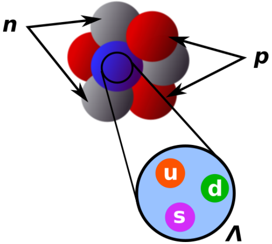
Precision calculations of p-shell hypernuclei and beyond

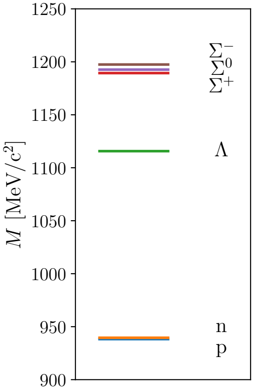
Marco Knöll

Progress in Ab Initio Nuclear Theory, TRIUMF, 2024

Introduction to hypernuclei



- systems with strangeness
- composed of nucleons and hyperons
- lifetime ≈ 260 ps
- use generalized NCSM



Introduction to hypernuclei

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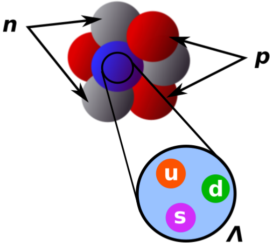

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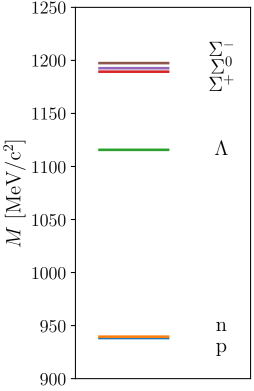

challenges for ab initio calculations

- strangeness as additional degree of freedom
- account for different rest masses of baryons
- require realistic baryonic interactions

The hypernuclear Hamiltonian from chiral EFT

$$H = T + V_{\mathrm{NN}} + V_{\mathrm{NNN}}$$

- V_{NN}: non-local nucleon-nucleon potential at $\mathrm{N}^3\mathrm{LO}$ — *EMN, Phys. Rev. C 96 024004 (2017)*
- V_{NNN}: non-local three-nucleon force at $\mathrm{N}^3\mathrm{LO}$ — *Hüther et al., Phys. Lett. B 808 135651 (2020)*

The hypernuclear Hamiltonian from chiral EFT

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$$H = T + V_{\mathrm{NN}} + V_{\mathrm{NNN}} + V_{\mathrm{YN}} + V_{\mathrm{YNN}}$$

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- V_{YN}: non-local hyperon-nucleon interaction at LO *Polinder et al., Nucl. Phys. A 779 244-266 (2006)*
- V_{YNN}: no initial interaction included but SRG induced forces are considered

The hypernuclear Hamiltonian from chiral EFT

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- V_{YNN}: no initial interaction included but SRG induced forces are considered
- ΔM: account for different rest masses

Hypernuclear no-core shell model

Gazda et al., Few-Body Syst. 55 857 (2014)
Wirth et al., Phys. Rev. C 97 064315 (2018)
Wirth, Roth, Phys. Rev. C 100 044313 (2019)

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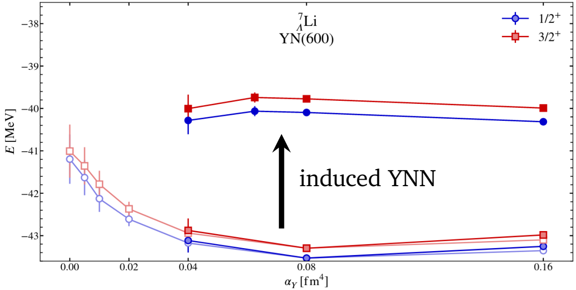
- expand Hamiltonian on finite Slater determinant basis and diagonalize
- include strangeness $\mathcal{S}$ in single-particle basis

  $|n(ls)jm_j, \mathcal{S}\, tm_t\rangle$

- access larger model spaces through importance measure (IT-NCSM)
- inclusion of SRG induced YNN forces is key for accurate description

modified from Wirth, Roth, Phys. Rev. Lett. 117 182501 (2016)



Outline

TECHNISCHE UNIVERSITÄT DARMSTADT

1st

YN interaction constrained on light hypernuclei

2nd

uncertainty quantification with artificial neural networks

3rd

towards ab initio calculations of medium-mass hypernuclei

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The hyperon-nucleon interaction

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YN @ LO — 5 LECs

Polinder et al., Nucl. Phys. A 779 244-266 (2006)

YN @ NLO — 23 (10) LECs

Haidenbauer et al., Eur. Phys. J. A 56 (3) 91 (2020)

YN @ N^2LO — 23 (10) LECs

Haidenbauer et al., Eur. Phys. J. A 59 (3) 63 (2023)

- 35 YN scattering data for S-waves + $B_\Lambda\left({}^3_\Lambda\mathrm{H}\right)$
- practically no data for P-waves

$\Rightarrow$ higher orders cannot be constrained on YN data only

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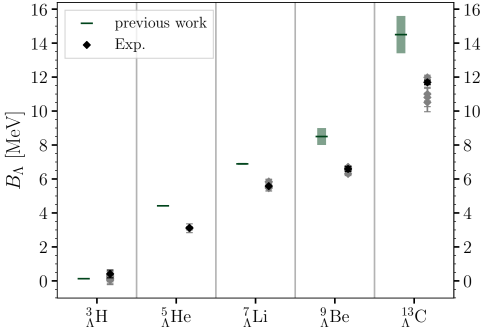

- 35 YN scattering data for S-waves + $B_\Lambda\left(^3_\Lambda\mathrm{H}\right)$
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$\Rightarrow$ higher orders cannot be constrained on YN data only

- hyperon in p-shell hypernuclei is overbound

$\Rightarrow$ use precise experimental ground-state and spectroscopic data as additional constraints

LEC sensitivity analysis

MK, Roth, Phys. Lett. B 846 138258 (2023)

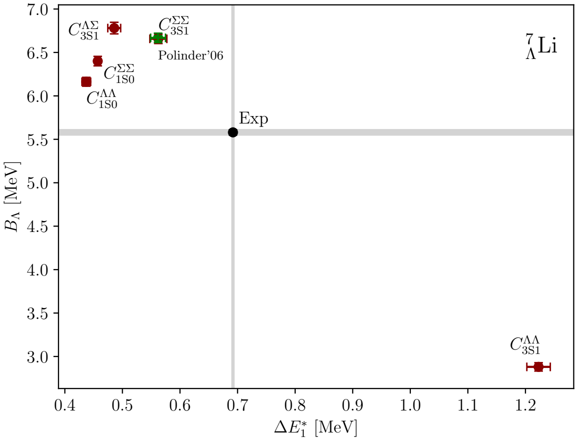
- 5 LECs associated with particle species and partial waves:

  $C^{\Lambda\Lambda}_{^1S_0}$, $C^{\Lambda\Lambda}_{^3S_1}$, $C^{\Sigma\Sigma}_{^1S_0}$, $C^{\Sigma\Sigma}_{^3S_1}$, $C^{\Lambda\Sigma}_{^3S_1}$

- vary single LECs by “natural” amounts

- most sensitive to $C^{\Lambda\Lambda}_{^3S_1}$ followed by $C^{\Lambda\Lambda}_{^1S_0}$

- limit optimization to these two LECs



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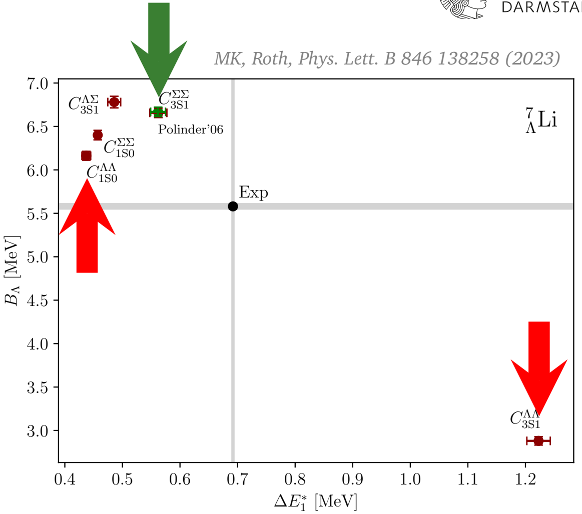
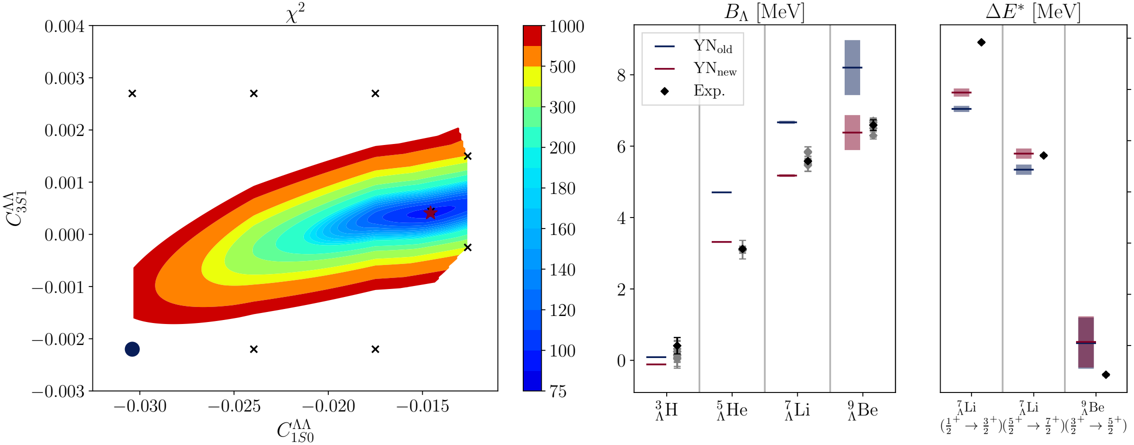
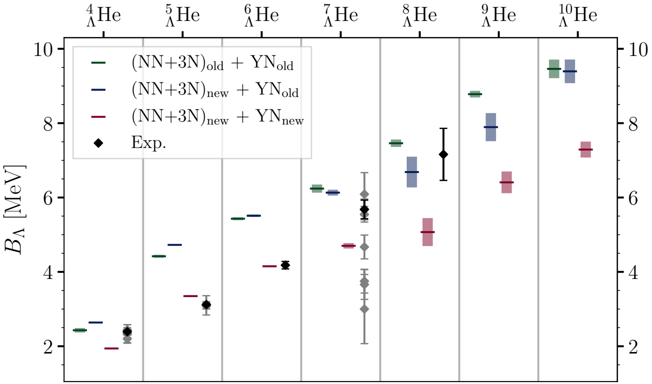


Optimization of LECs

MK, Roth, Phys. Lett. B 846 138258 (2023)



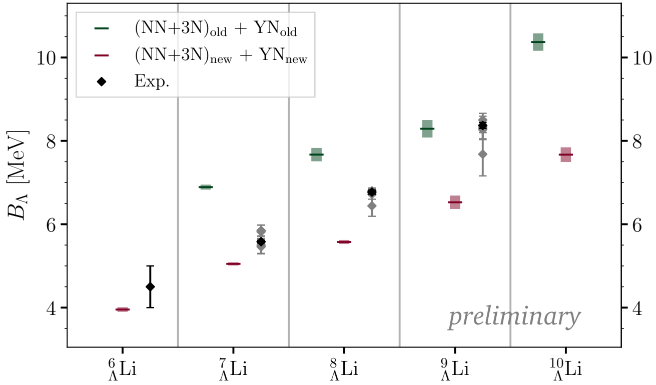
${}_{\Lambda}$He and ${}_{\Lambda}$Li isotopic chains



MK, Roth, Phys. Lett. B 846 138258 (2023)

- error bands indicate many-body uncertainties
- improved description of light isotopes
- dependence on nucleonic interaction
- difficult experimental situation

$_\Lambda$He and $_\Lambda$Li isotopic chains



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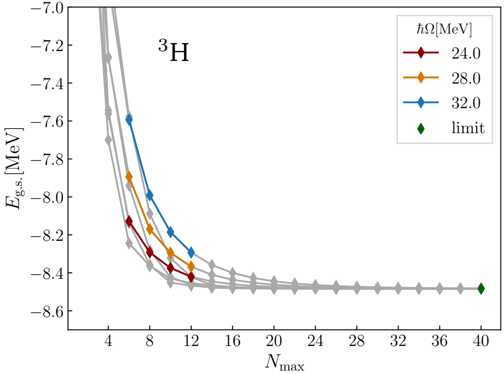
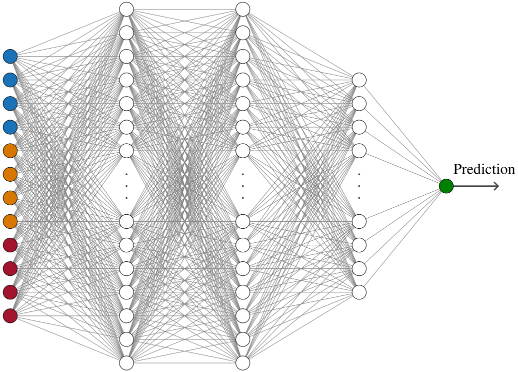
Model-space extrapolation with artificial neural networks

- prediction of converged value from sequences of calculations in accessible model spaces



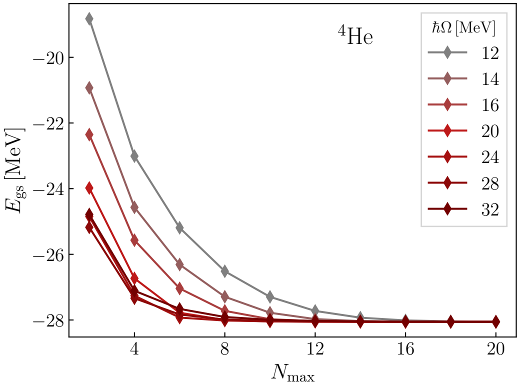
MK et. al., Phys. Lett. B 839 137781 (2023)



Wolfgruber, MK, Roth, Phys. Rev. C (submitted), arXiv:2310.05256 (2023)

Training an ANN

- train on converged Jacobi NCSM data for ^{2}H, ^{3}H, ^{4}He
- training includes 36 different realistic NN+3N interactions from chiral EFT



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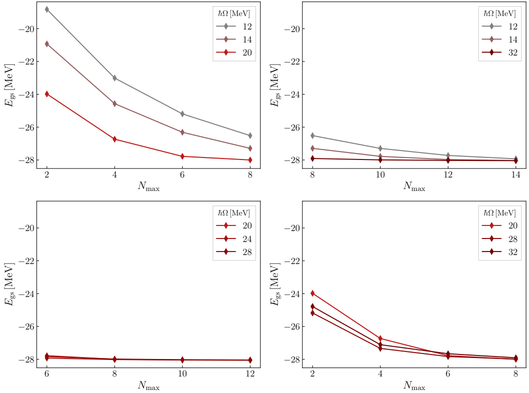
- train on converged Jacobi NCSM data for ^{2}H, ^{3}H, ^{4}He
- training includes 36 different realistic NN+3N interactions from chiral EFT
- construct all possible samples

735 samples



Training an ANN

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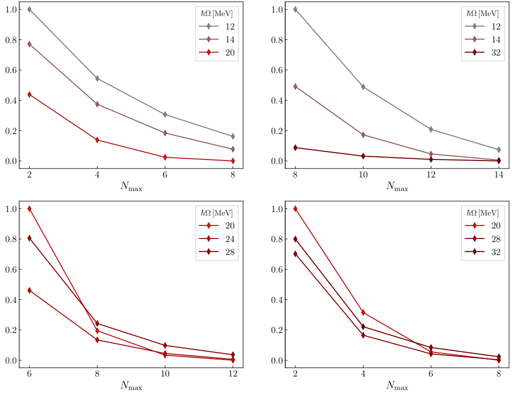
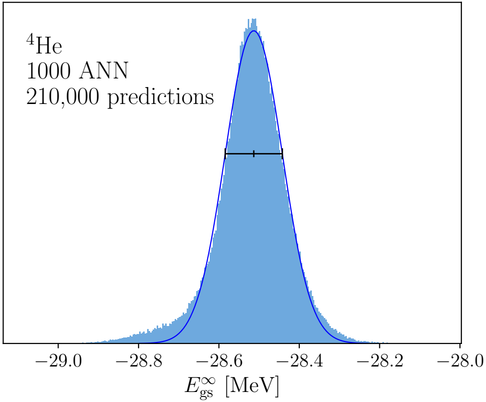
- train on converged Jacobi NCSM data for ^{2}H, ^{3}H, ^{4}He
- training includes 36 different realistic NN+3N interactions from chiral EFT
- construct all possible samples
- normalize data to $[0,1]$
- train until deviation from training data is minimal



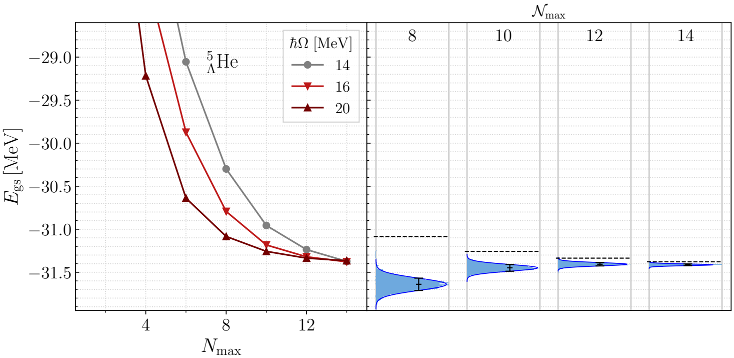
Statistical evaluation

- obtain multitude of predictions from
  - the construction of all possible evaluation samples
  - the evaluation with multiple ANNs
- fit Gaussian to extract mean prediction and uncertainty

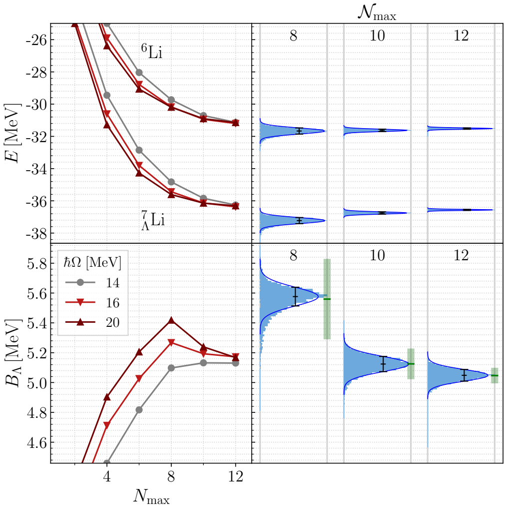


Application to hypernuclei

- universality through ANN interpolation capabilities
- convergence pattern dominated by nucleonic interaction
- direct application to hypernuclei
- sufficiently large model-spaces required



Predicting hyperon-separation energies



- $B_\Lambda = B\left({}^A_\Lambda\mathrm{X}\right) - B\left({}^{A-1}\mathrm{X}\right)$
- convergence behavior of B_Λ is not constrained
- neural network approach can be applied to difference-based observables
- sufficiently large model spaces required for accurate prediction

modified from MK, Roth, Phys. Lett. B 846 138258 (2023)

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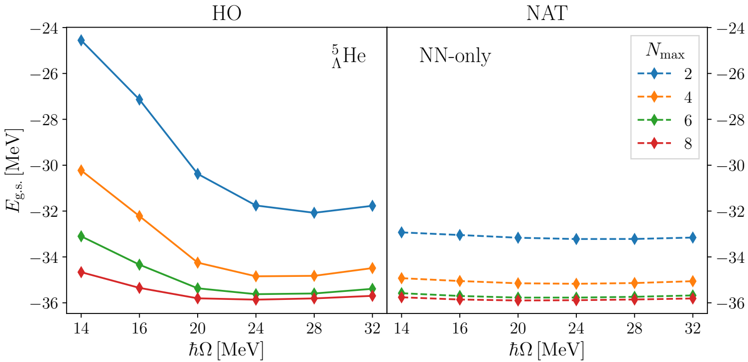
Hyperonic natural orbitals

- increased reach through improved convergence
- optimized basis from second-order corrected density matrix
- accelerated convergence rate in NCSM calculations
- frequency independence



In-medium no-core shell model

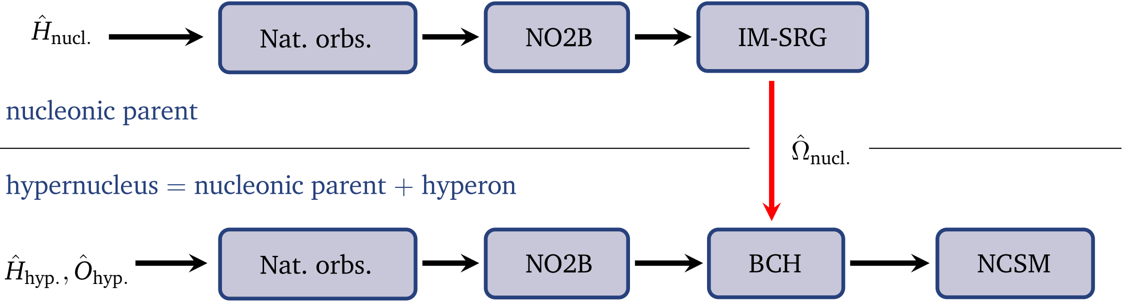
- employ IM-SRG evolved Hamiltonian in subsequent NCSM calculation
- preprocessing to speed up convergence
- use Magnus formalism

$$\frac{\mathrm{d}}{\mathrm{d}s}\hat{\Omega}(s) = \sum_{k=0}^{\infty} \frac{B_k}{k!} \left[\hat{\Omega}(s), \hat{\eta}(s)\right]_k$$

- evolve operators consistently via BCH series

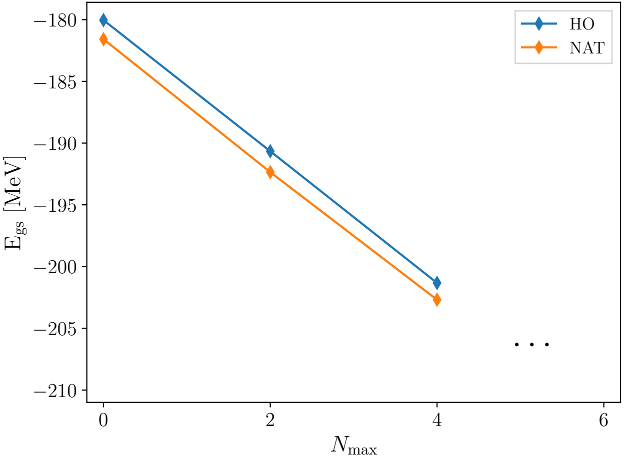
$$\hat{O}(s) = \sum_{k=0}^{\infty} \frac{1}{k!} \left[\hat{\Omega}(s), \hat{O}(0)\right]_k$$

Strange attachments to the IM-NCSM



Preliminary results for ${}^{17}_{\Lambda}$O

- NN-only
- limitation to small model spaces
- convergence for HO and NAT out of reach



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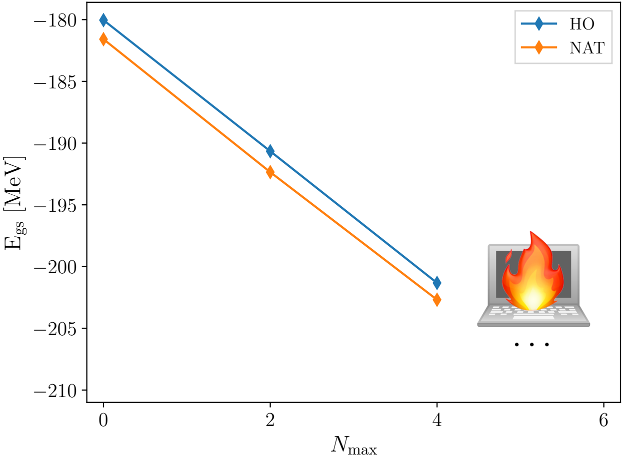

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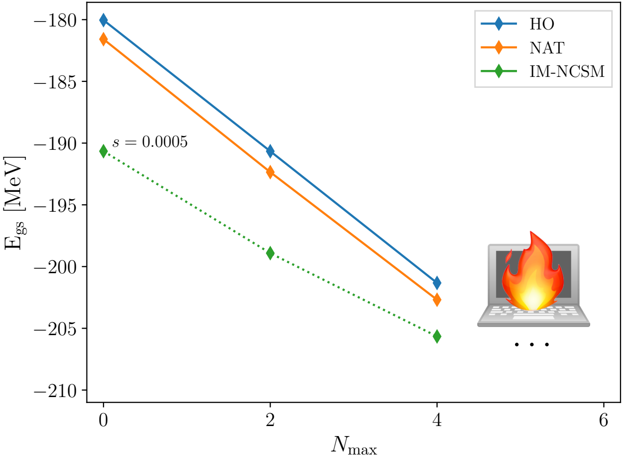

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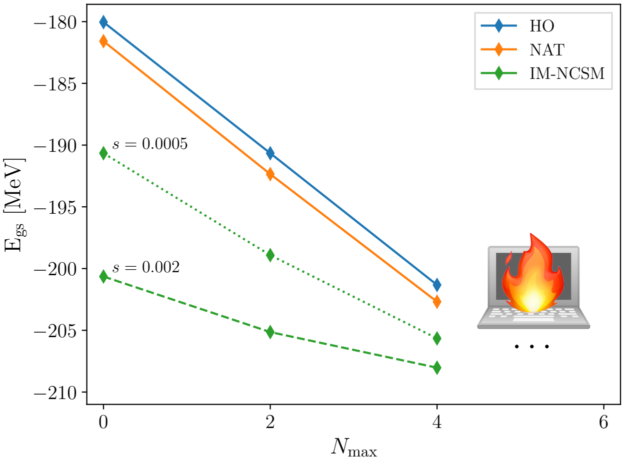

Preliminary results for $^{17}_{\Lambda}\mathrm{O}$

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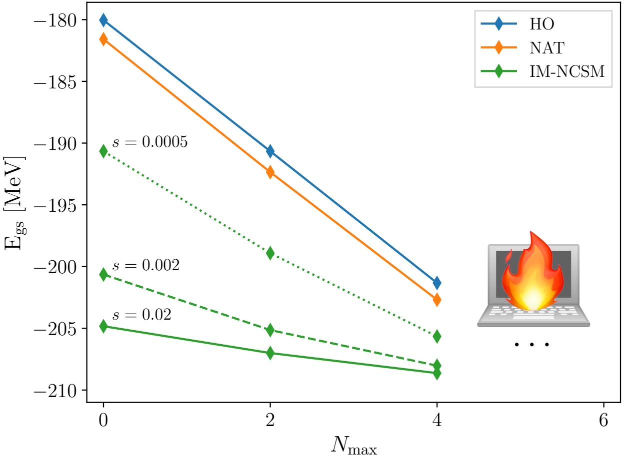
- limitation to small model spaces
- convergence for HO and NAT out of reach
- drastically faster convergence rate with increasing *s*
- enables ab initio calculations of medium-mass hypernuclei



Summary & outlook

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- main bottleneck: lack of high-quality scattering data
  $\Rightarrow$ poorly constrained YN interaction
- ground-state and spectroscopic data for p-shell hypernuclei yields additional constraints
  $\Rightarrow$ significantly improved description @ LO
- ANNs allow for reliable extrapolation and many-body uncertainty estimates
  $\Rightarrow$ crucial for precision calculations
- established convergence accelerators transfer to baryonic sector
- first steps towards ab intio description of medium-mass hypernuclei
  in the IM-NCSM framework
  $\Rightarrow$ much more work to be done …

Thank you for listening!

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- **thanks to my group and collaborators**

M. Agel
P. Falk
T. Gesser
K. Katzenmeier
P. Lehnung
L. Mertes
M. Müller
R. Roth
L. Wagner
C. Wenz
T. Wolfgruber

Hessisches Kompetenzzentrum für Hochleistungsrechnen

computing time

Bundesministerium für Bildung und Forschung

DFG

SFB 1245