

# *Ab initio* calculations on muon capture to probe neutrinoless double-beta decay

Lotta Jokiniemi (she/her)  
TRIUMF, Theory Department  
PAINT2024 Workshop  
29/02/2024





P. Navrátil



J. Kotila



K. Kravvaris

## Introduction

## Muon Capture from No-Core Shell Model

## Results

Muon capture on  ${}^6\text{Li}$

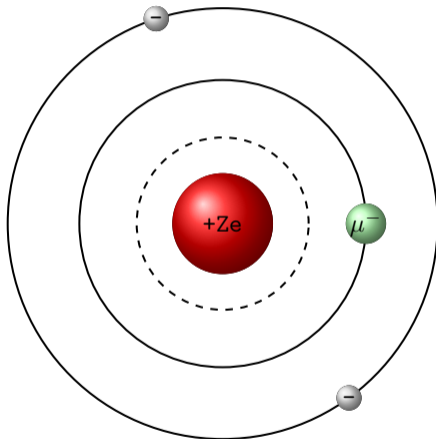
Muon capture on  ${}^{12}\text{C}$

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## Summary

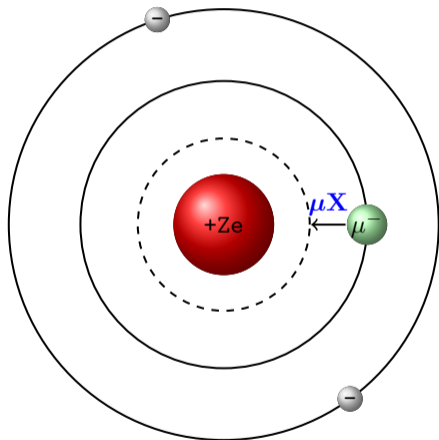
## Ordinary Muon Capture (OMC)

- A muon can replace an electron in an atom, forming a *muonic atom*



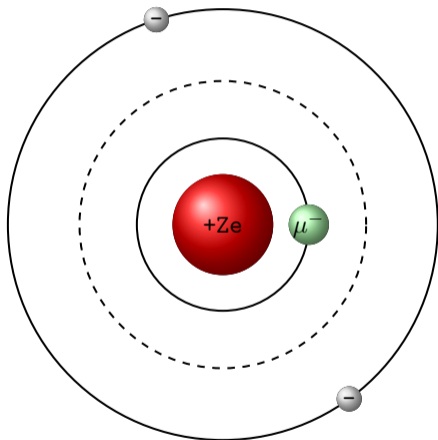
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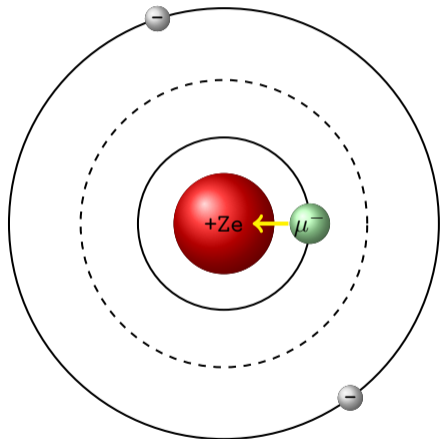
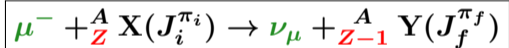
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  - ▶ Eventually bound on **the  $1s_{1/2}$  orbit**



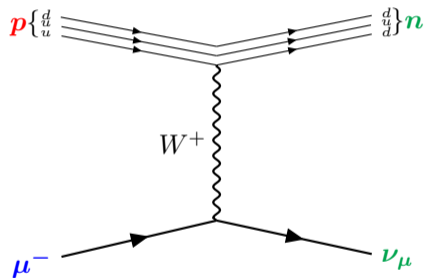
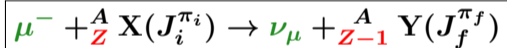
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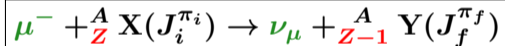
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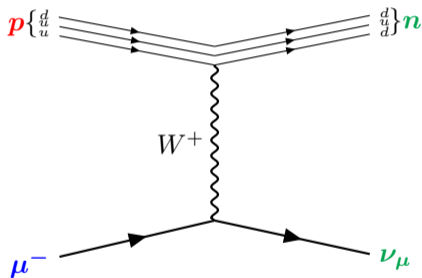
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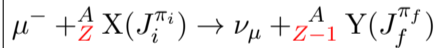
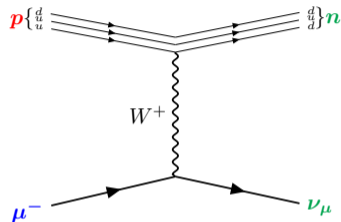
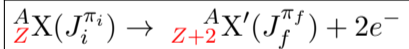
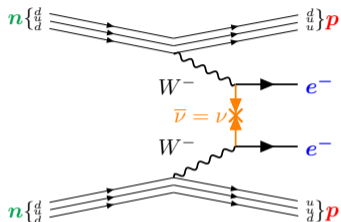
**Ordinary = non-radiative**

( Radiative muon capture (RMC): )

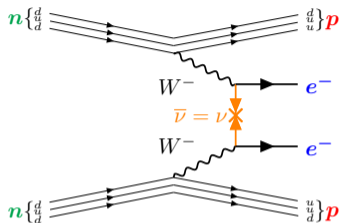
$$\mu^- + {}^A_Z X(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^A_{Z-1} Y(J_f^{\pi_f}) + \gamma$$



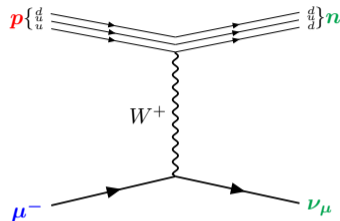
# $0\nu\beta\beta$ Decay vs. Muon Capture



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$${}^A_Z X(J_i^{\pi_i}) \rightarrow {}^{A}_{Z+2} X'(J_f^{\pi_f}) + 2e^-$$

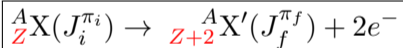
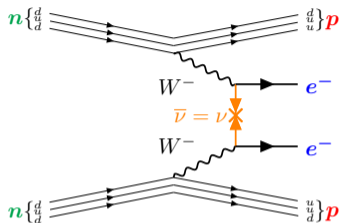


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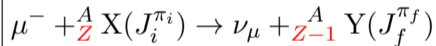
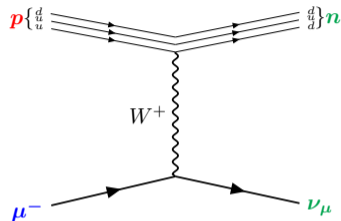
Both involve hadronic current:

$$j^{\alpha\dagger} = \bar{\Psi} \left[ g_V(q^2) \gamma^\alpha + i g_M(q^2) \frac{\sigma^{\alpha\beta}}{2m_p} q_\beta - g_A(q^2) \gamma^\alpha \gamma_5 - g_P(q^2) q^\alpha \gamma_5 \right] \Psi$$

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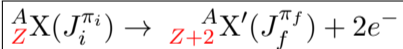
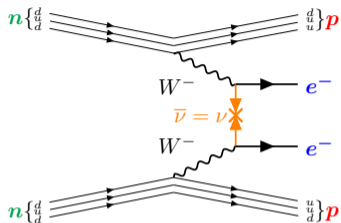
- $q \approx 1/|r_1 - r_2| \approx 100 - 200 \text{ MeV}$



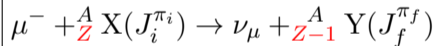
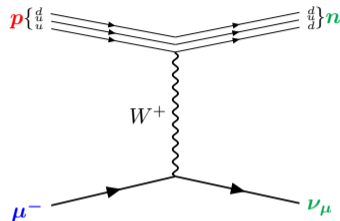
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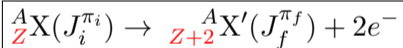
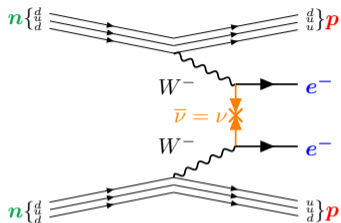


- $q \approx m_\mu + E_i - E_f \approx 100 \text{ MeV}$

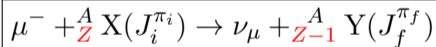
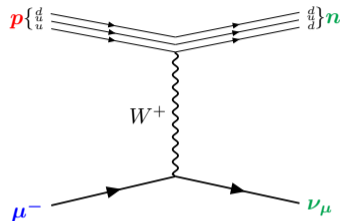
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- $q \approx 1/|r_1 - r_2| \approx 100 - 200 \text{ MeV}$
- **Yet hypothetical**

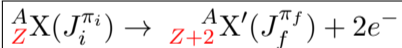
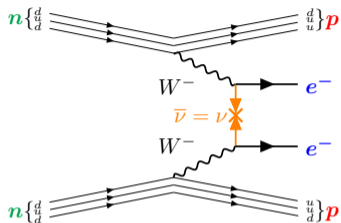


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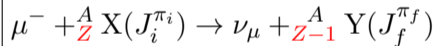
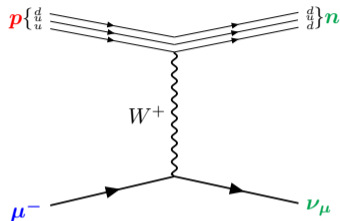
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- $q \approx m_\mu + E_i - E_f \approx 100 \text{ MeV}$
- **Has been measured!**

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- Interaction Hamiltonian  $\rightarrow$  capture rate:

$$W(J_i \rightarrow J_f) = \frac{1}{2J_i + 1} \left( 1 - \frac{q}{m_\mu + AM} \right) q^2 \sum_{\kappa u} |g_V M_V(\kappa, u) + g_M M_M(\dots) + g_A M_A(\dots) + g_P M_P(\dots)|^2$$

PHYSICAL REVIEW

VOLUME 118, NUMBER 2

APRIL 15, 1960

## Theory of Allowed and Forbidden Transitions in Muon Capture Reactions\*

MASATO MORITA

*Columbia University, New York, New York*

AND

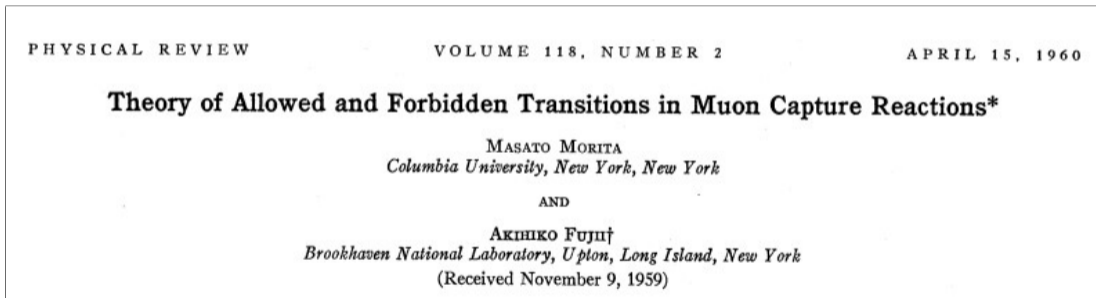
AKIHIKO FUJII†

*Brookhaven National Laboratory, Upton, Long Island, New York*

(Received November 9, 1959)

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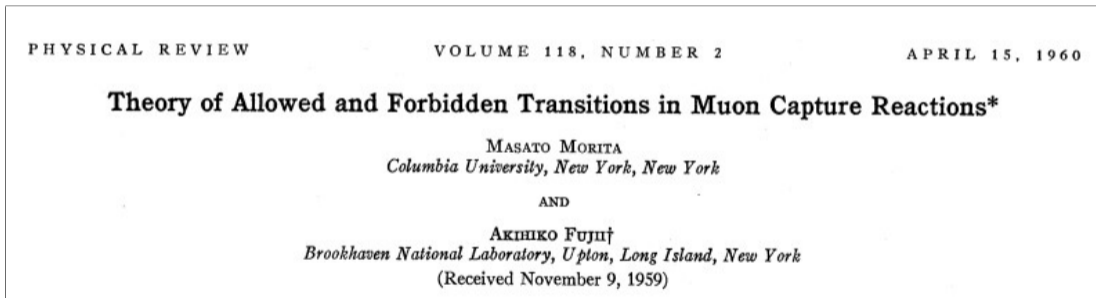
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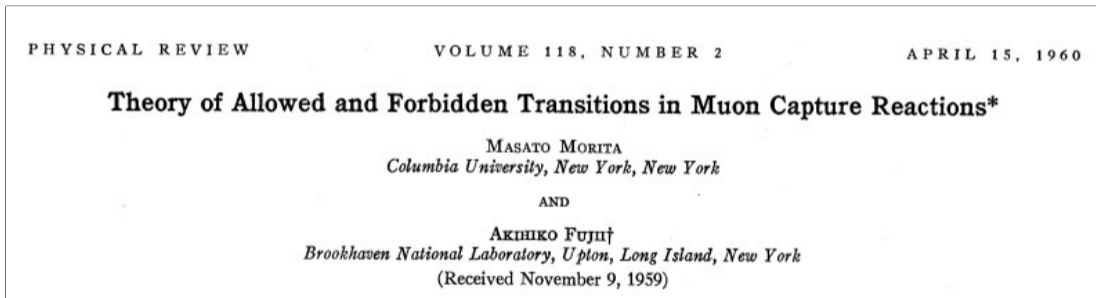
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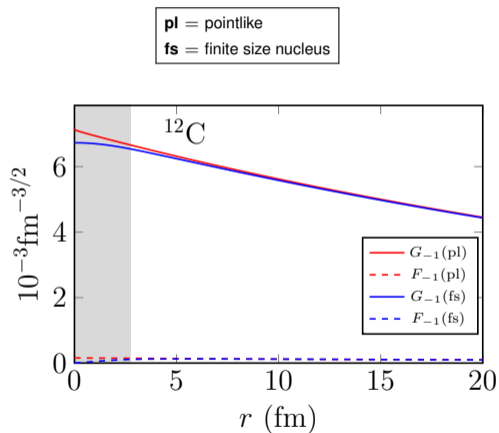
- + Realistic **bound-muon wave functions** solved from Dirac equations
- + Translationally invariant **nuclear wave functions** from no-core shell model
- + Approximate **two-body currents** via normal-ordering

## Bound-Muon Wave Functions

- Expand the muon wave function in terms of spherical spinors

$$\psi_{\mu}(\kappa, \mu; \mathbf{r}) = \psi_{\kappa\mu}^{(\mu)} = \begin{bmatrix} -i\mathbf{F}_{\kappa}(r)\chi_{-\kappa\mu} \\ \mathbf{G}_{\kappa}(r)\chi_{\kappa\mu} \end{bmatrix},$$

where  $\kappa = -j(j+1) + l(l+1) - \frac{1}{4}$   
 ( $\kappa = -1$  for the  $1s_{1/2}$  orbit)



LJ, Navrátil, Kotila, Kravvaris, work in progress

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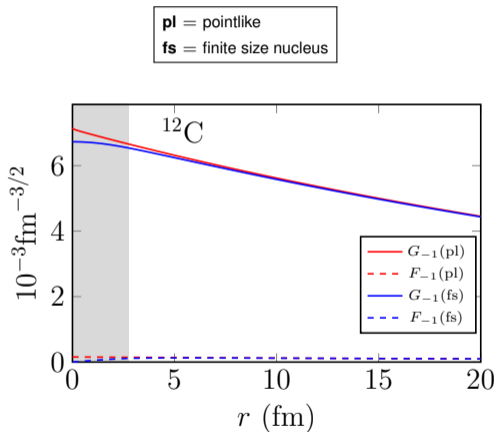
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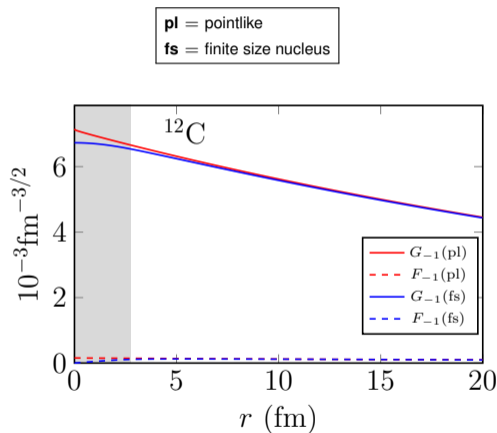
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$$\rightarrow \langle J_f || \sum_{s=1}^A \mathbf{G}_{-1}(\mathbf{r}_s) \mathcal{O}_s(q_s, r_s, \boldsymbol{\sigma}_s) || J_i \rangle$$



# Ab initio No-Core Shell Model (NCSM)

B. R. Barrett, P. Navrátil, J. P. Vary, *Progr. Part. Nucl. Phys.* **69**, 131 (2013)

- Solve nuclear many-body problem

$$H^{(A)}\Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = E^{(A)}\Psi^{(A)}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

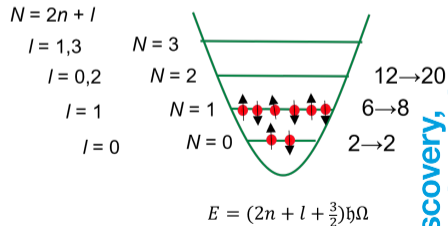
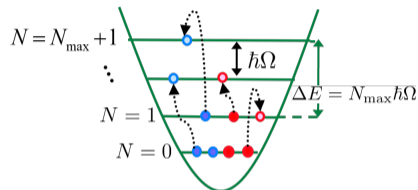


Figure courtesy of P. Navrátil



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$$H^{(A)} = \sum_{i=1}^A \frac{p_i^2}{2m} + \sum_{i<j=1}^A V^{2b}(\mathbf{r}_i - \mathbf{r}_j) + \sum_{i<j<k=1}^A V_{ijk}^{3b}$$

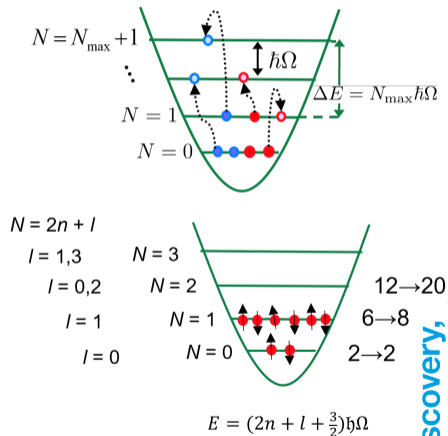


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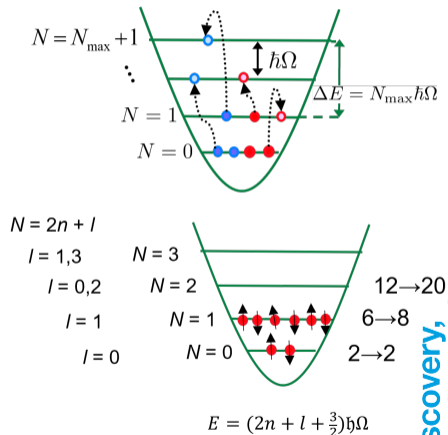


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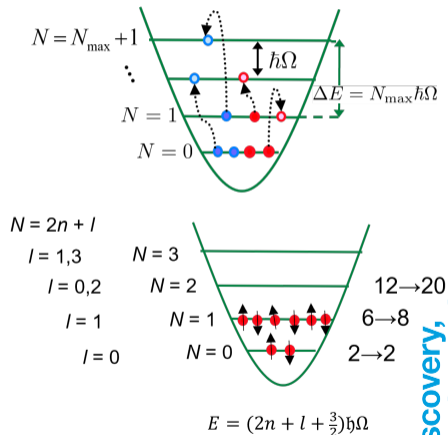
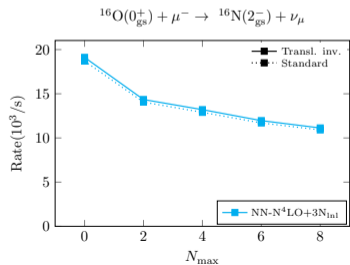
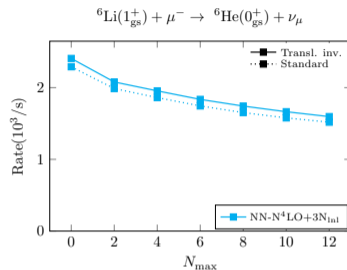


Figure courtesy of P. Navrátil

# Translationally Invariant Operators

- Operators depend on coordinates  $\mathbf{r}_s$  and  $\mathbf{p}_s$  w.r.t. the center of mass (CM) of the HO potential



*LJ, Navrátil, Kotila and Kravvaris, in progress*

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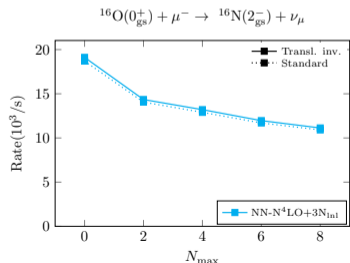
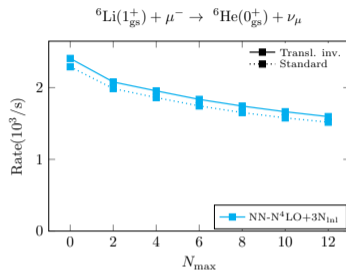
- Operators depend on coordinates  $\mathbf{r}_s$  and  $\mathbf{p}_s$  w.r.t. the center of mass (CM) of the HO potential
- We remove CM contamination as:

Navrátil, *Phys. Rev. C* **104**, 064322 (2021)

$$\begin{aligned}
 & (\Psi_f || \sum_{s=1}^A \hat{O}_s(\mathbf{r}_s - \mathbf{R}_{\text{CM}}, \mathbf{p}_s - \mathbf{P}) || \Psi_i) \\
 &= \sum_{pnp'n'} (n' || \hat{O}_s \left( -\sqrt{\frac{A-1}{A}} \boldsymbol{\xi}_s, -\sqrt{\frac{A-1}{A}} \boldsymbol{\pi}_s \right) || p') \\
 & \quad \times (M^u)_{n'p',np}^{-1} \frac{-1}{\sqrt{2u+1}} (\Psi_f || [a_n^\dagger \tilde{a}_p]_u || \Psi_i),
 \end{aligned}$$

where

$$\boldsymbol{\xi}_s = -\sqrt{A/(A-1)}(\mathbf{r}_s - \mathbf{R}_{\text{CM}}); \quad \boldsymbol{\pi}_s = -\sqrt{A/(A-1)}(\mathbf{p}_s - \mathbf{P})$$



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# Translationally Invariant Operators

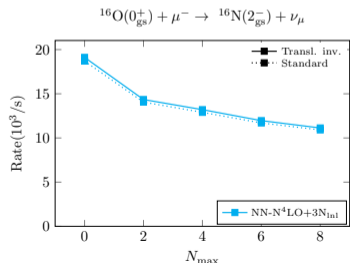
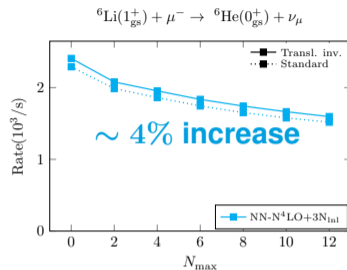
- Operators depend on coordinates  $\mathbf{r}_s$  and  $\mathbf{p}_s$  w.r.t. the center of mass (CM) of the HO potential
- We remove CM contamination as:

Navrátil, *Phys. Rev. C* **104**, 064322 (2021)

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LJ, Navrátil, Kotila and Kravvaris, in progress

# Translationally Invariant Operators

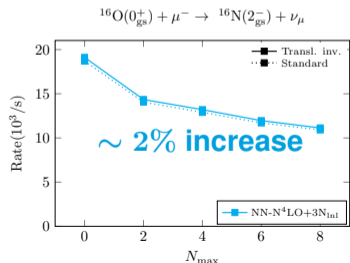
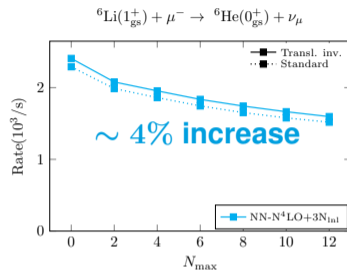
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Navrátil, *Phys. Rev. C* **104**, 064322 (2021)

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LJ, Navrátil, Kotila and Kravvaris, in progress

## Axial-Vector Two-Body Currents (2BCs)

- One-body currents

$$\mathbf{J}_{i,1b}^3 = \tau_i^- \left( g_A(q^2) \boldsymbol{\sigma}_i - \frac{g_P(q^2)}{2m_N} \mathbf{q} \cdot \boldsymbol{\sigma}_i \right)$$

+ two-body currents

$$\mathbf{J}_{i,2b}^{\text{eff}} = g_A \tau_i^- \left[ \delta a(\mathbf{q}^2) \boldsymbol{\sigma}_i + \frac{\delta a^P(\mathbf{q}^2)}{q^2} (\mathbf{q} \cdot \boldsymbol{\sigma}_i) \mathbf{q} \right]$$

*Hoferichter, Klos, Schwenk Phys. Lett. B 746, 410 (2015)*



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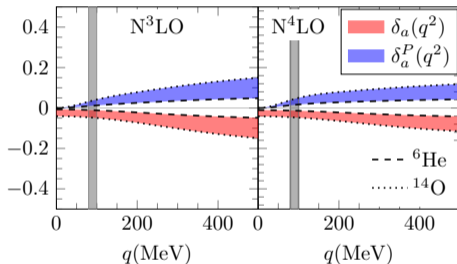
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*Hoferichter, Klos, Schwenk Phys. Lett. B 746, 410 (2015)*


- Two-body currents approximated by

$$\begin{cases} g_A(q^2, 2b) \rightarrow g_A(q^2) + g_A \delta a(q^2), \\ g_P(q^2, 2b) \rightarrow g_P(q^2) - \frac{2m_N g_A}{q} \delta a^P(q^2) \end{cases}$$



*LJ, Navrátil, Kotila, Kravvaris, work in progress*

## Dependency on the Harmonic-Oscillator Frequency



$$\Psi^{(A)} = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj} \Phi_{Nj}^{\text{HO}}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

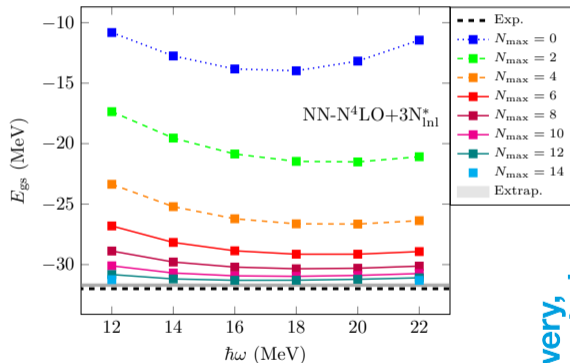
- The expansion depends on the HO frequency because of the  $N_{\max}$  truncation

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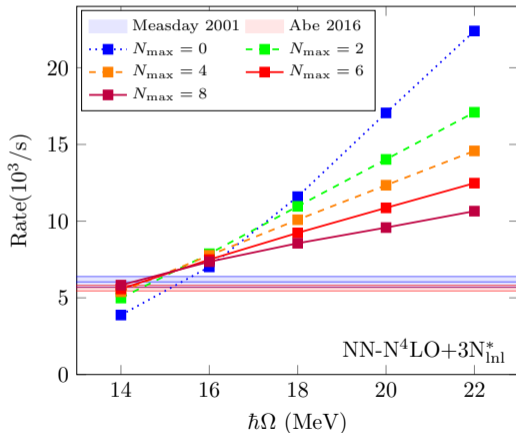
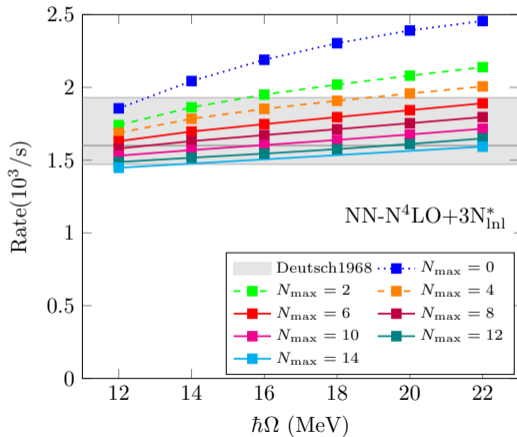
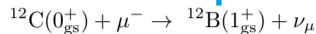
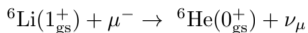
- The expansion depends on the HO frequency because of the  $N_{\max}$  truncation
  - ▶ **Increasing  $N_{\max}$  leads towards converged results**

Ground-state energy of  ${}^6\text{Li}$



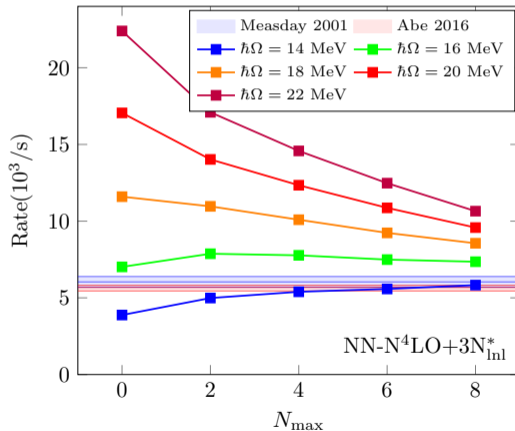
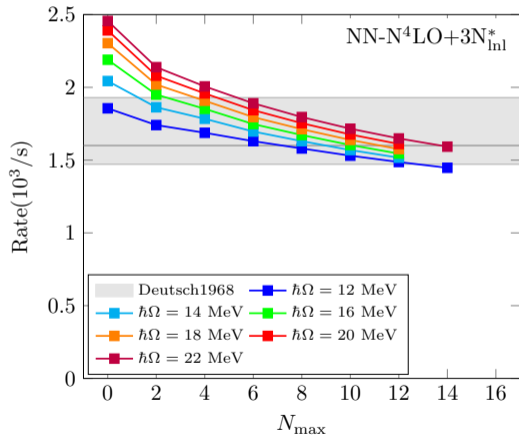
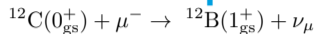
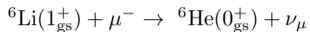
*LJ, Navrátil, Kotila, Kravvaris, work in progress*

# Harmonic-Oscillator Frequency Dependence of Muon Capture



LJ, Navrátil, Kotila and Kravvaris, in progress

# Harmonic-Oscillator Frequency Dependence of Muon Capture



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Muon capture on  ${}^6\text{Li}$

Muon capture on  ${}^{12}\text{C}$

Muon capture on  ${}^{16}\text{O}$

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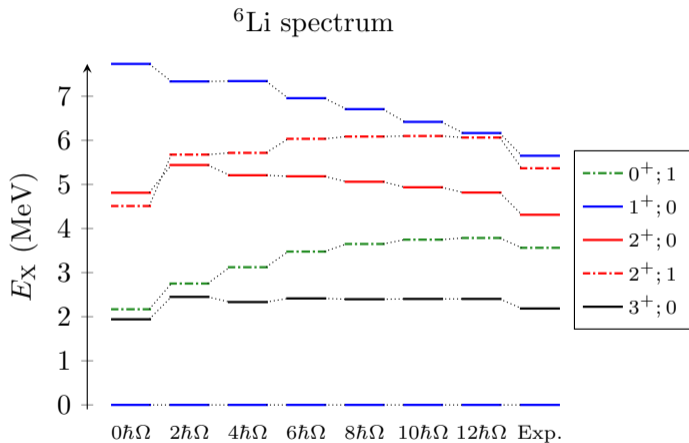
Muon capture on  ${}^6\text{Li}$

Muon capture on  ${}^{12}\text{C}$

Muon capture on  ${}^{16}\text{O}$

Summary

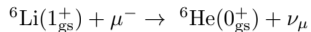
# Energy spectrum of ${}^6\text{Li}$



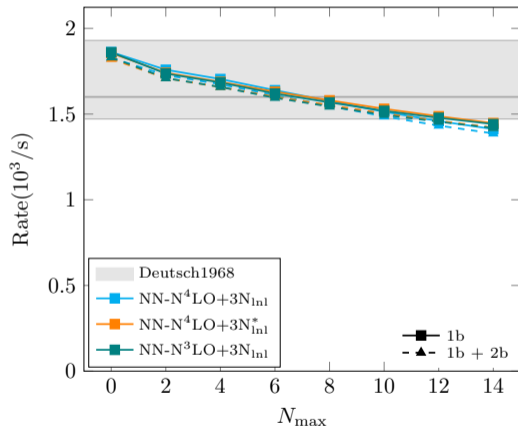
*LJ, Navrátil, Kotila, Kravvaris, in preparation*



# Capture Rates to the Ground State of ${}^6\text{He}$

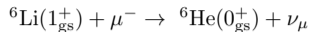


- NCSM slightly underestimating experiment



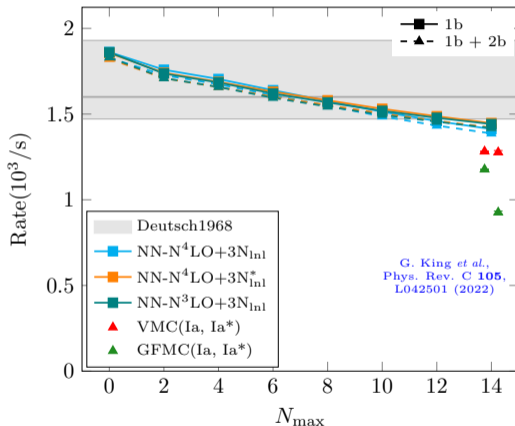
LJ, Navrátil, Kotila, Kravvaris, in preparation

# Capture Rates to the Ground State of ${}^6\text{He}$



- NCSM slightly underestimating experiment
- The results are consistent with the **variational (VMC)** and **Green's function Monte-Carlo (GFMC)** calculations

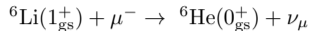
King *et al.*, Phys. Rev. C **105**, L042501 (2022)



G. King *et al.*,  
Phys. Rev. C **105**,  
L042501 (2022)

LJ, Navrátil, Kotila, Kravvaris, in preparation

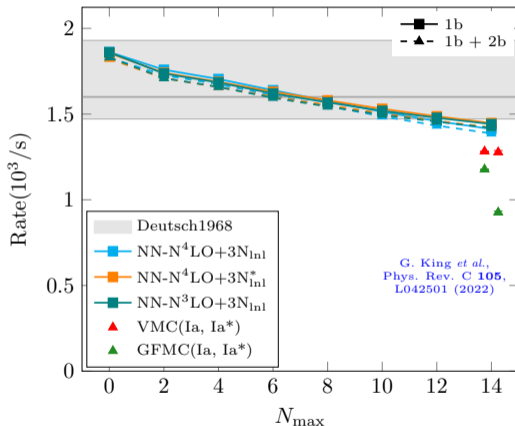
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King *et al.*, Phys. Rev. C **105**, L042501 (2022)

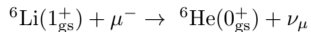
- ▶ Slow convergence likely due to cluster-structure



G. King *et al.*,  
Phys. Rev. C **105**,  
L042501 (2022)

LJ, Navrátil, Kotila, Kravvaris, in preparation

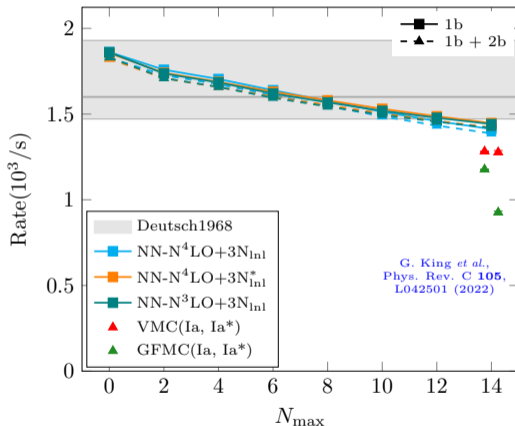
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King *et al.*, Phys. Rev. C **105**, L042501 (2022)

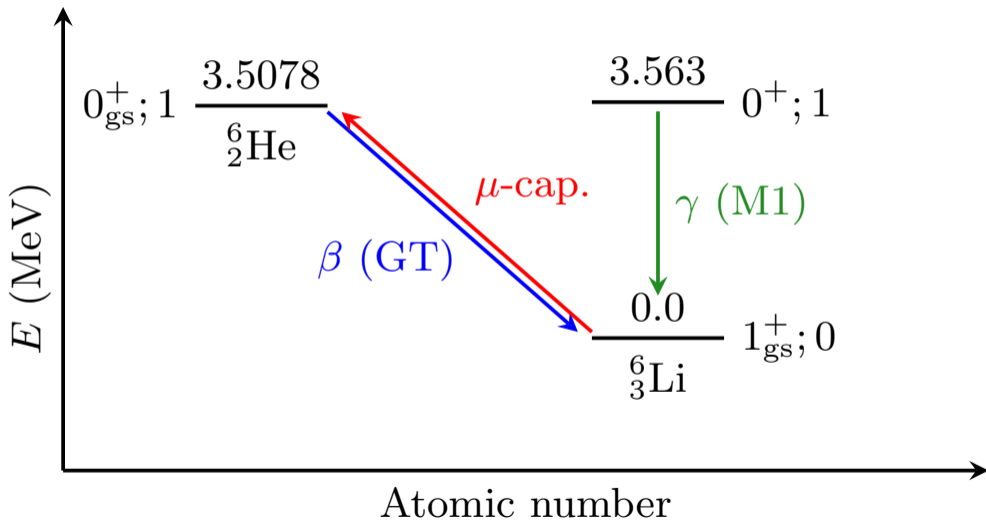
- ▶ Slow convergence likely due to cluster-structure
- ▶ NCSM with continuum (NCSMC) might give better results?



G. King *et al.*,  
Phys. Rev. C **105**,  
L042501 (2022)

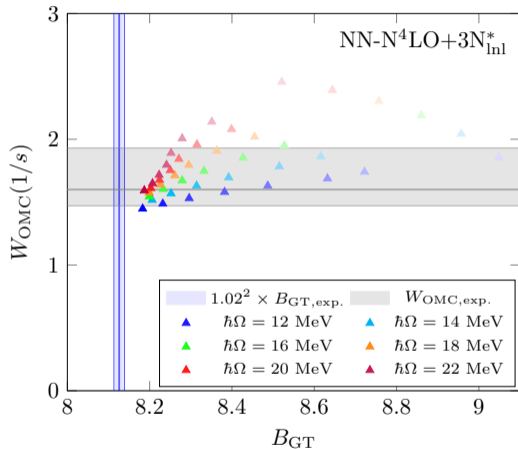
LJ, Navrátil, Kotila, Kravvaris, in preparation

## Correlations with Other Observables



# Correlations with Other Observables

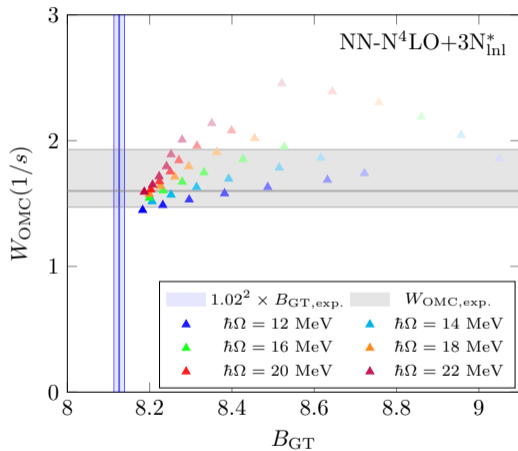
GT  $\beta$  decay:



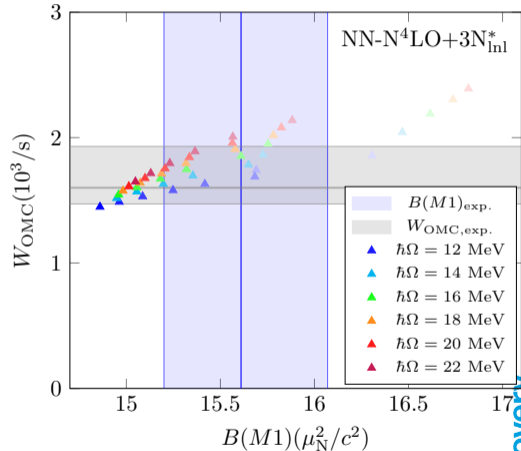
*LJ, Navrátil, Kotila and Kravvaris, in preparation*

# Correlations with Other Observables

GT  $\beta$  decay:



M1  $\gamma$  decay:



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Muon capture on  ${}^6\text{Li}$

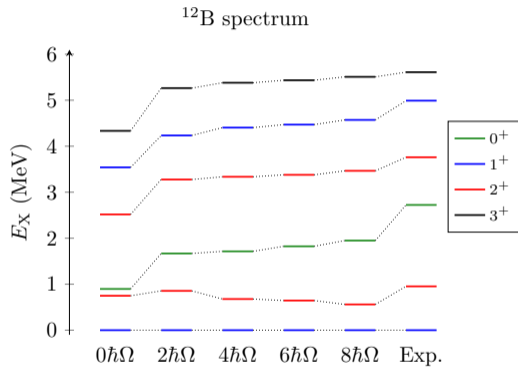
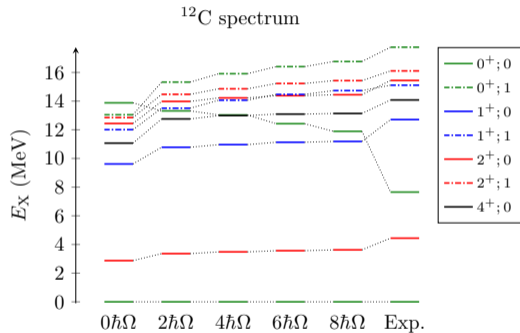
Muon capture on  ${}^{12}\text{C}$

Muon capture on  ${}^{16}\text{O}$

Summary

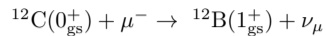


# Energy spectra of $^{12}\text{C}$ and $^{12}\text{B}$

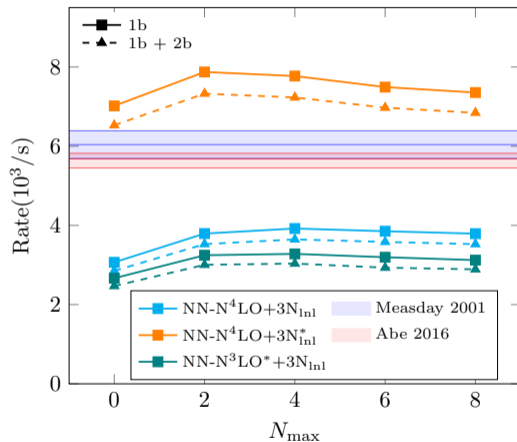


*LJ, Navrátil, Kotila, Kravvaris, in preparation*

# Capture Rates to the Ground State of $^{12}\text{B}$

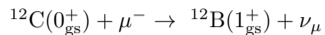


- Significant interaction dependence

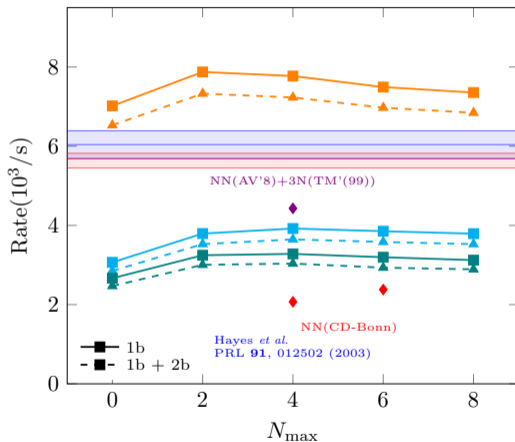


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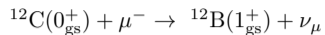


- Significant interaction dependence
  - ▶ The  $\text{NN-N}^4\text{LO}+3\text{N}_{\text{Inl}}^*$  interaction with the additional spin-orbit term most consistent with experiment



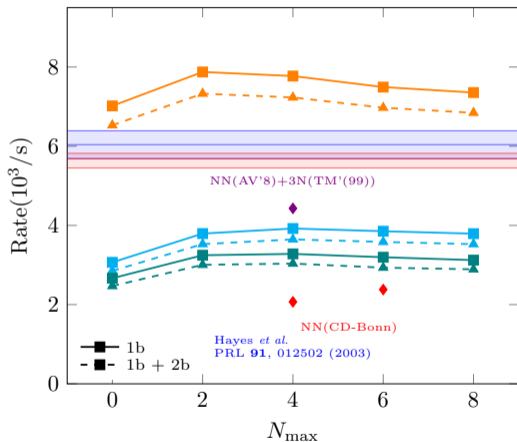
LJ, Navrátil, Kotila, Kravvaris, in preparation

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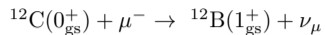
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Hayes *et al.*, *Phys. Rev. Lett.* **91**, 012502 (2003)



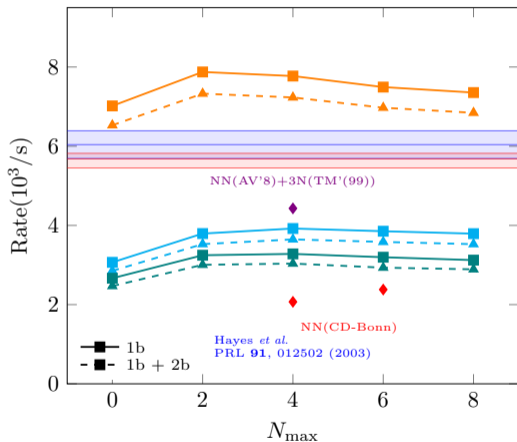
LJ, Navrátil, Kotila, Kravvaris, in preparation

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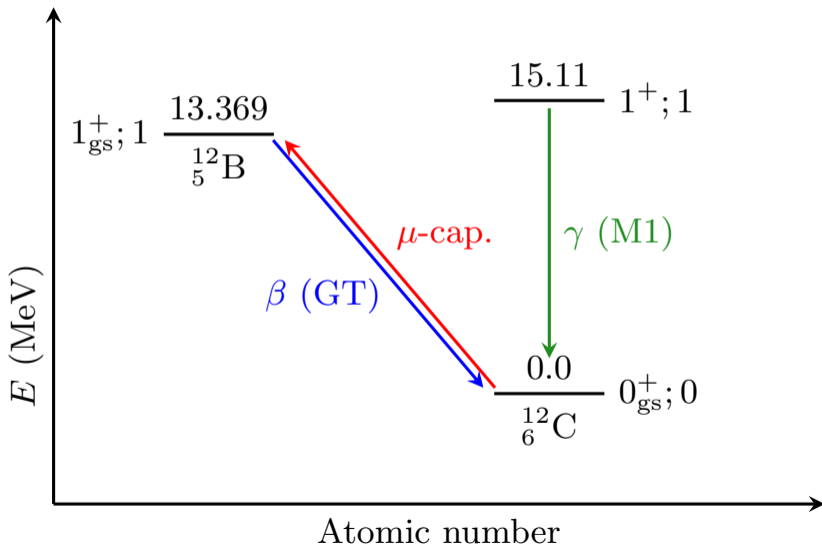
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  - ▶ The  $\text{NN-N}^4\text{LO}+3\text{N}_{\text{Inl}}^*$  interaction with the additional spin-orbit term most consistent with experiment
- The results can be compared against earlier NCSM calculations with phenomenological interactions
- 3-body forces essential to reproduce the measured rate

Hayes *et al.*, *Phys. Rev. Lett.* **91**, 012502 (2003)

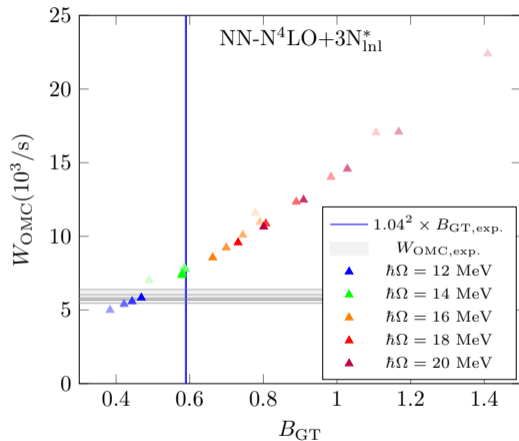


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## Correlations with Other Observables



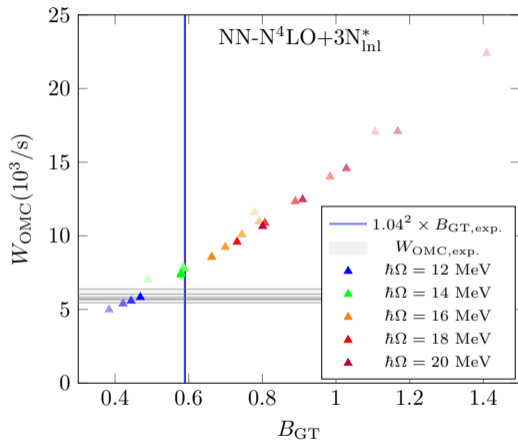
GT  $\beta$  decay:



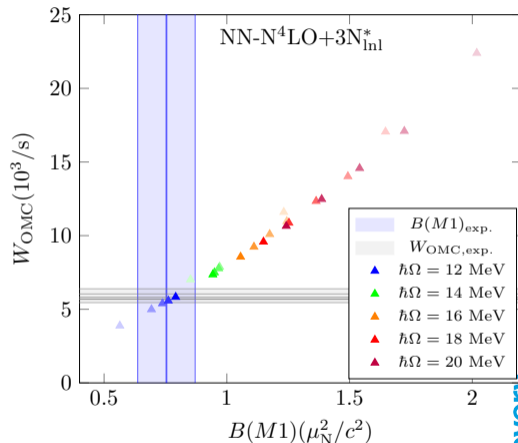
LJ, Navrátil, Kotila and Kravvaris, in progress

# Correlations with Other Observables

GT  $\beta$  decay:



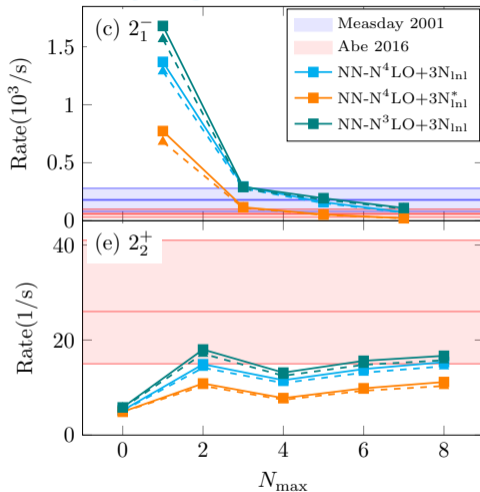
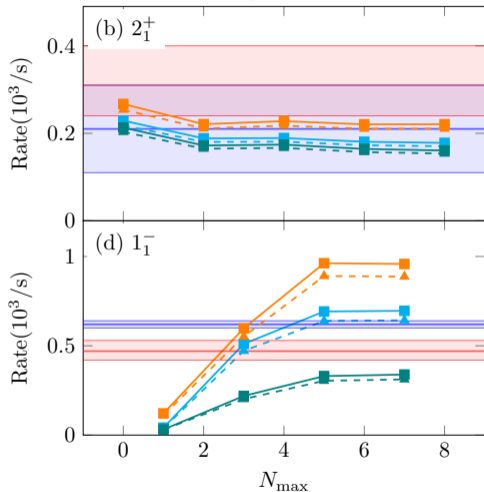
M1  $\gamma$  decay:



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# Capture Rates to Low-Lying States in $^{12}\text{B}$



*LJ, Navrátil, Kotila, Kravvaris, in preparation*

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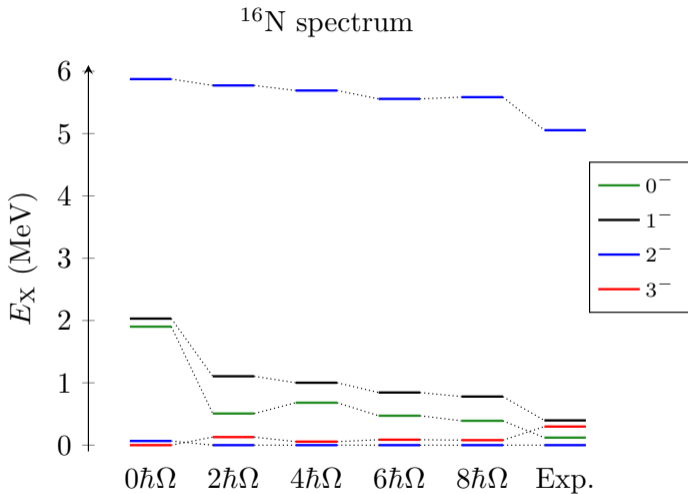
Muon capture on  ${}^6\text{Li}$

Muon capture on  ${}^{12}\text{C}$

Muon capture on  ${}^{16}\text{O}$

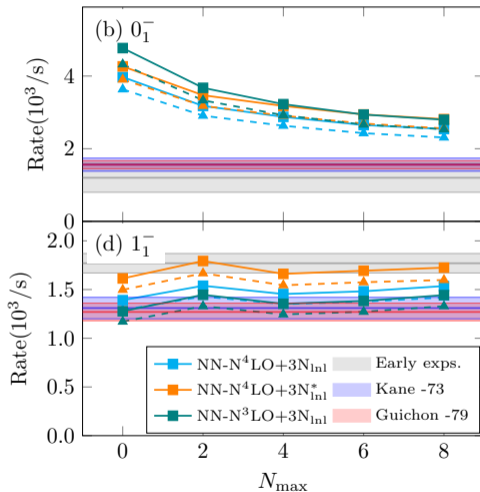
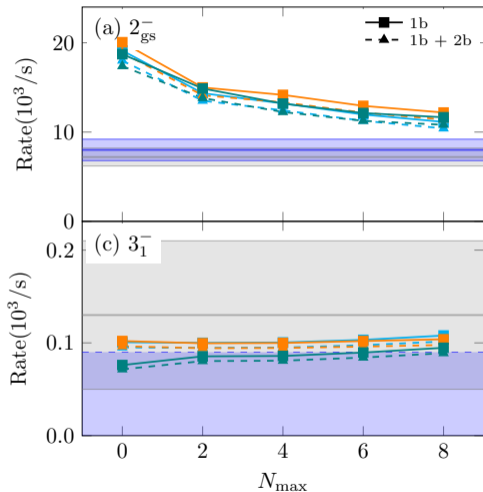
Summary

# Energy spectra of $^{16}\text{N}$



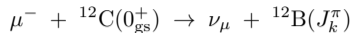
*LJ, Navrátil, Kotila, Kravvaris, in preparation*

# Capture Rates to Low-Lying States in $^{16}\text{N}$

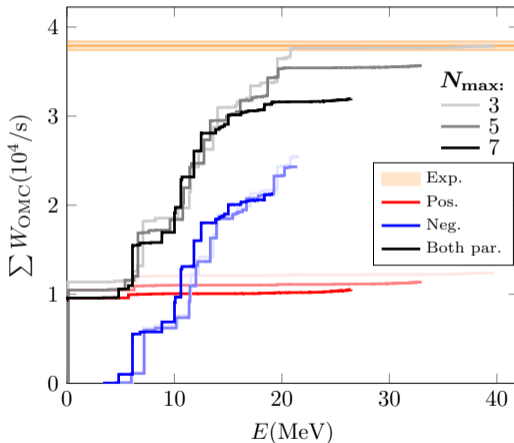


LJ, Navrátil, Kotila, Kravvaris, in preparation

# Total Muon-Capture Rates in $^{12}\text{B}$ and $^{16}\text{N}$

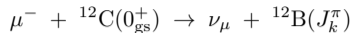


- Rates obtained summing over  $\sim 50$  final states of each parity

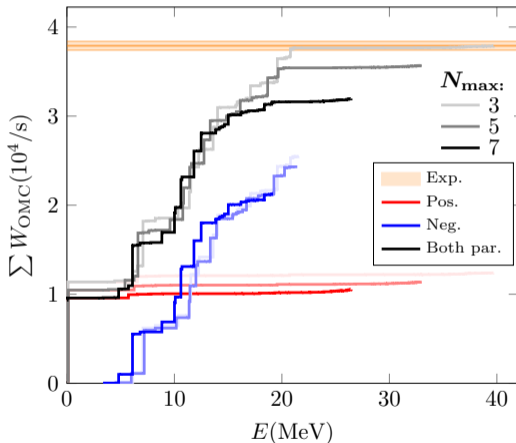


*LJ, Navrátil, Kotila, Kravvaris, in preparation*

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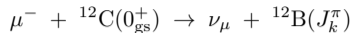


- Rates obtained summing over  $\sim 50$  final states of each parity
- Summing up **the rates up to  $\sim 20$  MeV**, we capture  **$\sim 85\%$  of the total rate** in both  $^{12}\text{B}$  and  $^{16}\text{N}$

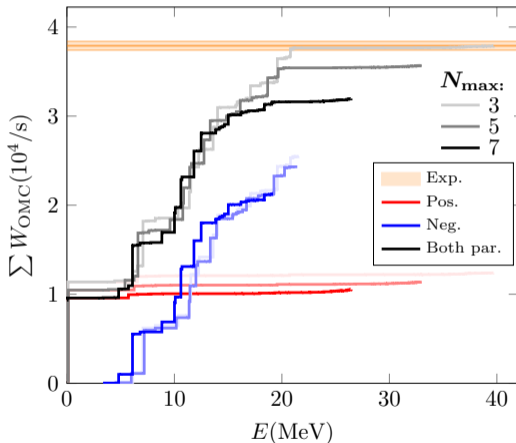


LJ, Navrátil, Kotila, Kravvaris, in preparation

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- Better estimation with the Lanczos strength function method ongoing (**see poster by D. Araujo**)



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Muon capture on  ${}^6\text{Li}$

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## Summary



- *Ab initio* muon-capture studies could shed light on nuclear electroweak currents at finite momentum exchange regime

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- No-core shell-model describes well partial muon-capture rates in light nuclei  ${}^6\text{He}$ ,  ${}^{12}\text{B}$  and  ${}^{16}\text{N}$
- Calculation of total capture rates currently in progress in NCSM

Thank you  
Merci



- Rates written in terms of reduced one-body matrix elements:

$$(\Psi_f || \sum_{s=1}^A \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) || \Psi_i) = -\frac{1}{\sqrt{2u+1}} \sum_{pn} (n || \hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s) || p) (\Psi_f || [a_n^\dagger \tilde{a}_p]_u || \Psi_i)$$

NME	$\hat{O}_{kwux}(\mathbf{r}_s, \mathbf{p}_s)$
$\mathcal{M}[0 w u]$	$j_w(qr_s) G_{-1}(r_s) \mathcal{Y}_{0wu}^{M_f - M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
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$\mathcal{M}[0 w u \pm]$	$[j_w(qr_s) G_{-1}(r_s) \mp \frac{1}{q} j_{w \mp 1}(qr_s) \frac{d}{dr_s} G_{-1}(r_s)] \mathcal{Y}_{0wu}^{M_f - M_i}(\hat{\mathbf{r}}_s) \delta_{wu}$
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## Axial-Vector Two-Body Currents (2BCs)

- One-body (1b) axial-vector currents given by

$$\mathbf{J}_{i,1b}^3 = \frac{\tau_i^3}{2} \left( g_A \boldsymbol{\sigma}_i - \frac{g_P}{2m_N} \mathbf{q} \cdot \boldsymbol{\sigma}_i \right),$$

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- Additional **pion-exchange, pion-pole, and contact** two-body (2b) currents

Hoferichter, Klos, Schwenk *Phys. Lett. B* **746**, 410 (2015)

$$\begin{aligned} \mathbf{J}_{12}^3 = & -\frac{g_A}{2F_\pi^2} [\tau_1 \times \tau_2]^3 \left[ c_4 \left( 1 - \frac{\mathbf{q}}{q^2 + M_\pi} \mathbf{q} \cdot \right) (\boldsymbol{\sigma}_1 \times \mathbf{k}_2) + \frac{c_6}{4} (\boldsymbol{\sigma}_1 \times \mathbf{q}) + i \frac{\mathbf{p}_1 + \mathbf{p}'_1}{4m_N} \right] \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{M_\pi^2 + k_2^2} \\ & - \frac{g_A}{F_\pi^2} \tau_2^3 \left[ c_3 \left( 1 - \frac{\mathbf{q}}{q^2 + M_\pi} \mathbf{q} \cdot \right) \mathbf{k}_2 + 2c_1 M_\pi^2 \frac{\mathbf{q}}{q^2 + M_\pi^2} \right] \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{k}_2}{M_\pi^2 + k_2^2} \\ & - d_1 \tau_1^3 \left( 1 - \frac{\mathbf{q}}{q^2 + M_\pi^2} \mathbf{q} \cdot \right) \boldsymbol{\sigma}_1 + (1 \leftrightarrow 2) - d_2 (\tau_1 \times \tau_2)^3 (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \left( 1 - \frac{\mathbf{q}}{q^2 + M_\pi^2} \mathbf{q} \cdot \right) \end{aligned}$$

where  $\mathbf{k}_i = \mathbf{p}'_i - \mathbf{p}_i$  and  $\mathbf{q} = -\mathbf{k}_1 - \mathbf{k}_2$



## Axial-Vector Two-Body Currents (2BCs)

- Approximate 2BCs by normal-ordering w.r.t. spin-isospin–symmetric reference state with  $\rho = 2k_F^3/(3\pi^2)$ :

Hoferichter, Menéndez, Schwenk, *Phys. Rev. D* **102**,074018 (2020)

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$$\mathbf{J}_{i,2b}^{\text{eff}} = \sum_j (1 - P_{ij}) \mathbf{J}_{ij}^3$$

$$\rightarrow \mathbf{J}_{i,2b}^{\text{eff}} = g_A \frac{\tau_i^3}{2} \left[ \delta a(\mathbf{q}^2) \boldsymbol{\sigma}_i + \frac{\delta a^P(\mathbf{q}^2)}{\mathbf{q}^2} (\mathbf{q} \cdot \boldsymbol{\sigma}_i) \mathbf{q} \right],$$

where

$$\delta a(\mathbf{q}^2) = -\frac{\rho}{F_\pi^2} \left[ \frac{c_4}{3} [3I_2^\sigma(\rho, \mathbf{q}) - I_1^\sigma(\rho, |\mathbf{q}|)] - \frac{1}{3} \left( c_3 - \frac{1}{4m_N} \right) I_1^\sigma(\rho, |\mathbf{q}|) - \frac{c_6}{12} I_{c6}(\rho, |\mathbf{q}|) - \frac{c_D}{4g_A \Lambda_\chi} \right],$$

$$\delta a^P(\mathbf{q}^2) = \frac{\rho}{F_\pi^2} \left[ -2(c_3 - 2c_1) \frac{m_\pi^2 \mathbf{q}^2}{(m_\pi^2 + \mathbf{q}^2)^2} + \frac{1}{3} \left( c_3 + c_4 - \frac{1}{4m_N} \right) I^P(\rho, |\mathbf{q}|) - \left( \frac{c_6}{12} - \frac{2}{3} \frac{c_1 m_\pi^2}{m_\pi^2 + \mathbf{q}^2} \right) I_{c6}(\rho, |\mathbf{q}|) \right. \\ \left. - \frac{\mathbf{q}^2}{m_\pi^2 + \mathbf{q}^2} \left( \frac{c_3}{3} [I_1^\sigma(\rho, |\mathbf{q}|) + I^P(\rho, |\mathbf{q}|)] + \frac{c_4}{3} [I_1^\sigma(\rho, |\mathbf{q}|) + I^P(\rho, |\mathbf{q}|) - 3I_2^\sigma(\rho, |\mathbf{q}|)] \right) - \frac{c_D}{4g_A \Lambda_\chi} \frac{\mathbf{q}^2}{m_\pi^2 + \mathbf{q}^2} \right]$$

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  - ▶ Working with  $A - 1$  Jacobi coordinates  $\xi_s = -\sqrt{A/(A - 1)}(\mathbf{r}_s - \mathbf{R}_{CM})$ :



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{\text{HO}}(\xi_1, \xi_2, \dots, \xi_{A-1})$$


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- ▶ Working with  $A$  single-particle coordinates and separating the center-of-mass motion:

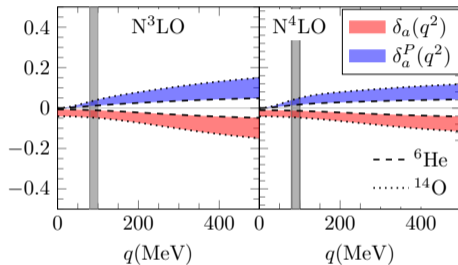


$$\Psi_{\text{SD}}^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Nj}^{\text{SD}} \Phi_{\text{SD } Nj}^{\text{HO}}(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = \Psi^A \Psi_{\text{CM}}(\mathbf{R}_{\text{CM}})$$

## Two-Body Currents

- Fermi-gas density  $\rho$  adjusted so that  $\delta_a(0)$  reproduces the effect of exact two-body currents in

*P. Gysbers et al., Nature Phys. 15, 428 (2019)*

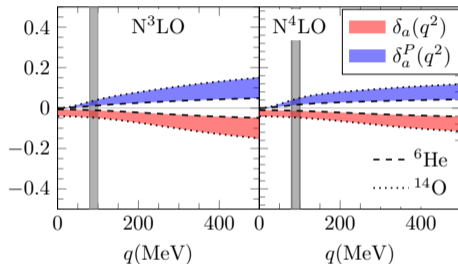


*LJ, Navrátil, Kotila and Kravvaris,  
in preparation*

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- Two-body currents typically **reduce** the OMC rates by  $\sim 1 - 2\%$  in  ${}^6\text{Li}$  and by  $\lesssim 10\%$  in  ${}^{12}\text{C}$  and  ${}^{16}\text{O}$

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