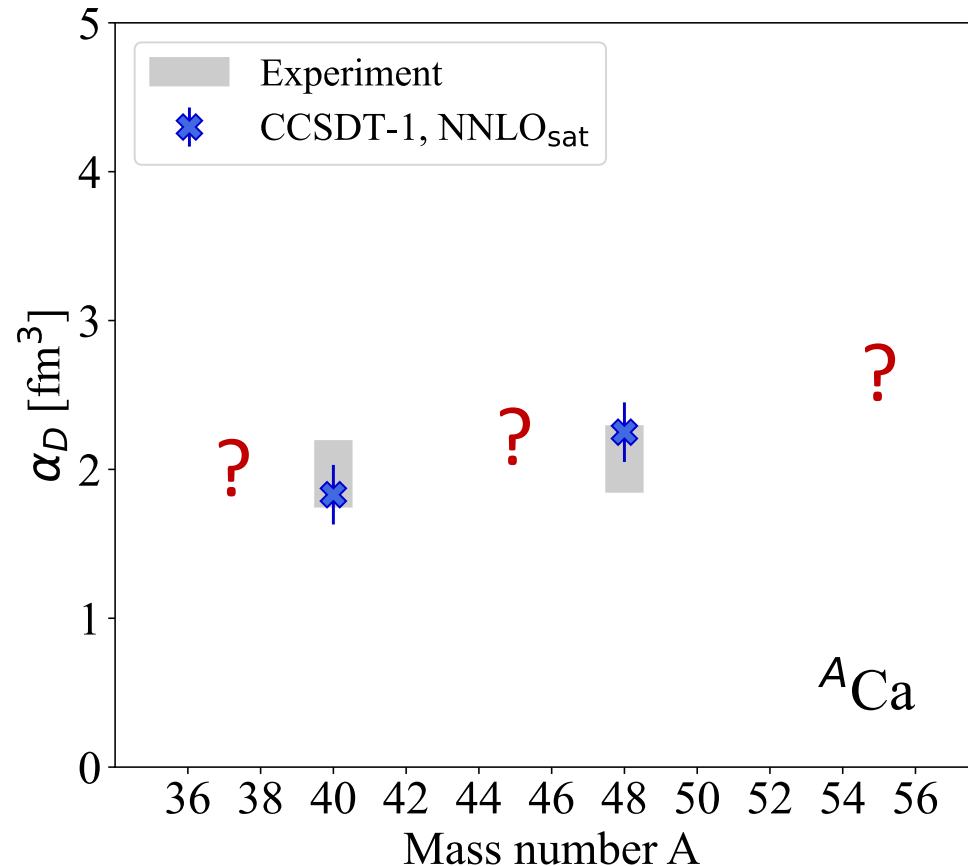
... and in the future, a FRIB400 upgrade will allow studies of  $\alpha_D$  for **very neutron-rich nuclei** with **Coulex experiments**.

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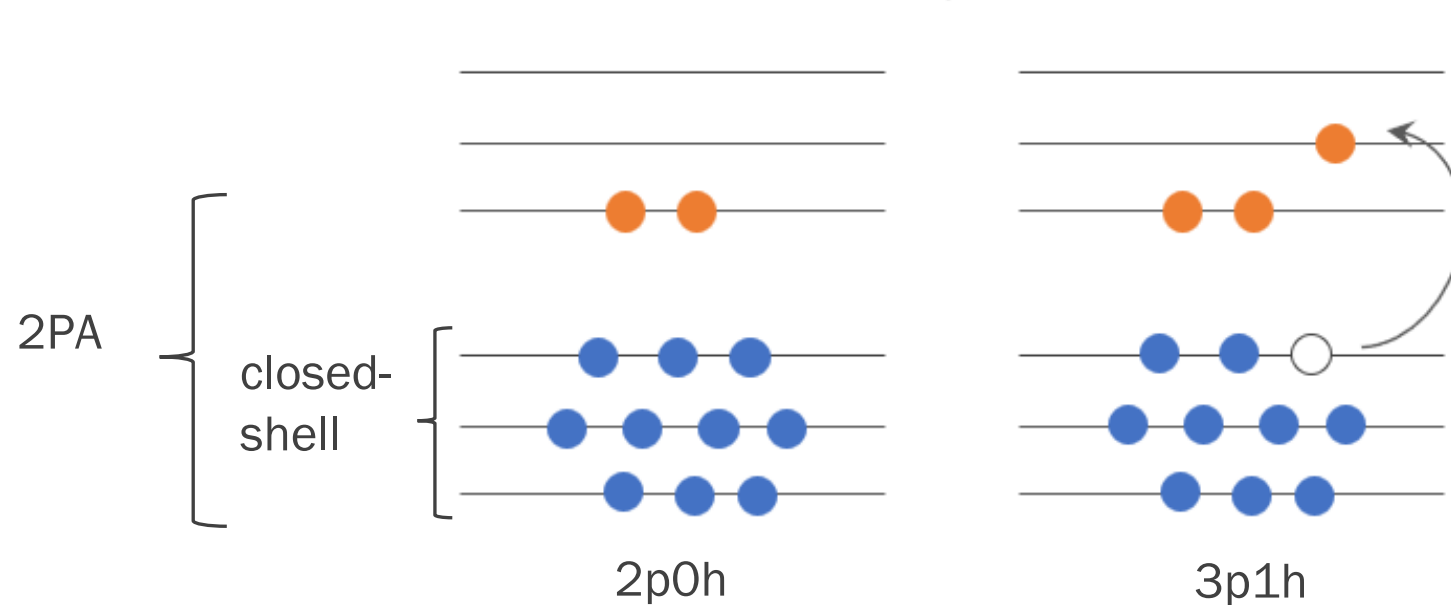
... and in the future, a FRIB400 upgrade will allow studies of  $\alpha_D$  for **very neutron-rich nuclei** with **Coulex experiments**.

**We need to extend the LIT-CC method beyond closed-shell nuclei!**

# LIT-CC for open-shell nuclei: the 2-Particle-Attached (2PA) case

$$\mathcal{R} = \frac{1}{2} \sum r^{ab} a_a^\dagger a_b^\dagger + \frac{1}{6} \sum r_i^{abc} a_a^\dagger a_b^\dagger a_c^\dagger a_i + \dots$$

$$|\Psi_R\rangle = \mathcal{R} |\Phi_0\rangle$$



# Many-body truncations in LIT-CC

To solve

$$(\bar{H} - E_0 - \sigma - i\Gamma) |\Psi_R\rangle = \bar{\Theta} |\Phi_0\rangle$$

we have **two CC expansions**, for ground and excited states.

In the closed-shell case we consider **excited states @CCSD** and **vary ground-state scheme**.

We then estimate the **many-body uncertainty** as:

$$\delta_{\alpha_D}^{CC} \approx \frac{\alpha_D^{CCSD} - \alpha_D^{CCSDT-1}}{2}$$



# What about the 2PA case?

In this case, for the moment we do not have access to orders higher than 3p1h...

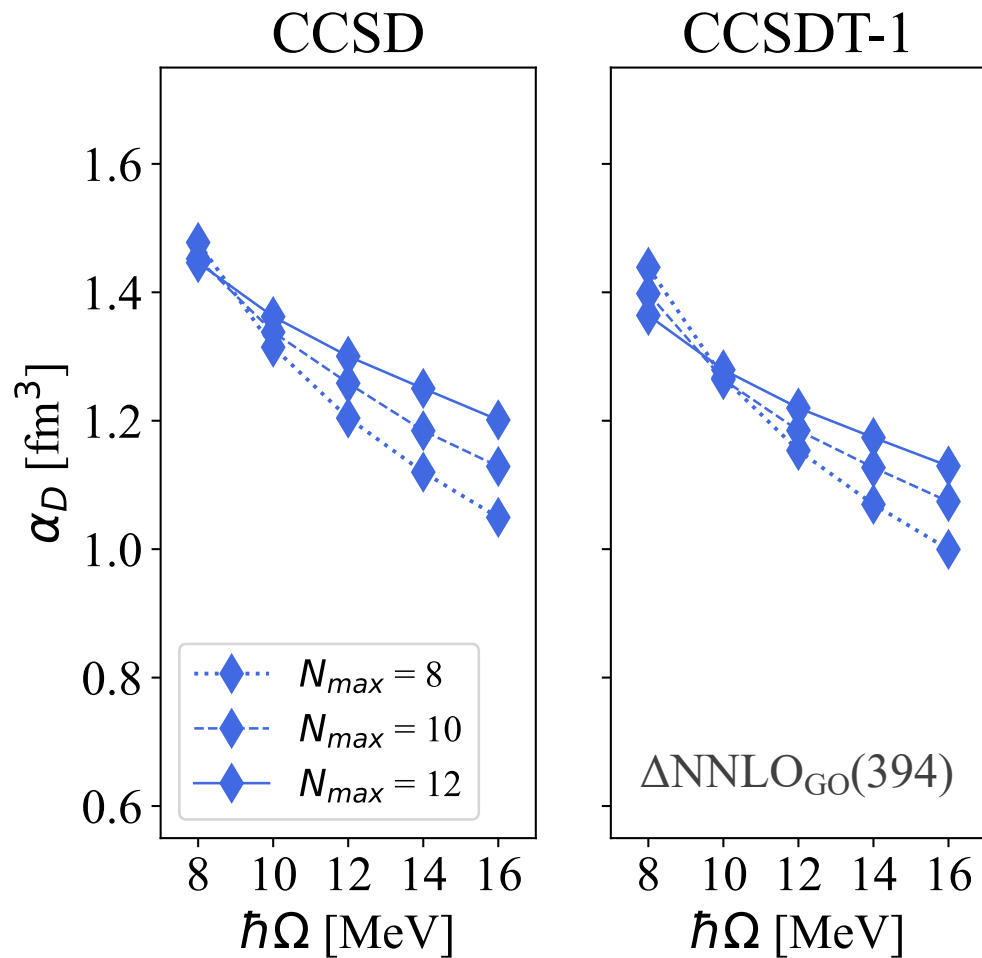
If we can't go higher, let's go lower!

We use the **3p1h scheme for excited states**, to keep as much correlations as possible in the LIT calculation, and we look at:

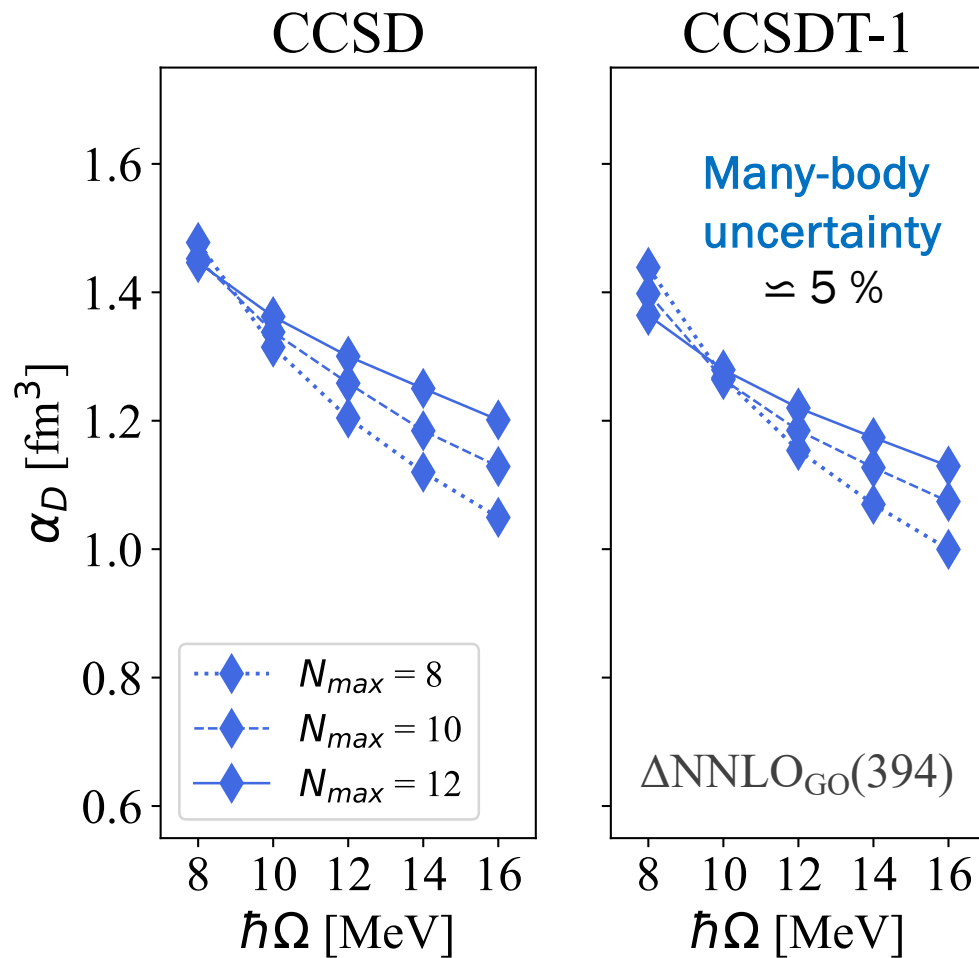
$$\delta_{\alpha_D}^{CC} \approx \frac{\alpha_D^{3p1h/3p1h} - \alpha_D^{2p0h/3p1h}}{2}$$



# Electric dipole polarizability of $^{24}\text{O}$

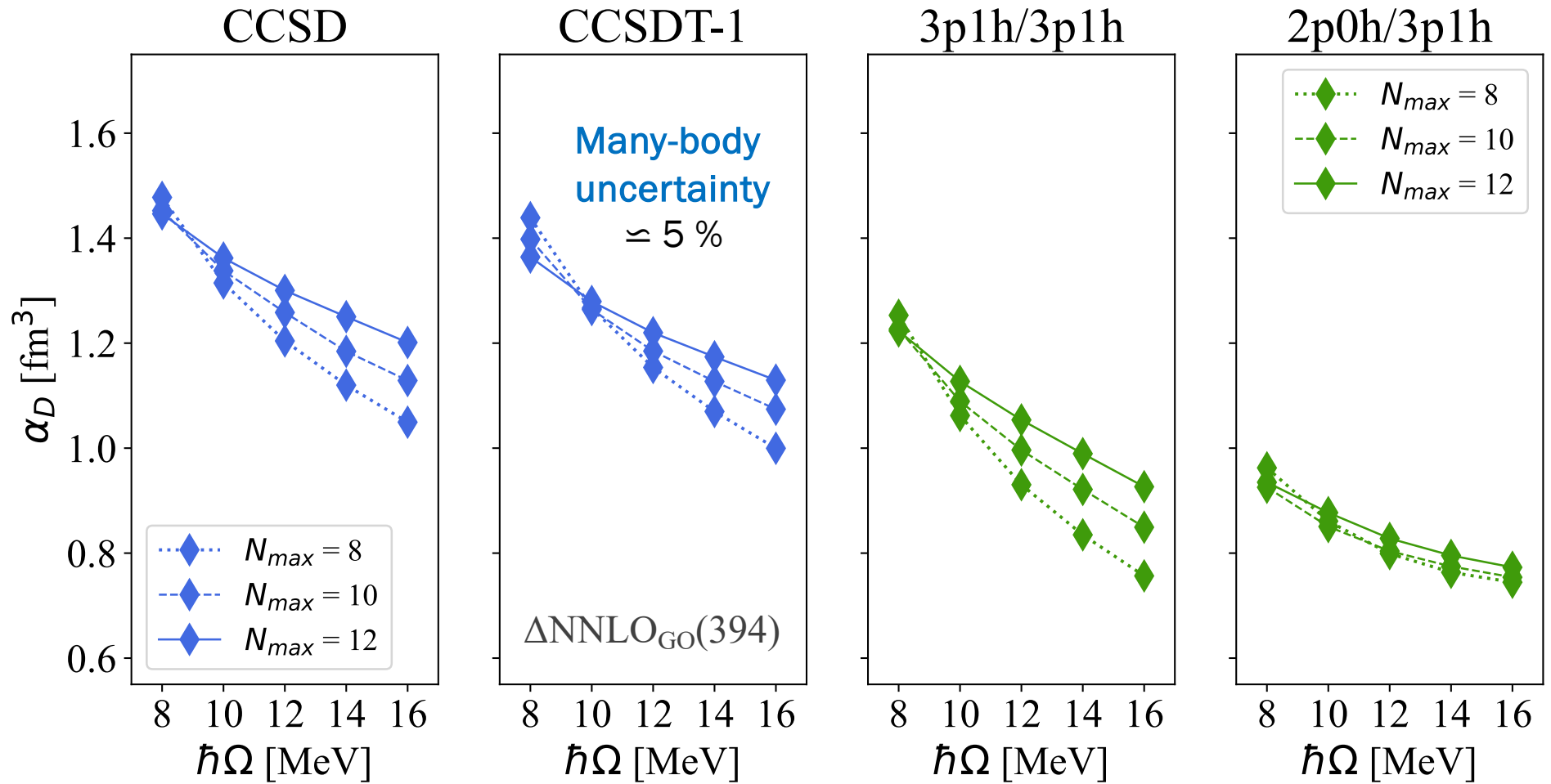


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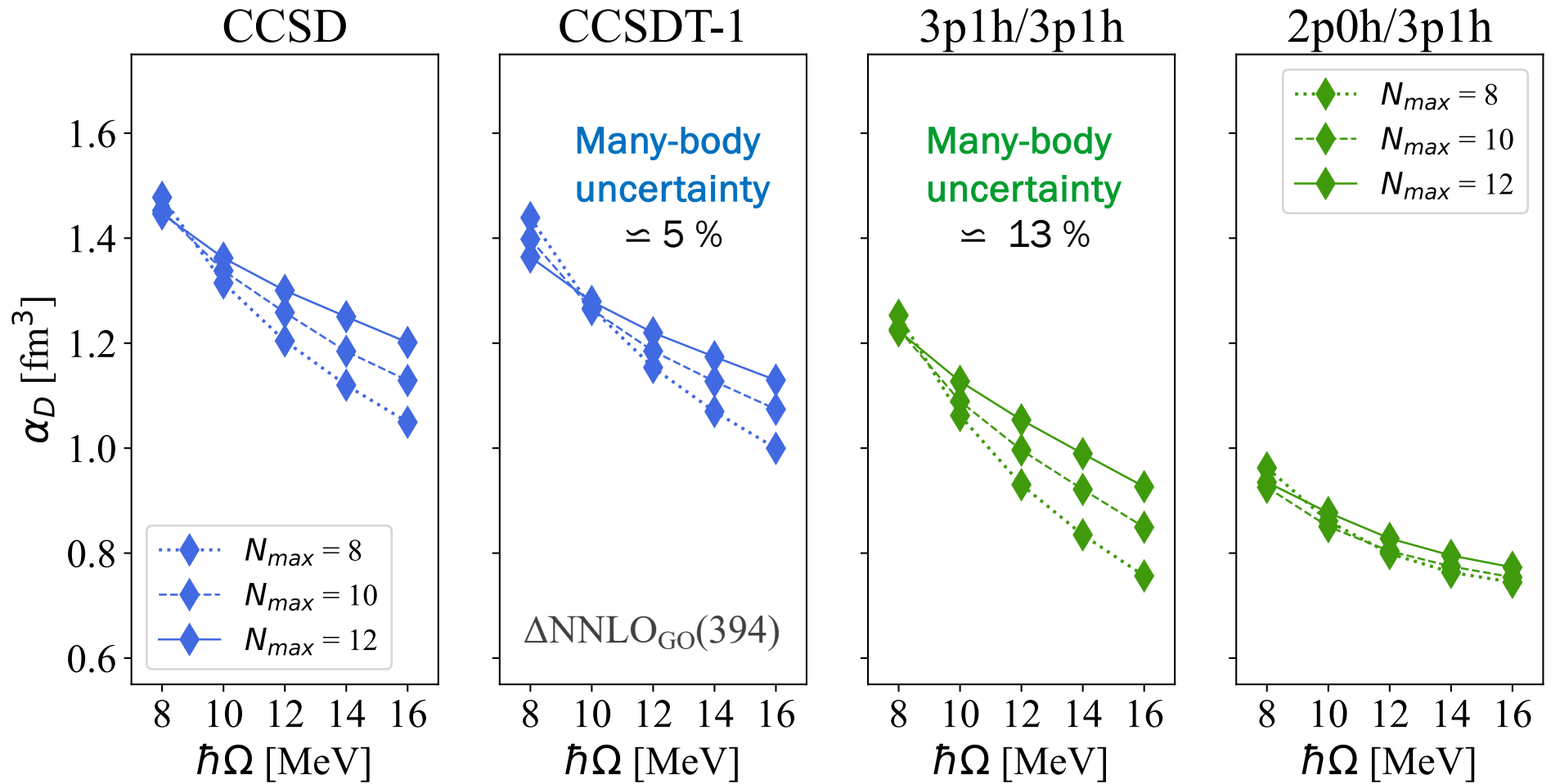




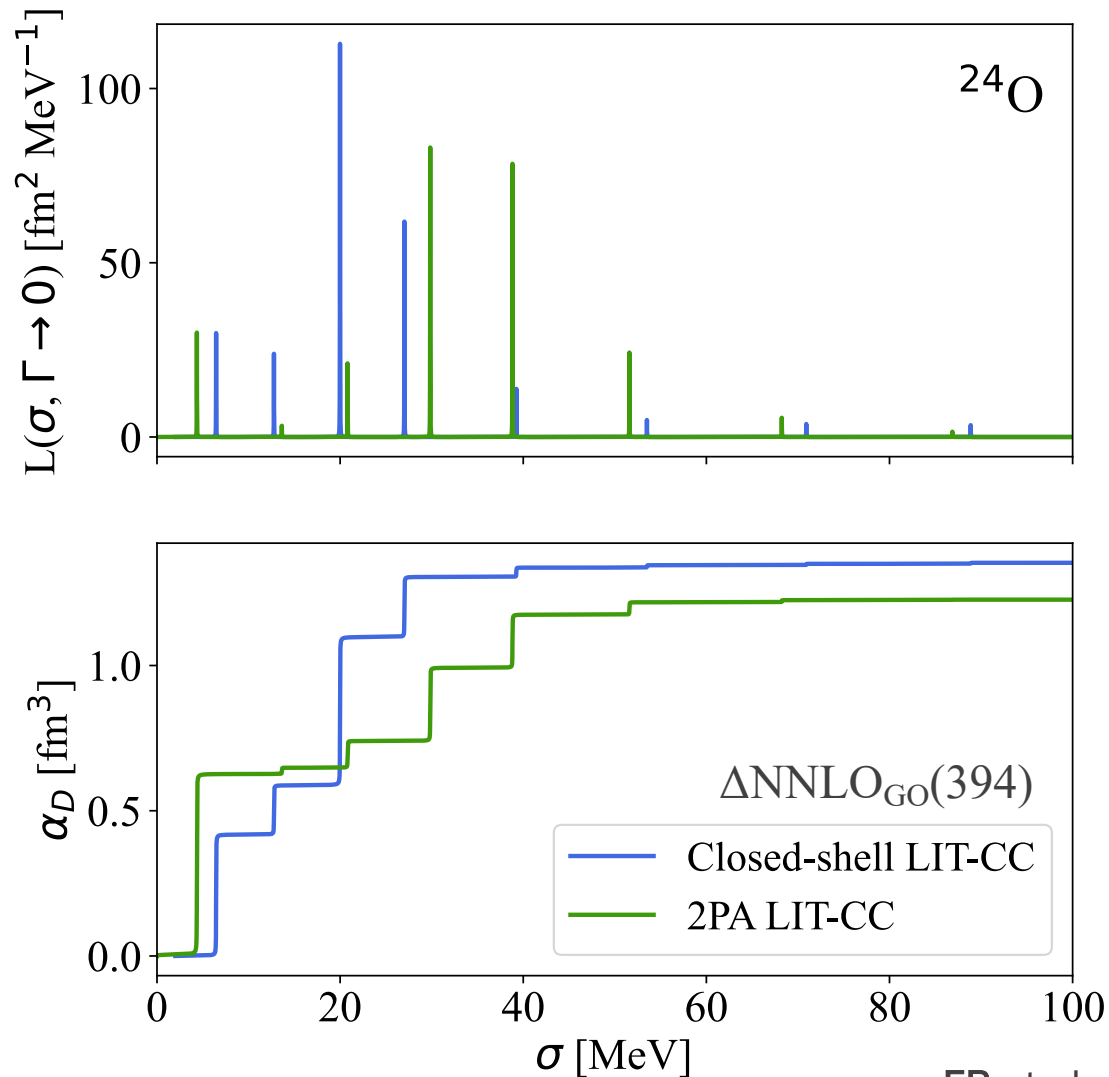
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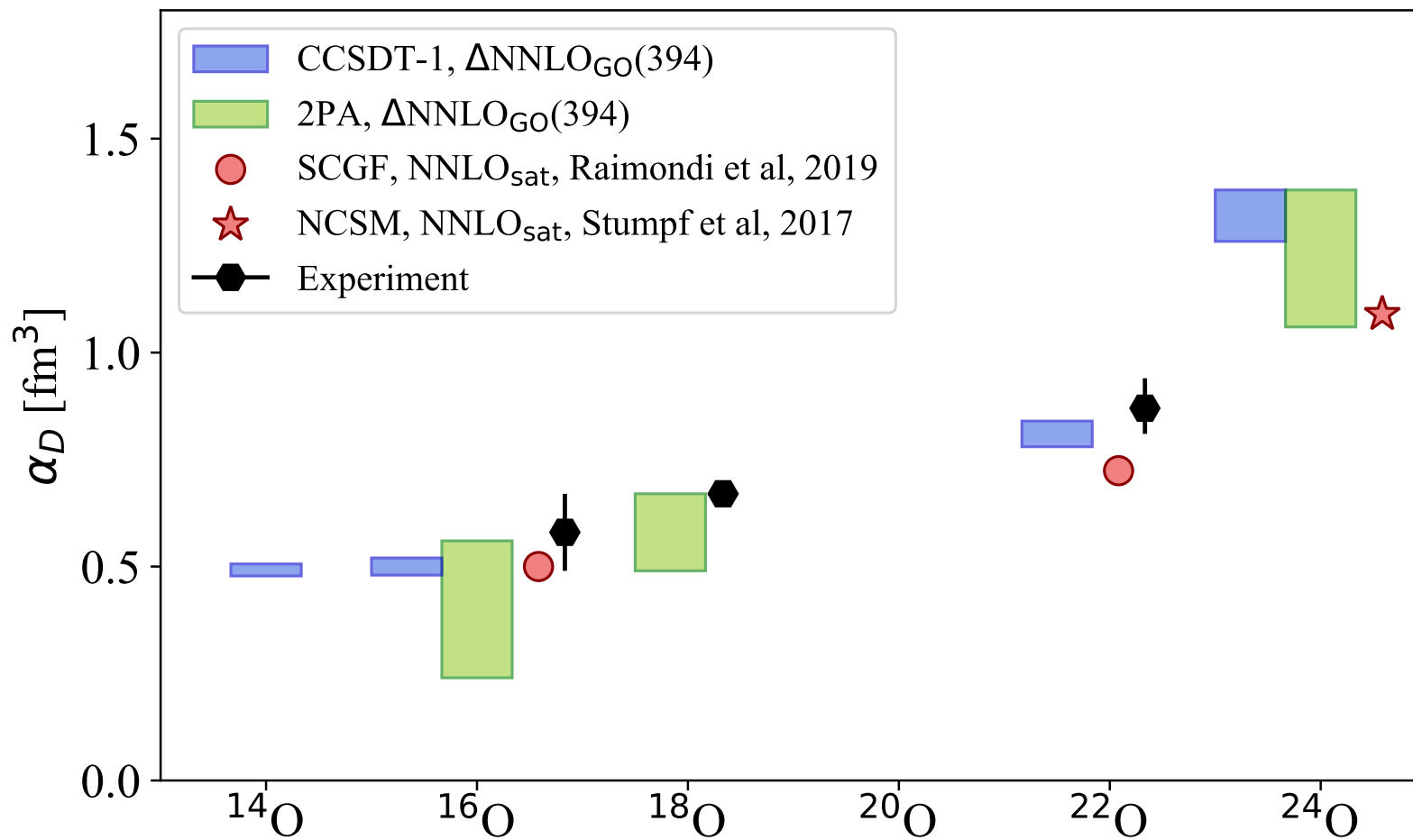


$$L(\sigma, \Gamma \rightarrow 0) = \int d\omega R(\omega) \delta(\omega - \sigma) = R(\sigma)$$

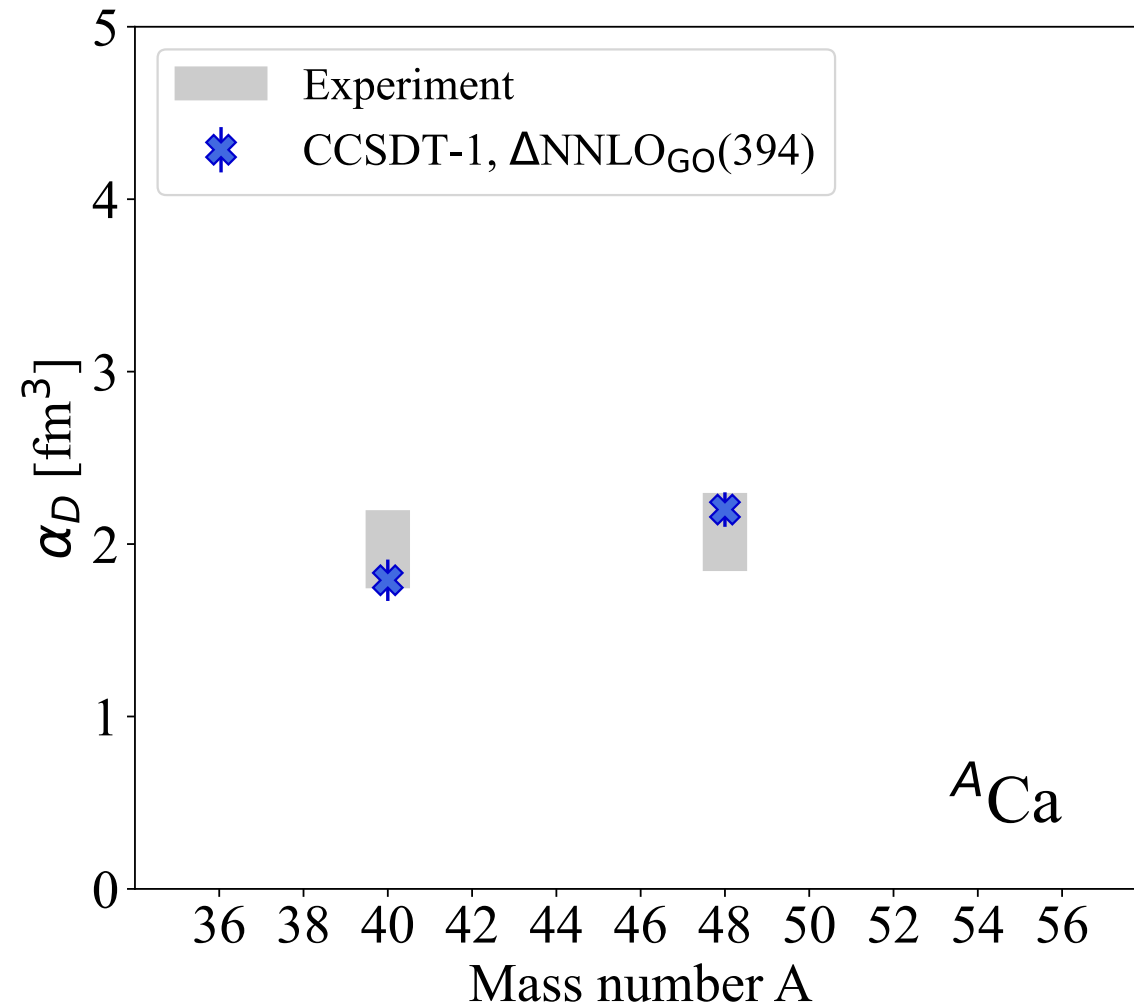


$$\alpha_D = 2\alpha \int d\sigma \frac{L(\sigma, \Gamma \rightarrow 0)}{\sigma}$$

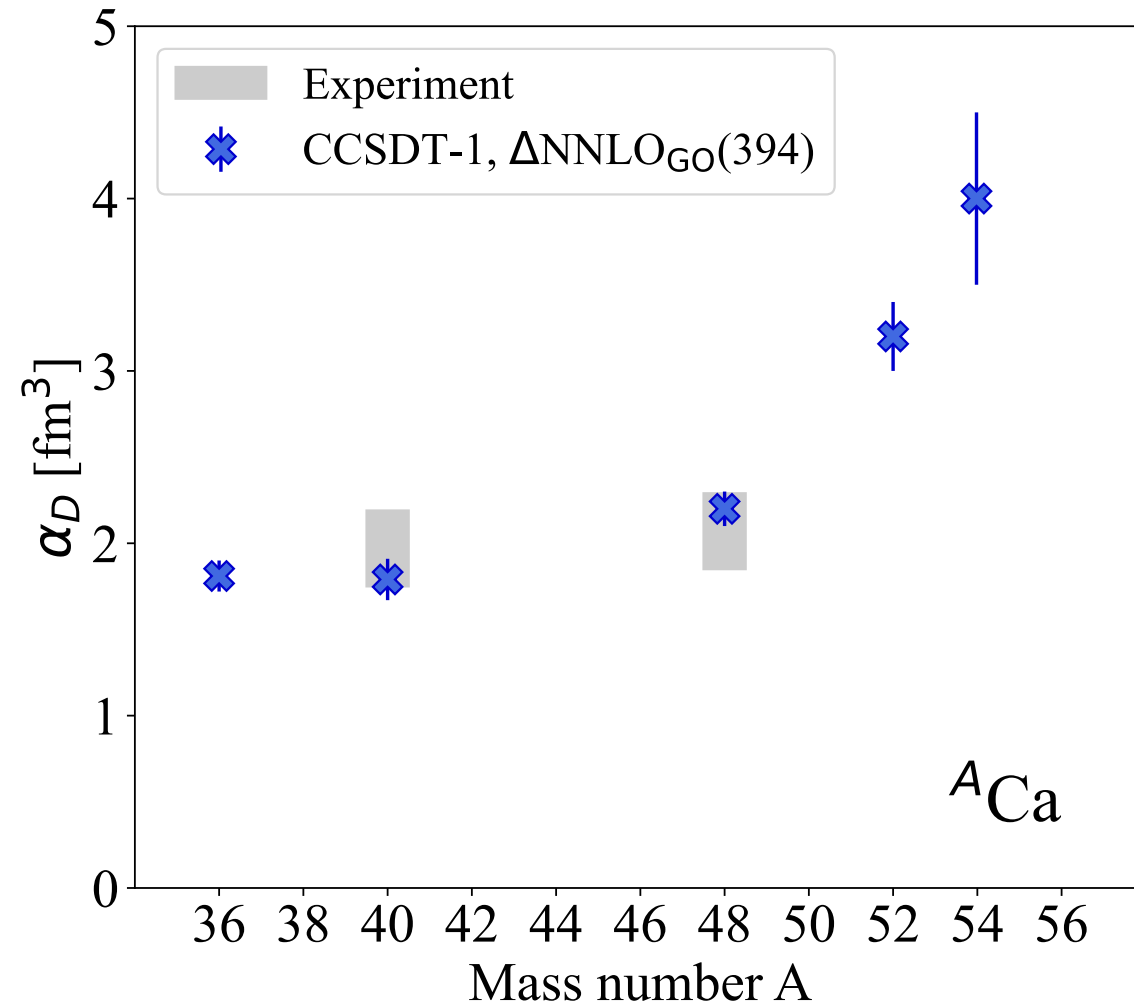
# $\alpha_D$ along oxygen isotopes



... and heavier:  $\alpha_D$  in calcium isotopes

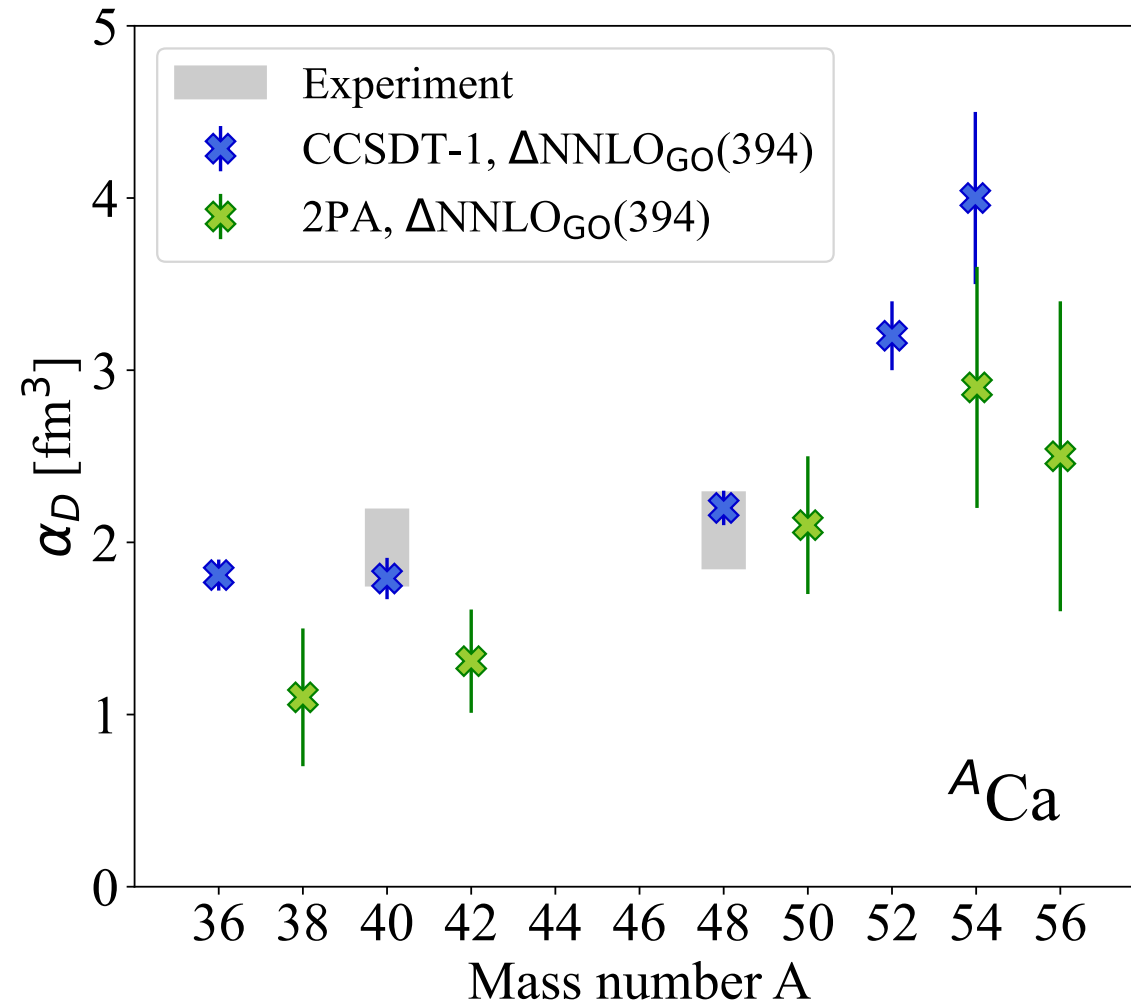


... and heavier:  $\alpha_D$  in calcium isotopes



FB et al., in preparation.

... and heavier:  $\alpha_D$  in calcium isotopes



# Conclusions

- ❑ Dipole polarizabilities provide a way to cast light on the **collective excitations of the nucleus** as well as to put **constraints on the nuclear symmetry energy**.
- ❑ We extended the reach of ab initio calculations of these electromagnetic observables to **nuclei in the vicinity of closed shells**, and investigated the **effect of many-body truncations**.
- ❑ Soon new experimental benchmarks for 2PA nuclei!

**Thank you for your attention!**