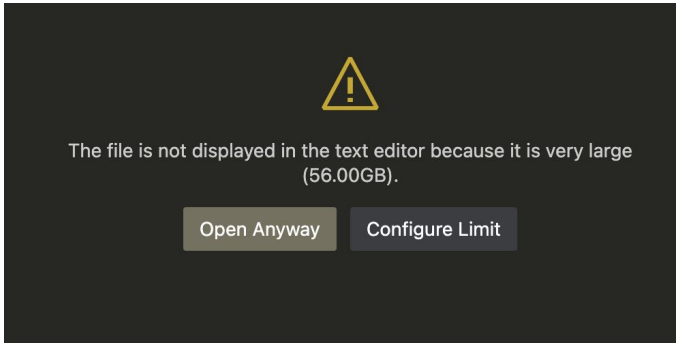
```python
def compute_partition_function(prbm, num_of_node_per_partition, device, batch_size=1024):
    # Calculate the total number of iterations
    total_states = 2 ** (num_of_node_per_partition * 4)

    # Determine batch size
    if batch_size > total_states:
        batch_size = total_states

    # Initialize the partition function's statistical value
    partition_stats = 0.0

    # Create a generator for all possible states
    all_states = itertools.product([0, 1], repeat=num_of_node_per_partition * 4)

    # Use tqdm to create a progress bar
    generator = tqdm(range(0, total_states, batch_size), desc='Computing partition function')

    for _ in generator:
        # Get the current batch of states
        states = [next(all_states) for _ in range(batch_size)]

        # Convert states into a tensor and send to the GPU
        states_tensor = torch.tensor(states, dtype=torch.float).to(device)

        # Split the state tensor to match each partition
        p_states = [states_tensor[:, i*num_of_node_per_partition:(i+1)*num_of_node_per_partition] for i in range(4)]

        # Compute the energy for the entire batch
        weight = prbm.weight_dict
        bias = prbm.bias_dict
        batch_energy = energy_exp(*p_states, weight, bias)  # Assuming energy_exp is defined elsewhere

        # Compute and update the partition function's statistical value
        exp_neg_energy = torch.exp(-batch_energy).sum().item()
        partition_stats += exp_neg_energy

        # Update the progress bar's postfix information (optional)
        generator.set_postfix(Partition=partition_stats)

    return partition_stats
```

Tried to compute exact partition function

Save all the state n=7: failed

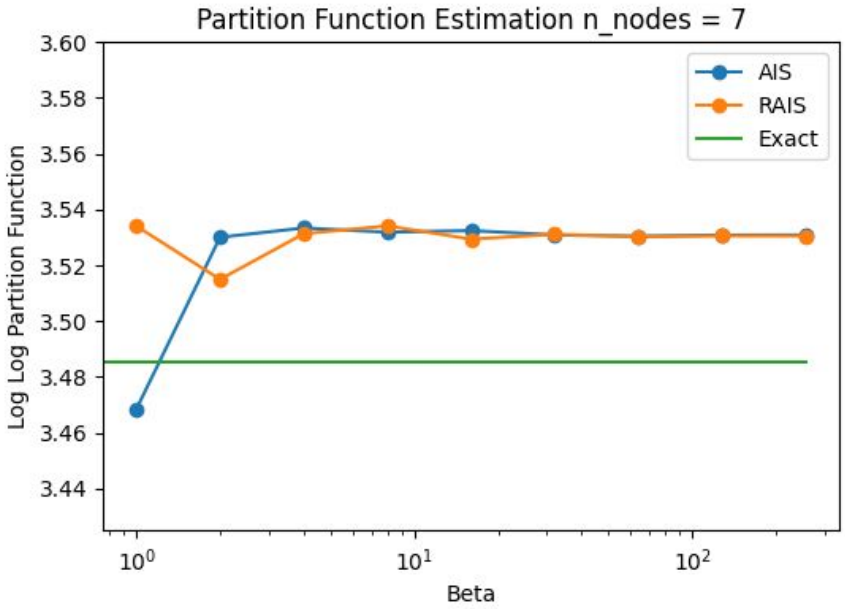


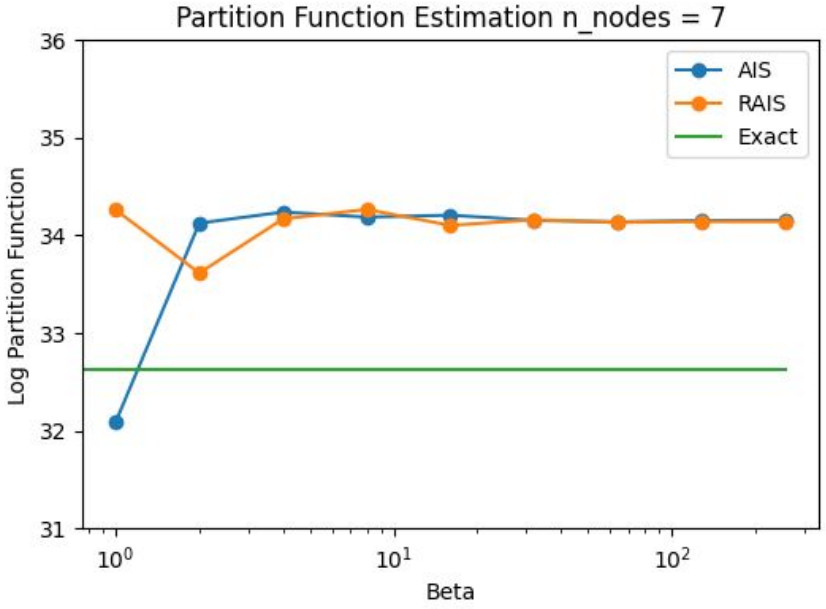
Direct Computing by Batches:

```
container_qml_v0.0.10.sif:[blazerjia@triumf-ml1:~/CaloQVAE$] python3 notebooks/rbm_list_generation.py -n 9 -b 81920 -d 4
<frozen importlib._bootstrap>:219: RuntimeWarning: scipy._lib.messagestream.MessageStream size changed, may indicate binary incompatibility. Expect
ed 56 from C header, got 64 from PyObject
Computing partition function:   0%|                                                                | 687/838861 [04:03<101:30:57,  2.29it/s, Partition=2.53e+11]
```

```
container_qml_v0.0.10.sif:[blazerjia@triumf-ml1:~/CaloQVAE$] python3 notebooks/rbm_list_generation.py -n 8 -b 8192 -d 4
<frozen importlib._bootstrap>:219: RuntimeWarning: scipy._lib.messagestream.MessageStream size changed, may indicate binary incompatibility. Expect
ed 56 from C header, got 64 from PyObject
Computing partition function:   0%|                                                                | 829/524288 [00:33<5:44:58, 25.29it/s, Partition=5.63e+10]
```

```
container_qml_v0.0.10.sif:[blazerjia@triumf-ml1:~/CaloQVAE$] python3 notebooks/rbm_list_generation.py -n 8 -b 81920 -d 4
<frozen importlib._bootstrap>:219: RuntimeWarning: scipy._lib.messagestream.MessageStream size changed, may indicate binary incompatibility. Expect
ed 56 from C header, got 64 from PyObject
Computing partition function:   1%|                                                                | 334/52429 [02:10<5:38:29,  2.57it/s, Partition=8.15e+8]
```


Partition Function Estimation n_nodes = 7
Log Log Partition Function
Beta
AIS
RAIS
Exact
3.60
3.58
3.56
3.54
3.52
3.50
3.48
3.46
3.44
10^0
10^1
10^2


Partition Function Estimation n_nodes = 7
Log Partition Function
Beta
AIS
RAIS
Exact
36
35
34
33
32
31
10^0
10^1
10^2