BEYOND THE STANDARD MODEL

LEC2A: SUSY, IN ACTION

Flip Tanedo
UC Riverside Particle Theory

29 JULY 2019

PHYSICS & ASTRONOMY

PI · SNOLAB · TRIUMF
TRISEP

flip.tanedo@ucr.edu

Unfinished Business

Last time: Why don’t we just introduce a right-handed neutrino into the Standard Model?

Unfinished Business

Last time: Why don't we just introduce a right-handed neutrino into the Standard Model?

It's not actually necessary!

MAJORANA MASS w/o ν_R

$$\frac{1}{\Lambda}|HL|^2 \supset \frac{v^2}{\Lambda}\nu_L\nu_L$$

By the way, I don't think RH neutrinos have a protected mass term

Review of Last Time

SUSY qualitatively

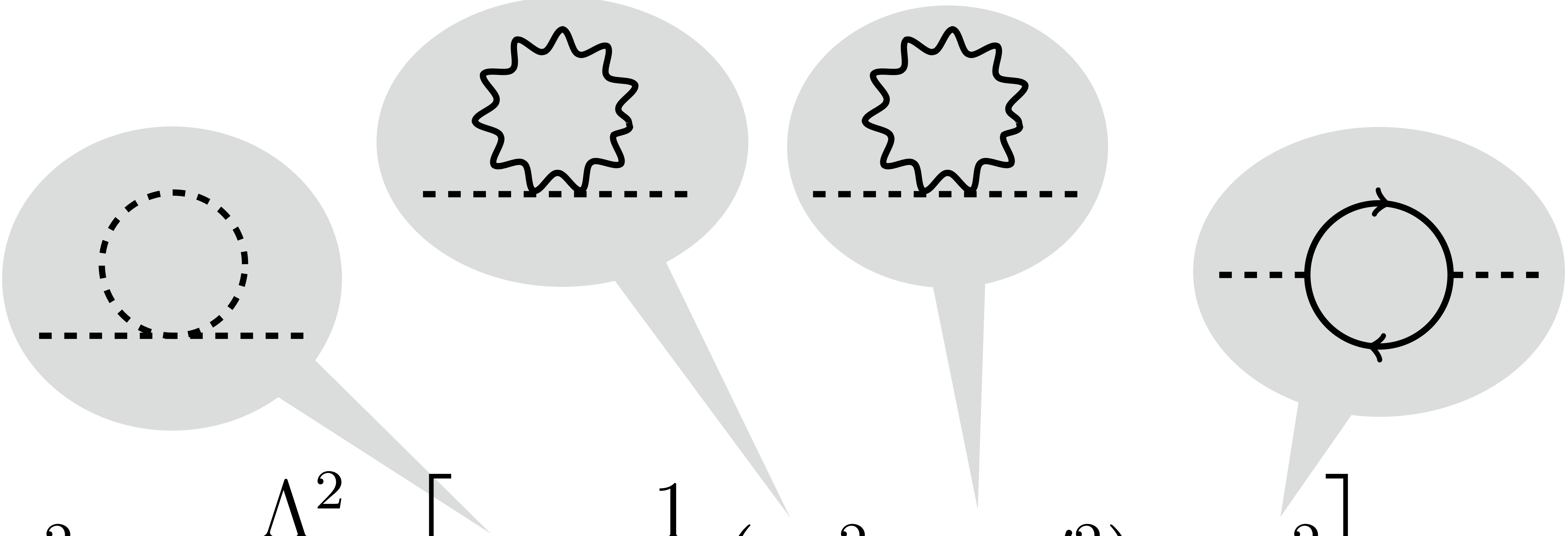
Hierarchy problem

$$\delta m_H^2 = \frac{\lambda_S}{16\pi^2}\left[\Lambda_{\text{UV}}^2 - 2m_S^2 \ln\left(\frac{\Lambda_{\text{UV}}}{m_S}\right) + (\text{finite})\right]$$

The Higgs is quadratically sensitive to the mass scale of any new physics that couples to it.

↑ nothing to do w/ REGULARIZATION!

quantum contributions to Higgs mass



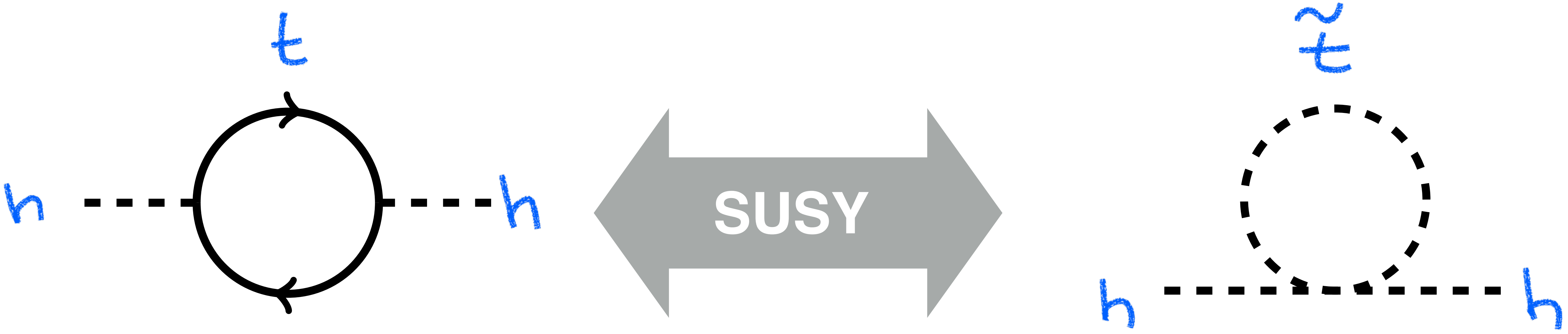
$$\delta m_H^2 = \frac{\Lambda^2}{32\pi^2}\left[6\lambda + \frac{1}{4}\left(9g^2 + 3g'^2\right) - y_t^2\right]$$

SCALARS

GAUGE

FERMIONS

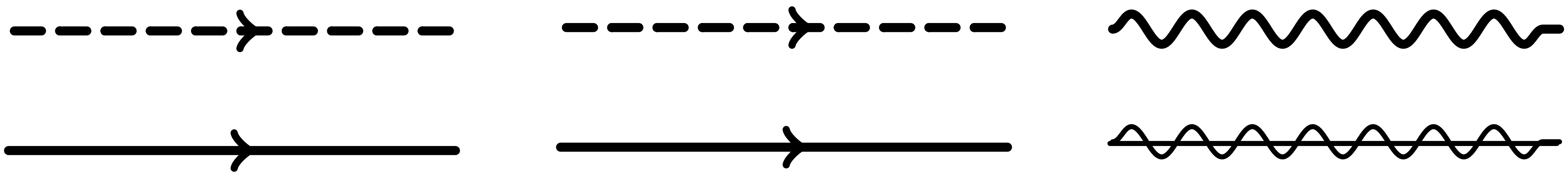
Cancellations in SUSY



additional particles
“just right” coupling

superpartners also contribute to Higgs mass

MSSM Particle Content



(s)fermions

(QUDLE)

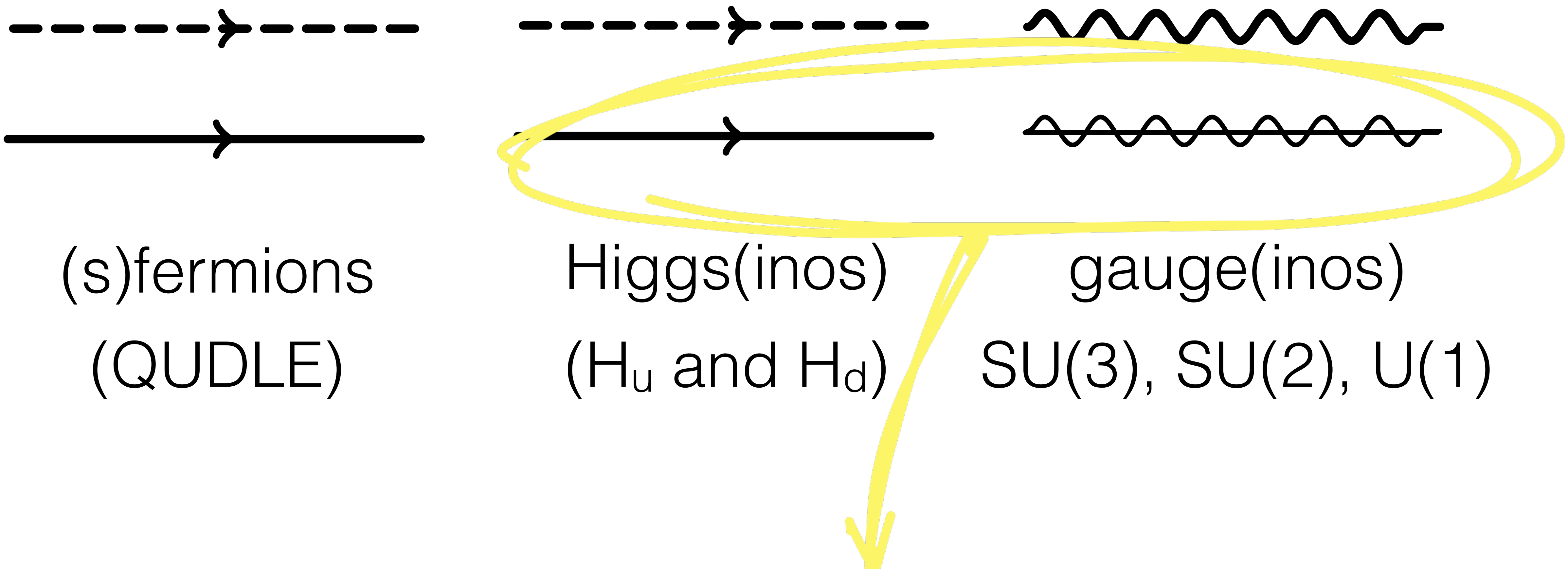
Higgs(inos)

(H_u and H_d)

gauge(inos)

SU(3), SU(2), U(1)

MSSM Particle Content



(s)fermions

(QUDLE)

Higgs(inos)

(H_u and H_d)

gauge(inos)

SU(3), SU(2), U(1)

CHARGINOS & NEUTRALINOS

Spectrum: sparticles are typically heavier.

flip.tanedo@ucr.edu

Interactions

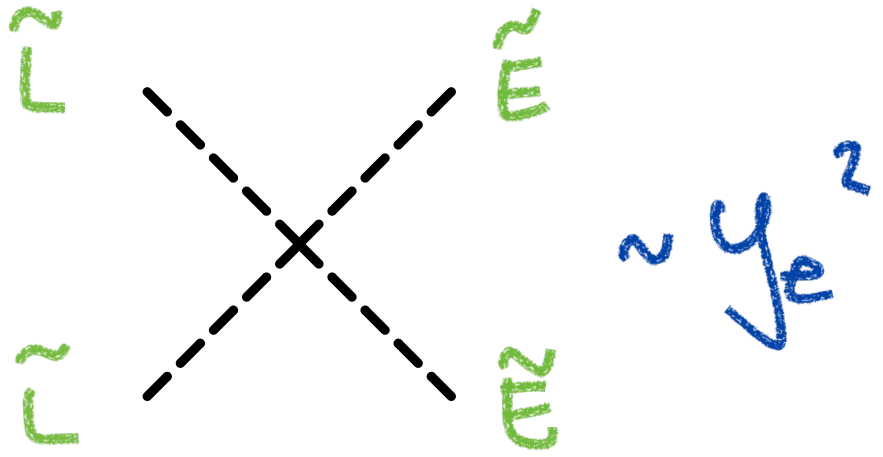
Crude estimate: take SM vertex, promote two lines to sparticles. (Sufficient for cocktail parties)

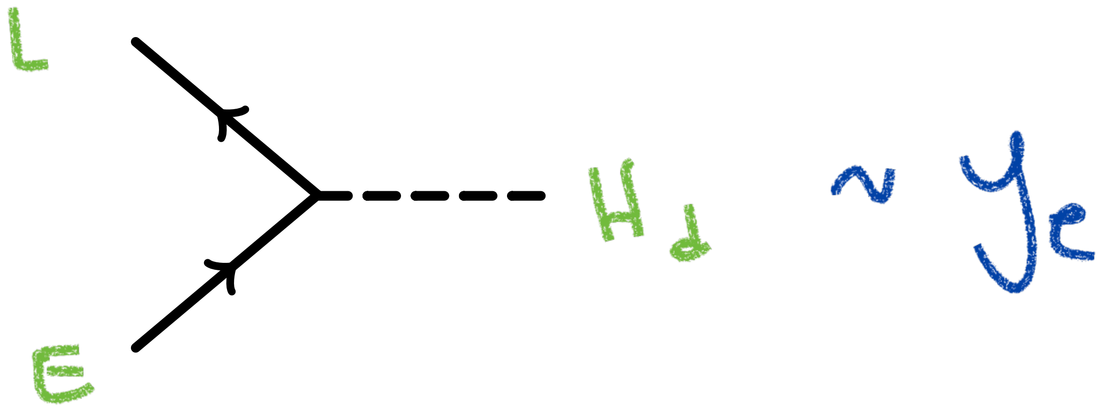
Systematic: supersymmetric action
e.g. superpotential

$$W^{(\text{good})} = y_u^{ij} Q^i H_u \bar{U}^j + y_d^{ij} Q^i H_d \bar{D} + y_e^{ij} L^i H_d \bar{E}^j + \mu H_u H_d$$

Superpotential

$$W^{(\text{good})} = y_u^{ij} Q^i H_u \bar{U}^j + y_d^{ij} Q^i H_d \bar{D} + \boxed{y_e^{ij}} \underset{A}{\underline{L^i}}\, \underset{C}{\underline{H_d}}\, \underset{B}{\underline{\bar{E}^j}} + \mu H_u H_d$$



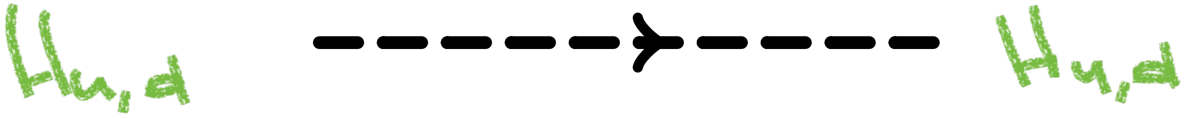


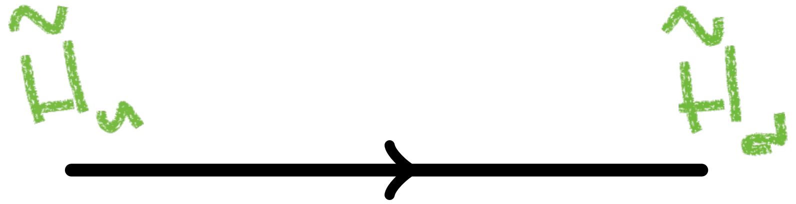
W = [coupling] A B C

gauge invariant combinations of superfields

Superpotential

$$W^{(\text{good})} = y_u^{ij} Q^i H_u \bar{U}^j + y_d^{ij} Q^i H_d \bar{D} + y_e^{ij} L^i H_d \bar{E}^j + \mu \underline{H_u} \, \underline{H_d}$$





Bilinear term gives mass.

More generally

$$\mathcal{L} = \underbrace{\overbrace{\int d^4\theta\, K(\Phi, \Phi^\dagger)}^{\text{real function}}}_{\text{Kähler potential}} + \underbrace{\overbrace{\left(\int d^2\theta\, W(\Phi) + \text{h.c.}\right)}^{\text{holomorphic function}}}_{\text{superpotential}}$$

Why quartic/Yukawa couplings are *just so*:
they come from the same object.

Kähler potential gives scalar quartics + derivative interactions. We'll stick to superpotential.

SUSY as a symmetry

where it comes from

Angular Momentum Algebra

$$[J_i, J_j] = i\epsilon_{ijk} J_k$$

Gives us representations of SU(2).
e.g. 3-vector has three components

Generalizes to Poincaré group.
e.g. spinor representation ("induced reps")

Poincaré: $P^\mu, M^{\mu\nu}$

For induced representations: see Weinberg QFT vol 1, chapter 2

flip.tanedo@ucr.edu

SUSY Algebra

$$\left\{ Q_\alpha \, , \, \bar{Q}_{\dot{\beta}} \right\} = 2\left(\sigma^\mu\right)_{\alpha\dot{\beta}} P_\mu$$

Fermionic generators, only allowed extension to spacetime symmetry with mass gap.

Exception to “no-go” theorerm (Coleman-Mandula)

For induced representations: see Weinberg QFT vol 1, chapter 2
For 2-component spinors: see Haber TASI lectures 1205.4076

flip.tanedo@ucr.edu

Action of symmetry on fields

$$g \;=\; \exp\left(i\left(\omega^{\mu\nu} M_{\mu\nu} + a^{\mu} P_{\mu} + \theta^{\alpha} Q_{\alpha} + \bar{\theta}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}\right)\right)$$

anticommuting

Write generators as differential operators, e.g.

$$\exp(-i a_{\mu} \mathcal{P}^{\mu})\, \varphi(x^{\mu}) \;=:\; \varphi(x^{\mu} - a^{\mu})$$

$$\Longrightarrow \quad \mathcal{P}_{\mu} \;=\; -i\partial_{\mu}$$

Translations in superspace

$$\begin{aligned}\mathcal{Q}_\alpha &= -i\,\frac{\partial}{\partial\theta^\alpha} - c\,(\sigma^\mu)_{\alpha\dot\beta}\,\bar\theta^{\dot\beta}\,\frac{\partial}{\partial x^\mu}\\ \bar{\mathcal{Q}}_{\dot\alpha} &= +i\,\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + c^*\,\theta^\beta\,(\sigma^\mu)_{\beta\dot\alpha}\,\frac{\partial}{\partial x^\mu}\end{aligned}$$

PICK $c=1$

Grassmann Numbers 101

$$\boxed{\{\theta, \theta\} = 0}$$

Taylor series truncates:

LINEAR!

$$f(\theta) \;=\; \sum_{k=0}^{\infty} f_k\,\theta^k \;=\; \boxed{f_0 \;+\; f_1\,\theta} + \; f_2\;\underbrace{\cancel{\theta^2}}_{0} + \underbrace{\dots}_{0}$$

"INTEGRATION" IS TRIVIAL!

$$\int \mathrm{d}\theta\,\frac{\mathrm{d}f}{\mathrm{d}\theta} \;:=\; 0 \qquad\qquad \int \mathrm{d}\theta\;\theta \;:=\; 1$$

$$\int \mathrm{d}\theta\,\left(f_0 \;+\; f_1\,\theta\right) \;=\; f_1 \;=\; \frac{\mathrm{d}f}{\mathrm{d}\theta}$$

Grassmann Numbers 102

Now provide spin indices:

cf DOTTED/UNDOTTED INDICES: $\alpha, \dot{\alpha} \in \{1,2\}$

$$\{\theta_\alpha, \bar{\theta}_{\dot{\beta}}\} = \{\theta_\alpha, \theta_\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = 0$$

$$\theta\theta \quad := \quad \theta^\alpha\,\theta_\alpha \qquad\qquad \bar{\theta}\bar{\theta} \quad := \quad \bar{\theta}_{\dot{\alpha}}\,\bar{\theta}^{\dot{\alpha}}$$

$$\int \mathrm{d}^2\theta\ \theta\theta \quad = \quad 1 \qquad\qquad \int \mathrm{d}^2\theta \int \mathrm{d}^2\bar{\theta}\ (\theta\theta)\,(\bar{\theta}\bar{\theta}) \quad = \quad 1$$

$$\int d^2\theta\,[\cdots + F(x)\theta^2 + \cdots] = F(x)$$

integrals are projections

Putting this all together

$$\Phi = \varphi(x) - i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\varphi(x) - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\varphi(x)$$
$$+\sqrt{2}\theta\eta + \frac{i}{\sqrt{2}}\theta^2\partial_{\mu}\eta\sigma^{\mu}\bar{\theta} + \sqrt{2}\theta^2 F(x)\,.$$

chiral superfield: complex scalar, Weyl fermion, auxiliary
transforms into itself under a SUSY rotation

Transformation of components

$$\begin{aligned}\delta\varphi &= \sqrt{2}\,\epsilon\psi \\ \delta\psi &= i\sqrt{2}\,\sigma^{\mu}\,\bar{\epsilon}\,\partial_{\mu}\varphi + \sqrt{2}\,\epsilon\,F \\ \delta F &= i\sqrt{2}\,\bar{\epsilon}\,\bar{\sigma}^{\mu}\,\partial_{\mu}\psi\,.\end{aligned}$$

Check degrees of freedom

complex scalar, Weyl fermion, auxiliary

Towards a SUSY'ic Lagrangian

$$\delta F = i\sqrt{2}\,\bar{\epsilon}\,\bar{\sigma}^{\mu}\,\partial_{\mu}\psi\,.$$

Transforms as a total derivative. Invariant in *S*.

Fact: products of XSF are XSF.

Rule: write gauge-invariant product of chiral superfields. Call that *W*. The auxiliary field component are good potential terms. (+h.c.)

The Wess-Zumino Model

The ϕ^4 theory of SUSY

crayon: an example of what SUSY calcs are like

The Wess-Zumino lagangian

$$W = \frac{m}{2}\Phi^2 + \frac{\lambda}{3}\Phi^3$$

WANT : $\mathcal{O}(\theta^2)$
TERM

$$\Phi = \varphi(x) - i\theta\sigma^\mu\bar\theta\partial_\mu\varphi(x) - \frac{1}{4}\theta^2\bar\theta^2\partial^2\varphi(x)$$
$$+\sqrt{2}\theta\eta + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\eta\sigma^\mu\bar\theta + \sqrt{2}\theta^2 F(x)\,.$$

complex scalar, Weyl fermion, auxiliary field

from 0907.0039

Superpotential to ordinary potential

$$\mathcal{L} = \int d^2\theta W(\Phi_i) + \cdots .$$

WANT : $\mathcal{O}(\theta^2)$ TERM

$$= \frac{1}{2}\frac{\partial^2 W}{\partial\Phi_i\partial\Phi_j}\eta_i\eta_j - \frac{\partial W}{\partial\Phi_i}F_i \, .$$

$$\Phi = \varphi(x) - i\theta\sigma^\mu\bar\theta\partial_\mu\varphi(x) - \frac{1}{4}\theta^2\bar\theta^2\partial^2\varphi(x)$$

$$+\sqrt{2}\theta\eta + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\eta\sigma^\mu\bar\theta + \sqrt{2}\theta^2 F(x) \, .$$

More generally

$$\mathcal{L} = \int d^4\theta\, K(\Phi_i) + \int d^2\theta W(\Phi_i) + \int d^2\bar{\theta} W(\Phi_i^\dagger)$$

$$= g^{ij}\left(\partial\varphi_i^*\partial\varphi_j + i\eta_i^*\partial_\mu\bar{\sigma}^\mu\eta_j + F_i^*F_j\right)$$

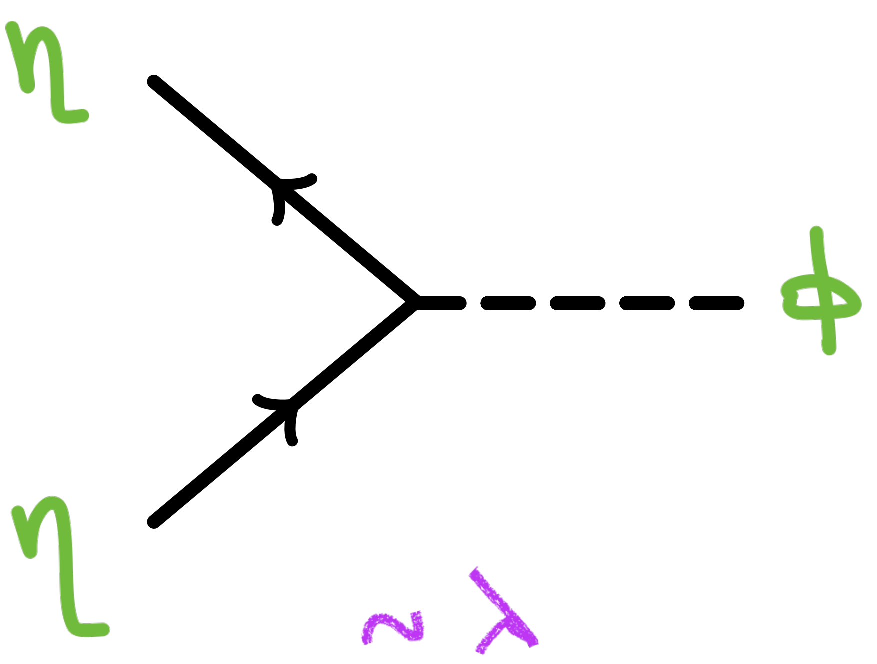
$$-\left(\frac{1}{2}\frac{\partial^2 W}{\partial\Phi_i\partial\Phi_j}\eta_i\eta_j - \frac{\partial W}{\partial\Phi_i}F_i + \text{h.c.}\right)$$

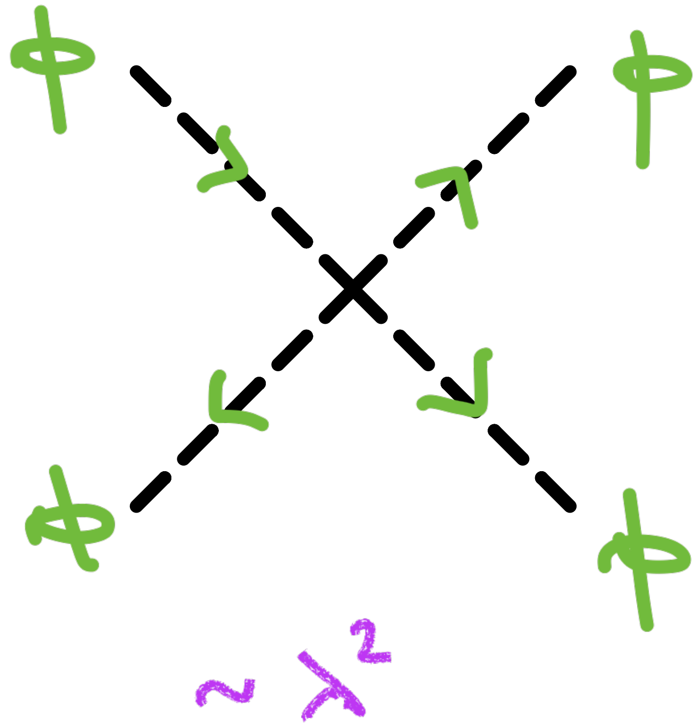
$$F \quad \text{EOM: } \partial\frac{\delta\mathcal{L}}{\delta(\partial F)} - \frac{\delta\mathcal{L}}{\partial F} = \boxed{F^* - \frac{\partial W}{\partial\phi} = 0}$$

Superpotential to ordinary potential

$$\mathcal{L} \supset -\frac{1}{2}\frac{\partial^2 W}{\partial\Phi\partial\Phi}\eta\eta + \frac{1}{2}\left|\frac{\partial W}{\partial\Phi}\right|^2 + \text{h.c.}$$

$W \supset \frac{\lambda}{3}\Phi^3$





Building a SUSY model

ingredients for the MSSM

Vector superfields & gauge invariance

$$\begin{aligned}\Phi &\mapsto \exp(iq\Lambda)\,\Phi \\ V &\mapsto V - \tfrac{i}{2}\left(\Lambda - \Lambda^\dagger\right)\end{aligned}$$

$$\Phi^\dagger \exp(2qV)\,\Phi$$

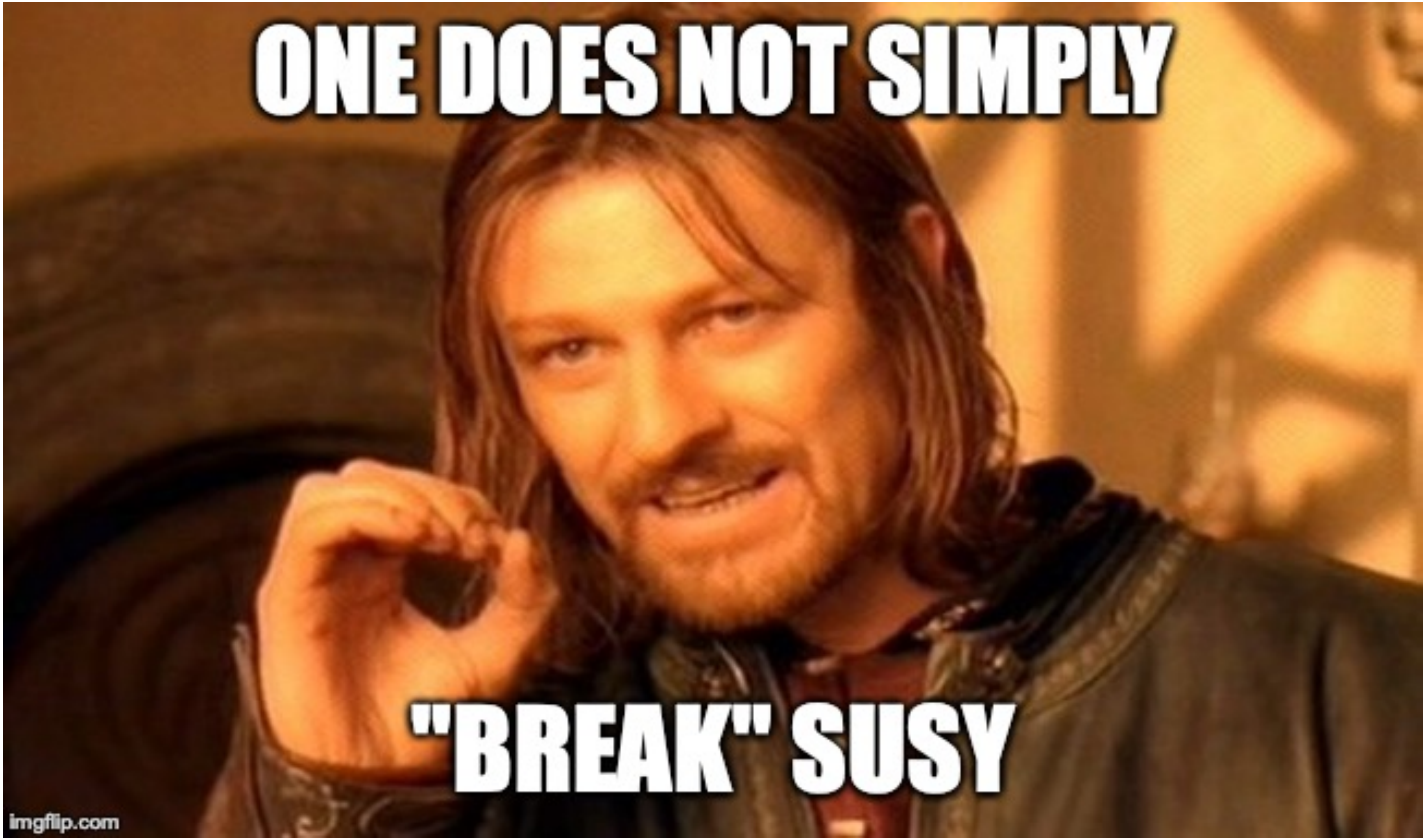
gauge invariant term
(Kähler potential)

$$V = -\,\theta\sigma^\mu\bar{\theta}V_\mu(x) + i\theta^2\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}^2\theta\lambda(x) + \frac{1}{2}\theta^2\bar{\theta}^2 D(x)$$

Vector superfield: force superfield

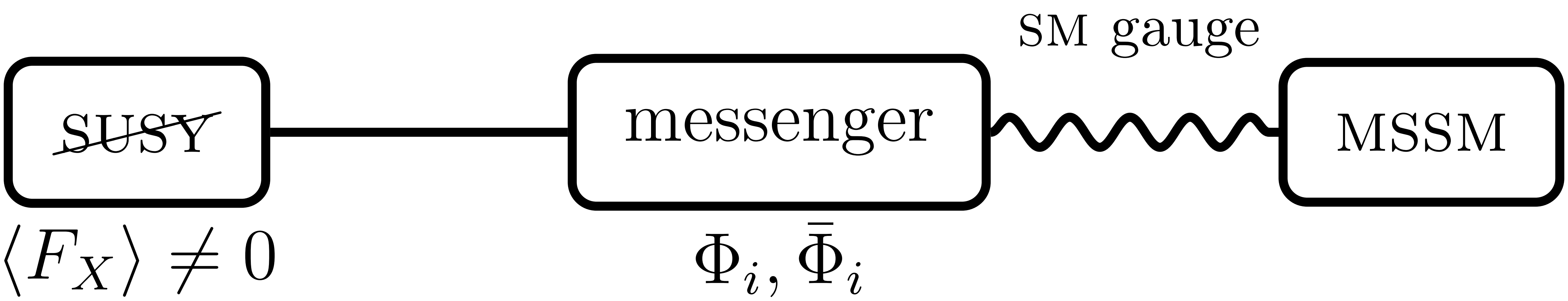
spin-1, Majorana fermion, auxiliary field

$$\mathrm{STr}\ m^2 = 0$$



https://imgflip.com/memegenerator/One-Does-Not-Simply

SUSY breaking



for e.g.

SM gauge

~~SUSY~~

messenger

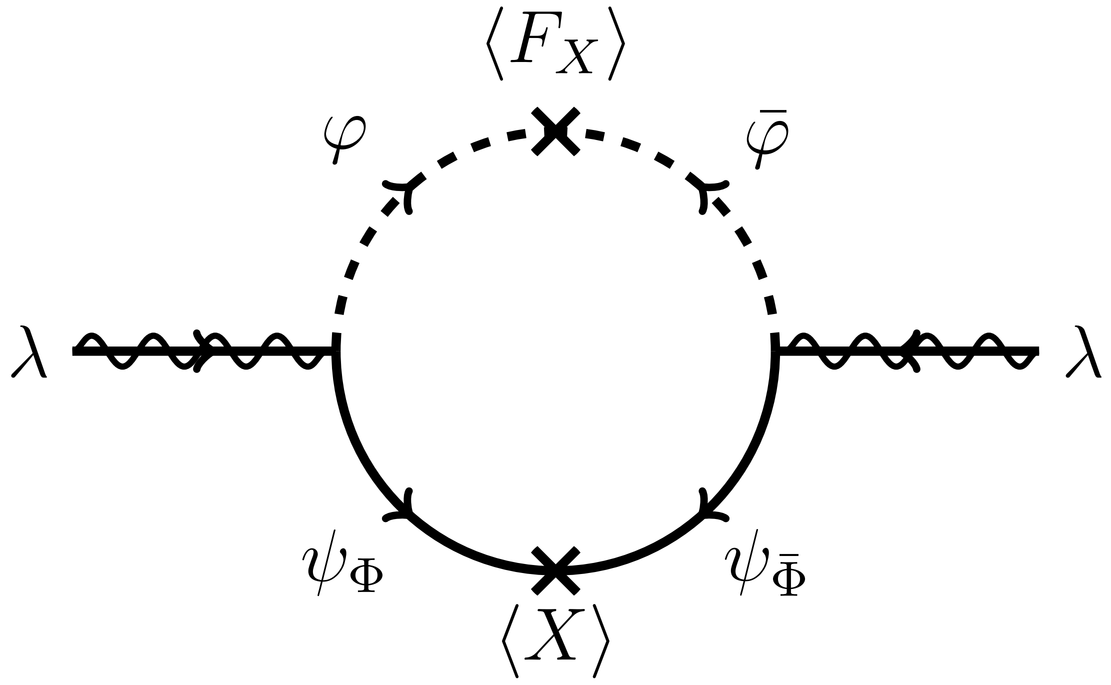
MSSM

$\langle F_X \rangle \neq 0$

$\Phi_i, \bar{\Phi}_i$

Typical assumption: SUSY is broken in a different **sector.**

Mediated to SM by additional fields.



Soft SUSY breaking effective terms

$$
\begin{aligned}
\mathcal{L}_{\text{soft}} = & -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B}\right) + \text{h.c.} \\
& -\left(a_u\widetilde{Q}H_u\widetilde{\bar{u}} + a_d\widetilde{Q}H_d\widetilde{\bar{d}} + a_e\widetilde{L}H_d\widetilde{\bar{e}}\right) + \text{h.c.} \\
& -\widetilde{Q}^\dagger m_Q^2\widetilde{Q} - \widetilde{L}^\dagger m_L^2\widetilde{L} - \widetilde{u}^\dagger m_u^2\widetilde{\bar{u}} - \widetilde{d}^\dagger m_d^2\widetilde{\bar{d}} - \widetilde{e}^\dagger m_e^2\widetilde{\bar{e}} - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d \\
& -\left(bH_uH_d + \text{h.c.}\right)\right) .
\end{aligned}
$$

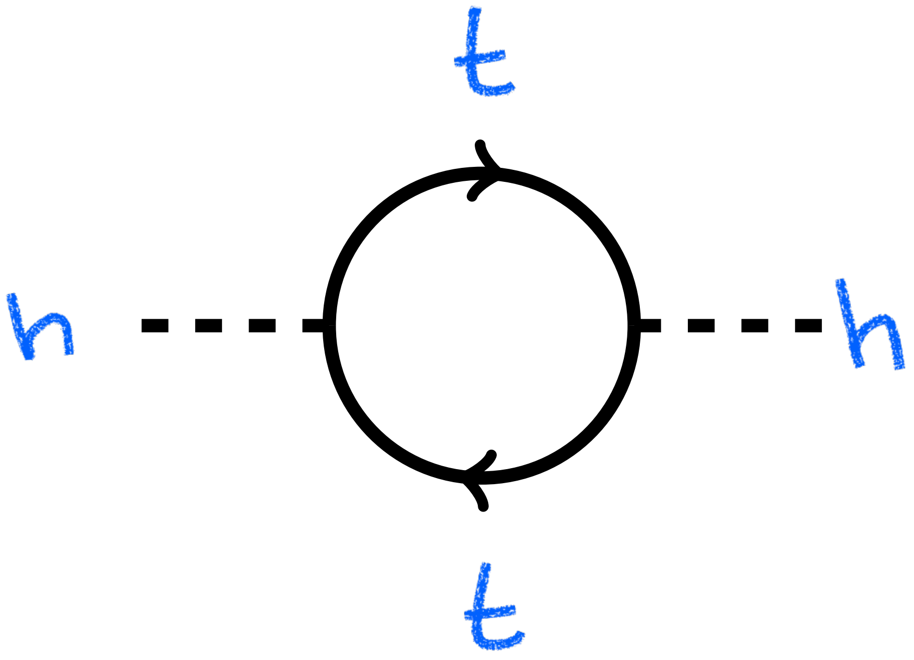
All terms that break SUSY but do not re-introduce a hierarchy between Higgs and Planck scale.

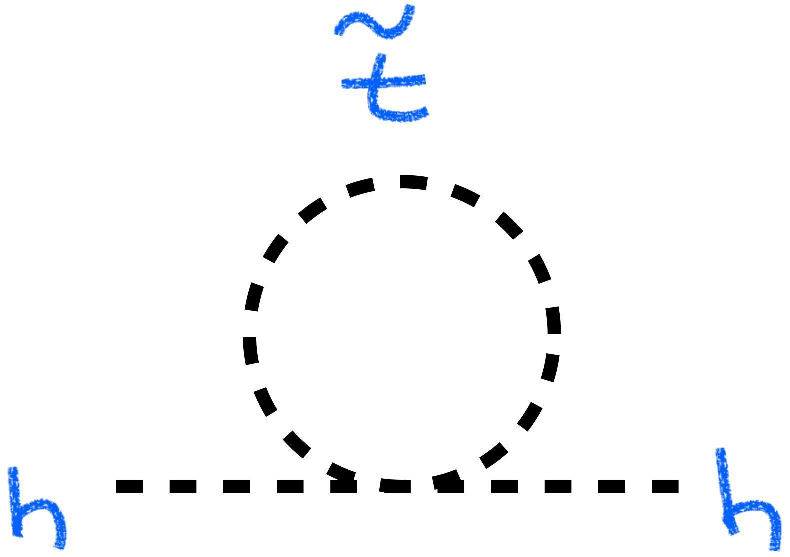
Specific SUSY breaking prescription predicts patterns in these terms.

Little Hierarchy

common to all BSM

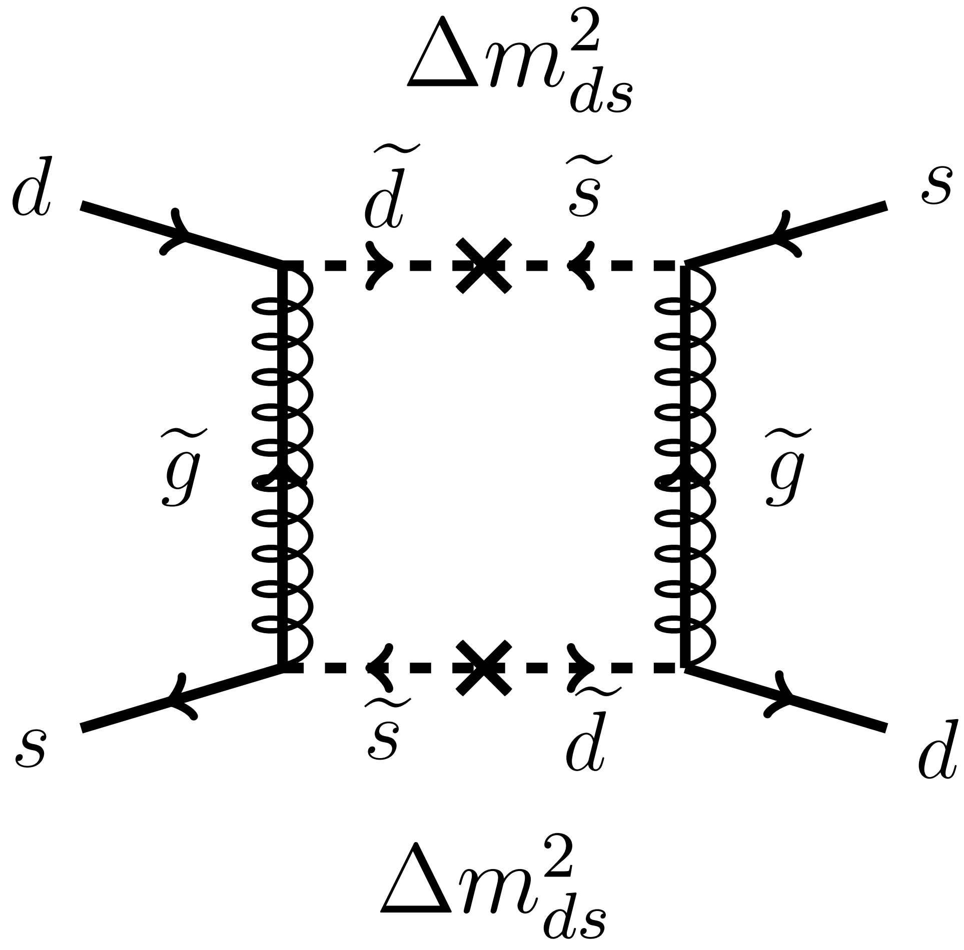
$$\Delta m^2_{H_u} = \frac{3y_t^2}{4\pi^2} m^2_{\tilde{t}} \ln\left(\frac{\Lambda_{\text{UV}}}{m_{\tilde{t}}}\right).$$





Flavor

common to all BSM



$$\mathcal{M}^{\text{MSSM}}_{K\bar{K}} \sim \alpha_{\mathsf{s}}^2 \left(\frac{\Delta m^2_{ds}}{m^2_{\text{SUSY}}}\right)^2 \frac{1}{m^2_{\text{SUSY}}}$$

$$\frac{\Delta m^2_{ds}}{m^2_{\text{SUSY}}} \lesssim 4 \cdot 10^{-3} \left(\frac{m_{\text{SUSY}}}{500\ \text{GeV}}\right)$$

Indirect constraints on sparticles…

often suggestive of flavor patterns (MFV)

Why theorists [still] love SUSY

Non-renormalization theorems
(Not true for Kahler potential)

Powerful non-perturbative results in gauge theory
e.g. electromagnetic duality

The “spherical cow” of field theory

see lectures by Strassler on SUSY

BEYOND THE STANDARD MODEL

LEC 2B : EXTRA DIMENSIONS & COMPOSITENESS

Flip Tanedo

UC Riverside Particle Theory

29 JULY 2019

flip.tanedo@ucr.edu

Extra Dimensions
& the Hierarchy Problem

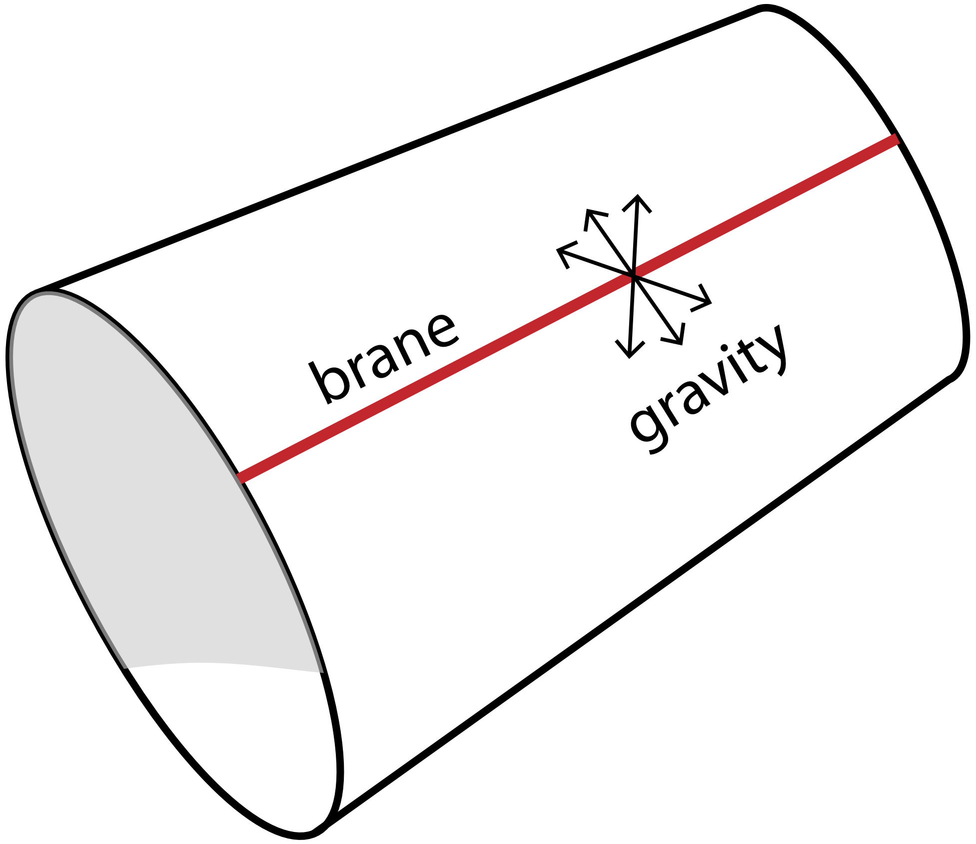
SUSY vs extra dimensions

$$\{x_\mu, \theta^\alpha, \bar{\theta}_{\dot{\alpha}}\}$$

$$\{x_\mu, z\}$$

$$\psi(x)$$

$$\varphi(x)$$



More particles, no cancellations

HIERARCHY PROBLEM

Why isn't **Higgs mass = Planck mass**?

largeness of M_{Pl} is
the weakness of gravity

$$G_N = \frac{1}{8\pi M_{\mathrm{Pl}}^2}$$

Maybe Planck mass isn't actually that big.

Field theory in 5D

$M \in 0,1,2,3,5$

$$S = \int d^5x\, \frac{1}{2}\partial_M\phi(x,y)\partial^M\phi(x,y)$$

$$= \int d^5x\, \frac{1}{2}\left[\partial_\mu\phi(x,y)\partial^\mu\phi(x,y) - \overbrace{(\partial_y\phi(x,y))^2}\right]$$

$\eta = (+,-,-,-,-)$

Assume y is compact; Fourier: $y \in (0, 2\pi)$

$$\phi(x,y) = \frac{1}{\sqrt{2\pi R}}\sum_{n=-\infty}^{\infty}\phi^{(n)}(x)\, e^{i\frac{n}{R}y}$$

KK Masses

$$S = \int d^5x \, \frac{1}{2} \left[\partial_\mu \phi(x,y) \partial^\mu \phi(x,y) - (\partial_y \phi(x,y))^2\right]$$

$$= \int d^4x \sum_{n>0} \left[\left(\partial_\mu \phi^{(n)}\right)^\dagger \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \left|\phi^{(n)}\right|^2\right]$$

$$\boxed{M_{(n)}^2 = \frac{n^2}{R^2}}$$

Gauge fields

$$A_M(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_n A_M^{(n)}(x)\, e^{i\frac{n}{R}y}.$$

$$\uparrow$$

$$A_M = (\underbrace{A_0, A_1, A_2, A_3,}_{A_\mu}\ \underset{\uparrow\ \text{4D SCALAR}}{A_5})$$

Dimensional Analysis

$$D_\mu = \partial_\mu - ig_5 A_\mu = \partial_\mu - i\frac{g_5}{\sqrt{2\pi R}} A_\mu^{(0)} + \cdots .$$

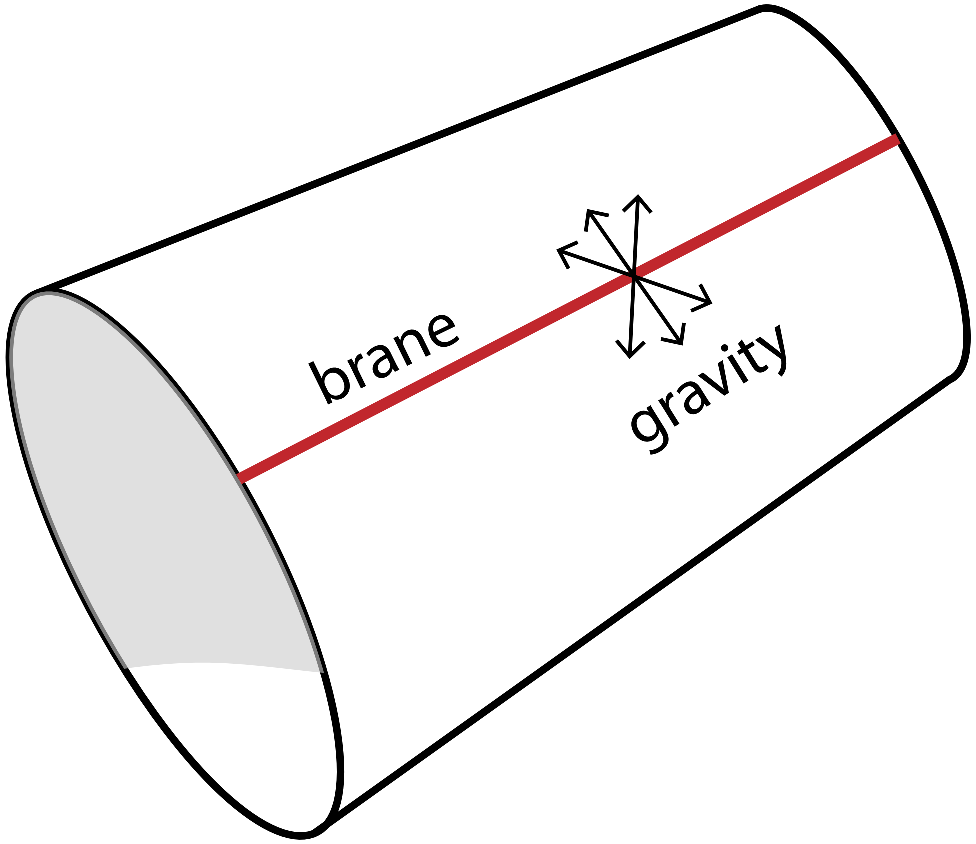
$$g_4 = \frac{g_5}{\sqrt{2\pi R}}$$

$$\boxed{g_4^2 = \frac{g_{(4+n)}^2}{\text{Vol}_n}}$$

volume suppression

$$A_M(x,y) = \frac{1}{\sqrt{2\pi R}} \sum_n A_M^{(n)}(x)\, e^{i\frac{n}{R}y}.$$

Braneworld

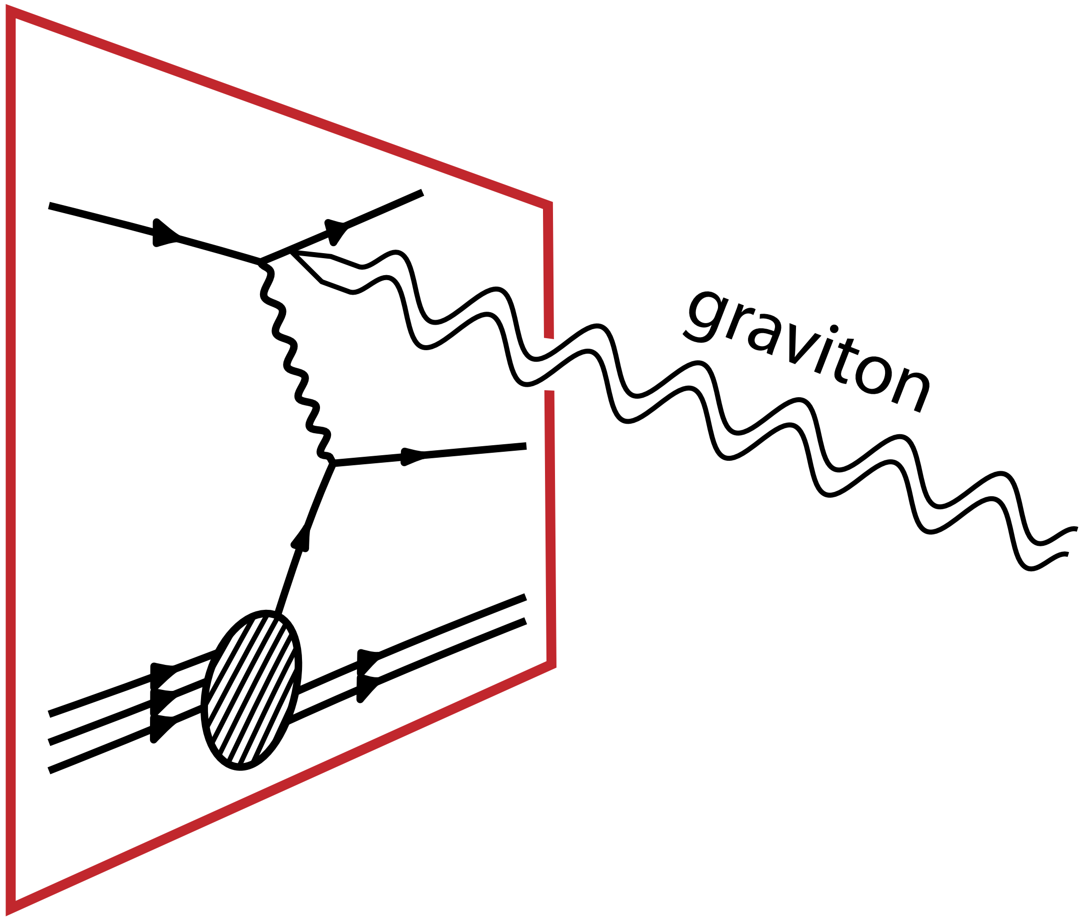


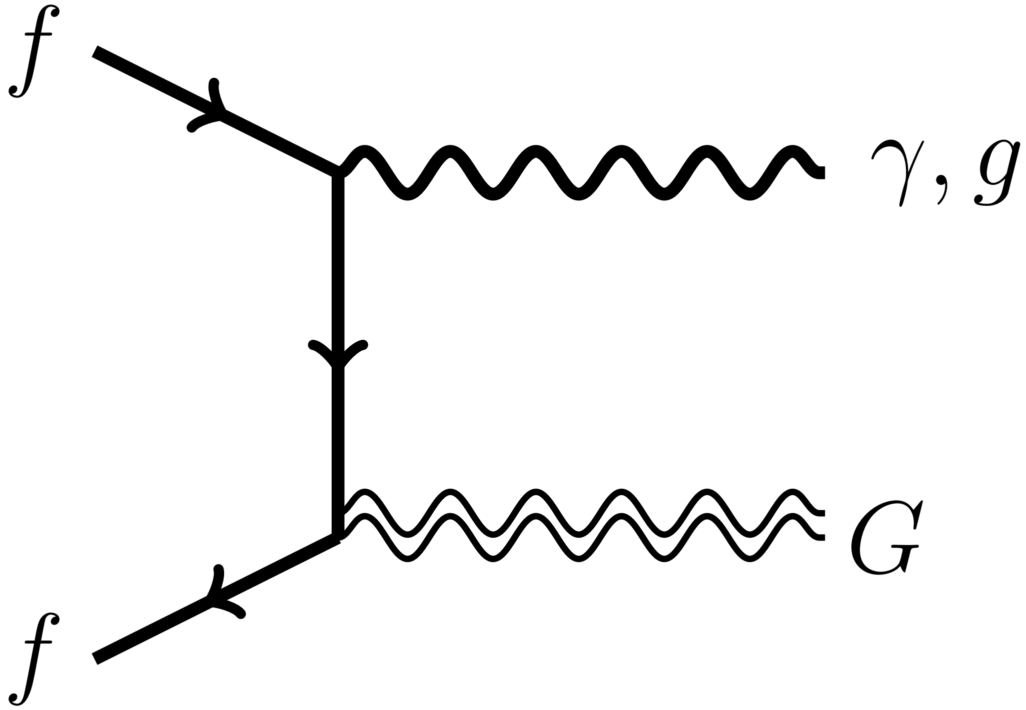
Fundamental scale is actually smaller than our perceived M_{Pl}

$$S_{(4+n)} = -M_{(4+n)}^{2+n} \int d^{4+n}x \, \sqrt{g} \, R_{(4+n)}$$

$$= -M_{(4+n)}^{2+n} V_n \int d^4x \, \sqrt{g_{(4)}} \, R_{(4)} + \cdots$$

Phenomenology





4D physics is the same, fields in the **bulk** have Kaluza-Klein resonances.

What if Standard Model fields were also in the bulk?

Compositeness
& the Hierarchy Problem

… really just an interlude about pions

Maybe the Higgs is like a pion.

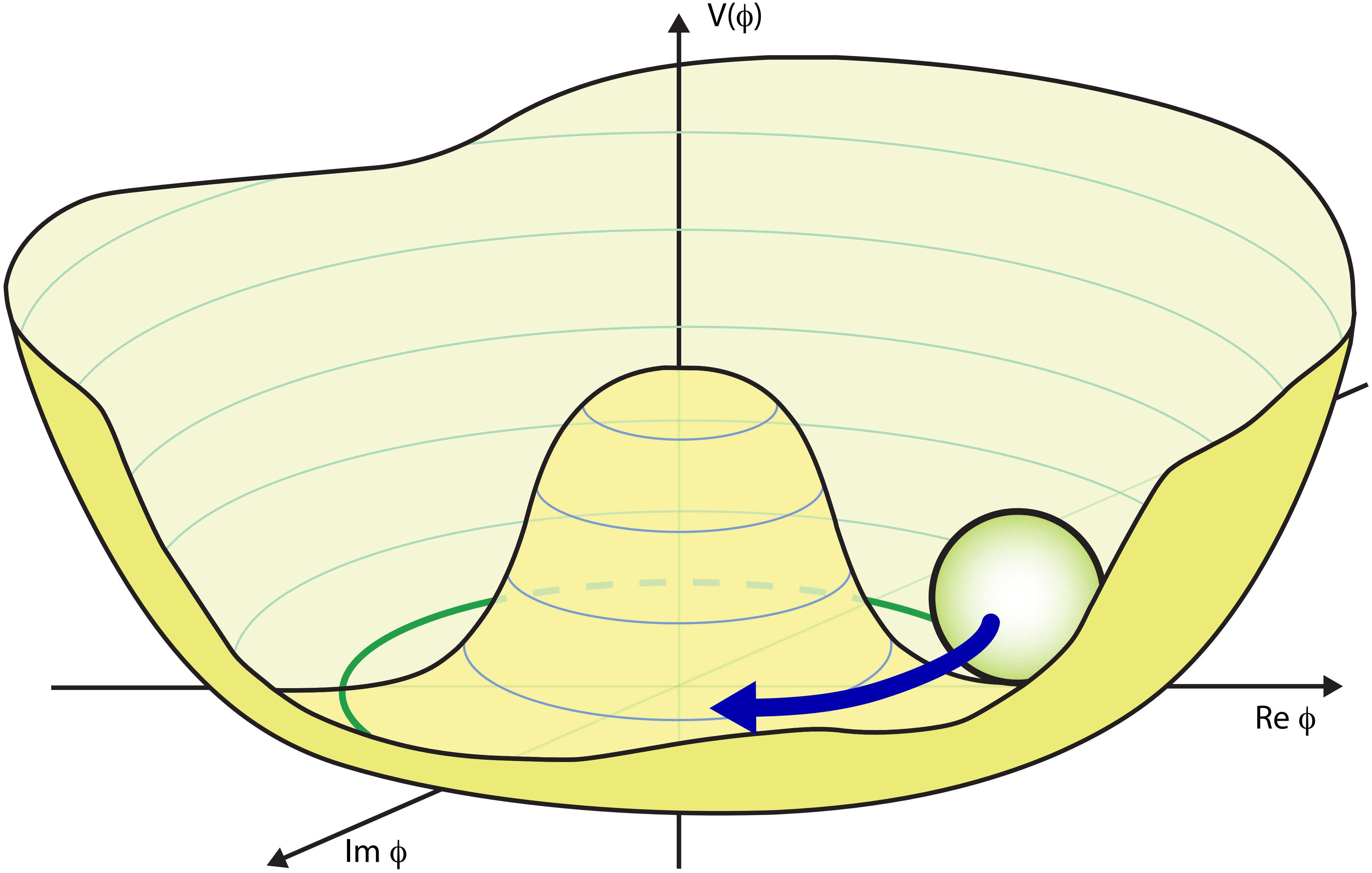
There's no *pion* hierarchy problem.

Why there's no π hierarchy problem

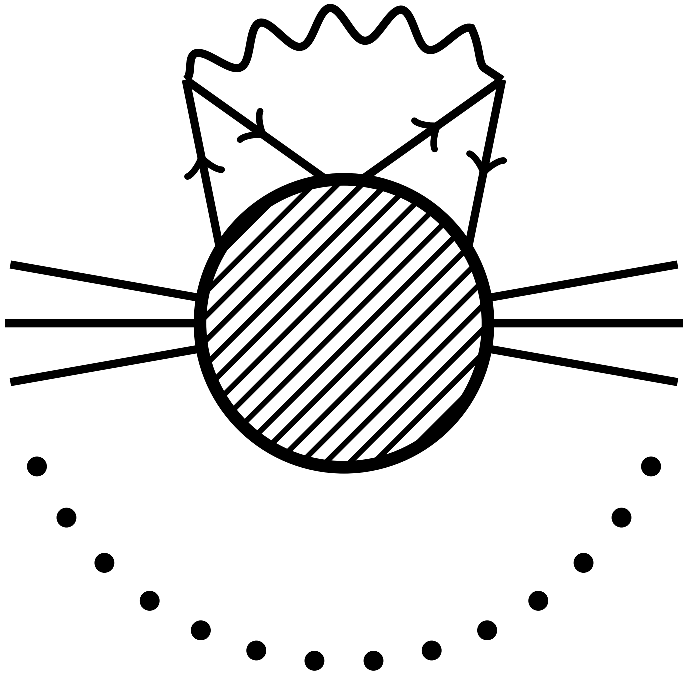
1. The pion is **composite.**
At small scales/high energies, it stops behaving like a scalar and starts behaving like two fermions.

2. The pion is a **goldstone boson**.
It is protected by a shift symmetry. (c.f. axion)

Exercise: what symmetry is broken spontaneously?
Exercise: what is breaking that symmetry?


V(ϕ)
Re ϕ
Im ϕ

Exercises



1. Why do pions have mass?

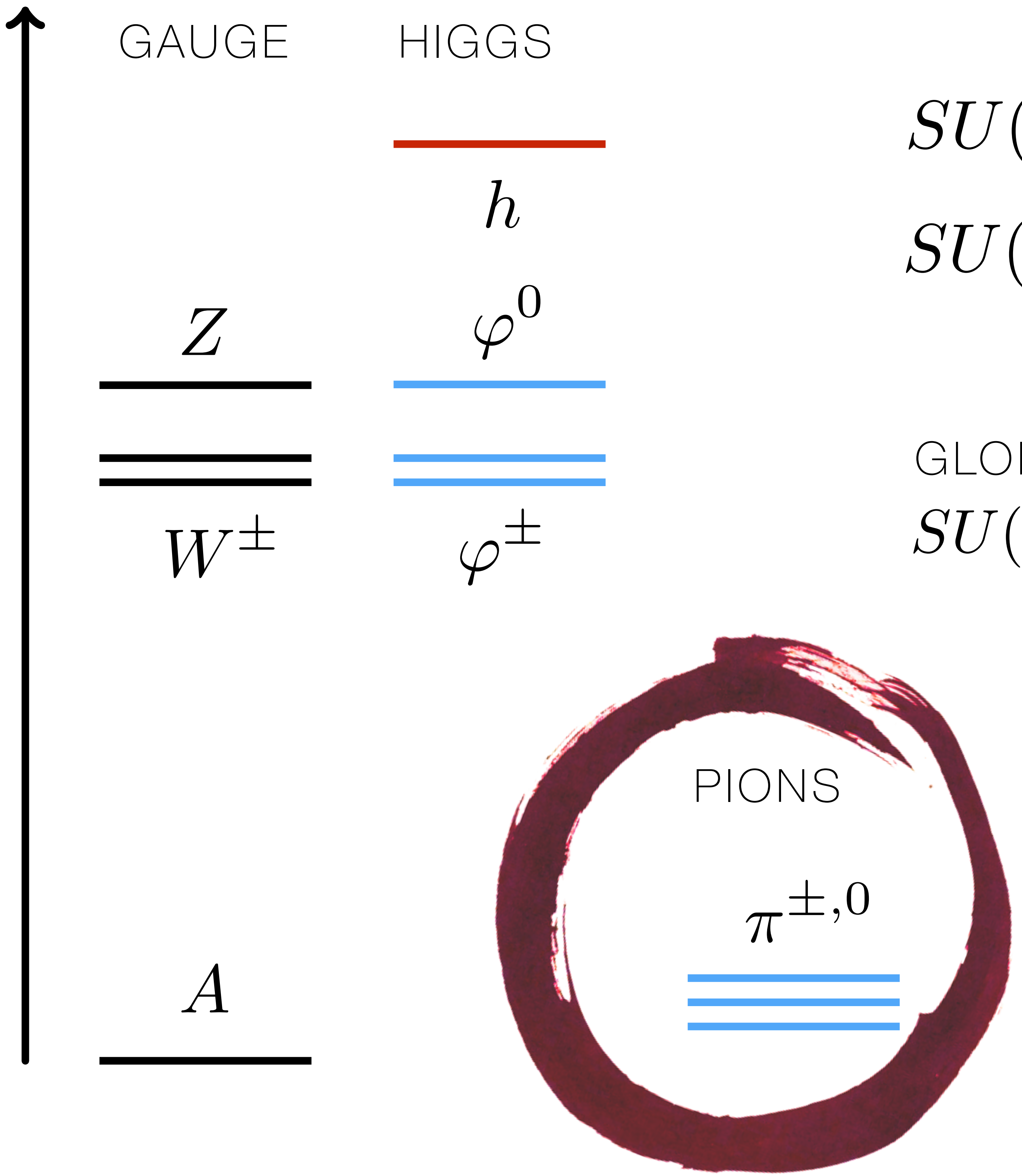
2. Why do some pions have charge?

3. Why are charged pions slightly heavier?

4. What about kaons?

5. Why don’t we have top mesons?

Pions in the Standard Model



$$SU(2)_L \times U(1)_Y \to U(1)_{\text{EM}}$$

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

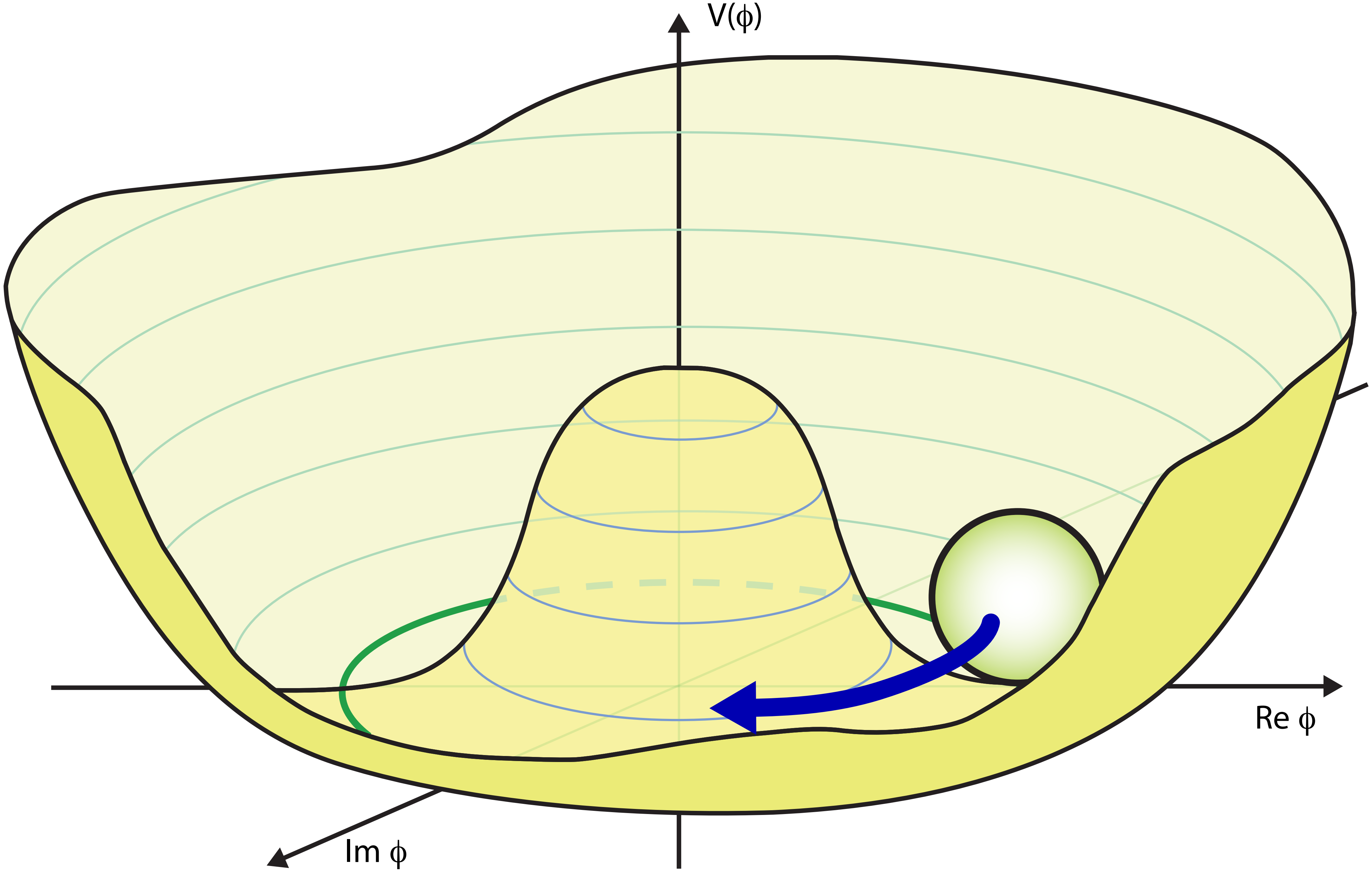
GLOBAL SYMMETRY

$$SU(2)_H \times SU(2)'_L \times U(1)_H$$

Mass from explicit breaking of global sym.

FROM QUARK MASSES

Pions as effective theory



Why XD ~ compositeness

A hypothetical conversation

Theorist

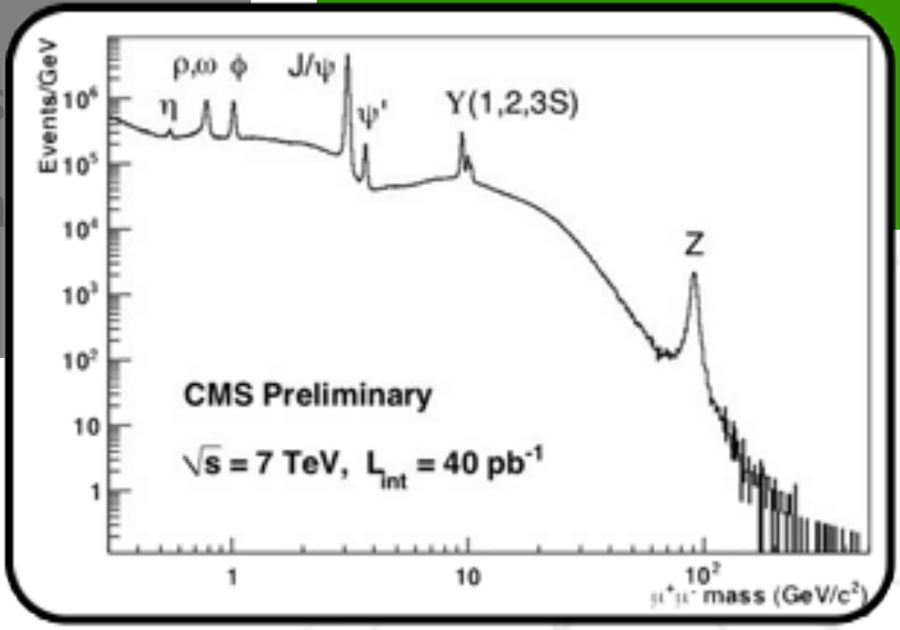
I have a new $\mathcal{L}_{\text{BSM}}$!

A tower of resonances coming from Kaluza-Klein excitations of fields living in an extra dimension. These include same-spin partners of the Standard Model fiel identified with the z ations. This solves h n by introducing a n metric...

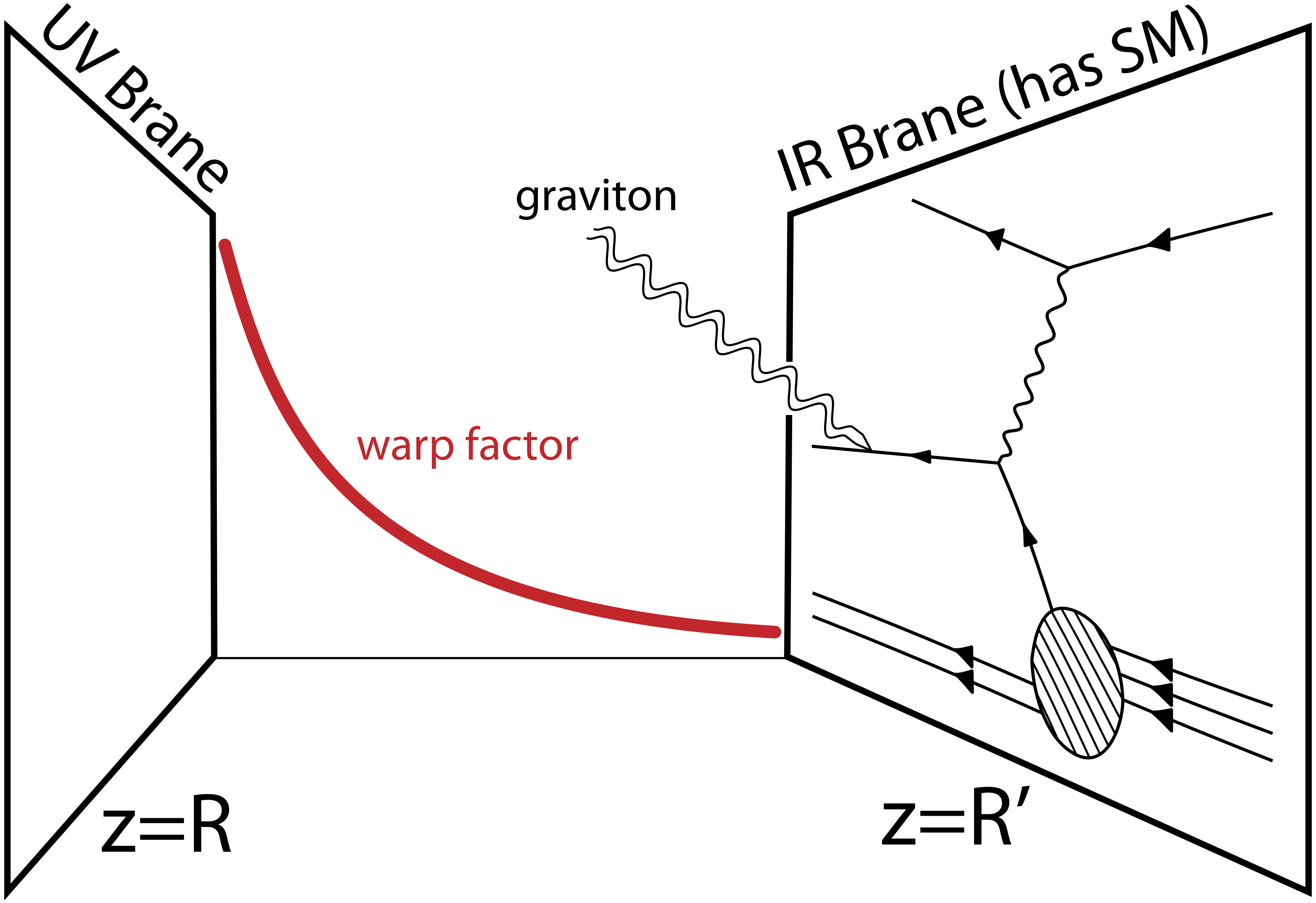
Experimentalist

Neat! What's the signal?

Oh, we already found that. It's QCD.



The Randall-Sundrum Model



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

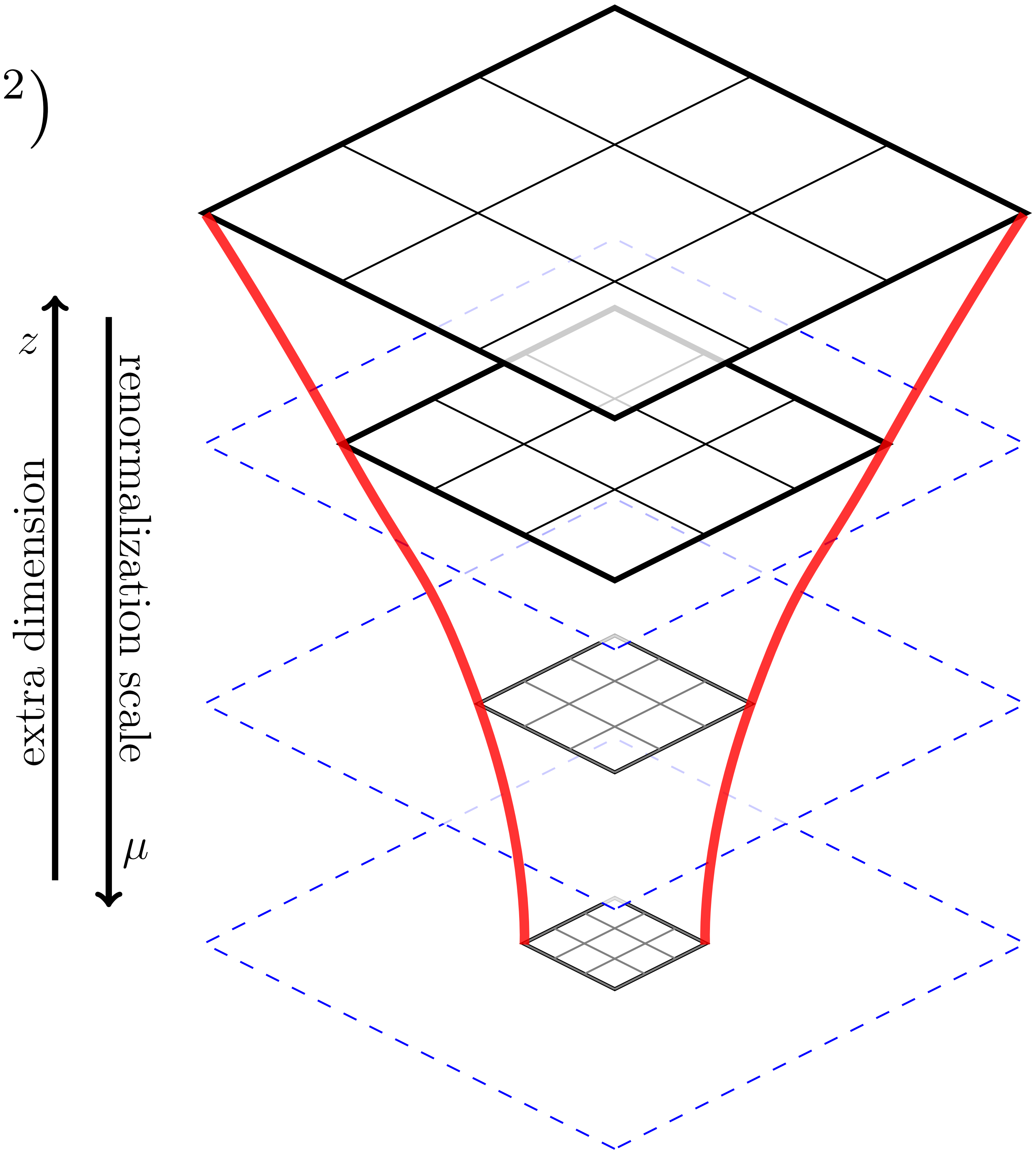
holography: geometerize RG flow

$$ds^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$x \to \alpha x$$

$$z \to \alpha z$$

$$\mu \frac{\partial}{\partial \mu} j_i(x,\mu) = \beta_i(j_j(x,\mu),\mu)$$



see e.g. Sundrum TASI lectures

explicit example: AdS/CFT

$$\mathrm{AdS}_5 \times S^5 \qquad \Longleftrightarrow \qquad \underset{\uparrow\ \text{MANY } Q\ \bar{Q}}{\underline{\mathcal{N} = 4}} \text{ super Yang-Mills.}$$

- The isometry of the S^5 space is $\mathrm{SO}(6) \cong \mathrm{SU}(4)$. This is precisely the R-symmetry group of the $\mathcal{N} = 4$ gauge theory.
- The isometry of the AdS_5 space is $\mathrm{SO}(4,2)$, which exactly matches the spacetime symmetries of a 4D conformal theory.

$$\frac{\overset{\text{RADIUS OF CURVATURE}\ \searrow}{R^4}}{\underset{\text{string length}\ \nearrow}{\ell^4}} = 4\pi \overset{\text{GAUGE}\ \downarrow}{g}^2 \overset{\swarrow\ \text{SU(N)}}{N}$$