Diagrammatic Monte Carlo for atomic nuclei

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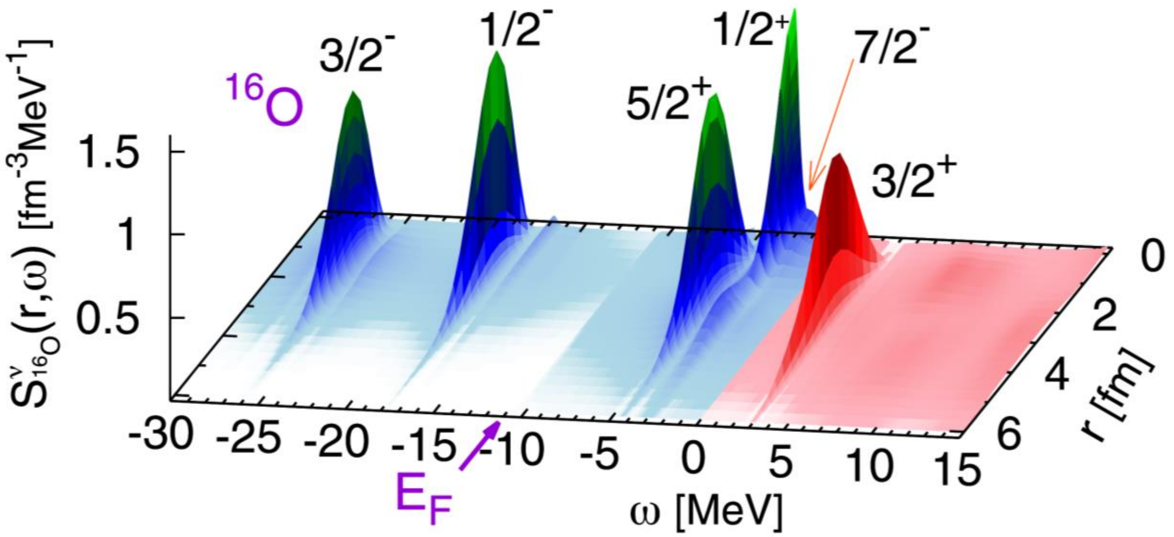
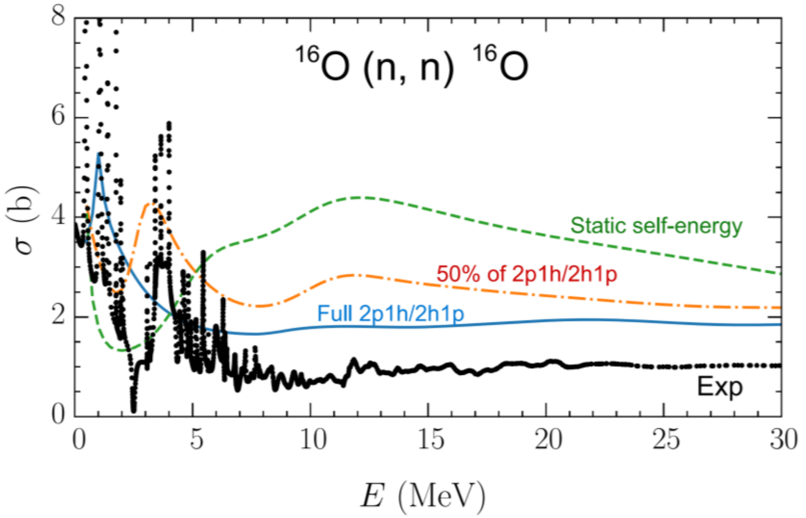
The Green's function

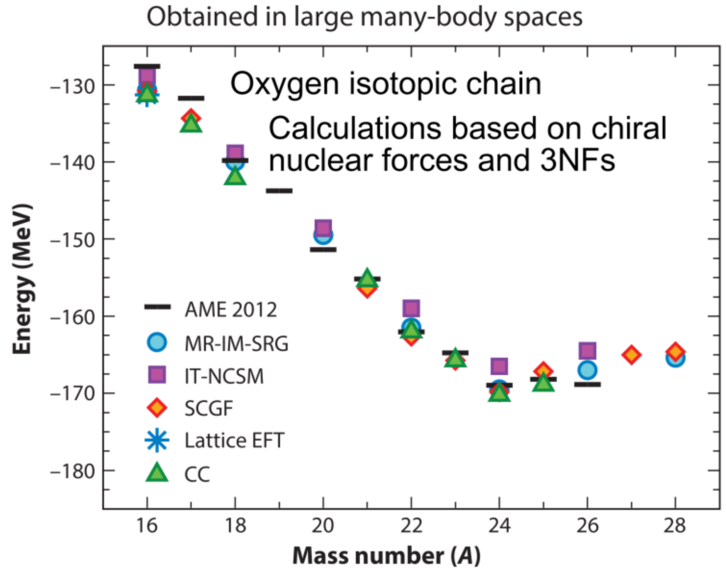
- Ground state energy
- Charge and matter density and radii
- Spectral function
- ***Reactions (optical potential)***



Source: Cipollone et al., 2015: *Phys. Rev. C*, 92, 014306



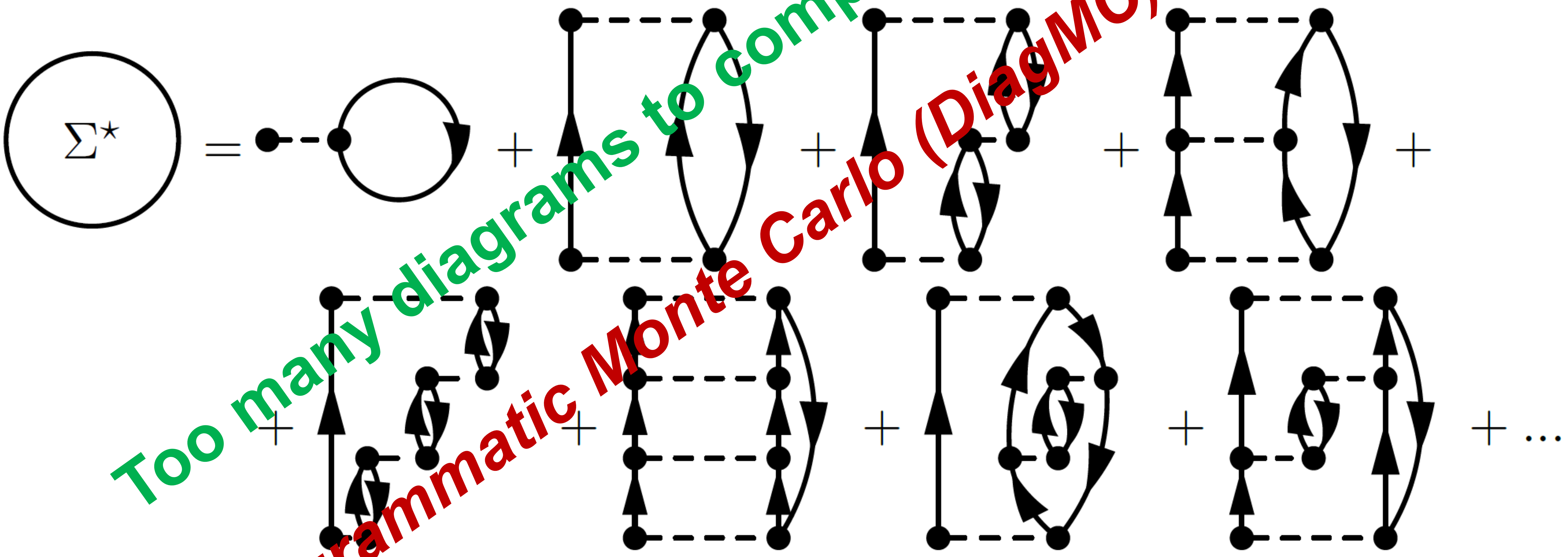
Source: Idini et al., 2019: *Phys. Rev. Lett.*, 123, 092501



Source: Hebeler et al., 2015: *Annu. Rev. Nucl. Part. Sci.*, 65:457-84

The Dyson equation

$$G_{\alpha\beta}(\omega) = G^{(0)}_{\alpha\beta}(\omega) + G^{(0)}_{\alpha\gamma}(\omega)\Sigma^{\star}_{\gamma\delta}(\omega)G_{\delta\beta}(\omega)$$



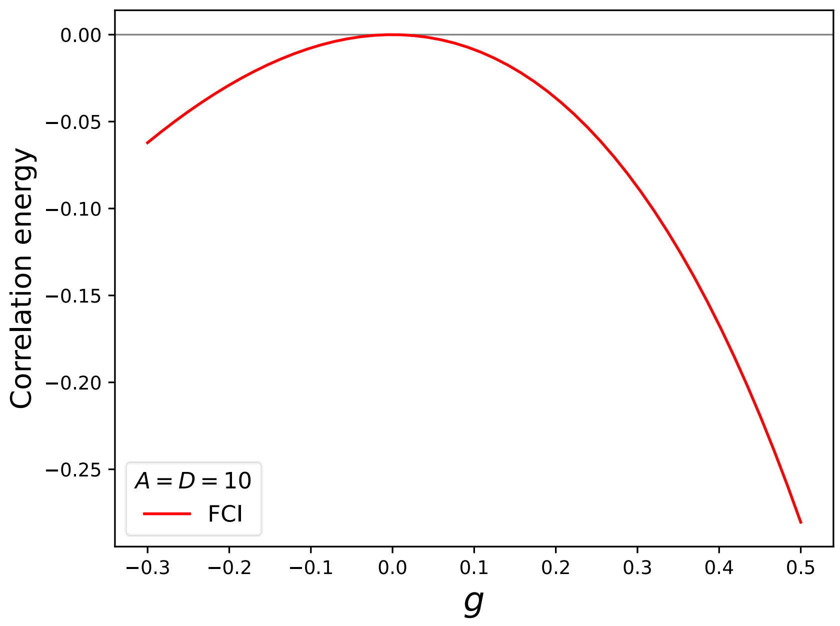
Too many diagrams to compute directly

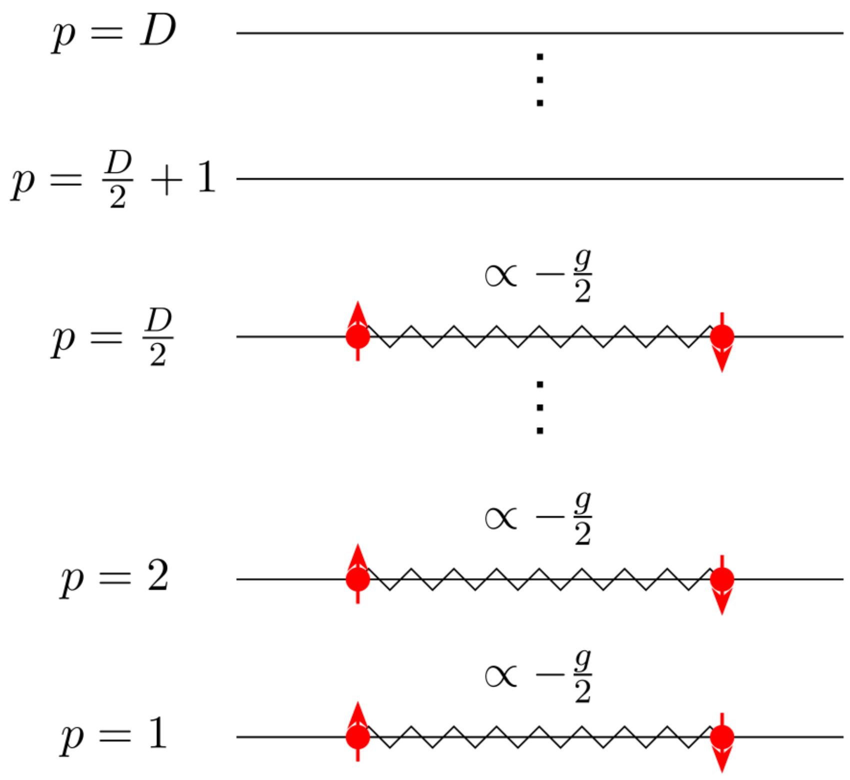
Diagrammatic Monte Carlo (DiagMC) sampling

The Richardson pairing model

$$H = \sum_{p=1}^{D} \sum_{s=\uparrow,\downarrow} (p-1) c_{ps}^{\dagger} c_{ps} - \frac{g}{2} \sum_{p,q=1}^{D} c_{p\uparrow}^{\dagger} c_{p\downarrow}^{\dagger} c_{q\downarrow} c_{q\uparrow}$$

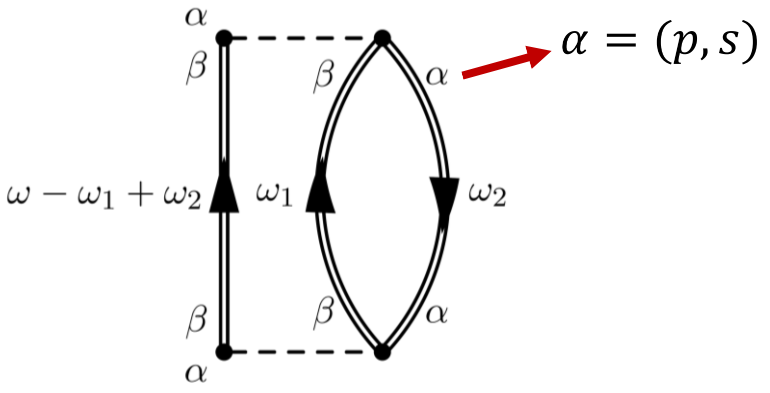
- $D/2$ pairs of particles.
- Simple but challenging due to a pure pairing interaction.
- Exactly solvable: used to benchmark many-body methods[1].





1. M. Hjorth-Jensen et al., 2017, An Advanced Course in Computational Nuclear Physics, Springer

Sampling Feynman diagrams



$$C = (\mathrm{T};\ \gamma_1, \ldots, \gamma_n;\ \omega_1, \ldots, \omega_m)$$

Topology (T)

Internal single-particle quantum numbers ($\gamma_1, \ldots, \gamma_n$)

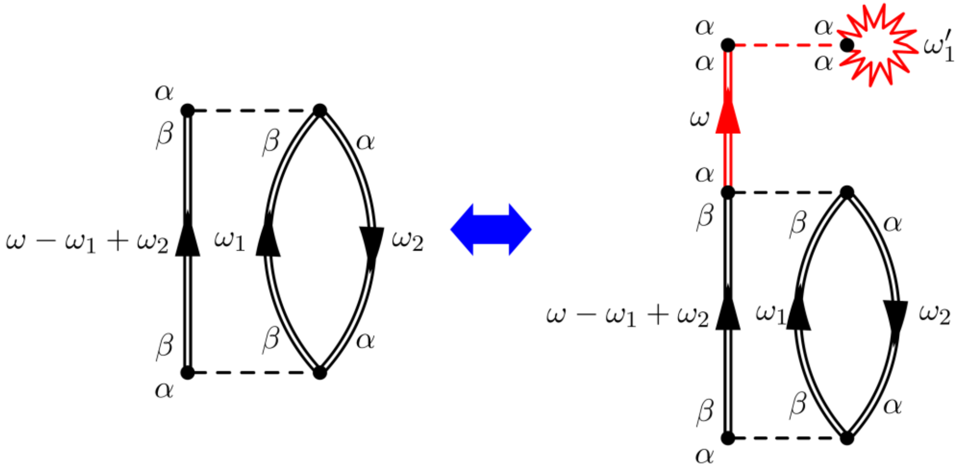
Internal frequencies ($\omega_1, \ldots, \omega_m$)

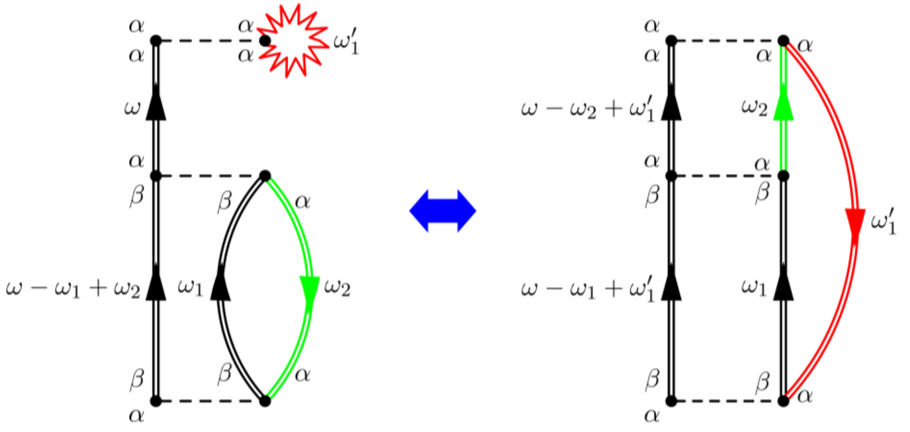
Sum only skeleton topologies

$$\Sigma^{\star}_{\alpha\alpha}(\omega) = \int dC\, D^{\omega}_{\alpha\alpha}(C) 1_{\mathrm{T}\in S_{\Sigma^\star}} = Z^{\omega}_{\alpha\alpha} \int dC\, \frac{|D^{\omega}_{\alpha\alpha}(C)|}{Z^{\omega}_{\alpha\alpha}} e^{i \arg[D^{\omega}_{\alpha\alpha}(C)]} 1_{\mathrm{T}\in S_{\Sigma^\star}}$$

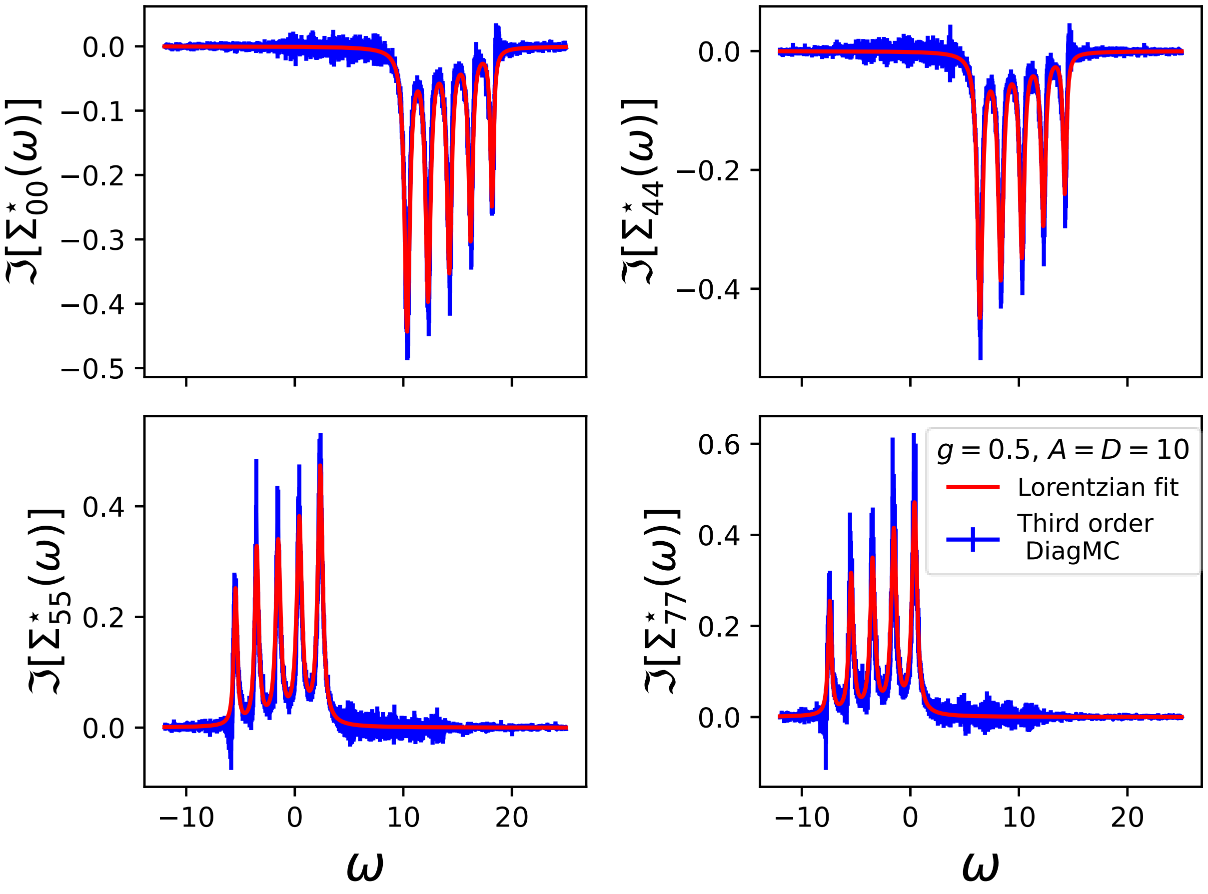
$$= Z^{\omega}_{\alpha\alpha} \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} e^{i \arg[D^{\omega}_{\alpha\alpha}(C_i)]} 1_{\mathrm{T}_i \in S_{\Sigma^\star}}$$

The self-energy of the Richardson model is diagonal

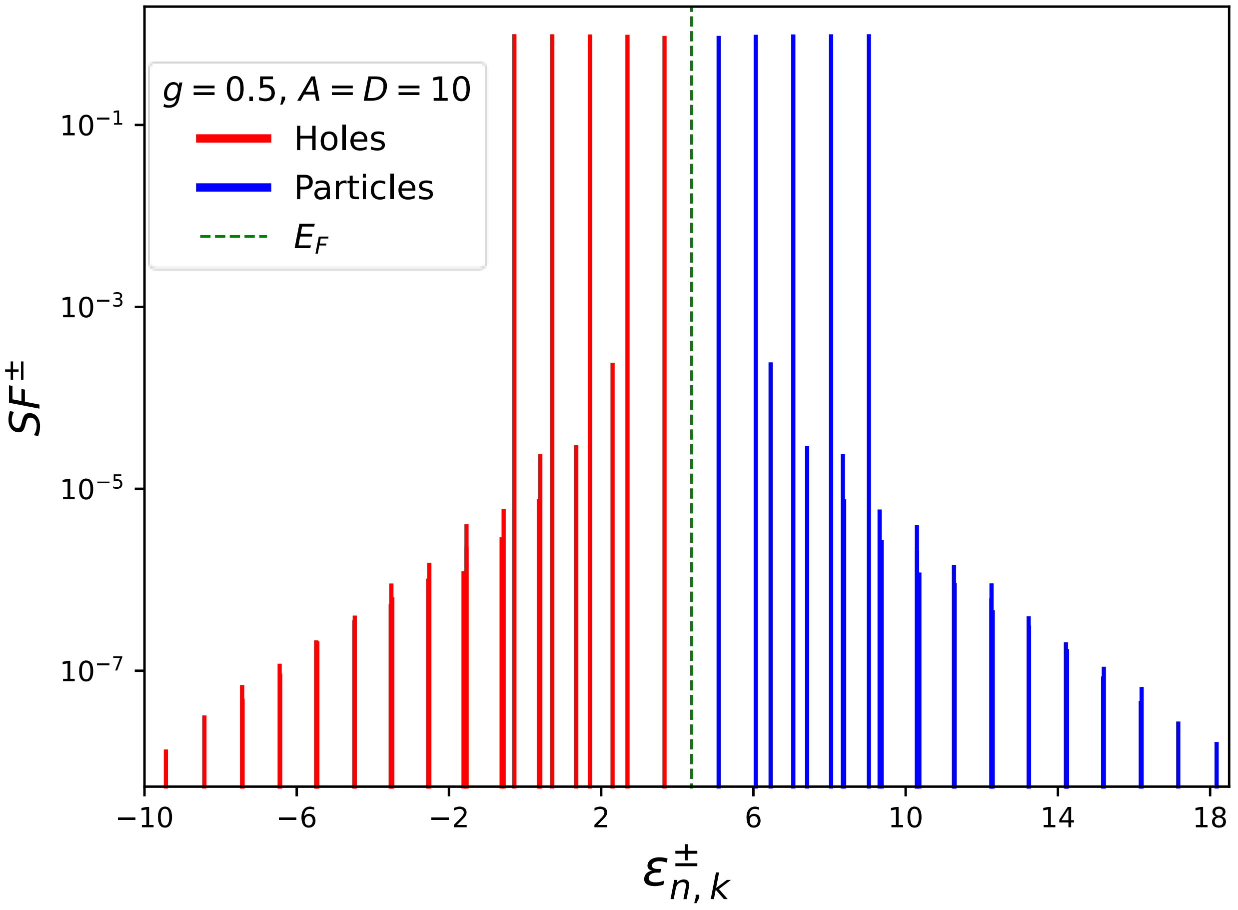




DiagMC results

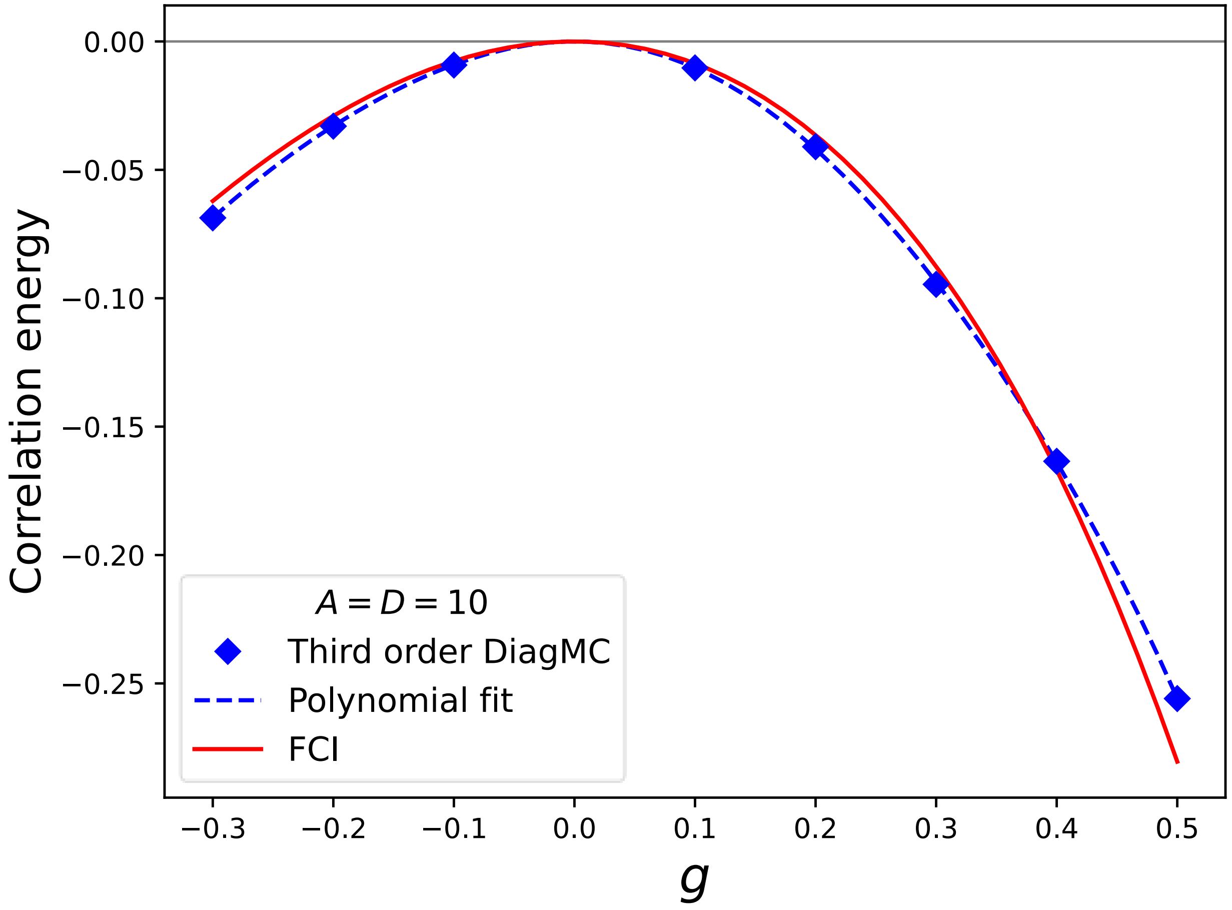


Imaginary part of components p = $0, 4, 5, 7$ of the diagonal self-energy $\Sigma^\star_{p\uparrow p\uparrow}$ at third order for $D = A = 10$ and $g = 0.5$.

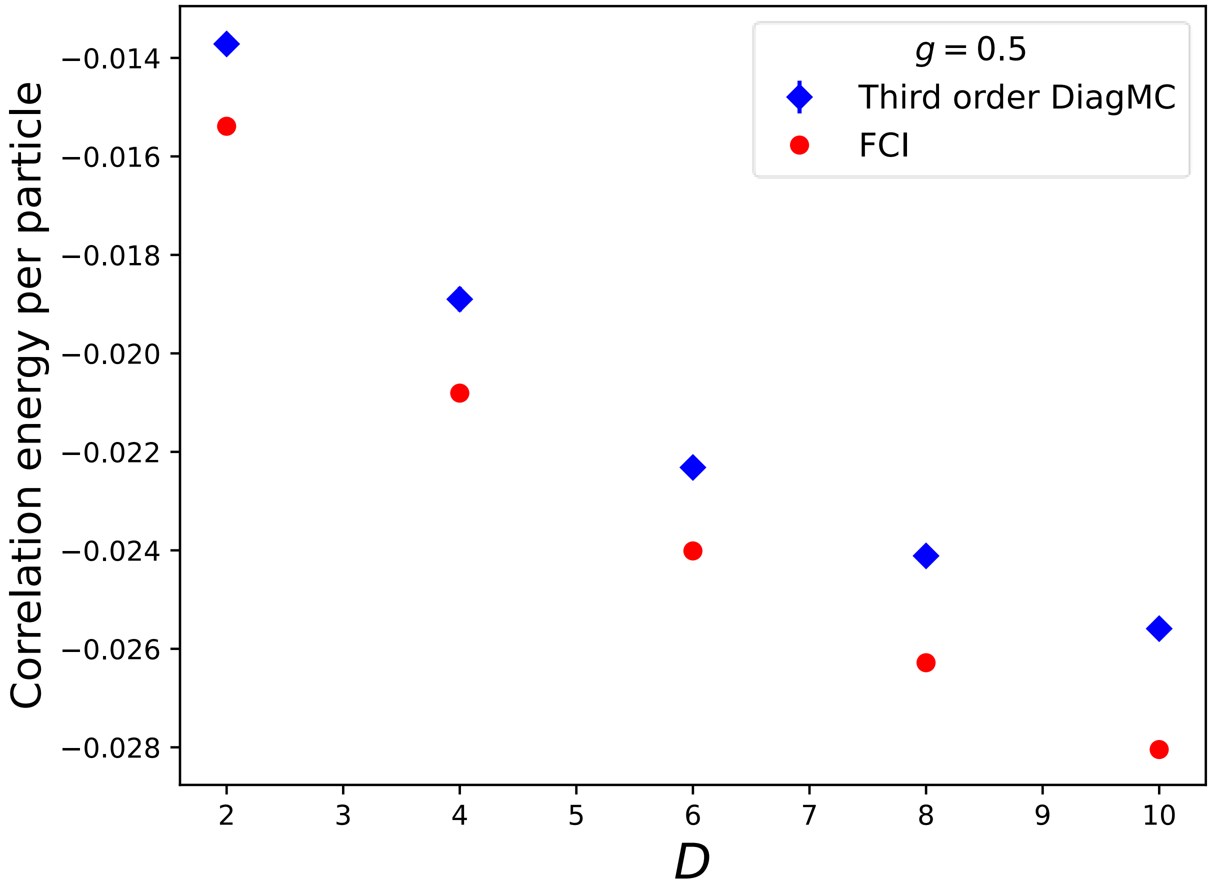


Spectral function at third order for $D = A = 10$ and $g = 0.5$.

Correlation energies

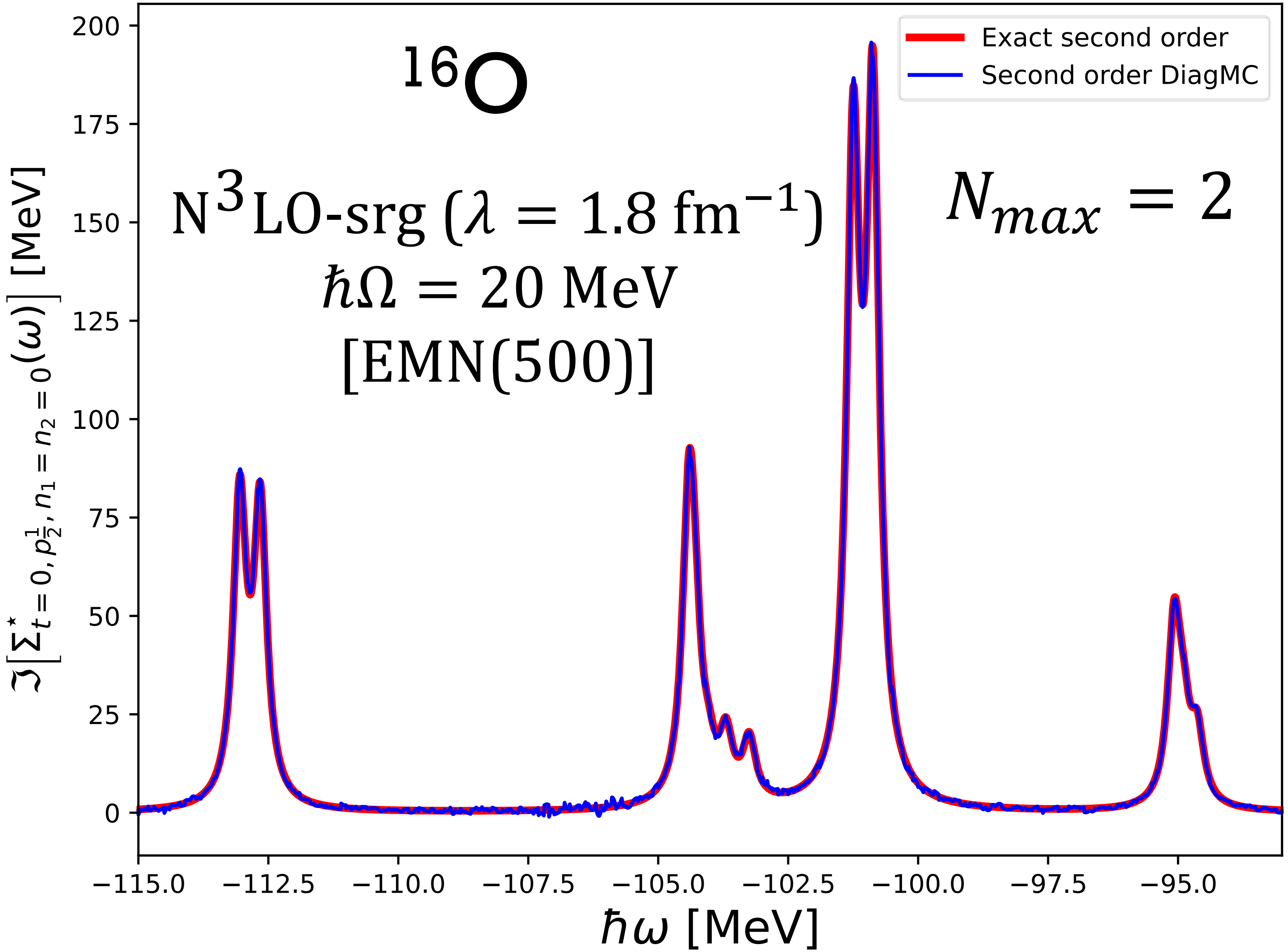


Correlation energy for $D = A = 10$ as a function of the coupling g.



Correlation energy per particle for $g = 0.5$ as a function of the model dimension D ($A = D$).

Nuclei in no-core model spaces



Imaginary part of the neutron $p\frac{1}{2}$ hole self-energy in ^{16}O.

Thank you!