TRISEP Tutorial 2

(Dated: July 23, 2019)

I. NEUTRAL CURRENT INTERACTIONS AND THE INVISIBLE WIDTH OF THE Z BOSON

The interactions of the Z boson with fermions comes from their covariant derivatives. Keeping only the neutral electroweak gauge fields (i.e., neglecting gluons and the $W^{\pm}$), the covariant derivative is

$$D_\mu = \partial_\mu + igT^3 W_\mu^3 + ig' Y_L B_\mu \tag{1}$$

for left-handed fermions, and

$$D_\mu = \partial_\mu + ig' Y_R B_\mu \tag{2}$$

for right-handed fermions. Note the following:

- The $SU(2)_L$ generator for W_μ^3 is $T^3 = \frac{1}{2}\sigma^3 = \mathrm{diag}(+\frac{1}{2}, -\frac{1}{2})$. Note that all the fields are eigenvalues of T^3: u_L and ν_L have $T^3 = +\frac{1}{2}$, while d_L and e_L have $T^3 = -\frac{1}{2}$.
- We attach the subscript $Y_{L,R}$ to the hypercharges Y for future convenience. For example, for the u quark $Y_L = \frac{1}{6}$ and $Y_R = \frac{2}{3}$, for the electron $Y_L = -\frac{1}{2}$ and $Y_R = -1$, etc.

The neutral current (NC) interaction is then

$$\mathcal{L}_{\mathrm{NC}} = \sum_{\text{fermions } \psi} \bar{\psi}\gamma^\mu \Big(\left(gT^3 W_\mu^3 + g' Y_L B_\mu\right) P_L + \left(g' Y_R B_\mu\right) P_R \Big)\psi \tag{3}$$

(a) Verify that $T^3 + Y_L = Y_R = Q$, where Q is electric charge, for each type of field: u, d, e, and ν.[1]

(b) Expressing W_μ^3 and B_μ in terms of Z_μ and A_μ, show that the neutral current interaction is

$$\mathcal{L}_{\mathrm{NC}} = \sum_{\text{fermions } \psi} \bar{\psi}\gamma^\mu \Big(\frac{g}{c_W}\left(T^3 P_L - Q s_W^2\right) Z_\mu + eQA_\mu \Big)\psi\,. \tag{4}$$

(c) Next, we will consider decays $Z \to \psi\bar{\psi}$. Let's start with the following interaction in a slightly different form from Eq. (4) and consider a generic Lagrangian:

$$\mathcal{L} = \bar{\psi}\gamma^\mu (g_L P_L + g_R P_R)\psi Z_\mu\,. \tag{5}$$

Neglecting the fermion mass ψ, show that the summed squared matrix element is

$$\sum_{spins} |\mathcal{M}|^2 = 2(g_L^2 + g_R^2) m_Z^2 \tag{6}$$

[1] For the neutrino ν, it is only important to note that $T^3 + Y_L = 0$ since there is no ν_R field in the minimal Standard Model where neutrinos are massless.

where the sum goes over all fermion spins and Z polarizations. Don't forget that summing over polarizations yields

$$\sum_{i=1}^{3} \varepsilon_\mu^{(i)}(k)\varepsilon_\nu^{(i)}(k)^* = -\eta_{\mu\nu} + \frac{k_\mu k_\nu}{m_Z^2} \tag{7}$$

(d) Following from part (c), show that the partial width for $Z \to \psi\bar{\psi}$ is

$$\Gamma(Z \to \psi\bar{\psi}) = \frac{m_Z}{24\pi}(g_L^2 + g_R^2)\,. \tag{8}$$

It is probably helpful to recall that the formula for partial width is

$$\Gamma(Z \to \psi\bar{\psi}) = \frac{1}{2m_Z}\int \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}\,(2\pi)^4\delta^4(k-p_1-p_2)\,\frac{1}{3}\sum_{\text{spins}}|\mathcal{M}|^2\,, \tag{9}$$

where the $k = (m_Z, 0, 0, 0)$ is the initial Z momentum at rest, $p_{1,2}$ are the outgoing $\psi, \bar{\psi}$ momenta, and the factor of $\frac{1}{3}$ comes from averaging over the initial three Z polarizations. The integral over the 2-body final state phase space is not difficult if you have done it before. The result of that 6-dimensional integral is

$$\int \frac{d^3p_1}{(2\pi)^3}\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3}\frac{1}{2E_2}\,(2\pi)^4\delta^4(k-p_1-p_2) = \frac{1}{8\pi} \tag{10}$$

assuming the fermions are massless and that the matrix element is rotationally invariant (which is the case in the Z rest frame after averaging over polarizations).

(e) Next, we're going to plug in some actual numbers to evaluate the total Z width Γ_Z. First, from Eq. (4), we have

$$g_L = \frac{g}{c_W}(T^3 - Qs_W^2)\,, \quad g_R = -\frac{g}{c_W}Qs_W^2\,. \tag{11}$$

Determine numerical values for g, s_W, c_W from the known measured quantities

$$\alpha_{\text{em}} = 1/137\,, \quad m_W = 80.4 \text{ GeV}\,, \quad m_Z = 91.2 \text{ GeV}\,. \tag{12}$$

Next, compute the the total Z width

$$\Gamma_Z = \sum_\psi \Gamma(Z \to \psi\bar{\psi}) \tag{13}$$

by summing over all possible fermions in the final state. The observed value is $\Gamma_Z = 2.4952 \pm 0.0023$ GeV. How does your value compare? (Hint: Don't forget about color for quarks.)

(f) The invisible branching fraction of the Z is observed to be $20.000 \pm 0.055\%$. In the Standard Model, invisible Z decays come from decays to the three neutrinos, since these are not observed in most particle detectors. It is customary to rephrase this as a constraint on the **number of neutrinos** as measured through Z decays:

$$N_\nu = \frac{\Gamma(Z \to \text{inv})}{\Gamma_Z^{1\nu}} \tag{14}$$

where $\Gamma(Z \to \text{inv}) = 0.4990 \pm 0.0014$ GeV is the *experimentally measured* invisible partial width for the Z boson, and $\Gamma_Z^{1\nu}$ is your *theoretical calculation* for Z to decay into *one* neutrino. How many neutrinos N_ν are there?

Lee and Weinberg (1977) suggested that an additional heavy neutrino could be a reasonable dark matter candidate for masses $m_\nu > 2$ GeV. Thus, you see that the Z boson excludes this possibility unless $m_\nu > m_Z/2$.