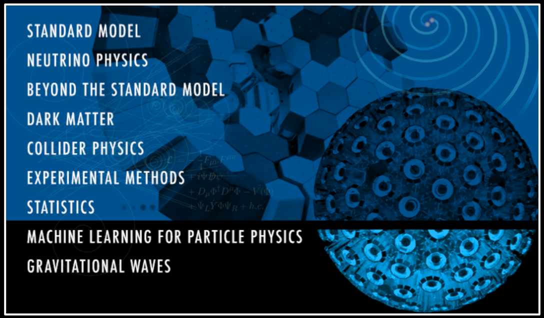


TRISEP lectures on Gravitational Waves

Djuna Croon, TRIUMF

Yesterday

THE QUADRUPOLE APPROXIMATION
BINARY MERGERS

EFE for a metric perturbation

$$\partial^\nu h_{\mu\nu} = 0$$

Lorentz gauge

$$\Box h_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

Metric perturbation

Source: energy momentum tensor

GW Solutions

- For far away, non-relativistic sources, we found

$$h_{ij}^{TT} = \left[h_{ij}^{TT}\right]_{\text{quad}} + \dots$$

$$\left[h_{ij}^{TT}\right]_{\text{quad}} = \frac{1}{r}\frac{2G}{c^4}\Lambda_{ij,kl}\ddot{M}_{kl}(t - r/c)$$

- Gravitational waves are generated by (huge) accelerated mass distributions with a nonzero mass quadrupole moment

Inspiral of a binary merger

Other assumptions:

- $m_1 = m_2$
- Circular orbits

- No backreaction,

$$h_+ = \frac{1}{r}\frac{4G\mu\omega^2 R^2}{c^4}\left(\frac{1+\cos^2\theta}{2}\right)\cos(2\omega t + 2\phi)$$

$$h_\times = \frac{2}{r}\frac{4G\mu\omega^2 R^2}{c^4}\cos\theta\sin(2\omega t + 2\phi)$$

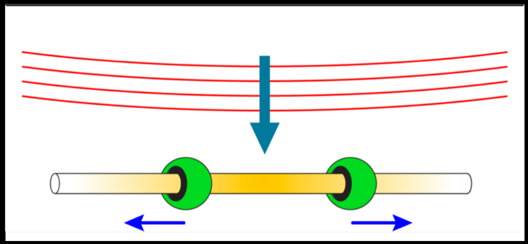
- But in reality there is backreaction

$$P_{\mathrm{GW}} = \dot{E}_{\mathrm{orbit}} \qquad E_{\mathrm{orbit}} = -G\frac{m_1 m_2}{2r}$$

The sticky bead revisited:

Why does the stick not stretch?

- No concept of a gravitational force in GR
- However, we can consider the "stretching" of the stick due to GW strain,



$$h = A\sin(\omega t)$$
$$L = L_0(1 + A\sin(\omega t))$$

- Molecular forces mediated by photons
- Compare ω_{GW} to ω in the material
  - Molecular vibrations, $\omega \sim 10^{14}\ \mathrm{Hz}$

Gravitational backreaction

- Orbital frequency: Kepler's 3rd law

$$\omega^2 = G_N \frac{m_1 + m_2}{r^3}$$

- GW emission drains energy from the system,

$$P_{\rm GW} = \dot{E}_{\rm orbit}$$

$$E_{\rm orbit} = E_{\rm kin} + E_{\rm pot} = -G \frac{m_1 m_2}{2r}$$

GW emission implies that the orbital radius decreases and the frequency increases

Chirp signal

- If the orbits are still quasi-circular, we can use what we have derived so far
- To find the frequency as a function of time, we can solve $P_{\mathrm{GW}} = \dot{E}_{\mathrm{orbit}}$

$$M_c = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}}$$

$$P_{\mathrm{gw}} = \frac{32}{5}\frac{c^5}{G}\left(\frac{G M_c \omega_{\mathrm{gw}}}{2c^3}\right)^{10/3}$$

$$f_{\mathrm{gw}}(t) = \frac{1}{\pi}\left(\frac{5}{256}\frac{1}{t}\right)^{3/8}\left(\frac{G M_c}{c^3}\right)^{-5/8}$$

Here t is defined as the time left until coalescence!

Gravitational wave

DETECTION

Detectors

- In general, detectors will measure,

$$h(t) = D^{ij} h_{ij}(t)$$

GW strain ↑ $h(t)$

Parametrizes detector geometry ↑ D^{ij}

Metric perturbation ↑ h_{ij}

- Detectors are only sensitive to strain in the direction of D^{ij}

Gravitational wave strain

- Characteristic strain: displacement of test masses in the gravitational field
  $\Delta L/L \sim D^{ij}h_{ij}$
- Note that this quantity falls off as r^{-1}

$$\left[h_{ij}^{TT}\right]_{\rm quad} = \frac{1}{r}\frac{2G}{c^4}\Lambda_{ij,kl}\ddot{M}_{kl}(t-r/c)$$

Q: Detector sensitivity to EM radiation falls of as r^{-2}, what is the difference?

Gravitational wave strain

- Characteristic strain: displacement of test masses in the gravitational field
  $\Delta L/L \sim D^{ij}h_{ij}$
- Note that this quantity falls off as r^{-1}

$$\left[h_{ij}^{TT}\right]_{\text{quad}} = \frac{1}{r}\frac{2G}{c^4}\Lambda_{ij,kl}\ddot{M}_{kl}(t-r/c)$$

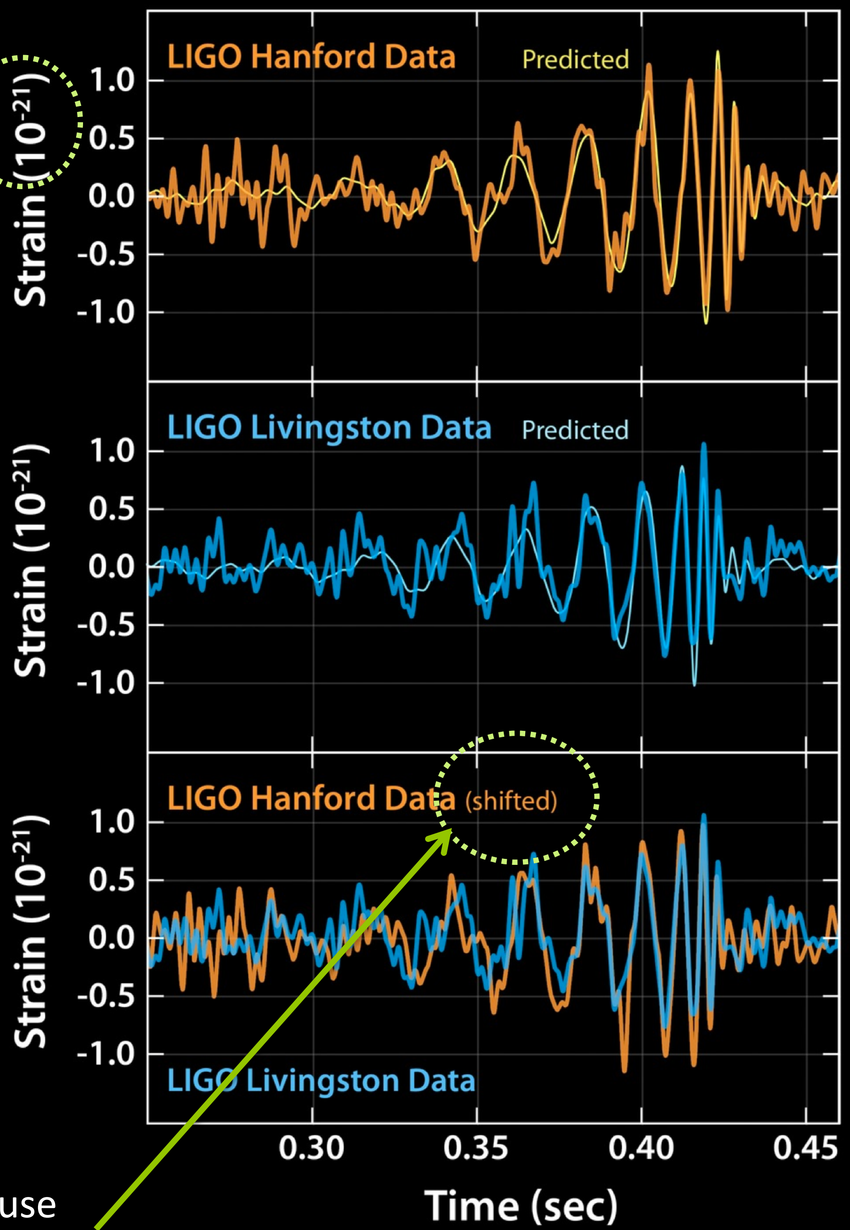
A: For EM radiation, you measure the deposited energy, which goes as the square of the amplitude.

BBH merger

The first LIGO detection (2015):

- 30 solar mass BHs
- 1.3 billion ly away

Q: Does LIGO probe a BNS merger for a longer or shorter period, and why?



Shifted and inverted (because of the different orientation) to cross-correlate

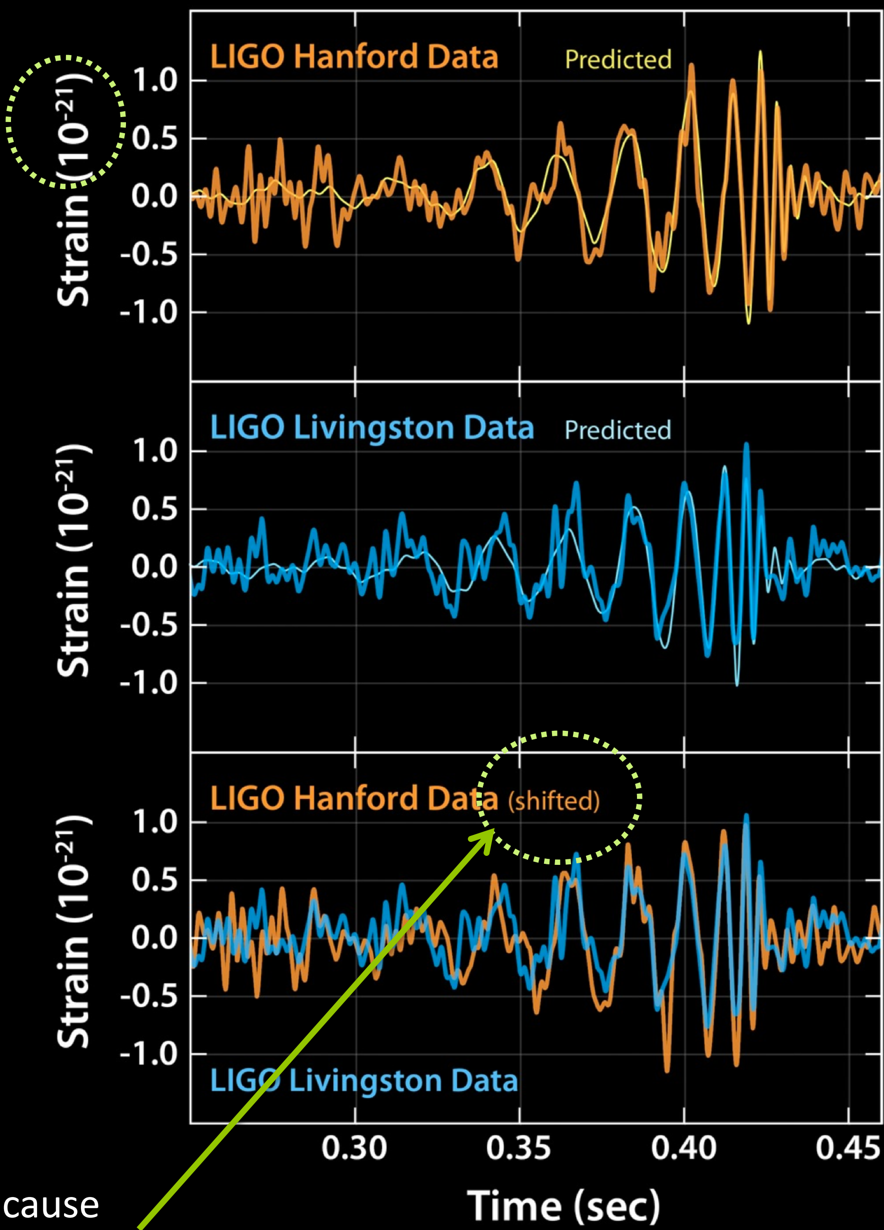
Source: ligo.caltech.edu

BBH merger

The first LIGO detection (2015):

- 30 solar mass BHs
- 1.3 billion ly away

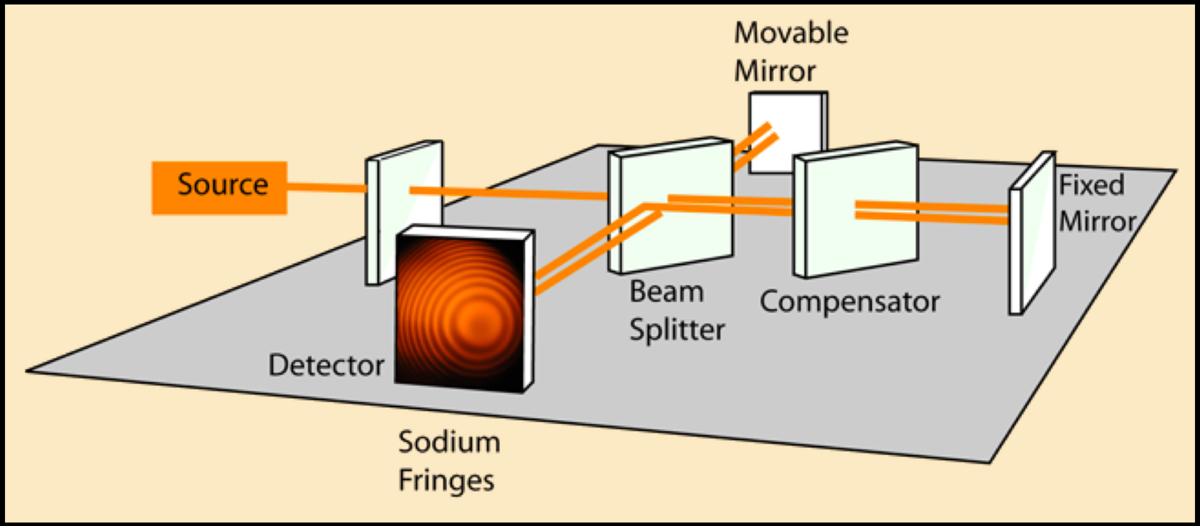
A: longer, as the binary loses energy (and hence radius) in proportion to the chirp mass



Shifted and inverted (because of the different orientation) to cross-correlate

Source: ligo.caltech.edu

Michelson interferometers



A Michelson interferometer, from Hyperphysics.com

LIGO and Virgo are Michelson interferometers with Fabry Perot cavities (and power recycling mirrors)

FP cavities effectively enlarge the arm length, to enhance the visibility of deviations

Interferometers

- Armlengths:
  - LIGO: $4\ \mathrm{km}$, but reflected 400x, such that the effective armlength = $1600\ \mathrm{km}$
  - LISA: $2.5\ \mathrm{million\ km}$
- The longer the arms, the smaller the frequencies the experiment probes

$$\lambda_{GW} \sim L \quad \rightarrow \quad \frac{c}{L_{\mathrm{LIGO}}} \sim 10^2\,\mathrm{Hz}$$

$$\rightarrow \quad \frac{c}{L_{\mathrm{LISA}}} \sim 10^{-1}\,\mathrm{Hz}$$

Signal and noise

- The signal will be something like,

$$s(t) = n(t) + h(t)$$

Detector output — Noise — Gravitational wave strain

- Unfortunately, $|h(t)| \ll |n(t)|$ is not unusual
- To dig the GW signal out of the noise, we can use the fact that they are uncorrelated

Noise

- If we assume the noise is stationary,

$$\langle \tilde{n}^*(f)\tilde{n}(f')\rangle \equiv \delta(f-f')\frac{1}{2}S_n(f)$$

This factor is here such that we can integrate over physical $f>0$

- $S_n(f)$ is the noise spectral density
- Alternative definition in terms of auto-correlation function of the source

Matched filtering

- Imagine we know the form of the GW strain $h(t)$ well,

$$s(t) = n(t) + h(t)$$

$$\int_0^{t_{\rm obs}} s(t)h(t)dt = \int_0^{t_{\rm obs}} n(t)h(t)dt + \int_0^{t_{\rm obs}} h^2(t)dt$$

In general, may use another *filter function*, optimized to pick out the GW strain

Oscillating, grows much slower with t_{obs}

Positive definite, so grows with t_{obs}

SNR

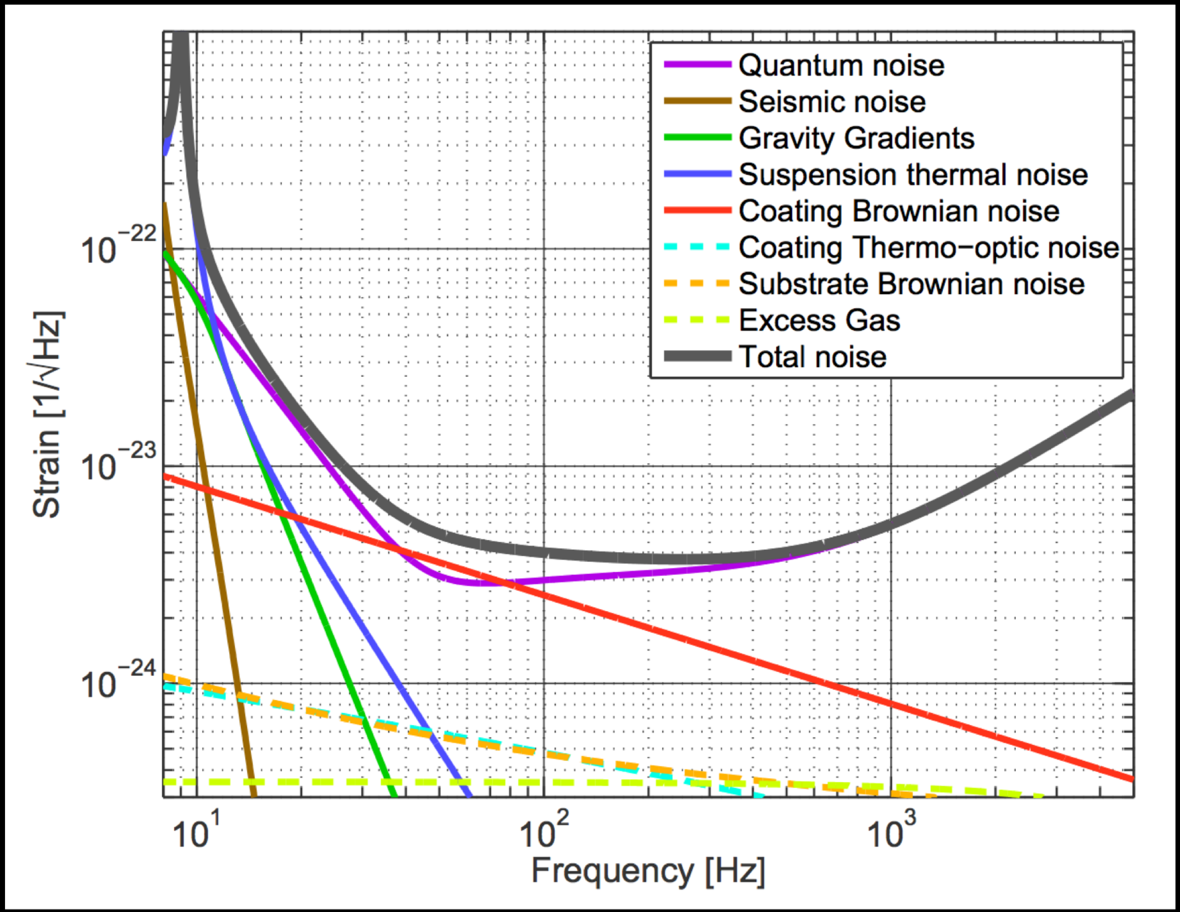
- The best matched filter function is

$$\tilde{K}(f) = \frac{\tilde{h}(f)}{S_n(f)}$$

- This then gives signal to noise ratio (SNR),

$$\left.\begin{aligned} S &= \int_{-\infty}^{\infty} df\, \tilde{h}(f)\tilde{K}^*(f) \\ N^2 &= \int_{-\infty}^{\infty} df\, \frac{1}{2} S_n(f) |\tilde{K}(f)|^2 \end{aligned}\right\} \left(\frac{S}{N}\right)^2 = 4\int_0^{\infty} df\, \frac{|\tilde{h}(f)|^2}{S_n(f)}$$

LIGO noise curve

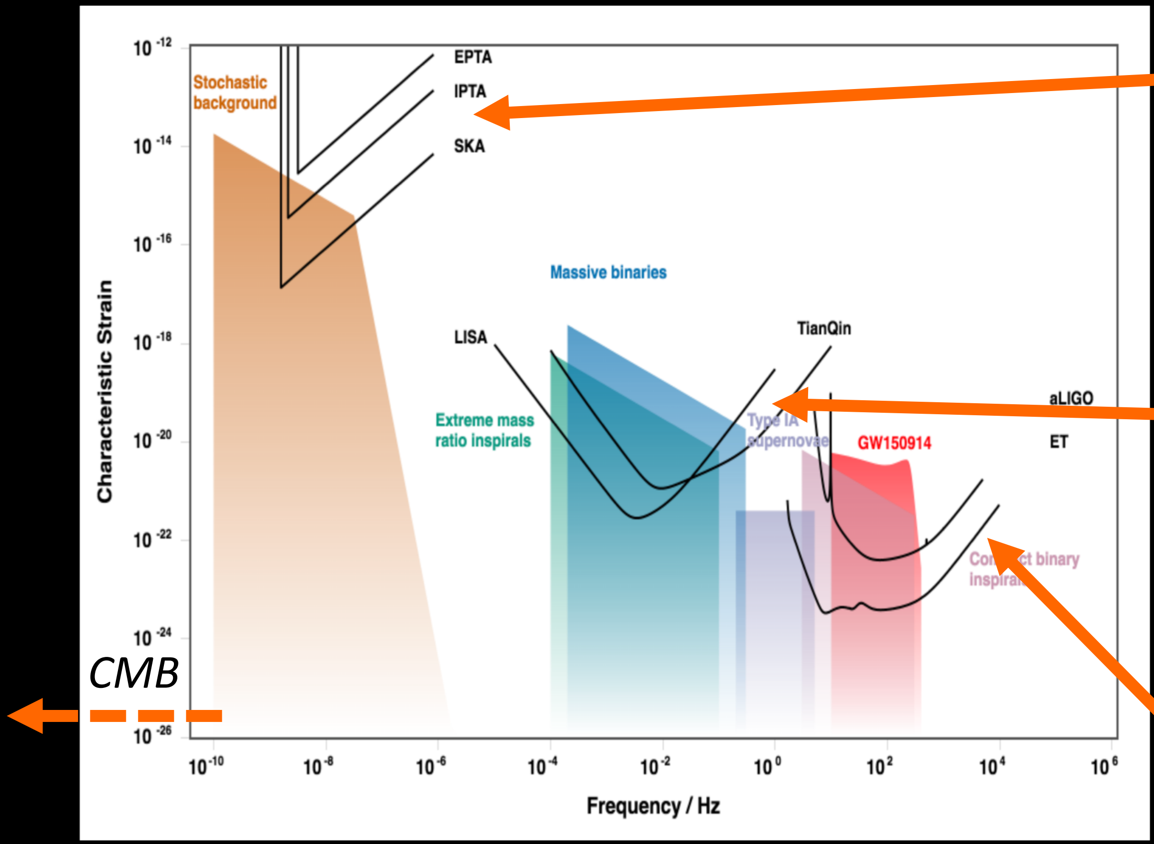


Quantum noise:
- Radiation pressure noise (small f)
- Photon shot noise (large f)

Nothing to do with optics, just with harmonic oscillations of the test masses and mirrors

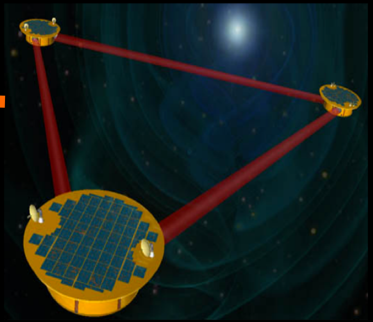
LIGO-T010075-v2

(Other) experiments



Moore, Cole and Berry, Class.Quant.Grav. 32 (2015)

Pulsar timing arrays



Space-based interferometers

Ground-based interferometers

Stochastic GW Backgrounds

- Plane wave decomposition:

$$h_{ab}(t, \vec{x}) = \int_{-\infty}^{\infty} df \int_{S^2} d^2\Omega_{\hat{k}} \sum_A e_{ab}^A(\hat{k}) h_A(f, \hat{k}) e^{2i\pi f(t - \hat{k}\cdot\vec{x}/c)}$$

Polarization tensor (A=+,x)

Plane waves

- Schematically, for an unpolarized and isotropic GRB

$$\int df' \langle h_A(f) h^*_{A'}(f') \rangle \propto \frac{h_c^2}{f} = \frac{3H_0^2}{2\pi^2} \frac{\Omega_{GW}(f)}{f^3}$$

GW fractional energy density

$$\Omega_{\mathrm{GW}}(f) \equiv \frac{f}{\rho_c} \frac{d\rho_{GW}}{df}$$

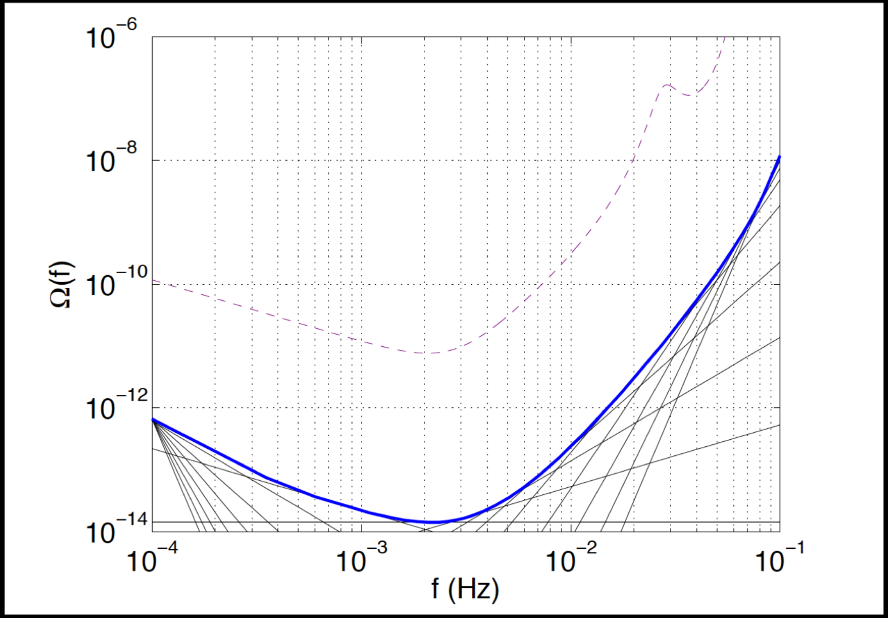
Power-law integrated sensitivity

Romano, Thrane, arXiv:1310.5300

- For power-law spectra,

$$\Omega_{GW}(f) = \Omega_\beta \left(\frac{f}{f_{\rm ref}}\right)^\beta$$

- Define the bandwidth of the detector (f_{min} , f_{max})
- For a set of indices β, calculate Ω_β (integration over f) such that the SNR has some fixed value

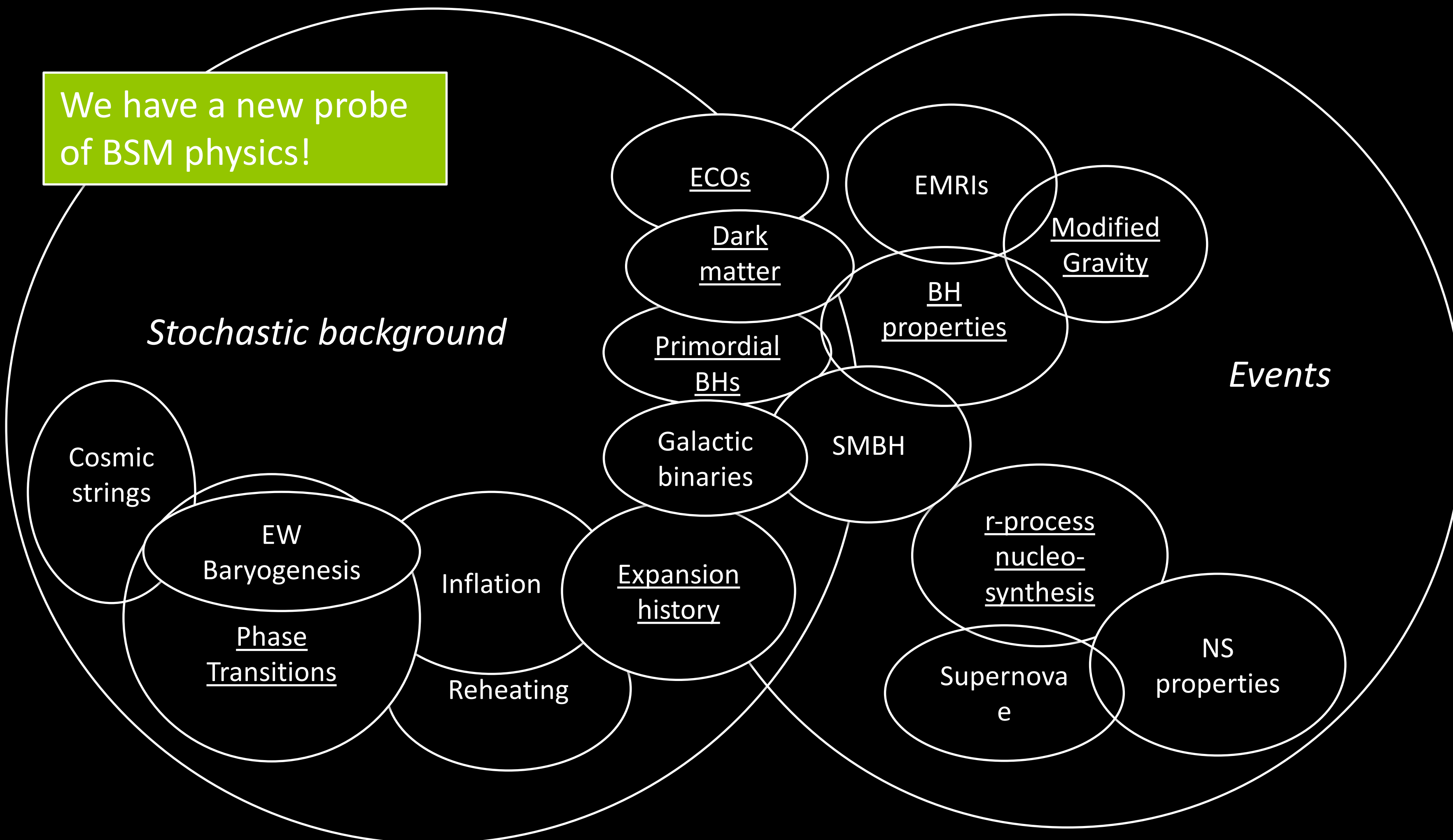


$$\left(\frac{S}{N}\right)^2 = 4\int_0^\infty df\,\frac{|\tilde{h}(f)|^2}{S_n(f)}$$

A brief look at

SCIENCE OPPORTUNITIES AND PROSPECTS

We have a new probe of BSM physics!



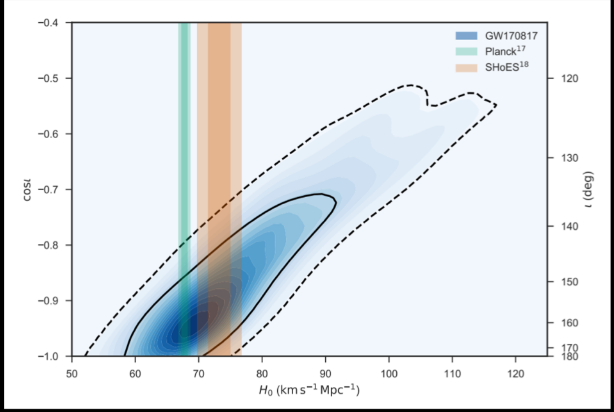
Standard sirens

- Gravitational waves give a distance
  - Interferometers measure the amplitude and the phase of the GW
  - The distance depends on M_c and r, but the phase only on M_c
- EM counterpart BNS merger revealed the host galaxy at $z = 0.009680 \pm 0.00079$
- For small z,

$$d_L(z) = \frac{z}{H_0} + \mathcal{O}(z^2)$$

Standard sirens

LIGO, VIRGO, DES, et al.
Nature 551 (2017) no.7678, 85-88



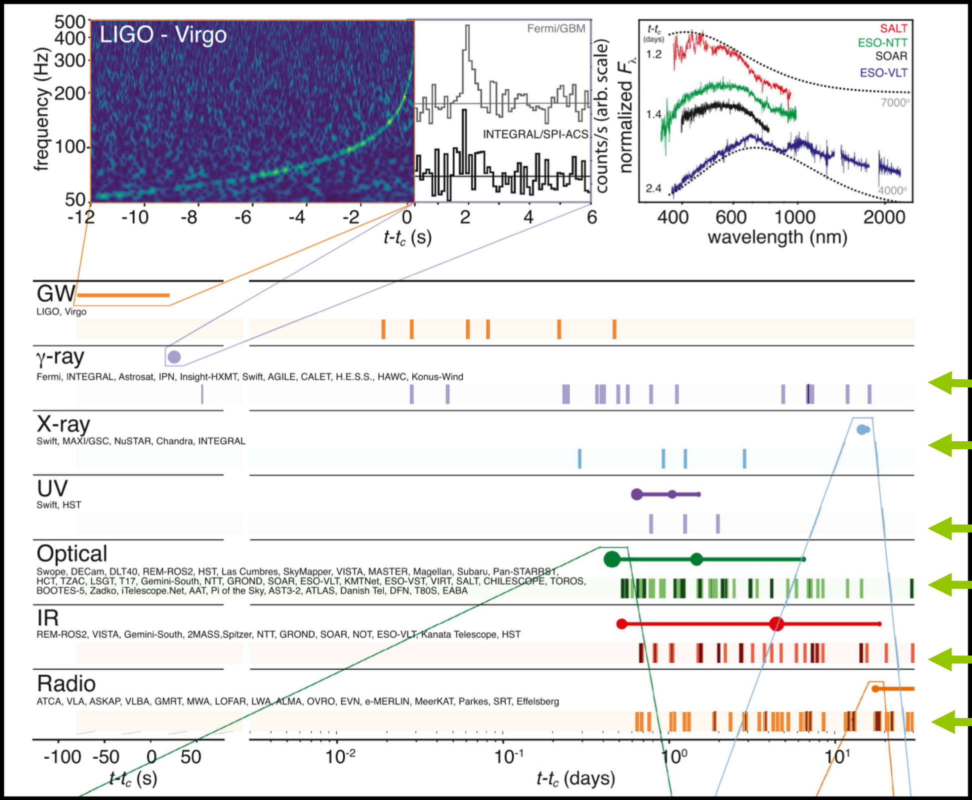
- This gives,

  $$H_0 = 70.0^{+12}_{-8.0}\,\mathrm{km\ s^{-1}Mpc^{-1}}$$

  - Luminosity and comoving distance differ only at order $\mathrm{v/c} \sim 1\%$
  - Compatible with (and independent of) earlier measurements such as Planck

- No “distance ladder” or prior on H_0

Multi-messenger astronomy with BNS-mergers



Kilonova: nucleosynthesis and subsequent decay of radioactive elements

- Accretion of a remnant disk following the merger
- Afterglow emission (only for viewing angles close to the jet axis)
- Decay of free neutrons in ejecta
- Lanthanide-free components of ejecta
- Lanthanide components of ejecta
- Bremsstrahlung from ionized gas

(GW170817) and (GRB 170817A), Astrophys.J. 848 (2017) no.2, L12

Kilonovas and heavy elements

- Largely consistent with predictions and simulations:
  - Spectrum
  - Luminosity
  - Timescales
  - Ejecta mass and velocity

Ejecta Type	$M_{ej}(M_\odot)$	$v_{ej}(c)$	Color	M_{ej} decreases with
Tidal Tails	$\sim 10^{-4} - 10^{-2}$	$0.15 - 0.35$	Red (NIR)	$q = M_2/M_1$
Polar Shocked	$\sim 10^{-4} - 10^{-2}$	$0.15 - 0.35$	Blue (visual)	$M_{rem}/M_{max}, R_{ns}$
Disk Outflows	$10^{-4} - 0.07$	$0.03 - 0.1$	Blue+Red	M_{rem}/M_{max}

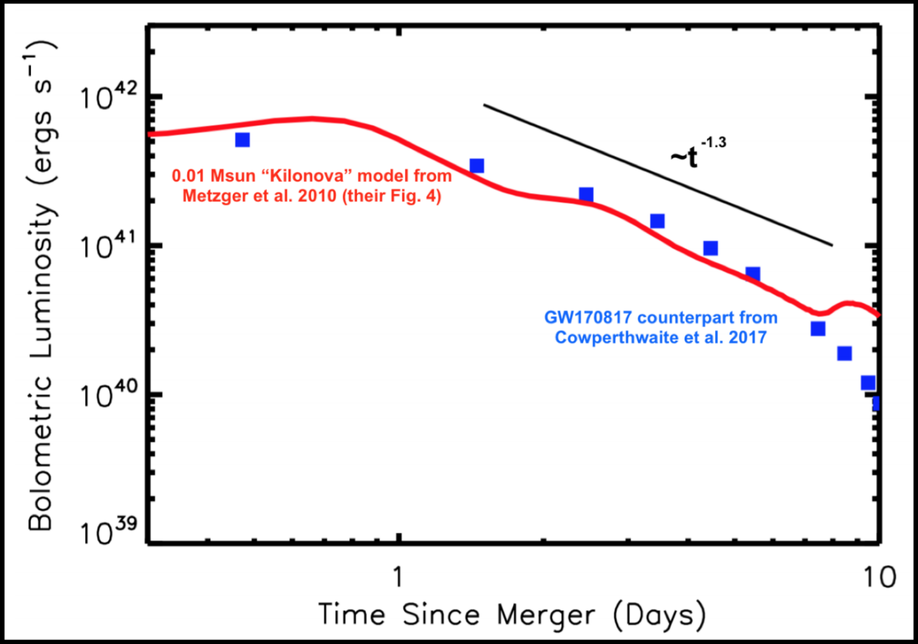
$$Y_e = \frac{n_p}{n_n + n_p}$$

- Nucleosynthesis: strong support for binary NS mergers as the dominant source of heavy r-process nuclei
  - Neutron-rich ejecta produce (heavy) lanthanide elements
  - $Y_e \lesssim 0.1$-0.2 consistent with the solar system abundance

A helpful overview:
Metzger, arXiv:1710.05931 [astro-ph.HE]

The first Kilonova observation

Metzger, arXiv:1710.05931 [astro-ph.HE]



Low level of lanthanide elements compared to theoretical curve

$$Y_e = \frac{n_p}{n_n + n_p}$$

Lighter r-process elements

Heavier r-process elements

Modified gravity

- Scalar-tensor theories

Tensor perturbation → $G^{\mu\nu}$

$$G^{\mu\nu} = C g^{\mu\nu} + D\phi^{,\mu}\phi^{,\nu} + E\phi^{;\mu\nu}$$

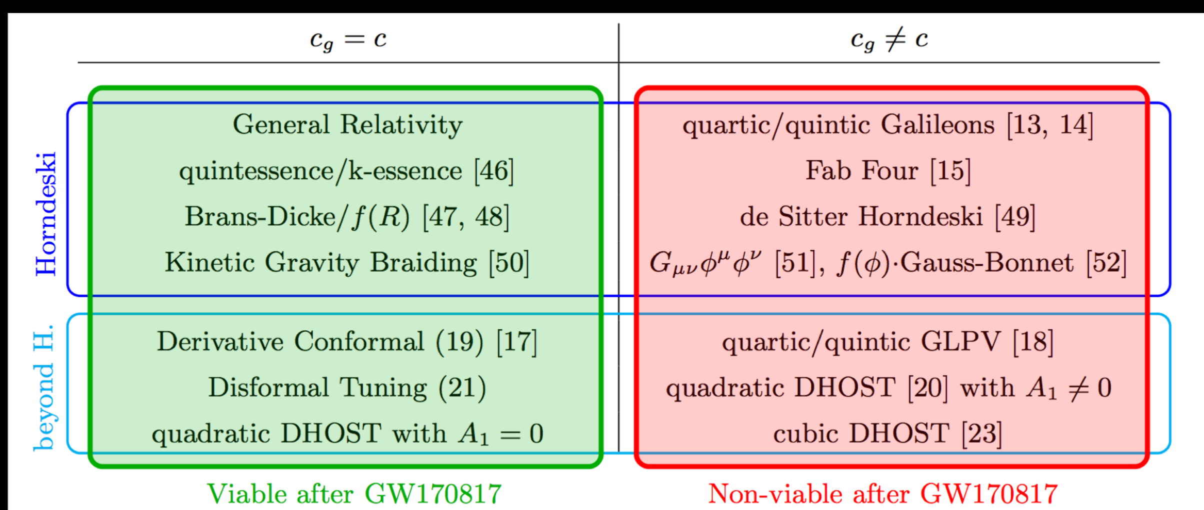
$$G^{\mu\nu}k_\mu k_\nu = 0$$

$$c_g^2 = \frac{\omega^2}{k^2}$$

- Multi-messenger signal:

$$|c_g/c - 1| \geq 5 \times 10^{-16}$$

Ezquiaga, Zumalacárregui
arXiv:1710.05901 [astro-ph.CO]



Post-Newtonian corrections

- For self-gravitating systems, when $v \ll c$ breaks down, so does the assumption that spacetime is flat
  - Must include higher multipoles
  - Must include GR corrections to the wave equation
- PN expansion in v/c (in the near region)
- Effects such as tidal forces come in at 5PN
  - Probe the NS EoS
  - Probe ECOs with smaller compactness

EMRIs/IMRIs

- Merger of a supermassive ($\sim 10^6\ \mathrm{M}_\odot$) or intermediate mass ($\sim 10^4\ \mathrm{M}_\odot$) BH and a solar mass object, probed by LISA
  - LISA can detect EMRIs up to z=4
  - Inspirals are slow: LISA typically probes 10^4-10^5 cycles
- Potential to probe black hole spacetime
  - Nonzero Love numbers imply tidal forces
  - For black holes these effects are absent, while for mimickers they are present

EMRIs/IMRIs

- Three main formation mechanisms with very different resulting orbits,
  1. Two-body relaxation -> eccentric orbits, inclined towards the BH spin
  2. Absorption of a binary star -> circular orbits, inclined towards the BH spin
  3. Star formation in the accretion disk -> circular orbits in the equatorial plane
- The orbits are also quite relativistic, $v/c \sim 0.3$

Stochastic Background (from binary mergers)

$$\Omega_{\rm GW}(f) \equiv \frac{f}{\rho_c}\frac{d\rho_{GW}}{df}$$

$$\Omega_{\rm GW}(f, M_*, f_{BBS}) = \frac{f}{\rho_c H_0}\int_0^{z_{max}} \frac{R_m(z, M_*, f_{\rm BBS})}{(1+z)\sqrt{\Omega_M(1+z)^3+\Omega_\Lambda}}\frac{dE}{df_s}\, dz$$

LIGO, PRL 120, 091101

Stochastic Background (from binary mergers)

$$\Omega_{\rm GW}(f) \equiv \frac{f}{\rho_c}\frac{d\rho_{GW}}{df}$$

Observed frequency

$f_s = (1+z)f$

Merger rate

$$\Omega_{\rm GW}(f, M_*, f_{BBS}) = \frac{f}{\rho_c H_0}\int_0^{z_{max}} \frac{R_m(z, M_*, f_{\rm BBS})}{(1+z)\sqrt{\Omega_M(1+z)^3+\Omega_\Lambda}}\frac{dE}{df_s}dz$$

Differential energy emitted by a single source

LIGO, PRL 120, 091101

The Merger Rate

$$R_m(t, M_*, f_{\rm BBS}) = \int_{\Delta t_{min}}^{\Delta t_{max}} R_{\rm BBS}(t - \Delta t, M_*) \, p(\Delta t) \, d\Delta t.$$

Binary formation rate

Usual Ansatz: the formation of binaries tracks the star formation rate

Time delay distribution

Probability that two stars initially separated by a are gravitationally bounded

The early Universe

Gravitational waves?

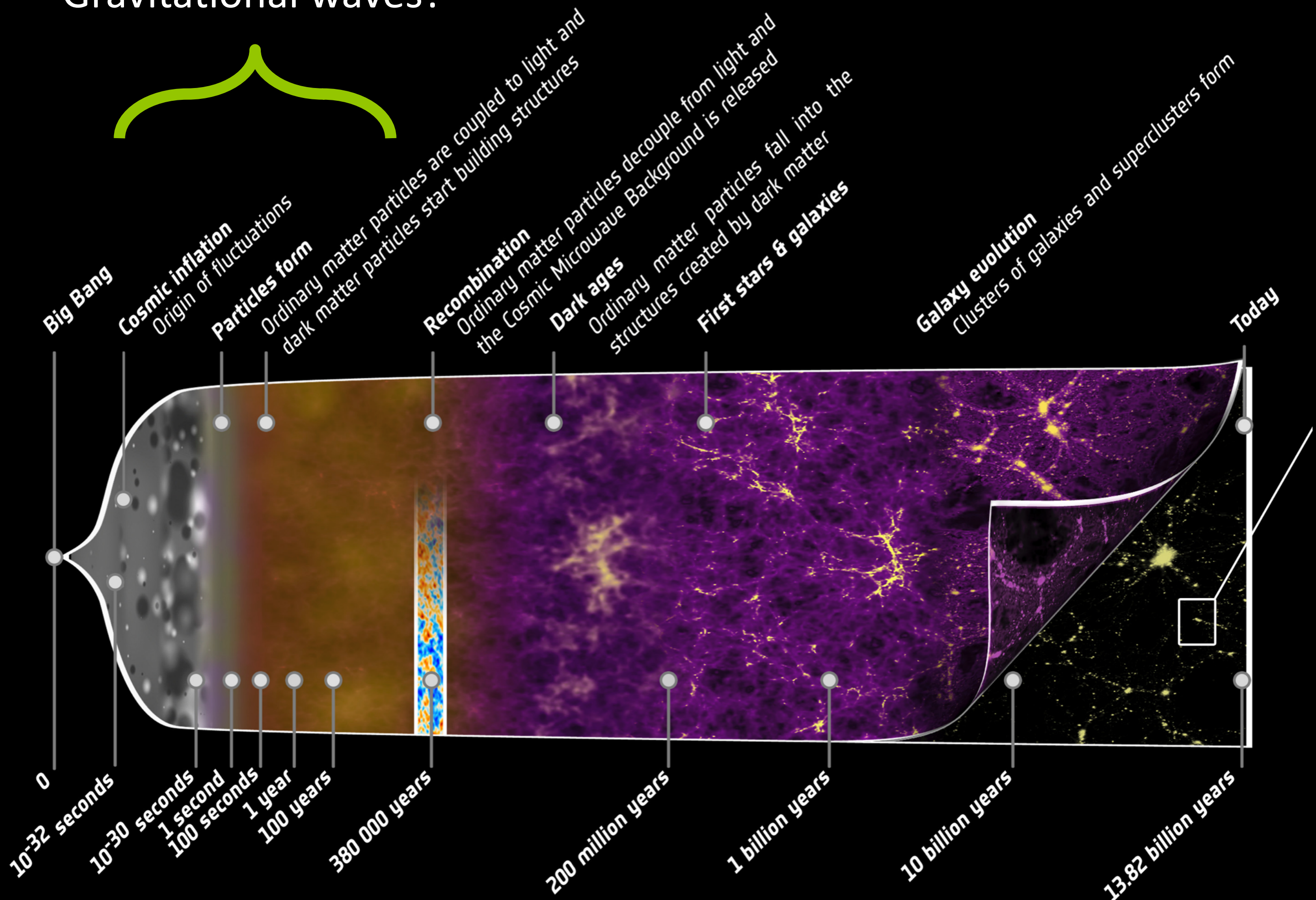


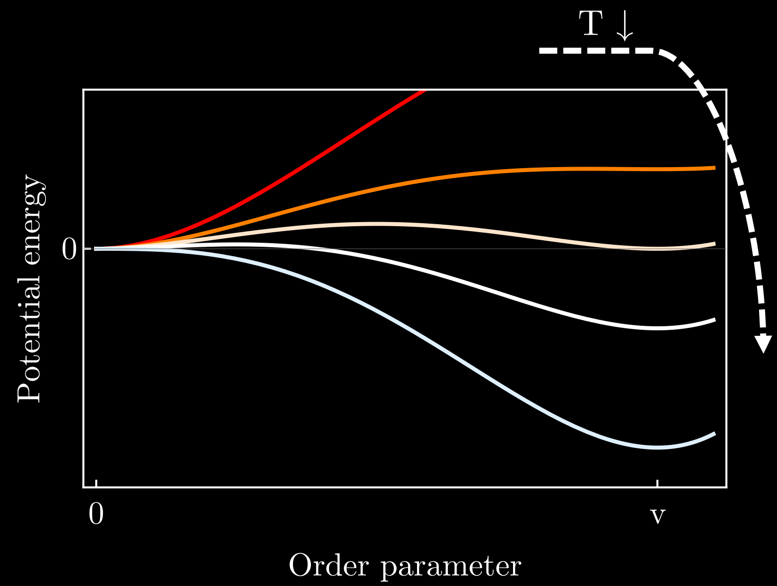
Image: ESA

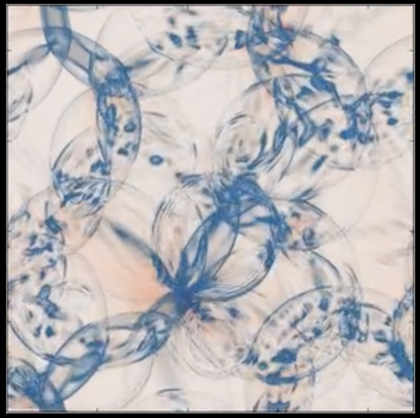
First order phase transitions

Change in vacuum state associated with the release of latent heat

Inhomogeneous and out-of-equilibrium

Nucleation of bubbles of ”true” vacuum described by instantons





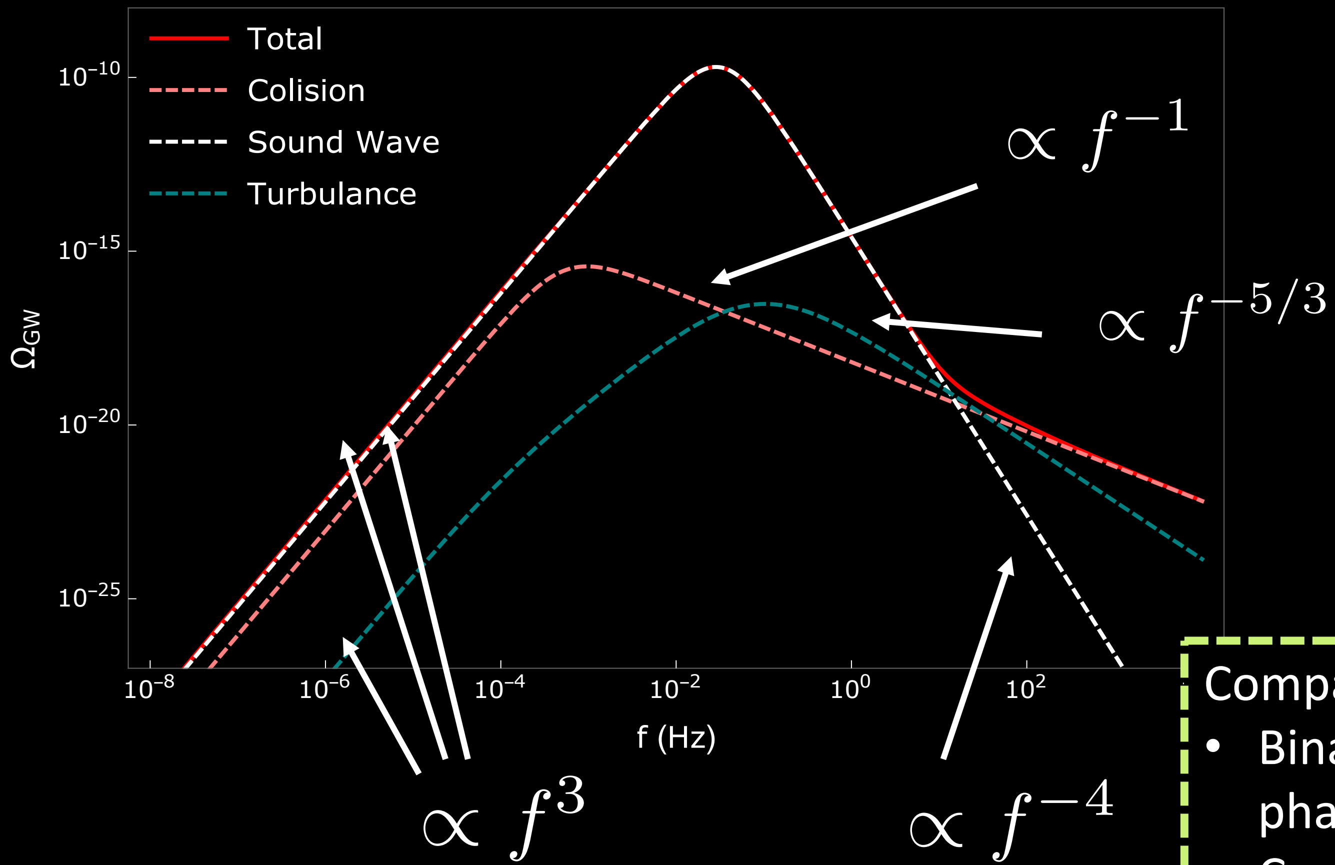
Snapshot from simulation: Daniel Cutting, private communication

What happens to the energy released by the phase transition?

It may dissipate as gravitational waves:

- Bubble collisions source GW
- Acoustic waves and turbulence in the plasma source GW

GW spectra from a SFOPT



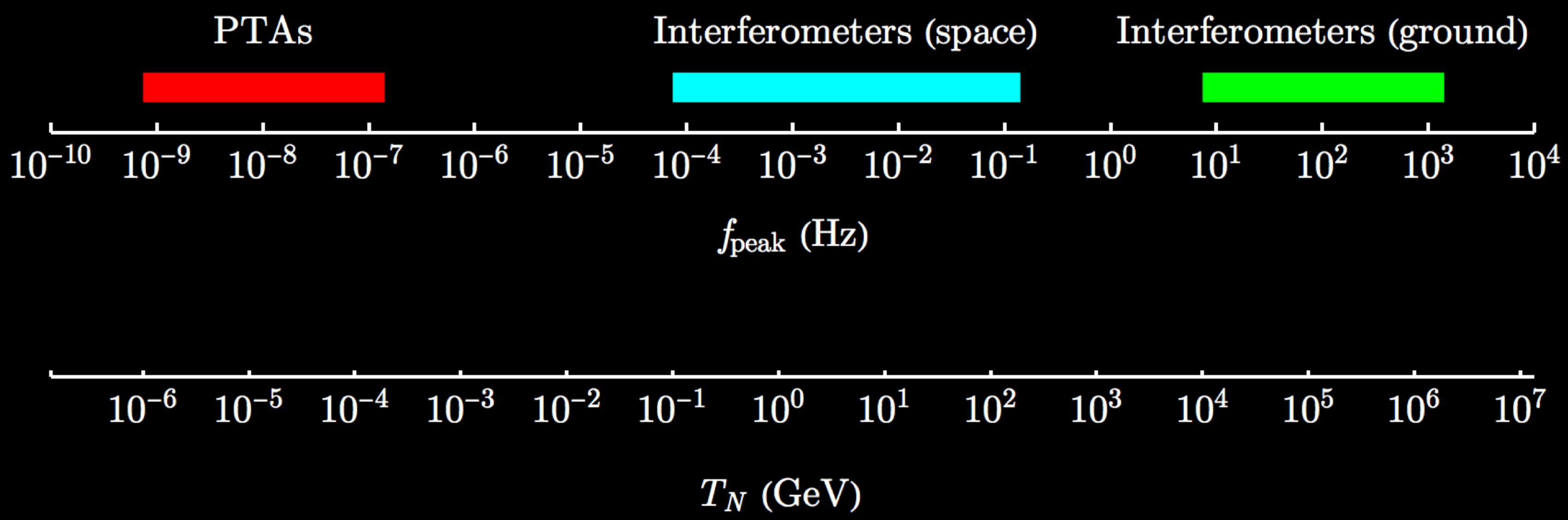
See for example:
Weir, [1705.01783]

Compare:

- Binary mergers: inspiral phase $\Omega \sim f^{2/3}$
- Cosmic strings $\Omega \sim f^{0}$

Typical scales (f_{peak})

$\beta/\mathrm{H}\sim10^3$, $\mathrm{v_w}\sim1$



Typical amplitudes

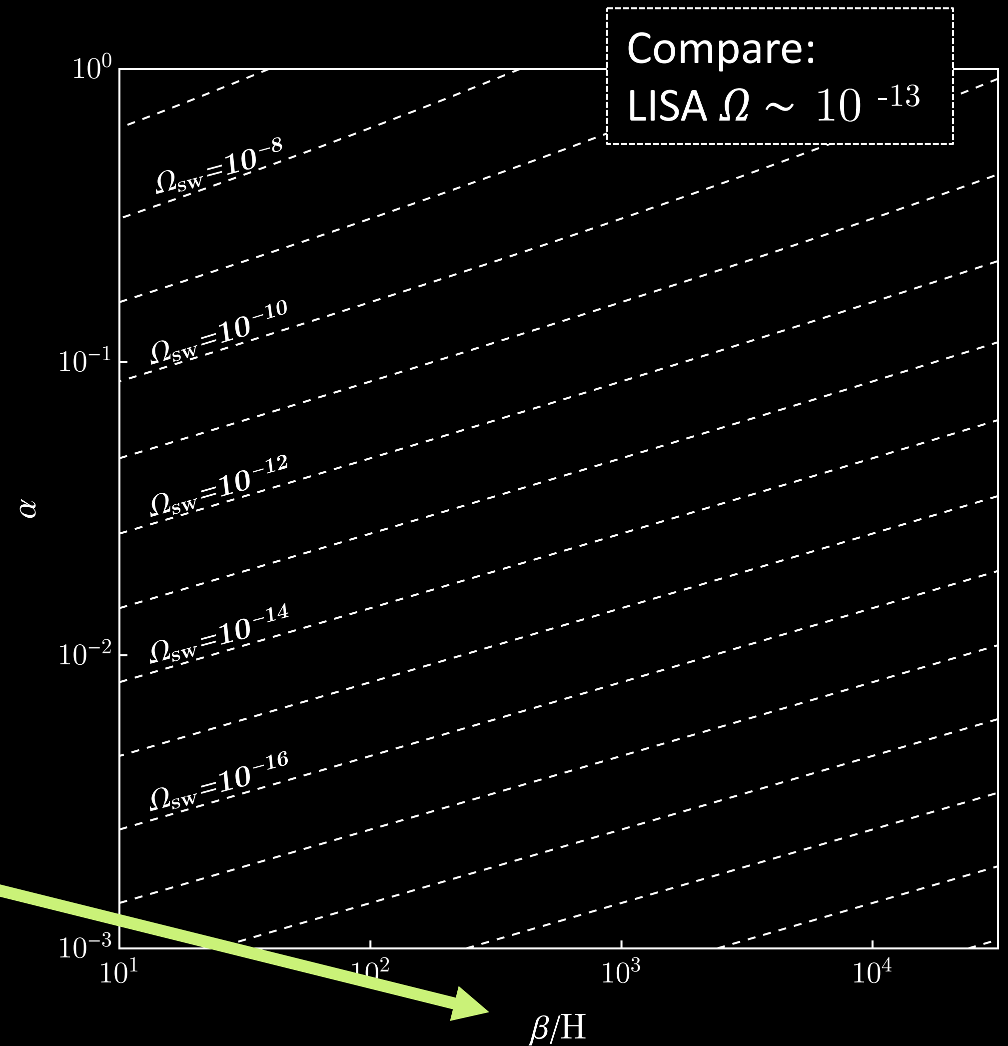
- A visible GW spectrum requires a large latent heat

$$\alpha \equiv \frac{\Delta\mathcal{L}}{\rho_{\rm rad}}$$

- Slower = better

$$\frac{\beta}{H} = \left[T\frac{d}{dT}\frac{S_E}{T}\right]_{T=T_N}$$

$v_w \sim 1$



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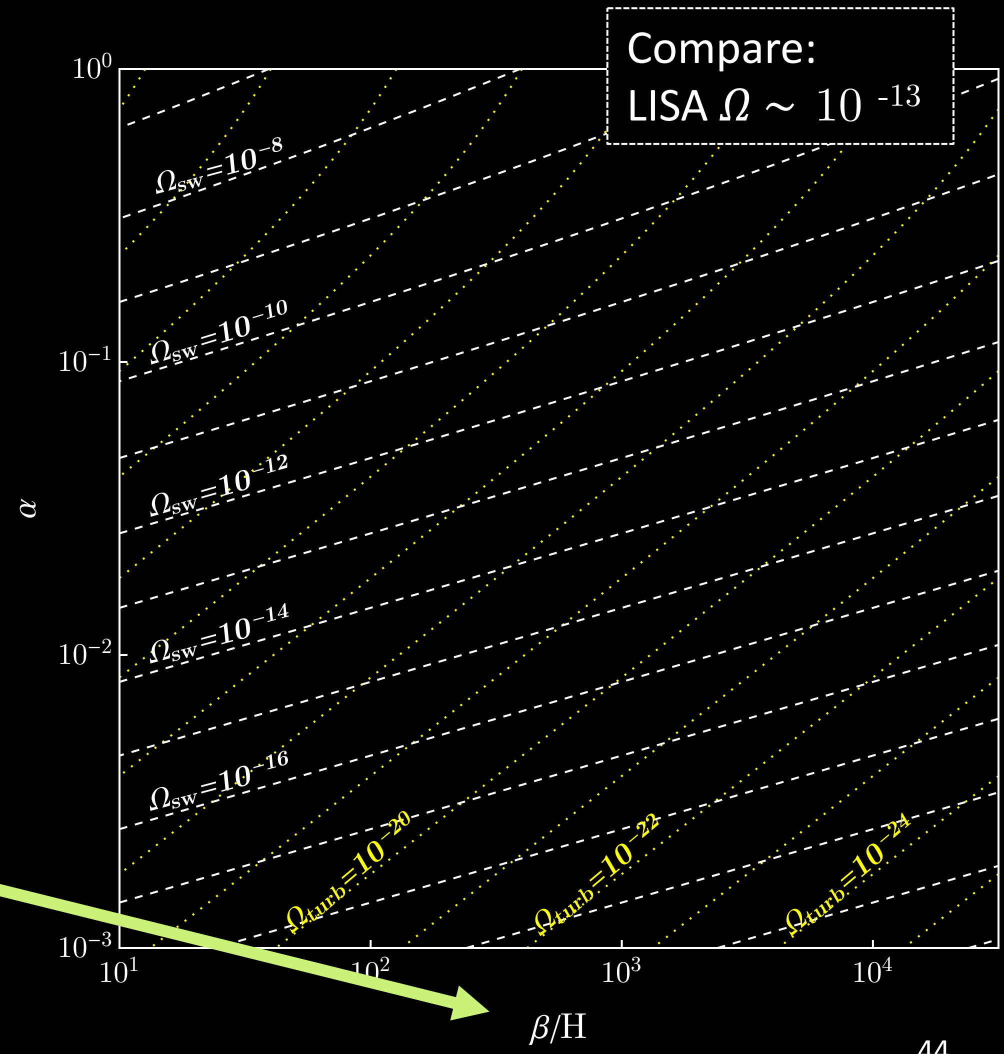


Dark matter

Lectures start next week!

- Dark matter interacts gravitationally:
  GW studies are a new opportunity for the phenomenology of dark sectors

Gravitational wave probes of dark matter: challenges and opportunities

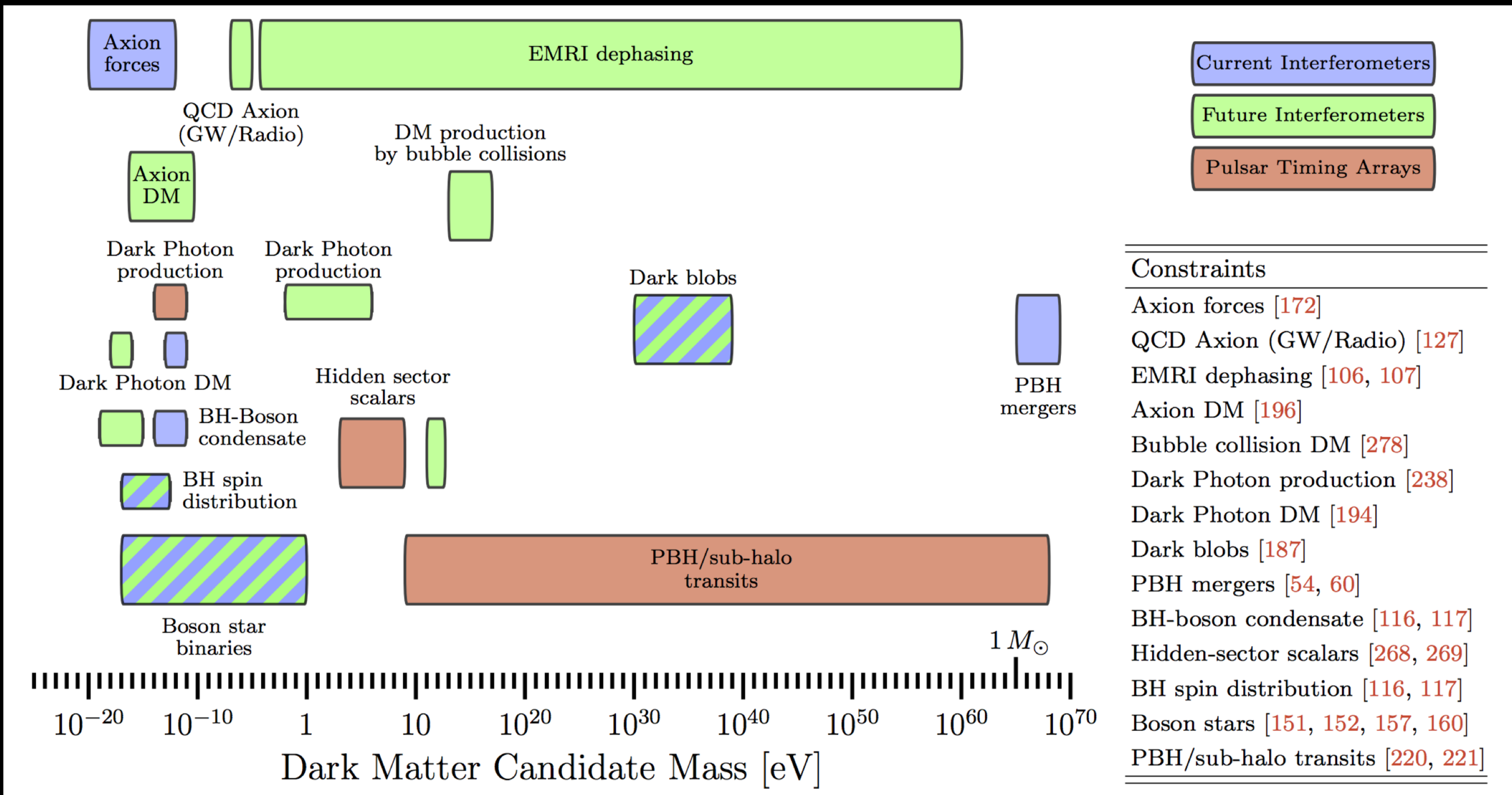
Gianfranco Bertone,[1,*] Djuna Croon,[2,†] Mustafa A. Amin,[3,‡] Kimberly K. Boddy,[4,§] Bradley J. Kavanagh,[1,¶] Katherine J. Mack,[5,‖] Priyamvada Natarajan,[6,**] Toby Opferkuch,[7,††] Katelin Schutz,[8,‡‡] Volodymyr Takhistov,[9,§§] Christoph Weniger,[1,¶¶] and Tien-Tien Yu[10,***]

arXiv: 1907.10610 (today!)

- GWs can probe DM candidates with many orders of magnitude in mass arXiv: 1907.10610

arXiv: 1907.10610

Dark matter



Resolvable mergers: modified inspirals

$$\Phi(t) = 2\pi \int dt\, f_{\rm GW}(t)$$

$$h(t) = A \left[\pi f_{{\rm GW}(t)}\right]^{2/3} \cos\left[\Phi(f_{\rm GW}(t)) + \varphi\right]$$

- Distance to the binary $(A \propto 1/r)$
- Inclination of the orbital plane
- Detector response
- Chirp mass

$$f_{\rm GW}(t) = \frac{\omega(t)}{\pi}$$

Exotic Compact Objects

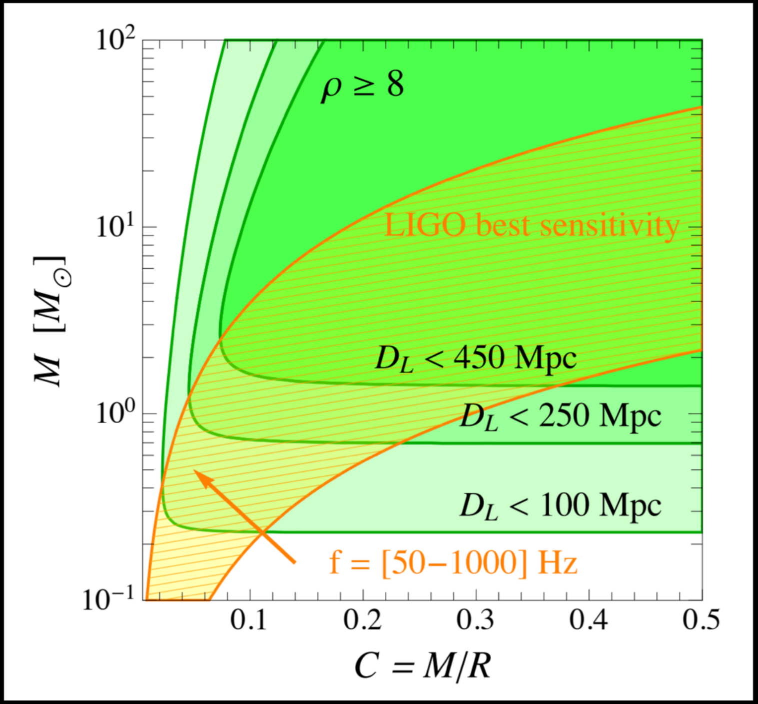
- Best detection prospects for $f_{\rm min} < f_{\rm ISCO} < f_{\rm max}$

- Defines an ECO sensitivity band

$$C_* = \frac{G_N M_*}{R_*}$$

$$f_{\rm ISCO} = \frac{C_*^{3/2}}{3^{3/2}\,\pi\, G_N (M_1 + M_2)}$$

- Important: masses and compactness of ECOs



Giudice, McCullough, Urbano 1605.01209

Primordial black holes

- Smoking gun signals
  - Stellar evolution: no BHs $< 1.4\ \mathrm{M}_\odot$
  - No astrophysical BH mergers with $\mathrm{z} > 40$ (which will be probed by ET and CE)

- Statistical evidence
  - PBH binaries could form abundantly before matter-radiation equality
  - PBHs have different spin distributions

- Incompatible with WIMPs!

Gravitational waves are a new opportunity for (astro-) particle physics and cosmology

Thanks for listening and enjoy the rest of TRISEP!