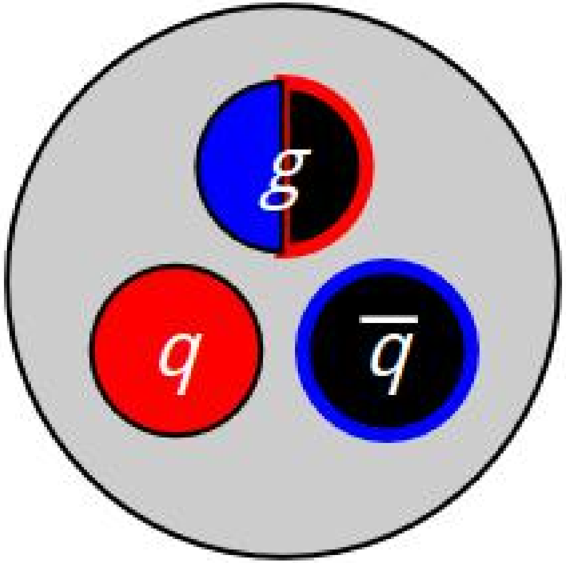
Meson-Hybrid Mixing in Vector (1^{--}) and Axial Vector (1^{++}) Charmonium

Derek Harnett

A. Palameta, J. Ho, D. Harnett, T. G. Steele, Phys. Rev. D97 (2018) 034001 [1707.00063]

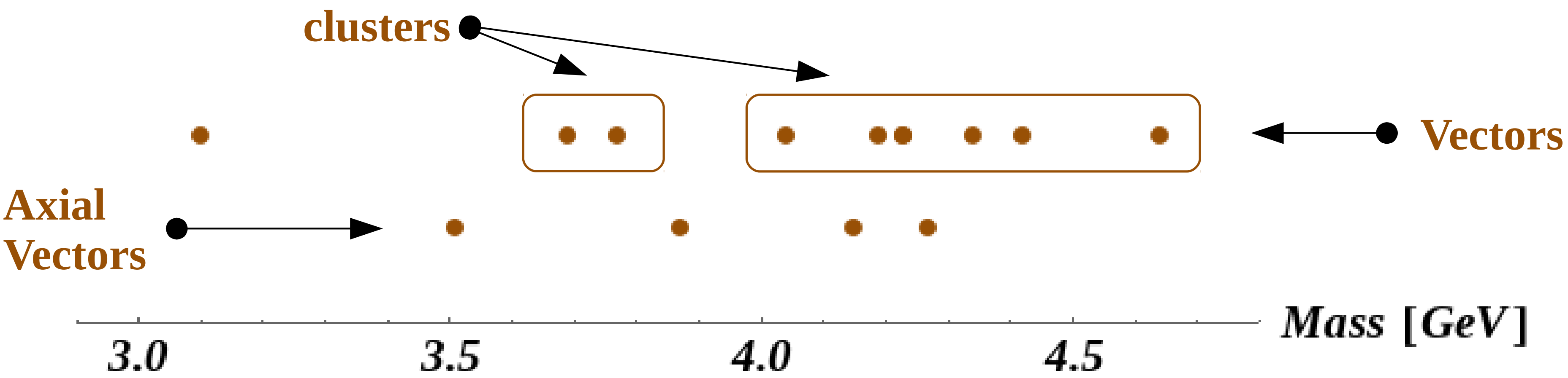
A. Palameta, D. Harnett, and T. G. Steele, Phys. Rev. D98 (2018) 074014 [1806.00157]

Hybrids are outside-the-quark-model hadrons with a constituent quark, antiquark, and gluon.



- test of confinement characterization
- not yet conclusively identified
- J^{PC} can be **exotic,** *e.g.*, {0^{+-}, 0^{--}, 1^{-+}}, or **non-exotic,** *e.g.*, {0^{++}, 0^{-+}, 1^{++}, 1^{--}, 1^{+-}}.
- For non-exotic J^{PC}, **hadron mixing** could be hampering hybrid detection and/or identification.

In the vector (1^{--}) and axial vector (1^{++}) charmonium-like channels, there are a several known resonances.



- In the vector channel, densely packed resonances get clustered.
- **We test each of these known resonances (or clusters) for meson-hybrid mixing using QCD Laplace sum-rules.**

Some closely related problems have been studied using both QCD sum-rules and lattice QCD.

- Matheus *et al.*, Phys. Rev **D80** (2009)
  - 1^{++} charmonium meson-$\bar{D}D^*$ mixing from QCD sum-rules
- Liu *et al.*, JHEP **07** (2012)
  - charmonium spectroscopy from lattice QCD
  - includes meson and hybrid operators
- Chen *et al.*, Phys. Rev. **D88** (2013)
  - 1^{++} charmonium hybrid-$\bar{D}D^*$ mixing from QCD sum-rules
- Padmanath, Lang, and Prelovsek, Phys. Rev. **D92** (2015)
  - 1^{++} charmonium spectroscopy from lattice QCD
  - includes meson, two-meson, and diquark-antidiquark operators

We study meson-hybrid mixing in charmonium using a two-point cross-correlator.

$$\Pi(q^2) = \frac{i}{D-1}\left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\int d^D x\, e^{iq\cdot x}\,\langle\Omega|\tau j_\mu^{(m)}(x) j_\nu^{(h)}(0)|\Omega\rangle$$

***D*=2+2ε, spacetime dimension**

charm quark

$$j_\mu^{(m)} = \begin{cases} \bar{c}\gamma_\mu c \text{ for } 1^{--} \\ \bar{c}\gamma_\mu\gamma_5 c \text{ for } 1^{++} \end{cases}$$

meson current

gluon field strength

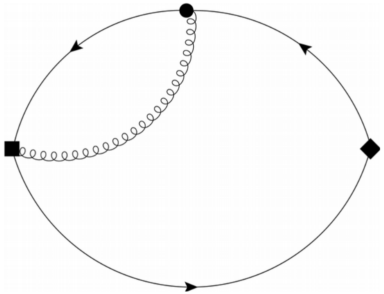
hybrid current

$$j_\nu^{(h)} = \begin{cases} g_s\bar{c}\gamma^\rho\gamma_5\frac{\lambda^a}{2}\left(\frac{1}{2}\epsilon_{\nu\rho\omega\eta}G^a{}_{\omega\eta}\right) c \text{ for } 1^{--} \\ g_s\bar{c}\gamma^\rho\frac{\lambda^a}{2}\left(\frac{1}{2}\epsilon_{\nu\rho\omega\eta}G^a{}_{\omega\eta}\right) c \text{ for } 1^{++} \end{cases}$$

We compute the cross-correlator within the operator product expansion.

$$\Pi^{\mathrm{QCD}}(q^2) =$$

perturbation theory



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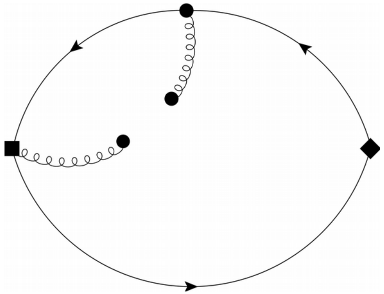
4d gluon condensate



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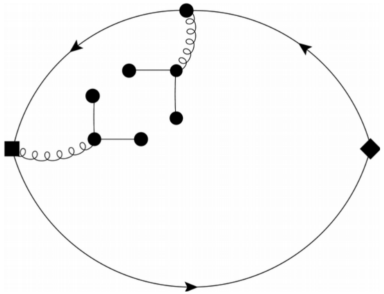
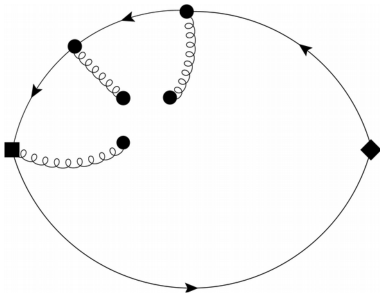
6d quark condensate



6d gluon condensate

$+$



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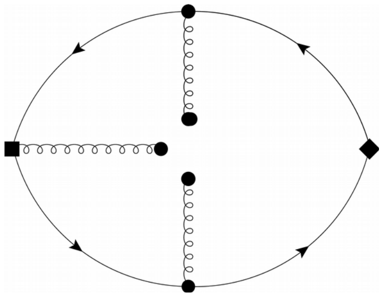

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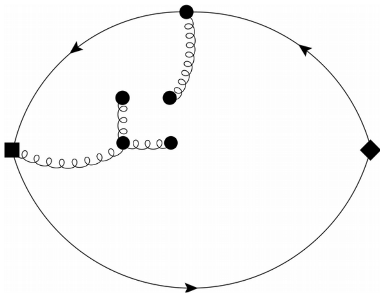


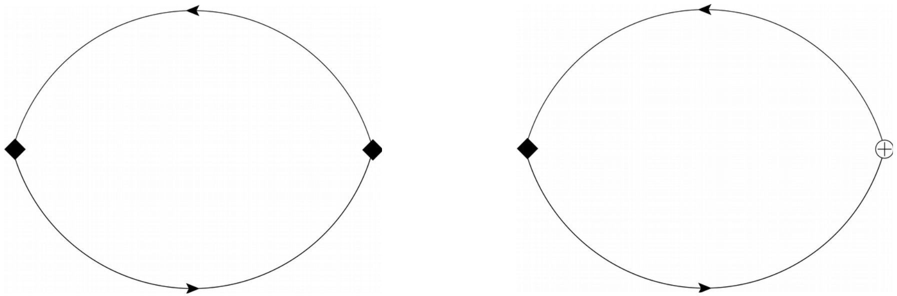
Perturbation theory has a nonlocal divergence eliminated through operator renormalization.

The vector and axial vector currents are renormalization-group invariant, but...

$$j_\nu^{(h)} \to j_\nu^{(h)} - \frac{5 g_s^2 m_c^2}{18\pi^2 \epsilon} (\bar{c}\gamma_\nu c) + \frac{g_s^2 m_c}{9\pi^2 \epsilon} (\bar{c} i D_\nu c) \ \text{ for } 1^{--},$$

$$j_\nu^{(h)} \to j_\nu^{(h)} - \frac{5 g_s^2 m_c^2}{18\pi^2 \epsilon} (\bar{c}\gamma_\nu \gamma_5 c) - \frac{g_s^2 m_c}{9\pi^2 \epsilon} (\bar{c} i \gamma_5 D_\nu c) \ \text{ for } 1^{++}.$$

renormalization-induced diagrams



covariant derivative

Dispersion relations relate QCD to hadron physics, *i.e.*, quark-hadron duality.

$$\Pi\left(Q^2\right) = \frac{Q^6}{\pi}\int_{t_0}^{\infty} \frac{\mathrm{Im}\Pi(t)}{t^3\left(t+Q^2\right)}\,\mathrm{d}t + \cdots \quad \text{for } Q^2 = -q^2 > 0$$

hadron production threshold

subtraction constants

$$\Pi(Q^2) \longrightarrow \Pi^{\mathrm{QCD}}(Q^2)$$

resonance content

continuum threshold

$$\mathrm{Im}\Pi(t) \to \rho^{\mathrm{had}}(t) + \theta(t-s_0)\mathrm{Im}\Pi^{\mathrm{QCD}}(t)$$

We model the resonance content as a sum of narrow and/or rectangular resonances.

number of resonances → n

$$\rho^{\text{had}}(t) = \sum_{i=1}^{n} \rho_i^{\text{had}}(t)$$

resonance width → Γ_i

$$\rho_i^{\text{had}}(t) = \begin{cases} \xi_i\,\delta(t - m_i^2),\ \ \Gamma_i = 0 \\ \frac{\xi_i}{2m_i\Gamma_i}\,\theta(t - m_i(m_i - \Gamma_i))\theta(m_i(m_i + \Gamma_i) - t),\ \ \Gamma_i \neq 0 \end{cases}$$

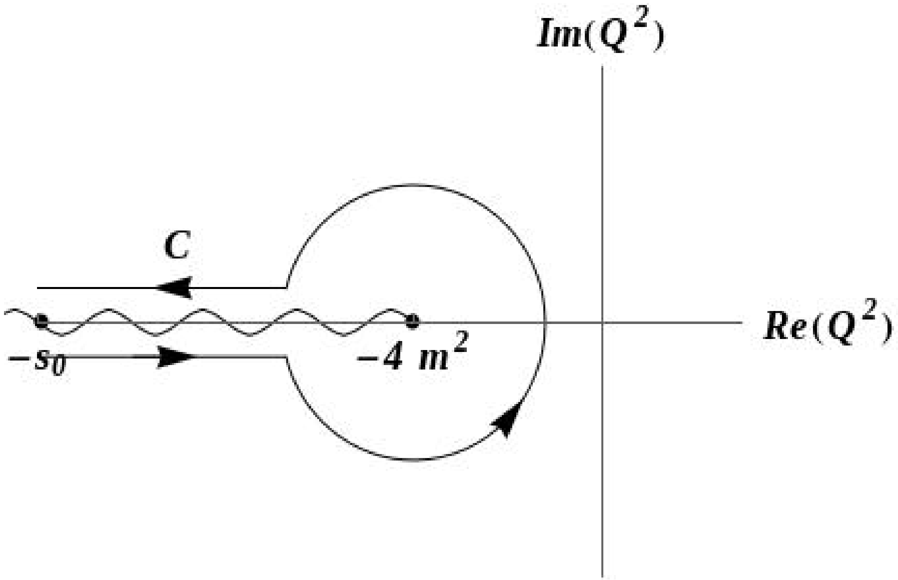
- The **mixing parameters, ξ_i,** are products of hadronic couplings.
- **A non-zero mixing parameter indicates coupling to both meson and hybrid currents.**

QCD Laplace sum-rules are transformed dispersion relations.

$$\mathcal{R}(\tau,\, s_0) \equiv \frac{1}{2\pi i}\int_C e^{Q^2\tau}\,\Pi^{\rm QCD}(Q^2)\,\mathrm{d}Q^2$$

$$= \int_{t_0}^{s_0} e^{-t\tau}\,\frac{1}{\pi}\rho^{\rm had}(t)\,\mathrm{d}t$$

subtracted Laplace sum-rules (LSRs)



Hadron parameters extracted as best-fit parameters between QCD and hadron physics.

Using QCD sum-rules, we input masses (and cluster widths) and extract mixing parameters.

Model	m_1 [GeV]	m_2 [GeV]	m_3 [GeV]	Γ_3 [GeV]
V1	3.10	–	–	–
V2	3.10	3.73	–	–
V3	3.10	3.73	4.30	–
V4	3.10	3.73	4.30	0.30
V5	3.10	–	4.30	–
V6	3.10	–	4.30	0.30

Model	m_1 [GeV]	m_2 [GeV]	m_3 [GeV]	m_4 [GeV]
A1	3.51	–	–	–
A2	3.51	3.87	–	–
A3	3.51	3.87	4.15	–
A4	3.51	3.87	4.15	4.27

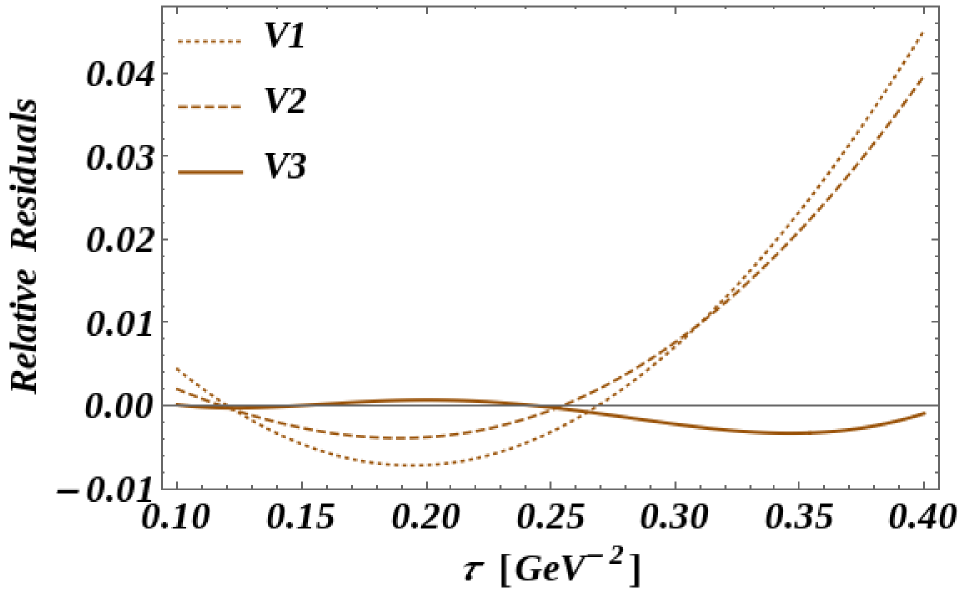
In the vector channel, our results favour a two-resonance scenario.

Model	s_0 [GeV^2]	$\frac{\chi^2}{\chi^2[\mathrm{V1}]}$	ζ [GeV^6]	$\frac{\xi_1}{\zeta}$	$\frac{\xi_2}{\zeta}$	$\frac{\xi_3}{\zeta}$
V1	12.5	1	0.51(2)	1	–	–
V2	13.9	0.73	0.73(4)	0.72(3)	0.27(3)	–
V3	24.1	0.038	2.9(3)	0.22(1)	–0.02(5)	0.76(3)
V4	24.2	0.038	3.0(3)	0.21(1)	–0.03(5)	0.76(3)
V5	23.7	0.042	2.7(3)	0.23(2)	–	0.77(2)
V6	23.6	0.047	2.7(3)	0.23(2)	–	0.77(2)

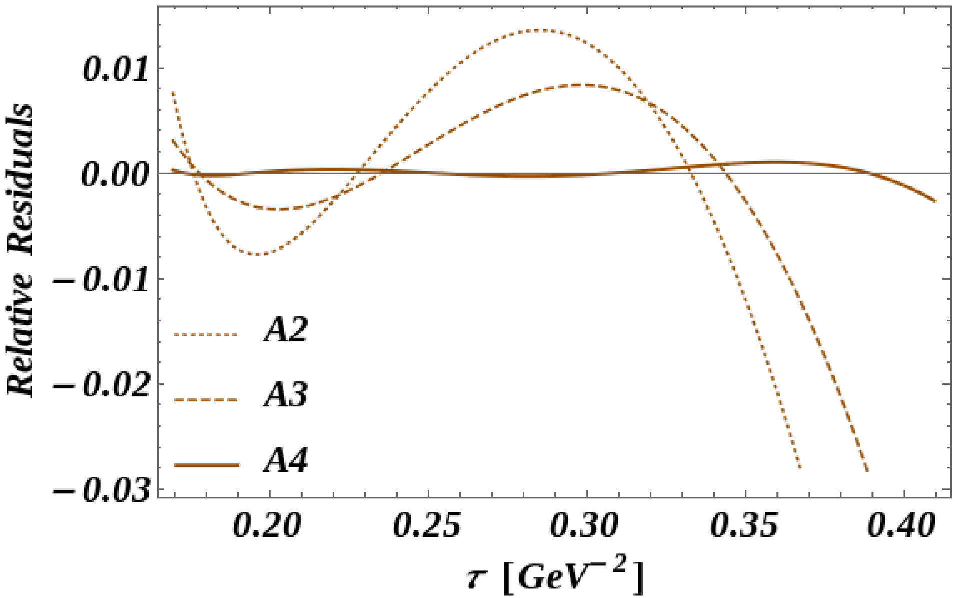
In the axial vector channel, our results favour a four-resonance scenario.

Model	$s_0[\text{GeV}^2]$	$\frac{\chi^2}{\chi^2[\text{A1}]}$	$\zeta[\text{GeV}^6]$	$\frac{\xi_1}{\zeta}$	$\frac{\xi_2}{\zeta}$	$\frac{\xi_3}{\zeta}$	$\frac{\xi_4}{\zeta}$
A1	18.8	1	0.18(1)	1	–	–	–
A2	28.8	0.0095	0.83(7)	0.47(2)	–0.53(2)	–	–
A3	18.8	0.0034	2.6(4)	0.21(2)	–0.45(1)	0.34(2)	
A4	31.7	7.3×10^{-6}	44(6)	0.03(1)	–0.16(1)	0.46(1)	–0.35(1)

In both channels, plots of relative residuals provide additional support for the favoured models.



Relative residuals vs. the Borel scale for Models V1—V3 in the vector channel.



Relative residuals vs. the Borel scale for Models A2—A4 in the axial vector channel.

We can draw some mainly qualitative conclusions about meson-hybrid mixing in charmonium.

Fits significantly improved by the inclusion of heavier states.

Vector Channel

- small hybrid component of the J/ψ
- no evidence for a hybrid component of the $\psi(2S)$ or $\psi(3770)$
- significant mixing around 4.3 GeV

Axial Vector Channel

- no evidence for a hybrid component of the $\chi_{c1}(1P)$
- weak mixing in the X(3872)
- significant mixing in the X(4140) and X(4274)

Acknowledgements

The correlator has QCD inputs: the strong coupling, charm quark mass, and condensates.

We use one-loop, $\overline{\text{MS}}$ running coupling and charm quark mass at four flavours.

$$\alpha_s(M_\tau) = 0.330 \pm 0.014$$

$$\bar{m}_c = (1.275 \pm 0.025)\ \text{GeV}$$

$$\langle \alpha G^2 \rangle = (0.075 \pm 0.020)\ \text{GeV}^4$$

$$\langle g^3 G^3 \rangle = ((8.2 \pm 1.0)\ \text{GeV}^2)\langle \alpha G^2 \rangle$$

$$\langle \bar{\psi}\psi \rangle = -(0.23 \pm 0.03)^3\ \text{GeV}^3$$

4d gluon condensate

6d gluon condensate

3d quark condensate?