Istituto Nazionale di Fisica Nucleare
Sezione di Napoli

Victoria, BC Canada Flavor Physics and CP Violation, FPCP 2019
6th May 2019

Theory perspectives on rare Kaon decays and CPV

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Closing in on the radiative weak chiral couplings Luigi Cappiello, Oscar Cata, GD arXiv: 1712.10270,,EPJC

Collaboration with Teppei Kitahara arXiv:1707.06999 PRL

Collaboration with Crivellin,A., Kitahara, T and Nierste, U. e-Print: arXiv:1703.05786 PRD

Collaboration with David Greynat and Marc Knecht arXiv:1812.00735 JHEP,+

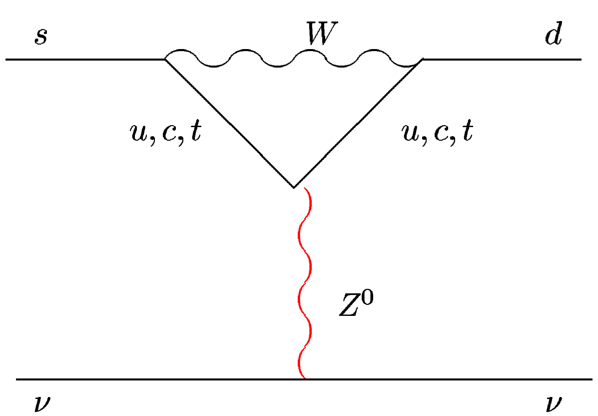
Outline

- $K \to \pi \nu \nu$
- $K_{S,L} \to \mu \mu$
- QCD, chiral perturbation theory and kaon decays

$K \to \pi\nu\overline{\nu}$

Why we need KOTO and NA62

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L\gamma_\mu d_L \quad \overline{\nu}_L\gamma^\mu\nu_L \times \left[\sum_{q=c,t} V^*_{qs}V_{qd}\, m_q^2\right]$$



$$\sim \quad \left[A^2\lambda^5\ (1-\rho-i\eta)m_t^2 + \lambda m_c^2\right]$$

SM $\underbrace{V-A \otimes V-A}$ Littenberg

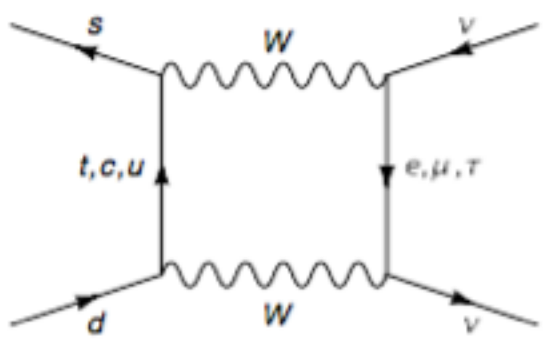
$\Downarrow$

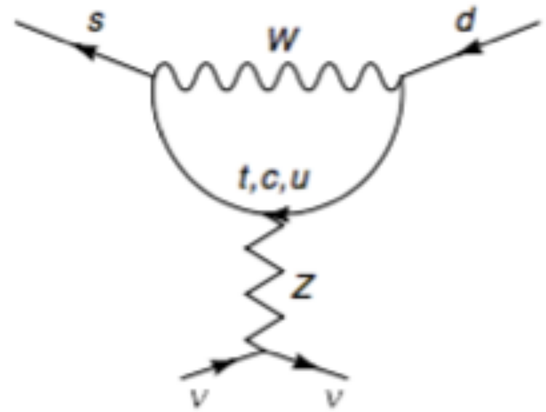
$$\Gamma(K_L \to \pi^0\nu\overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow\ J = A^2\lambda^6\eta \\ \text{Only } top \end{cases}$$

SM

Buchalla and Buras, hep-ph/9308272, Buras et al, 1503.02693.

$K^+ \to \pi^+ \nu \bar{\nu}$





Misiak, Urban; Buras, Buchalla; Brod, Gorbhan, Stamou`11, Straub

$$\lambda_q = V_{qd}{}^* V_{qs}$$

$$\mathcal{B}(K^+) \sim \kappa_+ \left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5} X_t \right)^2 + \left(\frac{\mathrm{Re}\lambda_c}{\lambda} (P_c + \delta P_{c,u}) + \frac{\mathrm{Re}\lambda_t}{\lambda^5} X_t \right)^2 \right]$$

$\downarrow$ K_{l3}

$30\% \pm 2.5\%$

LD

$$\mathcal{B}(K^\pm) = (8.82 \pm 0.8 \pm 0.3) \times 10^{-11}$$ TH

V_{cb} nonpert QCD

$$(1.73^{+1.15}_{-1.05}) \times 10^{-10}$$ E949

$$< 11 \cdot 10^{-10} \ 90\% \ \mathrm{CL}$$ NA62

Results

$$BR(K^+ \to \pi^+ \nu\overline{\nu}) < 11 \times 10^{-10} \text{ @ } 90\% \; CL$$

$$BR(K^+ \to \pi^+ \nu\overline{\nu}) < 14 \times 10^{-10} \text{ @ } 95\% \; CL$$

- One event observed in Region 2
- Full exploitation of the CLs method in progress
- The results are compatible with the Standard Model
- For comparison: $BR(K^+ \to \pi^+ \nu\overline{\nu}) = 28^{+44}_{-23} \times 10^{-11} \text{ @ } 68\% \; CL$

$$BR(K^+ \to \pi^+ \nu\overline{\nu})_{SM} = (8.4 \pm 1.0) \times 10^{-11}$$

$$BR(K^+ \to \pi^+ \nu\overline{\nu})_{exp} = (17.3^{+11.5}_{-10.5}) \times 10^{-11} \text{ (BNL, "kaon decays at rest")}$$

Prospects

- Processing of 2017 data on-going
  - ~ 20 times more data than the presented statistics
  - Expected reduction of upstream background
  - Methods to improve the reconstruction efficiency under study
- 2018 data taking under way
  - Further mitigation of the upstream background is expected
  - Processing in parallel with data taking
  - Final 2018 reprocessing expected beginning 2019
- Expect ~ 20 SM events from the 2017+2018 data sample. The analysis of this sample should provide:
  - Input to the European Strategy for Particle Physics
  - Solid extrapolation to the ultimate sensitivity of NA62 achievable after LS2

$K_L \to \pi^0 \nu \bar{\nu}$

$$B(K_L) = (3.14 \pm 0.17 \pm 0.06) \times 10^{-11}$$ TH

$$B(K_L) < 2.6 \times 10^{-8} \text{ at } 90\% \text{ C.L.}$$ E391a

Model-independent bound, based on SU(2) properties
dim-6 operators for $\bar{s}d\bar{\nu}\nu$ Grossman Nir

$$B(K_L) \leq \frac{\tau_L}{\tau_+} \times B(K^{\pm})_{\text{E949}} \leq 1.4 \times 10^{-9} \quad \text{at } 90\% C.L.$$

UV sensitivity

$$\mathcal{L} \sim \frac{1 - 0.3\ i}{(180\ \mathrm{TeV})^2}(\overline{s}_L \gamma_\mu d_L \overline{\nu}_L \gamma^\mu \nu_L)$$

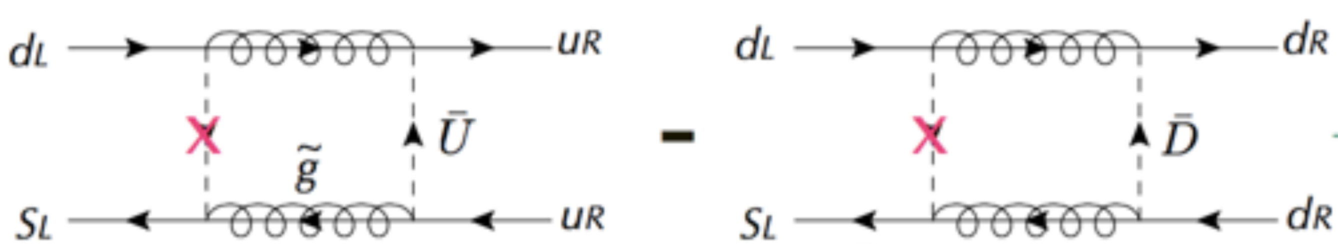
Kitahara,Nierste, Tremper PRL 16

ϵ' from isospin breaking

Kagan Neubert,99, Grossman, Kagan Neubert,99

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{1}{\sqrt{2}|\epsilon_K|_{\rm exp}} \frac{\omega_{\rm exp}}{({\rm Re}A_0)_{\rm exp}} \left(-{\rm Im}A_0 + \frac{1}{\omega_{\rm exp}} {\rm Im}A_2\right) \quad \text{where} \quad \frac{1}{\omega} \equiv \frac{{\rm Re}A_0}{{\rm Re}A_2} = 22.46 \text{ (exp.)}$$

Assuming a discrepancy 2.9 sigmas from SM



$\rightarrow$ EW penguin(Q_8)

$m_{\bar U} \neq m_{\bar D} \rightarrow$ contribute to ${\rm Im}A_2$

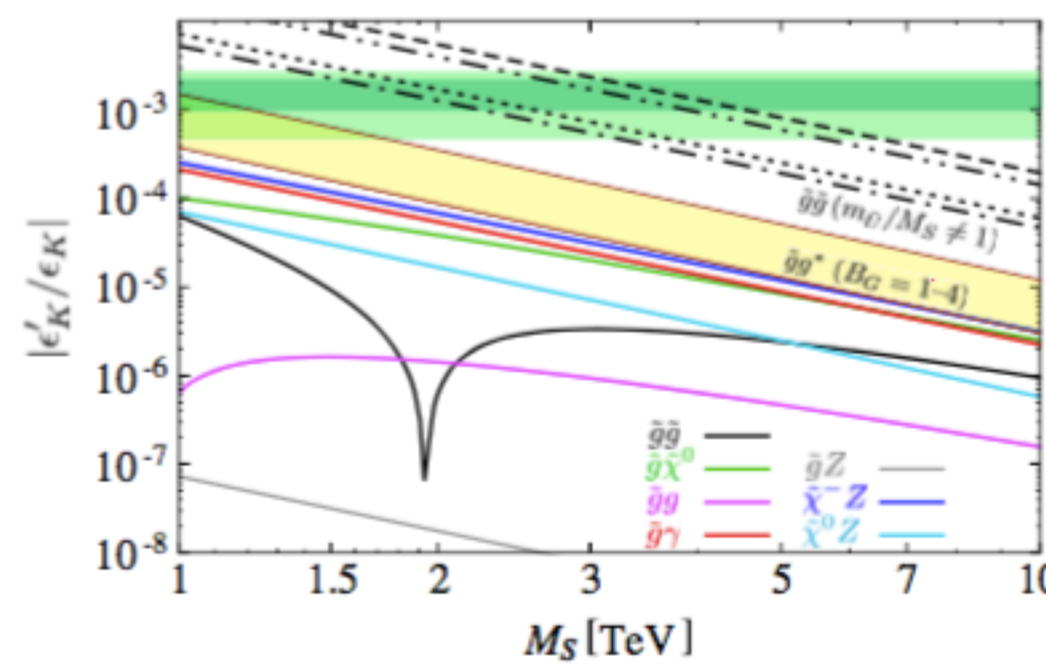
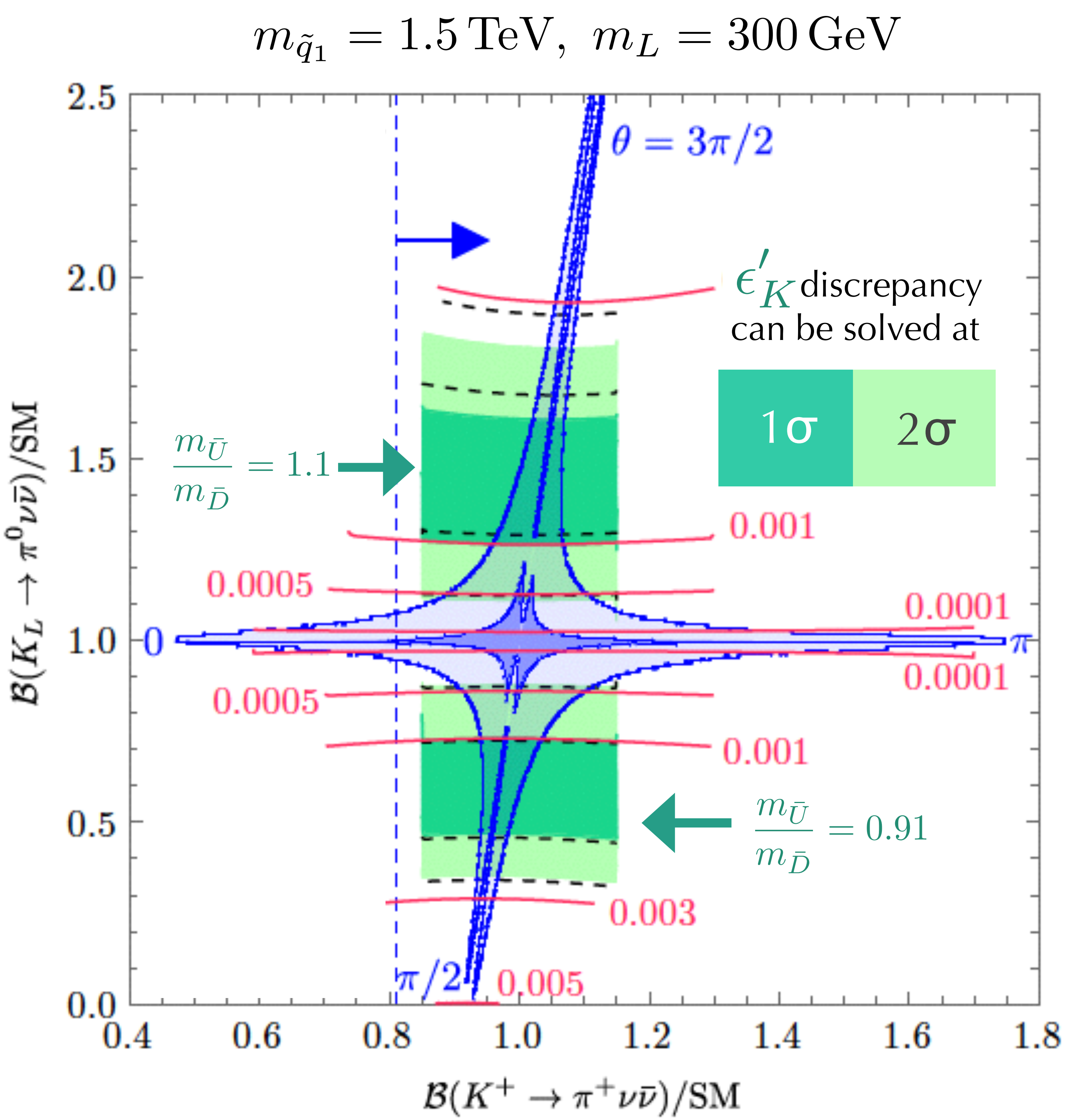


FIG. 3. Individual supersymmetric contributions to $|\epsilon'_K/\epsilon_K|$

B(K→πνν)

[Crivellin, D'Ambrosio, **TK**, Nierste, '17]



more than 10% mass shift of the gluino mass from $M_3 \simeq 1.45 M_S$ is possible in light of the constraint from ϵ_K

1-10 % mass shift of the gluino mass is possible

$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})/\mathrm{SM} \lesssim 2\,(1.2)$

$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})/\mathrm{SM} \lesssim 1.4\,(1.1)$

for a fine-tuning at the 1(10)% level

- $m_{\bar{U}}/m_{\bar{D}}$ determines a position of the green band
- Positive ϵ'_K predicts a strict correlation

$$\mathrm{sgn}\left[\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) - \mathcal{B}^{\mathrm{SM}}(K_L \to \pi^0 \nu\bar{\nu})\right] = \mathrm{sgn}\left[m_{\bar{U}} - m_{\bar{D}}\right]$$

$\mathrm{sgn}\left[m_{\bar{U}} - m_{\bar{D}}\right] \xrightarrow{\epsilon'_K} \arg\left[m^2_{Q12}\right]$

$\downarrow$

$\mathrm{sgn}\left[\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu}) - \mathcal{B}^{\mathrm{SM}}(K_L \to \pi^0 \nu\bar{\nu})\right]$

Further NA62 K Physics Program

Decay	Physics	Present limit (90% C.L.) / Result	NA62
$\pi^+\mu^+e^-$	LFV	1.3×10^{-11}	0.7×10^{-12}
$\pi^+\mu^-e^+$	LFV	5.2×10^{-10}	0.7×10^{-12}
$\pi^-\mu^+e^+$	LNV	5.0×10^{-10}	0.7×10^{-12}
$\pi^-e^+e^+$	LNV	6.4×10^{-10}	2×10^{-12}
$\pi^-\mu^+\mu^+$	LNV	1.1×10^{-9}	0.4×10^{-12}
$\mu^-\nu e^+e^+$	LNV/LFV	2.0×10^{-8}	4×10^{-12}
$e^-\nu\mu^+\mu^+$	LNV	No data	10^{-12}
π^+X^0	New Particle	$5.9\times10^{-11}\ m_{X^0}=0$	10^{-12}
$\pi^+\chi\chi$	New Particle	–	10^{-12}
$\pi^+\pi^+e^-\nu$	$\Delta S\neq\Delta Q$	1.2×10^{-8}	10^{-11}
$\pi^+\pi^+\mu^-\nu$	$\Delta S\neq\Delta Q$	3.0×10^{-6}	10^{-11}
$\pi^+\gamma$	Angular Mom.	2.3×10^{-9}	10^{-12}
$\mu^+\nu_h, \nu_h\to\nu\gamma$	Heavy neutrino	Limits up to $m_{\nu_h}=350\ MeV$	
R_K	LU	$(2.488\pm0.010)\times10^{-5}$	>×2 better
$\pi^+\gamma\gamma$	χPT	< 500 events	10^5 events
$\pi^0\pi^0e^+\nu$	χPT	66000 events	$O(10^6)$
$\pi^0\pi^0\mu^+\nu$	χPT	-	$O(10^5)$

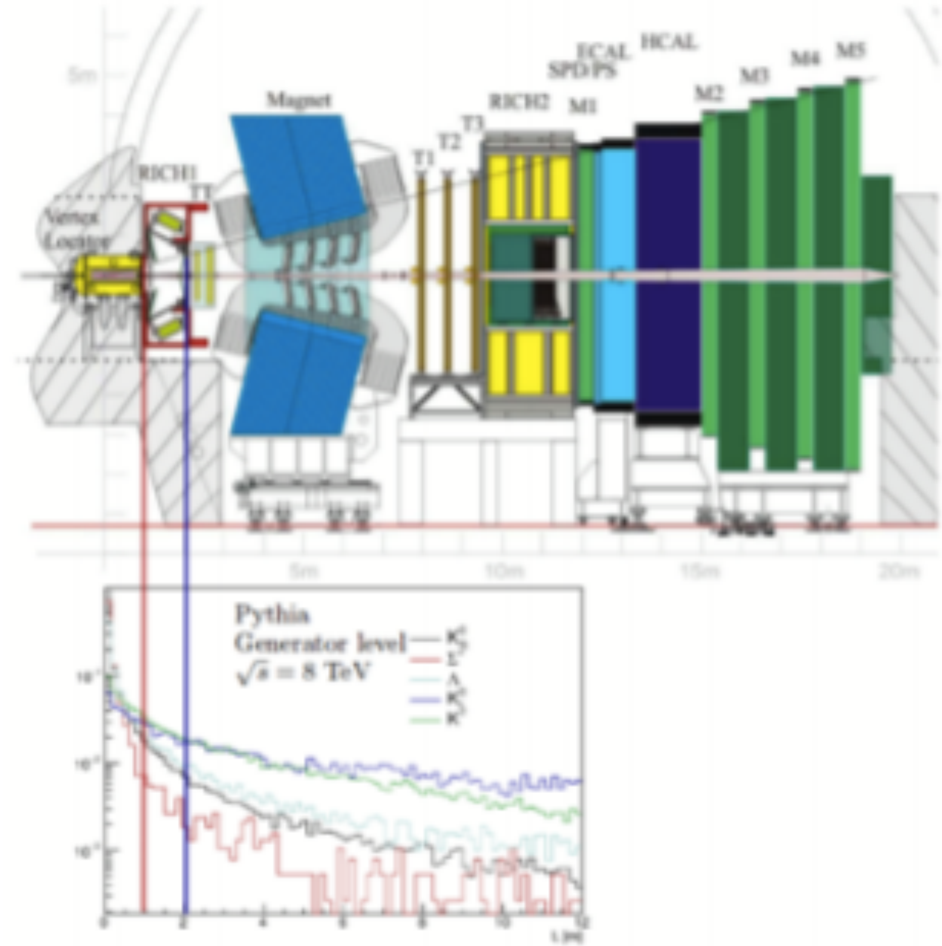
Rare Kaon decay program at LHCB

	PDG	Prospects
$K_S \to \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL	(LD)$(5.0 \pm 1.5) \cdot 10^{-12}$ NP $< 10^{-11}$
$K_S \to \mu\mu\mu\mu$	–	SM LD $\sim 2 \times 10^{-14}$
$K_S \to ee\mu\mu$	–	$\sim 10^{-11}$
$K_S \to eeee$	–	$\sim 10^{-10}$
$K_S \to \pi^0\mu\mu$	$(2.9 \pm 1.3) \cdot 10^{-9}$	$\sim 10^{-9}$
$K_S \to \pi^+\pi^-e^+e^-$	$(4.79 \pm 0.15) \cdot 10^{-5}$	SM LD $\sim 10^{-5}$
$K_S \to \pi^+\pi^-\mu^+\mu^-$	–	SM LD $\sim 10^{-14}$

Rare n Strange 2017: strange physics at LHCb

GD, Lewis Tunstall, Diego Martinez Santos,Veronika Chobanova,
Xabier Cid Vidal, Francesco Dettori, Marc-Olivier Bettler,
Teppei Kitahara,,Kei Yamamoto

Kaon in LHCb



- LHCb experiment has been designed for efficient reconstructions of ***b*** and ***c***
- Huge production of strangeness [$O(10^{13})/\text{fb}^{-1}$ K^0_S] is suppressed by its trigger efficiency [ε~1-2%@LHC Run-I, ε~18%@LHC Run-II]
- LHCb upgrade (LS2=Phase I upgrade, LS4=Phase II upgrade) could realize high efficiency for K^0_S [ε~90%@LHC Run-III] [M. R. Pernas, HL/HE LHC meeting, Fermilab, 2018]
- In LHC Run-III and HL-LHC, we could probe the *ultra* rare decay Br~$O(10^{-11\sim12})$

$$K_{L,S} \to \mu\mu$$

$K_L \to \mu\mu$

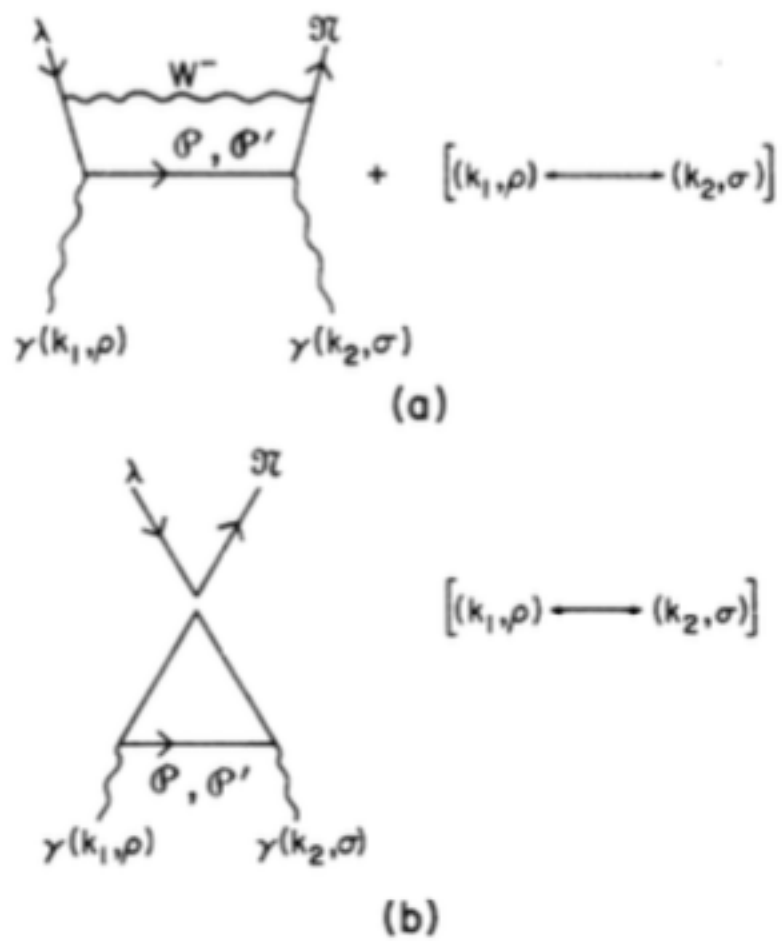


FIG. 7. Leading contributions to $\lambda + \bar{\mathfrak{N}} \to \gamma + \gamma$. To leading order in M_W^{-2}, the diagrams in (a) reduce to those of (b).

Gaillard Lee

$\Gamma(K_L^0 \to \mu^+\mu^-)/\Gamma(K_L^0 \to \pi^+\pi^-)$

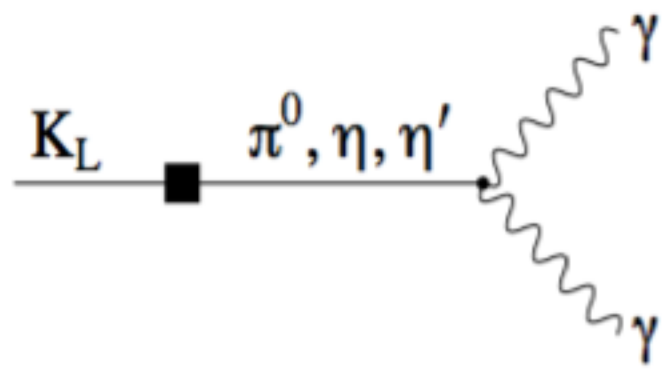
VALUE (10^{-6})	EVTS	DOCUMENT ID		TECN	C…
3.48 ± 0.05	**OUR AVERAGE**				
3.474 ±0.057	6210	AMBROSE	2000	B871	
3.87 ±0.30	179	[1] AKAGI	1995	SPEC	
3.38 ±0.17	707	HEINSON	1995	B791	
• • • We do not use the following data for averages, fits, limits, etc. • • •					
3.9 ±0.3 ±0.1	178	[2] AKAGI	1991B	SPEC	In …

$$\mathcal{B}(K_L \to \mu^+\mu^-)_{\rm exp} = (6.84 \pm 0.11) \times 10^{-9}$$

$K_L \to \gamma\gamma \,|_{\rm exp}$ known

Dispersive calculation: Re A, Im A

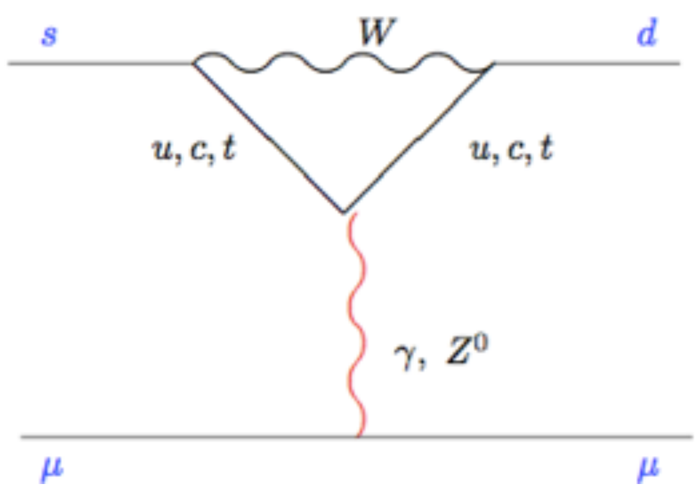
We do not know the sign of $A(K_L \to \gamma\gamma)$



$$A(K_L \to 2\gamma_\perp)_{O(p^4)} = A(K_L \to \pi^0 \to 2\gamma_\perp) + A(K_L \to \eta_8 \to 2\gamma_\perp)$$
$$= A(K_L \to \pi^0)A(\pi^0 \to 2\gamma_\perp)\left[\frac{1}{M_K^2 - M_\pi^2} + \frac{1}{3}\cdot\frac{1}{M_K^2 - M_8^2}\right] \simeq 0$$

Kaon Decays in the Standard Model
Vincenzo Cirigliano (Los Alamos), Gerhard Ecker, Helmut Neufeld (Vienna U.), Antonio Pich, Jorge Portoles, refs therein

$K_L \to \mu\mu$



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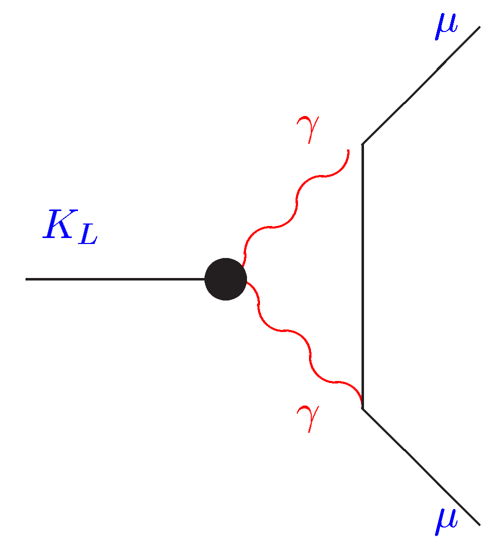


$$\frac{\Gamma(K_L \to \mu\bar{\mu})}{\Gamma(K_L \to \gamma\gamma)} \sim \quad |ReA|^2 + |ImA|^2$$

Absorptive calculation
model independent

27.14

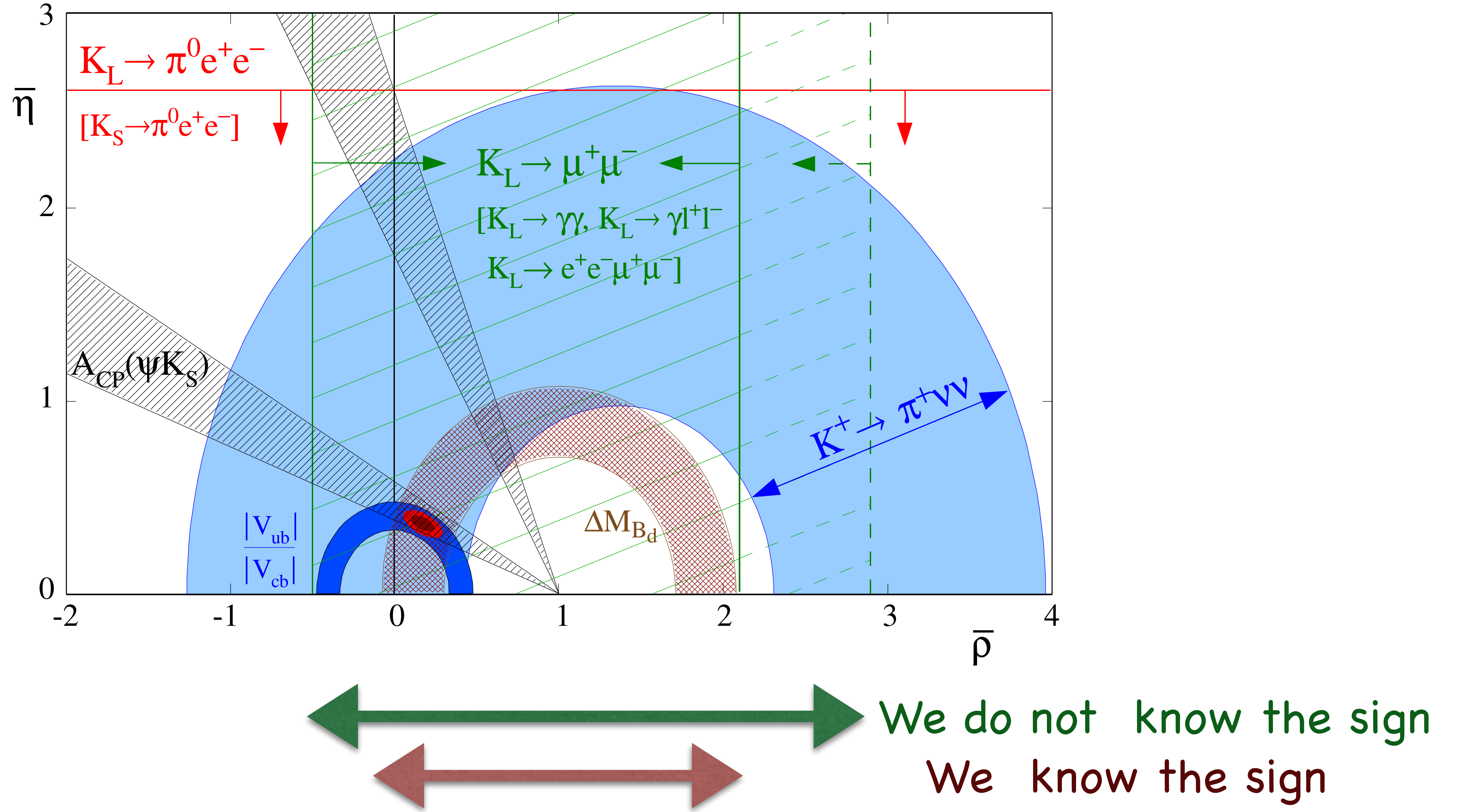
Subtracting from expt. the Absorptive contribution

$$0.98 \pm 0.55 = |ReA|^2 = (\chi_{\gamma\gamma}(M_\rho) + \chi_{\text{short}} - 5.12)^2$$

$$|\chi_{\text{short}}^{\text{SM}}| = 1.96(1.11 - 0.92\bar{\rho})$$

Isidori Unterdorfer

$K_L \to \mu\mu$: our sign ignorance



$K_S \to \mu\mu$

PHYSICAL REVIEW D VOLUME 10, NUMBER 3 1 AUGUST 1

Rare decay modes of the K mesons in gauge theories

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National Accelerator Laboratory, Batavia, Illinois 60510‡
(Received 4 March 1974)

Rare decay modes of the kaons such as $K\to\mu\bar{\mu}$, $K\to\pi\nu\bar{\nu}$, $K\to\gamma\gamma$, $K\to\pi\gamma\gamma$, and $K\to\pi e\bar{e}$ are of theoretical interest since here we are observing higher-order weak and electromagnetic interactions. Recent advances in unified gauge theories of weak and electromagnetic interactions allow in principle unambiguous and finite predictions for these processes. The above processes, which are "induced" $|\Delta S|=1$ transitions, are a good testing ground for the cancellation mechanism first invented by Glashow, Iliopoulos, and Maiani (GIM) in order to banish $|\Delta S|=1$ neutral currents. The experimental suppression of $K_L\to\mu\bar{\mu}$ and nonsuppression of $K_L\to\gamma\gamma$ must find a natural explanation in the GIM mechanism which makes use of extra quark(s). The procedure we follow is the following: We deduce the effective interaction Lagrangian for $\lambda+\mathfrak{N}\to l+\bar{l}$ and $\lambda+\bar{\mathfrak{N}}\to\gamma+\gamma$ in the free-quark model; then the appropriate matrix elements of these operators between hadronic states are evaluated with the aid of the principles of conserved vector current and partially conserved axial-vector current. We focus our attention on the Weinberg-Salam model. In this model, $K\to\mu\bar{\mu}$ is suppressed due to a fortuitous cancellation. To explain the small K_L-K_S mass difference and nonsuppression of $K_L\to\gamma\gamma$, it is found necessary to assume $m_{\wp}/m_{\wp'}\ll 1$, where $m_{\wp}$ is the mass of the proton quark and $m_{\wp'}$ the mass of the charmed quark, and $m_{\wp'}<5$ GeV. We present a phenomenological argument which indicates that the average mass of charmed pseudoscalar states lies below 10 GeV. The effective interactions so constructed are then used to estimate the rates of other processes. Some of the results are the following: $K_S\to\gamma\gamma$ is suppressed; $K_S\to\pi\gamma\gamma$ proceeds at a normal rate, but $K_L\to\pi\gamma\gamma$ is suppressed; $K_L\to\pi\nu\bar{\nu}$ is very much forbidden, and $K^+\to\pi^+\nu\bar{\nu}$ occurs with the branching ratio of $\sim 10^{-10}$; $K^+\to\pi^+e\bar{e}$ has the

Run1 data (3 fb^{-1})

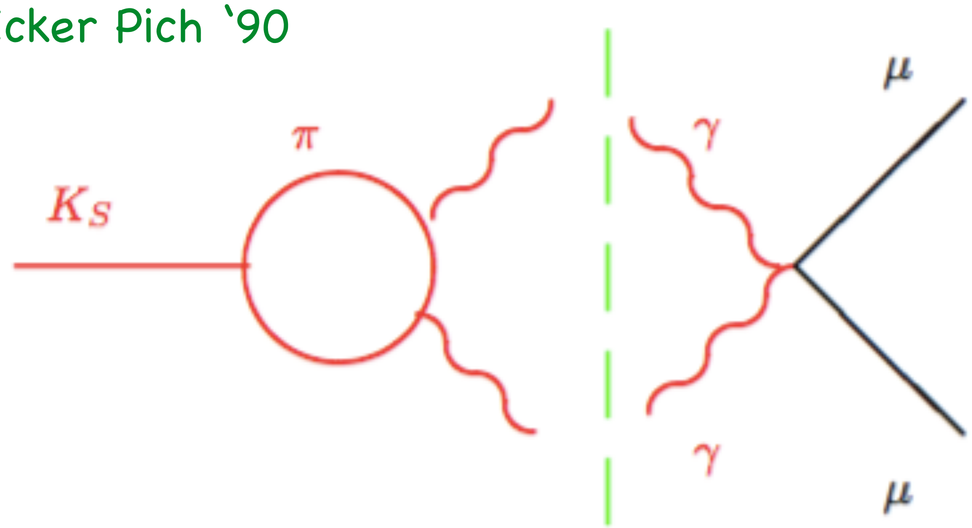
$\mathcal{B}(K_S^0\to\mu^+\mu^-) < 0.8(1.0)\times 10^{-9}$
90%, 95% CL
factor 11 improvement

VALUE (10^{-9})	CL%		DOCUMENT ID		TECN
< 9	90	1	AAIJ	2013G	LHCB
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$< 0.032\times 10^4$	90		GJESDAL	1973	ASPK
$< 0.7\times 10^4$	90		HYAMS	1969B	OSPK

[1] AAIJ 2013G uses 1.0 fb^{-1} of pp collisions at $\sqrt{s}=7$ TeV. They obtained B($K_S^0\to\mu^+\mu^-$) $< 11\times 10^{-9}$ at 95% C.L.

$K_S \to \mu\mu$

Ecker Pich '90



No CP conserving Short Distance due to Furry Theorem

Gaillard Lee

LD 5×10^{-12} 30% TH err

Short Distance

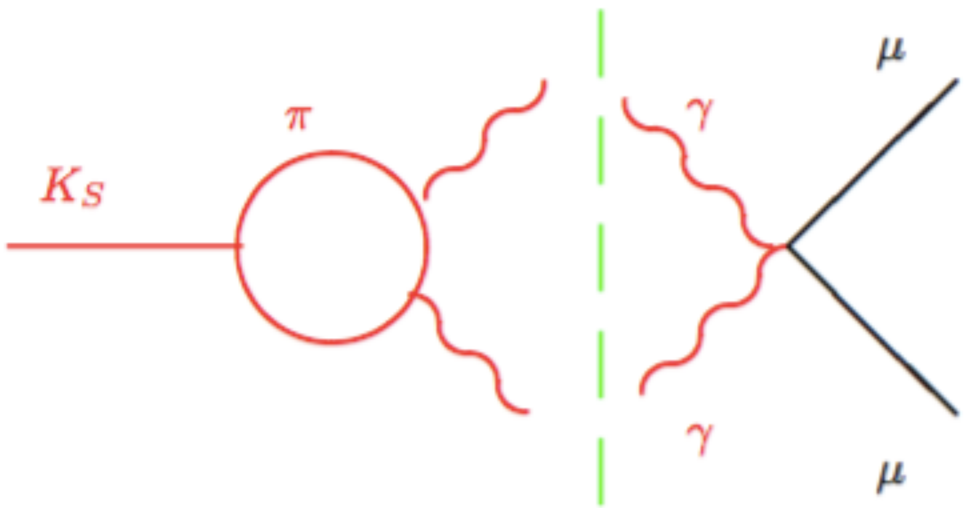
SM $10^{-5} |\Im(V_{ts}^* V_{td})|^2 \sim 10^{-13}$

NP few 10^{-11} allowed

LHCB

$< 8 \times 10^{-10}$ 90%CL

$K_S \to \mu\mu$: how to improve the THEORY error



Dispersive treatment of $K_S \to \gamma\gamma$ and $K_S \to \gamma l^+ l^-$

Gilberto Colangelo, Ramon Stucki, and Lewis C. Tunstall

LD 5×10^{-12} 20% TH err

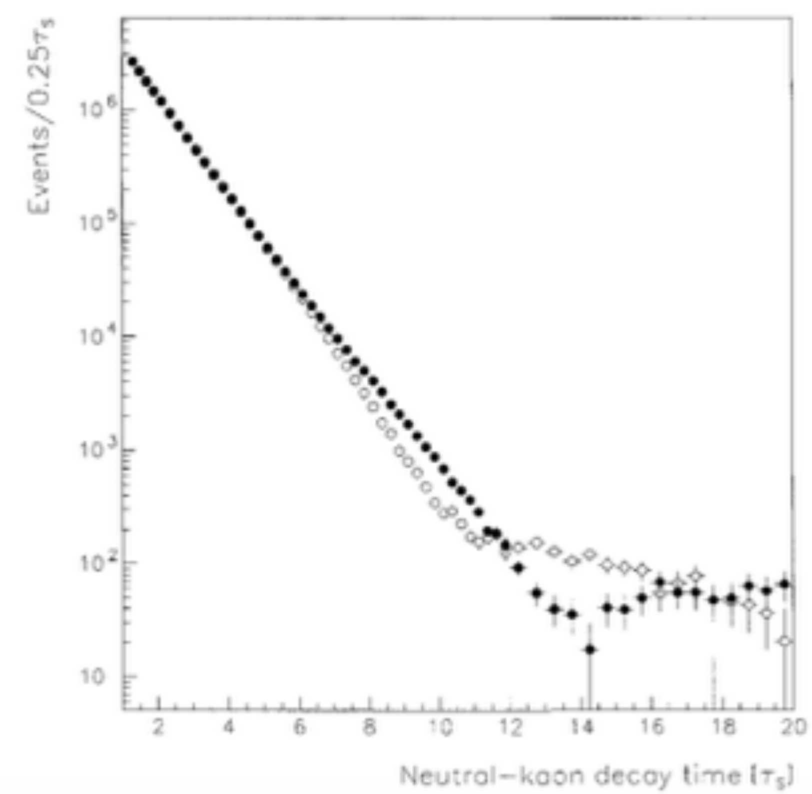
$$K_S \to \gamma\mu\mu$$
$$K_S \to \mu\mu\mu\mu$$
$$K_S \to ee\mu\mu$$
$$K_S \to \gamma\gamma$$

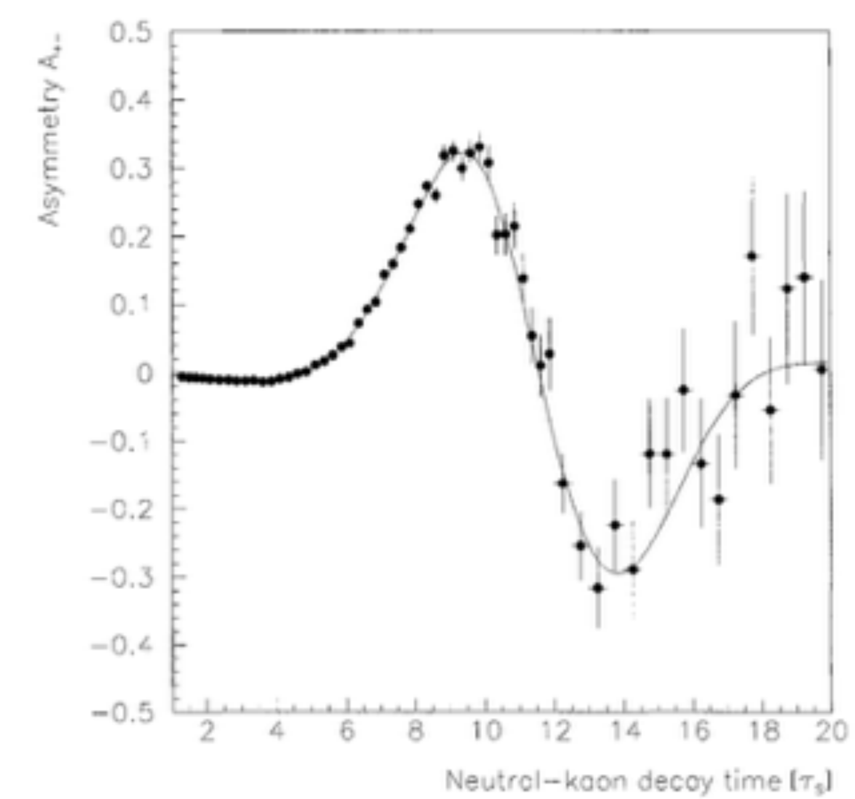
{

CPLEAR Flavor tagging

$$p\bar{p} \longrightarrow \begin{array}{l} K^-\pi^+K^0 \\ K^+\pi^-\overline{K}^0 \end{array}$$

$$\frac{R(\tau)}{\overline{R}(\tau)} \propto (1 \mp 2\mathrm{Re}(\varepsilon_L))(\mathrm{e}^{-\Gamma_S\tau} + |\eta_{+-}|^2\mathrm{e}^{-\Gamma_L\tau} \pm 2|\eta_{+-}|\mathrm{e}^{-\frac{1}{2}(\Gamma_S+\Gamma_L)\tau}\cos(\Delta m\tau - \phi_{+-}))$$





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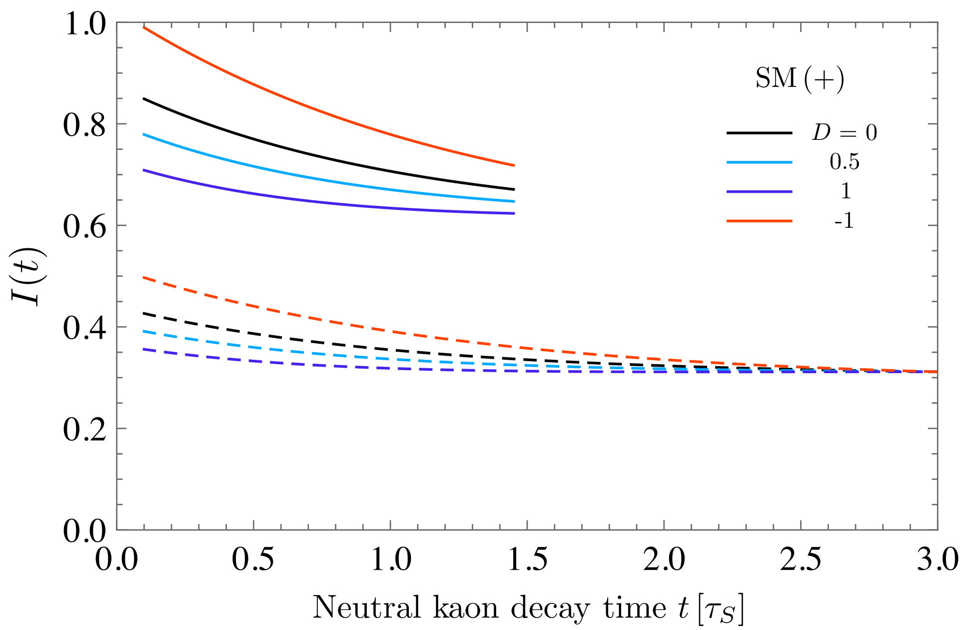
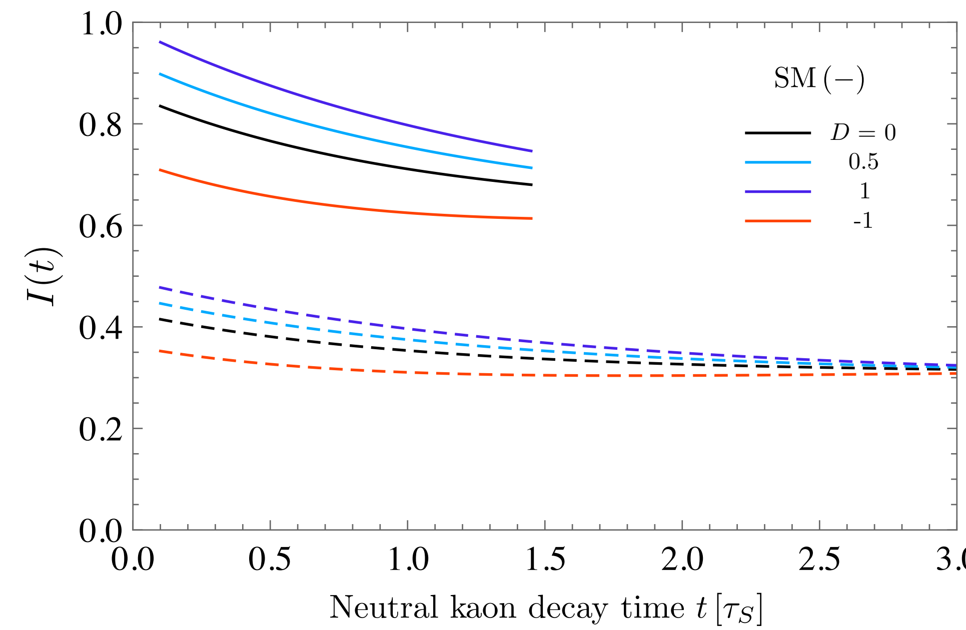
Can we study $K^0(t)$?

GD , Kitahara
1707.06999 PRL

$$pp \to K^0 K^- X$$

$$pp \to K^{*+} X \to K^0 \pi^+ X$$





$$|\overset{(-)}{K}{}^0(t)\rangle = \frac{1}{\sqrt{2}(1 \pm \bar{\epsilon})} \left[e^{-iH_S t} \left(|K_1\rangle + \bar{\epsilon}|K_2\rangle\right) \pm e^{-iH_L t} \left(|K_2\rangle + \bar{\epsilon}|K_1\rangle\right) \right]$$

$$D = \frac{K^0 - \overline{K}^0}{K^0 + \overline{K}^0}$$

- Short distance interfering with Large CP conserving LD contribution !

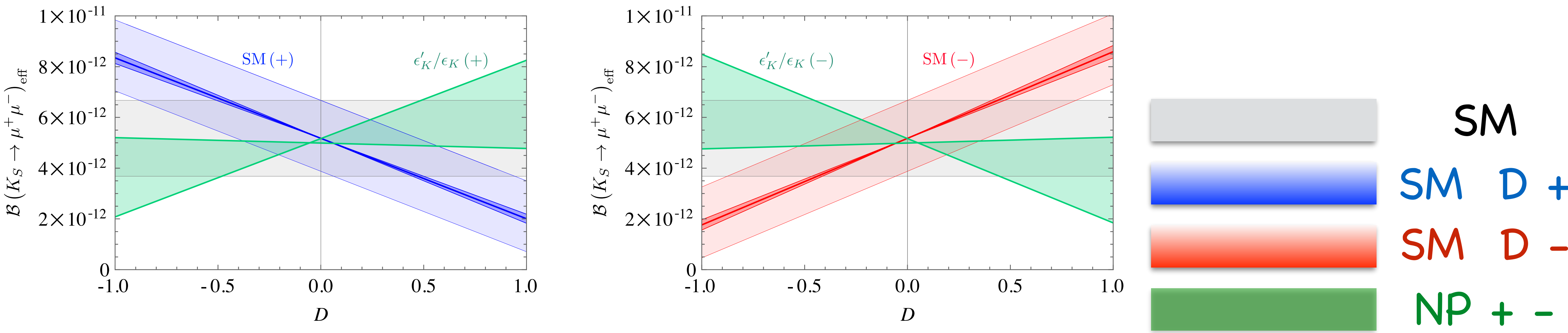
- We may be able to study the time evolution of K^0 by tracking the associated particles (K^-)

$$\sum_{\text{spin}} \mathcal{A}(K_1 \to \mu^+\mu^-)^* \mathcal{A}(K_2 \to \mu^+\mu^-)$$

$$\sim \mathrm{Im}[\lambda_t] y'_{7A} \left\{ A^{\mu}_{L\gamma\gamma} - 2\pi \sin^2\theta_W \left(\mathrm{Re}[\lambda_t] y'_{7A} + \mathrm{Re}[\lambda_c] y_c\right) \right\}$$

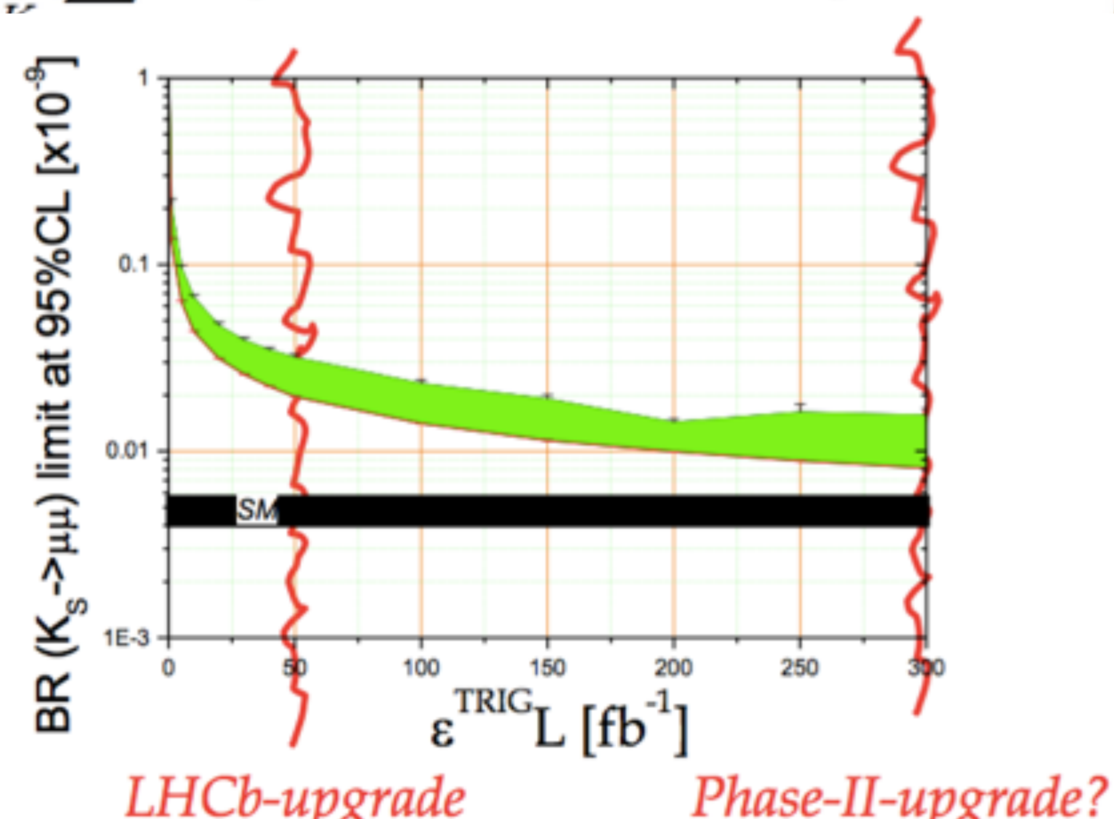
Short distance window

GD , Kitahara
1707.06999 PRL



modified Z-coupling model. ϵ'_K/ϵ

$$\mathcal{B}(K_S\to\mu^+\mu^-)_{\rm eff} = \tau_S \left[\int_{t_{min}}^{t_{max}} dt \left(\Gamma(K_1)e^{-\Gamma_S t} + \frac{D}{8\pi M_K}\sqrt{1-\frac{4m_\mu^2}{M_K^2}}\sum \mathrm{Re}\left[e^{-i\Delta M_K t}\mathcal{A}(K_1)^*\mathcal{A}(K_2)\right]e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\right)\varepsilon(t)\right] \times \left(\int_{t_{min}}^{t_{max}} dt\, e^{-\Gamma_S t}\varepsilon(t)\right)^{-1},$$



QCD and EFT

Chiral Perturbation theory

χPT effective field theory approach based on two assumptions

- π's Golstone bosons of $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$

$SU(3)_L \times SU(3)_R$ symm. $\mathcal{L}_{QCD}$ $m_q = 0$

- (chiral) power counting There is a small expansion parameter $p^2/\Lambda^2_{\chi SB}$

$\Lambda_{\chi SB} \simeq 4\pi F_\pi \sim 1.2$ GeV

Chiral sym. breaking through dim. parameter F_π, χ related to
$\langle 0|J_{5\mu}|\pi\rangle$, $\langle 0|q_L q_R|0\rangle$

$F_\pi \approx 93$ MeV

$$\mathcal{L}_S = \mathcal{L}_S^2 + \mathcal{L}_S^4 + \cdots = \frac{F_\pi{}^2}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi \to l\nu,\ \pi\pi \to \pi\pi,\ K \to \pi..} + \overbrace{\sum_i L_i O_i}^{K \to \pi..} + \cdots$$

Fantastic chiral prediction

$A_{\pi\pi} \sim (s - m_\pi^2)/F_\pi^2$

L_i Gasser Leutwyler coeff determined from expts.
O_i p^4 operator

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+ \to \pi^+ \gamma\gamma,\ K \to \pi l^+ l^-} + \cdots$$

Vector Meson Dominance in the strong sector

Ecker, Gasser, de Rafael, Pich

L_i	L_i expts	V	A	Total (Scalar incl.)	Total QCD rel. incl.
L_1	0.4 ± 0.3	0,6	0	0,6	0,9
L_2	1.4 ± 0.3	1,2	0	1,2	1,8
L_3	-3.5 ± 1.1	-3,6	0	-3,0	-4,9
L_4	-0.3 ± 0.5	0	0	0	0
L_5	1.4 ± 0.5	0	0	1,4	1,4
L_6	-0.2 ± 0.3	0	0	0	0
L_7	-0.4 ± 0.2	0	0	-0,3	-0,3
L_8	0.9 ± 0.3	0	0	0,9	0,9
L_9	6.9 ± 0.7	6,9	0	6,9	7,3
L_{10}	-5.5 ± 0.7	-10	4	-6,0	-5,5

QCD inspired relations relations

$$F_V = 2G_V = \sqrt{2} f_\pi$$

$$F_A = f_\pi$$

$$M_A = \sqrt{2} M_V$$

KSFR: $G_V = \sqrt{2}\ F_\pi$ determined by dominance of pion, V,A to recover QCD short distance constraints

$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}, \qquad L_9^V = \frac{F_V G_V}{2M_V^2}, \qquad L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

QCD inspired relations relations

$$L_1^V = L_2^V/2 = -L_3^V/6 = L_9^V/8 = -L_{10}^{V+A}/6 = f_\pi^2/(16M_V^2)$$

Not only a book-keeping but predictive already

π	2π	3π	N_i
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N_{14}^r - N_{15}^r$
$\pi^0\gamma^*\ (S)$	$\pi^0\pi^0\gamma^*\ (L)$		$2N_{14}^r + N_{15}^r$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$
	$\pi^+\pi^-\gamma\gamma\ (S)$		”
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
	$\pi^+\pi^-\gamma\ (S)$	$\pi^+\pi^0\pi^0\gamma$	”
		$\pi^+\pi^-\pi^0\gamma\ (L)$	”
		$\pi^+\pi^-\pi^0\gamma\ (S)$	$7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17})$
	$\pi^+\pi^-\gamma^*\ (L)$		$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma^*\ (S)$		$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$
	$\pi^+\pi^0\gamma^*$		$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma\ (L)$	$\pi^+\pi^-\pi^0\gamma\ (S)$	$N_{29} + N_{31}$
		$\pi^+\pi^+\pi^-\gamma$	”
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$
		$\pi^+\pi^-\pi^0\gamma\ (S)$	$5N_{29} - N_{30} + 2N_{31}$
		$\pi^+\pi^-\pi^0\gamma\ (L)$	$6N_{28} + 3N_{29} - 5N_{30}$

$K^+ \to \pi^+ l^+ l^-$

$K_S \to \pi^0 l^+ l^-$

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma,\, K\to\pi l^+ l^-} + \cdots$$

Weak chiral couplings

$K^{\pm} \to \pi^{\pm}\gamma^{*}$: $a_{+} = -0.578 \pm 0.016$ [3, 4]

$K_S \to \pi^{0}\gamma^{*}$: $a_S = (1.06^{+0.26}_{-0.21} \pm 0.07)$ [5, 6]

$K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$: $X_E = (-24 \pm 4 \pm 4)\ \text{GeV}^{-4}$ [7]

$K^{+} \to \pi^{+}\gamma\gamma$: $\hat{c} = 1.56 \pm 0.23 \pm 0.11$ [8] .

$$\mathcal{N}_E^{(1)} \equiv N_{14}^r - N_{15}^r = \frac{3}{64\pi^2}\left(\frac{1}{3} - \frac{G_F}{G_8}a_{+} - \frac{1}{3}\log\frac{\mu^2}{m_K m_\pi}\right) - 3L_9^r ;$$

$$\mathcal{N}_S \equiv 2N_{14}^r + N_{15}^r = \frac{3}{32\pi^2}\left(\frac{1}{3} + \frac{G_F}{G_8}a_S - \frac{1}{3}\log\frac{\mu^2}{m_K^2}\right) ;$$

$$\mathcal{N}_E^{(0)} \equiv N_{14}^r - N_{15}^r - N_{16}^r - N_{17} = -\frac{|\mathcal{M}_K| f_\pi}{2G_8} X_E ;$$

$$\mathcal{N}_0 \equiv N_{14}^r - N_{15}^r - 2N_{18}^r = \frac{3}{128\pi^2}\hat{c} - 3(L_9^r + L_{10}^r) ,$$

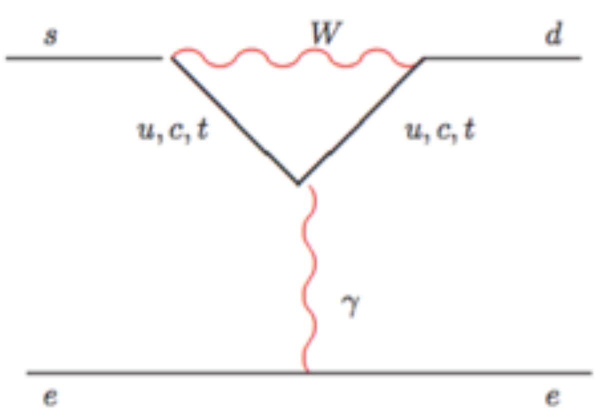
Decay mode	counterterm combination	expt. value
$K^{\pm} \to \pi^{\pm}\gamma^{*}$	$N_{14} - N_{15}$	$-0.0167(13)$
$K_S \to \pi^{0}\gamma^{*}$	$2N_{14} + N_{15}$	$+0.016(4)$
$K^{\pm} \to \pi^{\pm}\pi^{0}\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$	$+0.0022(7)$
$K^{\pm} \to \pi^{\pm}\gamma\gamma$	$N_{14} - N_{15} - 2N_{18}$	$-0.0017(32)$

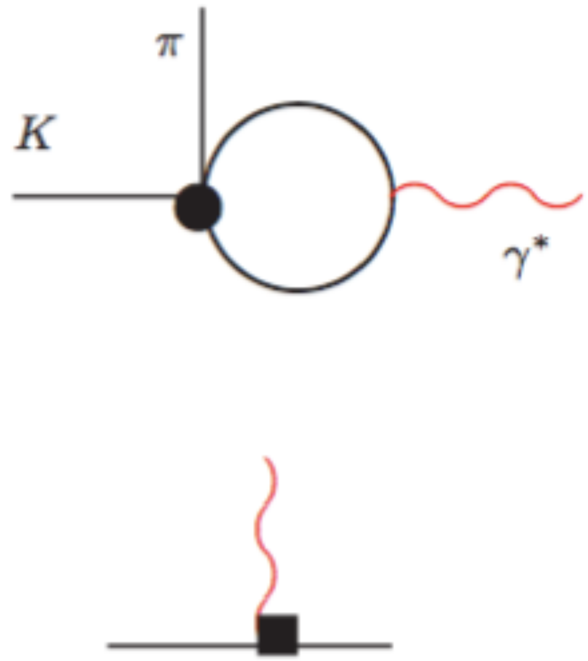
Luigi Cappiello, Oscar Cata,GD
arXiv:1712.10270,,EPJC

LFUV in Kaons

$$\frac{\Gamma(K^+ \to \pi^+ \mu^+ \mu^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)}$$

SD << LD





$$i \int d^4x e^{iqx} \langle \pi(p) | T \left\{ J^\mu_{\rm elm}(x) \mathcal{L}_{\Delta S=1}(0) \right\} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^\mu - (1-r_\pi^2) q^\mu \right]$$

$K^+ \to \pi^+ e^+ e^-$ $\qquad$ $K_S \to \pi^0 e^+ e^-$

- gauge+Lorentz inv. =>1 ff

$$W^i = G_F m_K^2 (a_i + b_i z) + W^i_{\pi\pi}(z)$$

$$i \int d^4x e^{iqx} \langle \pi(p) | T\left\{ J^\mu_{\rm elm}(x) \mathcal{L}_{\Delta S=1}(0) \right\} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^\mu - (1 - r_\pi^2) q^\mu \right]$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \qquad z = \frac{q^2}{m_K^2}$$

- Observables $\Gamma(K^+ \to \pi^+ e^+ e^-)$, $\Gamma(K^+ \to \pi^+ \mu \overline{\mu})$, slopes

- a_i $\quad O(p^4)$ $\qquad a_+ \sim N_{14} - N_{15}, \qquad a_S \sim 2N_{14} + N_{15}$ $\qquad$ Ecker, Pich, de Rafael
- b_i $\quad O(p^6)$ $\qquad$ G.D., Ecker, Isidori, Portoles

- $a_+, \ b_+$ in general not related to $a_S, \ b_S$ $\qquad$ Recent lattice determinations Christ et al.

averaging flavour

$$a_+^{\rm exp.} = -0.578 \pm 0.016$$
$$b_+^{\rm exp.} = -0.779 \pm 0.066$$

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

requires an unsubtracted dispersion relation

$$W(z)|_{\pi\pi} = \frac{1}{\pi}\int_0^\infty dx \frac{\mathrm{Abs}\, W(x/M_K^2)|_{\pi\pi}}{x - zM_K^2 - i0}$$

with

$$\frac{\mathrm{Abs}\, W(s/M_K^2)|_{\pi\pi}}{16\pi^2 M_K^2} = \theta(s-4M_\pi^2) \times \frac{s-4M_\pi^2}{s}\lambda_{K\pi}^{-1/2}(s) \times F_V^{\pi *}(s) \times f_1^{\pi^+\pi^-\to K^+\pi^-}(s)$$

Then a_+ and b_+ are given by spectral sum rules

$$G_F M_K^2 a_+|_{\pi\pi} = W(0)|_{\pi\pi} = \frac{1}{\pi}\int_0^\infty \frac{dx}{x}\,\mathrm{Abs}\, W(x/M_K^2)|_{\pi\pi}$$

and

$$\begin{aligned} G_F M_K^2 b_+|_{\pi\pi} &= W'(0)|_{\pi\pi} - \frac{1}{60}\left(\frac{M_K^2}{M_\pi^2}\right)^2\left(\alpha_+ - \beta_+\frac{s_0}{M_\pi^2}\right) \\ &= \frac{M_K^2}{\pi}\int_0^\infty \frac{dx}{x^2}\,\mathrm{Abs}\, W(x/M_K^2)|_{\pi\pi} - \frac{1}{60}\left(\frac{M_K^2}{M_\pi^2}\right)^2\left(\alpha_+ - \beta_+\frac{s_0}{M_\pi^2}\right) \end{aligned}$$

requires $F_V^{\pi *}(s)$ and $f_1^{\pi^+\pi^-\to K^+\pi^-}(s)$ beyond low-energy expansion

GD Greynat Knecht
arXiv:1812.00735 JHEP,+

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Simple approach: unitarize both using the inverse amplitude method

T. N. Truong, Phys Rev Lett 61, 2526 (1988)
A. Dobado et al, Phys Lett B 235, 134 (1990)
T. Hannah, Phys Rev D 55, 5613 (1997)
A. Dobado, J. R. Pelaez, Phys Rev D 56, 3057 (1997)
J. Nieves et al., Phys Rev D 65, 036002 (2002)

$$a_+|_{\pi\pi} = -(1.574^{+0.003}_{-0.020}) \qquad b_+|_{\pi\pi} = -(0.622^{+0.012}_{-0.017}) \qquad \text{for } \beta_+ = -0.85 \cdot 19^{-8}$$

note: position of the ρ resonance much too low for $\beta_+ = -2.88$... (phase goes through $\pi/2$ at $s \sim M_\rho^2/2$!)

$$a_+ = -1.58 + \begin{cases} -0.10 \div +0.03 & \text{NDR} \\ -0.14 \div +0.07 & \text{HV} \end{cases}$$

$$b_+ = -0.76 + \begin{cases} -0.04 \div +0.03 & \text{NDR} \\ -0.07 \div +0.03 & \text{HV} \end{cases}$$

Matching short distance

Collaboration with Crivellin, A Hoferichter, M and Tunstall,
Phys.Rev. D 2016

LFUV: Kaons

Channel	a_+	b_+	Reference
ee	-0.587 ± 0.010	-0.655 ± 0.044	E865
ee	-0.578 ± 0.016	-0.779 ± 0.066	NA48/2
$\mu\mu$	-0.575 ± 0.039	-0.813 ± 0.145	NA48/2

$$a_+^{\rm NP} = \frac{2\pi\sqrt{2}}{\alpha} V_{ud}V_{us}^* * C_{7V}^{\rm NP}$$

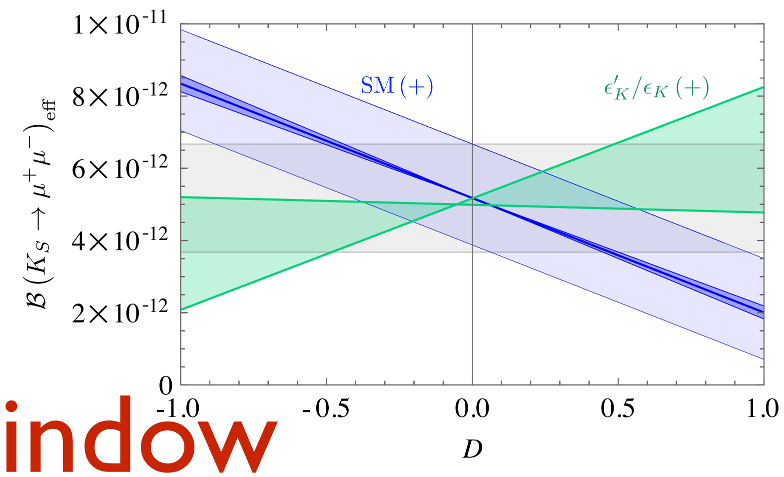
$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{ud}V_{us}^*} \quad \overset{MFV}{\Longrightarrow} \quad C_{9V}^{B,\mu\mu} - C_{9V}^{B,ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{td}V_{ts}^*} = -19 \pm 79$$

LHCB-NA62 PLEASE!!

High statistics: nominal # of decays 50 times greater than NA48/2

Conclusions

- Flavour anomalies: interplay with K->π***νν*** but 10% measurement needed!



- LHCB: K_S->μμ extraordinary result: interference effect!!!Short distance window

- weak chiral lagrangian

- LFUV in Kaons very useful

- Rich rare kaon program

Predicting $a_+,\ b_+$? Going beyond the low-energy expansion

Putting everything together

$$
\begin{aligned}
a_+ &= \int_0^\infty \frac{dx}{x}\,\frac{\rho_+^{\pi\pi}(x)}{G_F M_K^2} + \frac{f_+^{K^\pm\pi^\mp}(0)}{4\pi} \times 16\pi^2 \left(\frac{1}{\sqrt{2}} V_{us}^* V_{ud}\right) \left\{ \frac{C_{7V}(\nu)}{\alpha} + \sum_I C_I(\nu) \left[\xi_{00}^I - \xi_{01}^I \left(\ln\frac{M^2}{\nu^2} - \gamma_E \right) \right] \right\} \\
b_+ &= \int_0^\infty \frac{dx}{x^2}\,\frac{\rho_+^{\pi\pi}(x)}{G_F} + \frac{f_+^{K^\pm\pi^\mp}(0)}{4\pi} \times 16\pi^2 \left(\frac{1}{\sqrt{2}} V_{us}^* V_{ud}\right) \frac{\pi^2}{6} \frac{M_K^2}{M^2} \sum_I C_I(\nu)\, \xi_{01}^I \\
&\quad + \frac{f_+^{K^\pm\pi^\mp}(0)}{4\pi} \times \lambda_+ \frac{M_K^2}{M_\pi^2} \times 16\pi^2 \left(\frac{1}{\sqrt{2}} V_{us}^* V_{ud}\right) \left\{ \frac{C_{7V}(\nu)}{\alpha} + \sum_I C_I(\nu) \left[\xi_{00}^I - \xi_{01}^I \left(\ln\frac{M^2}{\nu^2} - \gamma_E \right) \right] \right\} \\
&\quad - \frac{1}{60} \left(\frac{M_K^2}{M_\pi^2}\right)^2 \left(\alpha_+ - \beta_+ \frac{s_0}{M_\pi^2} \right)
\end{aligned}
$$

$$
a_+ = -1.58 + \begin{cases} -0.10 \div +0.03 & \text{NDR} \\ -0.14 \div +0.07 & \text{HV} \end{cases}
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$$

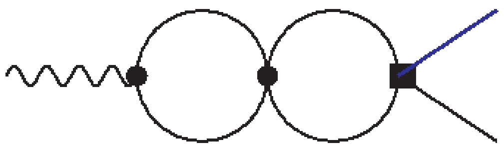
GD Greynat Knecht
work in progress

Data analized with several parameterisations of $W(z)$

Concentrate on "beyond one loop" (BOL) model

$$W_{\text{BOL}}(z) = G_F M_K^2 (a_+ + b_+ z) + \frac{8\pi^2}{3}\frac{M_K^2}{M_\pi^2}\left[\alpha_+ + \beta_+ \frac{M_K^2}{M_\pi^2}(z - z_0)\right]\left(1 + \frac{M_K^2}{M_V^2} z\right)\left[\frac{z - 4\frac{M_\pi^2}{M_K^2}}{z}\bar{J}_{\pi\pi}(zM_K^2) + \frac{1}{24\pi^2}\right]$$

G. D'Ambrosio et al., JHEP 9808, 004 (1998)



- Similar representation for $K_S \to \pi^0 \ell^+ \ell^-$
- α_+, β_+ from slope and curvature of $K^+ \to \pi^+\pi^+\pi^-$ amplitude (more on this later)

$$\alpha_+ = -20.84(74) \cdot 10^{-8}, \quad \beta_+ = -2.88(1.08) \cdot 10^{-8}$$

J. Bijnens et al. Nucl. Phys. B 648, 317 (2003)

- Only loops with pions, loops with kaons included in the polynomial part
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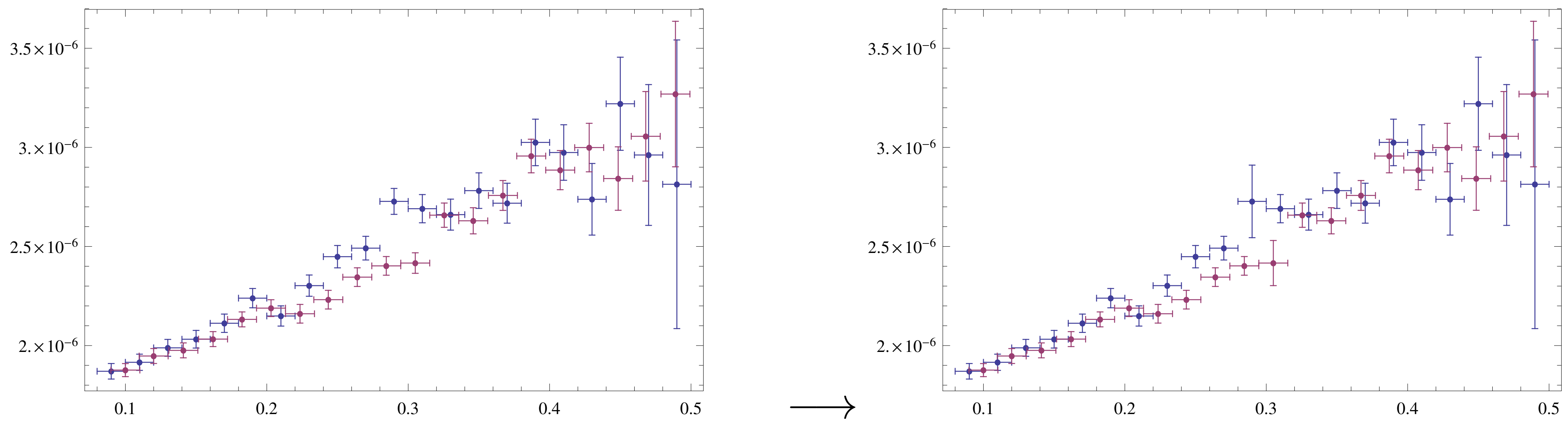
GD Greynat Knecht
work in progress

Combined fit (e^+e^-)

$\beta_+ \cdot 10^8$	a_+	b_+	χ^2/d.o.f
-3.96	$+0.483$	$+1.632$	86.7/39
	-0.598	-0.678	48.8/39
-2.88	$+0.489$	$+1.630$	60.4/39
	-0.592	-0.680	45.4/39
-1.80	$+0.495$	$+1.629$	74.8/39
	-0.585	-0.682	42.8/39



NA48/2 + E865

GD Greynat Knecht
work in progress

Fit to NA48/2 data ($\mu^+\mu^-$)

$\beta_+ \cdot 10^8$	a_+	b_+	$\chi^2/\text{d.o.f}$
-3.96	$+0.372$	$+2.102$	11.9/15
	-0.611	-0.746	15.9/15
-2.88	$+0.384$	$+2.081$	12.1/15
	-0.598	-0.768	15.2/15
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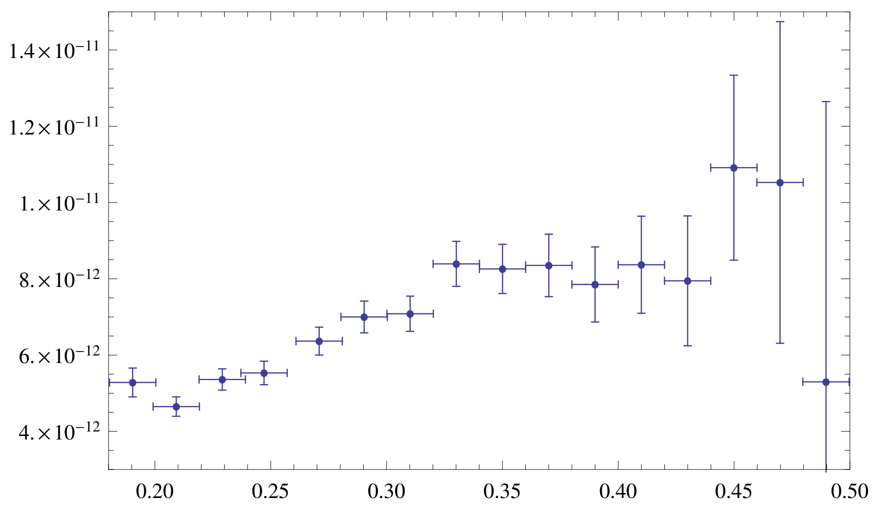

NA48/2

As in the NA48/2 data for the e^+e^- channel, the data show a slight preference for the positive solution

Robustness of determinations of a_+ and b_+

Impact of remaining two-loop contributions, not contained in $W_{\text{BOL}}(z)$

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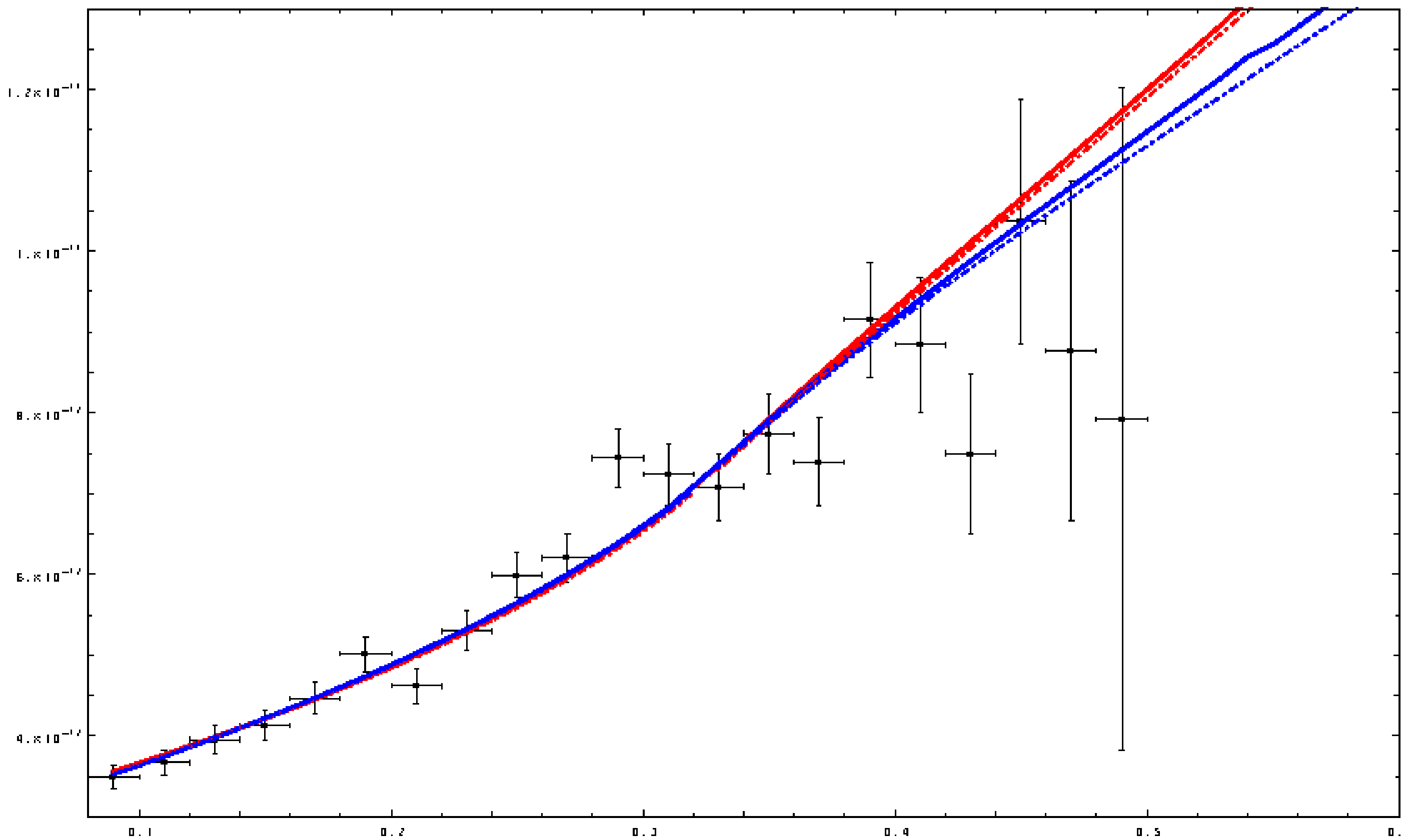
GD Greynat Knecht
work in progress

Comparing $W_{2\text{loop}}(z)$ and $W_{\text{BOL}}(z)$



solid lines: $|W_{2\text{loop}}(z)|^2$ full two loops

dash-dotted lines $|W_{\text{BOL}}(z)|^2$

red curves: $a_+ = -0.585, b_+ = -0.779, \beta_+ = -2.88 \cdot 10^{-8}$

blue curves: $a_+ = -0575, b_+ = -0.779, \beta_+ = -0.99 \cdot 10^{-8}$

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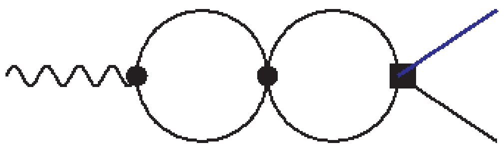

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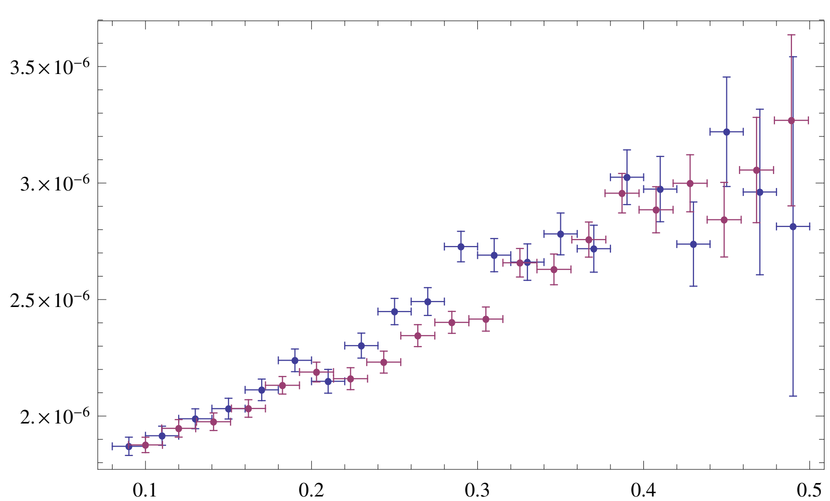

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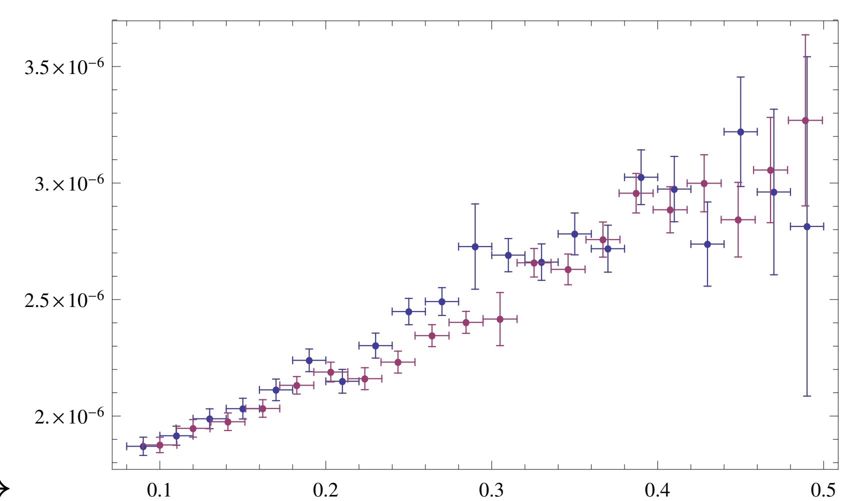

NA48/2 + E865

Rescale error bar of one data point in each set

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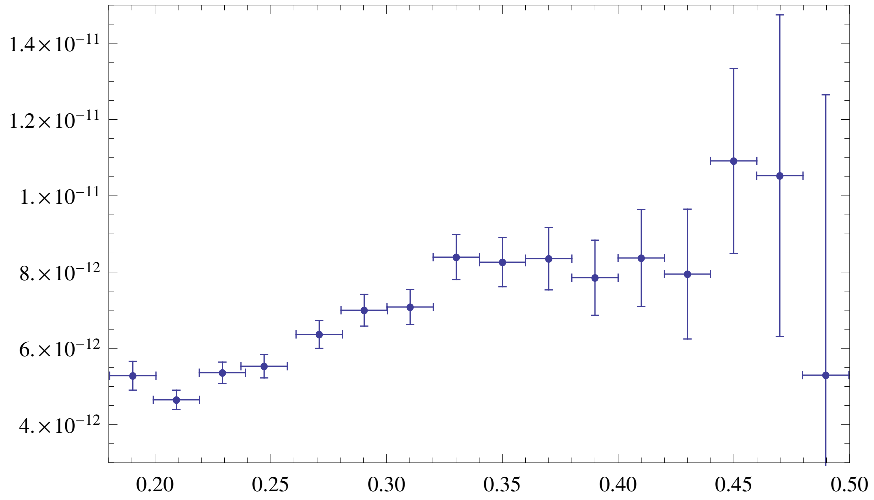

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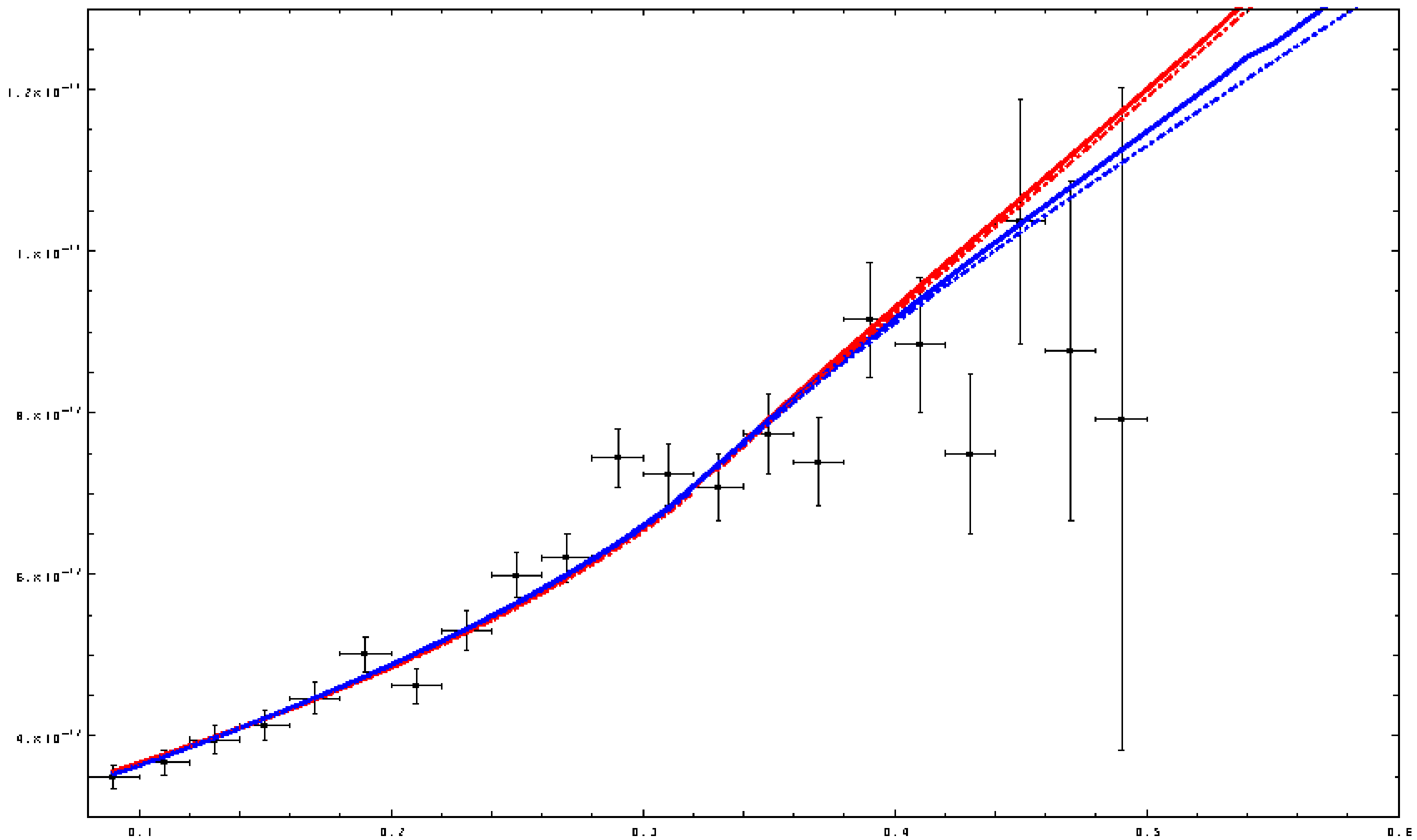
Comparing $W_{2\text{loop}}(z)$ and $W_{\text{BOL}}(z)$



solid lines: $|W_{2\text{loop}}(z)|^2$ full two loops

dash-dotted lines $|W_{\text{BOL}}(z)|^2$

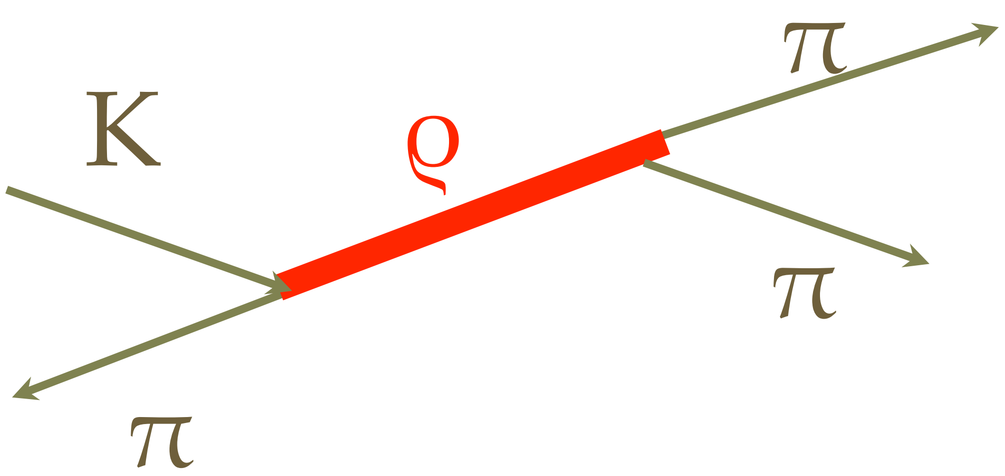
red curves: $a_+ = -0.585, b_+ = -0.779, \beta_+ = -2.88 \cdot 10^{-8}$

blue curves: $a_+ = -0575, b_+ = -0.779, \beta_+ = -0.99 \cdot 10^{-8}$

Hard Wall weak interactions: K->3pi

Luigi Cappiello, Oscar Cata and G.D.

In this channel there is a large VMD in the phenomenological slope



However this is proportional to $L_3 + 3/4\ L_9$

4D $L_3 + 3/4\ L_9 = 0$

5D $L_3 + 3/4\ L_9 \neq 0$ and in agreement with phenomenology

K->2 pi/3pi fit

Kambor Missimer Wyler, '90s

$$\mathcal{M}(K_L \to \pi^+\pi^-\pi^0) = \alpha_1 - \beta_1 u + (\zeta_1 + \xi_1)u^2 + \frac{1}{3}(\zeta_1 - \xi_1)v^2$$

$$\mathcal{M}(K_L \to \pi^0\pi^0\pi^0) = -3\alpha_1 - \zeta_1(3u^2 + v^2) ,$$

$$\mathcal{M}(K^+ \to \pi^+\pi^+\pi^-) = 2\alpha_1 + \beta_1 u + (2\zeta_1 - \xi_1)u^2 + \frac{1}{3}(2\zeta_1 + \xi_1)v^2 ,$$

$$\mathcal{M}(K^+ \to \pi^+\pi^0\pi^0) = -\alpha_1 + \beta_1 u - (\zeta_1 + \xi_1)u^2 - \frac{1}{3}(\zeta_1 - \xi_1)v^2 ,$$

$$\alpha_1 = \alpha_1^{(0)} - \frac{2g_8}{27 f_K f_\pi} m_K^4 \{(k_1 - k_2) + 24\mathcal{L}_1\} ,$$

$$\beta_1 = \beta_1^{(0)} - \frac{g_8}{9 f_K f_\pi} m_\pi^2 m_K^2 \{(k_3 - 2k_1) - 24\mathcal{L}_2\} ,$$

$$\zeta_1 = -\frac{g_8}{6 f_K f_\pi} m_\pi^4 \{k_2 - 24\mathcal{L}_1\} ,$$

$$\xi_1 = -\frac{g_8}{6 f_K f_\pi} m_\pi^4 \{k_3 - 24\mathcal{L}_2\} ,$$

Table 1

The values of the amplitudes in eqs. (4) and (5) obtained from fits to experiment are shown in the first two columns. Our value of $\delta_2 - \delta_0$ is obtained from $K \to 2\pi$ decays alone, while some additional constraints were used in ref. [8]. The $K \to 3\pi$ amplitudes $\alpha_1, ..., \xi_3'$ are in units of 10^{-8}. The results of lowest and next-to-lowest order chiral perturbation theory are displayed in the two columns to the right.

	Devlin and Dickey	Our fit	Lowest order	Order p^4
$a_{1/2}$ [keV]	0.4687 ± 0.0006	0.4699 ± 0.0012	0.4698	0.4698
$a_{3/2}$ [keV]	0.0210 ± 0.0001	0.0211 ± 0.0001	0.0211	0.0211
$\delta_2 - \delta_0$ (deg)	-45.6 ± 5	-61.5 ± 4	0	-29
α_1	91.46 ± 0.24	91.71 ± 0.32	74.0	91.8
α_3	-7.14 ± 0.36	-7.36 ± 0.47	-4.1	-7.6
β_1	-25.83 ± 0.41	-25.68 ± 0.27	-16.5	-25.6
β_3	-2.48 ± 0.48	-2.43 ± 0.41	-1.0	-2.5
γ_3	2.51 ± 0.36	2.26 ± 0.23	1.8	2.5
ζ_1	-0.37 ± 0.11	-0.47 ± 0.15	–	-0.6
ζ_3	–	-0.21 ± 0.08	–	-0.02
ξ_1	-1.25 ± 0.12	-1.51 ± 0.30	–	-1.5
ξ_3	–	-0.12 ± 0.17	–	-0.05
ξ_3'	–	-0.21 ± 0.51	–	-0.08
χ^2/DOF	12.8/3	10.3/2	4121/5	37/13

Vector meson dominance in $K \to 3\pi$

$$\mathcal{M}(K_L \to \pi^+\pi^-\pi^0) = \alpha_1 - \beta_1 u + (\zeta_1 + \xi_1)u^2 + \frac{1}{3}(\zeta_1 - \xi_1)v^2$$

$$\mathcal{M}(K_L \to \pi^0\pi^0\pi^0) = -3\alpha_1 - \zeta_1(3u^2 + v^2) \ ,$$

$$\mathcal{M}(K^+ \to \pi^+\pi^+\pi^-) = 2\alpha_1 + \beta_1 u + (2\zeta_1 - \xi_1)u^2 + \frac{1}{3}(2\zeta_1 + \xi_1)v^2 \ ,$$

$$\mathcal{M}(K^+ \to \pi^+\pi^0\pi^0) = -\alpha_1 + \beta_1 u - (\zeta_1 + \xi_1)u^2 - \frac{1}{3}(\zeta_1 - \xi_1)v^2 \ ,$$

$$u = \frac{s_3 - s_0}{m_\pi^2} \ , \qquad v = \frac{s_1 - s_2}{m_\pi^2} \ , \qquad s_i = (p_K - p_{\pi_i})^2 \ , \qquad s_0 = \frac{1}{3}\sum_{i=1}^{3} s_i \ .$$

$$\alpha_1 = \alpha_1^{(0)} - \frac{2g_8}{27 f_K f_\pi} m_K^4 \{(k_1 - k_2) + 24\mathcal{L}_1\} \ ,$$

$$\beta_1 = \beta_1^{(0)} - \frac{g_8}{9 f_K f_\pi} m_\pi^2 m_K^2 \{(k_3 - 2k_1) - 24\mathcal{L}_2\} \ ,$$

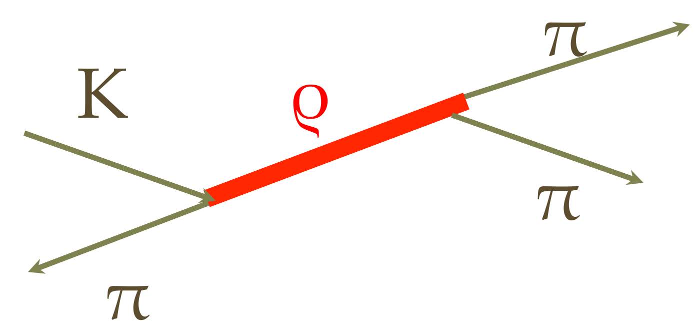
$$\zeta_1 = -\frac{g_8}{6 f_K f_\pi} m_\pi^4 \{k_2 - 24\mathcal{L}_1\} \ ,$$

$$\xi_1 = -\frac{g_8}{6 f_K f_\pi} m_\pi^4 \{k_3 - 24\mathcal{L}_2\} \ ,$$

Angular momentum decomposition

β_1 should be dominated by

ρ exchange



$$k_3 = 3(N_1 + N_2 - N_3)$$

It has VMD

Isidori, Pugliese
Ecker Kambor Wyler

We measure the slope, let's check theory predictions

In factorization

$$k_3/24 = 3(N_1 + N_2 - N_3)/24 = L_3 + 3/4 L_9$$

using L_i^{VMD}

TH VMD 0

in units 10^{-3}

$$\frac{k_3}{24} = \left(L_3 + \frac{3}{4} L_9\right) \overset{\text{expt}}{\sim} 1.7$$

using L_i^{exp}

5D 1.7

using L_i^{holo}

departures from KSFR

Luigi Cappiello, Oscar Cata and G.D.

Further NA62 K Physics Program

Decay	Physics	Present limit (90% C.L.) / Result	NA62
$\pi^+\mu^+e^-$	LFV	1.3×10^{-11}	0.7×10^{-12}
$\pi^+\mu^-e^+$	LFV	5.2×10^{-10}	0.7×10^{-12}
$\pi^-\mu^+e^+$	LNV	5.0×10^{-10}	0.7×10^{-12}
$\pi^-e^+e^+$	LNV	6.4×10^{-10}	2×10^{-12}
$\pi^-\mu^+\mu^+$	LNV	1.1×10^{-9}	0.4×10^{-12}
$\mu^-\nu e^+e^+$	LNV/LFV	2.0×10^{-8}	4×10^{-12}
$e^-\nu\mu^+\mu^+$	LNV	No data	10^{-12}
π^+X^0	New Particle	$5.9\times10^{-11}\ m_{X^0}=0$	10^{-12}
$\pi^+\chi\chi$	New Particle	–	10^{-12}
$\pi^+\pi^+e^-\nu$	$\Delta S\neq\Delta Q$	1.2×10^{-8}	10^{-11}
$\pi^+\pi^+\mu^-\nu$	$\Delta S\neq\Delta Q$	3.0×10^{-6}	10^{-11}
$\pi^+\gamma$	Angular Mom.	2.3×10^{-9}	10^{-12}
$\mu^+\nu_h,\nu_h\rightarrow\nu\gamma$	Heavy neutrino	Limits up to $m_{\nu_h}=350\ MeV$	
R_K	LU	$(2.488\pm0.010)\times10^{-5}$	>×2 better
$\pi^+\gamma\gamma$	χPT	< 500 events	10^5 events
$\pi^0\pi^0e^+\nu$	χPT	66000 events	$O(10^6)$
$\pi^0\pi^0\mu^+\nu$	χPT	-	$O(10^5)$

π	2π	3π	N_i
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N_{14}^r - N_{15}^r$
$\pi^0\gamma^*\ (S)$	$\pi^0\pi^0\gamma^*\ (L)$		$2N_{14}^r + N_{15}^r$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$
	$\pi^+\pi^-\gamma\gamma\ (S)$		"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
	$\pi^+\pi^-\gamma\ (S)$	$\pi^+\pi^0\pi^0\gamma$	"
		$\pi^+\pi^-\pi^0\gamma\ (L)$	"
		$\pi^+\pi^-\pi^0\gamma\ (S)$	$7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17})$
	$\pi^+\pi^-\gamma^*\ (L)$		$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma^*\ (S)$		$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$
	$\pi^+\pi^0\gamma^*$		$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma\ (L)$	$\pi^+\pi^-\pi^0\gamma\ (S)$	$N_{29} + N_{31}$
		$\pi^+\pi^+\pi^-\gamma$	"
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$
		$\pi^+\pi^-\pi^0\gamma\ (S)$	$5N_{29} - N_{30} + 2N_{31}$
		$\pi^+\pi^-\pi^0\gamma\ (L)$	$6N_{28} + 3N_{29} - 5N_{30}$

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma,\, K\to\pi l^+ l^-} + \cdots$$

OPE

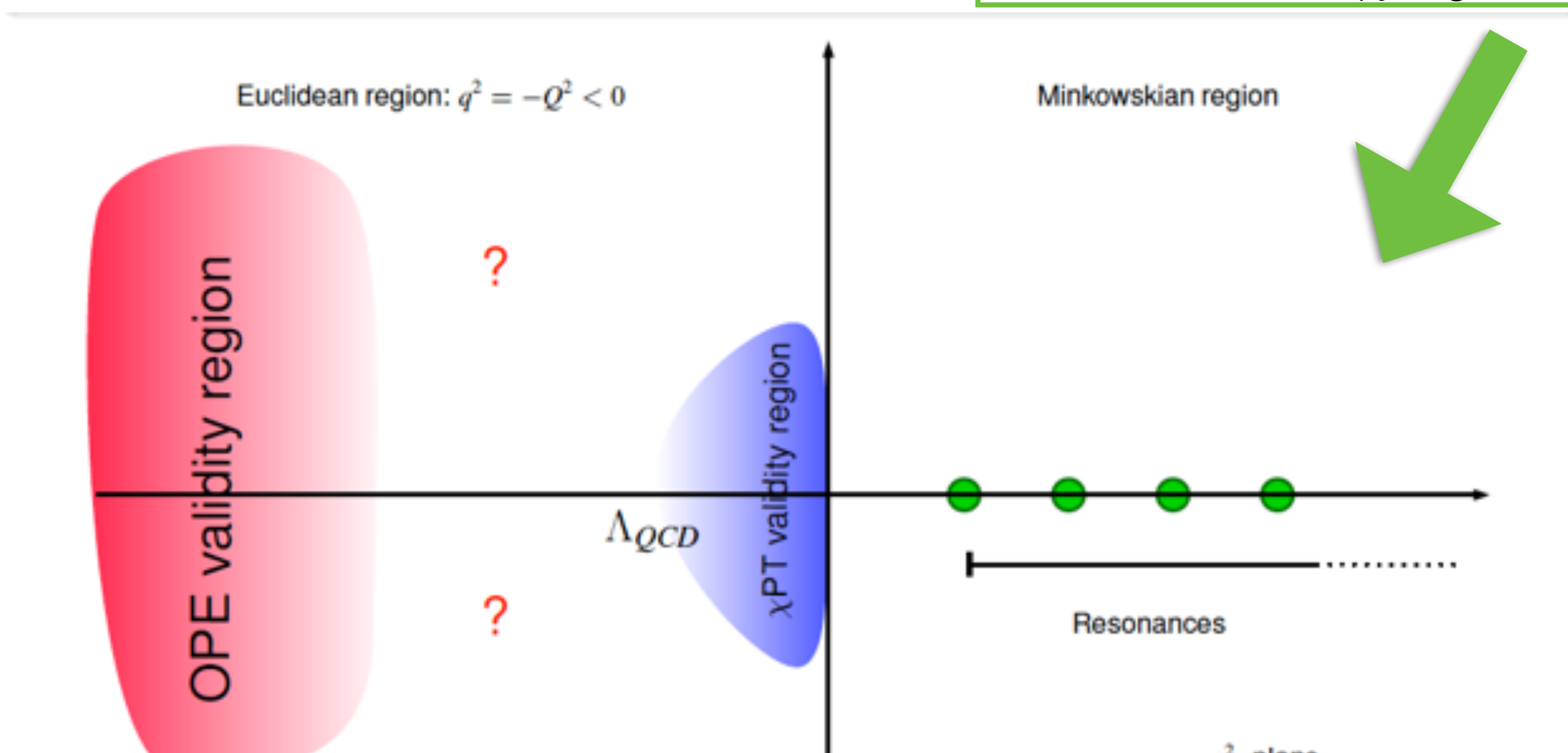
$$\Pi_V(Q^2) \underset{Q^2\to\infty}{\sim} \frac{N_c}{24\pi^2}\ln\left(\frac{\Lambda_V^2}{Q^2}\right) + \langle\mathcal{O}_2\rangle\frac{1}{Q^2} + \langle\mathcal{O}_4\rangle\frac{1}{Q^4} + \langle\mathcal{O}_6\rangle_V\frac{1}{Q^6}$$

$$\begin{cases} \langle\mathcal{O}_2\rangle = 0 \\ \langle\mathcal{O}_4\rangle = \frac{1}{12\pi}\,\alpha_s\,\langle G^2\rangle \\ \langle\mathcal{O}_6\rangle_V = -\frac{28\pi}{9}\,\alpha_s\,\langle\bar\psi\psi\rangle^2 \end{cases}$$

– Second Weinberg's sum rule –

Large Nc QCD

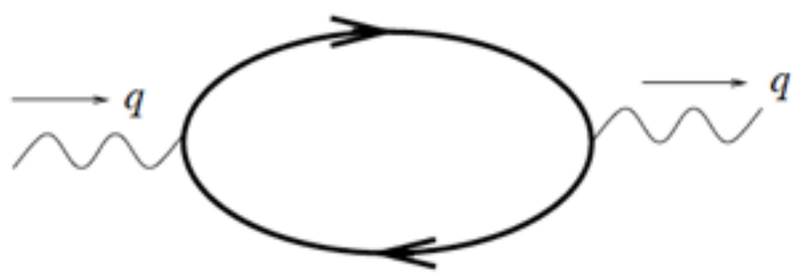
$$\Pi_V(Q^2) = \sum_{n=0}^{\infty}\frac{F_V(n)^2}{Q^2 + M_V(n)^2}$$



Holographic QCD at low energies

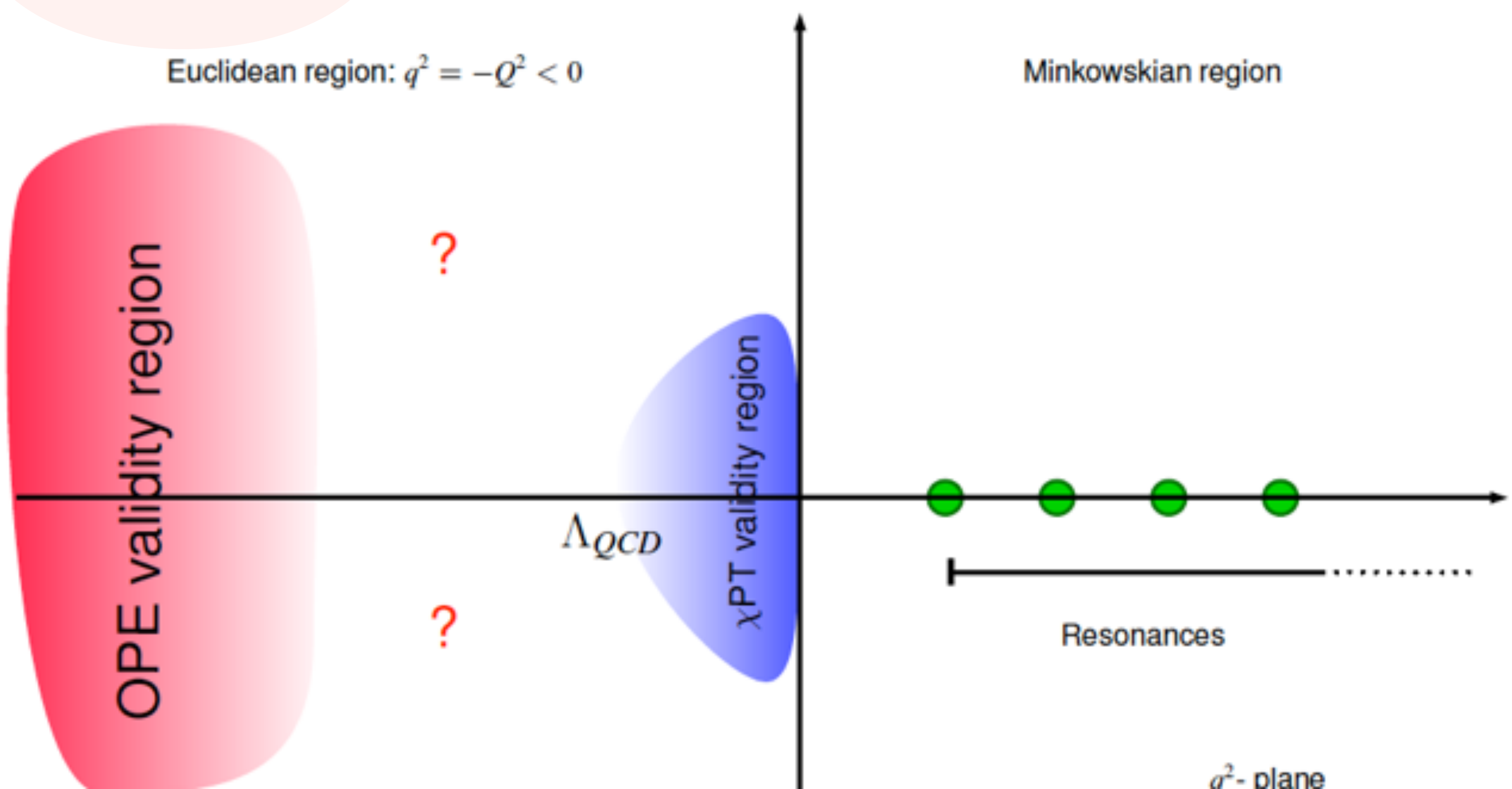
Large Nc QCD properties => conformal symmetry in 5d

$$\int d^4x\, e^{ip\cdot x}\langle J_\mu(x)J_\nu(0)\rangle = \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\Pi(p^2)$$

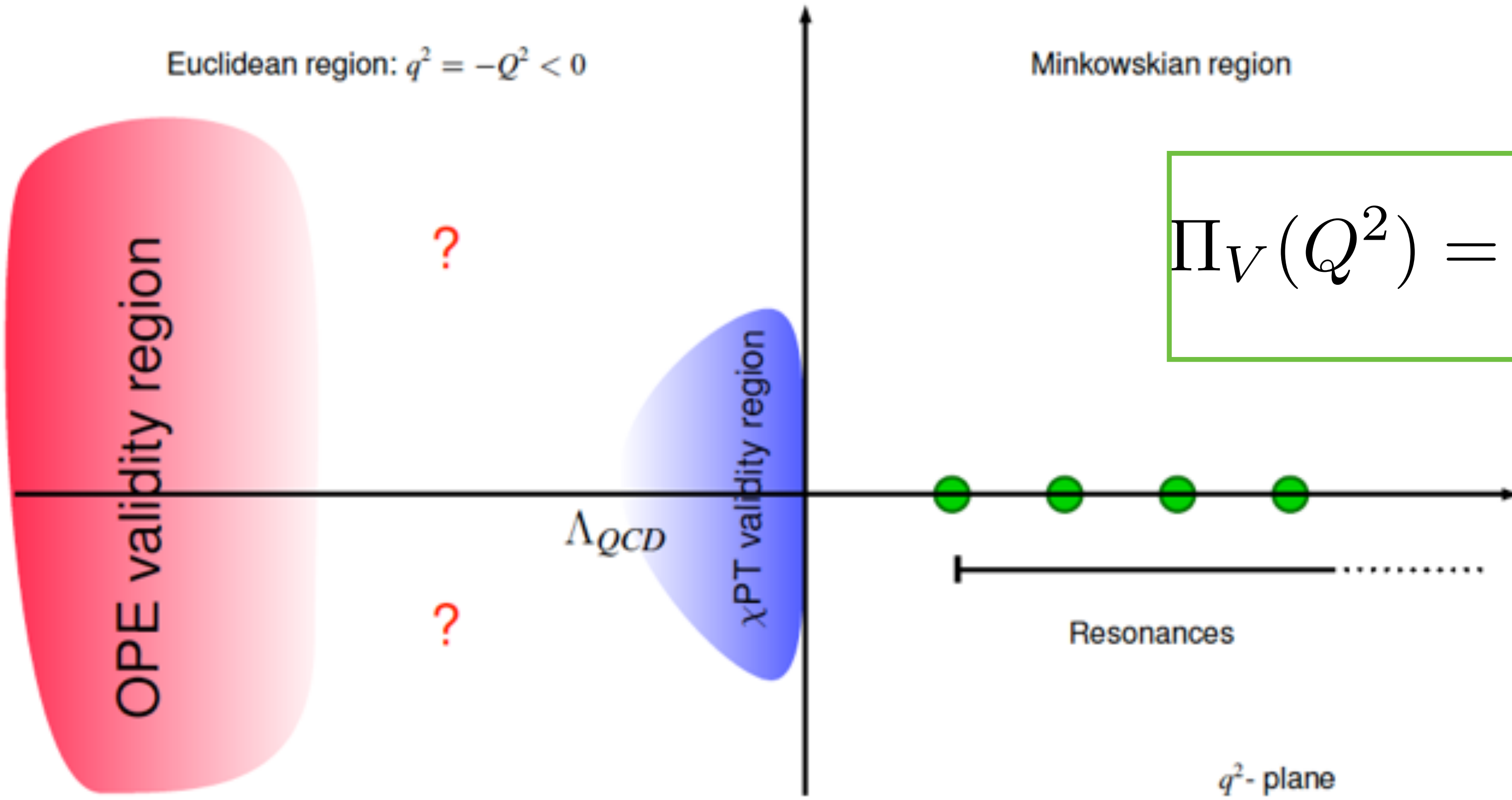


$$\Pi_V(Q^2) = -\frac{N_c}{12\pi^2}\ln Q^2 \qquad Q^2 \equiv -q^2$$

confinement: either IR cut-off or field modifying metric



Holographic QCD: Hard Wall



$$\Pi_V(Q^2) = \sum_{n=0}^{\infty} \frac{F_V(n)^2}{Q^2 + M_V(n)^2}$$

$$\Pi_V(Q^2) = -\frac{N_c}{12\pi^2} \ln Q^2 \quad Q^2 \equiv -q^2$$

→ IR very good, but

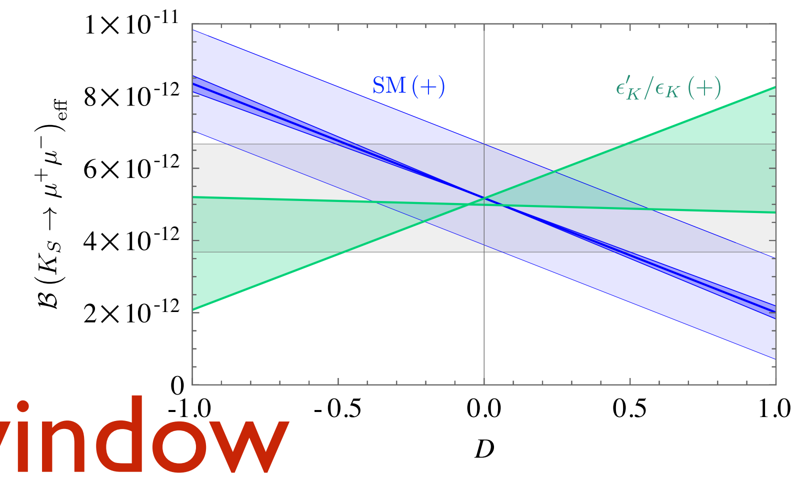
$$M_V^2(n) \sim n^2$$

HW ⇒

$$\Pi_V(q^2) = -\frac{4}{3}\frac{N_c}{(4\pi)^2}\left[\log\frac{q^2}{\mu^2} - \pi\frac{Y_0(qz_0)}{J_0(qz_0)}\right]$$

Conclusions

- Flavour anomalies: interplay with K->π***νν*** but 10% measurement needed!



- LHCB: K_S->μμ extraordinary result: interference effect!!!Short distance window

- weak chiral lagrangian

- LFUV in Kaons very useful

- Rich rare kaon program

Back up

Correlation with different flavor sectors

$\Lambda_{NP}^{b\to c,s} \sim \mathcal{O}(1, 100)$ TeV $\Rightarrow$ direct searches,
low-energy precision observables

GIM suppression and CKM suppression:

$$\mathcal{L}_{\rm eff} \supset -\frac{1-0.3\,i}{(180\ \mathrm{TeV})^2}(\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_L\gamma^\mu\nu_L) + \mathrm{h.c.}$$

Svjetlana Fajfer and Nejc Koˇsnik Luiz Vale Silva

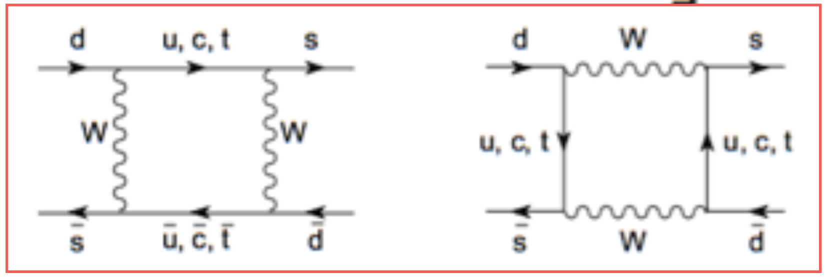
Flavour Problem

- the SM Yukawa structure

$$\mathcal{L}^{Y}_{SM} = \bar{Q} Y_D D H + \bar{Q} Y_U U H_c + \bar{L} Y_E E H + \text{h.c.}$$

FCNC

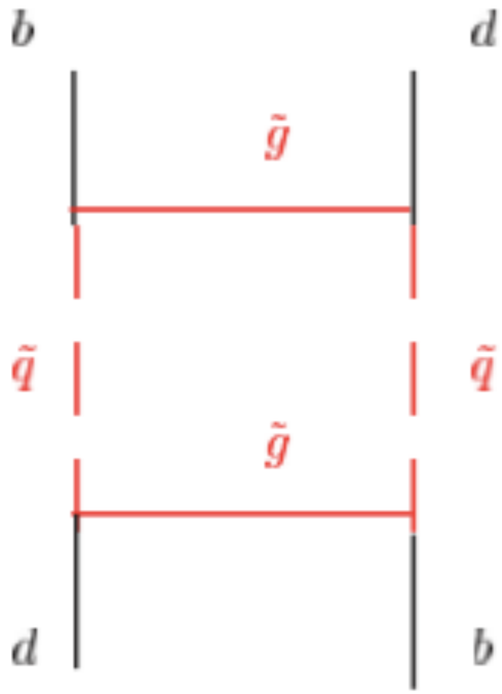
$$\mathcal{H}^{SM}_{\Delta F=2} \sim \frac{G_F^2 M_W^2}{16\pi^2}\left[\frac{(V^*_{td} m_t^2 V_{tb})^2}{v^4}(\bar{d}_L \gamma^\mu b_L)^2 + \frac{(V^*_{td} m_t^2 V_{ts})^2}{v^4}(\bar{d}_L \gamma^\mu s_L)^2\right] + \text{charm}$$



- Supersymmetry must be broken

$$-\mathcal{L}_{soft} = \tilde{Q}^\dagger m_Q^2 \tilde{Q} + \tilde{L}^\dagger m_L^2 \tilde{L} + \tilde{\bar{U}} a_u \tilde{Q}\ H_u + \ldots$$

- $m_Q^2, m_L^2, a_u, \ldots$ matrices in flavour space additional (to $Y_{u,d,l}$) non-trivial structures



$$\mathcal{H}^{\tilde g}_{\Delta F=2} \sim \frac{\alpha_s^2}{9M^2_{\tilde Q}}\left[(\delta^{LL}_{12})^2 \; (\bar s_L \gamma_\mu d_L)^2 + \cdots\right]$$

δ^{LL}_{12} departure from identity matrix m^2_Q

Gabrielli Masiero
Silvestrini

$$K - \bar K \Longrightarrow \quad \frac{(\delta^{LL}_{12})^2}{M^2_{\tilde Q}} \leq \frac{1}{(100\mathrm{TeV})^2} \Longrightarrow \text{Naturalness?}$$

obey some Flavour symmetry so that GIM is realized

$$m^2_Q \sim I$$

$$\mathcal{L}_{\Delta F=2} = \frac{C}{\Lambda^2_{MFV}}\left[\frac{(V^*_{td} m_t^2 V_{tb})^2}{v^4}(\bar d_L \gamma^\mu b_L)^2 + \frac{(V^*_{td} m_t^2 V_{ts})^2}{v^4}(\bar d_L \gamma^\mu s_L)^2\right]$$

Traditional solution

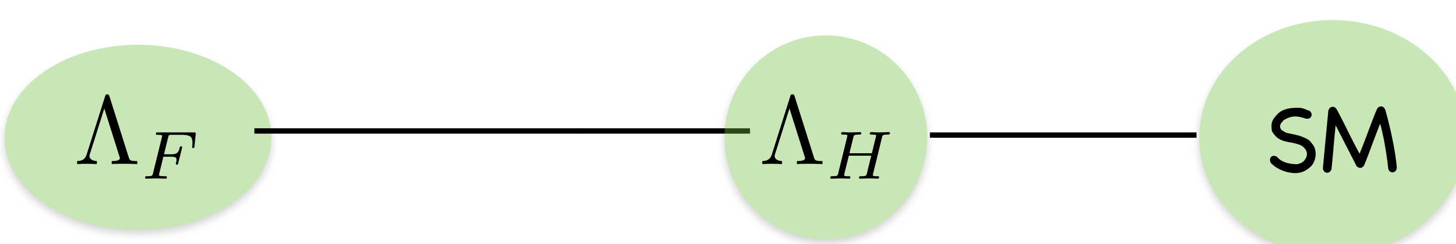
Problem already known since ’86 technicolour, (Chivukula Georg

susy (Hall Randall)

extra dimensions (Rattazzi Zafferoni)

G.D.,Giudice,Isidori,Strumia; A. Buras, Gambino,Silvestrini

Scale New Physics, stabilizing EW scale, Λ_H <<scale of the dynamical understanding of Flavor



$$\Lambda_H << \Lambda_F$$

CP violation in $K \to 2\pi$

$$A(K_L \to \pi^+\pi^-) \propto \epsilon + \epsilon'$$

$$A(K_L \to \pi^0\pi^0) \propto \epsilon - 2\epsilon'$$

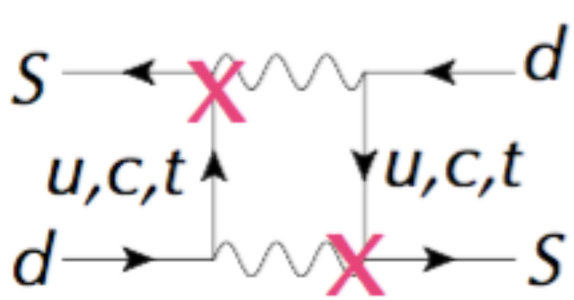
$\epsilon \sim \mathcal{O}(10^{-3})$ Christenson et al 64

$\epsilon' \sim \mathcal{O}(10^{-6})$ CERN NA31, Fermilab KTeV

$H_{\Delta S=2}$

Indirect CP violation

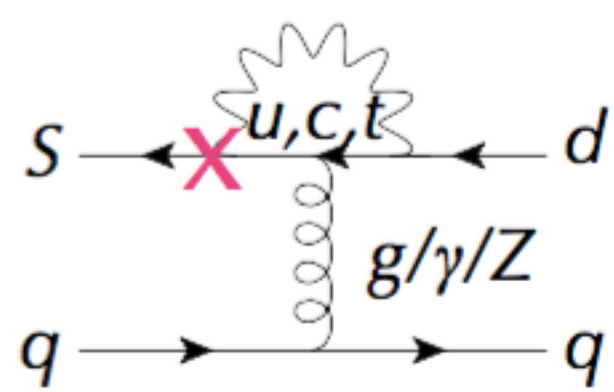
Kaon oscillation



$H_{\Delta S=1}$

Direct CP Violation

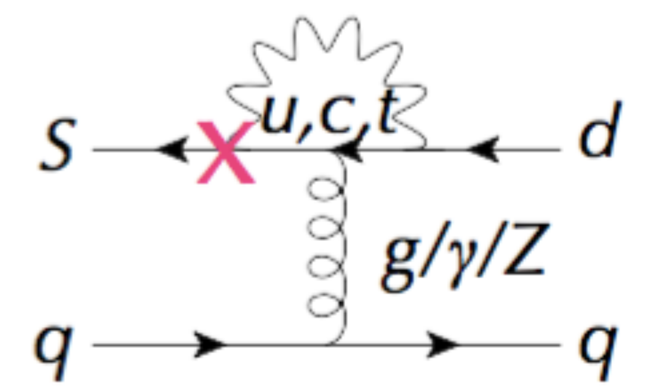
Penguin



$$\frac{\epsilon'_K}{\epsilon_K} = \frac{1}{\sqrt{2}|\epsilon_K|_{\rm exp}} \frac{\omega_{\rm exp}}{({\rm Re}A_0)_{\rm exp}} \left(-{\rm Im}A_0 + \frac{1}{\omega_{\rm exp}} {\rm Im}A_2 \right) \quad \text{where} \quad \frac{1}{\omega} \equiv \frac{{\rm Re}A_0}{{\rm Re}A_2} = 22.46 \text{ (exp.)}$$

gluon penguin Q_6

EW penguin Q_8



<O6> and <O8> have chiral enhancement factor

$$\langle Q_6(\mu)\rangle_0 = -4\left[\frac{m_K^2}{m_s(\mu)+m_d(\mu)}\right]^2 (F_K - F_\pi)\, B_6^{(1/2)}$$

$$\langle Q_8(\mu)\rangle_2 = \sqrt{2}\left[\frac{m_K^2}{m_s(\mu)+m_d(\mu)}\right]^2 F_\pi\, B_8^{(3/2)}.$$

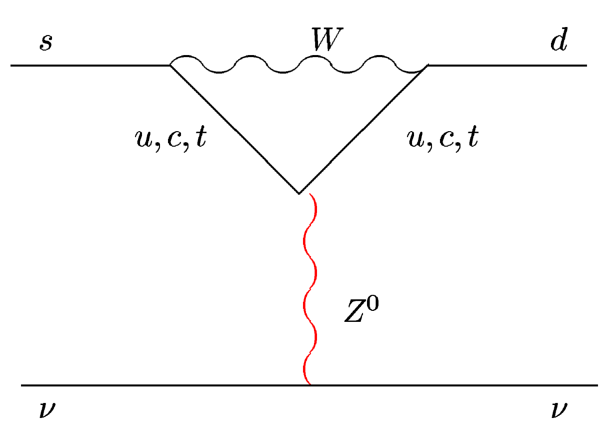
New lattice result 2015

Kei Yamamoto

$K \to \pi\nu\overline{\nu}$

Why we need to the experiments KOTO and NA62

$$A(s \to d\nu\overline{\nu})_{\rm SM} \sim \overline{s}_L\gamma_\mu d_L \quad \overline{\nu}_L\gamma^\mu\nu_L \times \left[\sum_{q=c,t} V^*_{qs}V_{qd}\, m_q^2\right]$$



$$\sim \quad \left[A^2\lambda^5\ (1-\rho-i\eta)m_t^2 + \lambda m_c^2\right]$$

SM $\underbrace{V-A \otimes V-A}$ Littenberg

$\Downarrow$

$$\Gamma(K_L \to \pi^0\nu\overline{\nu}) \quad \begin{cases} \text{CP violating} \\ \Rightarrow J = A^2\lambda^6\eta \\ \text{Only } top \end{cases}$$

SM

Buchalla and Buras, hep-ph/9308272, Buras et al, 1503.02693.

$K^+ \to \pi^+ \nu\bar{\nu}$

Misiak, Urban; Buras, Buchalla; Brod, Gorbhan, Stamou'11, Straub

$$B(K^+) \sim \kappa_+ \left[\left(\frac{\mathrm{Im}\lambda_\mathrm{t}}{\lambda^5} X_t \right)^2 + \left(\frac{\mathrm{Re}\lambda_\mathrm{c}}{\lambda} (P_c + \delta P_{c,u}) + \frac{\mathrm{Re}\lambda_\mathrm{t}}{\lambda^5} X_t \right)^2 \right]$$

$\lambda_q = V_{qd}{}^* V_{qs}$

- κ_+ from K_{l3}
- P_c: SD charm quark contribution (30%$\pm$2.5% to BR)
  LD $\delta P_{c,u} \sim 4 \pm 2\%$
- $B(K^\pm) = (8.82 \pm 0.8 \pm 0.3) \times 10^{-11}$ first error parametric (V_{cb}), second non-pert. QCD
- E949 $B(K^\pm) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$

K_L

$B(K_L) = (3.14 \pm 0.17 \pm 0.06) \times 10^{-11}$ vs

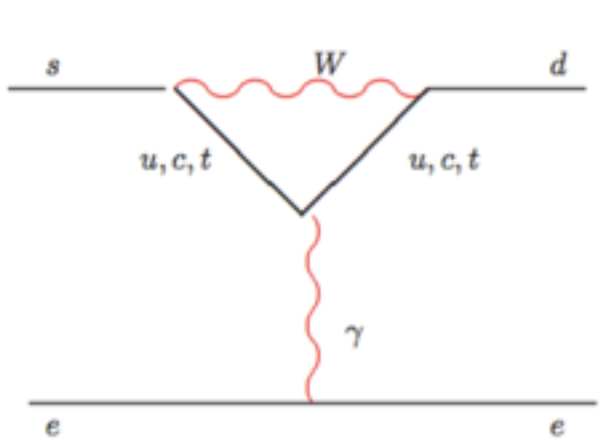
E391a $B(K_L) < 2.6 \times 10^{-8}$ at 90% C.L.

K_L Model-independent bound, based on $SU(2)$ properties dim-6 operators for $\bar{s}d\bar{\nu}\nu$ Grossman-Nir

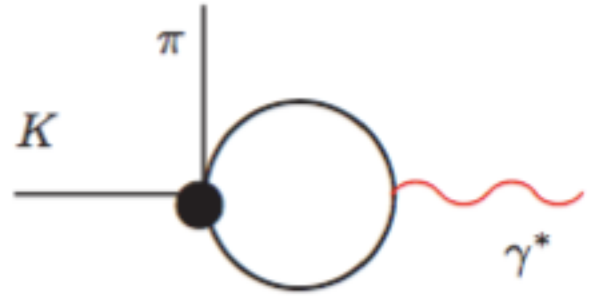
$$B(K_L) \leq \frac{\tau_L}{\tau_+} \times B(K^{\pm})_{\mathrm{E949}} \leq 1.4 \times 10^{-9} \quad \text{at } 90\% C.L.$$

	PDG	Prospects
$K_S \to \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL	$(\mathrm{LD})(5.0 \pm 1.5) \cdot 10^{-12}$ NP $< 10^{-11}$
$K_L \to \mu\mu$	$(6.84 \pm 0.11) \times 10^{-9}$	difficult : SD << LD
$K_S \to \mu\mu\mu\mu$	$-$	SM LD $\sim 2 \times 10^{-14}$
$K_S \to ee\mu\mu$	$-$	$\sim 10^{-11}$
$K_S \to eeee$	$-$	$\sim 10^{-10}$
$K_S \to \pi^+\pi^-\mu^+\mu^-$	$-$	SM LD $\sim 10^{-14}$

$K^+ \to \pi^+ e^+ e^-$ $K_S \to \pi^0 e^+ e^-$

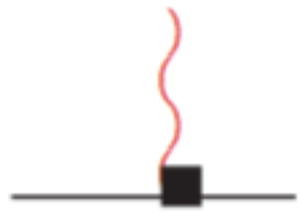


Short distance

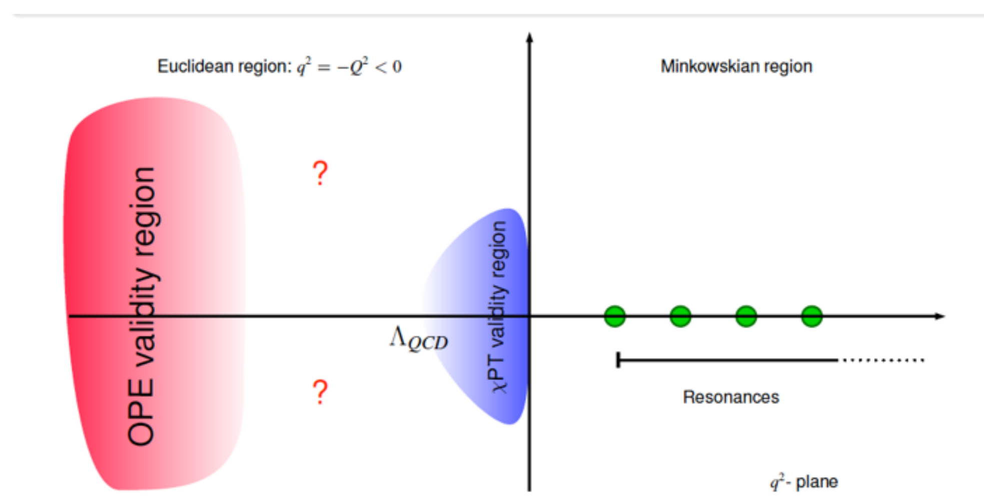


$O(p^4)$ ChPT

electrons and μ's in the final state



constant+ loop Ecker, Pich, de Rafael



'97 Initial data inconsistency e and μ's: LFV?

$K^+ \to \pi^+ e^+ e^-$ $K_S \to \pi^0 e^+ e^-$

- gauge+Lorentz inv. =>1 ff

$$i \int d^4x e^{iqx} \langle \pi(p) | T \{ J^\mu_{\rm elm}(x) \mathcal{L}_{\Delta S=1}(0) \} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^\mu - (1 - r_\pi^2) q^\mu \right]$$

$$W^i = G_F m_K^2 (a_i + b_i z) + W^i_{\pi\pi}(z)$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \qquad z = \frac{q^2}{m_K^2}$$

- Observables $\Gamma(K^+ \to \pi^+ e^+ e^-)$, $\Gamma(K^+ \to \pi^+ \mu \bar{\mu})$, slopes

- a_i $O(p^4)$ $a_+ \sim N_{14} - N_{15}$, $a_S \sim 2N_{14} + N_{15}$ Ecker, Pich, de Rafael
- b_i $O(p^6)$ G.D., Ecker, Isidori, Portoles

- $a_+, \ b_+$ in general not related to $a_S, \ b_S$ Recent lattice determinations Christ et al.

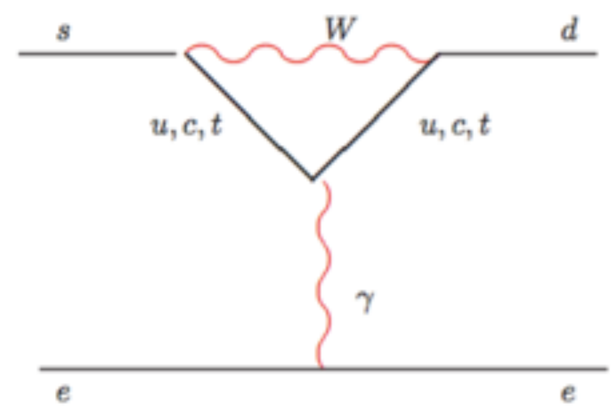
averaging flavour

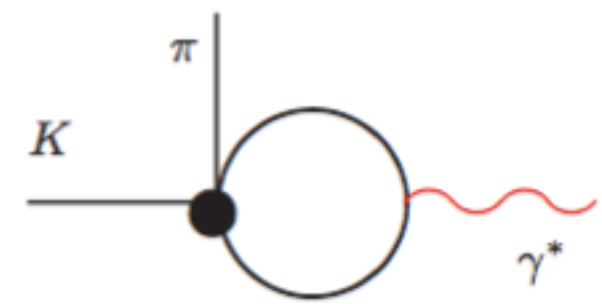
$$a_+^{\rm exp.} = -0.578 \pm 0.016$$
$$b_+^{\rm exp.} = -0.779 \pm 0.066$$

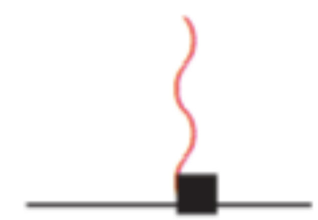
LFUV: Kaons

$$\frac{\Gamma(K^+ \to \pi^+ \mu^+ \mu^-)}{\Gamma(K^+ \to \pi^+ e^+ e^-)}$$

SM







$$W^i = G_F m_K^2 (a_i + b_i z) + W^i_{\pi\pi}(z)$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \qquad z = \frac{q^2}{m_K^2}$$

Collaboration with Crivellin, A Hoferichter, M and Tunstall,
Phys.Rev. D 2016

LFUV: Kaons

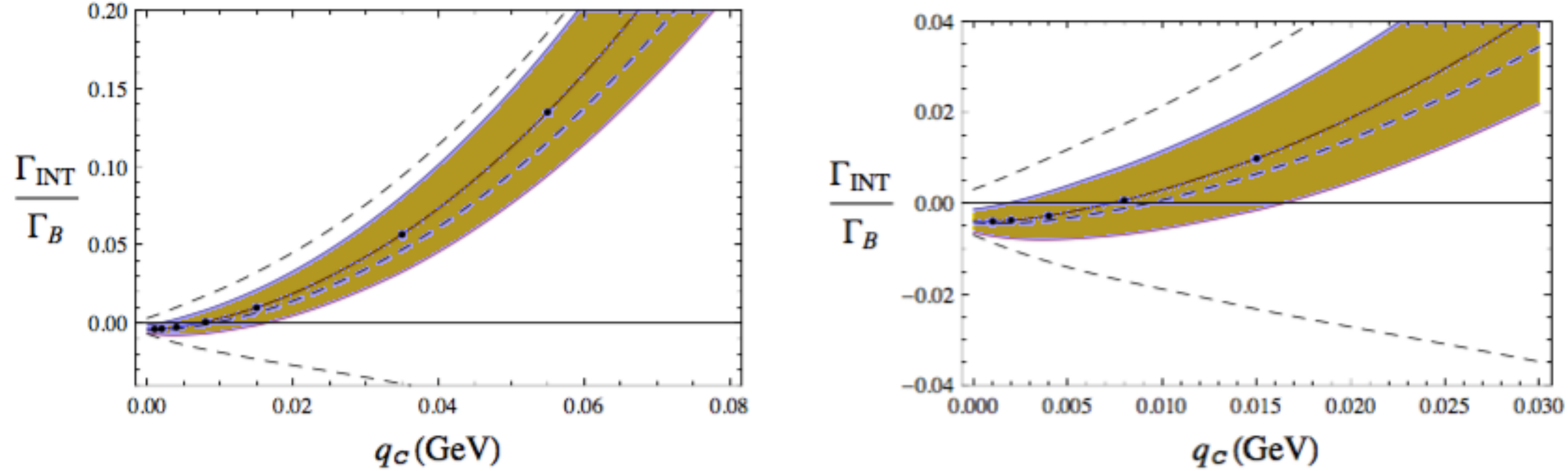
Channel	a_+	b_+	Reference
ee	-0.587 ± 0.010	-0.655 ± 0.044	E865
ee	-0.578 ± 0.016	-0.779 ± 0.066	NA48/2
$\mu\mu$	-0.575 ± 0.039	-0.813 ± 0.145	NA48/2

$$a_+^{\mathrm{NP}} = \frac{2\pi\sqrt{2}}{\alpha} V_{ud} V_{us}^* * C_{7V}^{\mathrm{NP}}$$

$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{ud} V_{us}^*} \quad \overset{MFV}{\Longrightarrow} \quad C_{9V}^{B,\mu\mu} - C_{9V}^{B,ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{td} V_{ts}^*} = -19 \pm 79$$

NA62 PLEASE!!

High statistics: nominal # of decays 50 times greater than NA48/2



q cut in minimum dilepton

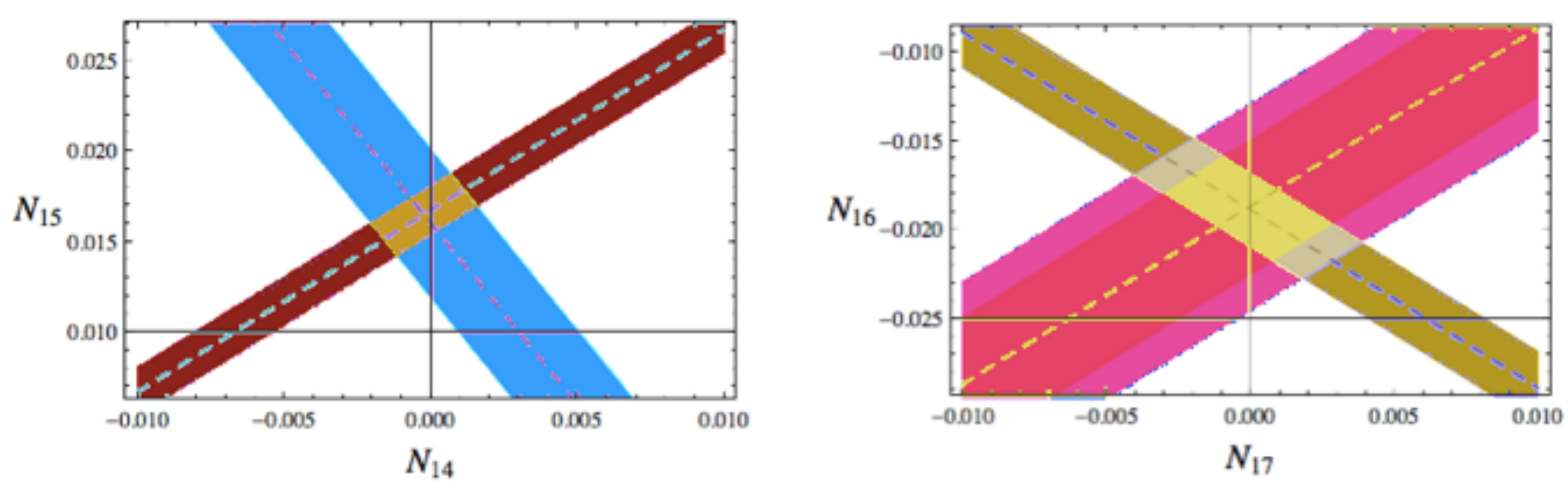


Figure 4: *Left panel: values of* N_{14} *and* N_{15} *as given by* $K^{\pm} \to \pi^{\pm}\gamma^{*}$ *(blue band) and* $K_S \to \pi^0\gamma^*$ *(violet band). Right panel: values for* N_{16} *and* N_{17} *extracted from* $K^{\pm} \to \pi^{\pm}\pi^0\gamma$ *(blue band) and* $K^{\pm} \to \pi^{\pm}\pi^0 e^+e^-$ *(yellow band) measurements. The latter is an educated estimate (see main text).*

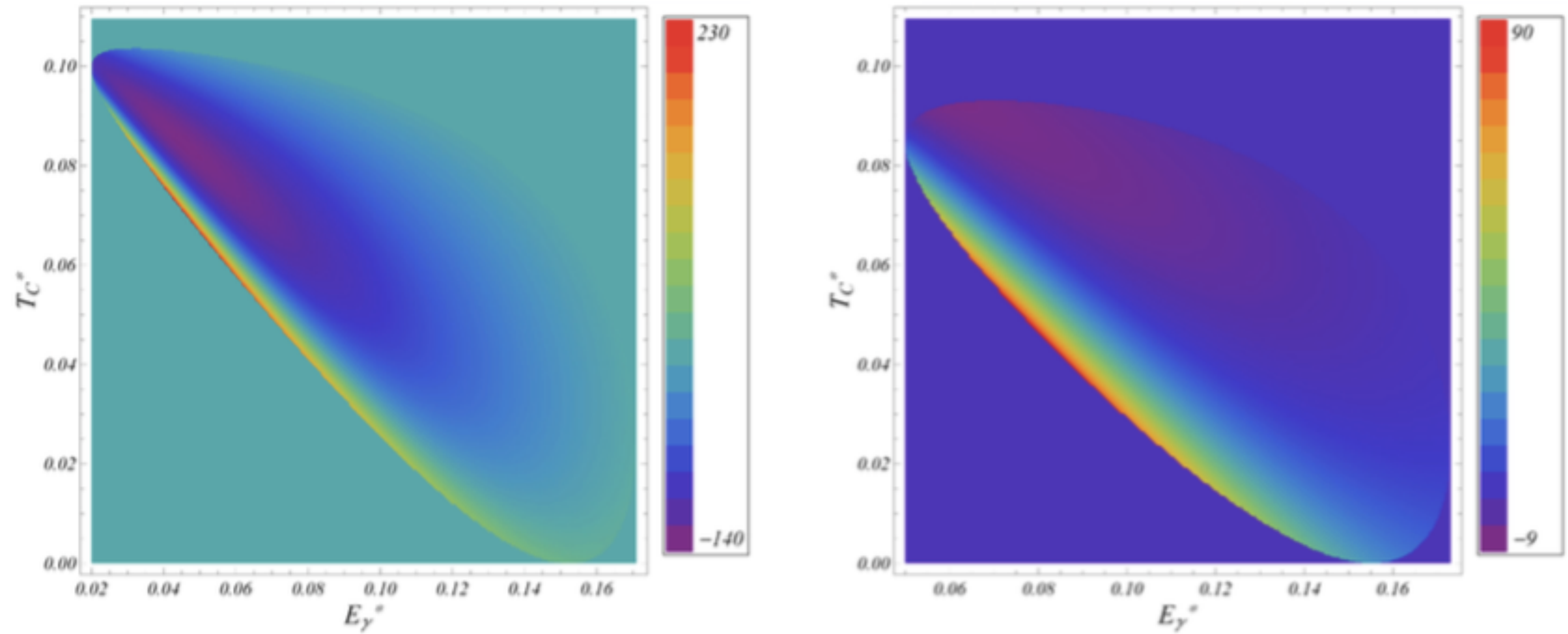


Figure 1: *Dalitz plots for the interference differential decay rate in the* (E_γ, T_c) *plane for* $q = 20$ *MeV (left panel) and* $q = 50$ *MeV (right panel). Numbers are given in units of* 10^{-20} GeV^{-1}. *The contour plot is 'spikier' the lower the* q *values are, a pattern mostly dictated by the structure of the Bremsstrahlung term.*

$$BR(K_S \to \pi^+\pi^-e^+e^-) = \underbrace{4.74 \cdot 10^{-5}}_{\text{Brems.}} + \underbrace{4.39 \cdot 10^{-8}}_{\text{Int.}} + \underbrace{1.33 \cdot 10^{-10}}_{\text{DE}}$$

q_c (MeV)	$10^8 \times \Gamma_{\mathcal{B}}$	$\left[\frac{\Gamma_{\mathcal{E}}}{\Gamma_{\mathcal{B}}}\right]^{-1}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]^{-1}_{(1,1,1)}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]^{-1}_{(1,0,1)}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]^{-1}_{(1,1,0)}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]^{-1}_{(0,1,1)}$
$2m_l$	418.27	1100	-253	-225	-115	216
2	307.96	821	-265	-226	-98	159
4	194.74	529	-363	-264	-78	101
8	109.60	304	1587	-850	-59	58
15	56.12	161	102	156	-43	31
35	15.50	50	18	21	-26	11
55	5.62	22	7	9	-18	5
85	1.37	8	3	4	-13	3
100	0.67	5	2	3	-11	2
120	0.24	3	1.6	2	-10	1.4
140	0.04	2	1.0	1.1	-8	0.9
180	0.003	1	0.7	0.8	-7	0.7

Table 2: *Branching ratios for the Bremsstrahlung and the relative weight of the electric and electric interference terms for different cuts in q, starting at q_{min} (first row) and ending at* 180 *MeV. To highlight the role of the different counterterms, the last columns show how the interference term changes when they are switched off one at a time.*

QCD at work: Short Distance expansion for weak interaction

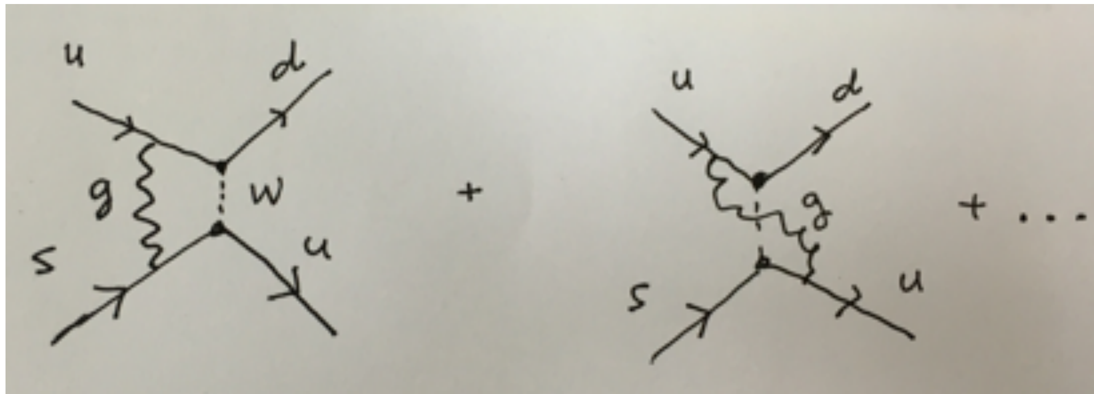
- Fermi lagrangian: description of the ΔS=1 weak lagrangian, in particular the explanation of ΔI =1/2 rule

$$\frac{A(K^+ \to \pi^+\pi^0)}{A(K_S \to \pi^+\pi^-)} \sim \frac{1}{22}$$

- Wilson suggestion (Feynman) , short distance expansion

$$-\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* C_- (\overline{s}_L \gamma^\mu u_L)(\overline{u}_L \gamma_\mu d_L)$$

- Gaillard Lee, Altarelli Maiani: right direction but not fully understood (Long distance?)



QCD at work, theoretical tools

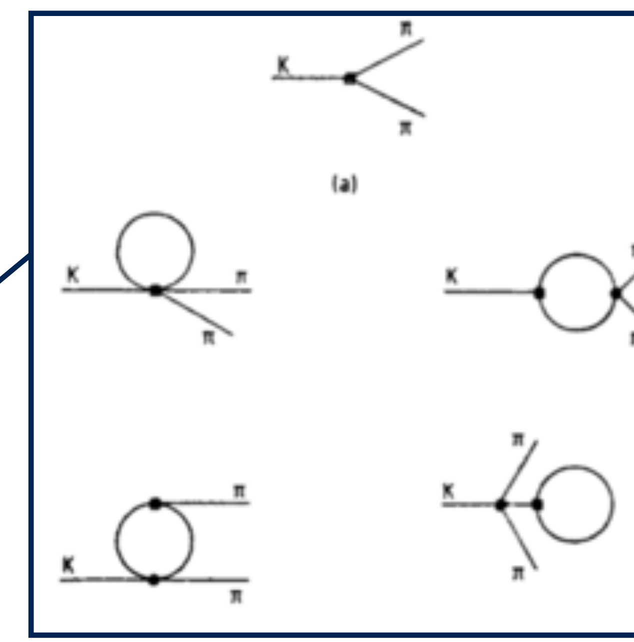
- analytic calculation 't Hooft, large Nc (it explains basic phenomenological facts of QCD, i.e. Zweig's rule) many implications: Skyrme model, VMD, Maldacena
- G. Parisi, `80s lattice: can we predict from QCD the proton mass at 10% level?
- Precise calculation of low energy QCD?

Bardeen Buras Gerard approach to K->ππ

Also evaluated ΔS=2 transitions, epsilon′ (Buras) and $\pi^+ - \pi^0$ mass diff.

Main idea: phys. amplitudes scale independent

Match SD with LD with a precise prescription for CT



CHPT+Large Nc

$$H_{\text{eff}} = \sum_i C_i(\mu)\ Q_i(\mu)$$

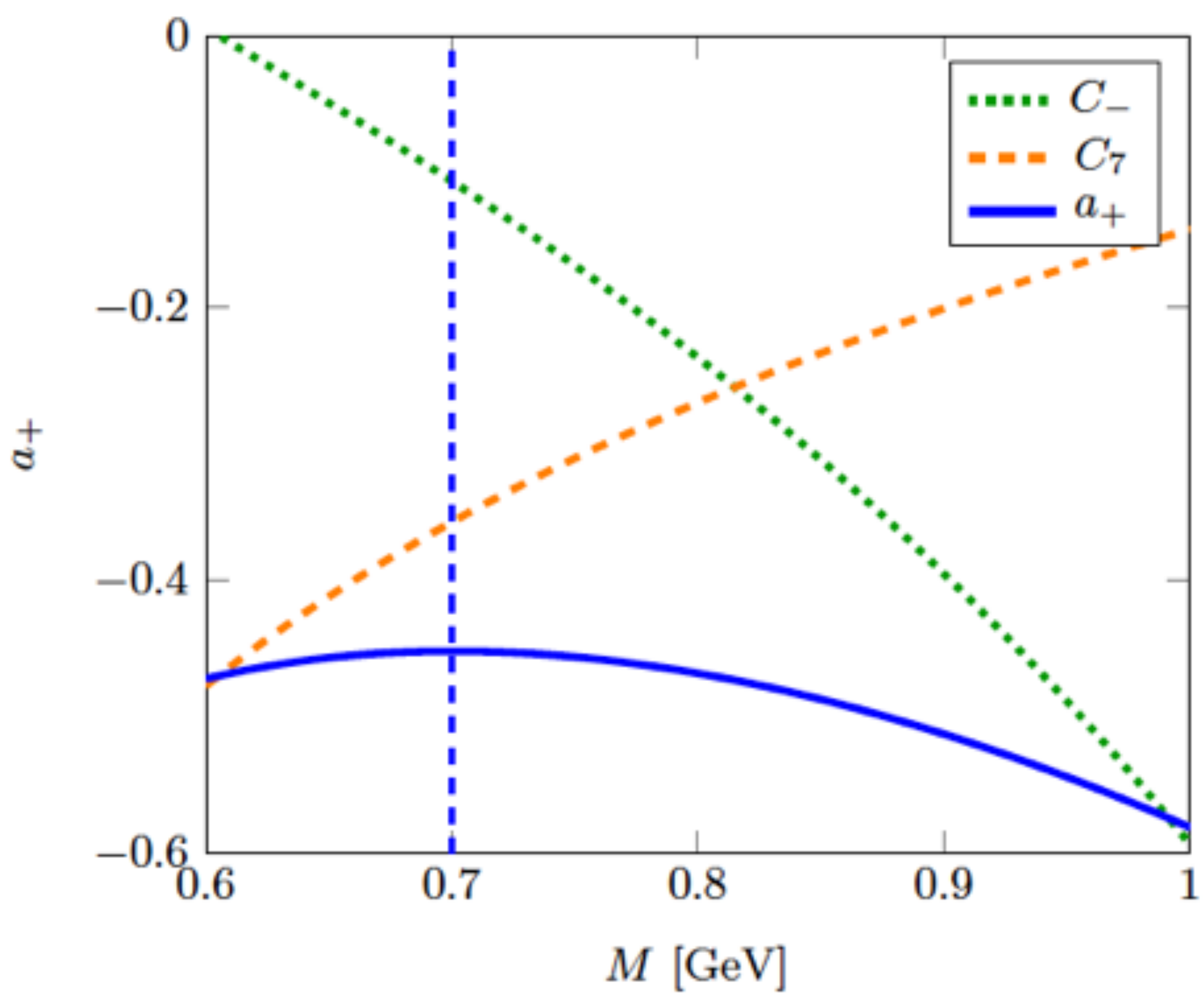
SD

Can we test somewhereelse the Bardeen Buras Gerard (BBG) approach?

Coluccio-Leskow, Estefania, GD, Greynat,David and Nath,Atanu

Matching a la BBG for $K^+ \to \pi^+ e^+ e^-$

Coluccio-Leskow,E. G.D ,Greynat, D and Nath, A



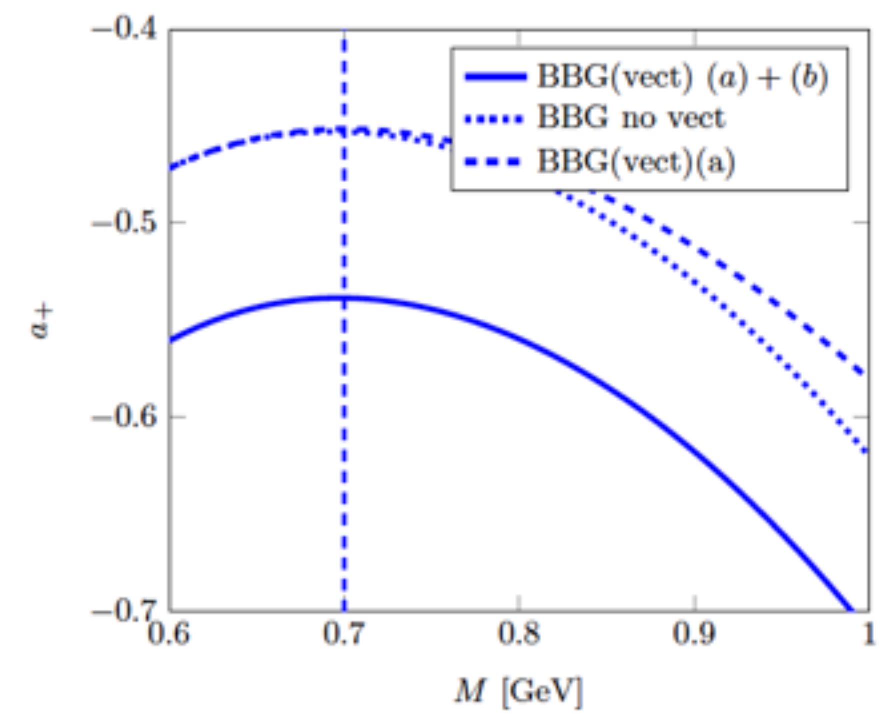
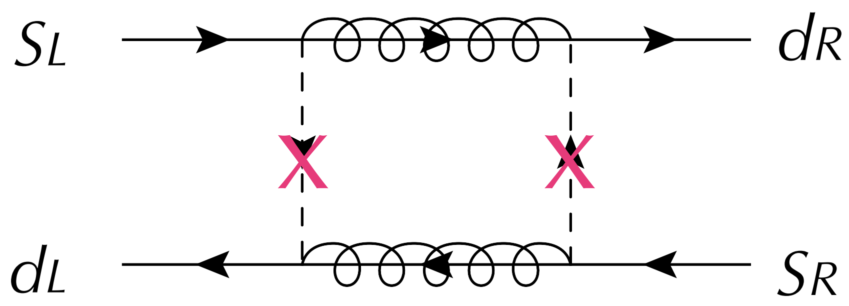


FIG. 5. a_+ as a function of M in the three different frameworks:'BBG no vect.' where vectors are not included, 'BBG(vect)(a)' represents the contribution coming only from diagrams (a) in Fig. 4 and 'BBG(vect) $(a)+(b)$' is the case where both (a) and (b) diagrams were included. The vertical line indicates the value $M = 0.7$ GeV.

Main Constraint: ϵ_K (ΔS=2, ID-CPV) cont.

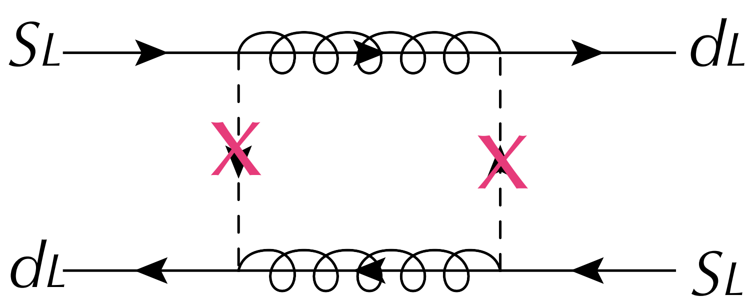
- The leading contribution is given by $\overline{d_L} s_L \overline{d_R} s_R$

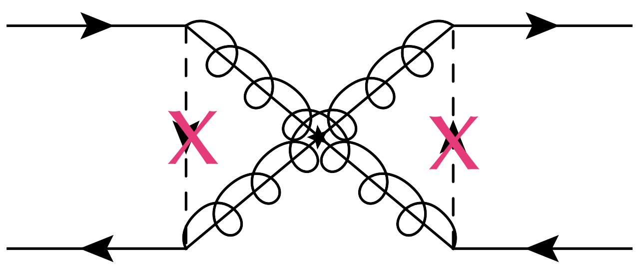


$$\propto \left(\frac{m_K}{m_s + m_d}\right)^2$$

this contribution is suppressed when $\Delta_{\bar{D},12} \simeq 0$

- The next contribution is given by $\overline{d_L} s_L \overline{d_L} s_L$





Crossed diagram gives **relatively negative** contributions

$m_{\tilde{g}} \simeq 1.5 m_{\tilde{q}}$: these contributions almost cancel out [Crivellin, Davidkov '10]

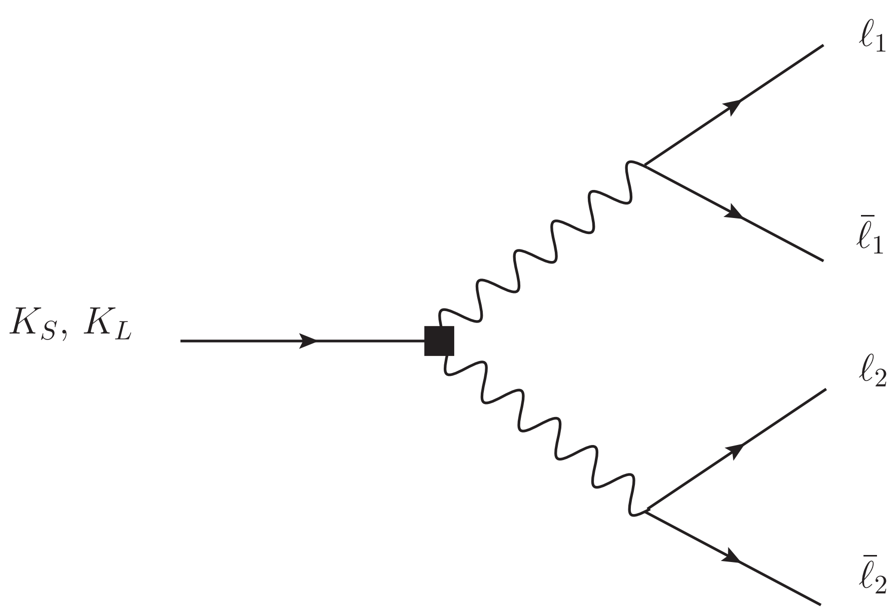
$m_{\tilde{g}} \gtrsim 1.5\ m_{\tilde{q}}$: suppressed by heavy gluing mass

Other interesting channels

$$K_S \to \mu\mu\mu\mu \quad - \quad \text{SM LD} \ \sim 2 \times 10^{-14}$$

$$K_S \to ee\mu\mu \quad - \quad \sim 10^{-11}$$

$$K_S \to eeee \quad - \quad \sim 10^{-10}$$



GD,Greynat, Vulvert