Recent achievements in developing ab initio optical potentials for nuclear reactions

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- Paolo Finelli
- Petr Navrátil
- Vittorio Somà

Outline

- Motivations
- The nucleon-nucleus optical potential within the multiple scattering theory
- Application to light nuclei with NCSM densities
- Application to medium-mass nuclei with SCGF densities
- Application to inelastic scattering & future challenges
- The nucleus-nucleus optical potential within the multiple scattering theory
- Summary & outlook

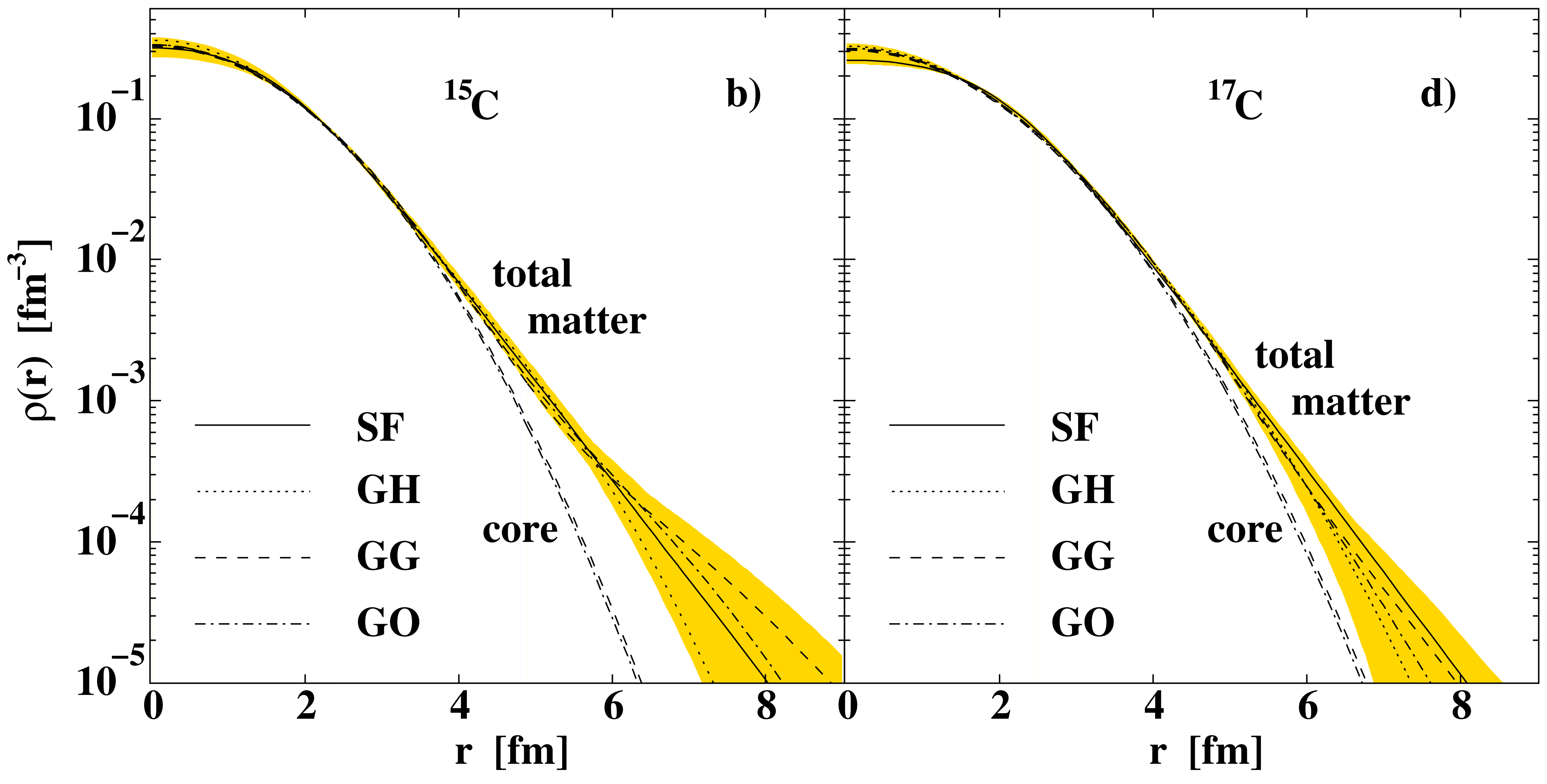
Motivations

- Increasing experimental efforts to develop the technologies necessary to study the elastic proton scattering in inverse kinematics
- Attempts to use such experiments to determine the matter distribution of nuclear systems at intermediate energies

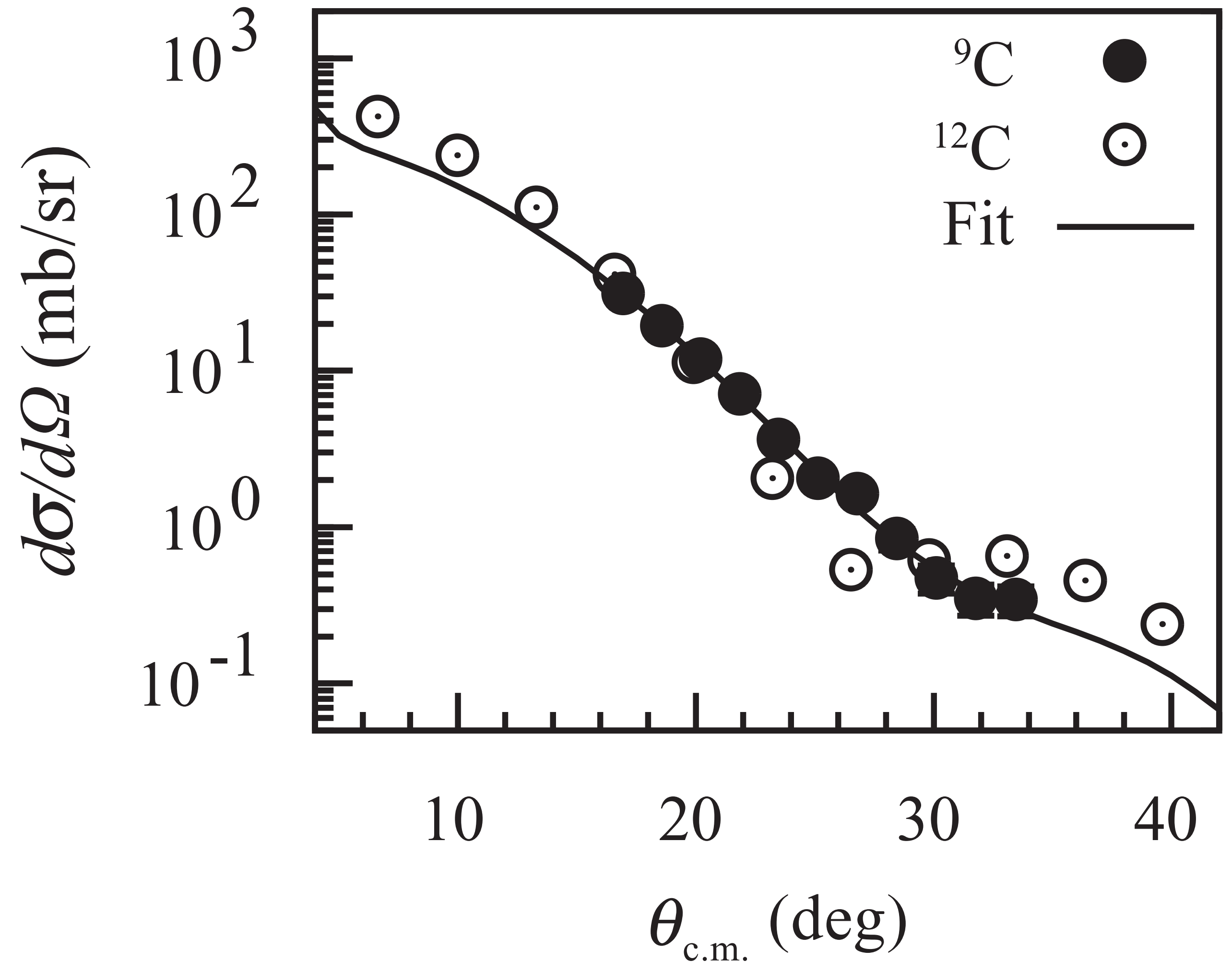
[Sakaguchi, Zenihiro, PPNP 97 (2017) 1–52]

- Measurements are not free from sizeable uncertainties
- The Glauber model is used to analyse the data
- An essential step in the data analysis is the subtraction of contributions from the inelastic scattering

Develop a microscopic approach to make reliable predictions for elastic and inelastic scattering



[Dobrovolsky et al., NPA 1008 (2021) 122154]



[Matsuda et al., PRC **87**, 034614 (2013)]

Optical potential

Phenomenological

Unfortunately, current used optical potentials for low-energy reactions are phenomenological and primarily constrained by elastic scattering data.

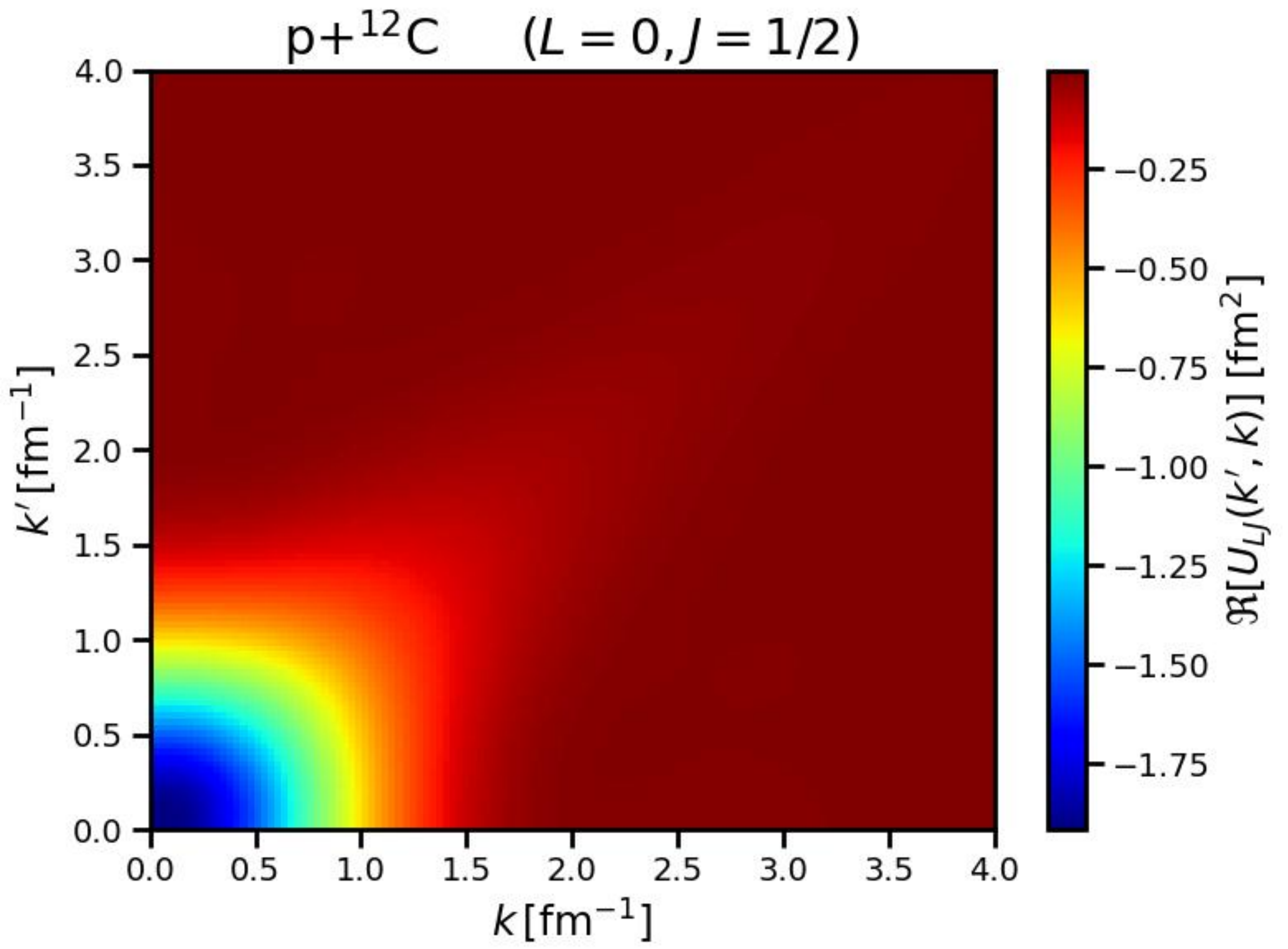
Unreliable when extrapolated beyond their fitted range in energy and nuclei

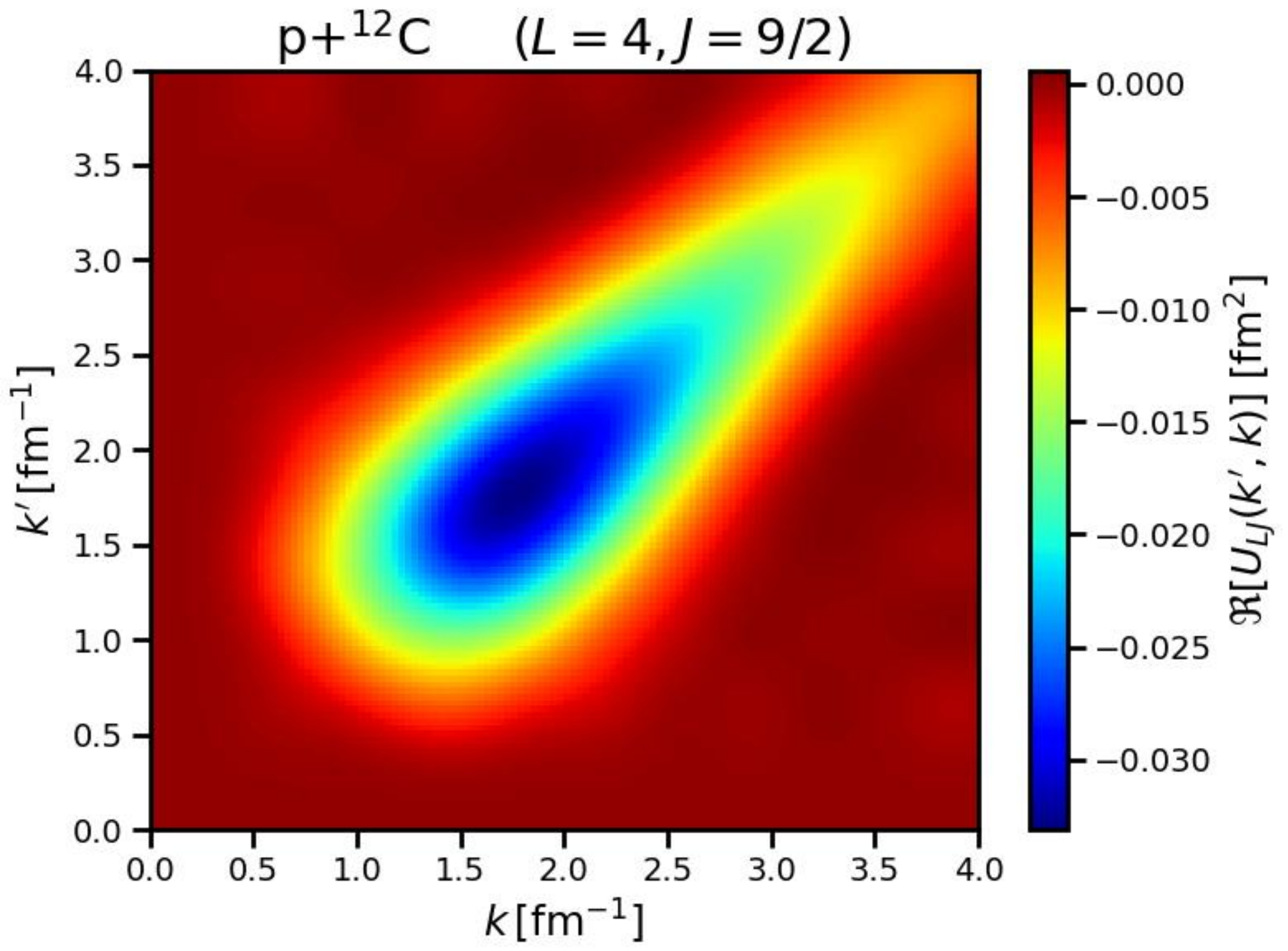
$$
\begin{aligned}
V(r) &= -V_R f_R(r) - iW_V f_V(r) \\
&+ 4a_{VD} V_D \frac{d}{dr} f_{VD}(r) + 4ia_{WD} \frac{d}{dr} f_{WD}(r) \\
&+ \frac{\lambda_\pi^2}{r} \left[V_{SO} \frac{d}{dr} f_{VSO}(r) + iW_{SO} \frac{d}{dr} f_{WSO}(r) \right] \vec{\sigma} \cdot \vec{l}
\end{aligned}
$$

Microscopic

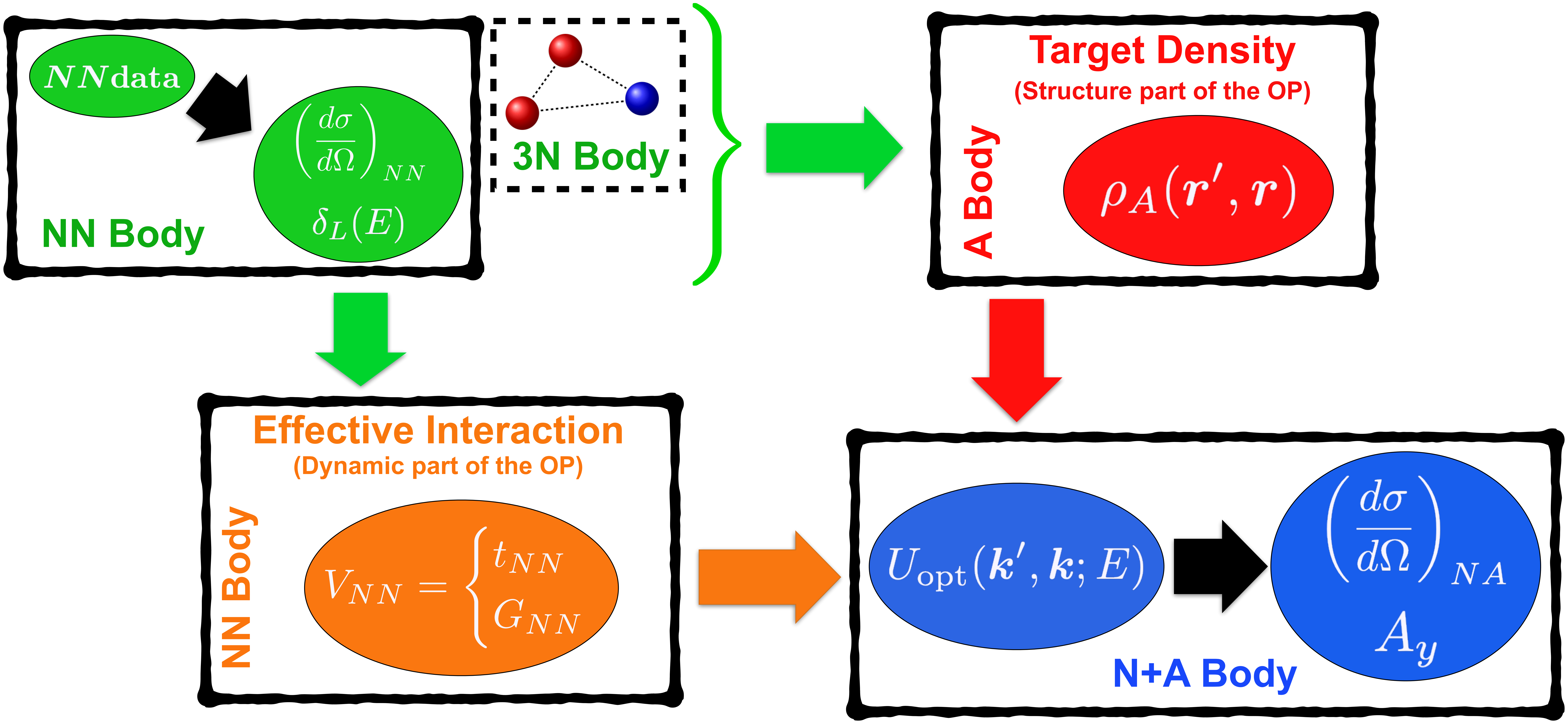
Existing microscopic optical potentials can be developed in a low- (Feshbach theory) or high-energy regime (Watson multiple scattering theory). Calculations are more difficult.

No fit to experimental data

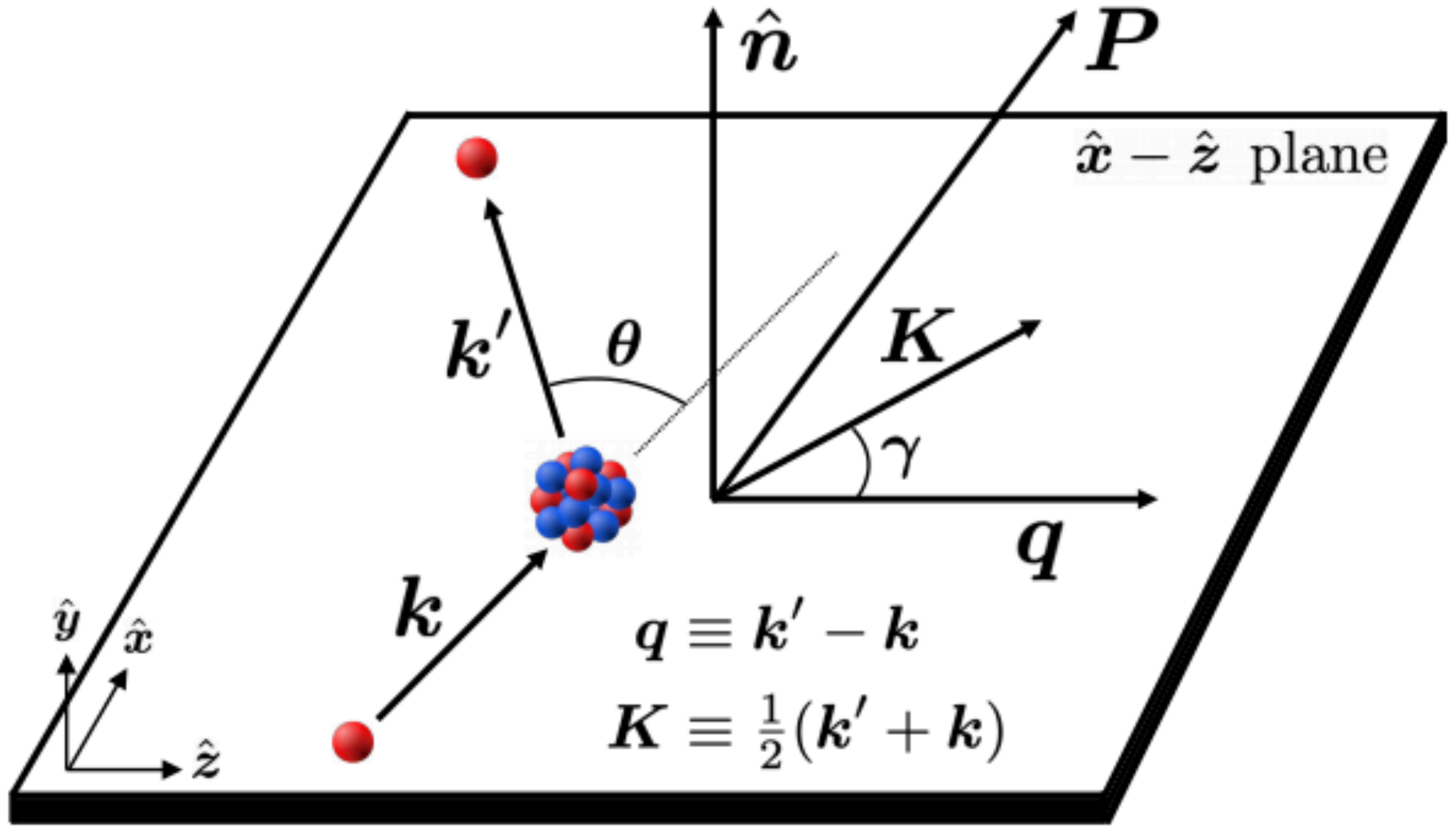




Road map to the optical potential



The first-order optical potential



Møller factor

$$t^{(NA)}_{\mathbf{p}N} = \eta\, t^{(NN)}_{\mathbf{p}N}$$

It imposes the Lorentz invariance of flux when we pass from the NA to the NN frame where the t matrices are evaluated

$$U_{\mathbf{p}}(\boldsymbol{q},\boldsymbol{K}) = \sum_{N=p,n}\int d\boldsymbol{P}\,\eta(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P})\,t_{\mathbf{p}N}(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P})\,\rho_N(\boldsymbol{q},\boldsymbol{P})$$

$\mathbf{p} = (p, n, \bar{p})$

Free two-body scattering matrix

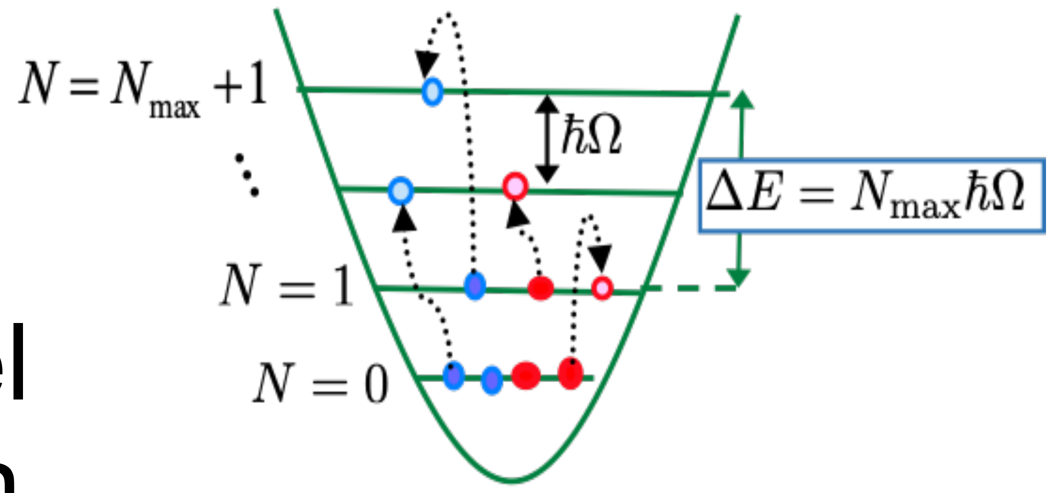
$$t_{0i} = v_{0i} + v_{0i}\, g_{0i}\, t_{0i}$$

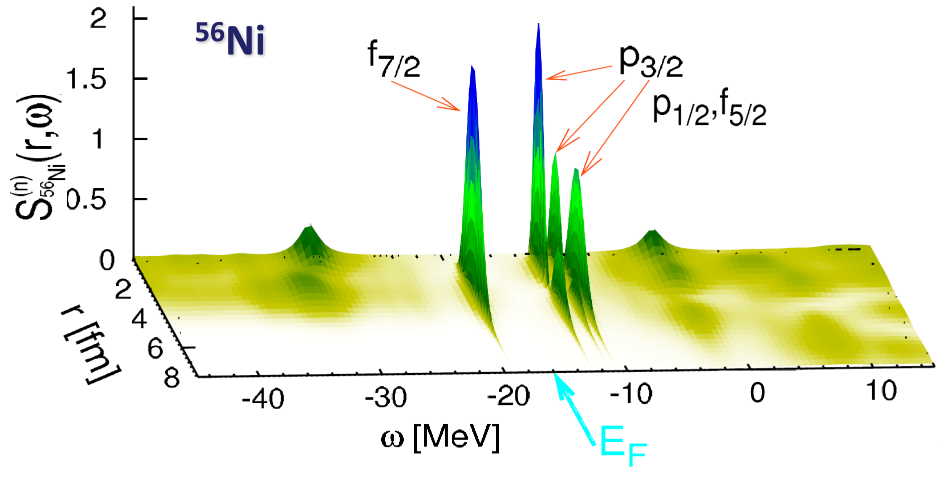
$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only **NN** interaction

Nonlocal one-body density

- Computationally expensive
- Obtained from the No-Core Shell Model or the Self-Consistent Green's Function
- Calculation performed with **NN** and **3N** interaction



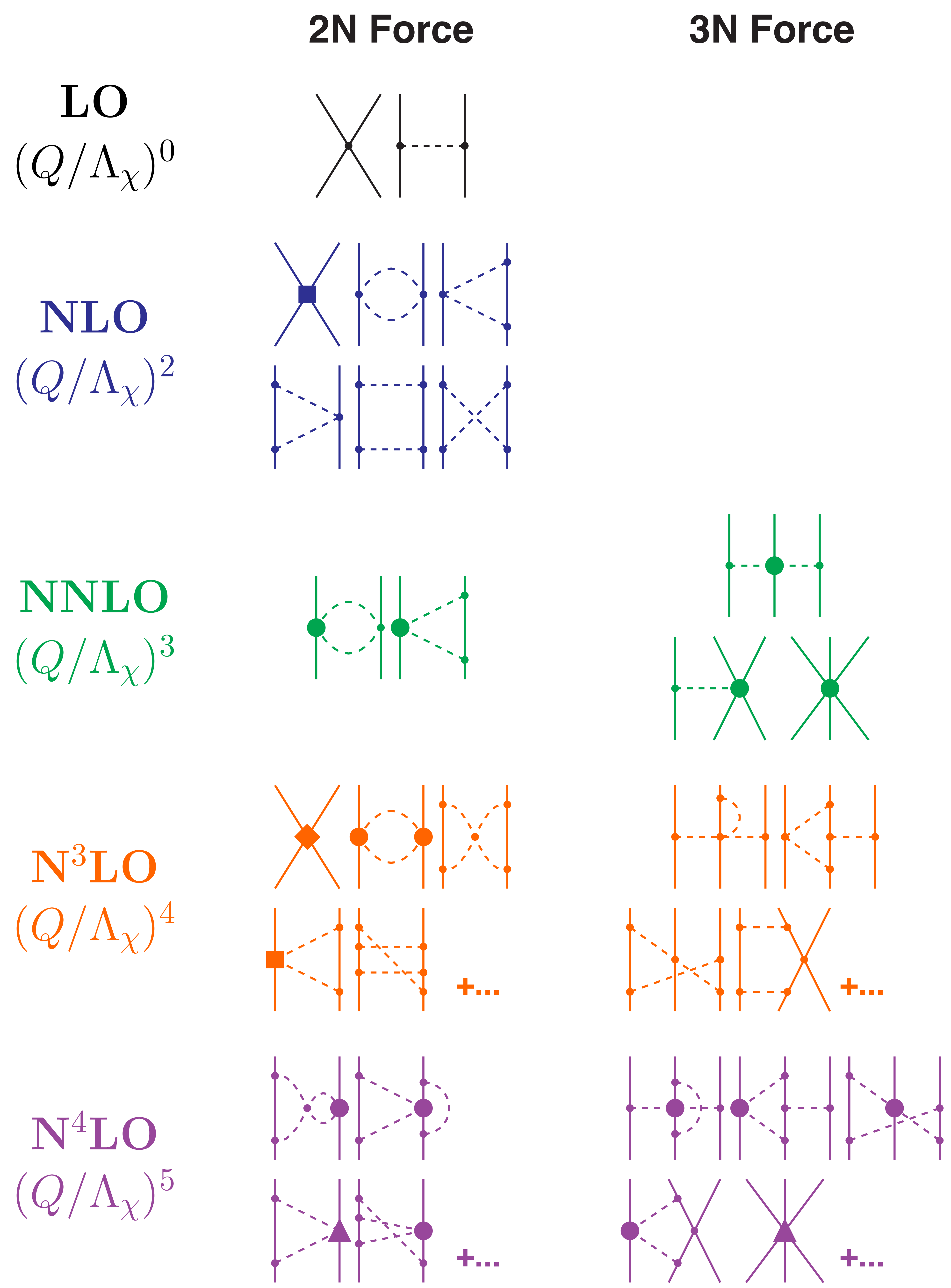


Chiral interactions

Advantages

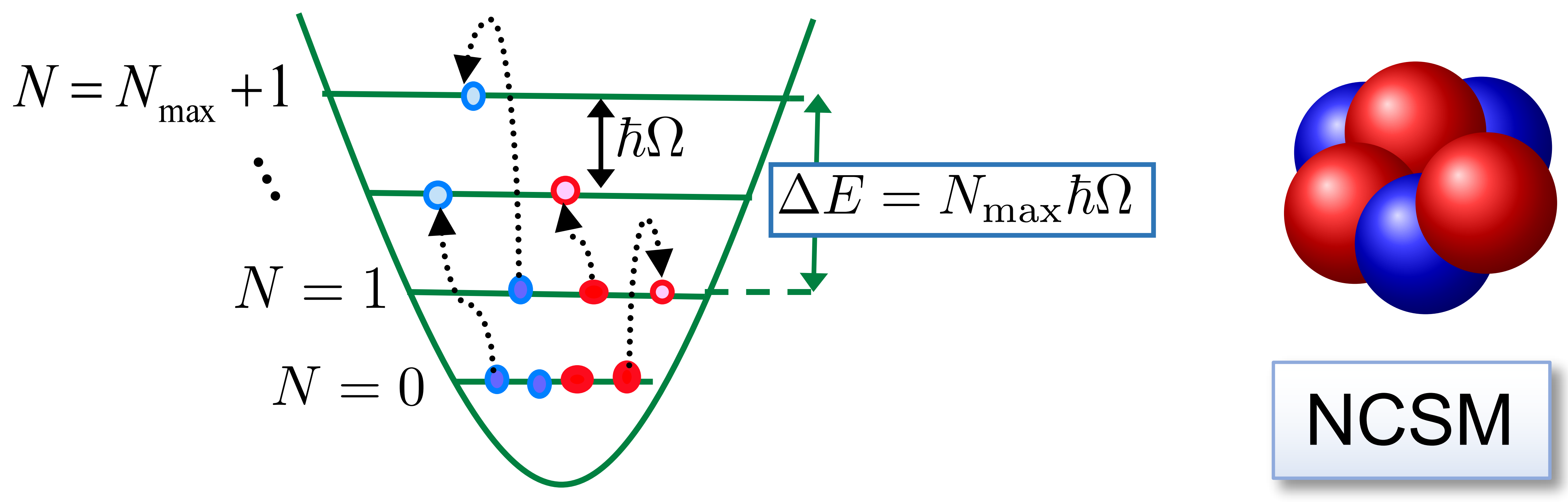
- QCD symmetries are consistently respected
- Systematic expansion (order by order we know exactly the terms to be included)
- Theoretical errors
- Two- and three-nucleon forces belong to the same framework

We use these interactions as the **only** input to calculate the **effective interaction** between projectile and target and the **target density**



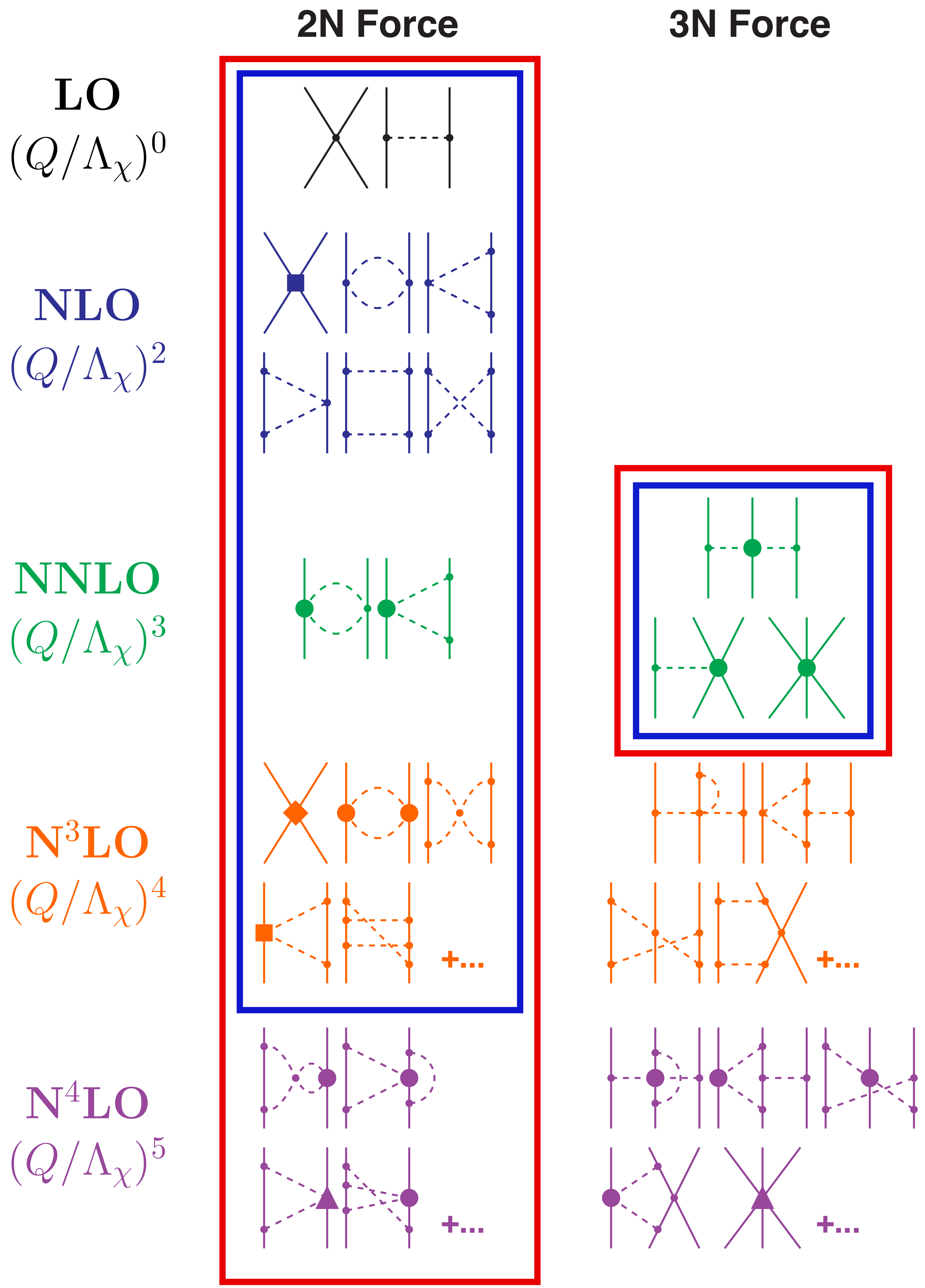
Chiral interactions for NCSM

No-Core Shell Model



In collaboration with P. Navrátil and M. Gennari (TRIUMF)

- **NN-N⁴LO + 3NInl (^{12}C, ^{16}O)**
  - N⁴LO: Entem et al., Phys. Rev. C **96**, 024004 (2017)
  - 3NInl: Navrátil, Few-Body Syst. **41**, 117 (2007)
  - c_D & c_E: Kravvaris et al., Phys. Rev. C **102**, 024616 (2020)
- **NN-N³LO + 3NInl (^{9,13}C, ^{6,7}Li, ^{10}B)**
  - N³LO: E&M, Phys. Rev. C **68**, 041001(R) (2003)
  - 3NInl: Navrátil, Few-Body Syst. **41**, 117 (2007)
  - c_D & c_E: Somà et al., Phys. Rev. C **101**, 014318 (2020)



Assessing the impact of the 3N interaction

General equation for the optical potential

$$U = (V_{NN} + V_{3N}) + (V_{NN} + V_{3N})G_0(E)QU$$

Treatment of the 3N force

[Holt et al., Phys. Rev. C **81**, 024002 (2010)]

$$V_{3N} = \frac{1}{2}\sum_{i=1}^{A}\sum_{\substack{j=1\\ j\neq i}}^{A} w_{0ij} \approx \sum_{i=1}^{A}\langle w_{0i}\rangle$$

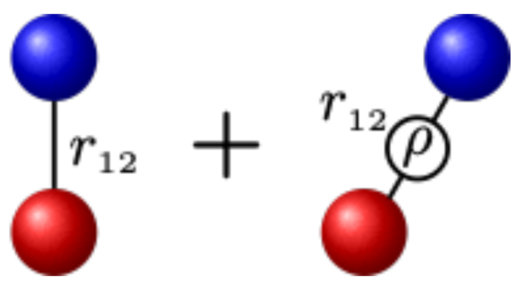
Density dependent

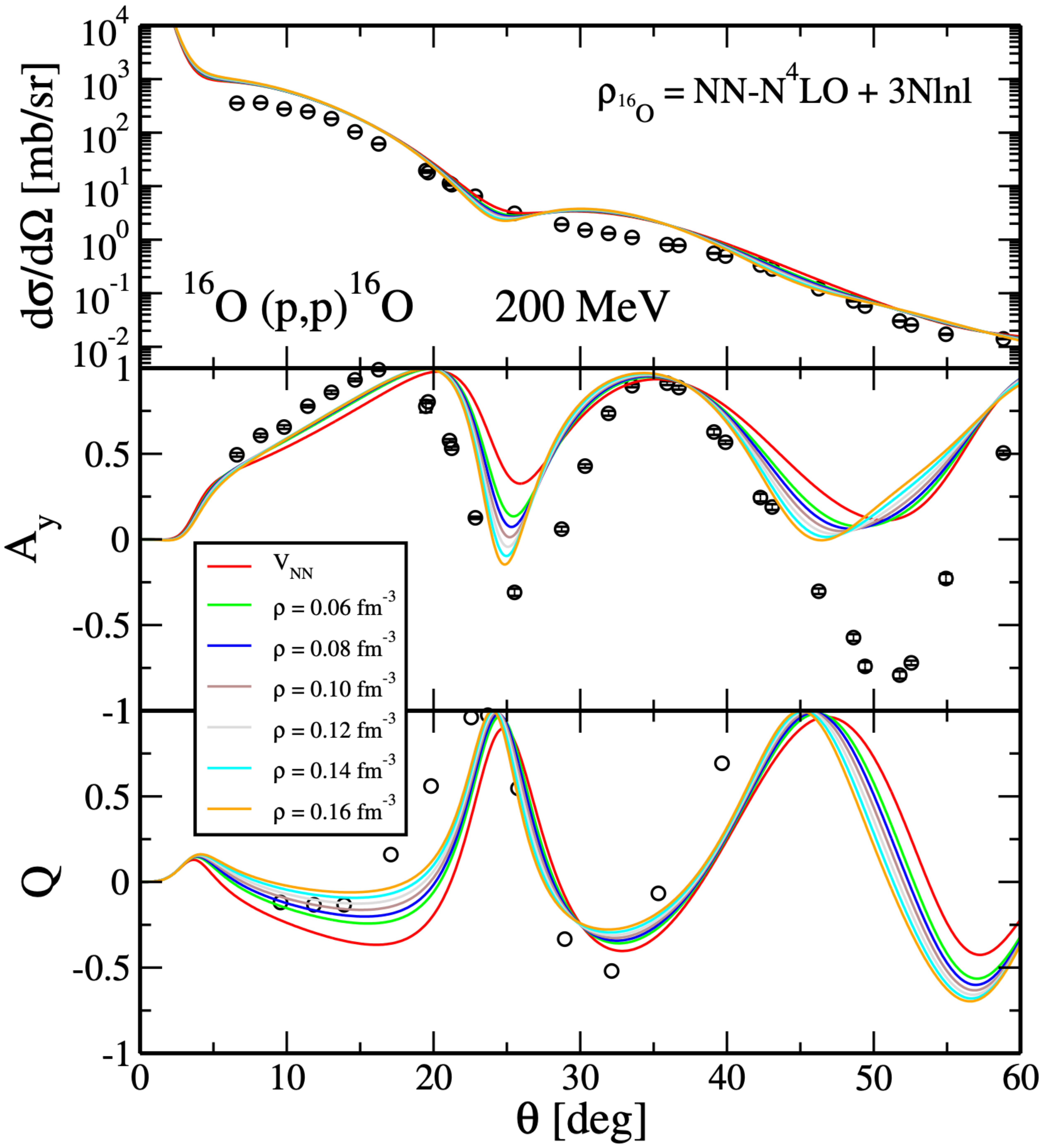
Modification of the *t* matrix

$$t_{0i} = v_{0i}^{(1)} + v_{0i}^{(2)} g_{0i}\, t_{0i}$$

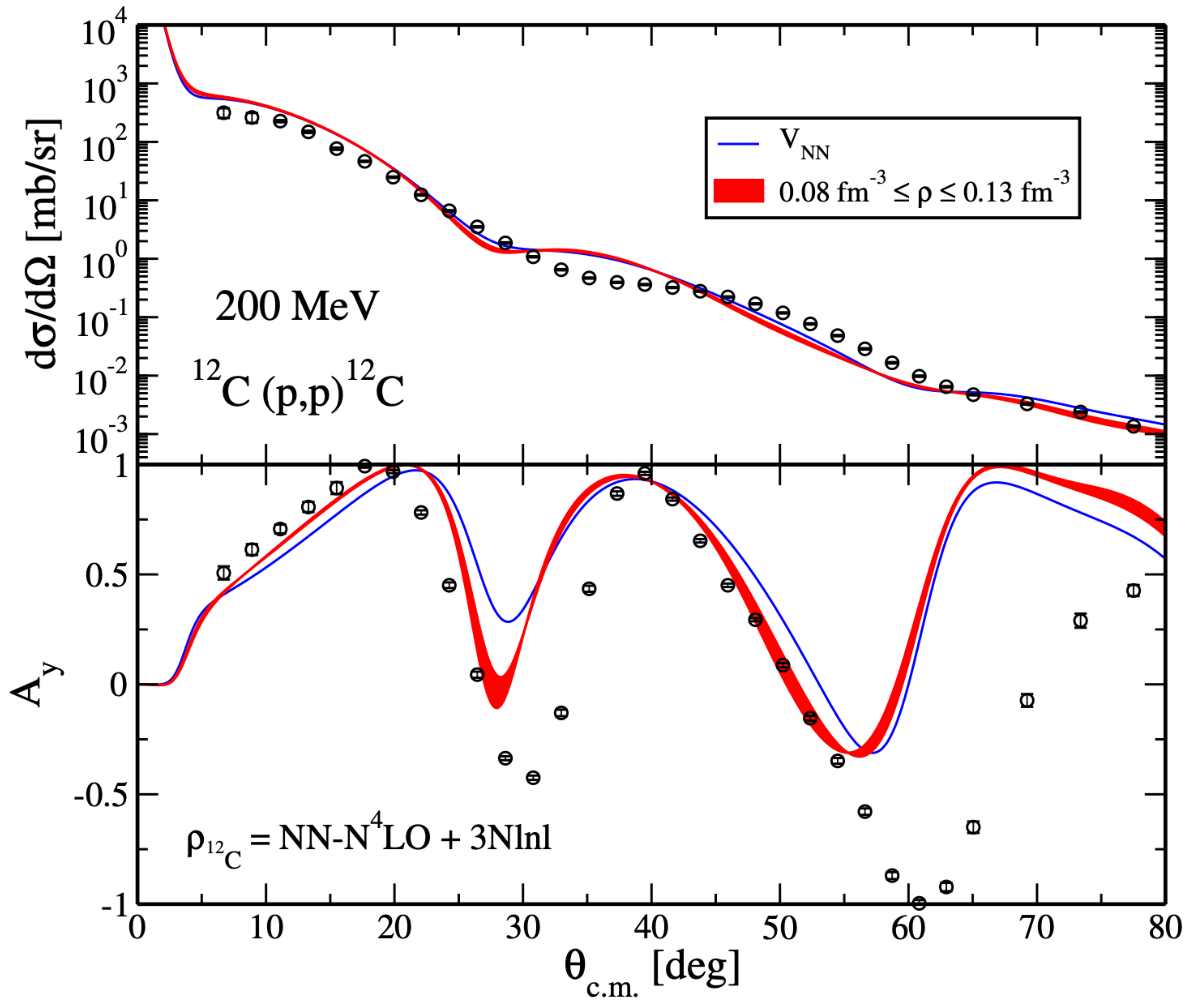
$$v_{0i}^{(1)} = v_{0i} + \tfrac{1}{2}\langle w_{0i}\rangle$$
$$v_{0i}^{(2)} = v_{0i} + \langle w_{0i}\rangle$$

$v =$

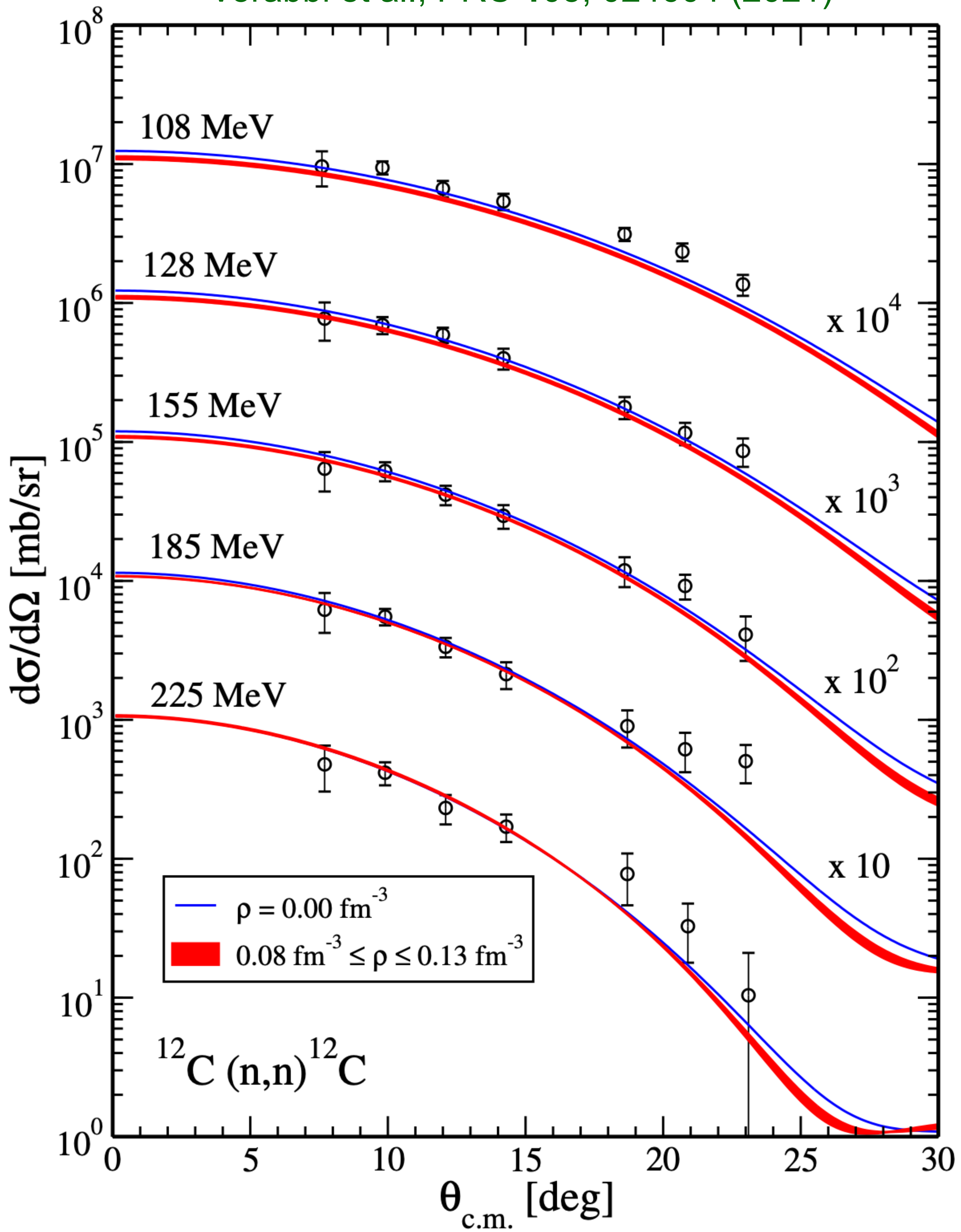




Assessing the impact of the 3N interaction



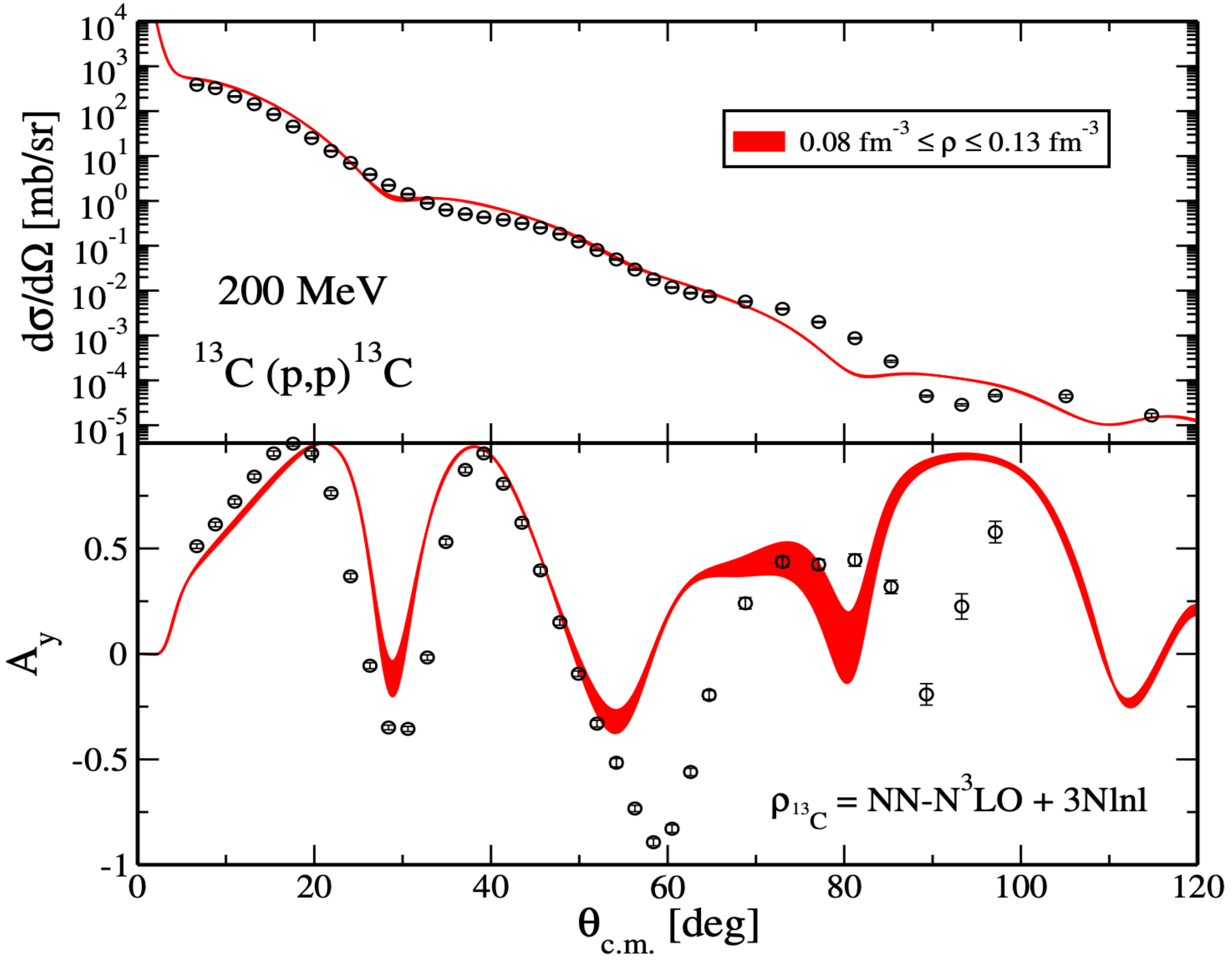
Vorabbi et al., PRC **103**, 024604 (2021)



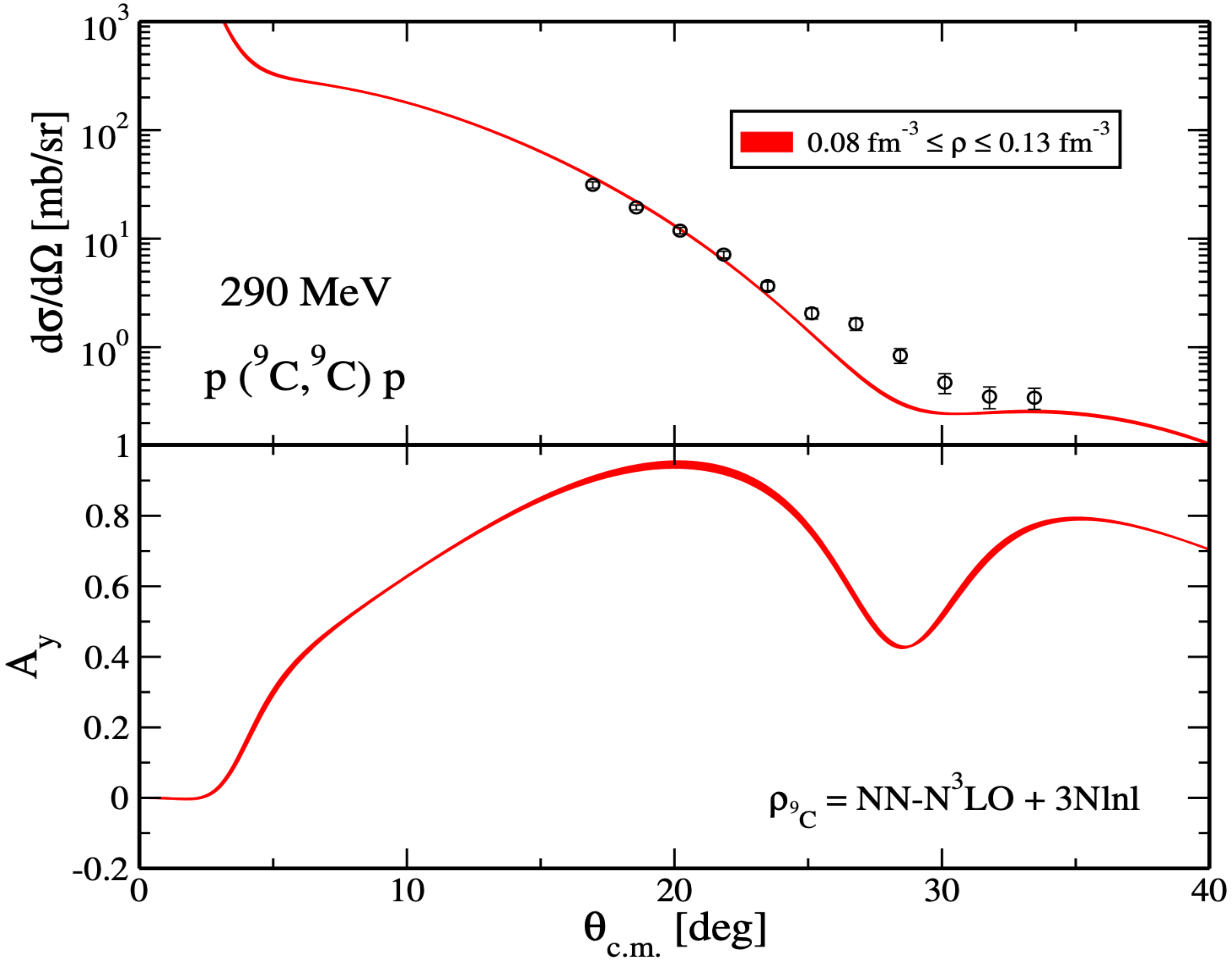
- For all nuclei we found very small contributions to the differential cross section
- The contributions to the spin observable are larger and they seem to improve the agreement with the data

Extension to non-zero spin targets

$J^\pi = 1/2^-$



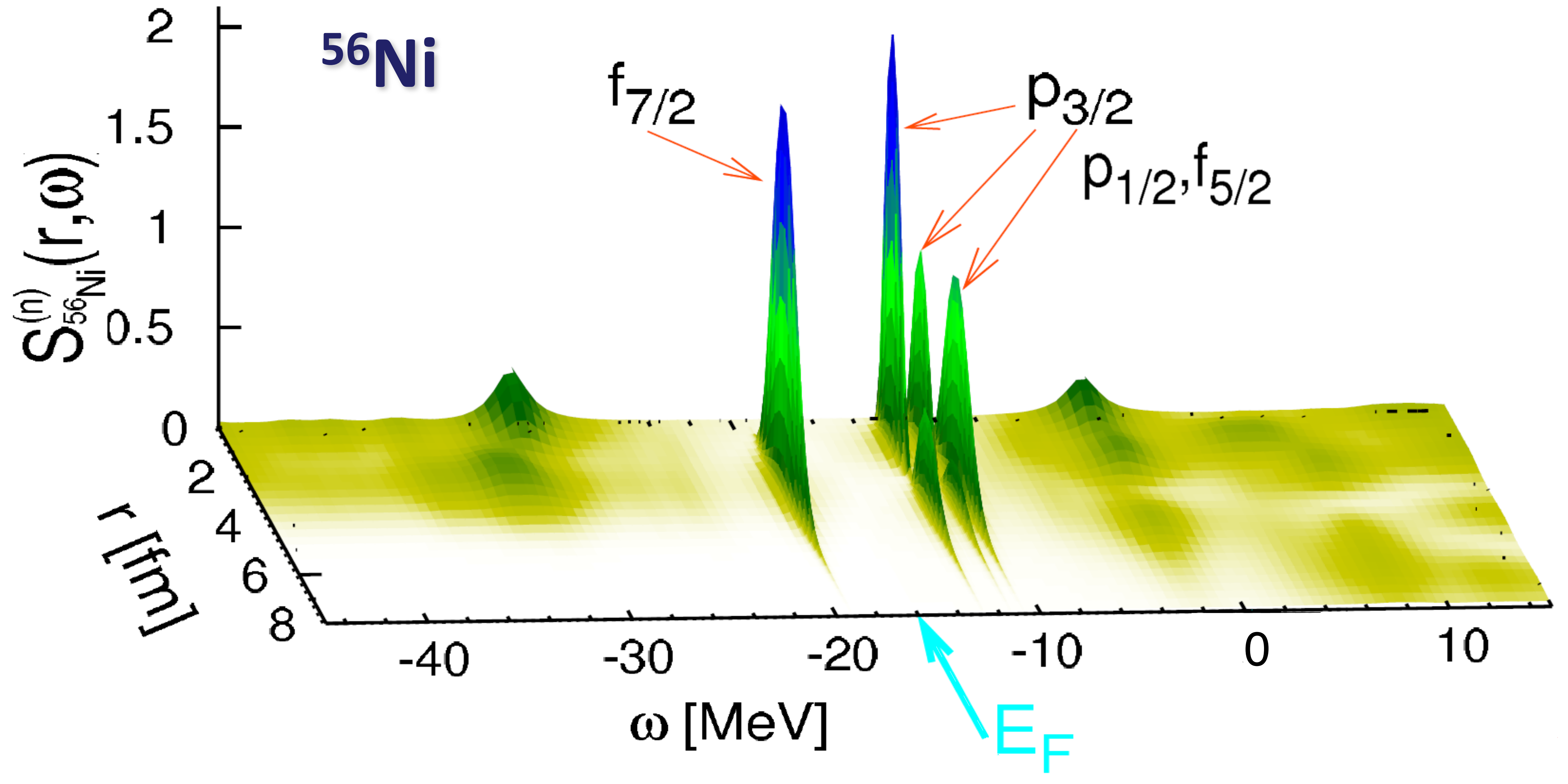
$J^\pi = 3/2^-$



[Vorabbi et al., Phys. Rev. C **105**, 014621 (2022)]

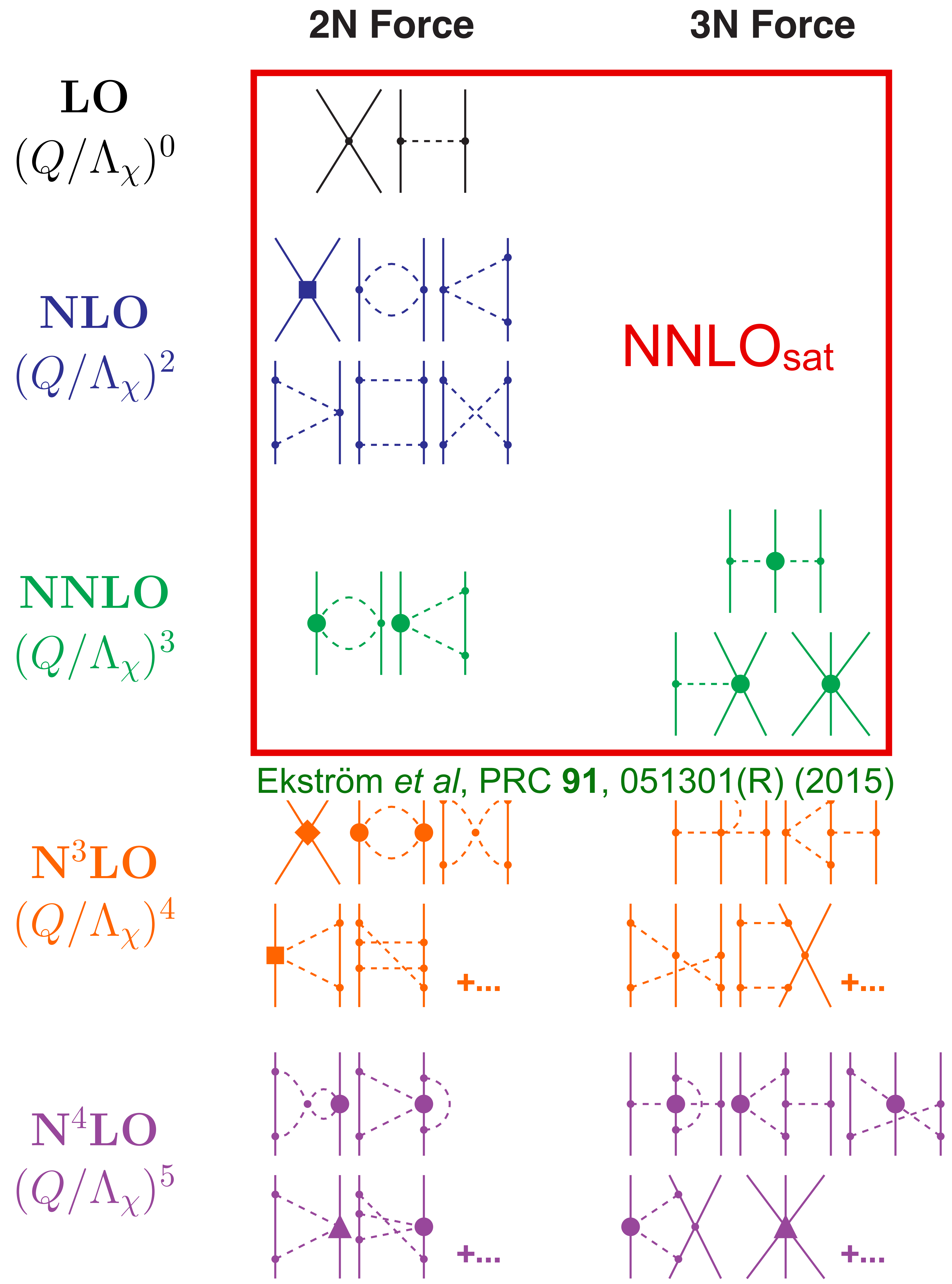
Chiral interactions for SCGF

Self Consistent Green's Function (SCGF)



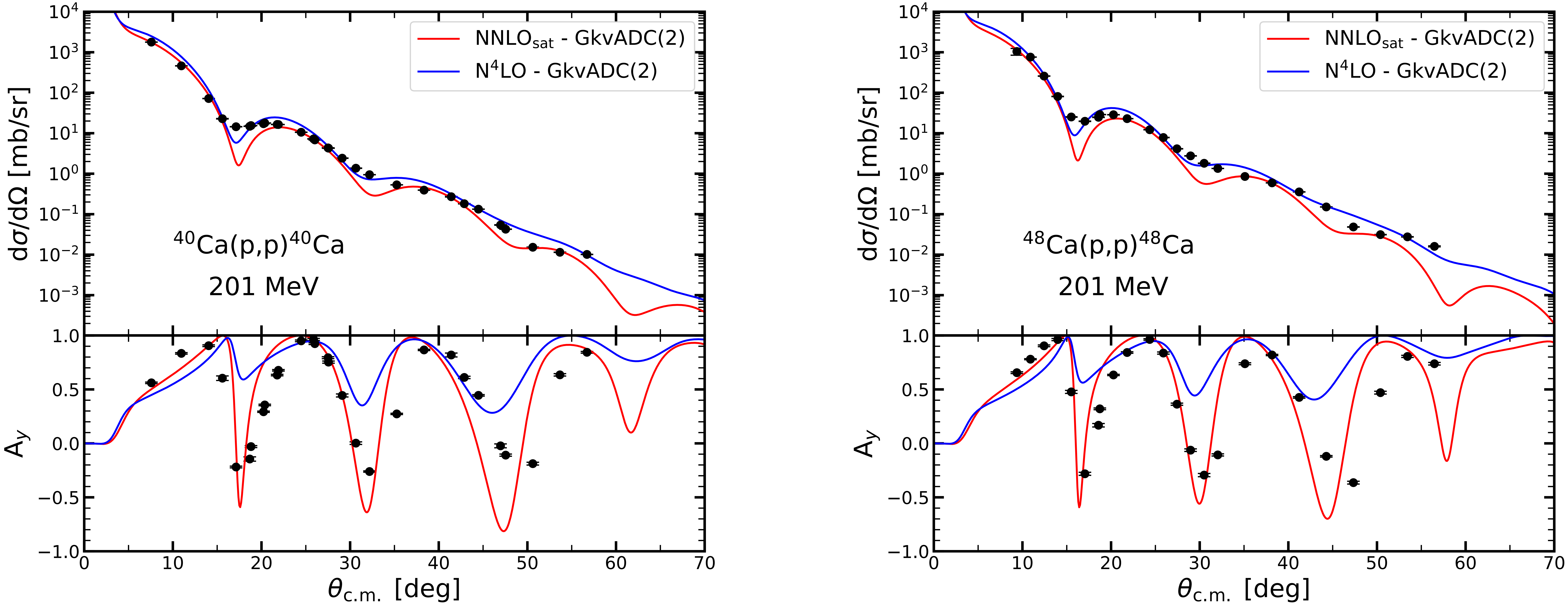
In collaboration with C. Barbieri (Milan) and V. Somà (Paris)
Somà, *SCGF Theory for Atomic Nuclei*, Frontiers **8** (2020) 340

$$U_{\mathbf{p}}(\boldsymbol{q},\boldsymbol{K}) = \sum_{N=p,n} \int d\boldsymbol{P}\, \eta(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P})\, t_{\mathbf{p}N}(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P})\, \rho_N(\boldsymbol{q},\boldsymbol{P})$$



Results for proton scattering off ^{40,48}Ca

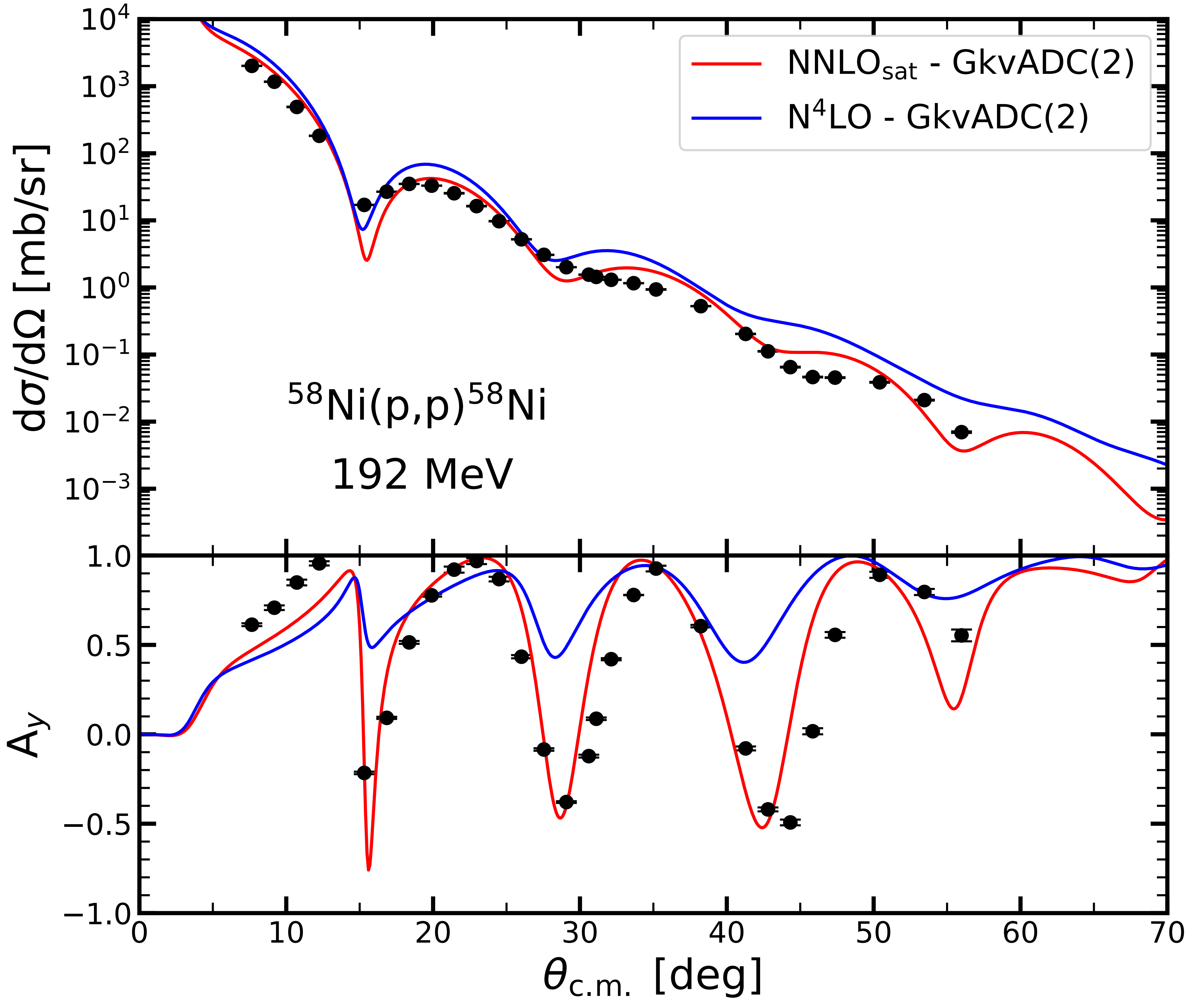
[Vorabbi et al., PRC **109**, 034613 (2024)]

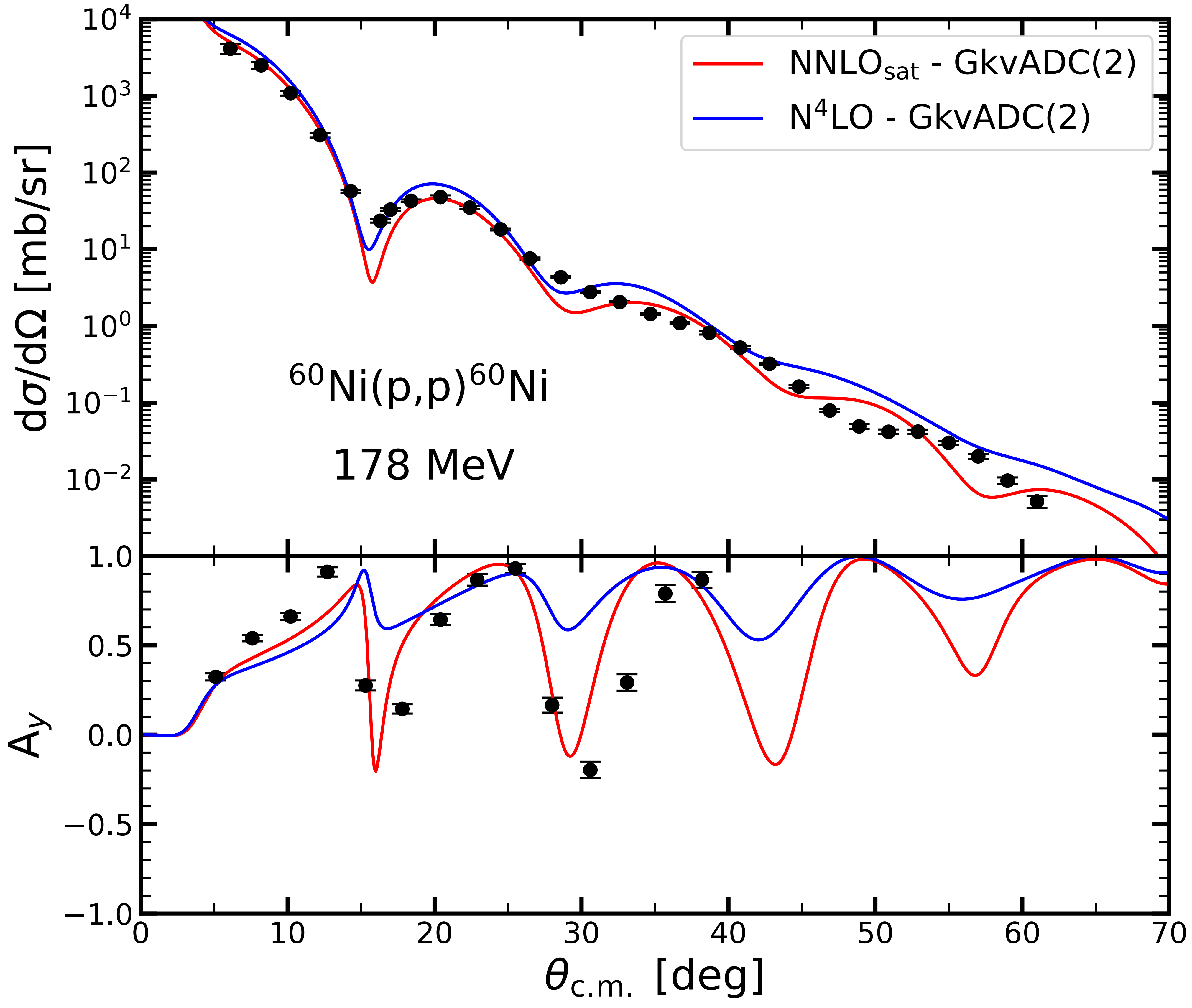


- First microscopic optical potential for calcium and nickel from *ab initio* densities
- For this comparison the densities are always computed with the $NNLO_{sat}$

Results for proton scattering off ^{58,60}Ni

[Vorabbi et al., PRC **109**, 034613 (2024)]

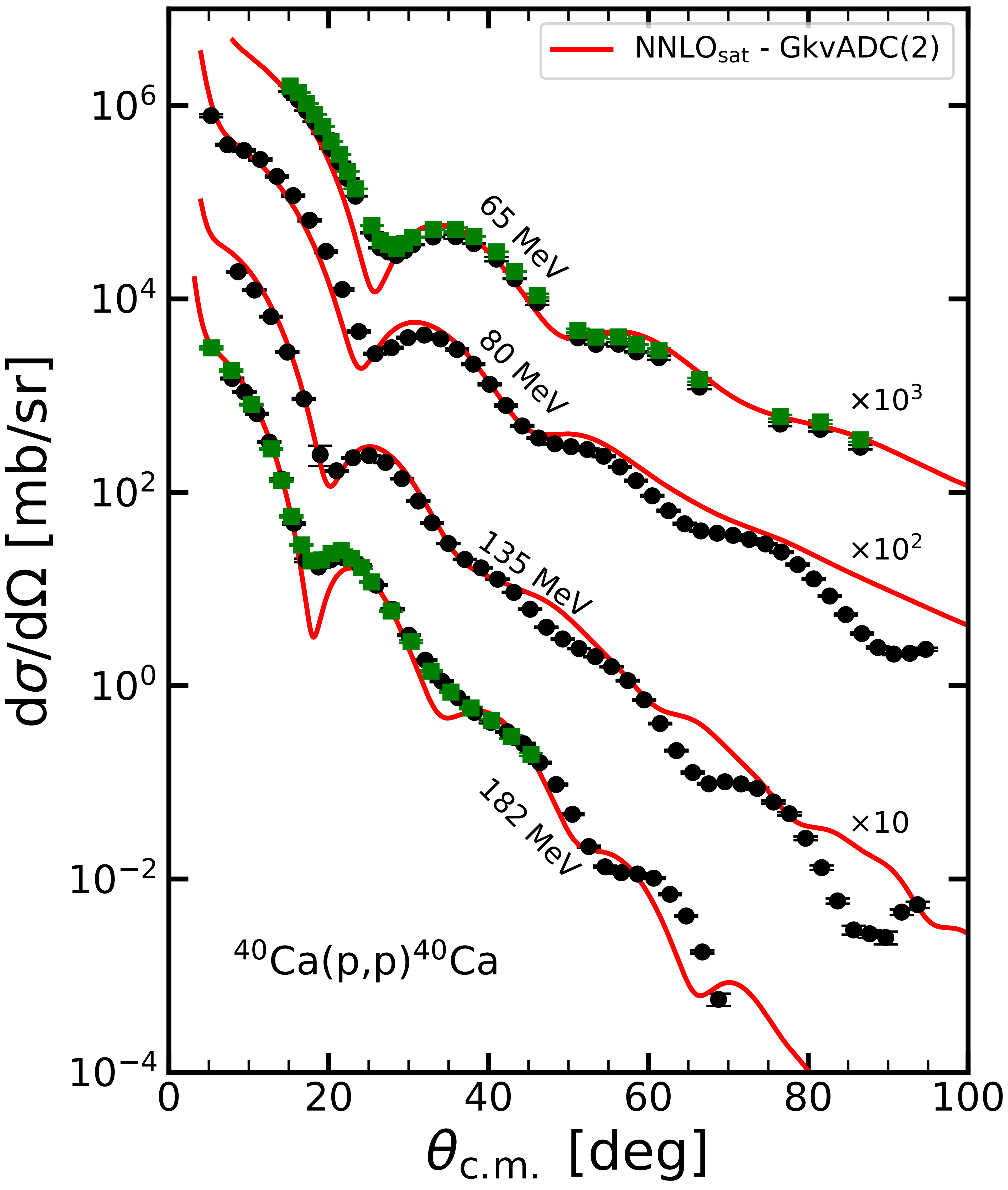




The data for the analysing power is remarkably well described!

(but remember that the NN potential does not reproduce the NN amplitudes)

Results for Calcium isotopic chain



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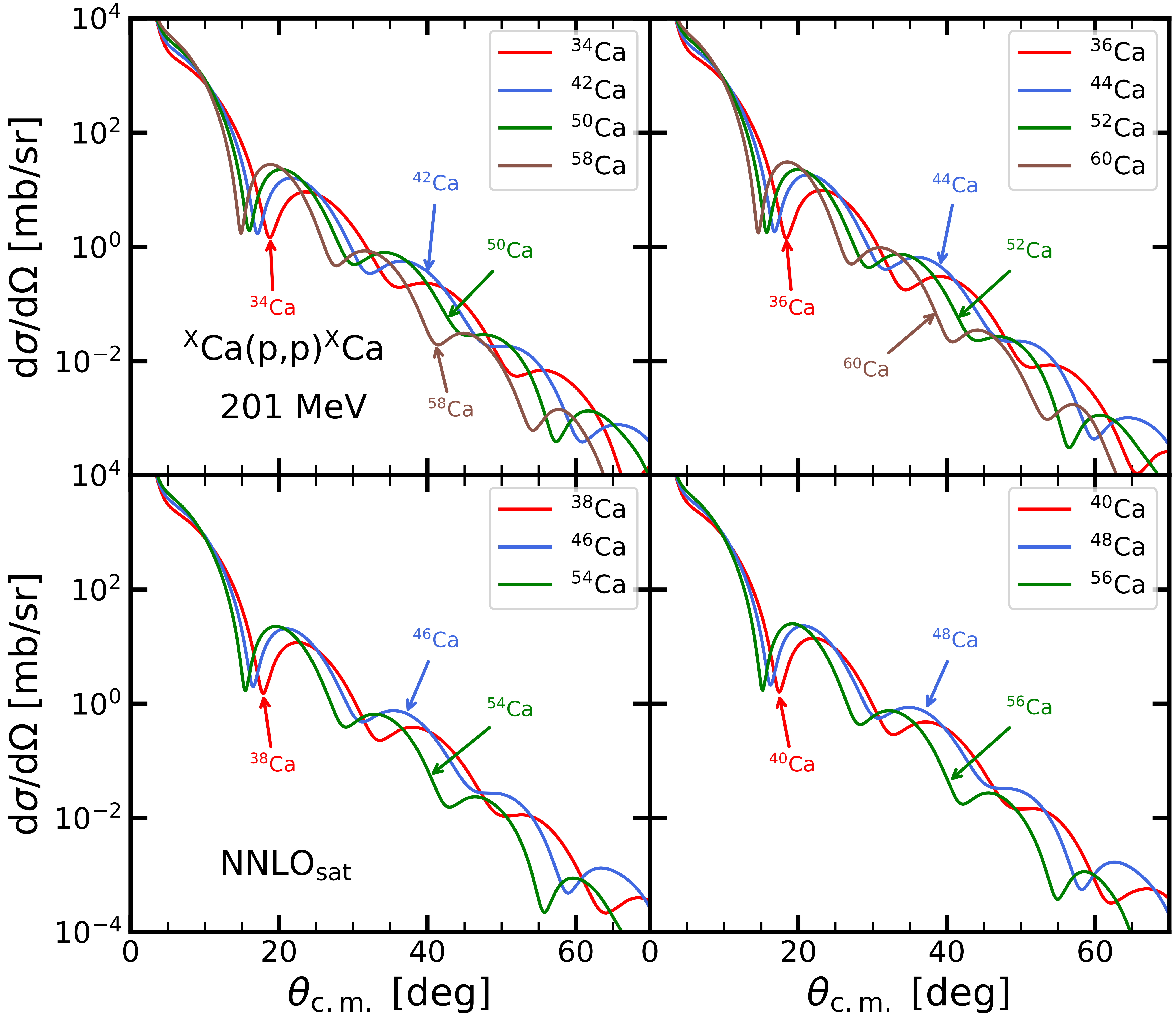


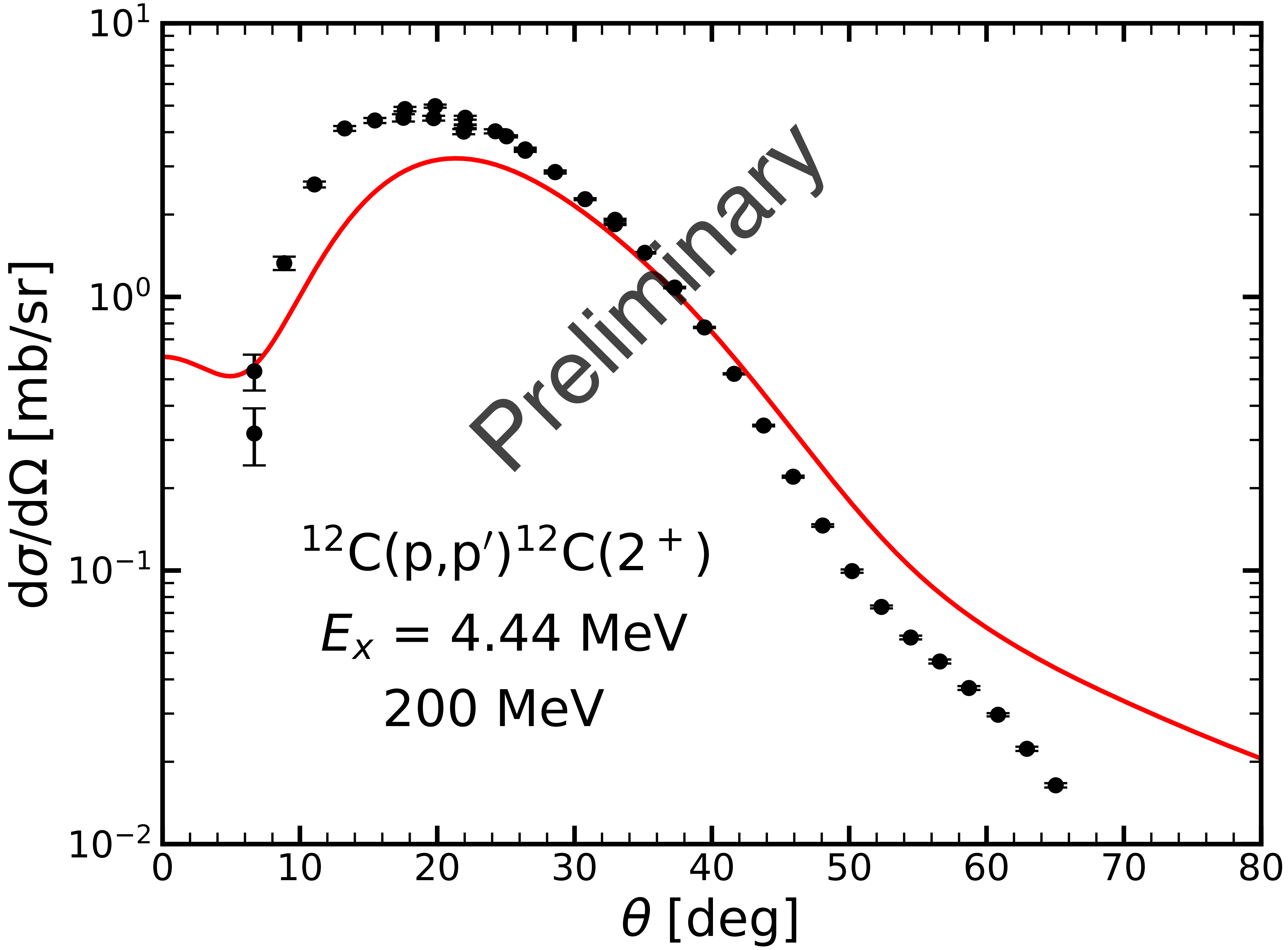
Distorted wave theory of inelastic scattering

The inelastic transition amplitude

[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1215 (1982)]
[Picklesimer, Tandy, Thaler, Phys. Rev. C **25**, 1233 (1982)]

$$T_{\rm inel}(\boldsymbol{k}_*, \boldsymbol{k}_0) = \int d\boldsymbol{r}' \int d\boldsymbol{r}\, \psi^\dagger(\boldsymbol{k}_*, \boldsymbol{r}')\, U_{\rm tr}(\boldsymbol{r}', \boldsymbol{r})\, \psi(\boldsymbol{k}_0, \boldsymbol{r})$$

- The optical potential used for the excited state is supposed to be different from that one used for the ground state
- With our approach we can distinguish the two states and calculate two different optical potentials
- The impact of this will be investigated in future works



Inclusion of medium effects

First-order term of the spectator expansion

$$\tau_{0i} = v_{0i} + v_{0i} G_0(E) \tau_{0i}$$

(A+1)-body propagator
The simplest approximation is

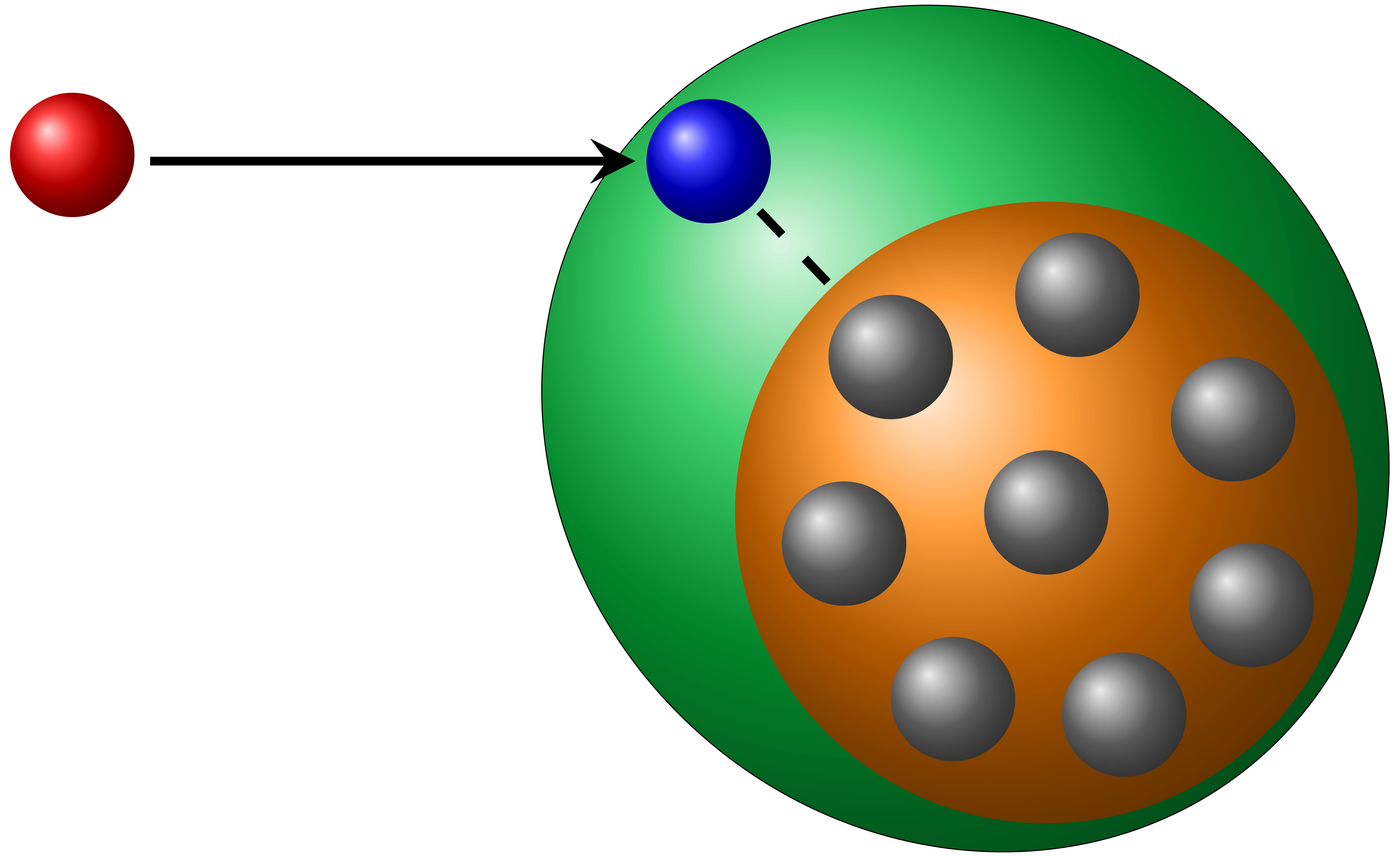
$$G_0(E) \approx g_0(E)$$

but there is not an intermediate one

Inclusion of medium effects

- Work has been done to include these effects at a mean-field level [Chinn *et al.*, PRC **52**, 1992 (1995)]
- We can use the SCGF to calculate the many-body propagator and the excitation spectrum

The first-order term is a 3-body problem



Optical potential for nucleus-nucleus elastic scattering

Møller factor

$$t_{NN}^{(AB)} = \eta\, t_{NN}^{(NN)}$$

It imposes the Lorentz invariance of flux when we pass from the AB to the NN frame where the t matrices are evaluated

Projectile density

Target density

$$U(\boldsymbol{q},\boldsymbol{K}) = \sum_{\alpha=n,p}\sum_{\beta=n,p}\int d\boldsymbol{P}\int d\boldsymbol{Q}\; \eta(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P},\boldsymbol{Q})\, t_{\alpha\beta}(\boldsymbol{q},\boldsymbol{K},\boldsymbol{P},\boldsymbol{Q};\mathcal{E})\, \rho_\alpha^{(\mathbb{P})}(\boldsymbol{q},\boldsymbol{P})\, \rho_\beta^{(\mathbb{T})}(\boldsymbol{q},\boldsymbol{Q})$$

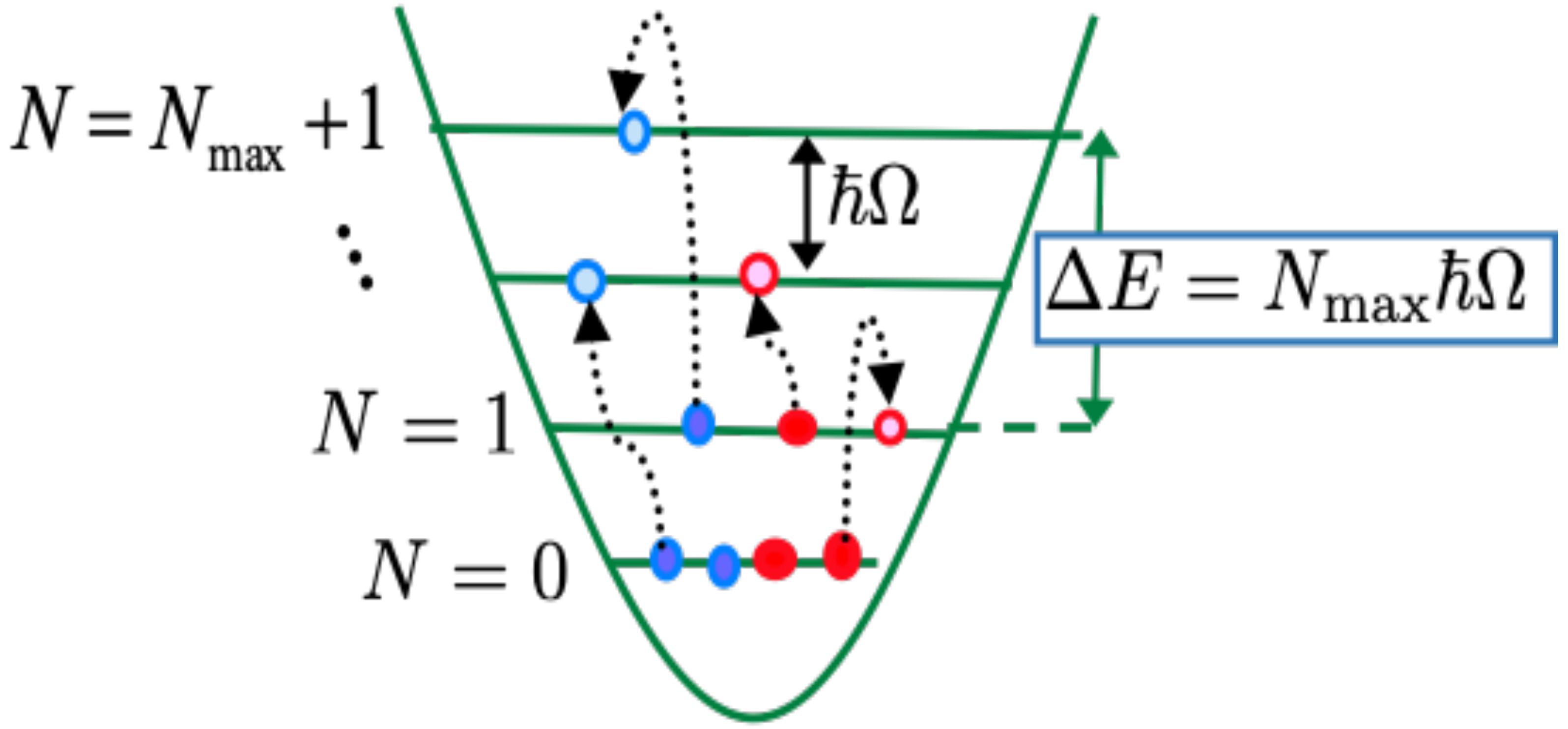
Free two-body scattering matrix

$$t_{0i} = v_{0i} + v_{0i}\, g_{0i}\, t_{0i}$$

$$g_{0i} = (E - h_0 - h_i + i\epsilon)^{-1}$$

- Simple one-body equation
- Can be solved easily
- Only **NN** interaction

Nonlocal one-body density



- Computationally expensive
- Obtained from the No-Core Shell Model
- Calculation performed with **NN** and **3N** interaction

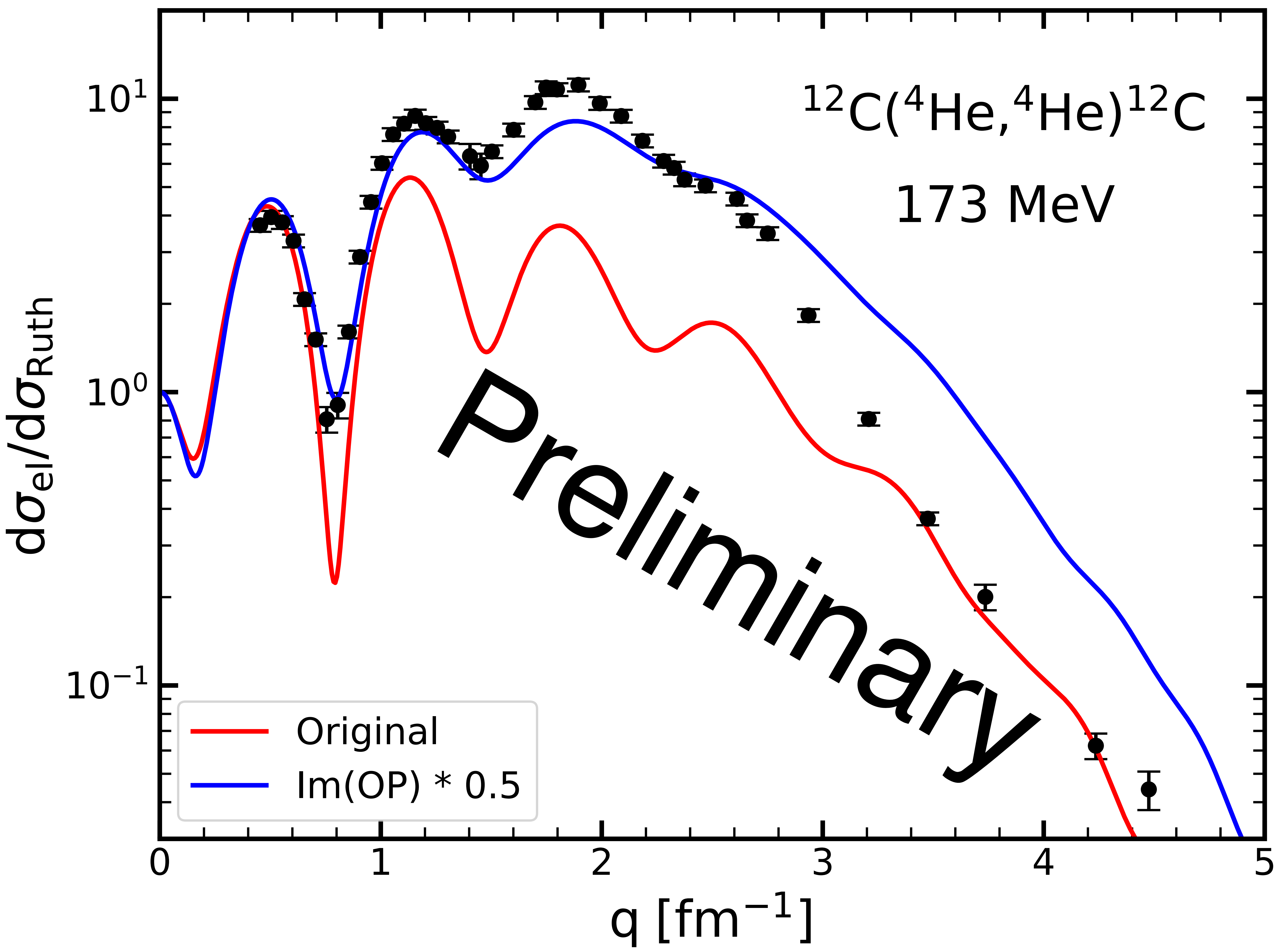
Results for elastic α-^{12}C scattering

Interesting results despite the approximations!
The optical potential seems too absorptive

How can we decrease the absorption?

- Inclusion of medium effects
- Introducing the energy dependence of the t matrix in the double-folding integral

$$t_{NN}(\boldsymbol{q}, \boldsymbol{K}, \boldsymbol{P}, \boldsymbol{Q}, \mathcal{E})$$



Summary & outlook

- ☑ The choice of the NN interaction is crucial to define the energy limits of applicability of the optical potential
- ☑ The combination of MST and SCGF looks promising for future calculations heavy systems
- ☑ Achieved a first step in the derivation of a nucleus-nucleus optical potential

- ☐ Extend the high- and low-energy limits of applicability of the optical potential
- ☐ Consistent treatment of the full 3N interaction
- ☐ Reducing the absorption in the nucleus-nucleus optical potential