



SAN DIEGO STATE  
UNIVERSITY

# Transition Sum Rules in the No-Core Shell Model

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# Strength Functions and Sum Rules

Strength function for a given transition operator

$$S(E_i, E_x) = \sum_f \delta(E_x + E_i - E_f) \left| \langle f | \hat{T} | i \rangle \right|^2$$

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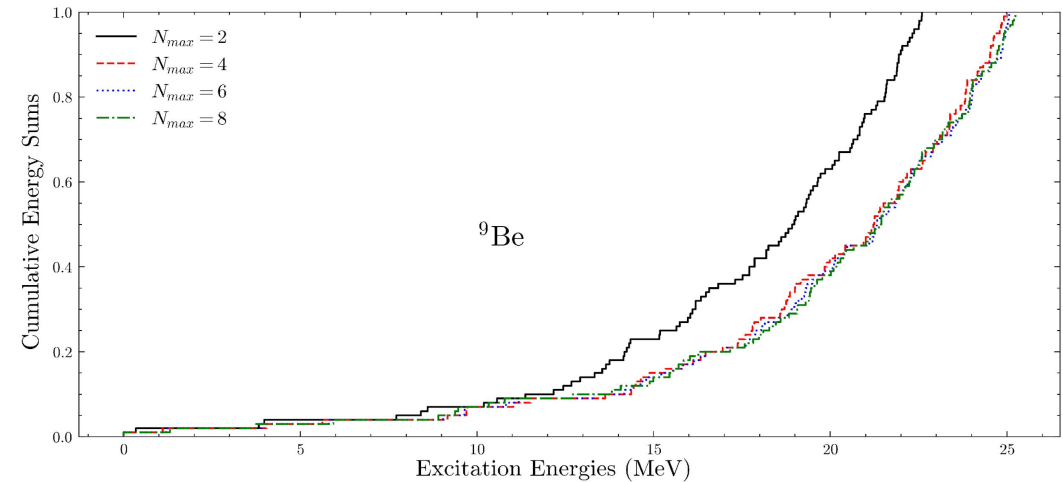
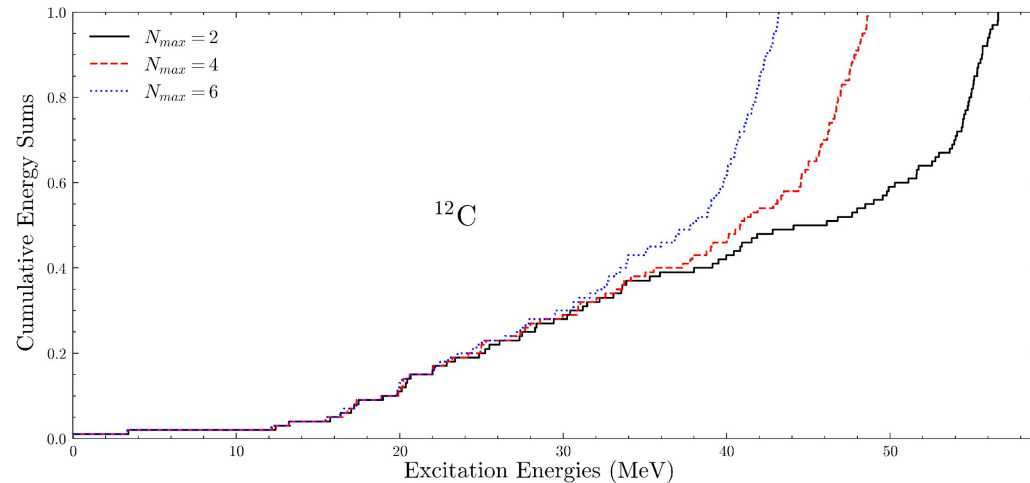
Non-energy weighted (NEWSR) and energy weighted (EWSR) sum rules can be expressed as expectation values of angular momentum scalars

$$S_0(E_i) = \langle i | \hat{T}^\dagger \hat{T} | i \rangle$$

$$S_1(E_i) = \frac{1}{2} \langle i | [[\hat{T}^\dagger, \hat{H}], \hat{T}] | i \rangle$$

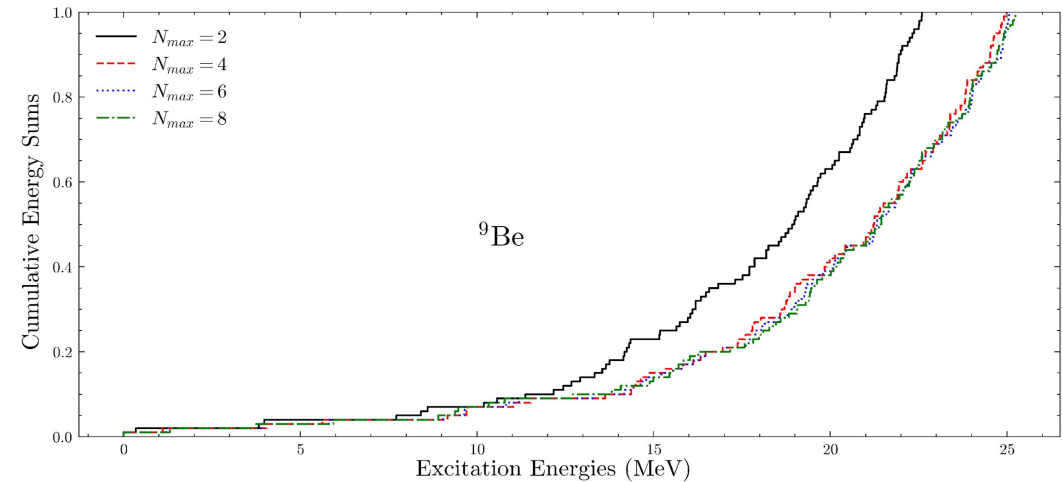
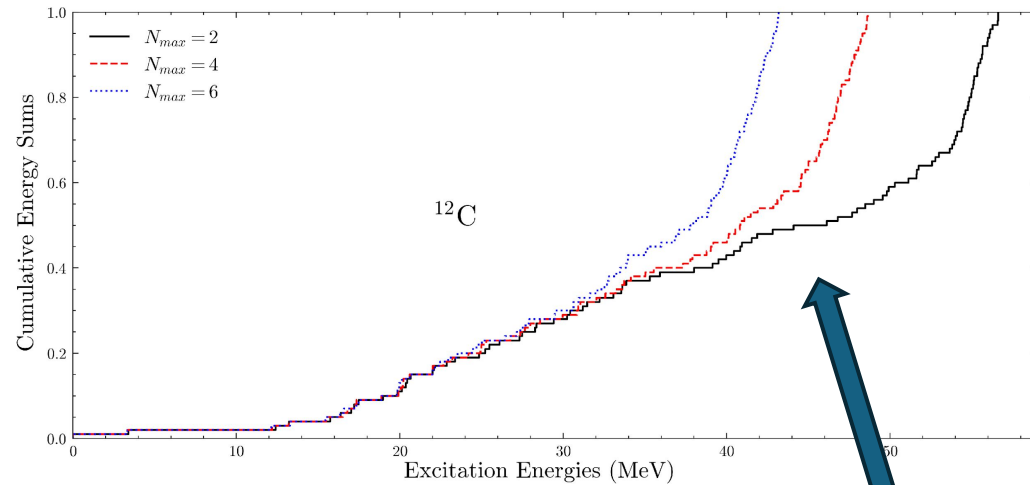
# No-Core Shell Model and Energy Spectra

- We use Entem-Machleidt matrix elements from N3LO Chiral EFT
- We compute 100 or so excited states in each nuclide
  - p-shell nuclei
  - Up to  $N_{\max} = 10$  for Li6;  $N_{\max} = 6$  for C12



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For even A nuclei, we reach the discretized continuum

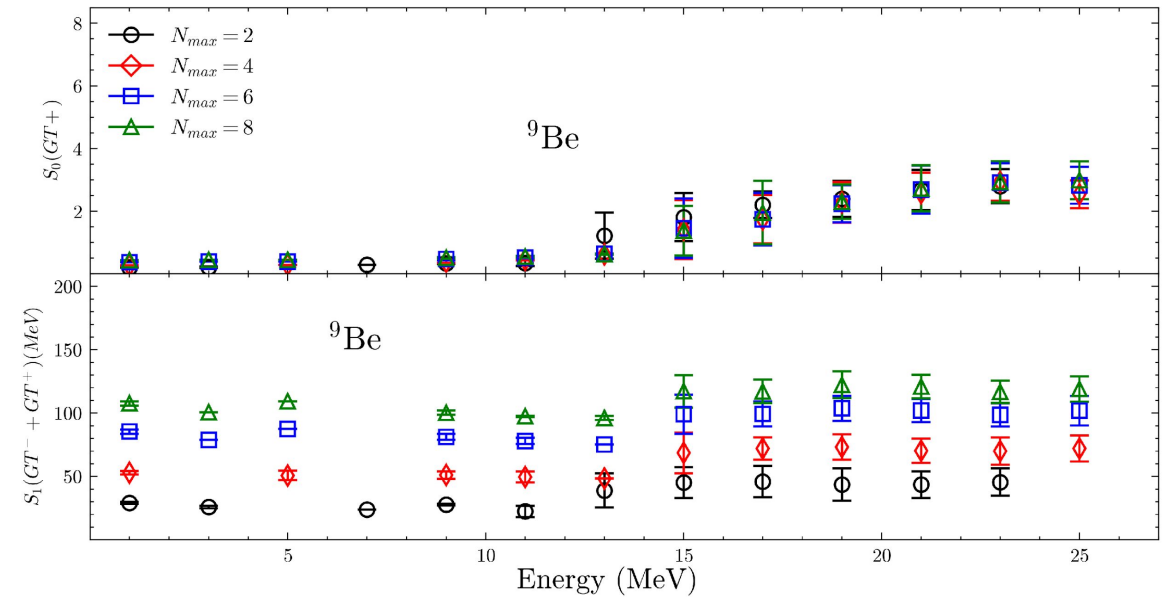
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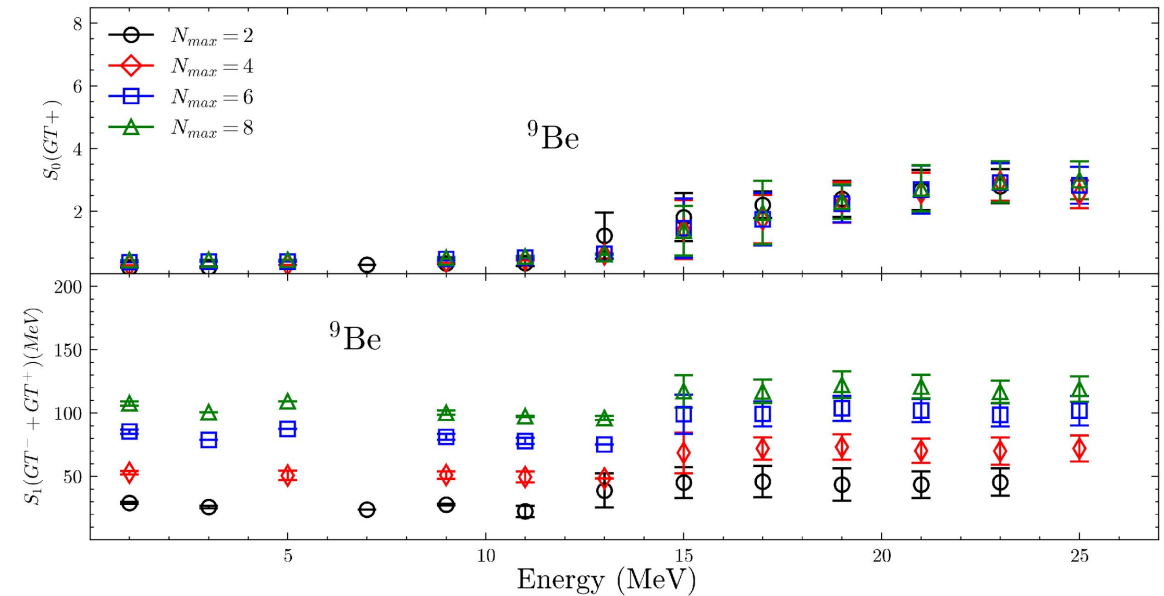
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- Most results don't converge but still can recover good systematic behavior
- We see different sensitivities to  $N_{max}$  truncations between EWSR and NEWSR, also between differences transitions

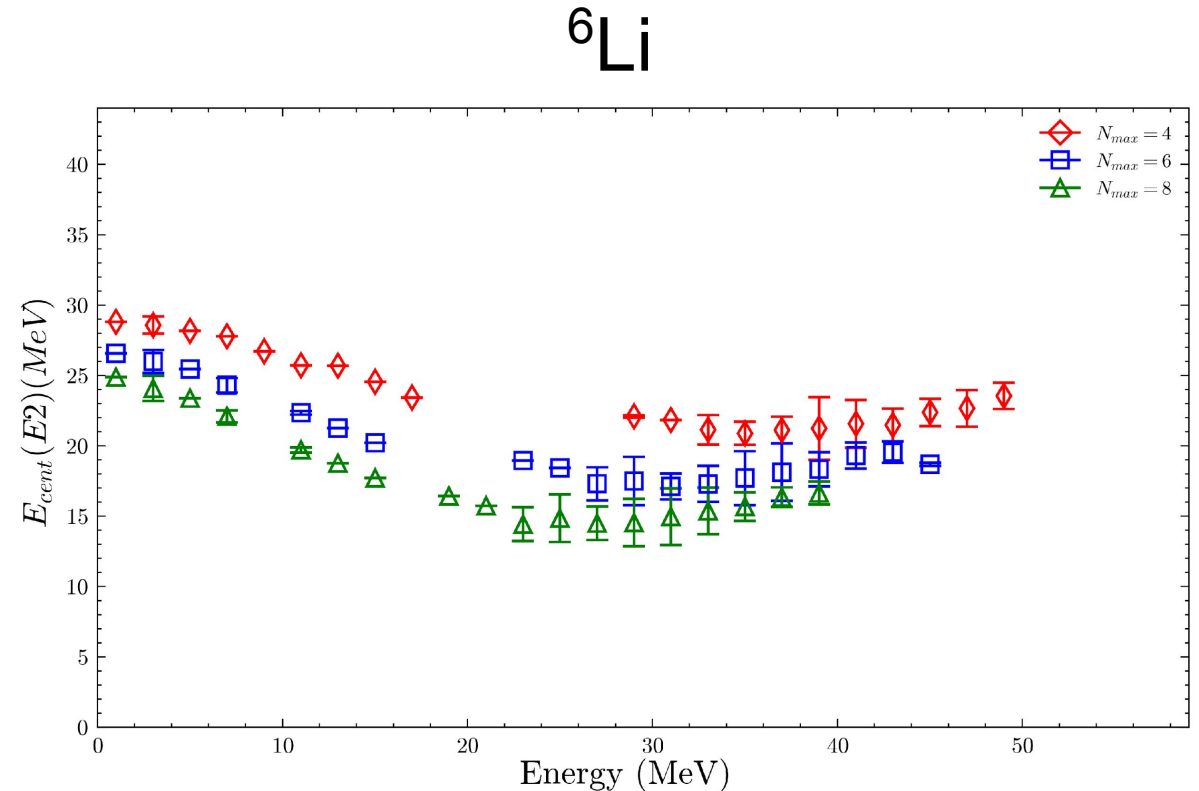
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# Centroids and Brink-Axel Hypothesis

- We can also take the energy centroids

$$E_{cent}(E_i) = S_1(\bar{E}_i)/S_0(E_i)$$

- With NEWSRs and centroids we can compare our results against the general Brink-Axel hypothesis:
  - The assumption that the strength distribution from any parent state is approximately constant, ie. Centroids and NEWSR are independent on initial energy

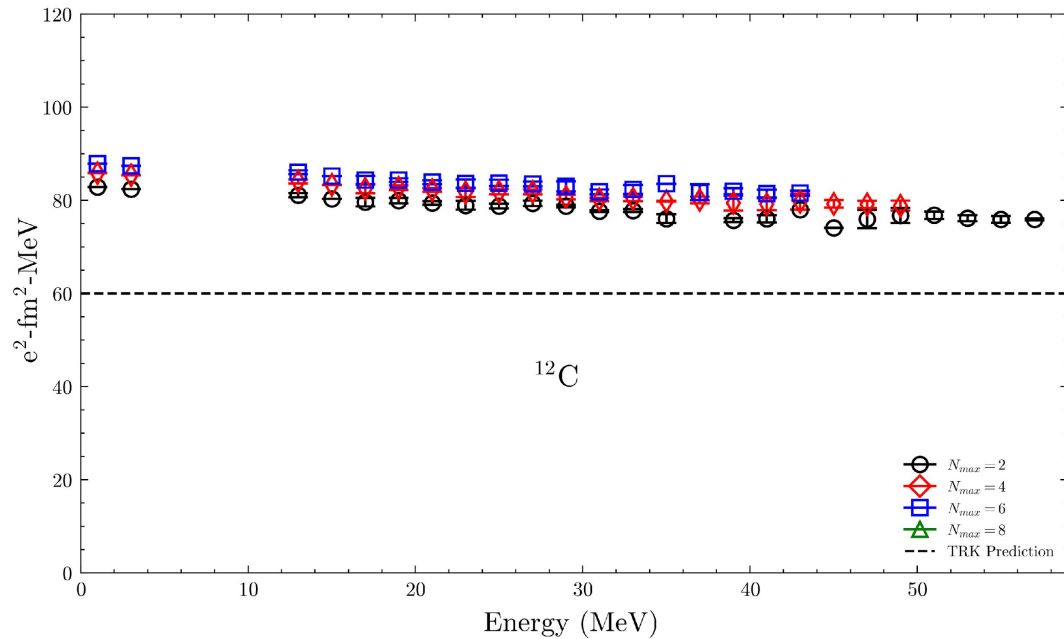


# Thomas-Reiche-Kuhn (TRK) Sum Rule

TRK Prediction for E1

$$S_1(E1) = (NZ/A)e^2\hbar^2/2m_N(1 + \kappa)$$

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- Comparing our E1 results to the bare TRK Sum rule shows a momentum dependence on the interaction ( $\kappa > 0$ )

Thank you!

Any Questions?