The Standard Model

Sean Tulin

stulin@yorku.ca

Recap from yesterday

Standard Model: Want renormalizable gauge theory

Problems:

- Masses for gauge bosons violate gauge symmetry and lead to nonrenormalizable theory
- Masses for fermions incompatible with chiral gauge interactions

Solution: Higgs mechanism to break gauge theory spontaneously

- Gauge boson and fermion masses from Higgs vacuum expectation value

Consequences:

- Residual scalar degree of freedom **h** (physical Higgs boson)
- Higgs boson couplings are fixed by mass

Recap from yesterday

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{scalar} + \mathcal{L}_{fermions} + \mathcal{L}_{Yukawa}$$

Recap from yesterday

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$$|D_\mu H|^2 = \ldots \frac{1}{2} m_Z^2 Z_\mu Z^\mu + m_W^2 W_\mu^+ W^{-\mu} + \ldots$$

where $m_W = \dfrac{gv}{2}$

$m_Z = \dfrac{gv}{2c_W}$

$m_\gamma = 0$

Recap from yesterday

From tutorial: set $H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$$D_\mu H = \frac{1}{\sqrt{2}}\left(\partial_\mu + \frac{i}{2} g \begin{pmatrix} W^3_\mu & \sqrt{2} W^+_\mu \\ \sqrt{2} W^-_\mu & -W^3_\mu \end{pmatrix} + \frac{i}{2} g' \begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix}\right)\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{iv}{2\sqrt{2}}\begin{pmatrix} g\sqrt{2}\, W^+_\mu \\ (g' B_\mu - g W^3_\mu) \end{pmatrix}$$

Recap from yesterday

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$$D_\mu H = \frac{1}{\sqrt{2}}\left(\partial_\mu + \frac{i}{2} g \begin{pmatrix} W_\mu^3 & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -W_\mu^3 \end{pmatrix} + \frac{i}{2} g' \begin{pmatrix} B_\mu & 0 \\ 0 & B_\mu \end{pmatrix}\right)\begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$= \frac{iv}{2\sqrt{2}} \begin{pmatrix} g\sqrt{2}\, W_\mu^+ \\ (g' B_\mu - g W_\mu^3) \end{pmatrix}$$

$$|D_\mu H|^2 = \frac{g^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{v^2}{8}\left(g' B_\mu - g W_\mu^3\right)^2$$

Recap from yesterday

$$|D_\mu H|^2 = \frac{g^2 v^2}{4} W^+_\mu W^{-\mu} + \frac{v^2}{8}\left(g' B_\mu - g W^3_\mu\right)^2$$

$$m_W^2 = \frac{g^2 v^2}{4}$$

$$m_W = \frac{g v}{2}$$

Recap from yesterday

$$|D_\mu H|^2 = \frac{g^2 v^2}{4} W^+_\mu W^{-\mu} + \frac{v^2}{8}\left(g' B_\mu - g W^3_\mu\right)^2$$

$$\frac{v^2}{8}\left(g^2+g'^2\right)\left(\frac{g'}{\sqrt{g^2+g'^2}} B_\mu - \frac{g}{\sqrt{g^2+g'^2}} W^3_\mu\right)^2$$

Recap from yesterday

$$|D_\mu H|^2 = \frac{g^2 v^2}{4} W^+_\mu W^{-\mu} + \frac{v^2}{8}\left(g' B_\mu - g W^3_\mu\right)^2$$

$$\frac{v^2}{8}(g^2+g'^2)\left(\underbrace{\frac{g'}{\sqrt{g^2+g'^2}}}_{\sin\theta_W} B_\mu - \underbrace{\frac{g}{\sqrt{g^2+g'^2}}}_{\cos\theta_W} W^3_\mu\right)^2$$

Recap from yesterday

$$|D_\mu H|^2 = \frac{g^2 v^2}{4} W^+_\mu W^{-\mu} + \frac{v^2}{8}\left(g' B_\mu - g W^3_\mu\right)^2$$

$$\swarrow$$

$$\frac{v^2}{8}(g^2+g'^2)\left(\cos\theta_W W^3_\mu - \sin\theta_W B_\mu\right)^2$$

Recap from yesterday

Only rotated field $Z_\mu = c_W W^3_\mu - s_W B_\mu$
gets mass.

Orthogonal field $A_\mu = s_W W^3_\mu + c_W B_W$
does not (photon)

$$\frac{v^2(g^2+g'^2)}{8} Z_\mu Z^\mu = \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

Recap from yesterday

$$|D_\mu H|^2 = m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

$$m_W = \frac{gv}{2} \qquad m_Z = \frac{\sqrt{g^2+g'^2}\, v}{2} = \frac{m_W}{c_W}$$

Recap from yesterday

$$|D_\mu H|^2 = m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

$$m_W = \frac{gv}{2} \qquad m_Z = \frac{\sqrt{g^2+g'^2}\, v}{2} = \frac{m_W}{c_W}$$

Note: $\cos\theta_W < 1$

so must have $m_W < m_Z$

Recap from yesterday

$$|D_\mu H|^2 = m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} m_Z^2 Z_\mu Z^\mu$$

$\Big\downarrow$ include Higgs boson h (just shift $v \to v+h$)

$$= \frac{1}{2}(\partial_\mu h)^2 + m_W^2 W_\mu^+ W^{-\mu}\left(1+\frac{h}{v}\right)^2 + \frac{1}{2} m_Z^2 Z_\mu Z^\mu \left(1+\frac{h}{v}\right)^2$$

Recap from yesterday

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{scalar} + \mathcal{L}_{fermions} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\bar{u}_L^i \not{W}^+ d_L^i - \frac{g}{\sqrt{2}}\bar{\nu}_L^i \not{W}^+ e_L^i$$

$$-\frac{g}{\sqrt{2}}\bar{d}_L^i \not{W}^- u_L^i - \frac{g}{\sqrt{2}}\bar{e}_L^i \not{W}^- \nu_L^i$$

(So far, W interactions only connect same generation)

Recap from yesterday

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{scalar} + \mathcal{L}_{fermions} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{NC} = \sum_{\text{fermions } f} -e Q_f \bar{f}^i \not{A} f^i$$

$$- \frac{g}{c_W} \bar{f}^i \not{Z} (T^3 P_L - Q_f s_W^2) f^i$$

(Explore in more detail in today's tutorial)

Goal for today

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{scalar} + \mathcal{L}_{fermions} + \underbrace{\mathcal{L}_{Yukawa}}_{\text{fermion masses}}$$

Recall: Fermion masses in the abelian Higgs model

$$\mathcal{L}_{\text{Yukawa}} = -y \overline{\Psi}_L \Psi_R \phi + \text{h.c.}$$

where fields have charges

ϕ : g

Ψ_L : g_L

Ψ_R : g_R

Need $g_L = g_R + g$

Recall: Fermion masses in the abelian Higgs model

$$\mathcal{L}_{\text{Yukawa}} = -y\,\overline{\Psi}_L \Psi_R \phi + \text{h.c.}$$

where fields have charges

$$\phi : \quad g = Q_\phi g \qquad (Q_\phi = +1)$$

$$\Psi_L : \quad g_L = Q_L g$$

$$\Psi_R : \quad g_R = Q_R g$$

Need $g_L = g_R + g$ or $Q_L = Q_R + Q_\phi$

How does this work in the Standard Model?

Need to generalize to $SU(3)_C \times SU(2)_L \times U(1)_Y$

Usual mass terms are forbidden
by $SU(2)_L \times U(1)_Y$ gauge symmetry

How does this work in the Standard Model?

Need to generalize to $SU(3)_C \times SU(2)_L \times U(1)_Y$

e.g. $\bar{u}u = \bar{u}_L u_R + \bar{u}_R u_L$

u_L have hypercharges $1/6$
u_R $2/3$

u_L transforms as part of $SU(2)_L$ doublet, while u_R does not

How does this work in the Standard Model?

Need to generalize to $SU(3)_C \times SU(2)_L \times U(1)_Y$

what Yukawa interactions are allowed?

We need to recall some group theory for SU(2)

Let η, ξ be two 2-component vectors (fundamental rep.) of $SU(2)$

$SU(2)$ transformation: $U \in SU(2)$
(2x2 matrix)

$$\eta \to U\eta$$

$$\xi \to U\xi$$

We need to recall some group theory for SU(2)

Two ways to make SU(2) invariant contraction of η & ξ:

$$\eta^\dagger \xi \qquad \text{and} \qquad \eta^T \varepsilon \xi$$

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

antisym. tensor

We need to recall some group theory for SU(2)

Two ways to make SU(2) invariant contraction of η & ξ:

$$\eta^\dagger \xi \qquad \text{and} \qquad \eta^T \varepsilon \xi$$

SU(2) $\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

$$\eta^\dagger \underbrace{U^\dagger U}_{\mathbb{1}} \xi = \eta^\dagger \xi \qquad\qquad \eta^T \underbrace{U^T \varepsilon\, U}_{\det(U)\varepsilon = \varepsilon} \xi = \eta^T \varepsilon \xi$$

We need to recall some group theory for SU(2)

Two SU(2) - invariant tensors:

Identity $\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Antisym. tensor $\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

Standard Model Lagrangian

Fermions	Quantum numbers ($SU(3)_C$, $SU(2)_L$, $U(1)_Y$)	
$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	$(3, 2, \frac{1}{6})$	quarks
u_R^i	$(3, 1, 2/3)$	
d_R^i	$(3, 1, -1/3)$	
$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$	$(1, 2, -1/2)$	leptons
e_R^i	$(1, 1, -1)$	

i = 1,2,3 labels **generation**. All fermions with same quantum numbers come in three copies.

Scalar

$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$	$(1, 2, 1/2)$	Higgs scalar doublet

Valid Yukawa interactions

$$H^\dagger \bar{d}_R Q_L \qquad \bar{u}_R Q_L^T \varepsilon H \qquad H^\dagger \bar{e}_R L_L$$

$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ acts on $SU(2)_L$ doublets

e.g. $Q_L^T \varepsilon H = (u_L, d_L) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$

Valid Yukawa interactions

$$H^\dagger \bar{d}_R Q_L \qquad \bar{u}_R Q_L^T \varepsilon H \qquad H^\dagger \bar{e}_R L_L$$

Invalid Yukawa interactions

$$H^\dagger \bar{d}_R L_L \qquad \bar{e}_R Q_L^T \varepsilon H \qquad H^\dagger \bar{u}_R L_L$$

$$\bar{d}_R Q_L^T \varepsilon H \qquad H^\dagger \bar{u}_R Q_L \qquad \bar{e}_R L_L^T \varepsilon H$$

Yukawa interactions can couple any two generations i,j

Most general Yukawa Lagrangian:

$$\mathcal{L}_{\text{Yukawa}} = - Y^{(u)}_{ij}\, \bar{u}^{i}_{R}\, Q^{j\,T}_{L}\, \varepsilon\, H$$

$$- Y^{(d)}_{ij}\, H^{\dagger}\, \bar{d}^{i}_{R}\, Q^{j}_{L}$$

$$- Y^{(e)}_{ij}\, H^{\dagger}\, \bar{e}^{i}_{R}\, L^{j}_{L}$$

$$+ \text{h.c.}$$

Let Higgs field get vev $H \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

$$\mathcal{L}_{Yukawa} = - Y^{(u)}_{ij} \, \bar{u}^i_R \, (u^j_L, d^j_L) \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

$$- Y^{(d)}_{ij} \left(0, \frac{v}{\sqrt{2}}\right) \bar{d}^i_R \begin{pmatrix} u^j_L \\ d^j_L \end{pmatrix}$$

$$- Y^{(e)}_{ij} \left(0, \frac{v}{\sqrt{2}}\right) \bar{e}^i_R \begin{pmatrix} \nu^j_L \\ e^j_L \end{pmatrix}$$

$$+ \text{h.c.}$$

Let Higgs field get vev $H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$$\mathcal{L}_{\text{Yukawa}} = -Y^{(u)}_{ij}\frac{v}{\sqrt{2}}\bar{u}^i_R u^j_L - Y^{(d)}_{ij}\frac{v}{\sqrt{2}}\bar{d}^i_R d^j_L$$

$$- Y^{(e)}_{ij}\frac{v}{\sqrt{2}}\bar{e}^i_R e^j_L + \text{h.c.}$$

Let Higgs field get vev $H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$

$$\mathcal{L}_{\text{Yukawa}} = -Y^{(u)}_{ij} \frac{v}{\sqrt{2}} \bar{u}^i_R u^j_L - Y^{(d)}_{ij} \frac{v}{\sqrt{2}} \bar{d}^i_R d^j_L$$

$$- Y^{(e)}_{ij} \frac{v}{\sqrt{2}} \bar{e}^i_R e^j_L + \text{h.c.}$$

We want fermion mass terms that are diagonal, but matrices $Y^{(u,d,e)}_{ij}$

three are arbitrary 3x3 complex matrices

We need to diagonalize

Hermitian matrix can be diagonalized with a unitary transformation

$$M \rightarrow U^{\dagger} M U = M_d = \begin{pmatrix} m_1 & & \\ & \ddots & \\ & & \end{pmatrix}$$

Arbitrary (non-Hermitian) square matrix can be diagonalized with a **biunitary** transformation with entries that are **real and positive**

$$M \rightarrow U^{\dagger} M V = M_d = \begin{pmatrix} m_1 & & \\ & \ddots & \\ & & \end{pmatrix}$$

U, V both unitary

Proof: M is square matrix

Note: $MM^\dagger$ is Hermitian and can be diagonalize by

$$U^\dagger M M^\dagger U = M_d^2 = \text{real, positive entries}$$

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Define M_d as positive sq. root of M_d^2

Proof: M is square matrix

Note: $MM^\dagger$ is Hermitian and can be diagonalize by

$$U^\dagger M M^\dagger U = M_d^2 = \text{real, positive entries}$$

Define M_d as positive sq. root of M_d^2

Then $U^\dagger M M^\dagger U M_d^{-1} = M_d$

Proof: M is square matrix

Note: $MM^\dagger$ is Hermitian and can be diagonalize by

$$U^\dagger M M^\dagger U = M_d^2 = \text{real, positive entries}$$

Define M_d as positive sq. root of M_d^2

Then
$$U^\dagger M \underbrace{M^\dagger U M_d^{-1}}_{\text{this our unitary matrix } V} = M_d$$

$$\rightarrow \quad U^\dagger M V = M_d$$

Just need to show $V = M^{\dagger} U M_d^{-1}$ is unitary

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$$V^{\dagger} V = \underbrace{M_d^{-1} U^{\dagger} M}_{V^{\dagger}} \underbrace{M^{\dagger} U M_d^{-1}}_{V}$$

Just need to show $V = M^{\dagger} U M_d^{-1}$ is unitary

$$V^{\dagger} V = M_d^{-1} \underbrace{U^{\dagger} M M^{\dagger} U}_{} M_d^{-1}$$

$$= M_d^{-1} M_d^{2} M_d^{-1} = 1$$

So both U, V unitary

Back to the Yukawa Lagrangian

$$\mathcal{L}_{Yukawa} = -\bar{u}_R M^{(u)} u_L \left(1+\frac{h}{v}\right)$$
$$-\bar{d}_R M^{(d)} d_L \left(1+\frac{h}{v}\right)$$
$$-\bar{e}_R M^{(e)} e_L \left(1+\frac{h}{v}\right) + h.c.$$

$$M^{(u,d,e)}_{ij} = Y^{(u,d,e)}_{ij} \frac{v}{\sqrt{2}}, \quad u_L = \begin{pmatrix} u_L^1 \\ u_L^2 \\ u_L^3 \end{pmatrix} \text{ etc.}$$

3x3 matrices in generation $(i,j = 1\ldots3)$

Now, diagonalize with biunitary transformation:

$$U_{u_R}^{\dagger} M^{(u)} U_{u_L} = m^{(u)} = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

$$U_{d_R}^{\dagger} M^{(d)} U_{d_L} = m^{(d)} = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}$$

$$U_{e_R}^{\dagger} M^{(e)} U_{e_L} = m^{(e)} = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}$$

U's are all unitary

Now, rotate the fermion fields to define a new basis, **mass eigenstate basis**

$$u_{L,R} = U_{u_{L,R}} \, u'_{L,R}$$

$\uparrow$ original basis (flavor basis)

$\uparrow$ mass basis

Now, rotate the fermion fields to define a new basis, **mass eigenstate basis**

$$u_{L,R} = U_{u_{L,R}} u'_{L,R}$$

$$\bar{u}_R M^{(u)} u_L + h.c. = \bar{u}'_R U^{\dagger}_{u_R} M^{(u)} U_{u_L} u'_L + h.c.$$

$$= \bar{u}'_R m^{(u)} u'_L + \bar{u}'_L m^{(u)} u'_R$$

$$= \bar{u}' m^{(u)} u'$$

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$$\bar{u}_R M^{(u)} u_L + h.c. = \bar{u}'_R U^{\dagger}_{u_R} M^{(u)} U_{u_L} u'_L + h.c.$$

$$= \bar{u}'_R m^{(u)} u'_L + \bar{u}'_L m^{(u)} u'_R$$

$$= \bar{u}' m^{(u)} u'$$

$$= (\bar{u}', \bar{c}', \bar{t}') \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u' \\ c' \\ t' \end{pmatrix}$$

We have usual Dirac mass terms

Now, rotate the fermion fields to define a new basis, **mass eigenstate basis**

$$u_{L,R} = U_{u_{L,R}} u'_{L,R}$$

$$d_{L,R} = U_{d_{L,R}} d'_{L,R}$$

$$e_{L,R} = U_{e_{L,R}} e'_{L,R}$$

For all fermions $\psi = u^{i\prime}, d^{i\prime}, e^{i\prime}$

$$\mathcal{L}_{Yukawa} = -\sum_{\psi} m_f \bar{\psi}\psi \left(1 + \frac{h}{v}\right)$$

Now, rotate the fermion fields to define a new basis, **mass eigenstate basis**

$$u_{L,R} = U_{u_{L,R}} u'_{L,R}$$

$$d_{L,R} = U_{d_{L,R}} d'_{L,R}$$

$$e_{L,R} = U_{e_{L,R}} e'_{L,R}$$

Since no mass term for the neutrinos, we are free to redefine the neutrino fields anyway we want

$$\nu_L = U_{e_L} \nu'_L$$

choose same rotation as e_L

We have changed our fields to the mass eigenstate basis

Need to check what happens to the gauge interactions

Neutral current interactions

$$\mathcal{L}_{NC} = \sum_{\text{fermion } \psi} -e Q_\psi \bar{\psi} \not{A} \psi$$

$$- \frac{g}{c_W} \bar{\psi} \not{Z} (T^3 P_L - Q_\psi s_W^2) \psi$$

Neutral current interactions

Consider u^i fields

$$\mathcal{L}_{NC} = \sum_{i=1}^{3} -Q_u\left(\bar{u}_L^i \gamma^\mu u_L^i + \bar{u}_R^i \gamma^\mu u_R^i\right) A_\mu$$

$$-\frac{g}{c_W}\left[\bar{u}_L^i \gamma^\mu u_L^i \left(\frac{1}{2} - Q_u s_W^2\right) + \bar{u}_R^i \gamma^\mu u_R^i \left(-Q_u s_W^2\right)\right] Z_\mu$$

Now transform to primed fields (mass eigenstate fields)

$$\text{e.g.} \sum_{i=1}^{3} \bar{u}_L^i \gamma^\mu u_L^i = \bar{u}_L \gamma^\mu u_L \qquad \left(\begin{array}{c} \text{Shorthand} \\ u = \begin{pmatrix} u^1 \\ u^2 \\ u^3 \end{pmatrix} \end{array} \right)$$

$$= \bar{u}_L' U_{u_L}^\dagger \gamma^\mu U_{u_L} u_L'$$

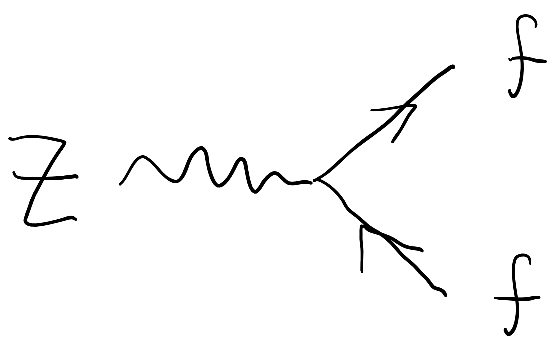
$$= \bar{u}_L' \gamma^\mu u_L' \qquad \text{since } U_{u_L}^\dagger U_{u_L} = 1$$

Now transform to primed fields (mass eigenstate fields)

Same for all other fields $u_R, d_{L,R}, e_{L,R}, \nu_L$

Because rotation of basis is **unitary**, the neutral current interactions are invariant

Neutral current was diagonal in original basis, still diagonal in mass eigenstate basis



$$= -i \frac{g}{c_W} \gamma^\mu \left(T_3 P_L - Q_f s_W^2 \right)$$

f in $\rightarrow$ f out

Charged current interaction

$$\mathcal{L}_{cc} = \sum_{i=1}^{3} -\frac{g}{\sqrt{2}} \left(\bar{u}_L^i \cancel{W}^+ d_L^i + \bar{\nu}_L^i \cancel{W}^+ e_L^i \right)$$

$$+ h.c.$$

Charged current interaction

$$\mathcal{L}_{cc} = \sum_{i=1}^{3} -\frac{g}{\sqrt{2}} \left(\bar{u}_L^i \not{W}^+ d_L^i + \bar{\nu}_L^i \not{W}^+ e_L^i \right) + h.c.$$

$$= -\frac{g}{\sqrt{2}} \left(\bar{u}_L' U_{u_L}^\dagger \not{W}^+ U_{d_L} d_L' + \bar{\nu}_L' U_{e_L}^\dagger \not{W}^+ U_{e_L} e_L' \right) + h.c.$$

shorthand

$$d = \begin{pmatrix} d^1 \\ d^2 \\ d^3 \end{pmatrix}$$

Charged current interaction

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left(\bar{u}_L' U_{u_L}^{\dagger} \not{W}^{+} U_{d_L} d_L' + \bar{\nu}_L' U_{e_L}^{\dagger} \not{W}^{+} U_{e_L} e_L'\right) + h.c.$$

Lepton CC interaction is still diagonal $U_{e_L}^{\dagger} U_{e_L} = \underline{1}$

- We defined our neutrino states to correspond to each charged lepton mass eigenstate

Charged current interaction

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left(\bar{u}'_L U^\dagger_{u_L} \not{W}^+ U_{d_L} d'_L + \bar{\nu}'_L U^\dagger_{e_L} \not{W}^+ U_{e_L} e'_L\right) + h.c.$$

Lepton CC interaction is still diagonal $U^\dagger_{e_L} U_{e_L} = \mathbb{1}$

- We defined our neutrino states to correspond to each charged lepton mass eigenstate

Quark CC interaction is not diagonal

$$U^\dagger_{u_L} U_{d_L} \neq \mathbb{1} \quad \text{in general}$$

Charged current interaction

$$V = U_{u_L}^{\dagger} U_{u_R}$$

Unitary matrix known as the Cabibbo-Maskawa-Kobayashi (CKM) matrix

Charged current interaction

$$V = U_{u_L}^{\dagger} U_{u_R}$$

Unitary matrix known as the Cabibbo-Maskawa-Kobayashi (CKM) matrix

Let's drop the primes from now on and forever more work in the mass eigenstate basis

$$\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}\left(\bar{u}_L^i V_{ij} \not{W}^{+} d_L^j + \bar{\nu}_L^i \not{W}^{+} e_L^i\right) + h.c.$$

V_{ij} : up-type $i \longleftrightarrow$ down-type j

Charged current interaction

$$V = U_{u_L}^{\dagger} U_{u_R}$$

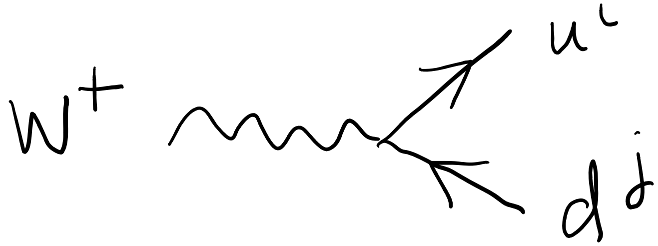
Unitary matrix known as the Cabibbo-Maskawa-Kobayashi (CKM) matrix

$$= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

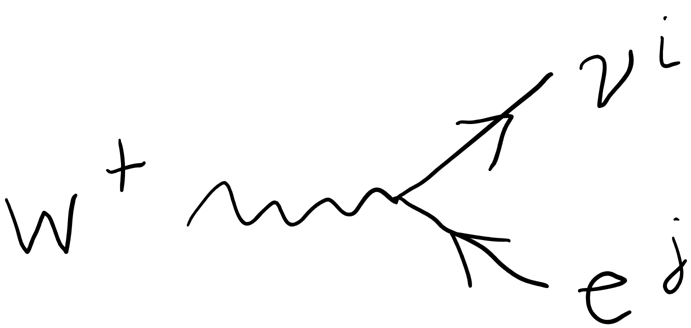
9 complex parameters

$\rightarrow$ but actually only 4 (real) parameters (since V unitary + 5 phases can be absorbed into the fields)

Charged current interaction



$$= -\frac{ig}{\sqrt{2}} V_{ij} \gamma^\mu P_L$$

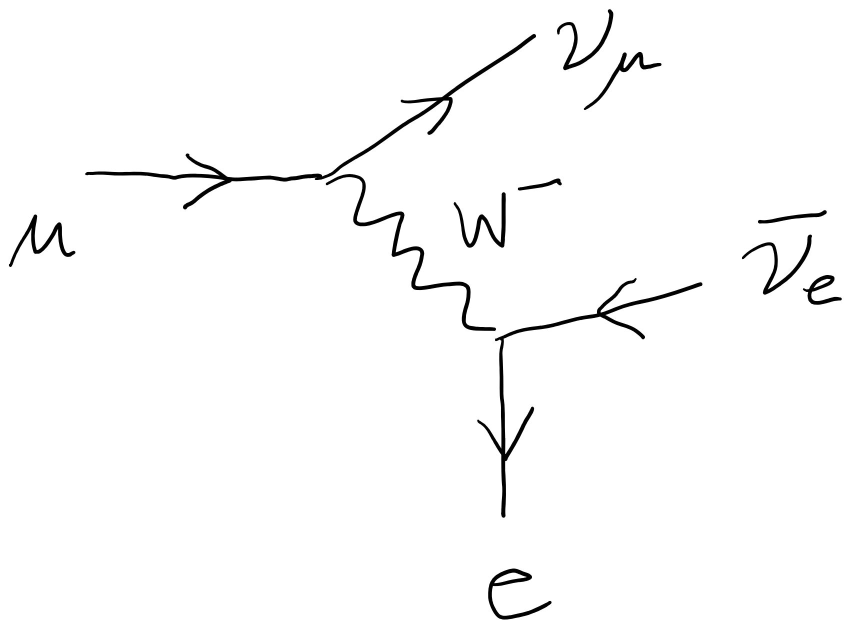


$$= -\frac{ig}{\sqrt{2}} \delta_{ij} \gamma^\mu P_L$$

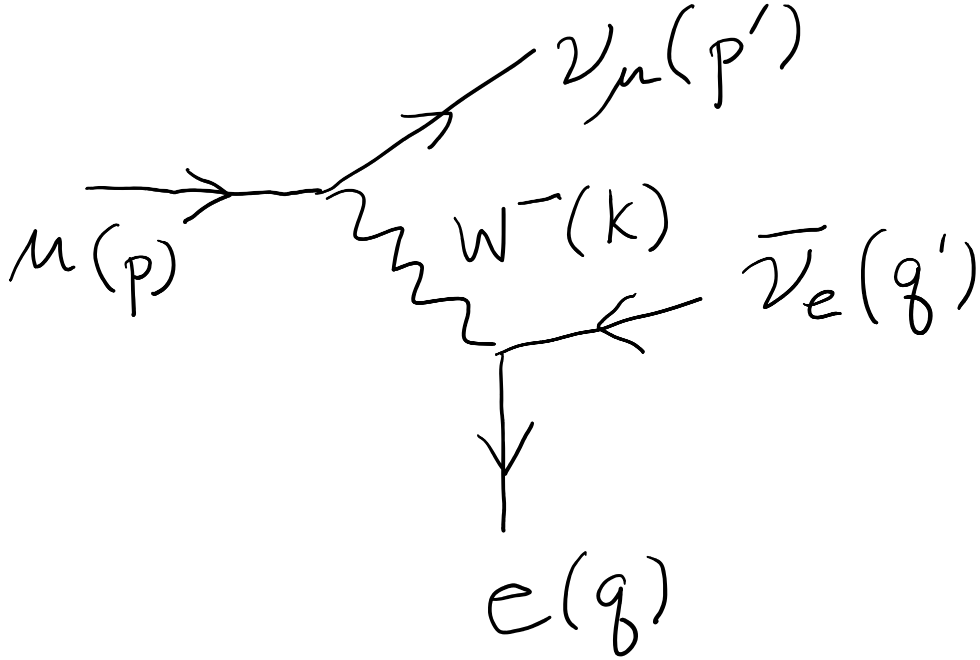
Effective theory of the weak interaction

At energies $E \ll m_W, m_Z$, don't need full $\mathcal{L}_{SM}$. Use effective Lagrangian where we "integrate out" the W, Z and express their interactions as nonrenormalizable operators

Example: μ decay



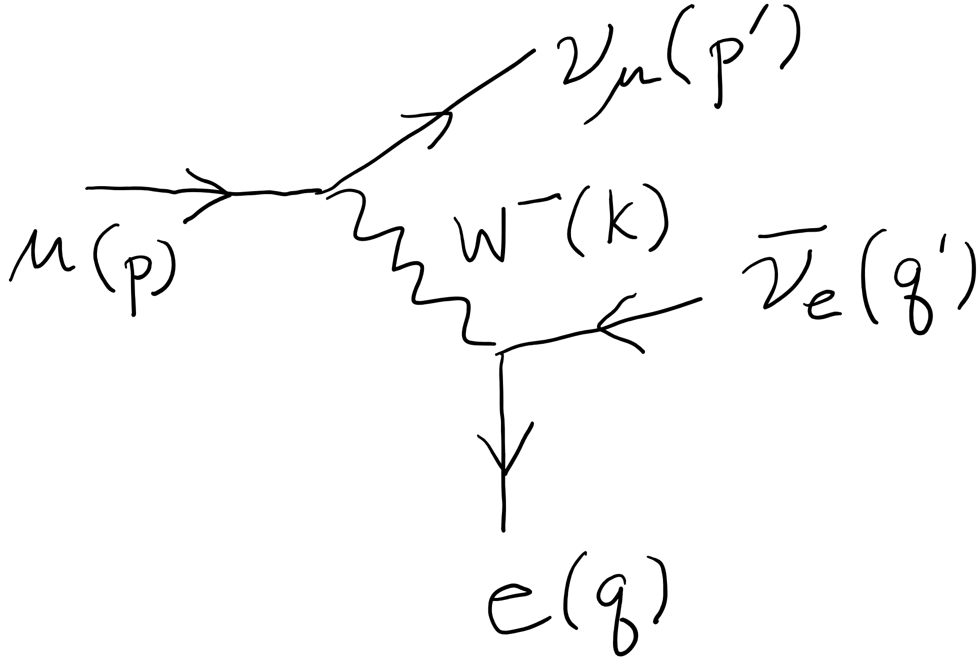
Example: μ decay



W boson momentum

$$k = p - p' = q + q'$$

Example: μ decay

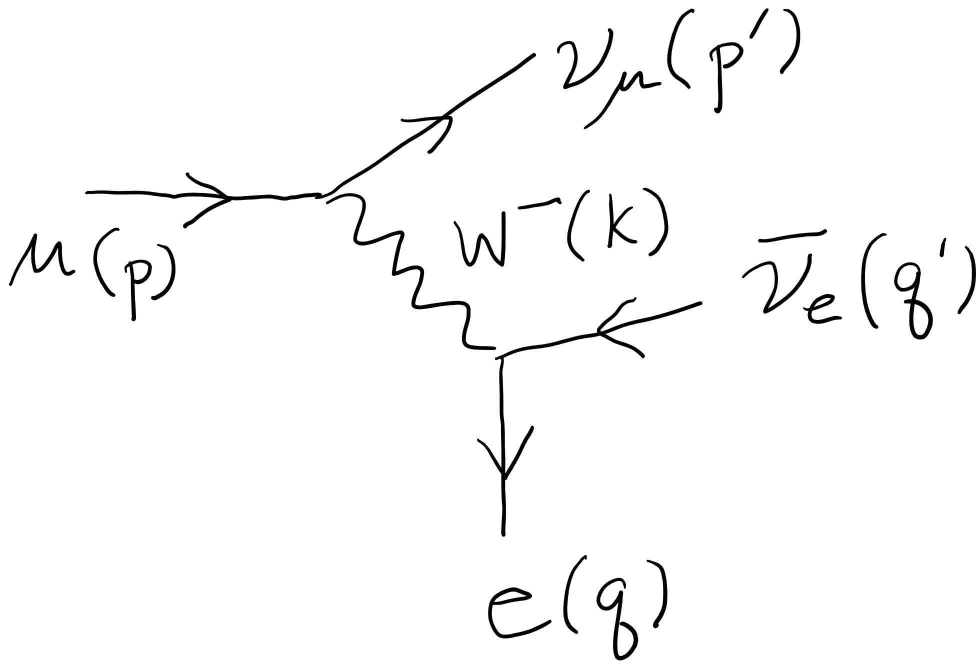


W boson momentum

$$k = p - p' = q + q'$$

$$i\mathcal{M} = \bar{u}_e\left(\frac{-ig}{\sqrt{2}}\right)\gamma^\mu P_L v_{\nu_e}\,\frac{-i\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_W^2}\right)}{k^2 - m_W^2}\,\bar{u}_{\nu_\mu}\left(\frac{-ig}{\sqrt{2}}\right)\gamma^\nu P_L u_\mu$$

Example: μ decay



W boson momentum

$$k = p - p' = q + q'$$

$k \sim m_\mu << m_W$

only keep leading terms

$$i\mathcal{M} = \bar{u}_e \left(\frac{-ig}{\sqrt{2}}\right)\gamma^\mu P_L v_{\nu_e} \frac{-i\left(\eta_{\mu\nu} - \cancel{\frac{k_\mu k_\nu}{m_W^2}}\right)}{\cancel{k^2} - m_W^2} \bar{u}_{\nu_\mu} \left(\frac{-ig}{\sqrt{2}}\right)\gamma^\nu P_L u_\mu$$

Example: μ decay

$$i\mathcal{M} = \frac{ig^2}{8m_W^2} \bar{u}_e \gamma^\nu (1-\gamma^5) v_{\nu_e} \bar{u}_{\nu_\mu} \gamma_\nu (1-\gamma^5) u_\mu$$

Example: μ decay

$$i\mathcal{M} = \frac{ig^2}{8m_W^2}\,\bar{u}_e\gamma^\nu(1-\gamma^5)v_{\nu_e}\;\bar{u}_{\nu_\mu}\gamma_\nu(1-\gamma^5)u_\mu$$

The same matrix element can be generated from a nonrenormalizable operator

$$\mathcal{L} = \frac{G_F}{\sqrt{2}}\,\bar{e}\gamma^\nu(1-\gamma^5)\nu_e\;\bar{\nu}_\mu\gamma_\nu(1-\gamma^5)\mu \quad + h.c.$$

where $\dfrac{G_F}{\sqrt{2}} = \dfrac{g^2}{8m_W^2}$ Fermi's constant

Fermi theory

Charge current interactions at low energy described by four fermion interactions [(V-A)-theory]

$$\mathcal{L}_{Fermi} = \frac{G_F}{\sqrt{2}} \sum_{i,j} \bar{e}^i \gamma^\mu (1-\gamma 5) \nu^i \; \bar{\nu}^j \gamma_\mu (1-\gamma^5) e^j$$

(leptonic)

$$+ \frac{G_F}{\sqrt{2}} \sum_{i,j,k} V_{ij} \; \bar{u}^i \gamma^\mu (1-\gamma^5) d^j \, \bar{e}^k \gamma_\mu (1-\gamma^5) \nu^k$$

(semi leptonic)

$$+ \frac{G_F}{\sqrt{2}} \sum_{ijkl} V_{ij} V^*_{kl} \; \bar{u}^i \gamma^\mu (1-\gamma 5) d^j \, \bar{d}^l \gamma_\mu (1-\gamma^5) u^k$$

(hadronic)

$+ h.c.$

Some take away messages

1. Universality of the weak interaction

All Fermi-theory interactions governed by only five parameters

G_F, CKM matrix (4 parameters)

Motivated weak force as a gauge theory

(Otherwise, every term $ijkl$ has its own coefficient)

Some take away messages

2. Higgs vev

Using $m_W = \dfrac{gv}{2}$, $\dfrac{G_F}{\sqrt{2}} = \dfrac{g^2}{8m_W^2} = \dfrac{1}{2v^2}$

$$\rightarrow \quad v = \left(\sqrt{2}\, G_F\right)^{-1/2}$$

Some take away messages

2. Higgs vev $v = \left(\sqrt{2}\, G_F\right)^{-1/2}$

$G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$ from μ decay

$\rightarrow \quad v = 246\ \mathrm{GeV}$

Some take away messages

2. Higgs vev $v = 246$ GeV

Note: alternative convention

$$H = \begin{pmatrix} 0 \\ v + \frac{h}{\sqrt{2}} \end{pmatrix} \text{ instead of } H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$\hookrightarrow$ $v = 174$ GeV

G_F same in all conventions

Standard Model: Is it a beautiful theory?

- Three gauge groups
- Three generations
- 19 free parameters

3 gauge couplings g, g', g_s

2 Higgs parameters $(\mu^2, \lambda) \xrightarrow{\text{or}} (m_h, G_F)$

9 Fermion masses $(m_e, m_\mu, m_\tau, m_u, m_c, m_t,$
$m_d, m_s, m_b)$

4 CKM matrix parameters

1 angle θ_{QCD} (seems to be $\approx$ zero)

Standard Model: Is it a beautiful theory?

The Standard Model is an ugly theory, except compared to all other Beyond the Standard Model theories.