Lattice QCD and current tensions in the Standard Model: $|V_{cb}|$ from $\bar{B} \to D^* \ell \bar{\nu}$ at non-zero recoil

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On behalf of the Fermilab/MILC collaborations

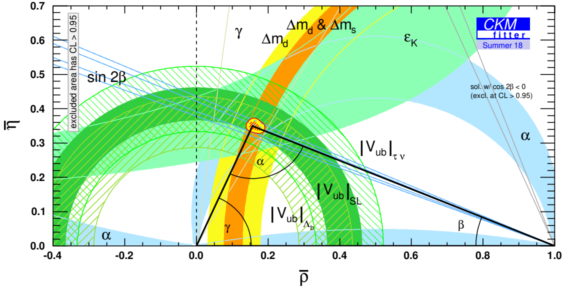
The V_{cb} matrix element: Tensions

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Determination	$\lvert V_{cb} \rvert \, (\cdot 10^{-3})$
Exclusive	39.2 ± 0.7
Inclusive	42.2 ± 0.8

PDG 2016

- Matrix must be unitary (preserve the norm)
- Based on CLN, motivated this work



The V_{cb} matrix element: Measurement from exclusive processes

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \underbrace{\frac{G_F^2 m_B^5}{48\pi^2}(w^2-1)^{\frac{1}{2}}P(w)\left|\eta_{ew}\right|^2}_{\text{Known factors}}\underbrace{\left|\mathcal{F}(w)\right|^2}_{\text{Theory}}\left|V_{cb}\right|^2$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative
- Can use effective theories (HQET) to say something about $\mathcal{F}$
  - Separate light (non-perturbative) and heavy degrees of freedom as $m_Q \to \infty$
  - $\lim_{m_Q\to\infty}\mathcal{F}(w) = \xi(w)$, which is the Isgur-Wise function
  - **We don't know what $\xi(w)$ looks like, but we know $\xi(1) = 1$**
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- Reduction in the phase space $(w^2-1)^{\frac{1}{2}}$ limits experimental results at $w \approx 1$
  - Need to extrapolate $\left|V_{cb}\right|^2\left|\eta_{ew}\mathcal{F}(w)\right|^2$ to $w = 1$
  - This extrapolation is done using well established parametrizations

The V_{cb} matrix element: The parametrization issue

All the parametrizations perform an expansion in the z parameter

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

- Boyd-Grinstein-Lebed (BGL)

  Phys. Rev. Lett. 74 (1995) 4603-4606

  Phys.Rev. D56 (1997) 6895-6911

  Nucl.Phys. B461 (1996) 493-511

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called *outer function* and must be computed for each form factor
  - Unitarity constrains $\sum_n |a_n|^2 \leq 1$

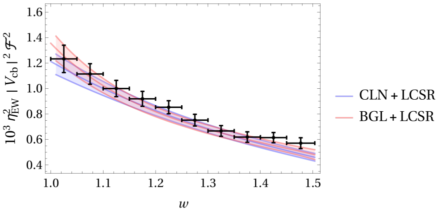
- Caprini-Lellouch-Neubert (CLN)

  Nucl. Phys. B530 (1998) 153-181

$$\mathcal{F}(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $\mathcal{F}(w)$: four independent parameters, one relevant at $w = 1$

The V_{cb} matrix element: The parametrization issue



From *Phys. Lett. B769 (2017) 441-445* using Belle data from arXiv:1702.01521 and the Fermilab/MILC'14 value at zero recoil

- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one

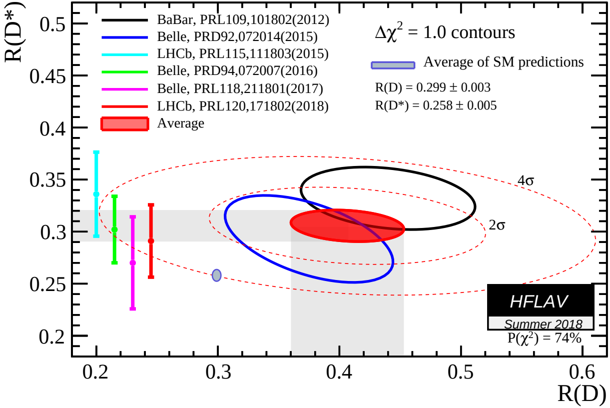
$$|V_{cb}| = 41.7 \pm 2.0(\times 10^{-3})$$

- Latest Belle dataset and Babar analysis seem to contradict this picture
  - From Babar's paper arXiv:1903.10002 **BGL is compatible with CLN and far from the inclusive value**
  - Belle's paper arXiv:1809.03290v3 finds **similar results in its last revision**
- The discrepancy inclusive-exclusive is not well understood
- Data at $w \gtrsim 1$ is **urgently needed** to settle the issue
- Experimental measurements perform badly at low recoil

We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$

The V_{cb} matrix element: Tensions in lepton universality

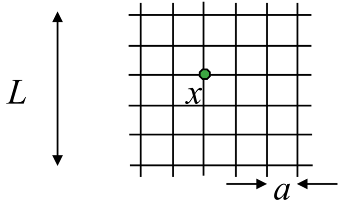
$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current $\approx 4\sigma$ tension with the SM

Introduction to Lattice QCD

$$\mathcal{L}_{QCD} = \sum_f \bar{\psi}_f \left(\gamma^\mu D_\mu + m_f\right) \psi_f + \frac{1}{4} \mathrm{tr} F_{\mu\nu} F^{\mu\nu}$$



- Discretize space-time in a computer
- Perform simulations approaching the physical limit
  - Finite lattice spacing $a \to 0$
  - Finite volume $L \to \infty$
  - $m_\pi \to m_\pi^{\mathrm{Phys}}$, $m_Q \to m_Q^{\mathrm{Phys}}$
- Extrapolate to physical conditions
- **Perform a systematic error analysis using EFTs**

- Use the path integral formulation and montecarlo simulations

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \, e^{-S}, \qquad S = \int d^4x \, \mathcal{L}_{QCD}(\bar{\psi}, \psi, A)$$

Calculating V_{cb} on the lattice: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,\left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\ \ \rho\sigma}v_B^\rho v_{D^*}^\sigma h_V(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,\left|\bar{B}(p_B)\right\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}\left(1+w\right)h_{A_1}(w) - v_B^\nu\left(v_B^\mu h_{A_2}(w) + v_{D^*}^\mu h_{A_3}(w)\right)\right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

Calculating V_{cb} on the lattice: Formalism

- Helicity amplitudes

$$H_{\pm} = \sqrt{m_B\, m_{D^*}}(w+1)\left(h_{A_1}(w) \mp \sqrt{\frac{w-1}{w+1}}h_V(w)\right)$$

$$H_0 = \sqrt{m_B\, m_{D^*}}(w{+}1)m_B\left[(w-r)h_{A_1}(w) - (w-1)\left(r\, h_{A_2}(w) + h_{A_3}(w)\right)\right]/\sqrt{q^2}$$

$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)h_{A_1}(w) + (wr-1)h_{A_2}(w) + (r-w)h_{A_3}(w)\right]$$

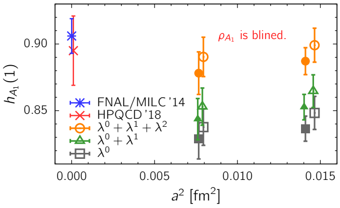
- Form factor in terms of the helicity amplitudes

$$\chi(w)\,|\mathcal{F}|^2 = \frac{1-2wr+r^2}{12 m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

Calculating V_{cb} on the lattice: Available calculations

Zero recoil

- HPQCD'17
  $|\mathcal{F}(1)| = 0.895 \pm 0.010 \pm 0.024$
  PRD**97**, 054502 (2018)
- Fermilab/MILC'14
  $|\mathcal{F}(1)| = 0.906 \pm 0.004 \pm 0.012$
  PRD**89**, 114504 (2014)
- LANL/SWME In progress



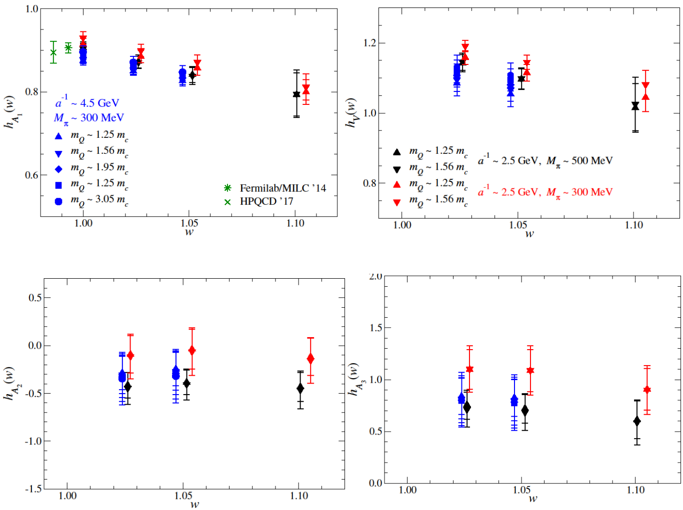
Preliminary results taken from Y. Jang slides during the KEK-FF 2019 meeting

Non-zero recoil

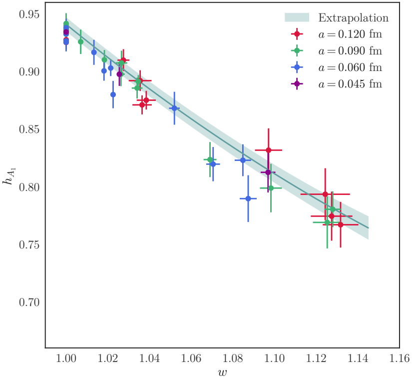
- JLQCD In progress
  - Uses domain wall fermions
  - ~ 8 different ensembles, lowest pion mass ~ 230 MeV
  - Unphysical b quark masses, extrapolation errors
- Fermilab/MILC In progress
  - Uses Staggered asqtad light + Fermilab heavy
  - 15 different ensembles, lowest pion mass ~ 180 MeV
  - Physical b quark mass, discretization and matching errors

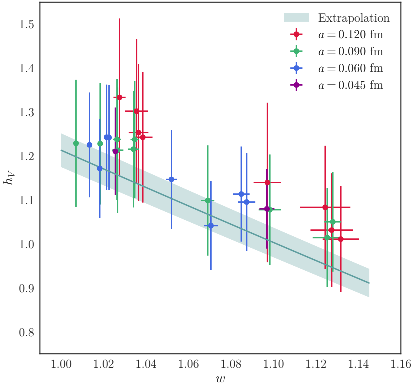
Two different approaches with very different systematics

Results: JLQCD



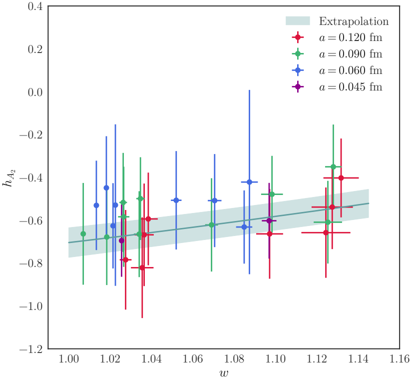
- Preliminary results taken from T. Kaneko's talk during the KEK-FF 2019 conference.

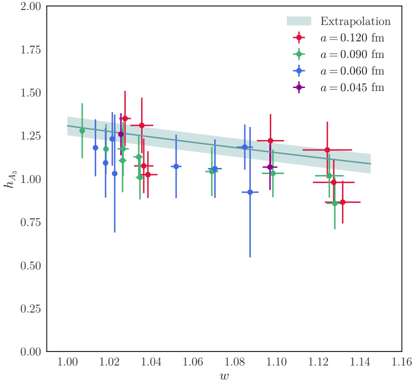




- Preliminary **blinded** results, joint fit $p - \text{value} = 0.36$

Results: Chiral-continuum fits





- Preliminary **blinded** results, joint fit $p - \mathrm{value} = 0.36$

Analysis: Preliminary error budget

Source	h_V (%)	h_{A_1} (%)	h_{A_2} (%)	h_{A_3} (%)
Statistics	1.1	0.4	4.9	1.9
Isospin effects	0.0	0.0	0.6	0.3
χPT/cont. extrapolation	**1.9**	**0.7**	**6.3**	**2.9**
Matching	*1.5*	*0.4*	*0.1*	*1.5*
Heavy quark discretization	*1.4**	*0.4**	*5.8**	*1.3**

*Estimate, currently subject of intensive study

- **Bold** marks errors to be reduced/removed when using HISQ for light quarks
- *Italic* marks errors to be reduced/removed when using HISQ for heavy quarks
  - Heavy HISQ would introduce new extrapolation errors

- We are adding a preliminary 2% error coming from HQ discretization to our form factors in our fits

Analysis: z-Expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w) h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j$$

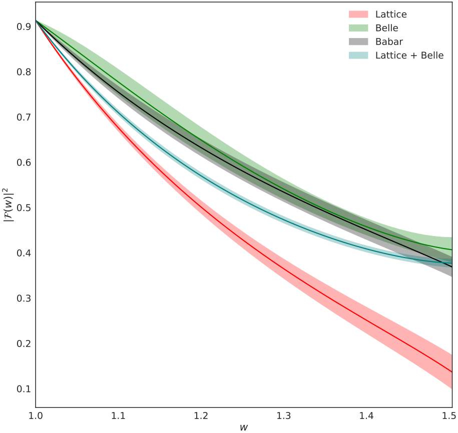
$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

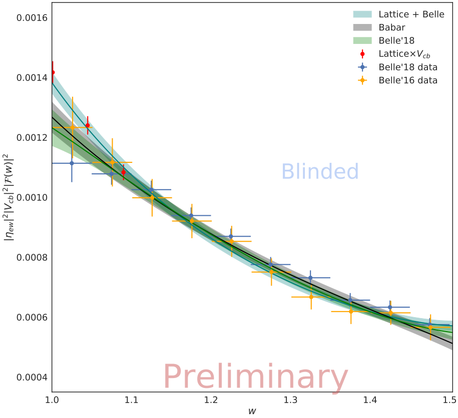
$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- Constraint $(1+w) m_B^2 (1-r) \mathcal{F}_1(z = z_{\mathrm{Max}}) = (1+r) \mathcal{F}_2(z = z_{\mathrm{Max}})$
- BGL (weak) unitarity constraints (all HISQ will use strong constraints)

$$\sum_j a_j^2 \leq 1, \qquad \sum_j b_j^2 + c_j^2 \leq 1, \qquad \sum_j d_j^2 \leq 1$$

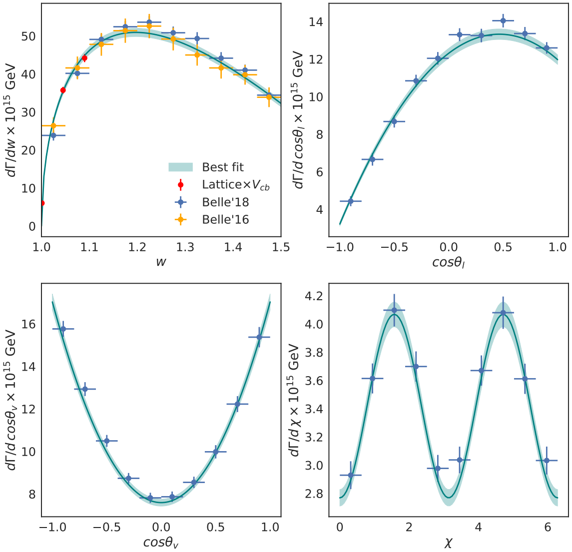
Results: Pure-lattice prediction and joint fit





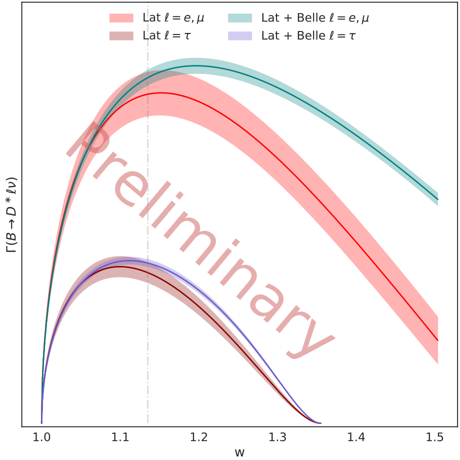
- Lattice + Belle'18 BGL $p-\text{value} \sim O(10^{-5})$
- Lattice only BGL $p-\text{value} = 0.56$, Belle'18 BGL $p-\text{value} = 0.09$
- We are carefully reviewing the latest developments

Results: Angular bins

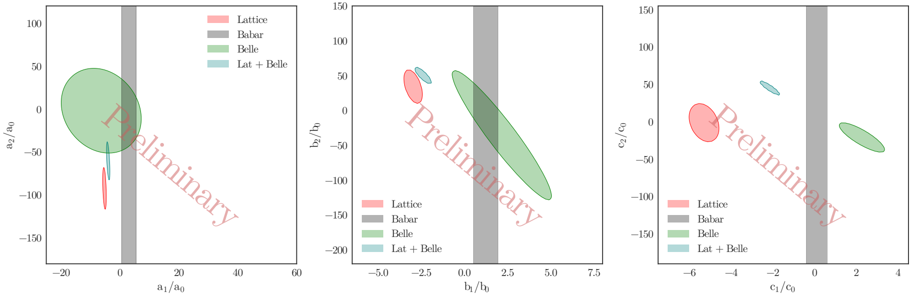


Results: $R(D^*)$

- Pure lattice QCD prediction of $R(D^*)$
- Includes constraint $\mathcal{F}_1(w_{\text{Max}}) = \frac{1+r}{(1+w)m_B^2(1-r)}\mathcal{F}_2(w_{\text{Max}})$



Results: Tensions in the BGL coefficients



- The b_j represent the small recoil behavior $\sim h_{A_1}$
- The c_j represent the large recoil behavior $\sim H_0$

Conclusions

What to expect

- Errors might not be improved compared to previous lattice estimations
- The main new information of this analysis won't come from the zero-recoil value, but from the slope
- Main sources of errors of our form factors are
    - χPT-continuum extrapolation
    - HQ discretization
    - Matching
- **We need to understand better the current lattice and Belle data**
- At this stage, no ETA for this paper (a few crosscheck remain)

The future

- Well established roadmap to reduce errors in our calculation with newer lattice ensembles
- Our next analysis will be joint $B \to D$ and $B \to D^*$ to benefit from strong unitarity constraints
- This roadmap is to be followed in other processes involving other CKM matrix elements

Analysis: What happens if I use CLN?

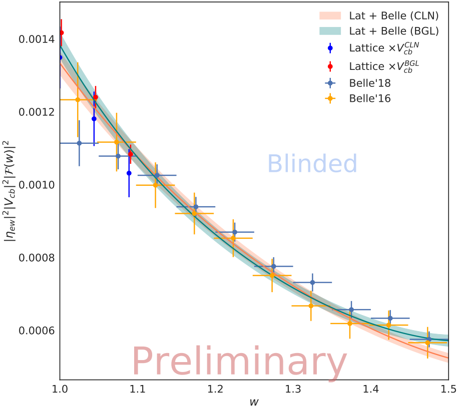
- CLN is much more constraining than BGL, using only 4 fit parameters

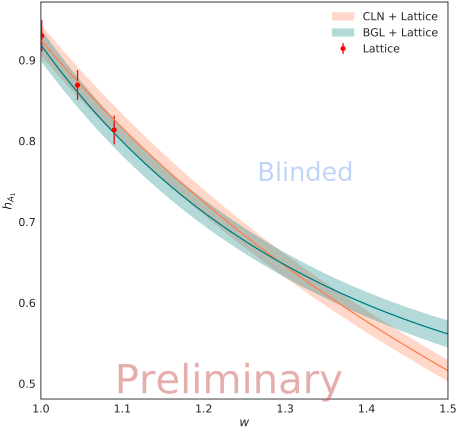
$$h_{A_1}(w) = h_{A_1}(1)\left[1 - 8\rho^2 z + \left(53\rho^2 - 15\right)z^2 - \left(231\rho^2 - 91\right)z^3\right]$$
$$R_1(w) = R_1(1) - 0.12\,(w-1) + 0.05\,(w-1)^2$$
$$R_2(w) = R_2(1) + 0.11\,(w-1) - 0.06\,(w-1)^2$$

with

$$R_1(w) = \frac{h_{A_1}(w)}{h_V(w)}$$
$$R_2(w) = \frac{\frac{m_{D^*}}{m_B}h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)}$$

Analysis: What happens if I use CLN?





- Lattice + Belle'18 CLN $p - \text{value} =\sim O(10^{-13})$
- Prediction for h_{A_1} very constrained in the CLN parametrization