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Perturbative computation of neutron-proton scattering observables using χ EFT up to N^3 LO

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Introduction

- Make predictions of nuclear properties from first principles.
 - A model: χ EFT
 - A power counting (PC): Hierarchy of importance among interactions.
 - Data: To constrain unknown interaction strengths (LECs).
- Singular attraction \implies
 1. Keep cutoff finite.
 2. Demand cutoff independence
 \implies need to modify PC.

[A. Nogga et al., Phys. Rev. C 72, \(2005\)](#)

Modified power counting

- Need promoted counterterms in attractive triplet NN partial waves.
- Corrections beyond leading order (LO) need to be treated perturbatively.
- One-pion exchange treated perturbatively for ($l > 1$).

The Long & Yang PC

order	potential	non-perturbative (at LO) channels	purely perturbative channels	cumulative # LECs
LO	$V^{(0)}$	$V_{1\pi}^{(0)} + V_{ct}^{(0)}$	0	4
NLO	$V^{(1)}$	$V_{ct}^{(1)}$	$V_{1\pi}^{(0)}$	6
N ² LO	$V^{(2)}$	$V_{2\pi}^{(2)} + V_{ct}^{(2)}$	0	19
N ³ LO	$V^{(3)}$	$V_{2\pi}^{(3)} + V_{ct}^{(3)}$	$V_{2\pi}^{(2)}$	33

B. Long, C. J. Yang,
[Phys. Rev. C **84**, \(2011\)](#),
[Phys. Rev. C **85**, \(2012\)](#),
[Phys. Rev. C **86**, \(2012\)](#)

B. Long and U. van Kolck,
[Ann. Phys. **323**, \(2008\)](#)

Perturbative amplitudes

- LO amplitude is computed by solving the Lippmann-Schwinger equation:

$$T^{(0)} = V^{(0)} + V^{(0)} G_0^+ T^{(0)}$$

- Sub-leading amplitudes are computed perturbatively:

$$T^{(1)} = \Omega_-^\dagger V^{(1)} \Omega_+,$$

$$T^{(2)} = \Omega_-^\dagger \left(V^{(2)} + V^{(1)} G_1^+ V^{(1)} \right) \Omega_+,$$

$$T^{(3)} = \Omega_-^\dagger \left(V^{(3)} + V^{(2)} G_1^+ V^{(1)} + V^{(1)} G_1^+ V^{(2)} + \right. \\ \left. + V^{(1)} G_1^+ V^{(1)} G_1^+ V^{(1)} \right) \Omega_+$$

$$\Omega_+ = \mathbf{1} + G_0^+ T^{(0)},$$

$$\Omega_-^\dagger = \mathbf{1} + T^{(0)} G_0^+,$$

Outline of our work

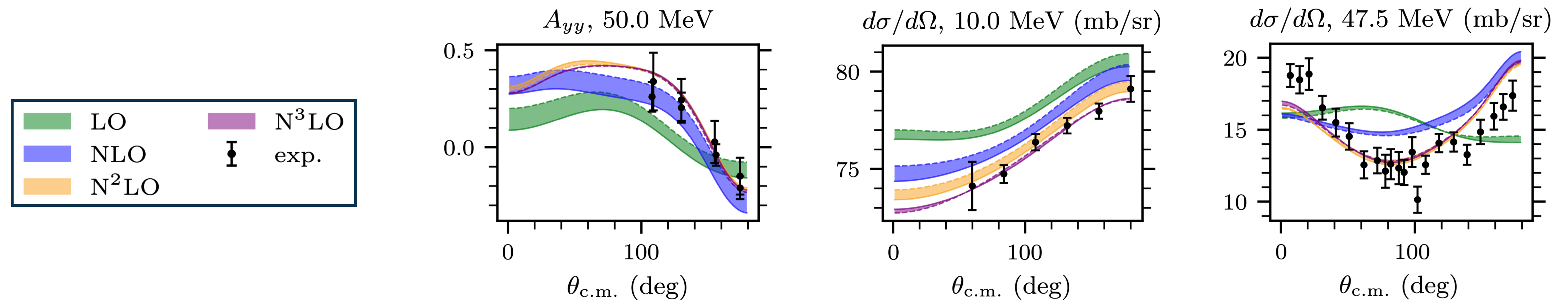
- We are interested in studying the predictive power of the Long & Yang PC.
 - Confirm that scattering observables can be described.
 - Use data to infer LECs and construct quantitative potentials.

[OT, E. May, A. Ekström, and C. Forssén, Phys. Rev. C **108**, \(2023\)](#)

[OT, A. Ekström, and C. Forssén, arXiv:2402.15325, \(2024\)](#)

- Study the convergence and breakdown of Δ -less and Δ -full χ EFT.
- Compute and predict bound-state properties in nuclei.

Neutron-proton scattering observables



- Conclusions:
 - Works sufficiently well to warrant a more detailed inference.
 - Hints that the breakdown scale can be as low as $\sim 200 - 300$ MeV.

Perturbative computation of neutron-proton observables using χ EFT up to N³LO

Oliver Thim, Andreas Ekström, Christian Forssén

Introduction

A well-defined power counting in χ EFT is crucial for:

- connecting quantum chromodynamics to properties of nuclei.
- accurate estimates of the EFT truncation error.

In several studies, e.g., Nogga *et al.* [1], the canonical power counting due to Weinberg has been questioned due to the cutoff dependence induced by the singular potentials. Long & Yang [2] proposed and investigated neutron-proton phase shifts up to N³LO in a renormalization-group invariant modified Weinberg power counting based on (i) promoted counterterms and (ii) a perturbative treatment of sub-leading potentials. Furthermore, Yang *et al.* [3] studied ground-state energies in selected nuclei up to $A = 16$ to NLO in this modified power counting.

We are interested in studying the predictive power of this group invariant formulation of χ EFT. We focus on quantifying uncertainty and in particular the EFT truncation error. In Thim *et al.* '23 we used **History Matching** and **Inference** at LO from neutron-proton scattering data to determine the joint probability distribution of the LECs at LO from neutron-proton scattering data. This poster summarizes new results where neutron-proton scattering observables are computed up to N³LO. We also present an analysis of low-energy theoretical effective range parameters in the 1S_0 and 3S_1 partial waves.



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Thank you!

Scattering amplitudes

- Potential contributions in the power counting we employ read:

order potential

LO	$V^{(0)}$
NLO	$V^{(1)}$
N ² LO	$V^{(2)}$
N ³ LO	$V^{(3)}$

• LO amplitude, $T^{(0)}$, is computed by solving the Lippmann-Schwinger equation:

$$T^{(0)} = \Omega_+^\dagger V^{(0)} \Omega_+$$

$$T^{(1)} = \Omega_+^\dagger (V^{(1)} + V^{(0)} G_1^+ V^{(0)}) \Omega_+$$

$$T^{(2)} = \Omega_+^\dagger (V^{(2)} + V^{(0)} G_1^+ V^{(1)} + V^{(1)} G_1^+ V^{(0)} + V^{(0)} G_1^+ V^{(0)} G_1^+ V^{(0)}) \Omega_+$$

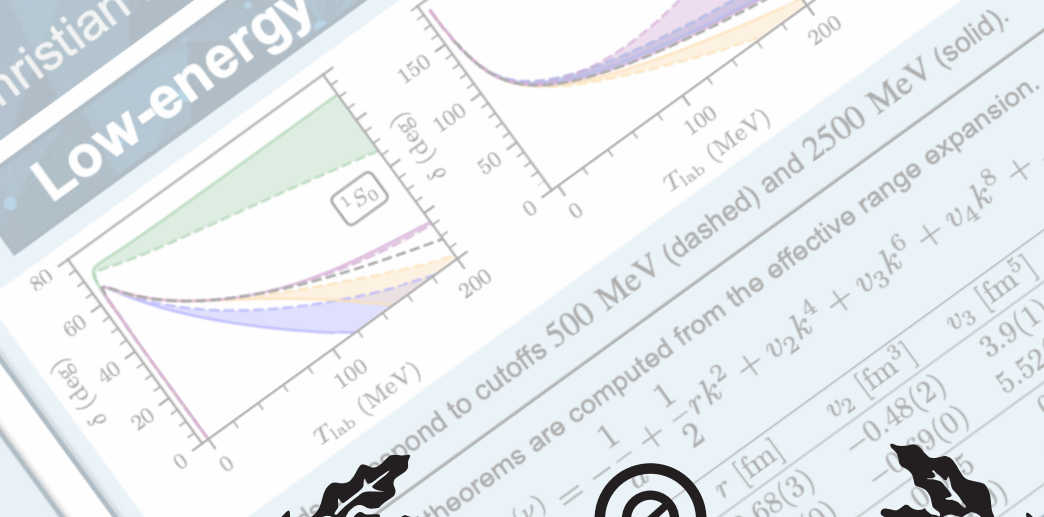
$$T^{(3)} = \Omega_+^\dagger (V^{(3)} + V^{(0)} G_1^+ V^{(2)} + V^{(1)} G_1^+ V^{(1)} + V^{(0)} G_1^+ V^{(0)} G_1^+ V^{(1)} + V^{(0)} G_1^+ V^{(0)} G_1^+ V^{(0)} G_1^+ V^{(0)}) \Omega_+$$

• Sub-leading orders are computed perturbatively order-by-order. For example, shifts are computed perturbatively order-by-order. For example, channels up to NLO we get

$$T_{ij}^{(0)} = 1 - i\pi m_N k T_{ij}^{(0)sj}$$

$$T_{ij}^{(1)} = 1 - i\pi m_N k T_{ij}^{(1)sj} \exp(-2i\delta_{ij}^{(0)})$$

Low-energy theorems



Phase shifts correspond to cutoffs 500 MeV (dashed) and 2500 MeV (solid).

Low-energy theorems are computed from the effective range expansion:

$1 + F(\nu)$	$1 + \frac{1}{2} r^2 k^2 + v_2 k^4 + v_3 k^6 + v_4 k^8 + \dots$
r [fm]	$2.68(3)$
v_2 [fm ³]	$-0.48(2)$
v_3 [fm ⁵]	$5.52(0)$
v_4 [fm ⁷]	0.67
v_4 [fm ⁷]	$0.67(2)$
v_4 [fm ⁷]	$-19.6(5)$
v_4 [fm ⁷]	$-30.6(5)$
v_4 [fm ⁷]	-3.94
v_4 [fm ⁷]	$-4.0(9)$

• The relative difference in the total cross section between (i) integrating the differential cross section and (ii) using the optical theorem.

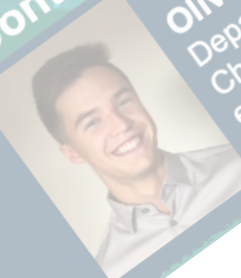
• This quantifies the breaking of unitarity in the perturbative calculation.

$$(S^{(v)})^\dagger S^{(v)} \approx 1 - c(Q/\Lambda_b)^{v+1}$$

References

- [1] A. Nogga, R. G. E. Timmermans, and U. van Kolck, Phys. Rev. C **72** (2005)
- [2] B. Long and C. J. Yang, Phys. Rev. C **84** (2011), **85** (2012), **86** (2012)
- [3] C. J. Yang, A. Ekström, C. Forssén, and G. Hagen, Phys. Rev. C **59** (1999)
- [4] T. D. Cohen, J. M. Hansen, E. Amaro, E. Ruiz Arriola, J. Phys. G **43**(11) (2016)
- [5] R. Navarro Pérez, J. E. Amaro, E. Ruiz Arriola, J. Phys. G **43**(11) (2016)

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