



Nuclear physics for the precise extraction of V_{ud}

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Outline

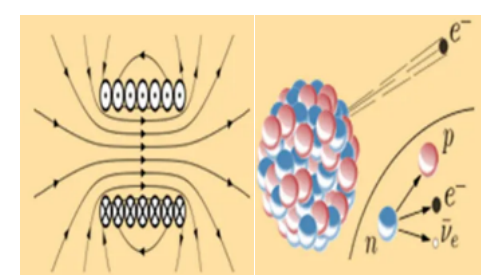
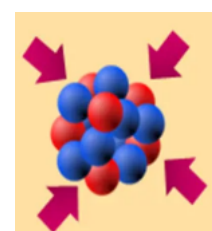
1. Precision frontier and the CKM unitarity
2. V_{ud} from beta decays
3. Inputs at tree-level
 - Weak decay form factors
 - Isospin-breaking correction
4. Inputs at loop-level
 - Nucleus-independent radiative corrections
 - Nucleus-dependent radiative corrections
5. Summary and outlook

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Standard Model of Elementary Particles

three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III	
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	$\approx 124.97 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0
QUARKS	u up	c charm	t top	g gluon
	$\approx 4.7 \text{ MeV}/c^2$	$\approx 96 \text{ MeV}/c^2$	$\approx 4.18 \text{ GeV}/c^2$	0
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	d down	s strange	b bottom	γ photon
	$\approx 0.511 \text{ MeV}/c^2$	$\approx 105.66 \text{ MeV}/c^2$	$\approx 1.7768 \text{ GeV}/c^2$	$\approx 91.19 \text{ GeV}/c^2$
	-1	-1	-1	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	e electron	μ muon	τ tau	Z Z boson
LEPTONS	$< 1.0 \text{ eV}/c^2$	$< 0.17 \text{ MeV}/c^2$	$< 18.2 \text{ MeV}/c^2$	$\approx 80.39 \text{ GeV}/c^2$
	0	0	0	± 1
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson



$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

Strong (QCD)

Electroweak

Image credit: Wikipedia

Two Ultimate Goals in Nuclear and Particle Physics (my opinion)

To understand
QCD

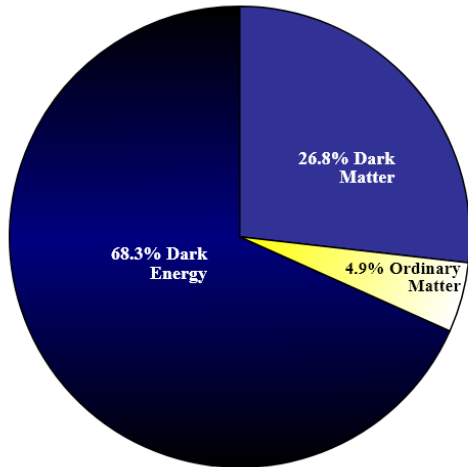
To discover
BSM physics in
EW sector



Petronas Twin Towers, Kuala Lumpur, Malaysia

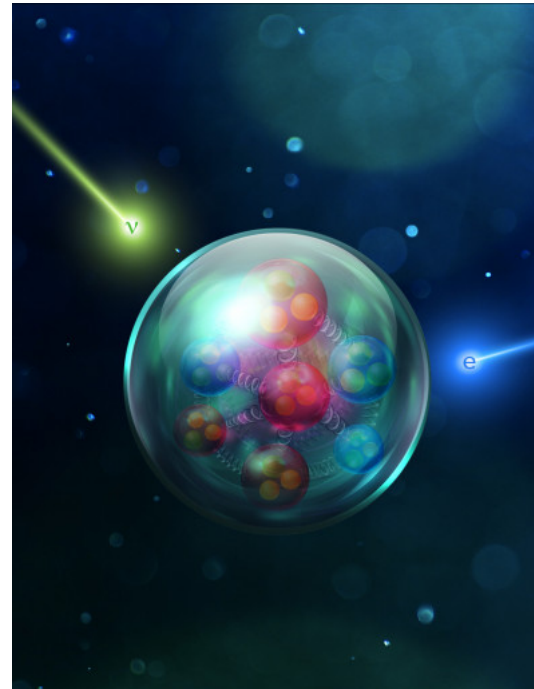
Many unresolved problems call for **physics beyond the Standard Model (BSM)** !

Image credit: Wikipedia

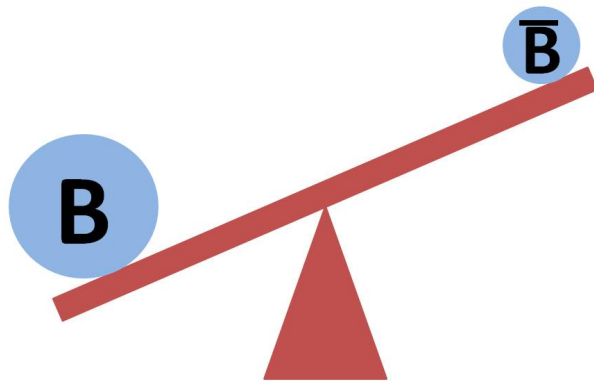


What is the origin of dark energy and dark matter?

Image credit: Jefferson Lab

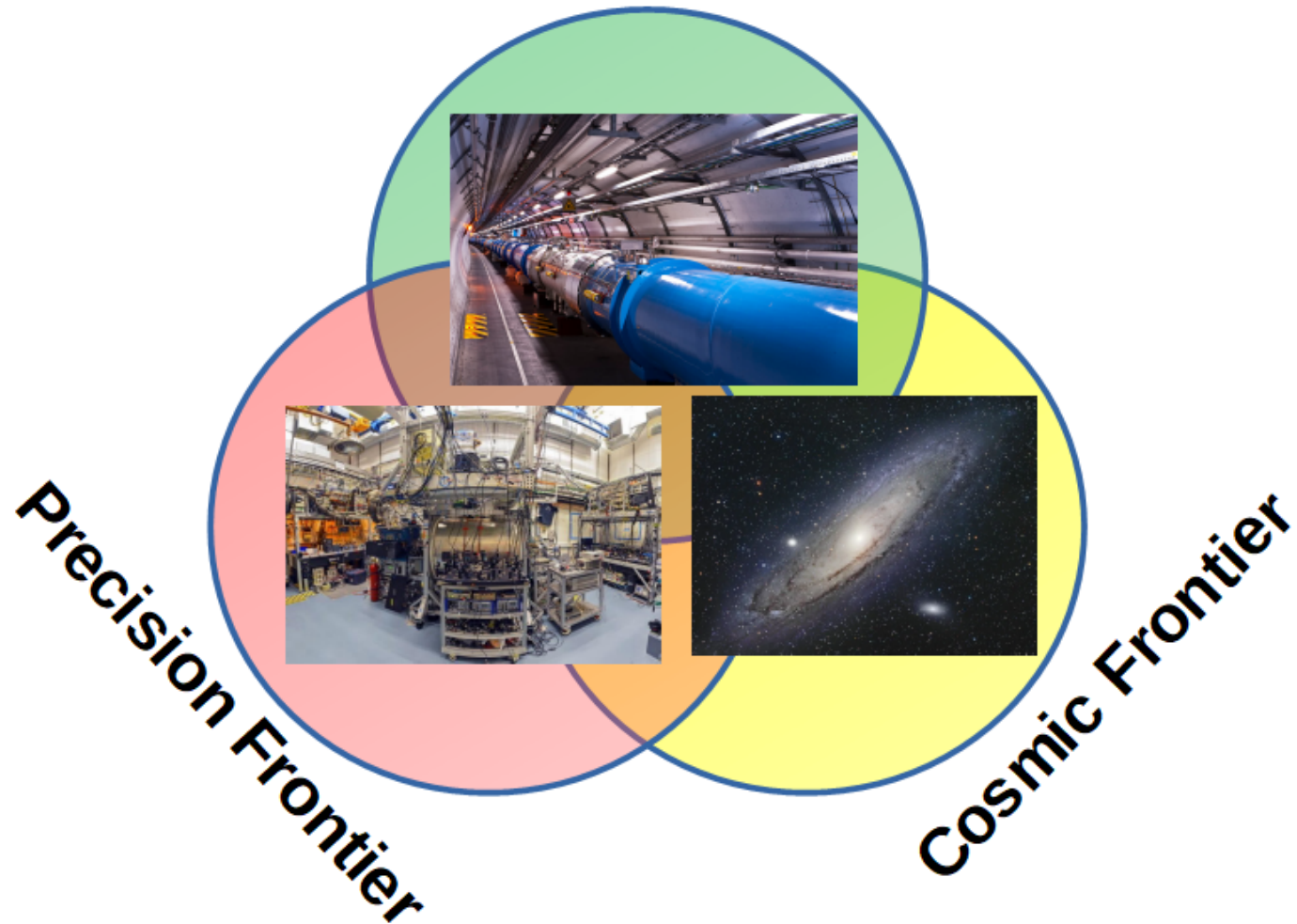


What is the nature of the neutrino mass?



Why is there much more matter than antimatter in the observed universe?

Energy Frontier



Precision Frontier: Measure things very precisely, and look for their **deviations** from SM prediction!

Precision Frontier



Standard Model Prediction



Experimental Result

Precision Frontier



Standard Model Prediction



Experimental Result

Precision Frontier



Standard Model Prediction



Experimental Result

Example: Charged weak decays and CKM unitarity

Massive quarks ==> Generation mixing

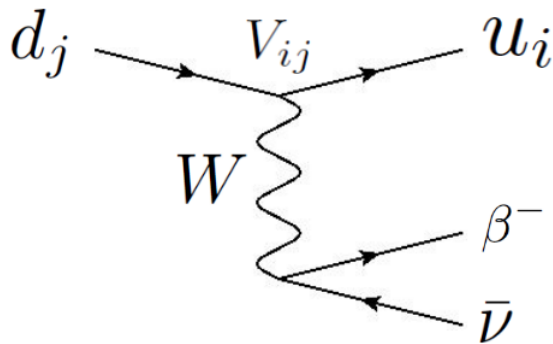
Leptons	Quarks	u up	c charm	t top
		d down	s strange	b bottom
		ν_e e- Neutrino	ν_μ μ - Neutrino	ν_τ τ - Neutrino
	e electron	μ muon	τ tau	
	I	II	III	
	The Generations of Matter			

$$\Psi_{d,f} = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_f = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_m$$

Cabibbo-Kobayashi-Maskawa (CKM) matrix

Three generations of quarks and leptons





Weak interaction universality \implies
 Unitarity of the measured CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Can be tested at **0.01%** level! Probes new physics at the scale:

$$\left(\frac{v_H}{\Lambda_{\text{BSM}}} \right)^2 \sim 0.01\% \implies \Lambda_{\text{BSM}} \sim 20 \text{ TeV}$$

Competitive to high-energy experiments!

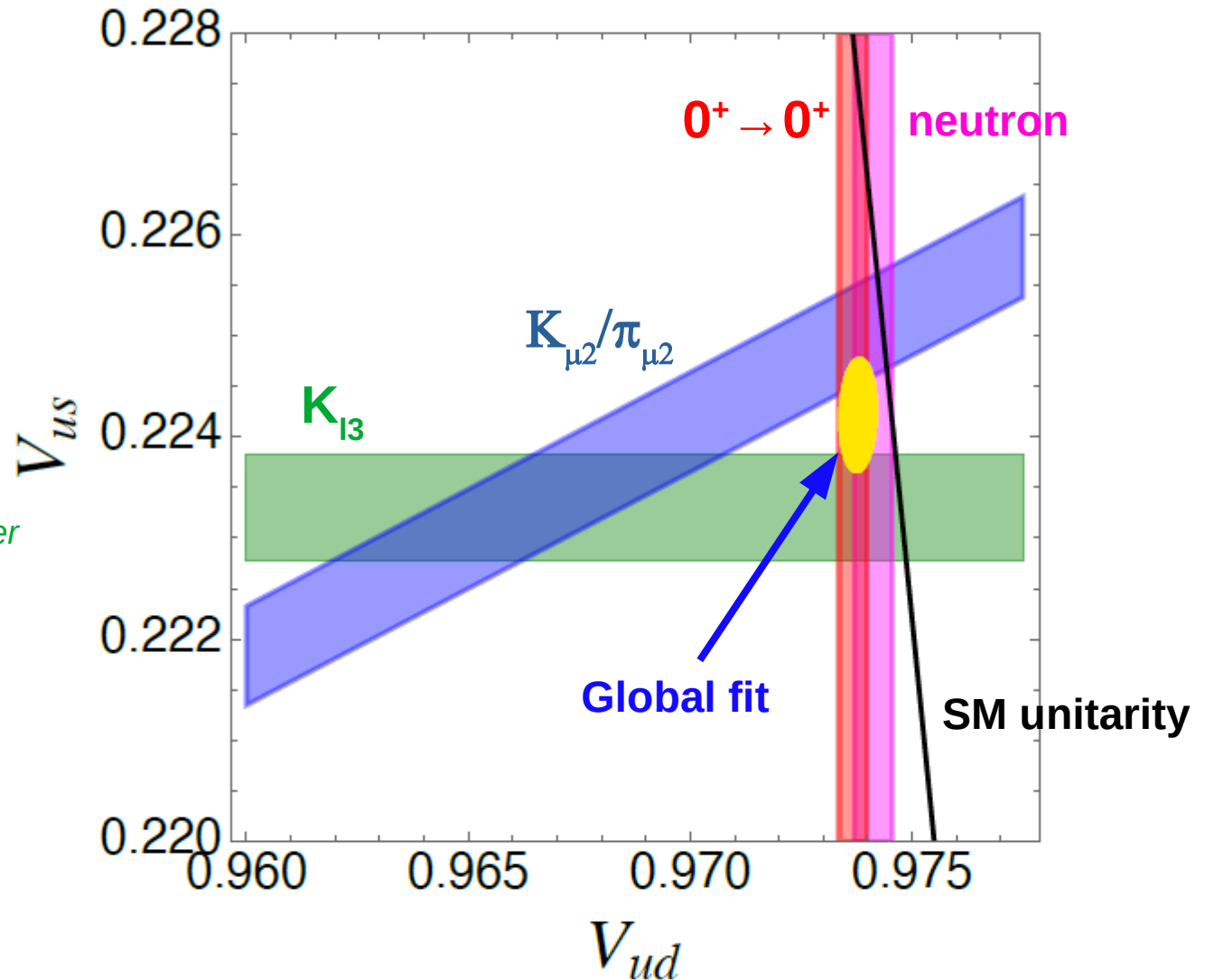
“First-row CKM unitarity”:

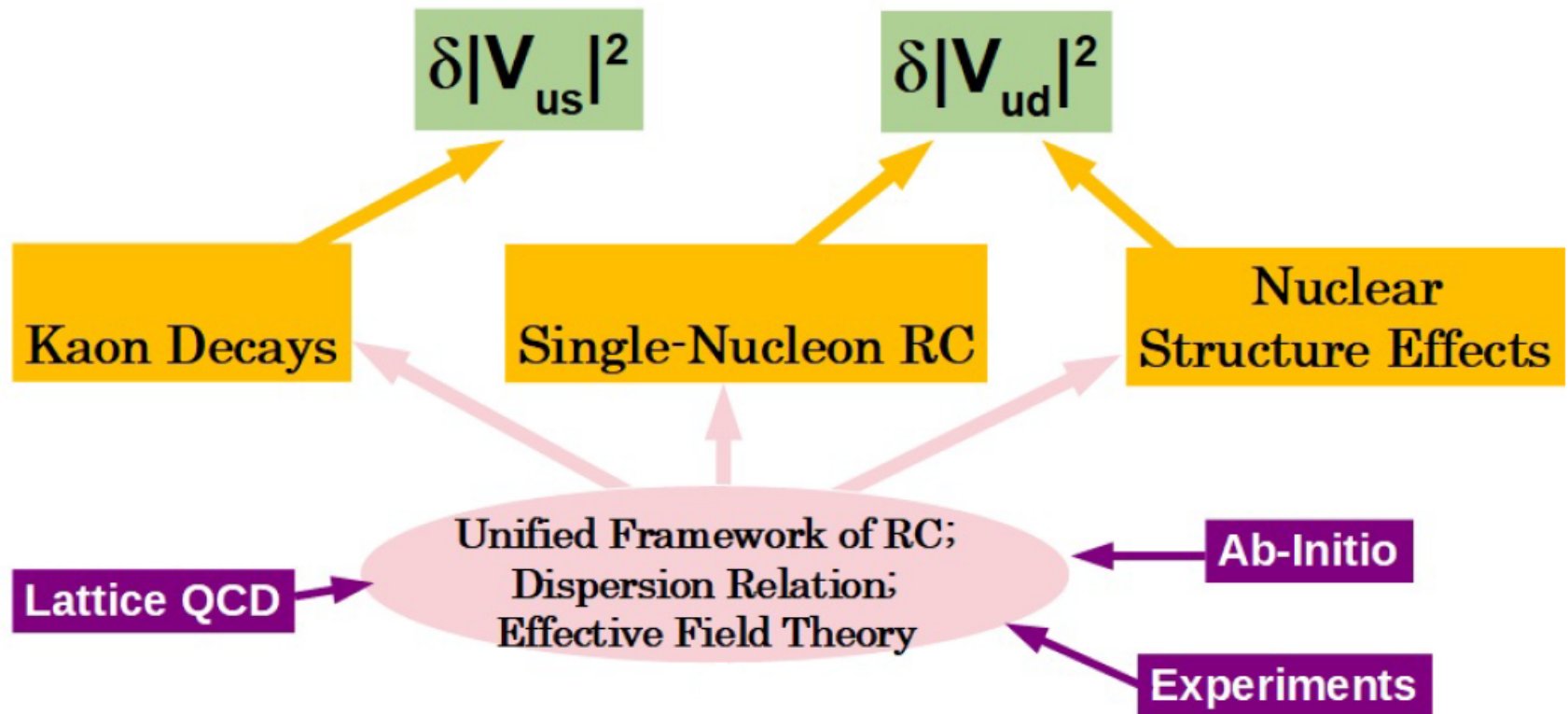
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Global fit:

2.8σ deficit

*Cirigliano, Crivellin, Hoferichter
and Moulson, 2023 PLB*





$$|V_{ud}|^2 \sim 0.95 \quad \implies \quad \text{precision goal} \sim 0.01\%$$

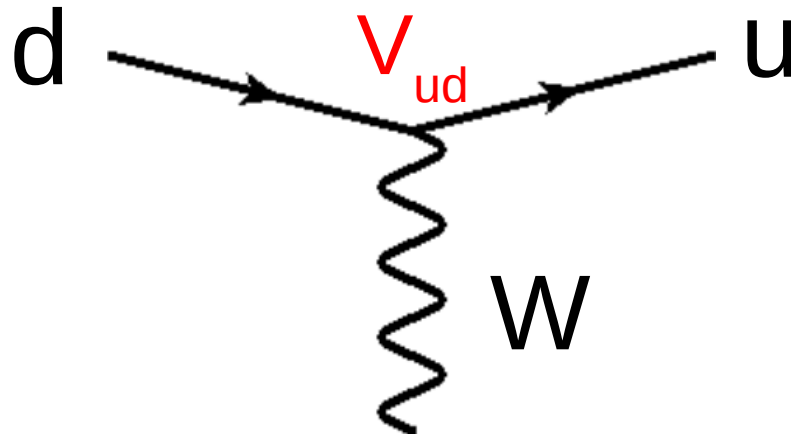
$$|V_{us}|^2 \sim 0.05 \quad \implies \quad \text{precision goal} \sim 0.1\%$$

Perfect example of strong-electroweak interplay

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V_{ud} : The most precisely-studied CKM matrix element



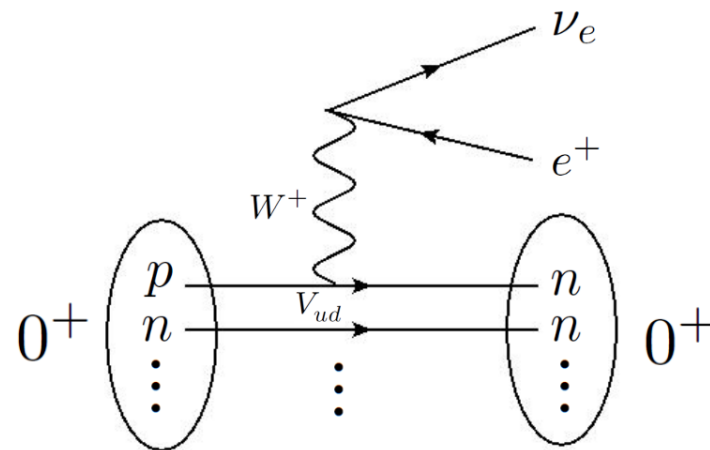
Pion: $\pi^+ \rightarrow \mu^+ \nu_\mu$ or $\pi^0 e^+ \nu_e$

Neutron: $n \rightarrow p e \nu_e$

Nucleus: $i \rightarrow f e \nu_e$

“Superallowed” beta decays of $I=1, J^p=0^+$ nuclei

$$i(0^+) \rightarrow f(0^+) + e^+ + \nu_e$$



Provides the **best measurement of V_{ud}** :

- Tree-level amplitude is (almost) known
- **23** measured transitions
- **15** with ft-precision better than **0.23%**

Master Formula

$$|V_{ud}|^2 = \frac{\pi^3 \ln 2}{G_F^2 m_e^5 ft(1 + \delta'_R)(1 + \Delta_R^V)(1 + \delta_{NS} - \delta_C)}$$

Master Formula

Experimental Inputs:

$$|V_{ud}|^2 = \frac{\pi^3 \ln 2}{G_F^2 m_e^5 ft (1 + \delta'_R)(1 + \Delta_R^V)(1 + \delta_{NS} - \delta_C)}$$

Partial half-life

Fermi's constant

Statistical rate function

Master Formula

Theoretical Inputs:

$$|V_{ud}|^2 = \frac{\pi^3 \ln 2}{G_F^2 m_e^5 ft (1 + \delta'_R) (1 + \Delta_R^V) (1 + \delta_{NS} - \delta_C)}$$

Isospin-breaking correction to Fermi matrix element

Statistical rate function

“outer” radiative correction

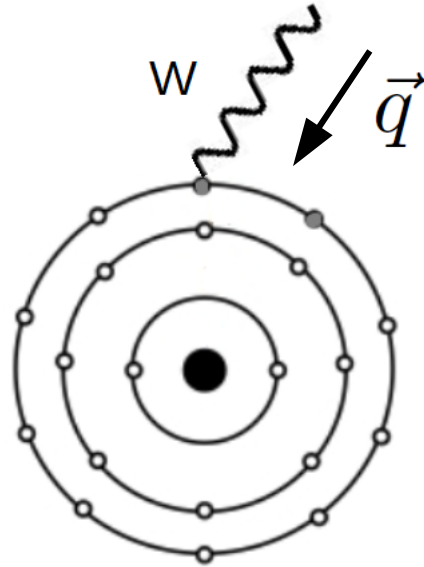
Single-nucleon radiative correction

Nuclear structure effects in radiative correction

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Superallowed decays at tree level



Charged weak decay matrix element:

$$f_+(\vec{q}^2) = M_F(1 + \bar{f}_+(\vec{q}^2))$$

Fermi matrix element

q^2 -dependence

Treatment of q^2 -dependence:

1. Conventional shell-model calculation

Hardy and Towner, 2005 PRC

$$\langle f | O | i \rangle = \sum_{\alpha\beta} \underbrace{\langle \alpha | O | \beta \rangle}_{\text{single-nucleon transition M.E}} \underbrace{\langle f | a_{\alpha}^{\dagger} a_{\beta} | i \rangle}_{\text{One-body density M.E}}$$

single-nucleon states

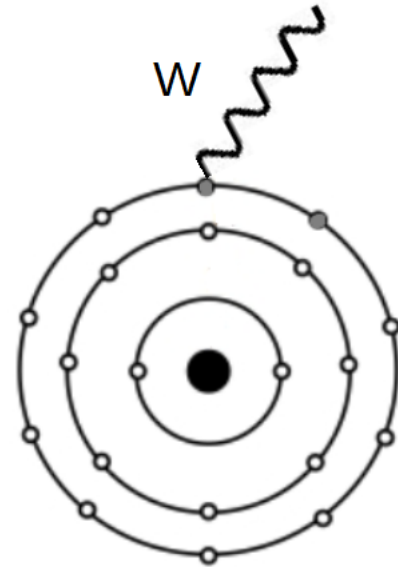
Difficult to quantify theory uncertainty!

Treatment of q^2 -dependence:

2. Relate it to a distribution

$$\bar{f}_+(\vec{q}^2) \xleftrightarrow{\text{Fourier transform}} \rho_{\text{CW}}(r)$$

$\rho_{\text{CW}}(r)$: Distribution of “**active**” nucleons eligible for weak transitions in a nucleus



Treatment of q^2 -dependence:

2. Relate it to a distribution

$$\bar{f}_+(\vec{q}^2) \xleftrightarrow{\text{Fourier transform}} \rho_{\text{CW}}(r)$$

“Semi” data-driven approach:

*Wilkinson, 1993 Nucl.Inst.Meth.Phys.Res.A;
Hayen et al., 2018 RMP*

$$\rho_{\text{CW}}(r) = \rho_{\text{ch}}(r) + \delta\rho(r)$$

Nuclear charge
distribution (data)

Assume small,
estimate using
shell model

Treatment of q^2 -dependence:

2. Relate it to a distribution

$$\bar{f}_+(\vec{q}^2) \xleftrightarrow{\text{Fourier transform}} \rho_{\text{CW}}(r)$$

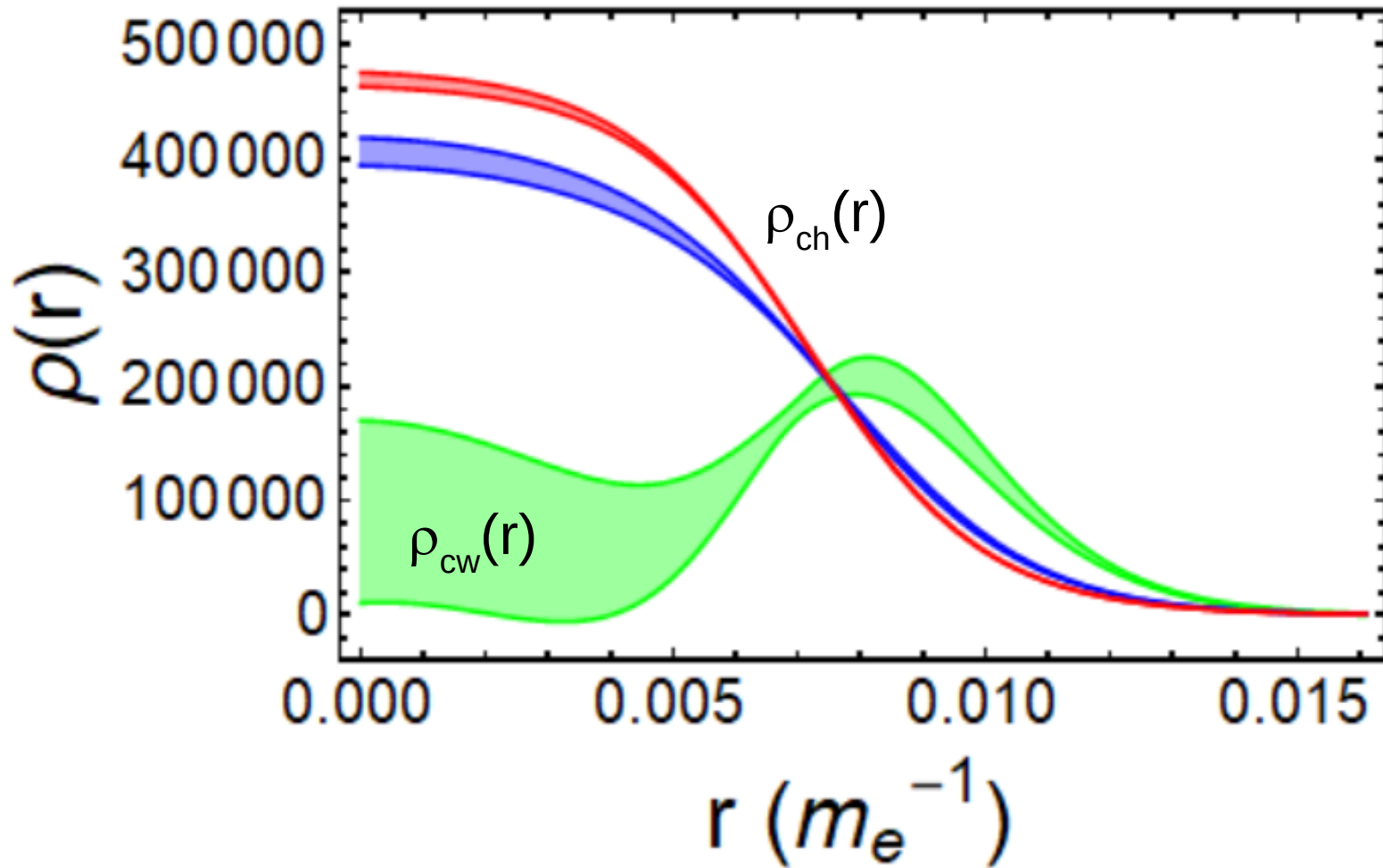
Fully data-driven approach:

Relate to **a pair of nuclear charge distributions** using **CVC**

Hostein, 1974 RMP; CYS, 2023 PRL

$$\begin{aligned} \rho_{\text{CW}}(r) &= \rho_{\text{ch},1}(r) + Z_0 (\rho_{\text{ch},0}(r) - \rho_{\text{ch},1}(r)) \\ &= \rho_{\text{ch},1}(r) + \frac{Z_{-1}}{2} (\rho_{\text{ch},-1}(r) - \rho_{\text{ch},1}(r)) \end{aligned}$$

$A=22$



“Semi” data-driven approach not well-justified!

Measured nuclear charge radii

A	$\langle r_{\text{ch},-1}^2 \rangle^{1/2}$ (fm)	$\langle r_{\text{ch},0}^2 \rangle^{1/2}$ (fm)	$\langle r_{\text{ch},1}^2 \rangle^{1/2}$ (fm)
10	${}^{10}_6\text{C}$	${}^{10}_5\text{B(ex)}$	${}^{10}_4\text{Be: } 2.3550(170)^a$
14	${}^{14}_8\text{O}$	${}^{14}_7\text{N(ex)}$	${}^{14}_6\text{C: } 2.5025(87)^a$
18	${}^{18}_{10}\text{Ne: } 2.9714(76)^a$	${}^{18}_9\text{F(ex)}$	${}^{18}_8\text{O: } 2.7726(56)^a$
22	${}^{22}_{12}\text{Mg: } 3.0691(89)^b$	${}^{22}_{11}\text{Na(ex)}$	${}^{22}_{10}\text{Ne: } 2.9525(40)^a$
26	${}^{26}_{14}\text{Si}$	${}^{26m}_{13}\text{Al: } 3.130(15)^f$	${}^{26}_{12}\text{Mg: } 3.0337(18)^a$
30	${}^{30}_{16}\text{S}$	${}^{30}_{15}\text{P(ex)}$	${}^{30}_{14}\text{Si: } 3.1336(40)^a$
34	${}^{34}_{18}\text{Ar: } 3.3654(40)^a$	${}^{34}_{17}\text{Cl}$	${}^{34}_{16}\text{S: } 3.2847(21)^a$
38	${}^{38}_{20}\text{Ca: } 3.467(1)^c$	${}^{38m}_{19}\text{K: } 3.437(4)^d$	${}^{38}_{18}\text{Ar: } 3.4028(19)^a$
42	${}^{42}_{22}\text{Ti}$	${}^{42}_{21}\text{Sc: } 3.5702(238)^a$	${}^{42}_{20}\text{Ca: } 3.5081(21)^a$
46	${}^{46}_{24}\text{Cr}$	${}^{46}_{23}\text{V}$	${}^{46}_{22}\text{Ti: } 3.6070(22)^a$
50	${}^{50}_{26}\text{Fe}$	${}^{50}_{25}\text{Mn: } 3.7120(196)^a$	${}^{50}_{24}\text{Cr: } 3.6588(65)^a$
54	${}^{54}_{28}\text{Ni: } 3.738(4)^e$	${}^{54}_{27}\text{Co}$	${}^{54}_{26}\text{Fe: } 3.6933(19)^a$
62	${}^{62}_{32}\text{Ge}$	${}^{62}_{31}\text{Ga}$	${}^{62}_{30}\text{Zn: } 3.9031(69)^b$
66	${}^{66}_{34}\text{Se}$	${}^{66}_{33}\text{As}$	${}^{66}_{32}\text{Ge}$
70	${}^{70}_{36}\text{Kr}$	${}^{70}_{35}\text{Br}$	${}^{70}_{34}\text{Se}$
74	${}^{74}_{38}\text{Sr}$	${}^{74}_{37}\text{Rb: } 4.1935(172)^b$	${}^{74}_{36}\text{Kr: } 4.1870(41)^a$

Shell model Data-driven

Transition	t (ms)	$(ft)_{\text{HT}}$ (s)	$(ft)_{\text{new}}$ (s)
$^{18}\text{Ne} \rightarrow ^{18}\text{F}$	21630 ± 590	2912 ± 79	2912 ± 80
$^{22}\text{Mg} \rightarrow ^{22}\text{Na}$	7293 ± 16	3051.1 ± 6.9	3050.4 ± 6.8
$^{26}\text{Si} \rightarrow ^{26m}\text{Al}$	2969.0 ± 5.4	3052.2 ± 5.6	3050.7 ± 5.6
$^{34}\text{Ar} \rightarrow ^{34}\text{Cl}$	896.55 ± 0.81	3058.0 ± 2.8	3057.1 ± 2.8
$^{38}\text{Ca} \rightarrow ^{38m}\text{K}$	574.8 ± 1.1	3062.8 ± 6.0	3062.2 ± 5.9
$^{42r}\text{Ti} \rightarrow ^{42}\text{Sc}$	433 ± 12	3090 ± 88	3085 ± 86
$^{50}\text{Fe} \rightarrow ^{50}\text{Mn}$	205.8 ± 4.7	3099 ± 71	3098 ± 72
$^{54}\text{Ni} \rightarrow ^{54}\text{Co}$	144.9 ± 2.3	3062 ± 50	3063 ± 49
$^{26m}\text{Al} \rightarrow ^{26}\text{Mg}$	$6351.24^{+0.55}_{-0.54}$	3037.61 ± 0.67	3036.5 ± 1.0
$^{34}\text{Cl} \rightarrow ^{34}\text{S}$	$1527.77^{+0.47}_{-0.44}$	$3049.43^{+0.95}_{-0.88}$	3048.0 ± 1.1
$^{38m}\text{K} \rightarrow ^{38}\text{Ar}$	925.42 ± 0.28	3051.45 ± 0.92	3050.5 ± 1.1
$^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$	681.44 ± 0.26	3047.7 ± 1.2	3045.0 ± 2.7
$^{50}\text{Mn} \rightarrow ^{50}\text{Cr}$	283.68 ± 0.11	3048.4 ± 1.2	3046.1 ± 3.6
$^{54}\text{Co} \rightarrow ^{54}\text{Fe}$	$193.495^{+0.086}_{-0.063}$	$3050.8^{+1.4}_{-1.1}$	$3051.3^{+1.7}_{-1.4}$
$^{74}\text{Rb} \rightarrow ^{74}\text{Kr}$	65.201 ± 0.047	3082.8 ± 6.5	3086 ± 11

Isospin symmetry breaking (ISB) correction

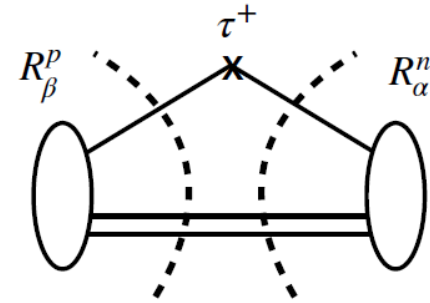
Full Fermi matrix element:

$$\begin{aligned} M_F^2 &= |\langle f | \tau_+ | i \rangle|^2 \\ &= (M_F^0)^2 (1 - \delta_C) \end{aligned}$$

Isospin limit

ISB correction,
primarily Coulomb (C)

(0.1-1)%



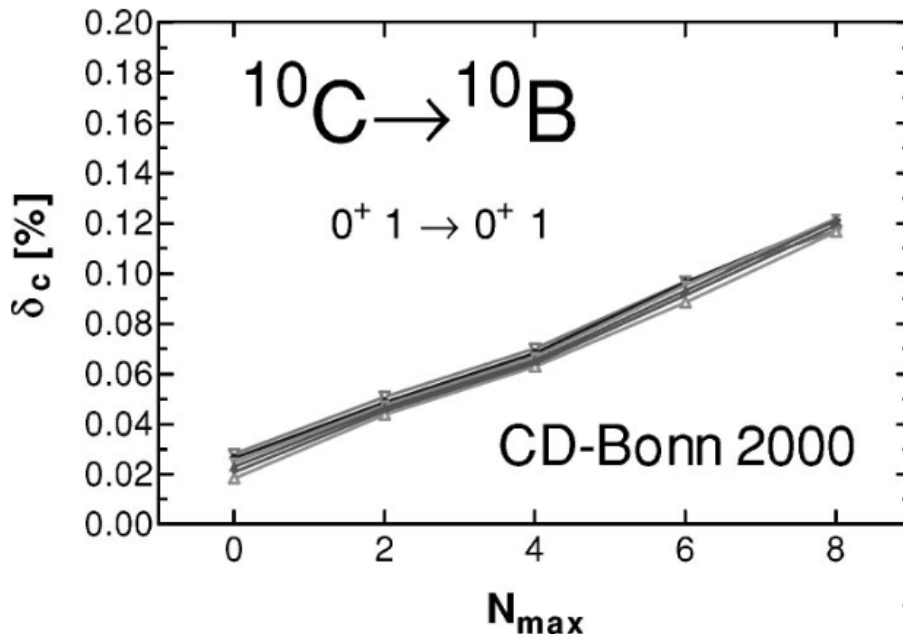
Symmetry broken due to:

1. Proton's electric charge
2. $M_p \neq M_n$

6 decades of δ_C -calculations
show huge model-dependence!

Ab-initio study with no-core shell
model (NCSM) returned
non-convergent result!

Caurier, Navratil, Ormand and Vary, 2002 PRC



Intruder states effects

Transitions	δ_C (%)				
	WS	DFT	HF	RPA	Micro
$^{26m}\text{Al} \rightarrow ^{26}\text{Mg}$	0.310	0.329	0.30	0.139	0.08
$^{34}\text{Cl} \rightarrow ^{34}\text{S}$	0.613	0.75	0.57	0.234	0.13
$^{38m}\text{K} \rightarrow ^{38}\text{Ar}$	0.628	1.7	0.59	0.278	0.15
$^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$	0.690	0.77	0.42	0.333	0.18
$^{46}\text{V} \rightarrow ^{46}\text{Ti}$	0.620	0.563	0.38	/	0.21
$^{50}\text{Mn} \rightarrow ^{50}\text{Cr}$	0.660	0.476	0.35	/	0.24
$^{54}\text{Co} \rightarrow ^{54}\text{Fe}$	0.770	0.586	0.44	0.319	0.28

(Selected model results)

*See also talks by Calvin Johnson,
Mark Caprio, Anna McCoy,
Patrick Fasano...*

Similar convergence issue in other ISB-observables:

1. Quadrupole moments in mirror nuclei

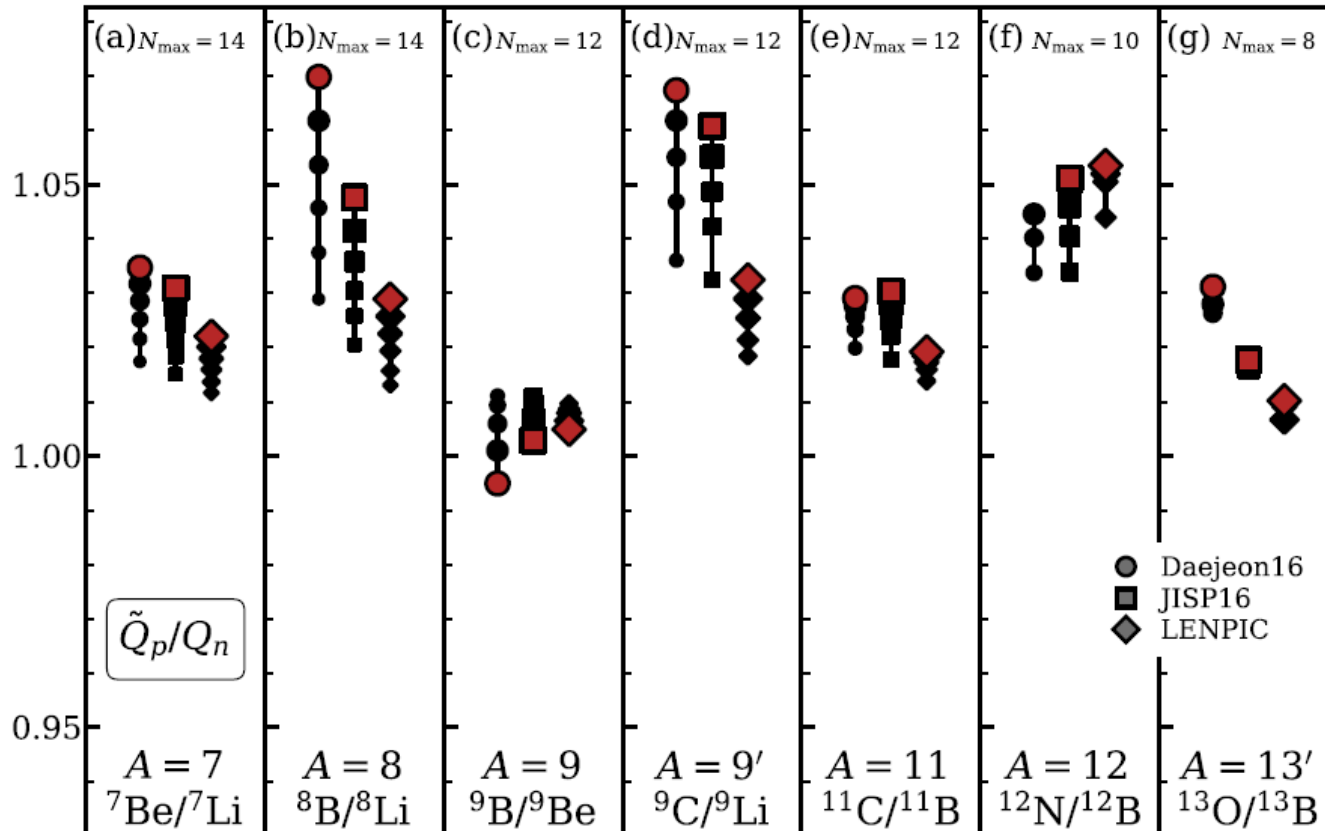
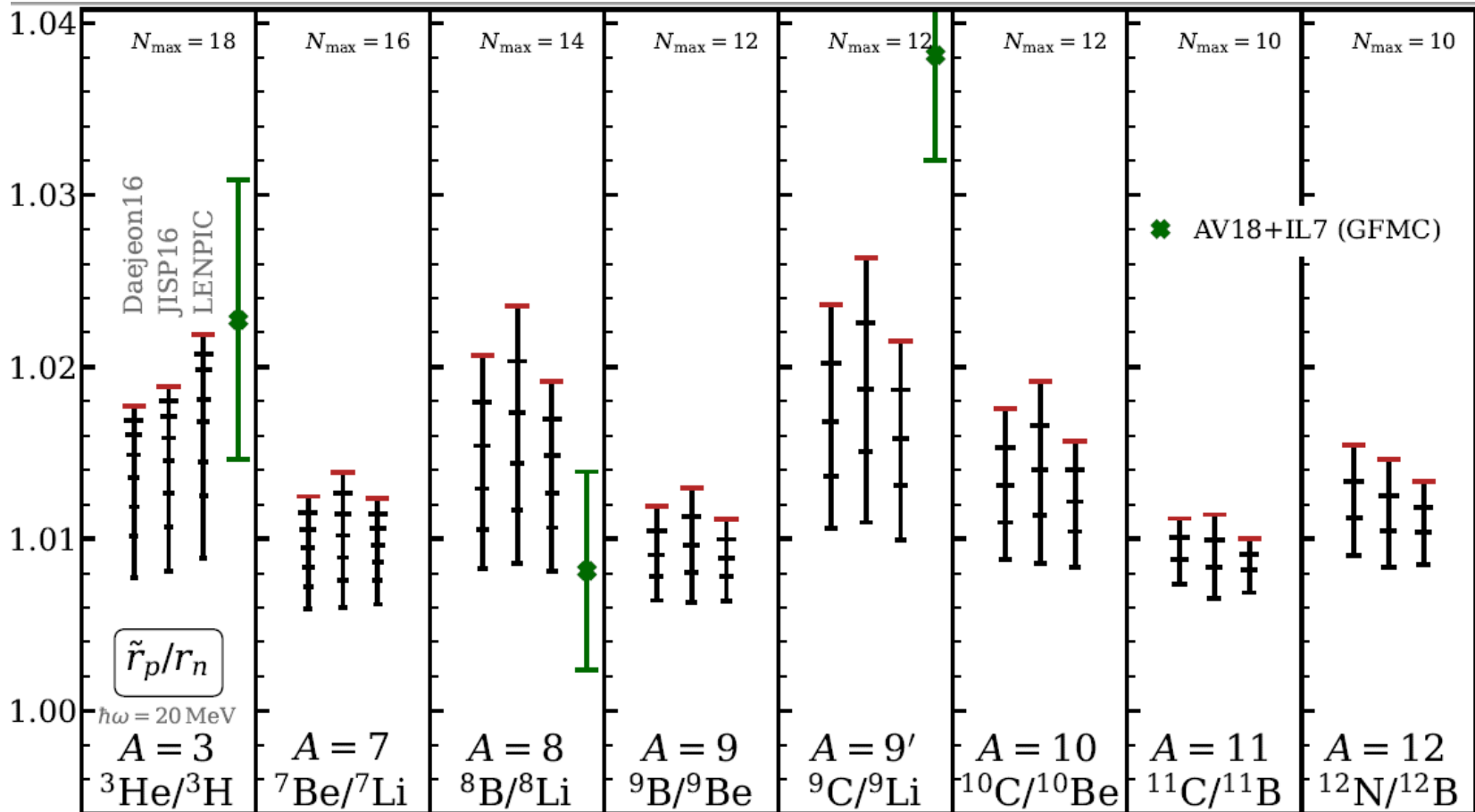


FIG. 7. Ratio of Q_p in one member of mirror pair to Q_n in the other, calculated with the Daejeon16 (circles), JISP16 (squares), and LENPIC (diamonds) interactions at fixed $\hbar\omega$ (15, 20, and 25 MeV, respectively, for the three interactions). Calculated values are shown for successive even values of N_{max} (increasing symbol size), from $N_{\text{max}} = 4$ to the maximum value for that mirror pair, indicated at top.

Similar convergence issue in other ISB-observables:

2. RMS radii in mirror nuclei



Mark Caprio, preliminary results

Perturbative representation of ISB:

Miller and Schwenk, 2008 PRC; 2009 PRC

$$H = H_0 + \textcircled{V} \leftarrow \text{ISB potential}$$

Energy eigenstates: $H|n\rangle = E_n|n\rangle$, $H_0|n\rangle = E_n^0|n\rangle$

Wigner-Brillouin perturbation theory:

$$|n\rangle = \sqrt{Z_n} \left[|n\rangle + \frac{1}{E_n - \Lambda_n H \Lambda_n} \Lambda_n V |n\rangle \right]$$

Normalization factor

Projection operator: $\Lambda_n = 1 - |n\rangle\langle n|$

Perturbative expansion makes underlying physics more transparent!

Correction to M_F starts from **second order** in ISB interaction:

Behrends and Sirlin, 1960 PRL; Ademollo and Gatto, 1964 PRL

$$\delta_C \approx -\frac{d}{dz} \left\{ \left[(i|VG(z)V|i) + (f|VG(z)V|f) - \frac{2}{M_F^0} (f|VG(z)\tau_+V|i) \right] - g.s. \right\}_{z=E_g^0}$$

ground-state contribution

Depends on **off-diagonal** matrix elements of V (unlike mass-splitting)

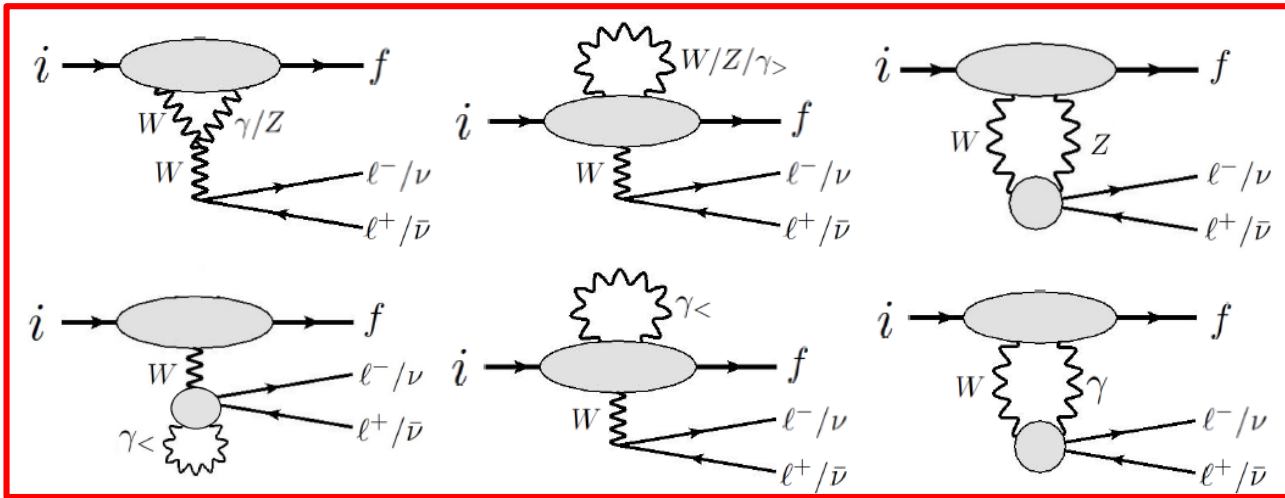
Nuclear Green's function: $G(z) \equiv \frac{1}{z - H_0}$

See Michael Gennari's talk

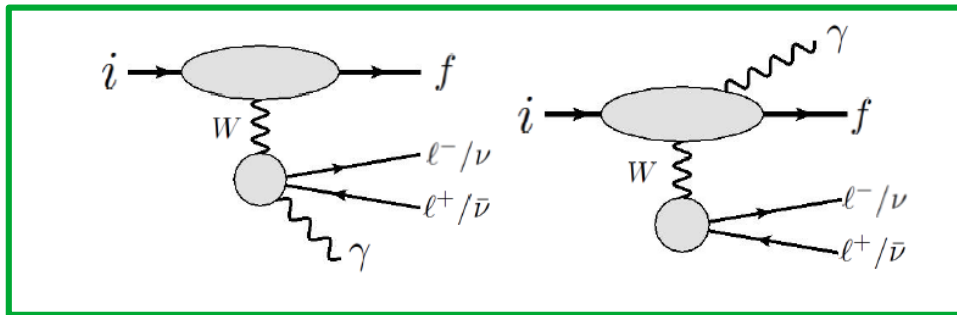
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Radiative Corrections



Loop corrections



Bremsstrahlung corrections

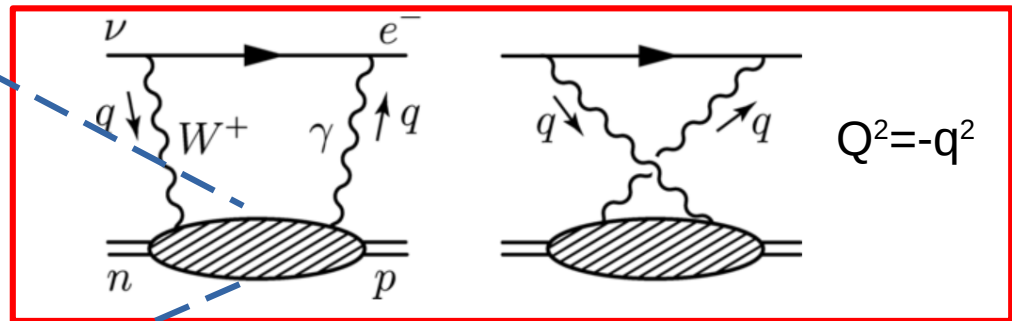
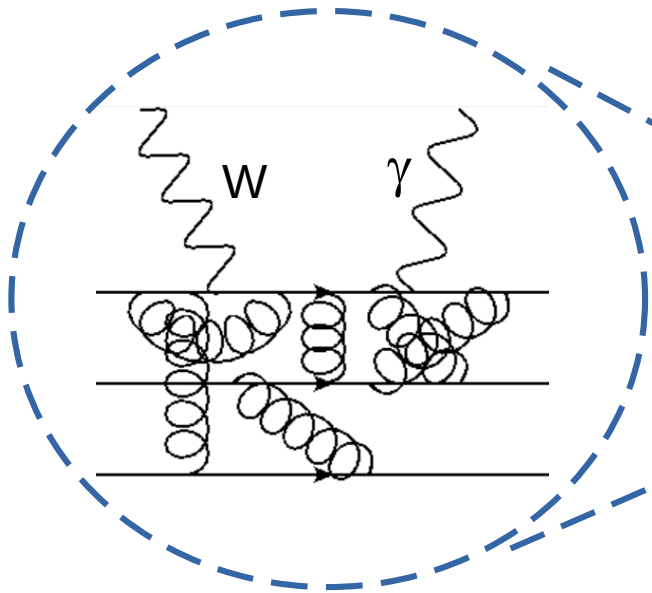
$$\text{Full R.C.} = \Delta_R^V + \delta_{\text{NS}}$$

Nucleus-independent

Nucleus-dependent

Nucleus-independent RC

Single-nucleon γW -box diagram

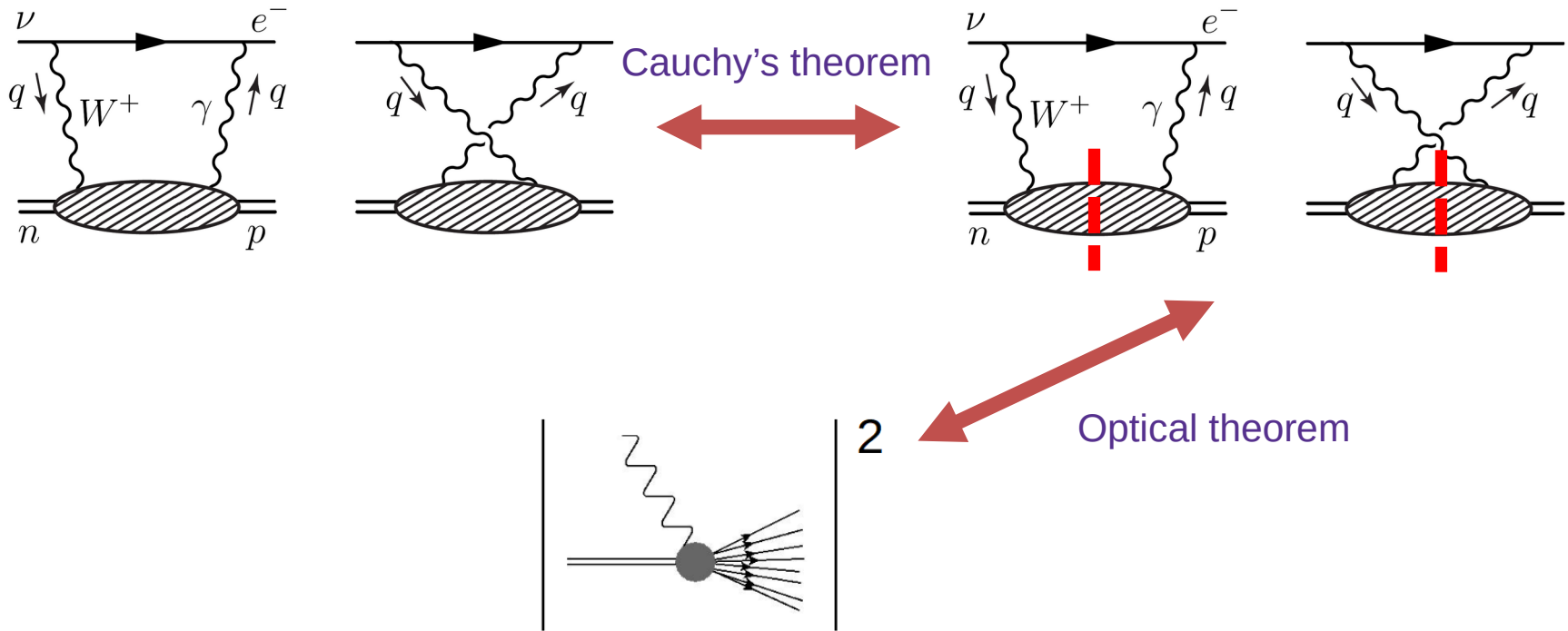


Large uncertainty due to **non-perturbative QCD** at $Q^2 \sim 1 \text{ GeV}^2$.

Methods to evaluate:

1. Dispersion relation (DR) --- relate the loop integral to experimentally-measurable **structure functions**

*CYS, Gorchtein, Patel and Ramsey-Musolf, 2018 PRL;
Shiells, Blunden and Melnitchouk, 2021 PRD*

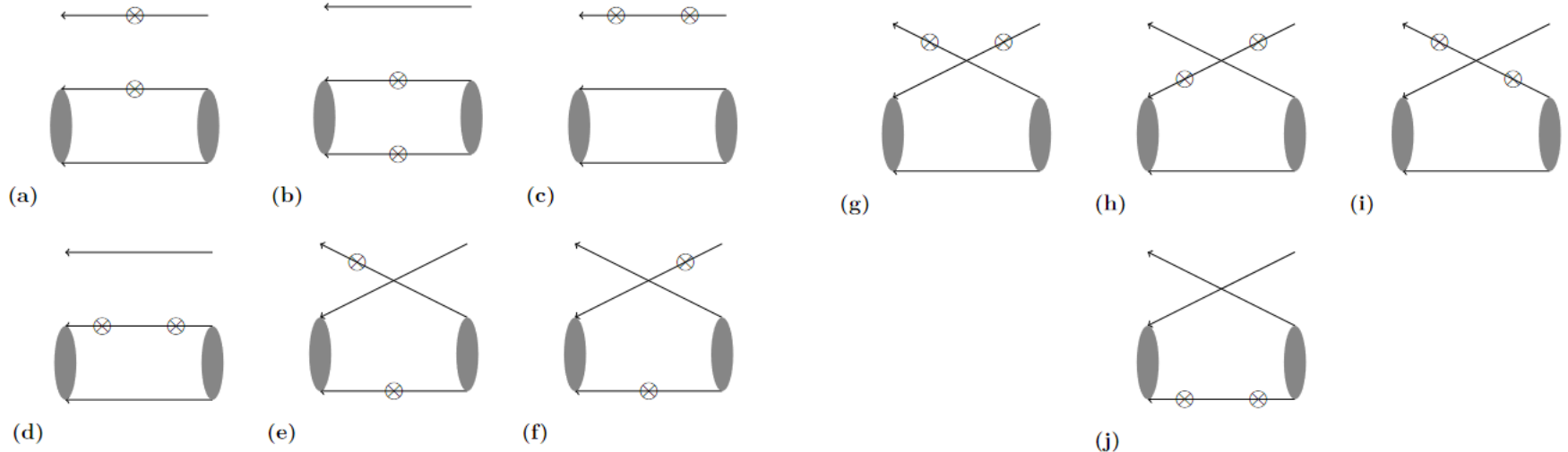
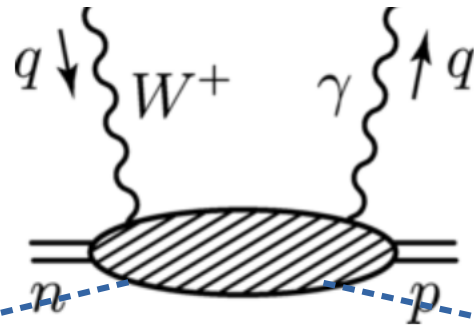


Data: $\nu(\bar{\nu})p \rightarrow e^\pm X$ structure functions

Methods to evaluate:

2. Lattice QCD --- Compute the shaded blob directly

Feng, Gorchtein, Jin, Ma and CYS, 2020 PRL; Ma et al., 2308.16755

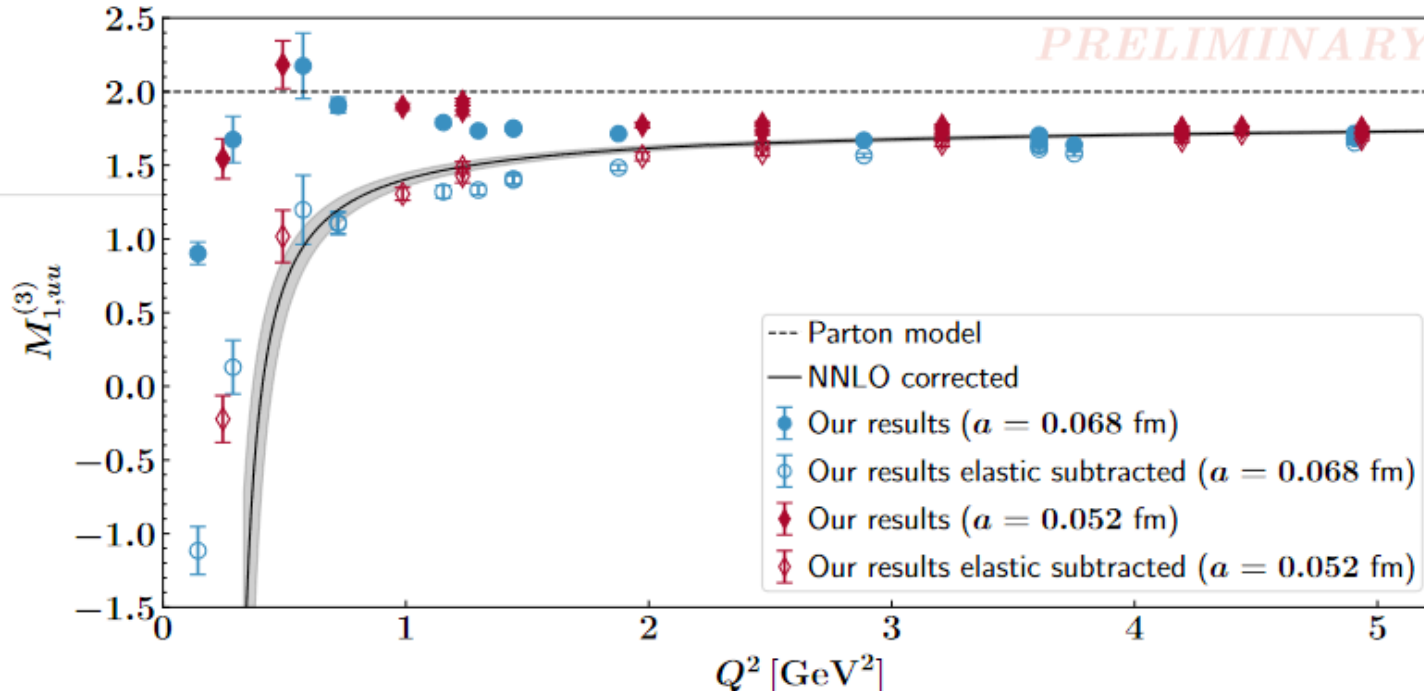


Methods to evaluate:

2. Lattice QCD --- Structure function from Feynman-Hellmann theorem *Can et al., 2402.00255*

$$S(\lambda) = S_0 + \lambda_1 \int d^4z \cos(q \cdot z) \mathcal{J}_\mu(z) + \lambda_2 \int d^4y \sin(q \cdot y) \mathcal{J}_\nu^A(y),$$

$$\frac{\mathcal{F}_3(\omega, Q^2)}{\omega} = \frac{Q^2}{\mathbf{q}_2} \left. \frac{\partial^2 E_{N_\lambda}(\mathbf{p})}{\partial \lambda_1 \partial \lambda_2} \right|_{\lambda=0}$$



*Kadir Utku Can,
INT Workshop
INT-24-87W*

Different evaluations of the nucleus-independent RC:

Pre-2018

Method	Δ_R^V
Phenomenological	0.02361(38)
DR with neutrino data (1)	0.02467(22)
DR with neutrino data (2)	0.02471(18)
DR with indirect lattice data	0.02477(24)
Non-DR (1)	0.02426(32)
Non-DR (2)	0.02473(27)
Lattice	0.02439(19)

Marciano and Sirlin, 2006 PRL

CYS, Gorchtein, Patel and Ramsey-Musolf, 2018 PRL

Shiells, Blunden and Melnitchouk, 2021 PRD

CYS, Feng, Gorchtein and Jin, 2020 PRD

Czarnecki, Marciano and Sirlin, 2019 PRD

Hayen, 2021 PRD

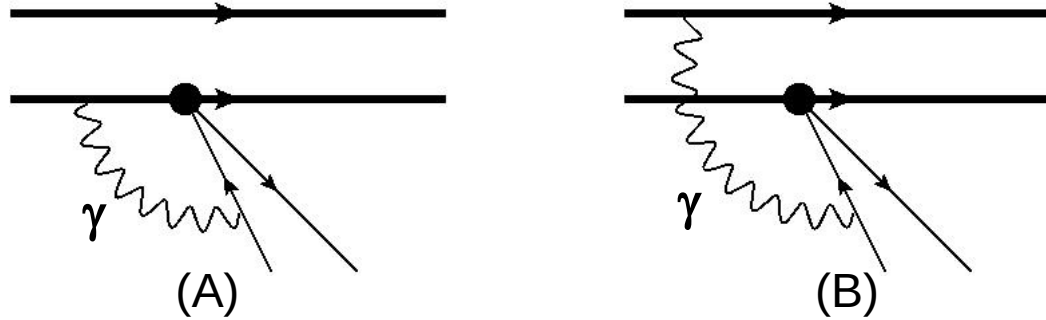
Ma et al., 2308.16755

Post-2018

$$\Delta_R^V \uparrow \implies |V_{ud}| \downarrow$$

Nucleus-dependent inner RC, δ_{NS}

Classical viewpoint:



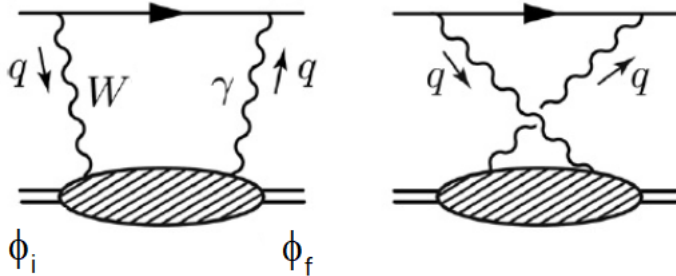
Type A: Nuclear medium effect, “quenched” couplings
Type B: Two-nucleon effects

Both computed with NR nuclear models!

*Jaus and Rasche, 1990 PRD;
Barker et al., 1992 NPA;
Towner, 1992 NPA, 1994 PLB*

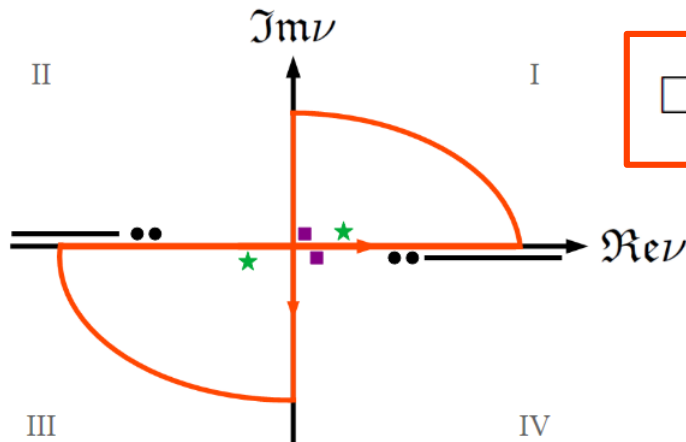
Modern viewpoint:

δ_{NS} in terms of **the difference between the nuclear and nucleon γW -box diagram**



$$\square_{\gamma W}^{\text{nucl.}} = \square_{\gamma W}^n + \underbrace{[\square_{\gamma W}^{\text{nucl.}} - \square_{\gamma W}^n]}_{\sim 1/2 \delta_{NS}}$$

Wick-rotation of the loop integral:

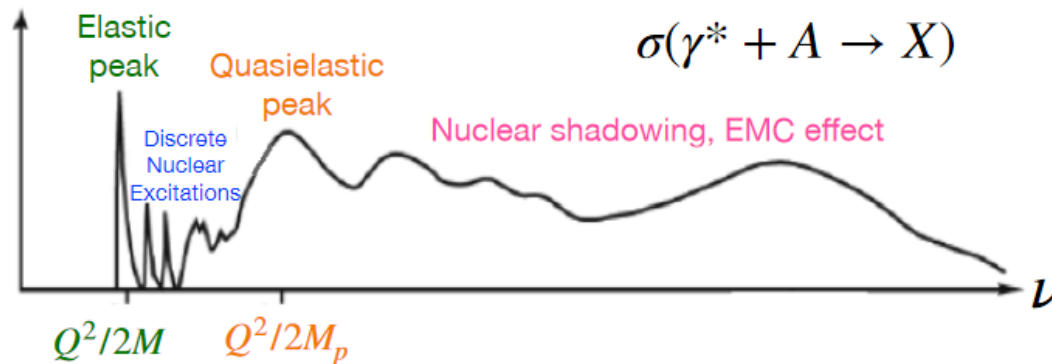
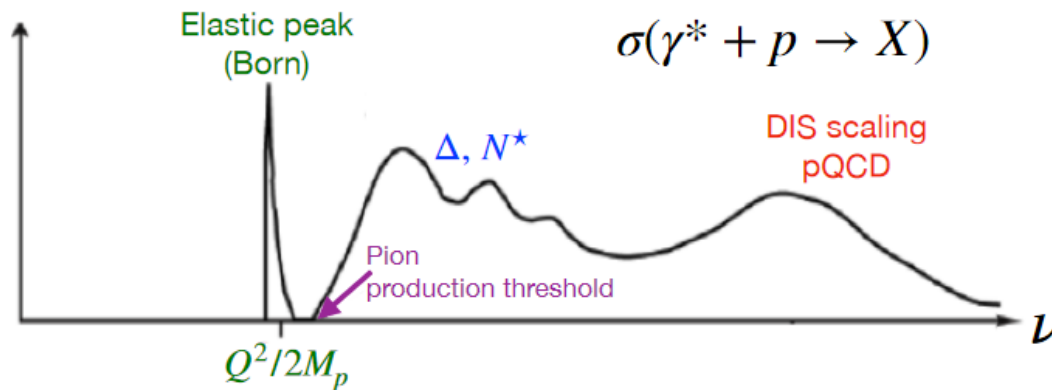


$$\square_{\gamma W}^{\text{nucl.}} = \underbrace{(\square_{\gamma W}^{\text{nucl.}})_{\text{Wick}}}_{\text{Analytic to } E_e} + \underbrace{(\square_{\gamma W}^{\text{nucl.}})_{\text{res},e}}_{\text{Non-analytic to } E_e} + \underbrace{(\square_{\gamma W}^{\text{nucl.}})_{\text{res},\text{nucl}}}_{\text{Non-analytic to } E_e}$$

The first two terms probe the **nuclear response function**:

$$R^{xy}(\nu, Q^2) \sim \sum_X \delta(M + q_0 - E_X) \langle f | J_{\text{em}}^x | X \rangle \langle X | J_W^y | i \rangle$$

See Francesca Bonaiti's talk

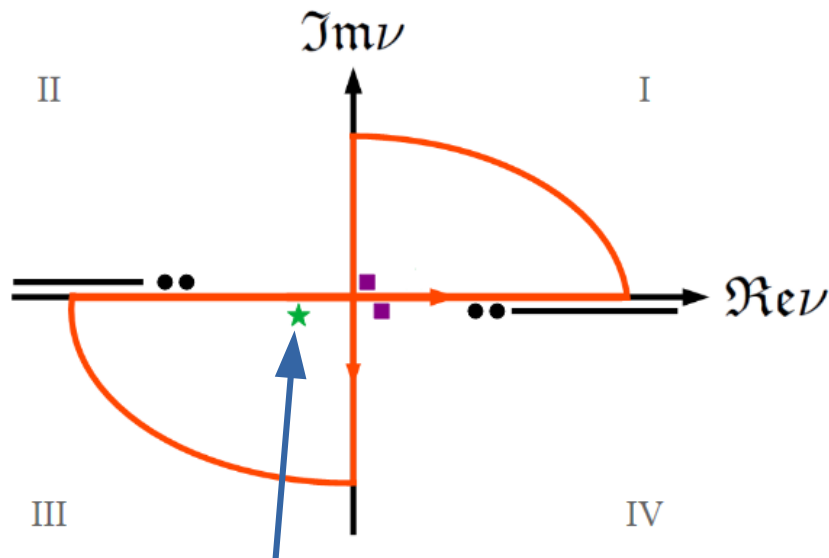
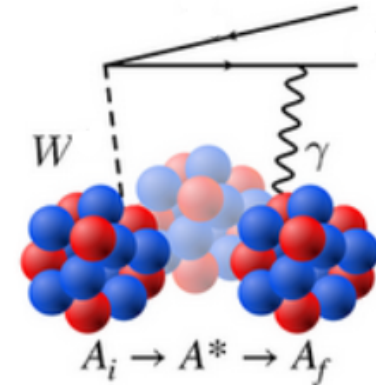
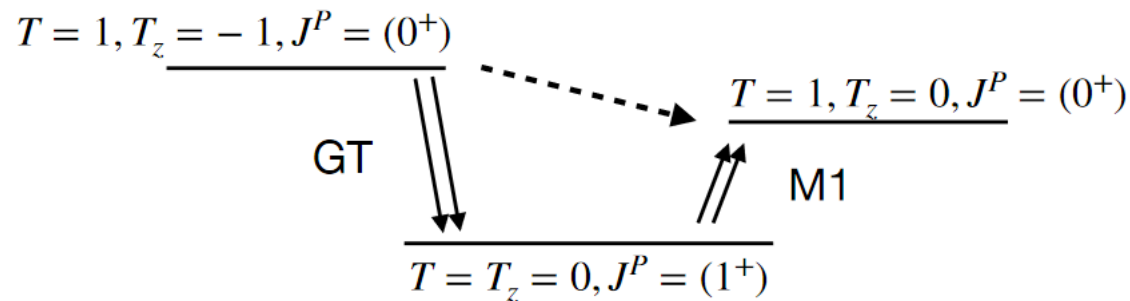


Can be studied using ab-initio methods! *See Michael Gennari's talk*

Residue contribution

Mehdi Drissi, Michael Gennari, Petr Navratil

If the daughter nucleus is an **excited state**:



A **residue contribution** from pole in 3rd quadrant

Residue due to a low-lying $J^P=1^+$ state:

$$\Re s T_3 \propto \underbrace{\langle f(0^+) | J_{\text{em}}^x(\vec{q}) | f(1^+) \rangle}_{\text{M1-transition}} \underbrace{\langle f(1^+) | J_W^y(-\vec{q}) | i(0^+) \rangle}_{\text{GT-transition}}$$

Matrix elements can be inferred from **M1** and **GT**-transition rates!

	GT, $\log_{10} ft$ (s)	M1, $t_{1/2}$
$^{10}\text{C} \rightarrow ^{10}\text{B}$	3.0426(7)	4.9(2.1) fs
$^{14}\text{O} \rightarrow ^{14}\text{N}$	7.279(8)	68(3) fs
$^{18}\text{Ne} \rightarrow ^{18}\text{F}$	3.091(4)	1.77(31) fs
$^{22}\text{Mg} \rightarrow ^{22}\text{Na}$	3.64	19.6(7) ps
$^{30}\text{S} \rightarrow ^{30}\text{P}$	4.322(11)	96(10) fs

From IAEA / NNDC website

Motivations for future experiments!

Ab-initio calculation? See Thomas Papenbrock's talk

Outline

1. Precision frontier and the CKM unitarity
2. V_{ud} from beta decays
3. Inputs at tree-level
 - Weak decay form factors
 - Isospin-breaking correction
4. Inputs at loop-level
 - Nucleus-independent radiative corrections
 - Nucleus-dependent radiative corrections
5. **Summary and outlook**

Current quoted value:

$$|V_{ud}|_{0^+} = 0.97361(5)_{\text{exp}}(6)_{\delta'_R}(4)_{\delta_C}(28)_{\delta_{\text{NS}}}(10)_{\text{RC}}$$

Gorchtein and CYS, 2311.00044

Included:

- (Averaged) new calculations of Δ_R^V
- Preliminary re-evaluation of δ_{NS}

NOT Included:

- Fully data-driven determination of ft -values
- Ab-initio calculation of δ_C and δ_{NS}

Result is preliminary, use with care!!

Valuable inputs in the future:

Experiment:

- Nuclear charge radii
- Nuclear M1-transition rates
- Nucleon/nuclear structure functions
- ...

Lattice QCD:

- Independent evaluation of nucleon structure functions
- ...

Ab-initio calculation:

- Controlled studies of δ_C and δ_{NS}
- ...