The Standard Model

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Goal for today:

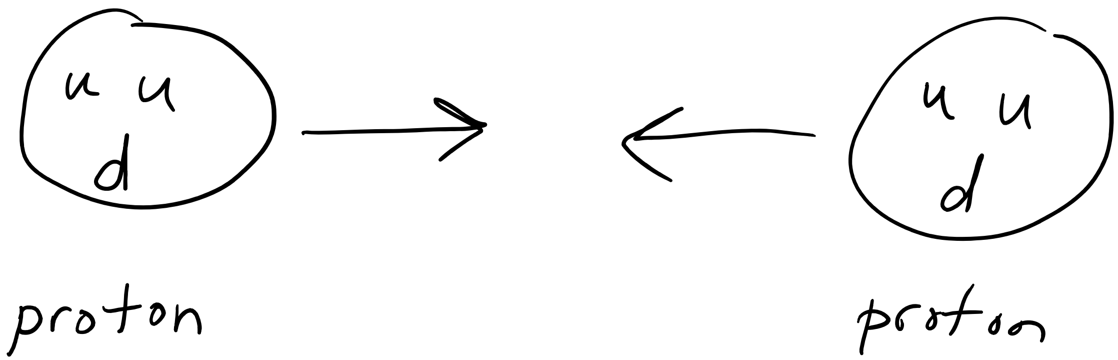
Work out some of the experimental consequences of the 125 GeV Higgs boson

How is h produced at colliders?

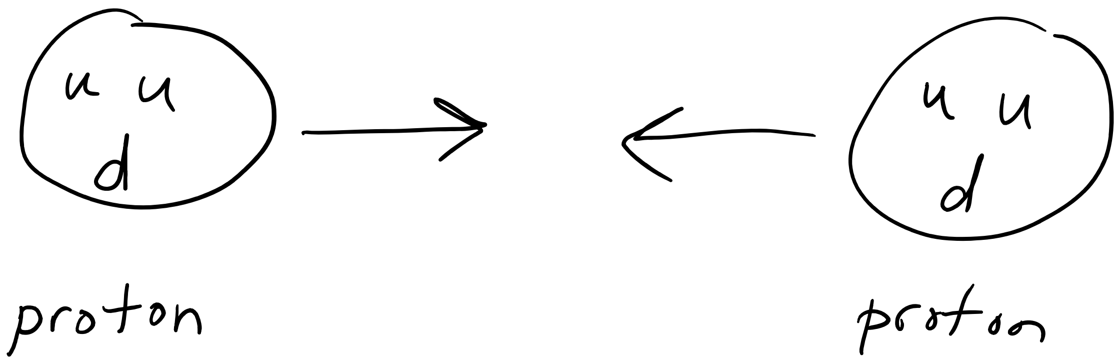
How does h decay?

Before we get to the Higgs, take a long digression into **deep inelastic scattering**

Higgs bosons at hadron colliders (LHC)



Higgs bosons at hadron colliders (LHC)



Initial states is are bound states of QCD, which is not possible to solve analytically

Given that it takes a supercomputer to calculate any of the **static** properties of the proton (lattice QCD), how can we hope to calculate what happens during a collision?

We need to introduce an idea called **factorization**

This is the key feature of QCD that will allow us to calculate what happens in high-energy collisions involving hadrons

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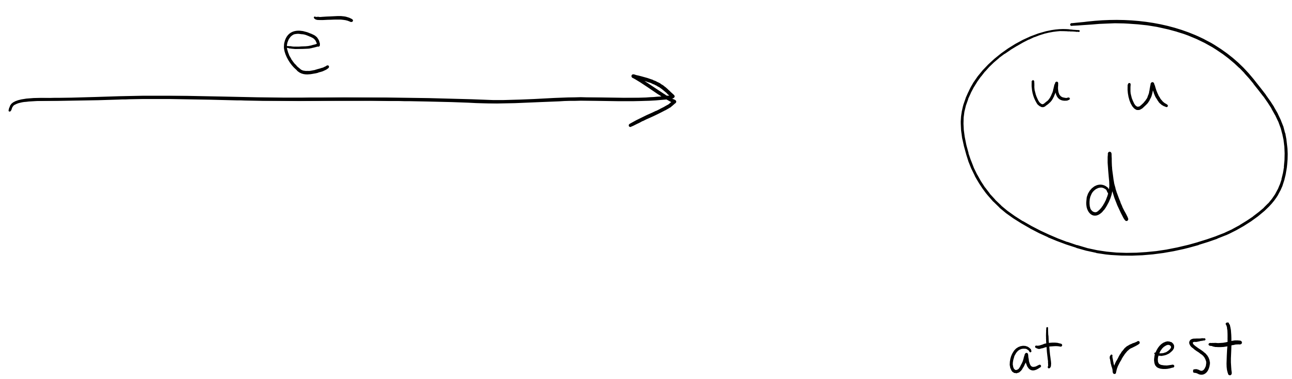
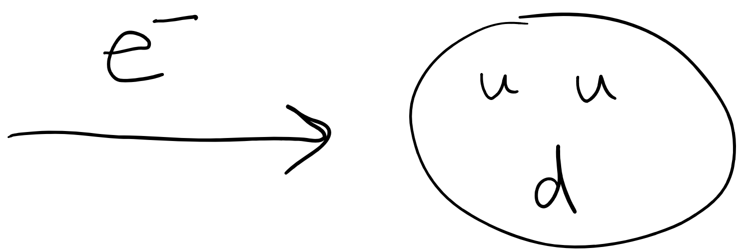
Start with a simpler type of collider: e^- beam on a fixed target



Deep inelastic scattering (DIS)

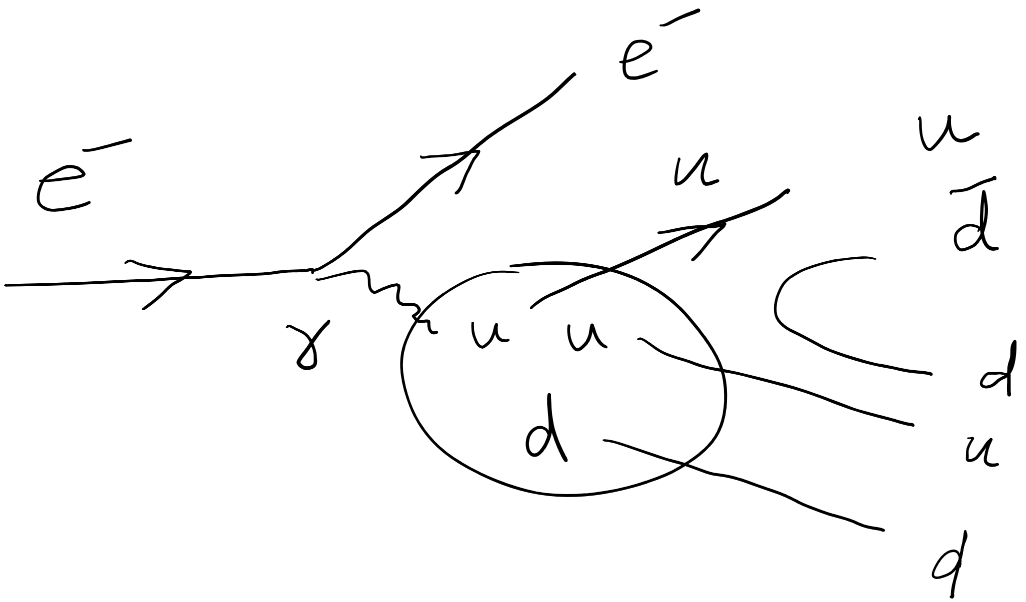


Electron interacts with quarks electromagnetically

No matter the energy, can never liberate a free quark from the proton (**confinement**)

Only produce more hadrons via fragmentation

Deep inelastic scattering (DIS)



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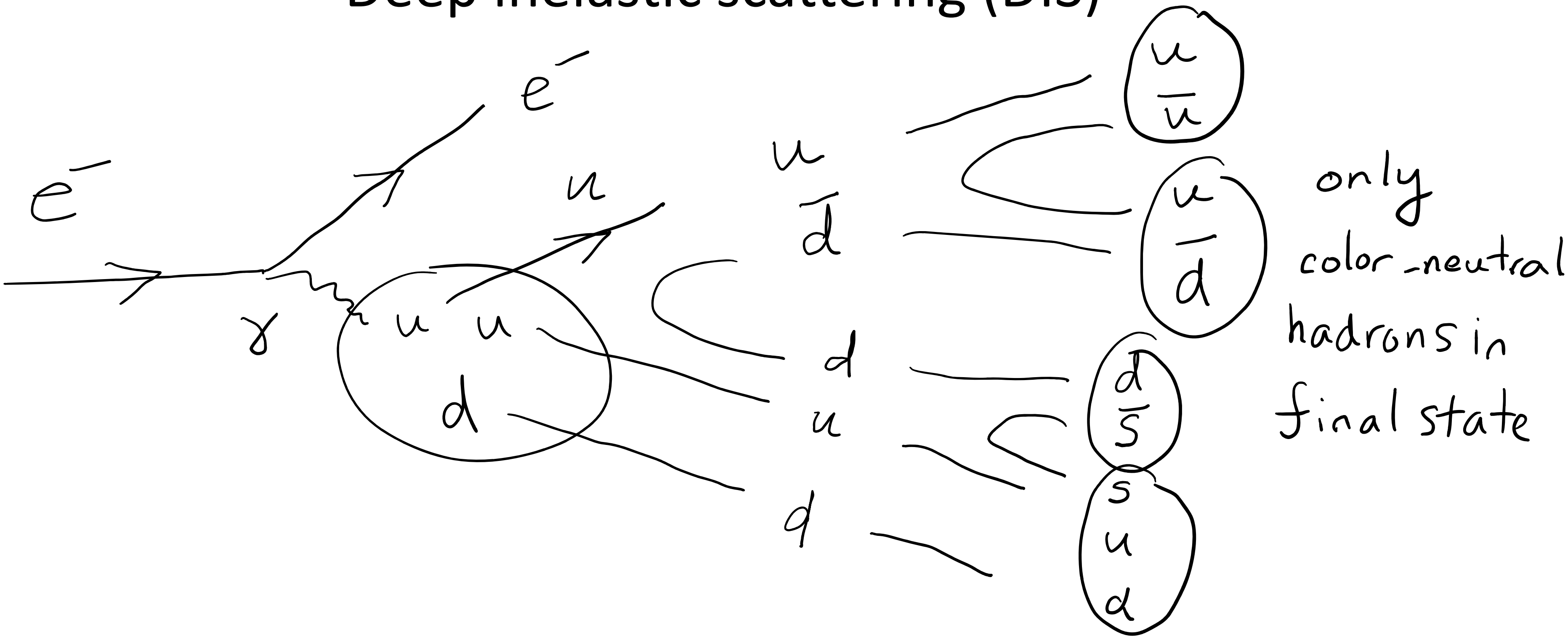


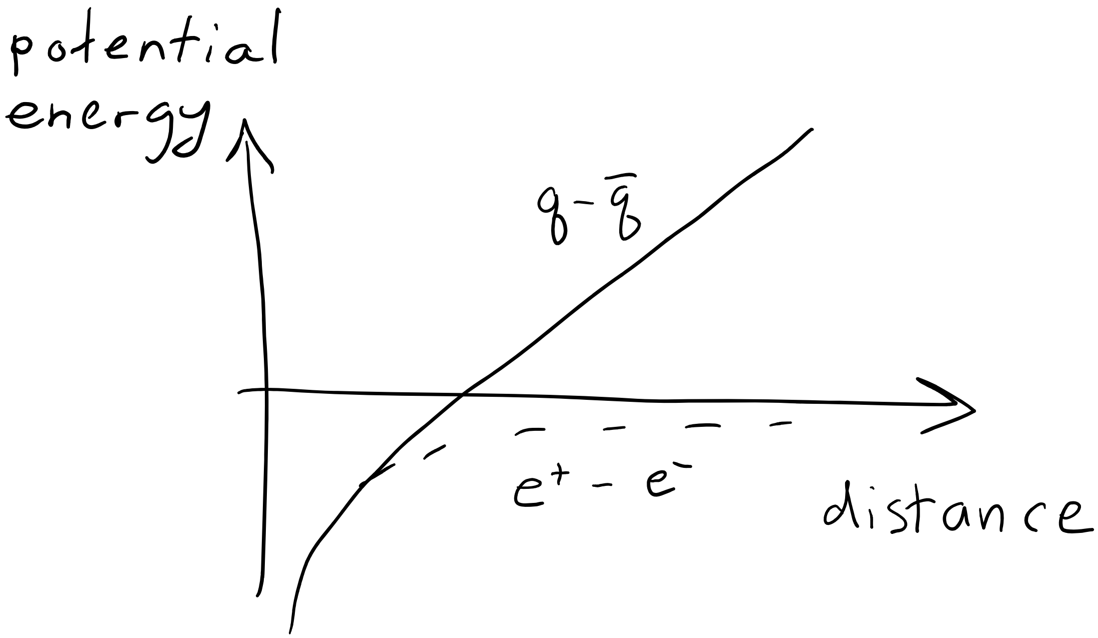
Electron interacts with quarks electromagnetically

No matter the energy, can never liberate a free quark from the proton (**confinement**)

Only produce more hadrons via process of **hadronization**

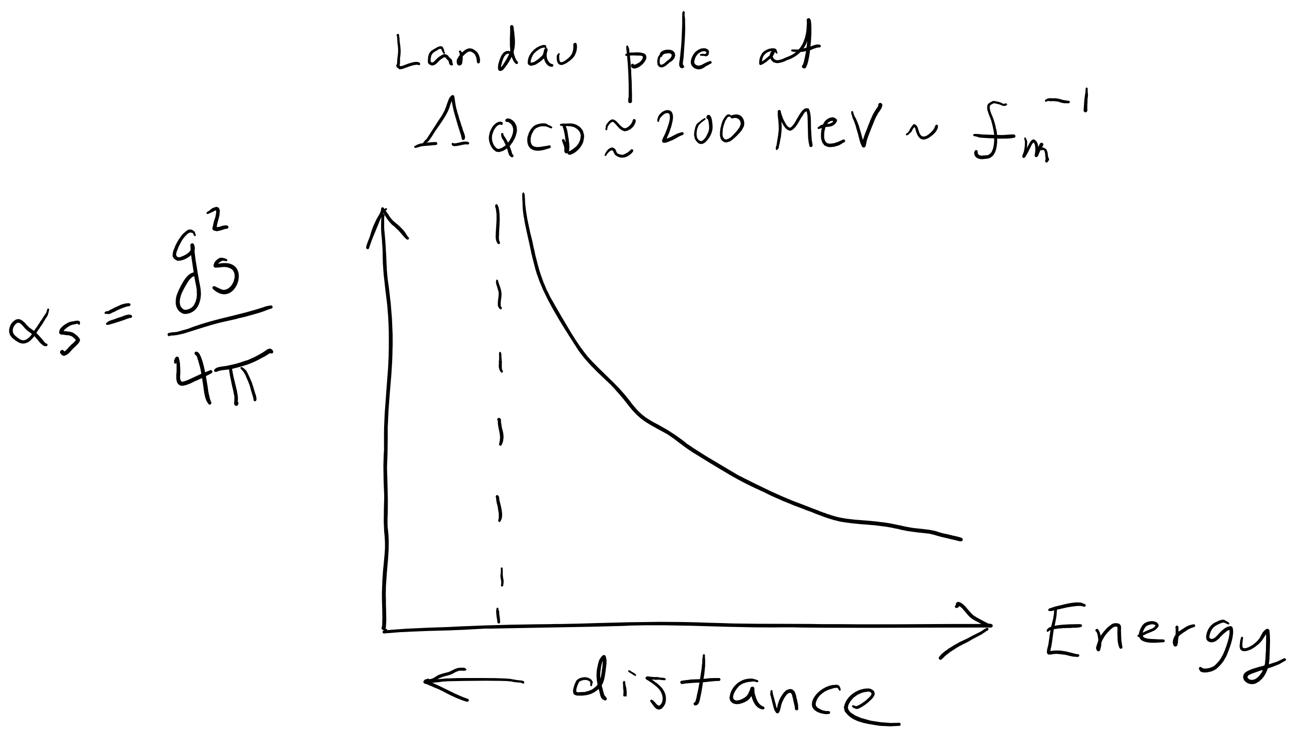
How do we even know hadrons are made from quarks?

Confinement



Energetically favorable to create extra $q\bar{q}$ pair

Confinement

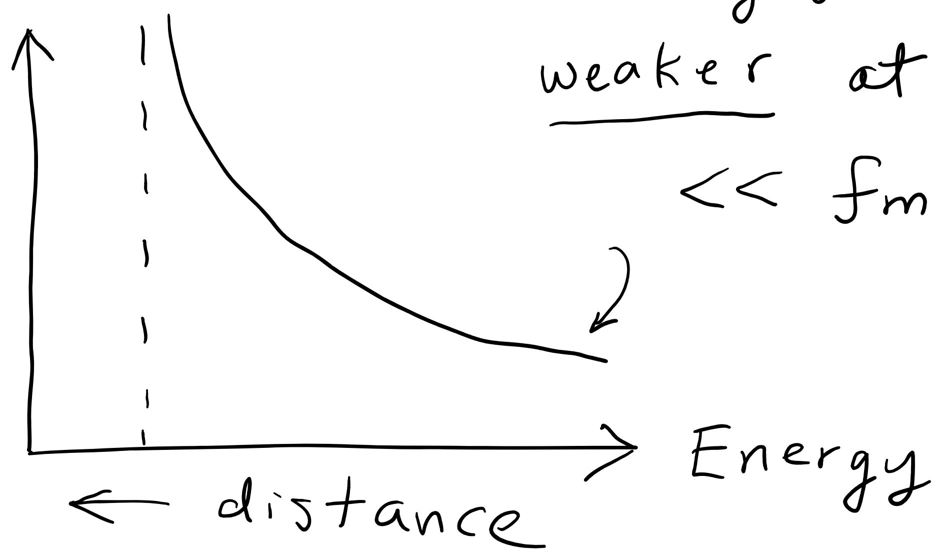


α_s becomes large at low energies (nonperturbative regime of QCD)

Asymptotic freedom

$$\alpha_s = \frac{g_s^2}{4\pi}$$

Strong force becomes <u>weaker</u> at distances $\ll$ fm



α_S becomes small at high energies (QCD is perturbative)

Factorization

What happens at small distances (<< fm) isn't affected by dynamics over large distances (> fm)

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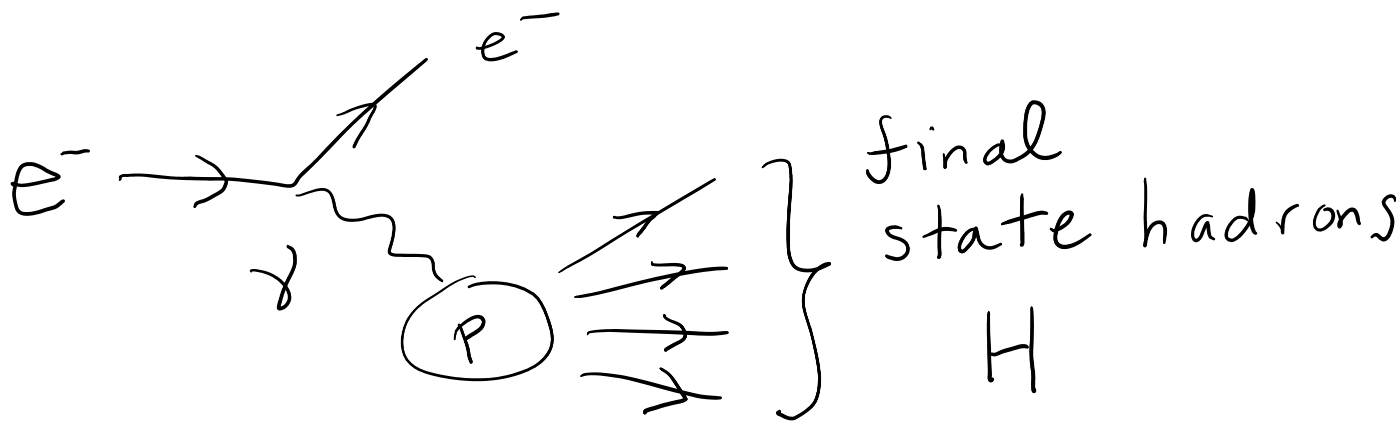
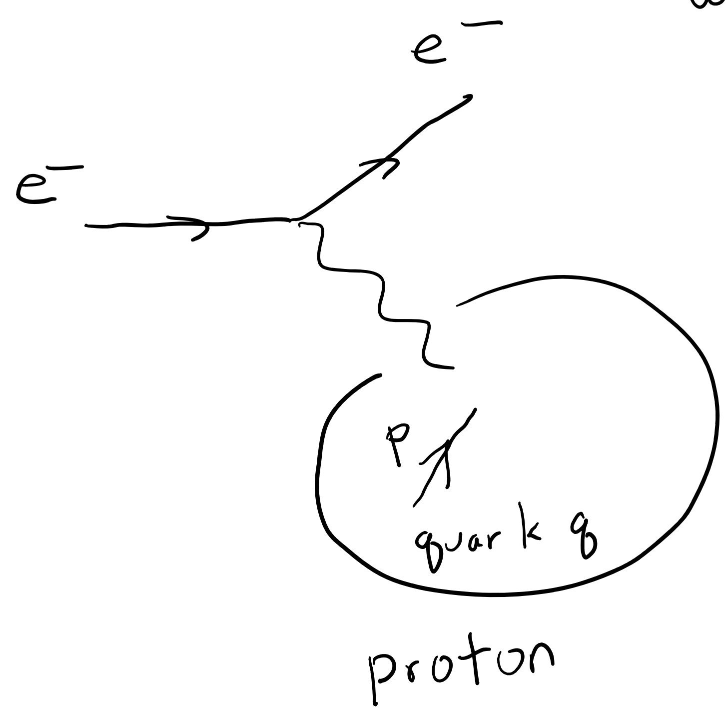


$$\sigma(e^- p \to e^- H)$$

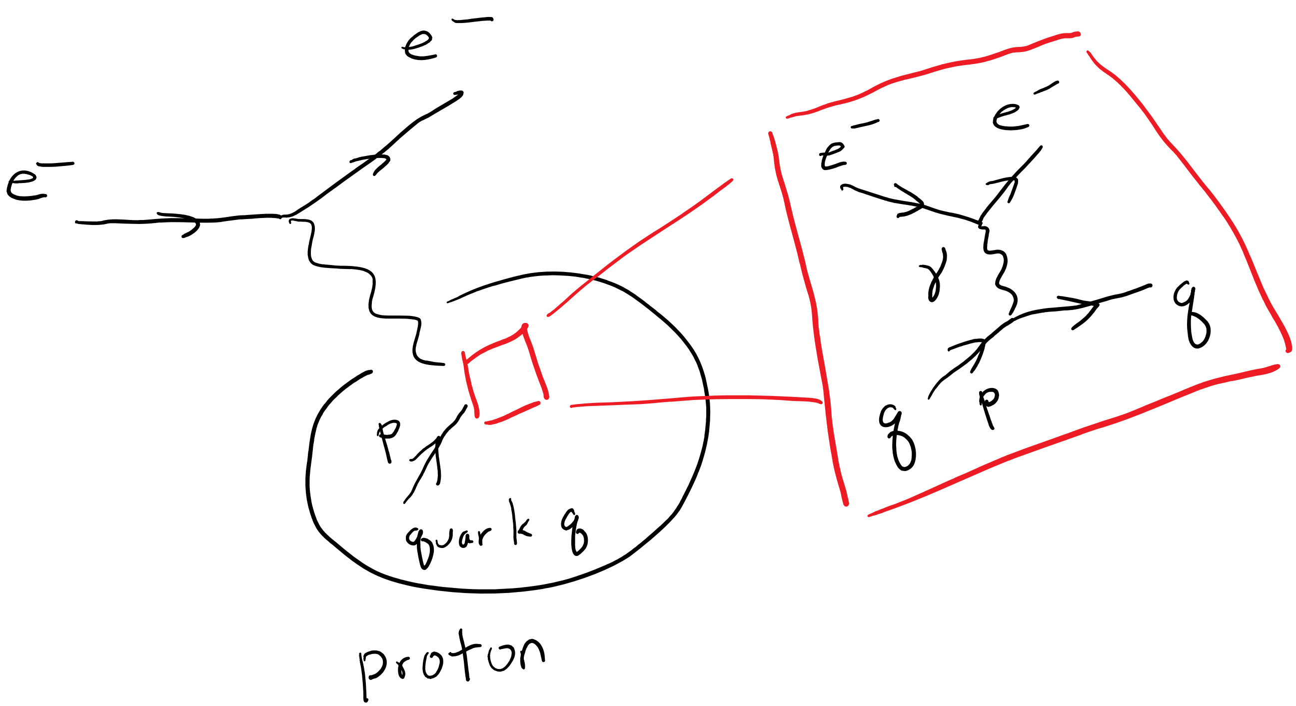
Factorization

$$\sigma(e^- p \to e^- H) = \underbrace{\mathrm{Prob}(q,p)}_{\substack{\text{probability of}\\ \text{finding quark } q\\ \text{with momentum } p}}$$



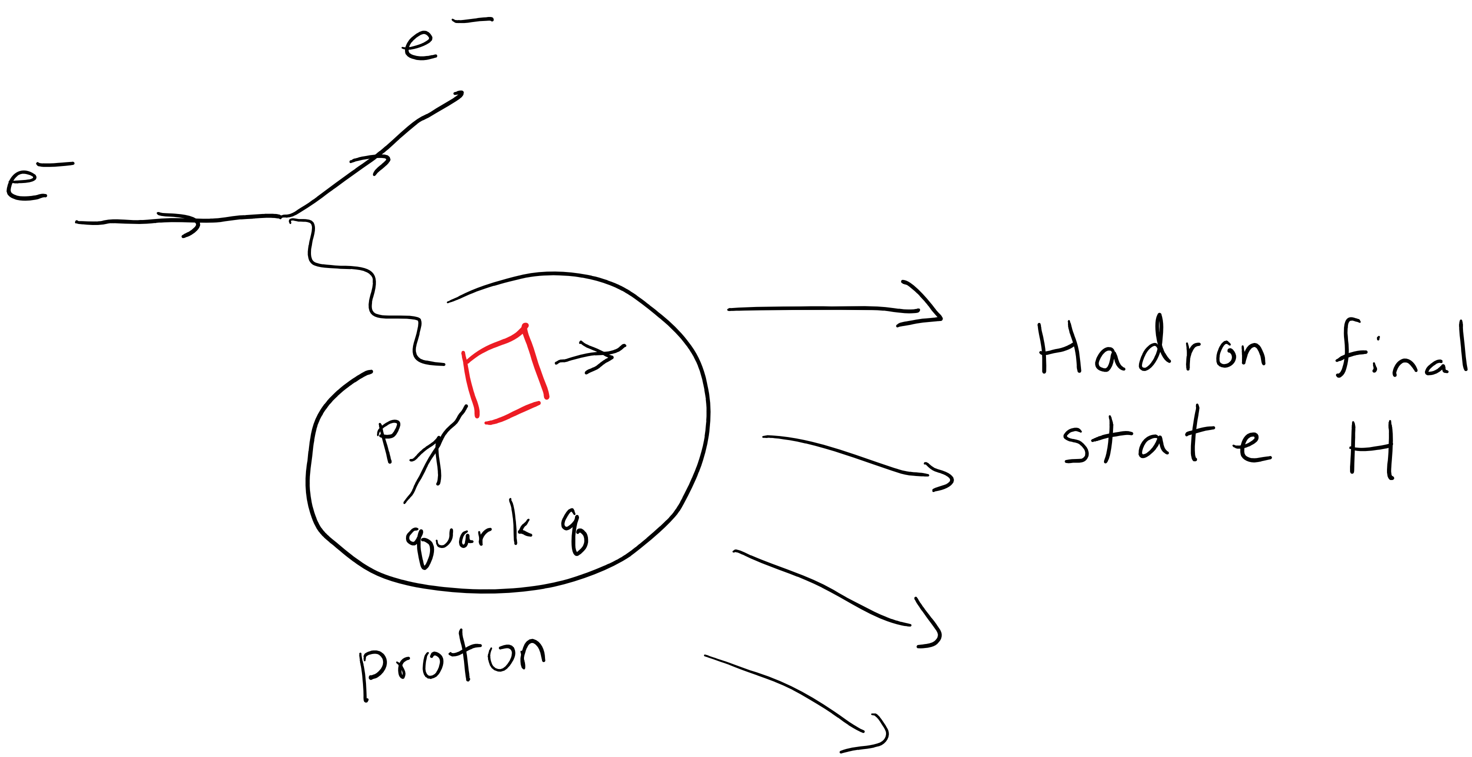
Factorization

$$\sigma(e^- p \to e^- H) = \text{Prob}(q, p) \times \underbrace{\sigma(e^- q \to e^- q)}_{\text{short distance physics}}$$



Factorization

$$\sigma(e^- p \to e^- H) = \text{Prob}(q,p) \times \sigma(e^- q \to e^- q) \times \underbrace{\text{Prob}(H)}_{\substack{\text{probability}\\ \text{to hadronize}\\ \text{into } H}}$$



Factorization

$$\sigma(e^- p \to e^- H) = \sum_{q,p} \text{Prob}(q,p) \times \sigma(e^- q \to e^- q) \times \text{Prob}(H)$$

Sum over all quarks & integrate over momentum p

Factorization

$\sigma(e^- p \to e^- H)$ **Exclusive** cross section to given final hadron state H

Easier to consider **inclusive** cross section summed over all possible final hadron states H

$$\sigma(e^- p \to e^- X) = \sum_H \sigma(e^- p \to e^- H)$$

$$= \sum_{q,p} \mathrm{Prob}(q,p)\, \sigma(e^- q \to e^- q) \sum_H \mathrm{Prob}(H)$$

Factorization

$\sigma(e^- p \to e^- H)$ **Exclusive** cross section to given final hadron state H

Easier to consider **inclusive** cross section summed over all possible final hadron states H

$$\sigma(e^- p \to e^- X) = \sum_H \sigma(e^- p \to e^- H)$$

$$= \sum_{q,p} \text{Prob}(q,p)\, \sigma(e^- q \to e^- q) \underbrace{\sum_H \text{Prob}(H)}_{\text{Total probability to hadronize} = 1}$$

Factorization

$$\sigma(e^- p \to e^- X) = \sum_{q,p} \mathrm{Prob}(q,p)\, \hat{\sigma}(e^- q \to e^- q)$$

X = all possible hadronic final states

Factorization

$$\sigma(e^- p \to e^- X) = \sum_{q,p} \underbrace{\mathrm{Prob}(q,p)}_{\text{large distance physics}} \; \underbrace{\hat{\sigma}(e^- q \to e^- q)}_{\text{short distance physics}}$$

large distance physics → can be measured

short distance physics → compute from perturbation theory

Factorization

$$\sigma(e^- p \to e^- X) = \sum_{q,p} \underbrace{\text{Prob}(q,p)}_{\substack{\text{large distance} \\ \text{physics}}} \; \underbrace{\hat{\sigma}(e^- q \to e^- q)}_{\substack{\text{short distance} \\ \text{physics}}}$$

large distance physics → can be measured

short distance physics → compute from perturbation theory

This is known as the **parton model**. Partons are the constituents inside hadrons: quarks and gluons.

Hats denote parton level quantities:

σ = hadron-level

$\hat{\sigma}$ = parton-level

Deep inelastic scattering

We're going to compute $\sigma(e^- p \to e^- X)$ in two ways

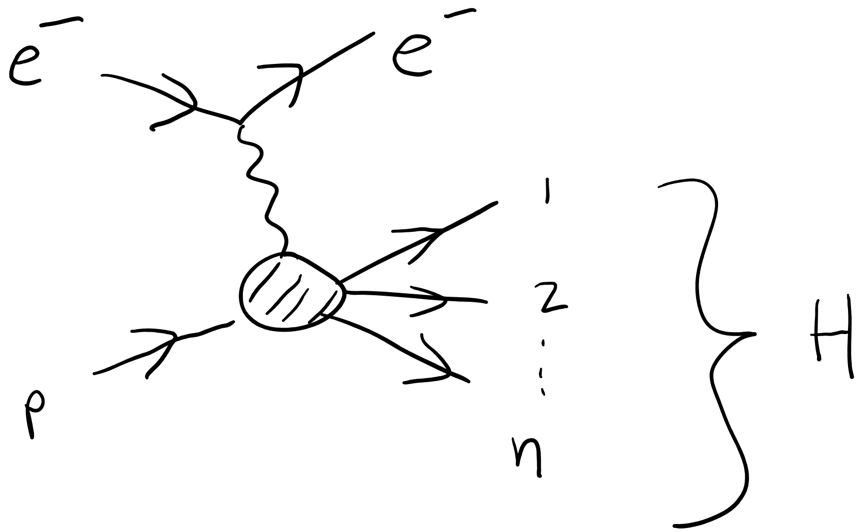
1. Assume nothing (more general)

2. Parton model using factorization

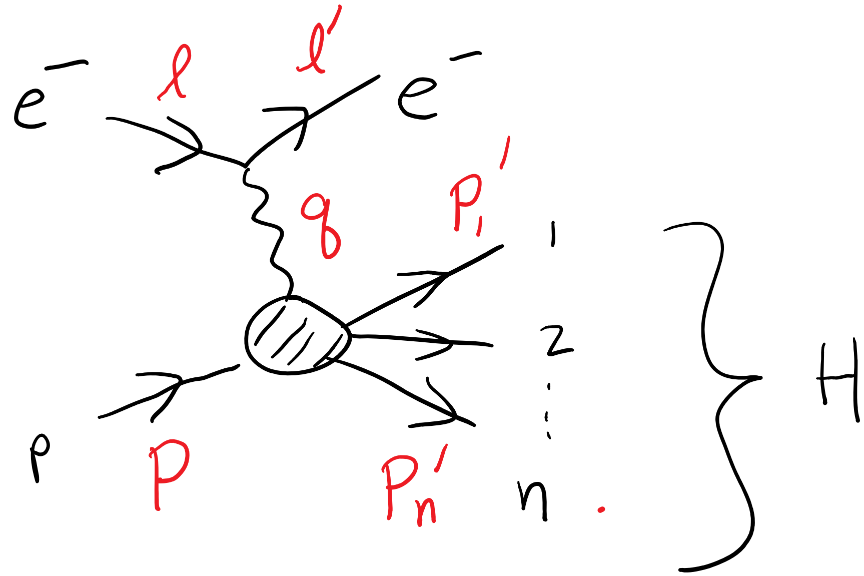
Show proton is made of spin-½ constituents

(Assume high energy, neglect electron mass)

Deep inelastic scattering: part 1



Deep inelastic scattering: part 1



4-momenta

ℓ^μ = incoming e^-

ℓ'^μ = outgoing e^-

P^μ = incoming proton

$$P'^\mu = \sum_{i=1}^{n} P_i'^\mu$$

= total outgoing hadron momenta

$q^\mu = \ell^\mu - \ell'^\mu$ = photon

CM energy

$$s = (P+\ell)^2 \simeq 2P\cdot\ell$$

for $\sqrt{s} \gg m_p, m_e$

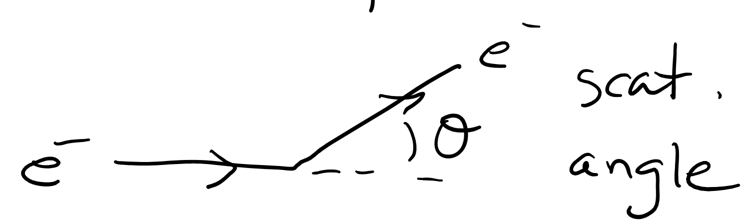
Some kinematic variables

(1) $Q^2 = -q^2 =$ momentum transfer (squared)

Note: $Q^2 > 0$

$$Q^2 = -(l - l')^2 = 2\,l \cdot l'$$

$$= 2E_l E_{l'} (1 - \cos\theta) > 0$$



Some kinematic variables

(2) $\nu = \dfrac{P \cdot q}{m_p}$

Note: "lab frame" proton at rest

$P = (m_p, 0, 0, 0)$

$$\nu = \frac{P \cdot (\ell - \ell')}{m_p} = E_\ell - E_{\ell'}$$

= energy transfer in lab frame

Some kinematic variables

(3) $$y = \frac{P \cdot q}{P \cdot l}$$

In lab frame:

$$y = \frac{m_p (E_l - E_l')}{m_p E_l} = 1 - \frac{E_l'}{E_l}$$

fractional energy transfer

Some kinematic variables

(4) $x = \dfrac{-q^2}{2P\cdot q}$ "Bjorken x"

Some kinematic variables

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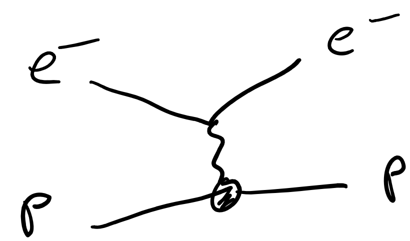
Consider elastic scattering



Final state H = initial state = proton

Some kinematic variables

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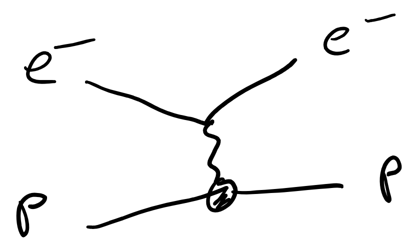
Consider elastic scattering



Final state H = initial state = proton

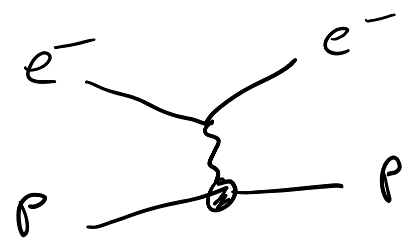
$$P'^2 = m_p^2 = (P + l - l')^2 = (P+q)^2$$

$$= m_p^2 + q^2 + 2P\cdot q$$

Some kinematic variables

(4) $$x = \frac{-q^2}{2P\cdot q}$$ "Bjorken x"

Consider elastic scattering



Final state H = initial state = proton

$$q^2 + 2P\cdot q = 0 \longrightarrow x = \underline{1}$$

Some kinematic variables

(4) $$x = \frac{-q^2}{2P\cdot q}$$ "Bjorken x"

Elastic scattering: $P'^2 = m_p^2 \rightarrow x = \underline{1}$

Inelastic scattering: $P'^2 > m_p^2$

$$m_p^2 + q^2 + 2P\cdot q > m_p^2 \rightarrow x < \underline{1}$$

Some kinematic variables

(4) $$x = \frac{-q^2}{2P\cdot q} \quad \text{"Bjorken } x\text{"}$$

Elastic scattering: $P'^2 = m_p^2 \rightarrow x = \underline{1}$

Inelastic scattering: $P'^2 > m_p^2$

$$m_p^2 + q^2 + 2P\cdot q > m_p^2 \rightarrow x < \underline{1}$$

Also $q^2 < 0 \rightarrow x > 0$

Some kinematic variables

(4) $$x = \frac{-q^2}{2P\cdot q}$$ "Bjorken x"

Elastic scattering: $x = 1$

Inelastic scattering: $0 < x < 1$

Counting kinematic variables

Recall: Mandelstam variables s, t, u

for $2 \to 2$ scattering

Not all independent $s + t + u = \sum_i m_i^2$

s is fixed by experimental setup

t (or u) is remaining kinematic variable to be integrated over

Counting kinematic variables

Elastic scattering $e^- p \to e^- p$

<u>one</u> kinematic variable t

Counting kinematic variables

Elastic scattering $e^- p \rightarrow e^- p$

<u>one</u> kinematic variable t

or any <u>one</u> of Q^2, y, ν

They are all related

$$t = -Q^2 , \quad \nu = \frac{Q^2}{2m_p} , \quad y = \frac{2m_p \nu}{s}$$

(and $x = 1$)

Counting kinematic variables

Inelastic scattering: $e^- p \to e^- H$

Two kinematic variables:

e.g. x and Q^2

then $\nu = \dfrac{Q^2}{2 m_p x}$, $y = \dfrac{Q^2}{x s}$

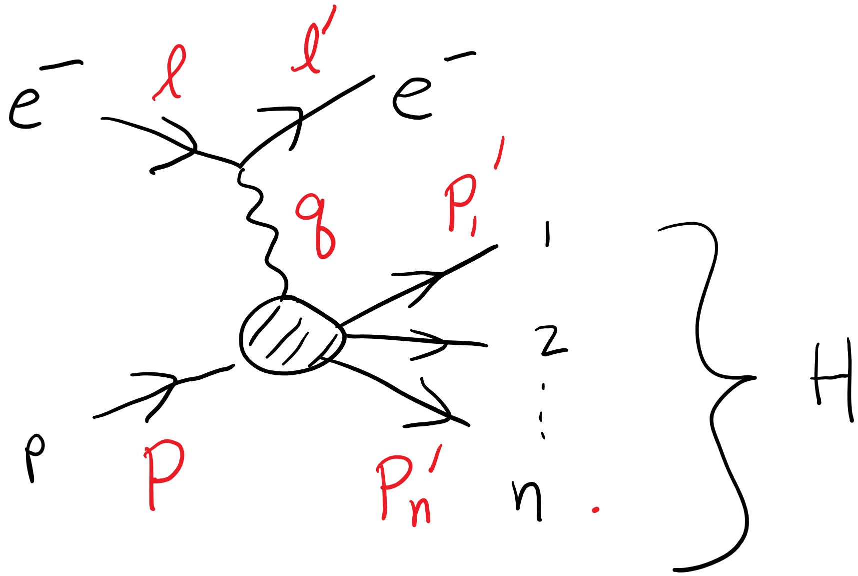
Counting kinematic variables

Elastic scattering: **one** independent kinematic variable

Inelastic scattering: **two** independent kinematic variables

(Not including center-of-mass energy *s*)

Deep inelastic scattering: part 1



$$\sigma(e^- p \to e^- X)$$

$$= \sum_H \frac{1}{2E_\ell 2E_p |v_{rel}|} \int \frac{d^3\ell'}{(2\pi)^3} \frac{1}{2E_\ell'} \prod_{i=1}^{n} \int \frac{d^3 P_i'}{(2\pi)^3} \frac{1}{2E_{p_i'}}$$

$$\times (2\pi)^4 \delta^4(\ell + P - \ell' - \sum_i P_i') \frac{1}{4} \sum_{spins} |\mathcal{M}(e^- p \to e^- H)|^2$$

Deep inelastic scattering: part 1

Matrix element:

$$i\mathcal{M}(e^-p \to e^- H)$$

$$= \bar{u}(\ell')\, ie\gamma^\mu u(\ell)\, \frac{-i}{q^2} \langle H |\, ie j_\mu^{em} \,| p \rangle$$

$$j_\mu^{em} = e \sum_q \bar{q}\, Q_q \gamma^\mu q = \text{EM current operator}$$

Deep inelastic scattering: part 1

Matrix element:

$$i\mathcal{M}(e^- p \to e^- H)$$

$$= \bar{u}(l')\, ie\gamma^\mu u(l)\, \frac{-i}{q^2} \underbrace{\langle H |\, ie J_\mu^{em} \,| p \rangle}_{\text{We can't compute this}}$$

Deep inelastic scattering: part 1

$$\frac{1}{4}\sum_{\text{spins}} \left|\mathcal{M}(e^- p \to e^- H)\right|^2$$

$$= \frac{e^4}{q^4}\,\frac{1}{4}\,\mathrm{Tr}\left[\not{k}'\gamma^\mu \not{k}'\gamma^\nu\right]\langle H|\,j^{em}_\mu\,|p\rangle\langle p|\,j^{em}_\nu\,|H\rangle$$

Deep inelastic scattering: part 1

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}(e^- p \to e^- H)|^2$$

$$= \frac{e^4}{q^4}\,\frac{1}{4}\,\mathrm{Tr}\left[\not{k}'\gamma^\mu \not{k}'\gamma^\nu\right] \underbrace{\langle H| j^{em}_\mu |p\rangle \langle p| j^{em}_\nu |H\rangle}_{\text{still can't compute this}}$$

Deep inelastic scattering: part 1

$$\frac{1}{4}\sum_{\text{spins}} \left|\mathcal{M}(e^- p \to e^- H)\right|^2$$

$$= \frac{e^4}{q^4} \underbrace{\frac{1}{4}\mathrm{Tr}\left[\not{l}'\gamma^\mu \not{l}'\gamma^\nu\right]}_{\frac{1}{2}L^{\mu\nu}} \langle H | j^{em}_\mu | p \rangle \langle p | j^{em}_\nu | H \rangle$$

$$\frac{1}{2}L^{\mu\nu} = l_\mu l'_\nu + l_\nu l'_\mu - l\cdot l'\,\eta_{\mu\nu}$$

Deep inelastic scattering: part 1

$$\sigma(e^- p \to e^- X) = \frac{1}{2E_\ell} (2\pi) \int \frac{d^3\ell'}{(2\pi)^3} \frac{1}{2E_{\ell'}} \frac{e^4}{q^4} L^{\mu\nu}$$

$$\times \frac{1}{2E_p} \sum_H \prod_{i=1}^{n} \int \frac{d^3P_i'}{(2\pi)^3} \frac{1}{2E_{P_i'}} (2\pi)^3 \delta^4(\ell + P - \ell' - P')$$

$$\times \frac{1}{2} \sum_{\text{spins}} \langle H | j_\mu^{em} | p \rangle \langle p | j_\nu^{em} | H \rangle$$

Deep inelastic scattering: part 1

$$\sigma(e^- p \to e^- X) = \frac{1}{2E_\ell}(2\pi)\int \frac{d^3\ell'}{(2\pi)^3}\frac{1}{2E_{\ell'}}\frac{e^4}{q^4}L^{\mu\nu}$$

$$\times \frac{1}{2E_p}\sum_H \prod_{i=1}^{n}\int \frac{d^3P_i'}{(2\pi)^3}\frac{1}{2E_{P_i'}}(2\pi)^3\,\delta^4(\ell+P-\ell'-P')$$

$$\times \frac{1}{2}\sum_{\text{spins}}\langle H|j_\mu^{em}|p\rangle\langle p|j_\nu^{em}|H\rangle$$

$W_{\mu\nu}(P,q)$ parametrize our ignorance

Deep inelastic scattering: part 1

$$\sigma(e^- p \to e^- X) = \frac{1}{2E_\ell}(2\pi)\int \frac{d^3\ell'}{(2\pi)^3}\frac{1}{2E_{\ell'}}\frac{e^4}{q^4}L^{\mu\nu}W_{\mu\nu}$$

Unknown hadronic physics in $W_{\mu\nu}$

Deep inelastic scattering: part 1

$W_{\mu\nu}(P, q)$ restricted by Lorentz invariance

How do we construct two index object?

$\eta_{\mu\nu} \quad P_\mu P_\nu \quad q_\mu q_\nu \quad P_\mu q_\nu \quad q_\mu P_\nu \quad \varepsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta$

Deep inelastic scattering: part 1

$W_{\mu\nu}(P, q)$ restricted by Lorentz invariance

How do we construct two index object?

$$\eta_{\mu\nu} \quad P_\mu P_\nu \quad \cancel{q_\mu q_\nu} \quad \cancel{P_\mu q_\nu} \quad \cancel{q_\mu P_\nu} \quad \varepsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta$$

vanish when contracted with $L^{\mu\nu}$

Deep inelastic scattering: part 1

$W_{\mu\nu}(P, q)$ restricted by Lorentz invariance

How do we construct two index object?

$$\eta_{\mu\nu} \quad P_\mu P_\nu \quad \cancel{q_\mu q_\nu} \quad \cancel{P_\mu q_\nu} \quad \cancel{q_\mu P_\nu} \quad \cancel{\varepsilon_{\mu\nu\alpha\beta} P^\alpha q^\beta}$$

parity-violating
not allowed
for EM interaction

Deep inelastic scattering: part 1

$W_{\mu\nu}(p, q)$ restricted by Lorentz invariance

How do we construct two index object?

$$\eta_{\mu\nu} \quad p_\mu p_\nu \quad \cancel{q_\mu q_\nu} \quad \cancel{p_\mu q_\nu} \quad \cancel{q_\mu p_\nu} \quad \cancel{\varepsilon_{\mu\nu\alpha\beta} p^\alpha q^\beta}$$

$$W_{\mu\nu} = -\frac{F_1}{2m_p}\eta_{\mu\nu} + \frac{F_2}{m_p^2 \nu} p_\mu p_\nu$$

Deep inelastic scattering: part 1

$F_{1,2}$ parametrize our ignorance

Deep inelastic scattering: part 1

$F_{1,2}(Q^2, x)$ functions of kinematic variables

Deep inelastic scattering: part 1

$$\sigma(e^- p \to e^- X) = \frac{1}{2E_\ell} (2\pi) \int \frac{d^3\ell'}{(2\pi)^3} \frac{1}{2E_{\ell'}} \frac{e^4}{q^4} L^{\mu\nu} W_{\mu\nu}$$

(1) Now plug in $W_{\mu\nu} \longrightarrow F_{1,2}$

(2) Change of variables

$$\int \frac{d^3\ell'}{(2\pi)^3} \frac{1}{2E_{\ell'}} \to \frac{m_p \nu}{8\pi^2} \int dx\, dy$$

(3) Some algebra

Deep inelastic scattering: part 1

$$\sigma(e^- p \to e^- X) = \int dx\, dy\, \frac{2\pi \alpha_{em}^2 S}{Q^4}$$

$$\times \left(x y^2 F_1(Q^2, x) + 2 F_2(Q^2, x)(1-y) \right)$$

Deep inelastic scattering: part 1

$$\sigma(e^- p \to e^- X) = \int dx\, dy\, \frac{2\pi \alpha_{em}^2 S}{Q^4}$$

$$\times \left(x y^2 F_1(Q^2, x) + 2 F_2(Q^2, x)(1-y) \right)$$

Double-differential cross section

$$\frac{d^2\sigma}{dx\, dy} = \frac{2\pi \alpha_{em}^2 S}{Q^4} \left(x y^2 \underbrace{F_1(Q^2, x)} + 2 \underbrace{F_2(Q^2, x)} (1-y) \right)$$

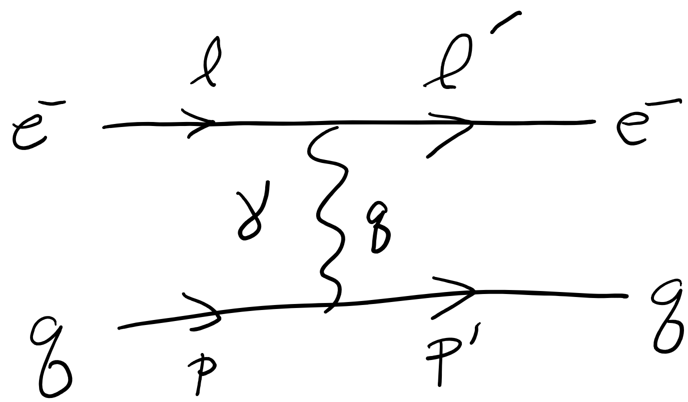
can be measured experimentally

Deep inelastic scattering: part 2

Repeat the same calculation in the parton model

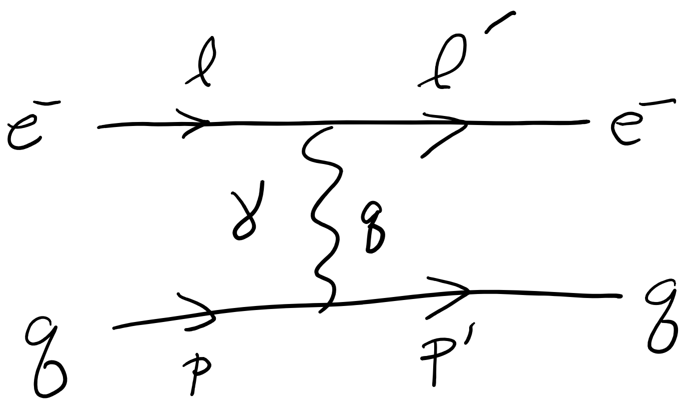
$$\sigma(e^- p \to e^- X) = \sum_{q,p} \mathrm{Prob}(q,p)\, \underbrace{\hat{\sigma}(eq \to eq)}_{\text{calculate in perturbation theory}}$$

Deep inelastic scattering: part 2



Assume e^-, q massless

Deep inelastic scattering: part 2

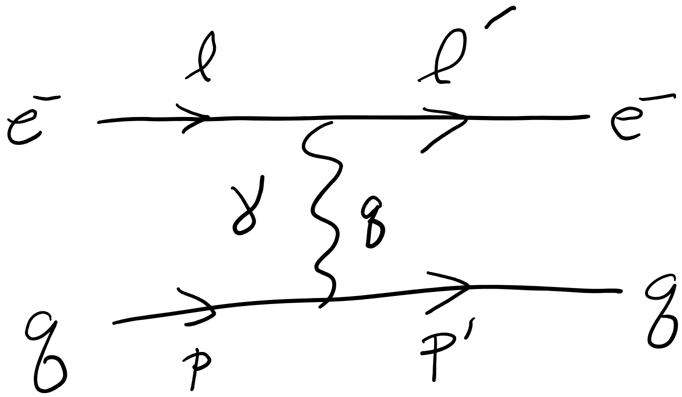


Assume e, q massless

$$\frac{1}{4}\sum |\mathcal{M}|^2 = \frac{1}{4}\frac{e^4}{q^4} Q_q^2 \,\mathrm{Tr}\left[\not{\ell}'\gamma^\mu \not{\ell}\gamma^\nu\right]\mathrm{Tr}\left[\not{p}'\gamma_\mu \not{p}\gamma_\nu\right]$$

$$= \frac{4e^4}{q^4} Q_q^2 \left(\ell\cdot p\,\ell'\cdot p' + \ell\cdot p'\,\ell'\cdot p\right)$$

Deep inelastic scattering: part 2



Assume e^-, q massless

Mandelstam variables:

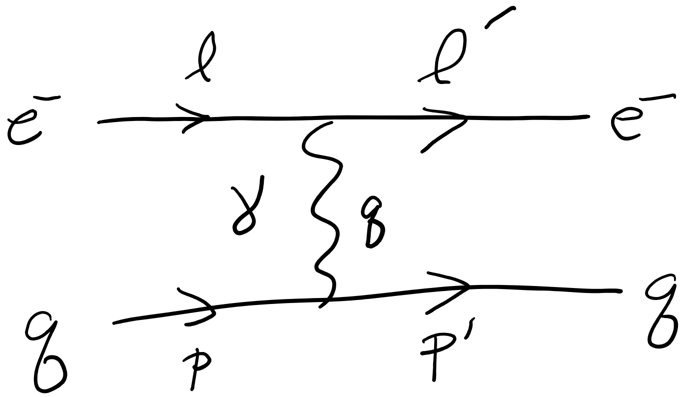
$$\hat{s} = (\ell + p)^2 = (\ell' + p')^2$$

$$\hat{t} = (\ell - \ell')^2 = q^2$$

$$\hat{u} = (\ell - p')^2 = (\ell' - p)^2$$

(hats ($\wedge$) for parton level)

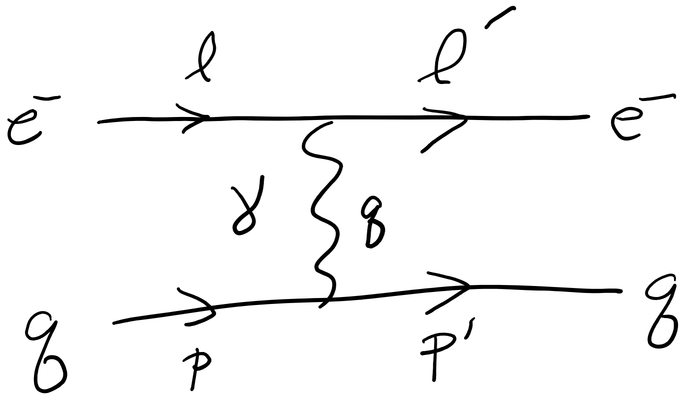
Deep inelastic scattering: part 2



Assume e^-, q massless

$$\frac{1}{4}\sum |\mathcal{M}|^2 = \frac{2e^4 Q_q^2}{\hat{t}^2}\left(\hat{s}^2 + \hat{u}^2\right)$$

Deep inelastic scattering: part 2



Assume e^-, q massless

$$\frac{1}{4}\sum|\mathcal{M}|^2 = \frac{2e^4 Q_q^2}{\hat{t}^2}\left(\hat{s}^2+\hat{u}^2\right)$$

$$\frac{d\hat{\sigma}}{d\hat{t}} = \frac{2\pi\alpha_{em}^2 Q_q^2}{\hat{s}^2\hat{t}^2}\left(\hat{s}^2+\hat{u}^2\right)$$

Parton-level differential cross section with respect to our **one** kinematic variable

Deep inelastic scattering: part 2

Now we need to connect parton-level quantities to the hadron-level quantities that may be observed experimentally

$\hat{t} = q^2 = -Q^2$ (same for both)

Deep inelastic scattering: part 2

Now we need to connect parton-level quantities to the hadron-level quantities that may be observed experimentally

p = 4-momentum of initial quark

P = 4-momentum of initial proton

Deep inelastic scattering: part 2

Now we need to connect parton-level quantities to the hadron-level quantities that may be observed experimentally

p = 4-momentum of initial quark

P = 4-momentum of initial proton

Suppose quark carries fraction x of initial proton momentum

$p^\mu = x P^\mu$ $\quad$ x = momentum fraction of struck quark

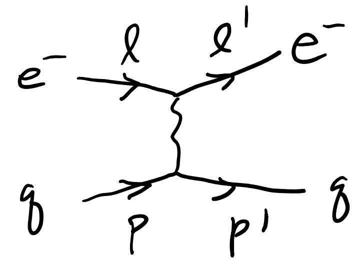
Deep inelastic scattering: part 2

Now we need to connect parton-level quantities to the hadron-level quantities that may be observed experimentally

Solve for x:

Note: massless quarks

$$p^2 = p'^2 = 0$$



$$p'^2 = (p + \ell - \ell')^2$$

$$= p^2 + 2p \cdot q + q^2$$

Deep inelastic scattering: part 2

Now we need to connect parton-level quantities to the hadron-level quantities that may be observed experimentally

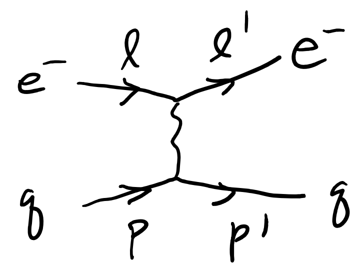
Solve for x:

Note: massless quarks

$$p^2 = p'^2 = 0$$

$$\overset{0}{\cancel{p'^2}} = (p + \ell - \ell')^2$$

$$= \overset{0}{\cancel{p^2}} + 2p \cdot q + q^2$$



Deep inelastic scattering: part 2

Now we need to connect parton-level quantities to the hadron-level quantities that may be observed experimentally

Solve for x:

$$\Rightarrow \quad 0 = 2p\cdot q + q^2 = 2x\,P\cdot q + q^2$$

$$x = \frac{-q^2}{2P\cdot q}$$

same Bjorken x!

different physical interpretation

Deep inelastic scattering: part 2

Now we need to connect parton-level quantities to the hadron-level quantities that may be observed experimentally

Also: $s = (\ell + P)^2 = 2\ell \cdot P$ in high energy limit

$$\hat{s} = (\ell + p)^2 = 2\ell \cdot p = 2\ell \cdot P x$$

$$= x s$$

$$\hat{t} = -y x s$$

$$\hat{u} = (1-y) x s$$

Deep inelastic scattering: part 2

Differential cross section becomes

$$\frac{d\hat{\sigma}}{dy} = \frac{2\pi \alpha_{em}^2 Q_q^2}{Q^4}\left(y^2 + 2(1-y)\right)$$

Deep inelastic scattering: part 2

Parton calculation for inclusive cross section

$$\sigma(e^- p \to e^- X) = \sum_{q,p} \underbrace{\mathrm{Prob}(q,p)}_{} \, \hat{\sigma}(eq \to eq)$$

$f_q(x)\,dx$ = probability of finding quark q in proton with momentum fraction from x to $x+dx$

Deep inelastic scattering: part 2

Parton calculation for inclusive cross section

$$\sigma(e^- p \to e^- X) = \sum_q \int_0^1 dx \, f_q(x) \, \hat{\sigma}(eq \to eq)$$

$f_q(x)$ is the **parton distribution function** (pdf)

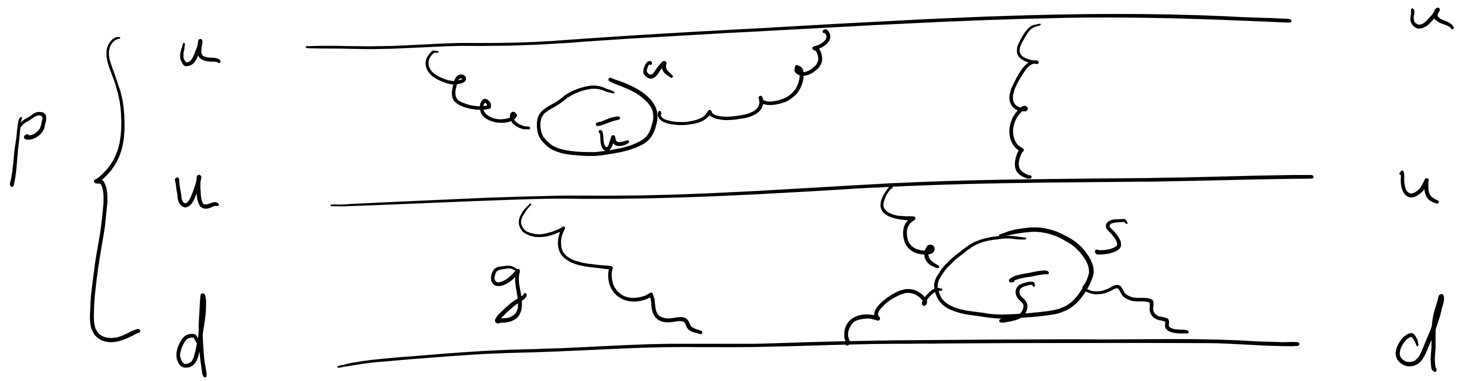
Parton distribution functions

Necessary ingredient for hadron collider calculations

Cannot be calculated from QCD and must be measured experimentally

Parton distribution functions

Which quarks do we include in the sum for the proton?



uud = valence quarks

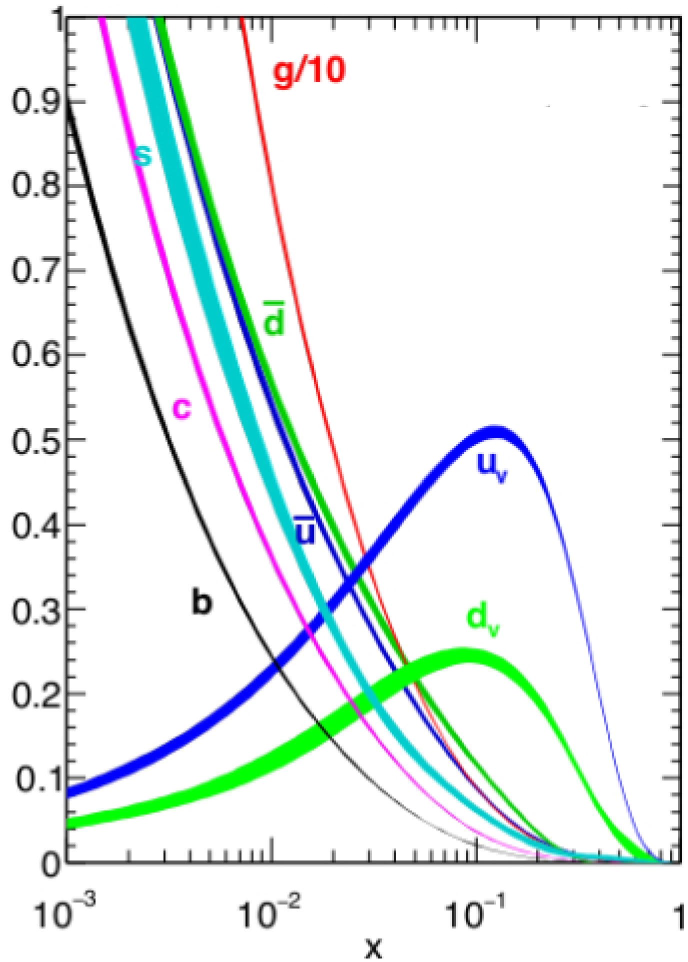
$q\bar{q}$ pairs that fluctuate from vacuum

= sea quarks

Parton distribution functions

Which quarks do we include in the sum for the proton? All quarks & antiquarks

$xf(x)$



PDG

High momentum partons are mostly valence quarks

Low momentum partons are mostly gluons (not important for EM scattering)

Heavy quarks do exist inside the proton

Deep inelastic scattering: part 2

Differential cross section

$$\frac{d\sigma(e^- p \to e^- X)}{dx} = \sum_q f_q(x)\, \hat{\sigma}(e^- q \to e^- q)$$

Deep inelastic scattering: part 2

Differential cross section

$$\frac{d\sigma(e^- p \to e^- X)}{dx} = \sum_q f_q(x)\, \hat{\sigma}(e^- q \to e^- q)$$

Double-differential cross section

$$\frac{d^2\sigma(e^- p \to e^- X)}{dx\, dy} = \sum_q f_q(x)\, \frac{d\hat{\sigma}}{dy}$$

Now let's compare our two results:

General calculation (no information about QCD put in):

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha_{em}^2 S}{Q^4}\left(x y^2 F_1(Q^2,x) + 2F_2(Q^2,x)(1-y)\right)$$

Parton-level calculation (based on factorization):

$$\frac{d^2\sigma}{dx\,dy} = \sum_q f_q(x)\,\frac{2\pi\alpha_{em}^2 Q_q^2\, x S}{Q^4}\left(y^2 + 2(1-y)\right)$$

Now let's compare our two results:

$$x F_1(x, Q^2) = F_2(x, Q^2)$$

$$= \sum_q Q_q^2 \, x \, f_q(x)$$

$$= x \Big[\left(\tfrac{2}{3}\right)^2 \big(f_u(x) + f_{\bar{u}}(x)\big)$$

$$+ \left(\tfrac{-1}{3}\right)^2 \big(f_d(x) + f_{\bar{d}}(x)\big)$$

$$+ \dots \Big]$$

Important results:

1. General result: $F_{1,2}$ functions of x & Q^2

Parton calculation: $F_{1,2}$ functions of x only, not Q^2

Bjorken scaling

Important results:

1. General result: $F_{1,2}$ functions of x & Q^2

   Parton calculation: $F_{1,2}$ functions of x only, not Q^2

   Bjorken Scaling

Deep inelastic scattering is ultimately **elastic** scattering off of the constituent partons. Secretly only **one** independent kinematic variable because it is elastic.

Important results:

2. $x F_1(x) = F_2(x)$ Callan-Gross relation

Holds only if partons are spin-$\frac{1}{2}$

If spin-0 instead have $F_1(x) = 0$

Important results:

3. Scaling violations

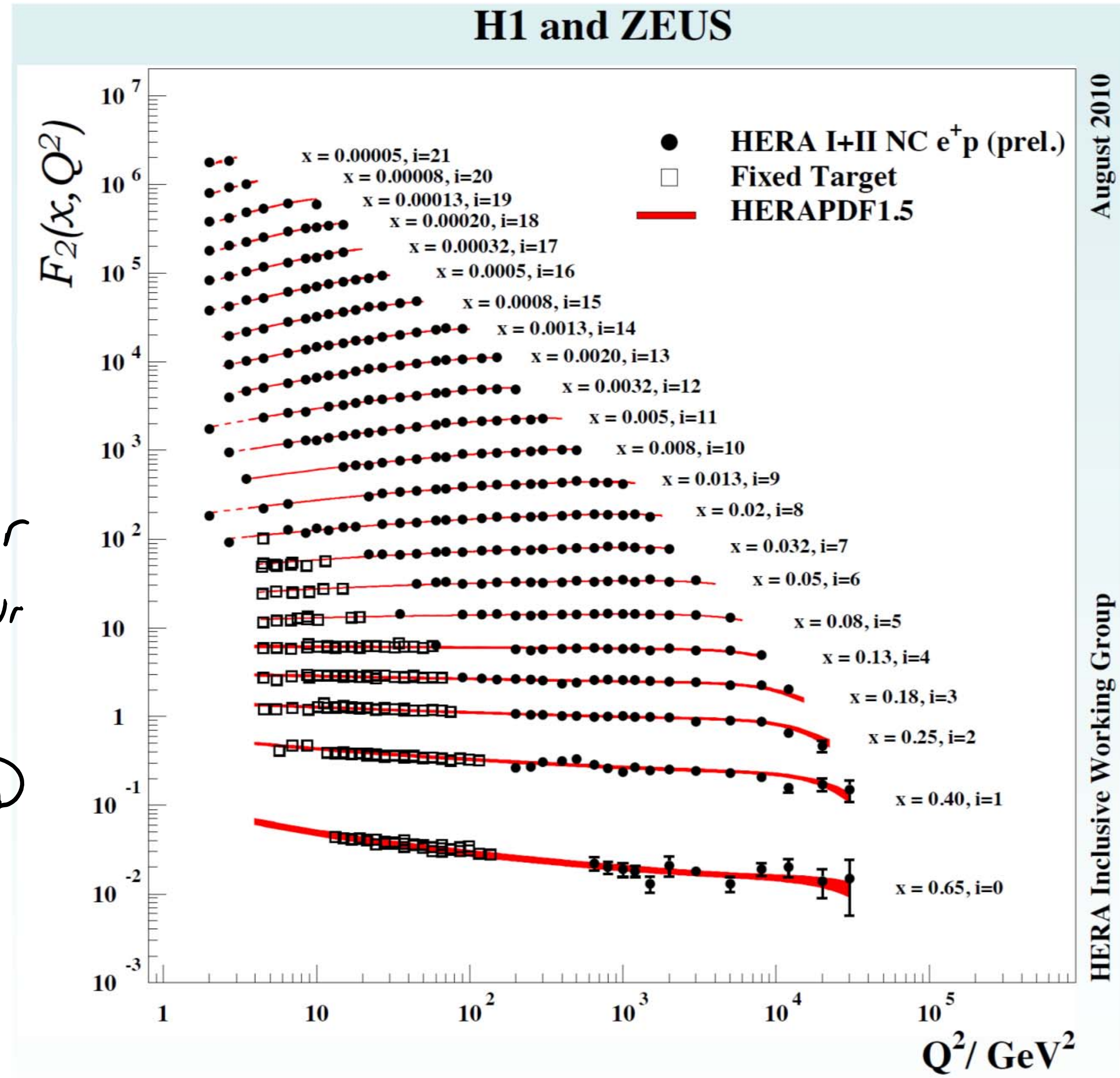
Leading order:

$F_{1,2}$ constant in Q^2

Beyond leading order

"Scaling violations"

Calculable in QCD



Summary: deep inelastic scattering (DIS)

DIS experiments done at SLAC in late 1960s—early 1970s were one of key pieces of evidence for quarks

- Bjorken scaling: manifestation of elastic scattering off of constituent particles in proton (less independent kinematic variables)

- Callan-Gross relation: partons are spin-1/2

- Scaling violations: QCD prediction verified experimentally

Decay and production of the Higgs boson

Higgs interactions relevant for decay

$$\mathcal{L} = -\frac{m_\psi}{v}\bar{\psi}\psi h + \frac{2m_W^2}{v}W^+_\mu W^{-\mu} h + \frac{m_Z^2}{v} Z_\mu Z^\mu h$$

Higgs boson has largest couplings to most massive particles (t,W,Z), but two-body decays $h \to t\bar{t},\, W^+W^-,\, ZZ$ are all forbidden since $m_h = 125$ GeV

This is actually very lucky! We can study not only these largest couplings (via higher order processes), but also more suppressed couplings to e.g. b,τ

Higgs decays to fermions

General expression: $\Gamma(h \to \psi\bar{\psi}) = \dfrac{m_\psi^2 m_h}{8\pi v^2}$

($\times N_c$ if ψ is a quark)

$BR(h \to \tau\bar{\tau}) \simeq 6.3\%$

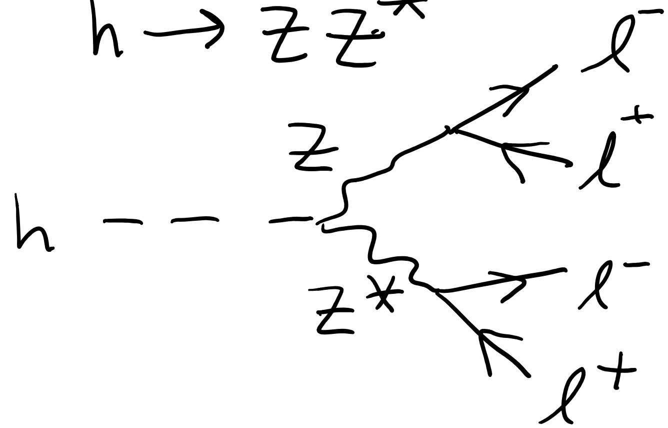
$BR(h \to b\bar{b}) \simeq 58\%$

$BR(h \to c\bar{c}) \simeq 2.9\%$

Higgs decays to gauge bosons (tree-level)

$h \to W^+W^-, ZZ$ allowed if one (or both) gauge bosons are off-shell

e.g. $h \to ZZ^*$



4 leptons (e^+e^- or $\mu^+\mu^-$)

$BR(h \to ZZ^*) \simeq 2.6\%$

$BR(h \to WW^*) \simeq 22\%$

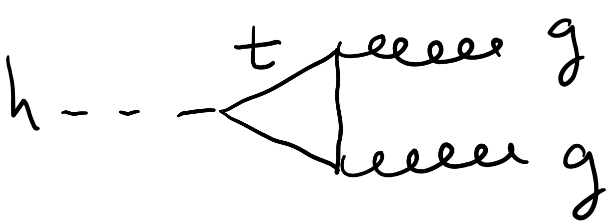
Higgs decays to gauge bosons (one-loop)

Gluons and photons are massless — no tree-level Higgs coupling

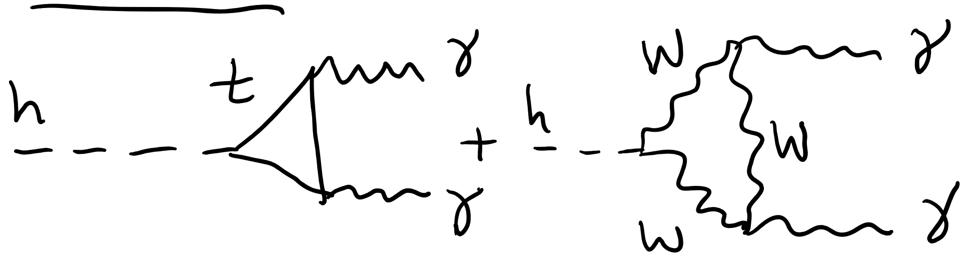
But couple at via loops

Higgs decays to gauge bosons (one-loop)

$\underline{h \to gg}$



$\underline{h \to \gamma\gamma}$



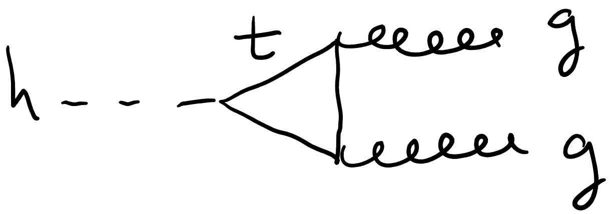
$BR(h \to gg) \simeq 8.6\%$

$BR(h \to \gamma\gamma) \simeq 0.15\%$

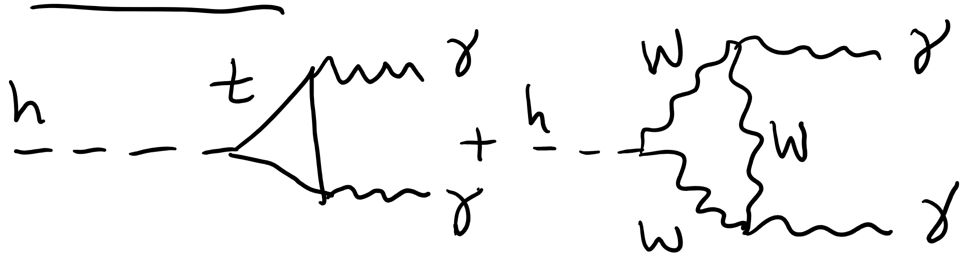
(inverse process $gg \to h$ is important for production at LHC)

Higgs decays to gauge bosons (one-loop)

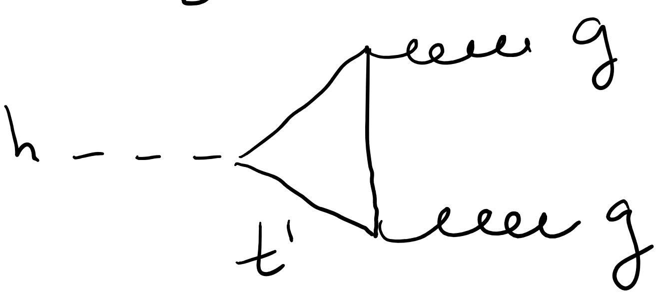
$\underline{h \to gg}$



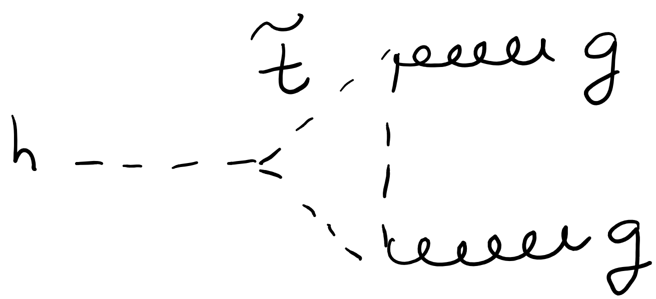
$\underline{h \to \gamma\gamma}$



very sensitive to Beyond the SM physics



4th generation



Supersymmetric partners

Higgs boson production

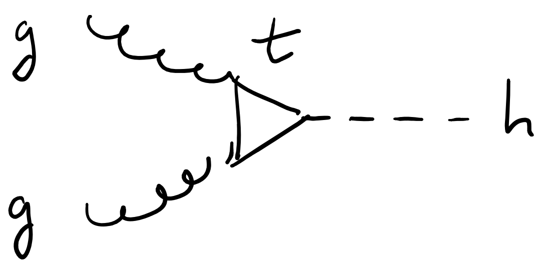
LHC: pp collider at $\sqrt{s} = 7 \ldots 13$ TeV

What is the inclusive cross section $pp \to h X$?

Higgs boson production channels

(In order of importance)

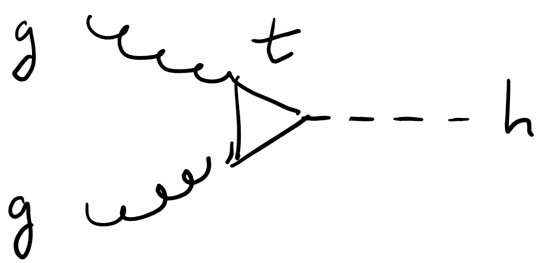
① gluon-gluon fusion (ggF)



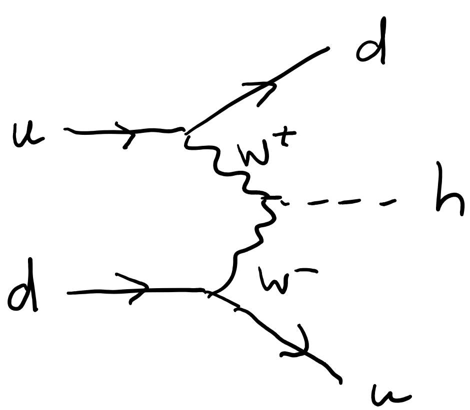
Higgs boson production channels

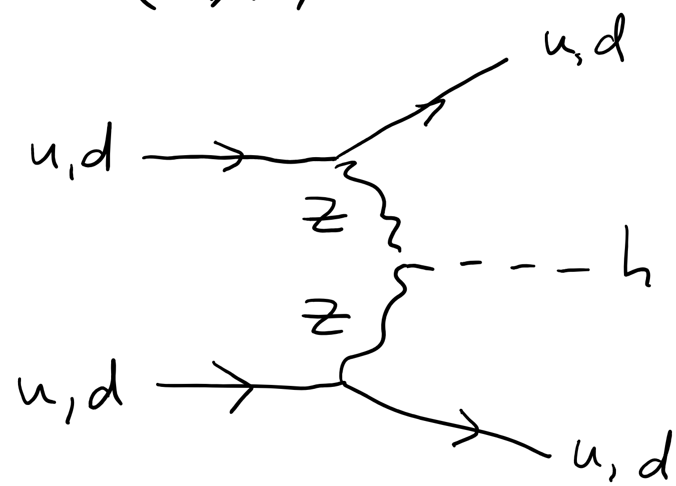
(In order of importance)

1. gluon-gluon fusion (ggF)



2. vector boson fusion (VBF)

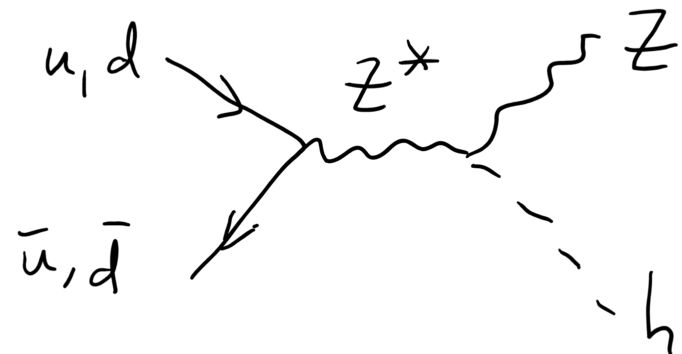




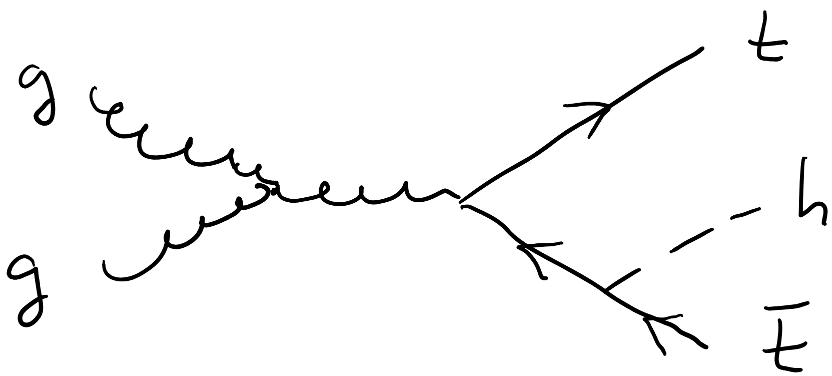
Higgs boson production channels

(In order of importance)

③ Vector boson associated production (WH, ZH)



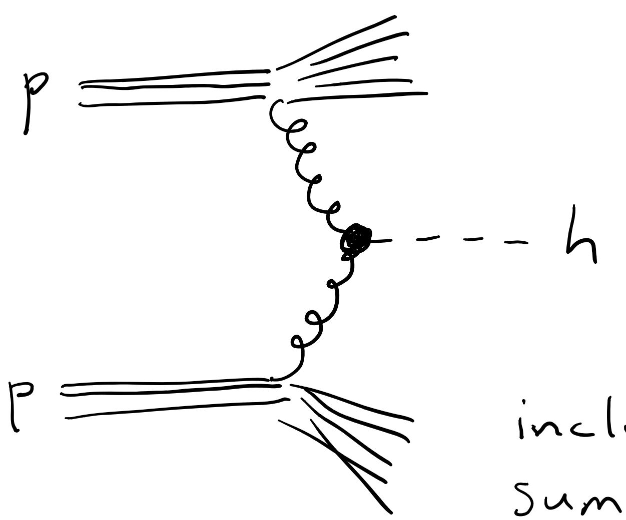
④ top associated production (ttH)



Higgs production cross section: gluon-gluon fusion

parton model:

$$\sigma(pp \to hX) = \int_0^1 dx_1\, f_g(x_1) \int_0^1 dx_2\, f_g(x_2)\, \hat{\sigma}(gg \to h)$$



f_g = gluon pdf

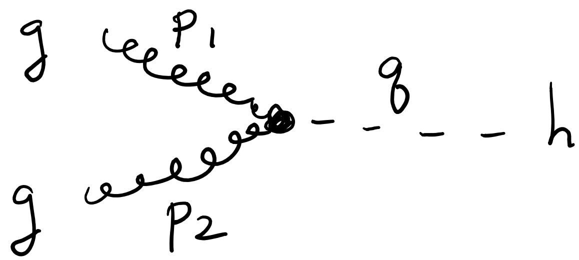
$x_{1,2}$ = fraction of proton momentum carried by struck gluon

inclusive

sum over all hadronic final states.

Higgs production cross section: gluon-gluon fusion

parton-level cross section:



$$\hat{\sigma}(gg\to h) = \frac{1}{4E_1E_2|v_{rel}|}\int\frac{d^3q}{(2\pi)^3}\frac{1}{2E_q}(2\pi)^4\delta^4(p_1+p_2-q)$$

$$\times\left(\frac{1}{8\cdot 2}\right)^2\sum_{\substack{\text{spins}\\\text{colors}}}|\mathcal{M}(gg\to h)|^2$$

Higgs production cross section: gluon-gluon fusion

Trick: use the decay rate

$$\Gamma(h \to gg) = \frac{1}{32\pi m_h} \sum_{\substack{\text{spins} \\ \text{colors}}} |\mathcal{M}(h \to gg)|^2$$

Higgs production cross section: gluon-gluon fusion

Trick: use the decay rate

T-reversal invariance

$$\Gamma(h \to gg) = \frac{1}{32\pi m_h} \sum_{\substack{\text{spins} \\ \text{colors}}} |\mathcal{M}(gg \to h)|^2$$

Higgs production cross section: gluon-gluon fusion

Trick: use the decay rate

T-reversal invariance

$$\Gamma(h \to gg) = \frac{1}{32\pi m_h} \sum_{\substack{\text{spins}\\ \text{colors}}} |\mathcal{M}(gg \to h)|^2$$

$$\hat{\sigma}(gg \to h) = \frac{\pi^2 \Gamma(h \to gg)}{8 m_h} \delta(\hat{s} - m_h^2)$$

Higgs production cross section: gluon-gluon fusion

Now relate parton-level quantities to hadron-level quantities

$P_{1,2}^{\mu}$ = initial proton momenta

$p_{1,2}^{\mu}$ = initial parton (gluon) momenta

$$= x_{1,2} \, P_{1,2}^{\mu}$$

Higgs production cross section: gluon-gluon fusion

Now relate parton-level quantities to hadron-level quantities

$P_{1,2}^{\mu}$ = initial proton momenta

$p_{1,2}^{\mu}$ = initial parton (gluon) momenta

$\quad = x_{1,2} \, P_{1,2}^{\mu}$

Now relate $\hat{s}$ to s:

$$\hat{s} = (p_1 + p_2)^2 = 2 p_1 \cdot p_2$$

$$s = (P_1 + P_2)^2 = 2 P_1 \cdot P_2$$

Higgs production cross section: gluon-gluon fusion

Now relate parton-level quantities to hadron-level quantities

$P^\mu_{1,2}$ = initial proton momenta

$p^\mu_{1,2}$ = initial parton (gluon) momenta

$= x_{1,2} \, P^\mu_{1,2}$

Now relate $\hat{s}$ to s:

$$\hat{s} = (p_1 + p_2)^2 = 2 p_1 \cdot p_2 \rightarrow \hat{s} = x_1 x_2 s$$

$$s = (P_1 + P_2)^2 = 2 P_1 \cdot P_2$$

Higgs production cross section: gluon-gluon fusion

$$\sigma(pp \to hX) = \int_0^1 dx_1\, f_g(x_1) \int_0^1 dx_2\, f_g(x_2)$$

$$\times \frac{\pi^2\, \Gamma(h \to gg)}{8 m_h}\, \delta(x_1 x_2 s - m_h^2)$$

Higgs production cross section: gluon-gluon fusion

$$\sigma(pp \to hX) = \int_0^1 dx_1\, f_g(x_1) \int_0^1 dx_2\, f_g(x_2)$$

$$\times \frac{\pi^2\, \Gamma(h \to gg)}{8 m_h} \underbrace{\delta(x_1 x_2 s - m_h^2)}_{\frac{1}{s x_1}\, \delta\left(x_2 - \frac{m_h^2}{s x_1}\right)}$$

Higgs production cross section: gluon-gluon fusion

$$\sigma(pp \to hX) = \int_0^1 dx_1\, f_g(x_1) \int_0^1 dx_2\, f_g(x_2)$$

$$\times \frac{\pi^2\, \Gamma(h \to gg)}{8 m_h} \underbrace{\delta(x_1 x_2 s - m_h^2)}_{\frac{1}{s x_1}\delta\left(x_2 - \frac{m_h^2}{s x_1}\right)}$$

$$= \int_{m_h^2/s}^{1} \frac{dx_1}{x_1} f_g(x_1)\, f_g\left(\frac{m_h^2}{s x_1}\right) \frac{\pi\, \Gamma(h \to gg)}{8 m_h s}$$

Higgs production cross section: gluon-gluon fusion

$$\sigma(pp \to hX) = \int_0^1 dx_1\, f_g(x_1) \int_0^1 dx_2\, f_g(x_2)$$

$$\times \frac{\pi^2 \Gamma(h \to gg)}{8 m_h} \underbrace{\delta(x_1 x_2 s - m_h^2)}_{\frac{1}{s x_1} \delta\left(x_2 - \frac{m_h^2}{s x_1}\right)}$$

$$= \int_{m_h^2/s}^{1} \frac{dx_1}{x_1} f_g(x_1)\, f_g\!\left(\frac{m_h^2}{s x_1}\right) \frac{\pi\, \Gamma(h \to gg)}{8 m_h s}$$

Higgs production cross section: gluon-gluon fusion

To go any further, you can download parton distribution functions and evaluate the integral numerically

$\sqrt{s}$	$\sigma(pp \to hX)$
7 TeV	10.5 pb
8 TeV	11.4 pb
13 TeV	14.8 pb

Higgs production cross section: gluon-gluon fusion

To go any further, you can download parton distribution functions and evaluate the integral numerically

$\sqrt{s}$	leading order in α_s $\sigma(pp \to hX)$	NNLO $\sigma(pp \to hX)$
7 TeV	10.5 pb	15.1 pb
8 TeV	11.4 pb	19.3 pb
13 TeV	14.8 pb	43.9 pb

(eventhough α_s is perturbative, many more diagrams at higher order)

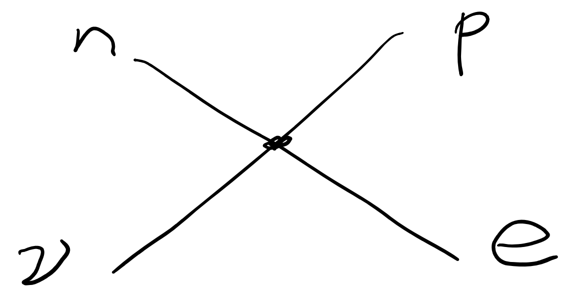
Higgs boson: triumph for both experiment and theory

Brief history of the weak interaction:

1899 Radioactive β-decay discovered by Becquerel

1930 ν proposed by Pauli to conserve
E, $\vec{J}$ in β-decay

1933 Fermi theory
Unknown correct
Dirac structure



1956-7 Parity violation in β-decay
theory: Lee & Yang
expt: Wu

Higgs boson: triumph for both experiment and theory

Brief history of the weak interaction:

1958: V–A theory Marshak, Sudarshan, Feynman, Gell-Man

1960's: Standard Model proposed
Glashow, Salam, Weinberg

w/ "Higgs mechanism" invented by
Higgs, Brout, Englert, Guralnik, Hagen, Kibble

1983: W, Z bosons discovered at CERN

2012: h discovered at CERN

Higgs boson: triumph for both experiment and theory

Taking seriously issues of gauge theory and renormalizability led to the prediction of new particles

Gauge bosons $W^{\pm}, Z$

Higgs boson h

A brand new type of particle – a **fundamental scalar boson**