Multiphonon excitations in dark matter direct detection: beyond the spherical ~~cow~~ guinea pig

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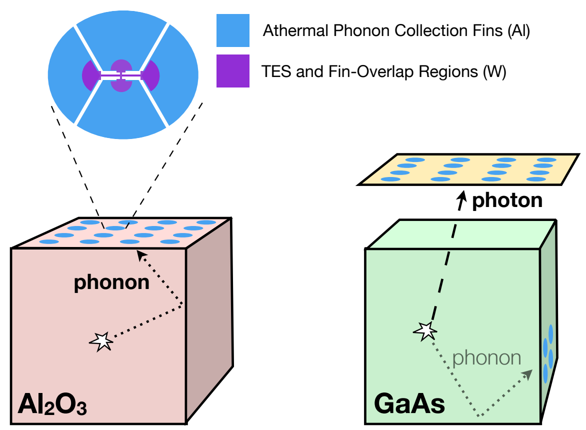
UCSD

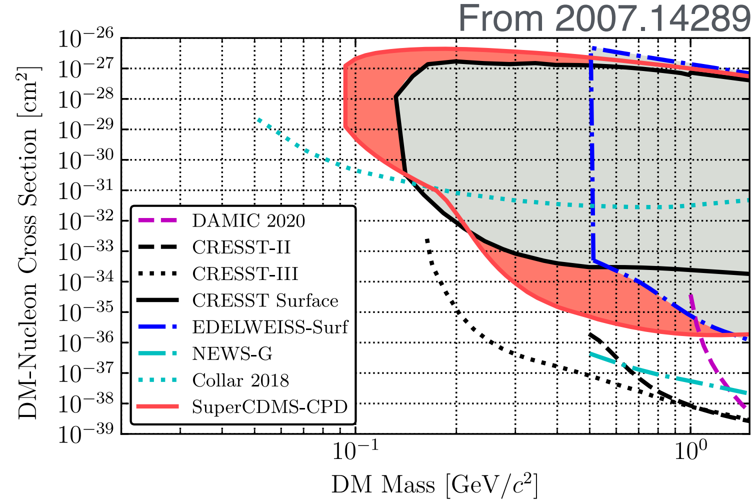
GUINEAPIG Workshop
August 20, 2024

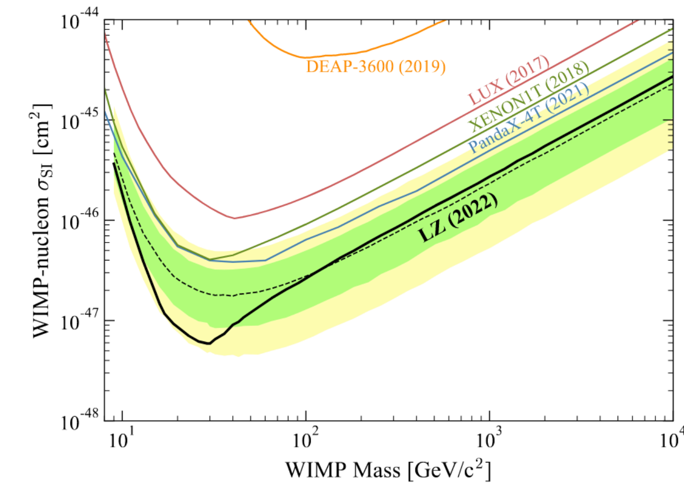
DM-nucleus scattering in crystals

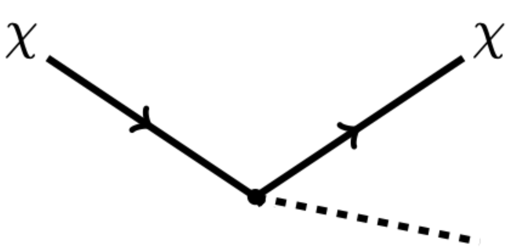


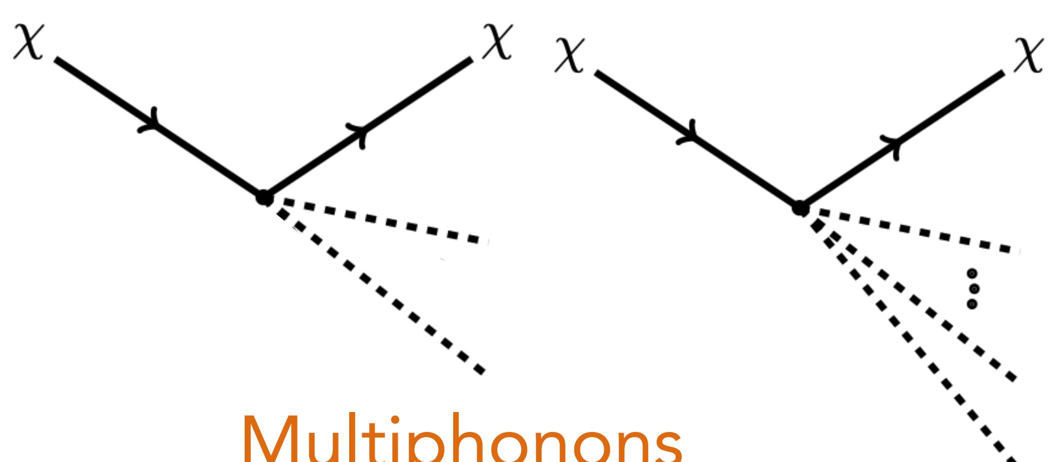
From TESSERACT white paper

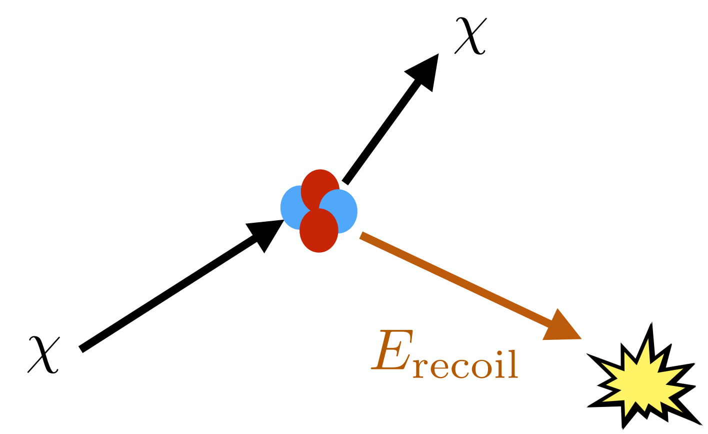
From 2007.14289











Single phonon excitation

Multiphonons

Nuclear recoils

keV MeV GeV TeV

Dark matter mass

DM-nucleus scattering in crystals: spherical guinea pig style

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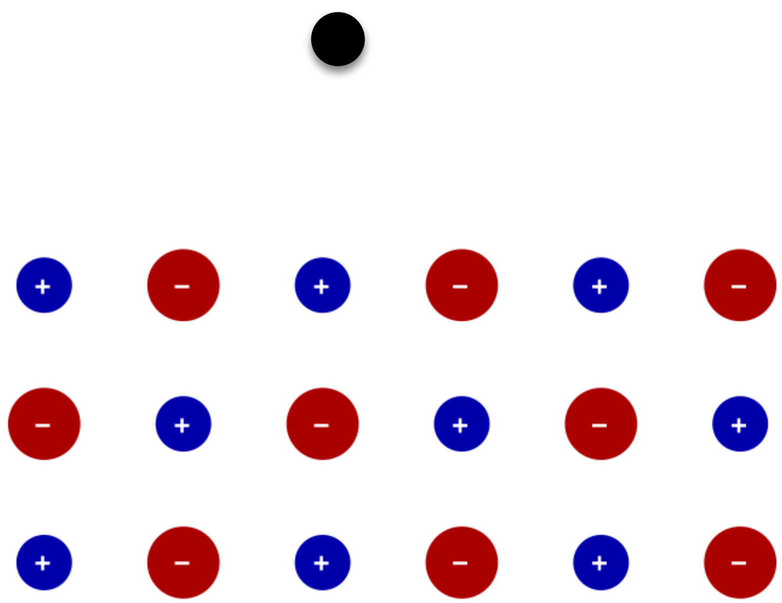
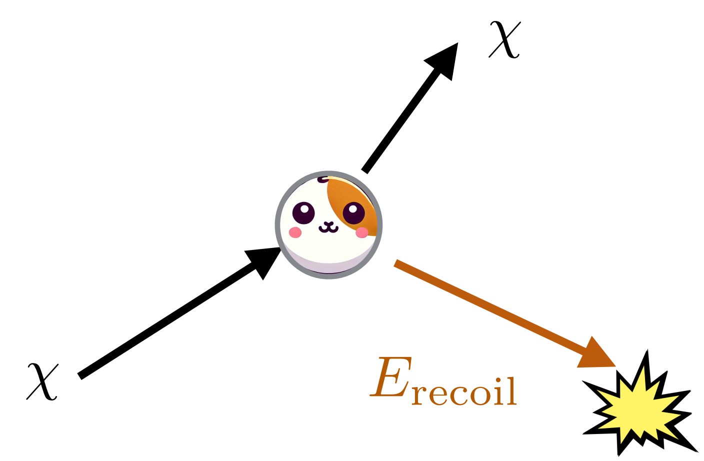
DM-nucleus scattering in crystals: spherical guinea pig style





Single phonon excitation: scatter against the whole lattice

Momentum transfer

$$q \ll q_{\rm BZ} = \frac{2\pi}{a} \sim \text{keV}$$

and energy deposition

$$\omega \sim \bar{\omega}_{\rm phonon} < 0.2\,\text{meV}$$

Nuclear recoils: scatter against a single atom

Momentum transfer

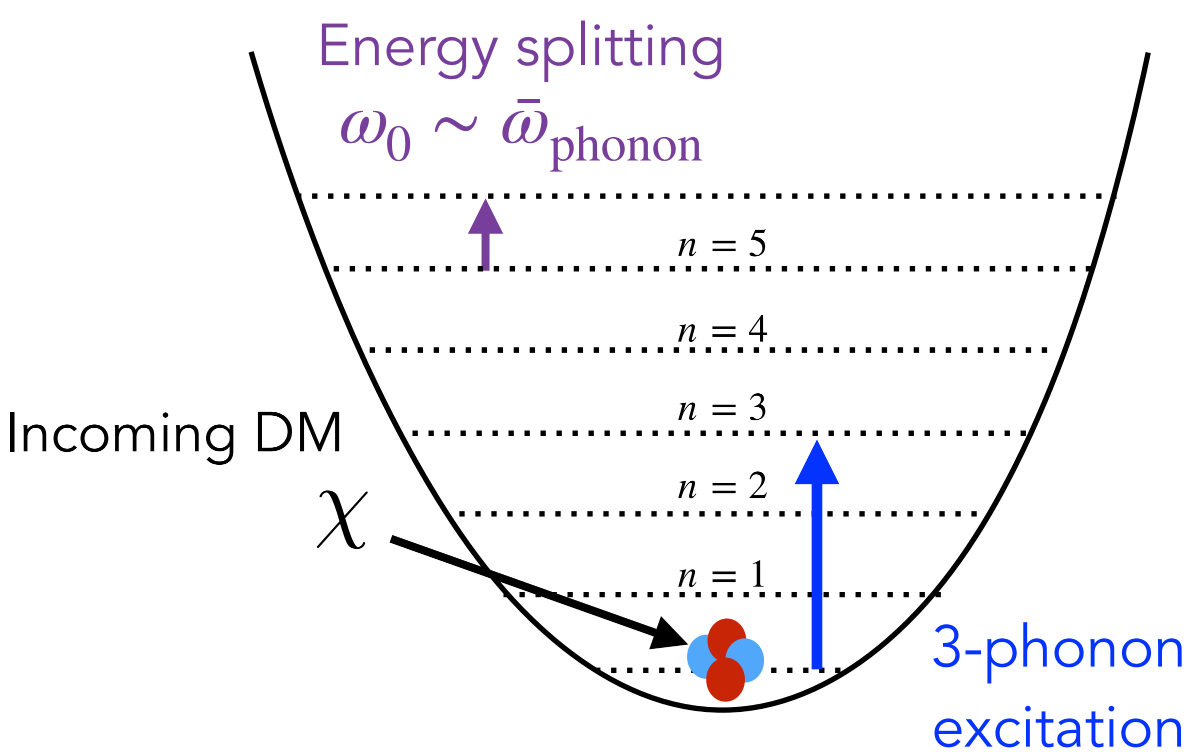
$$q \gg \sqrt{2m\bar{\omega}_{\rm phonon}} \sim 50\,\text{keV}$$

and energy deposition

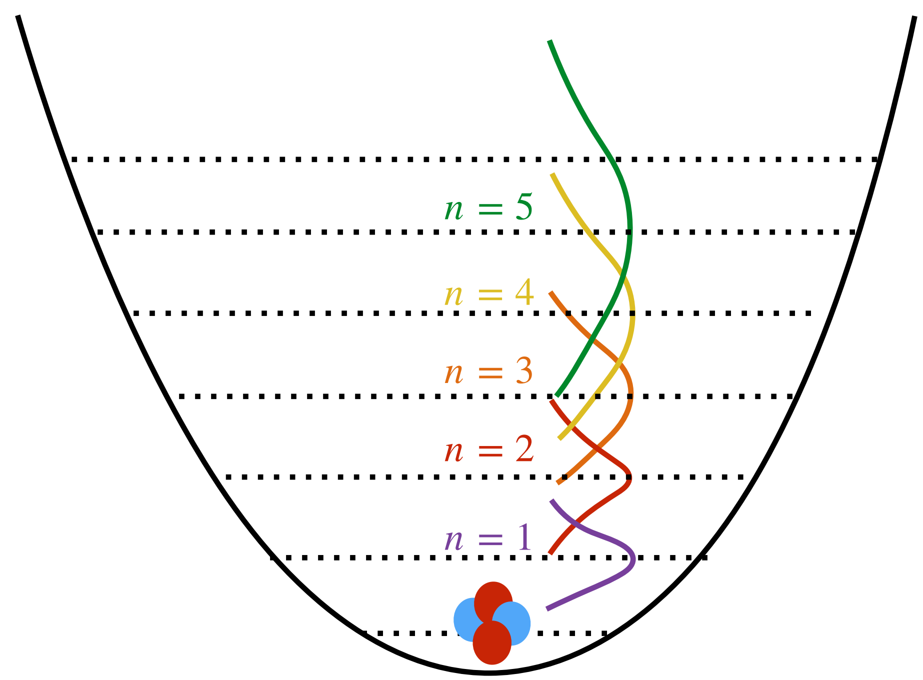
$$\omega \gg \bar{\omega}_{\rm phonon}$$

Multiphonon scattering

A first approximation: scattering off a nucleus in a bound **harmonic oscillator** potential



Phonons in a crystal exist in a continuum! An excellent second approximation:

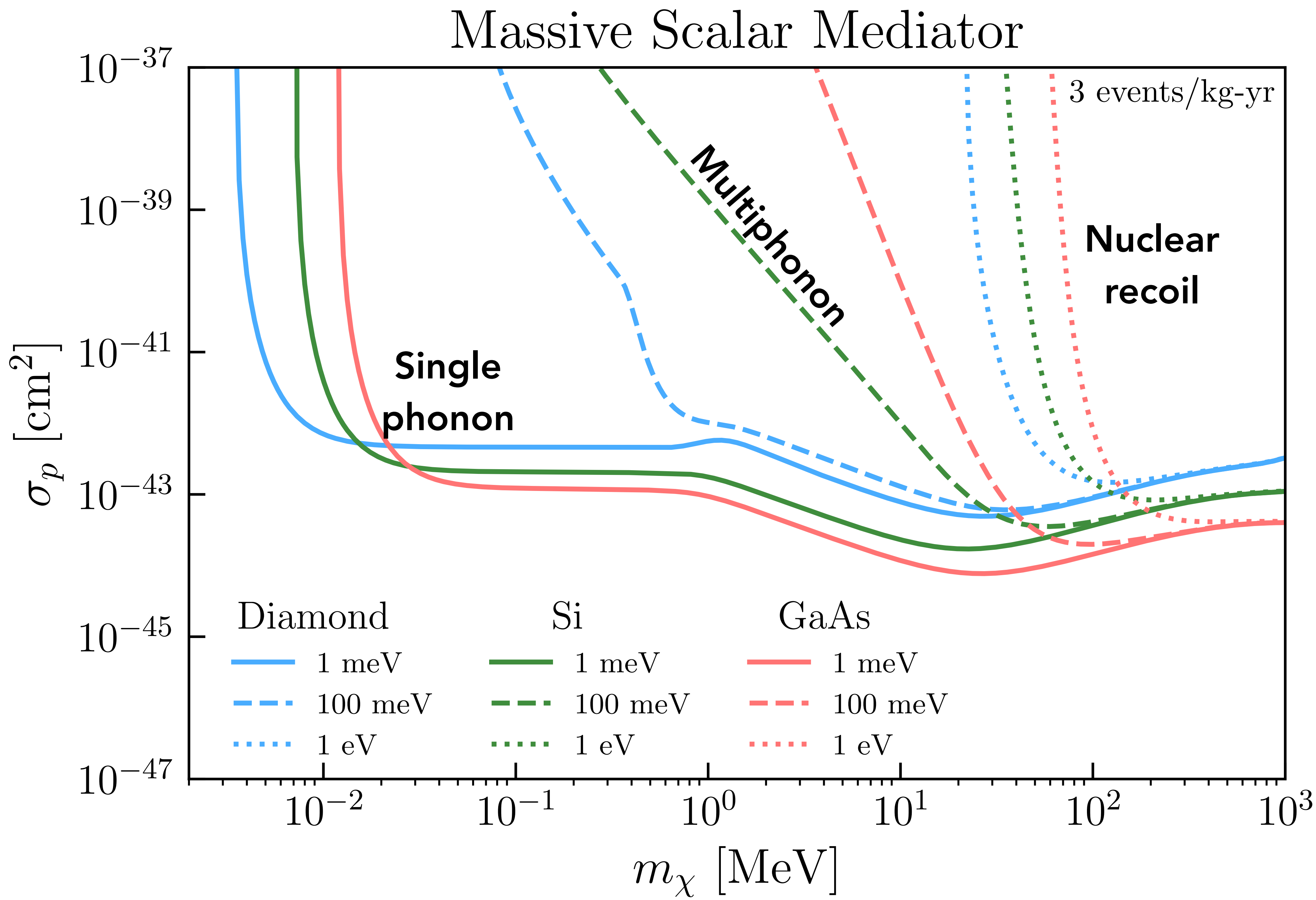


Account for density of states $D(\omega)$:

$$\frac{\delta(\omega - \omega_0)}{\omega_0} \to \frac{D(\omega)}{\omega}$$

Knapen, Kozaczuk, TL 2011.09496
Kahn, Krnjaic, Mandava 2011.09477
Campbell-Deem, Knapen, TL, Villarama 2205.02250
TL, Shen, Sholapurkar, Villarama 2309.10839

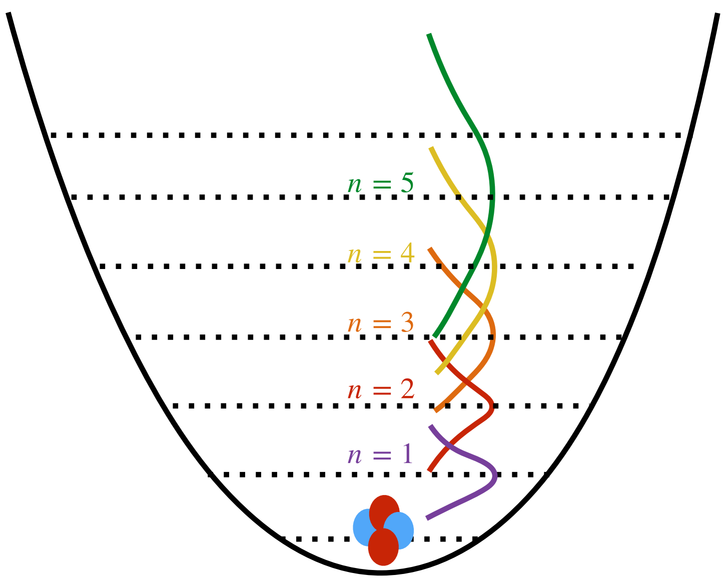
A unified picture of low-energy nuclear recoils in crystals



Campbell-Deem, Knapen, TL, Villarama 2205.02250

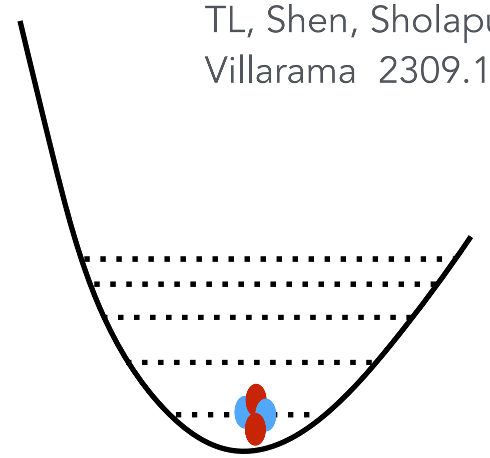
Multiphonon scattering

Spherical Guinea Pig Approach



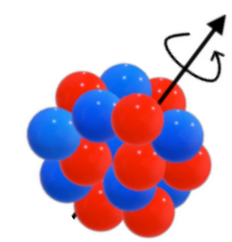
Anharmonic effects

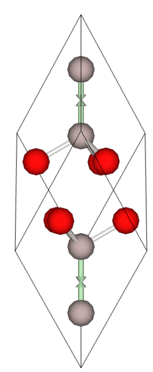
TL, Shen, Sholapurkar, Villarama 2309.10839



Spin-dependent scattering

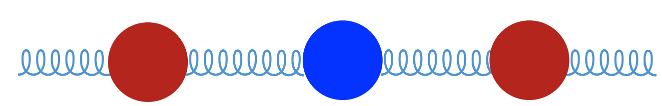
In progress with Knapen, Gori, Munbodh, Suter





Anisotropic crystals & daily modulation

Stratman, TL (to appear)



Incoherent approximation

In progress with Fang, Lin

DM scattering rate

DM-mediator form factor

$$\frac{d\sigma}{d^3\mathbf{q}\,d\omega} \propto \sigma_{\chi p}\,|\tilde{F}_{\rm med}(q)|^2\,S(\mathbf{q},\omega)\,\delta(\omega - \mathbf{q}\cdot\mathbf{v} + \tfrac{q^2}{2m_\chi})$$

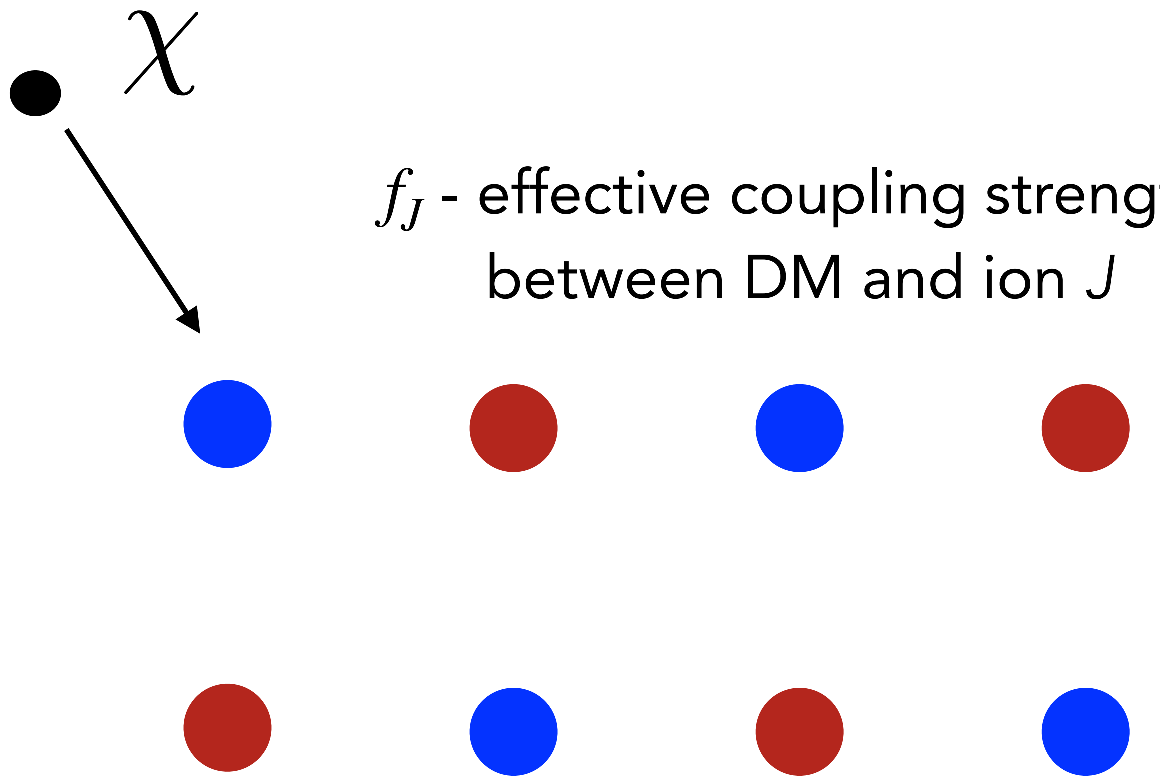
Dynamic structure factor captures response of target

For free nuclei and spin-independent interactions:

$$S(\mathbf{q},\omega) \propto A_N^2\,\delta\left(\omega - \frac{q^2}{2m_N}\right)$$

Goal: understand response function $S(\mathbf{q},\omega)$ from the single phonon to the nuclear recoil regime

DM-nucleus interaction



f_J - effective coupling strength between DM and ion J

Short range SI interaction

$$\sigma_{\chi p} = 4\pi b_p^2$$

Scattering potential in Fourier space

$$V(\mathbf{q}) \propto b_p \sum_J f_J \, e^{i\mathbf{q}\cdot\mathbf{r}_J}$$

$$S(\mathbf{q},\omega) \equiv \frac{2\pi}{V} \sum_f \left| \sum_J \langle \Phi_f | f_J e^{i\mathbf{q}\cdot\mathbf{r}_J} |0\rangle \right|^2 \delta\left(E_f - \omega\right)$$

$$= \frac{1}{V} \sum_{J,\,J'}^{N} f_J f_{J'}^* \int_{-\infty}^{\infty} dt \, \langle e^{-i\mathbf{q}\cdot\mathbf{r}_{J'}(0)} e^{i\mathbf{q}\cdot\mathbf{r}_J(t)} \rangle e^{-i\omega t}$$

Contains correlations between different atoms → single phonon excitations

Theory of neutron scattering: Squires 1996, Schober 2014

Dynamic structure factor

Phonons appear through positions of ions:

$$\mathbf{r}_J(t) = \mathbf{r}_J^0 + \mathbf{u}_J(t)$$

↑

Quantized phonon field given in terms of phonon dispersions $\omega_{\mathbf{q}}$ and eigenvectors $\mathbf{e}_{\mathbf{q}}$

Single phonon contribution has been studied extensively in literature, with $\omega_{\mathbf{q}}$, $\mathbf{e}_{\mathbf{q}}$ calculated from first principles approaches

$$S^{n=1}(\mathbf{q},\omega) \sim \sum_{J,J'} f_J f_{J'} \int dt \, \langle \mathbf{q}\cdot\mathbf{u}_J(0) \; \mathbf{q}\cdot\mathbf{u}_{J'}(t)\rangle e^{-i\omega t}$$

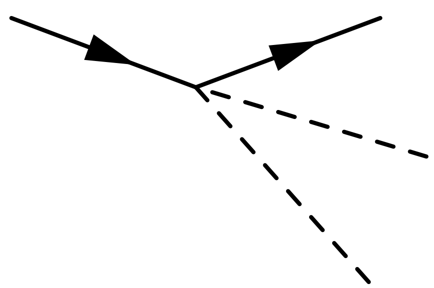
Griffin, Knapen, TL, Zurek 1807.10291; Griffin, Inzani, Trickle, Zhang, Zurek 1910.10716
Griffin, Hochberg, Inzani, Kurinsky, TL, Yu 2020; Coskuner, Tickle, Zhang, Zurek 2102.09567

Dynamic structure factor

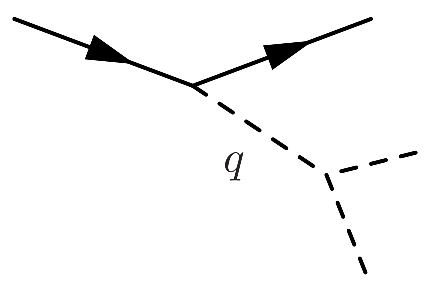
Expansion in $q^2/(M_N\omega)$ (and anharmonic interactions):

$$S(\mathbf{q},\omega) = \text{(0-phonon)} + \text{(1-phonon)} + \text{(2-phonon)} + \cdots$$

Harmonic



Anharmonic



Quickly becomes more complicated to evaluate for more than 1 phonon

Analytic approximations are valuable to make progress

The spherical guinea pig

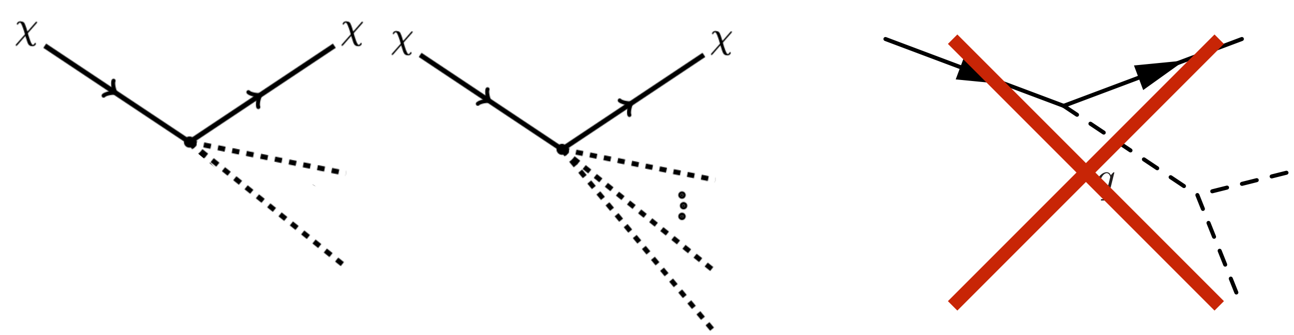
Neglect interference terms in structure factor:

Incoherent approximation

$$S(\mathbf{q},\omega) \approx \frac{1}{V}\sum_{J}^{N}(f_J)^2 \int_{-\infty}^{\infty} dt\, \langle e^{-i\mathbf{q}\cdot\mathbf{u}_J(0)} e^{i\mathbf{q}\cdot\mathbf{u}_J(t)}\rangle e^{-i\omega t}$$

Good approx for $q \gg q_{BZ} \sim \mathrm{keV}$

Harmonic crystal



Isotropic crystal

Phonon density of states $D(\omega)$ does not depend on direction of phonon mode

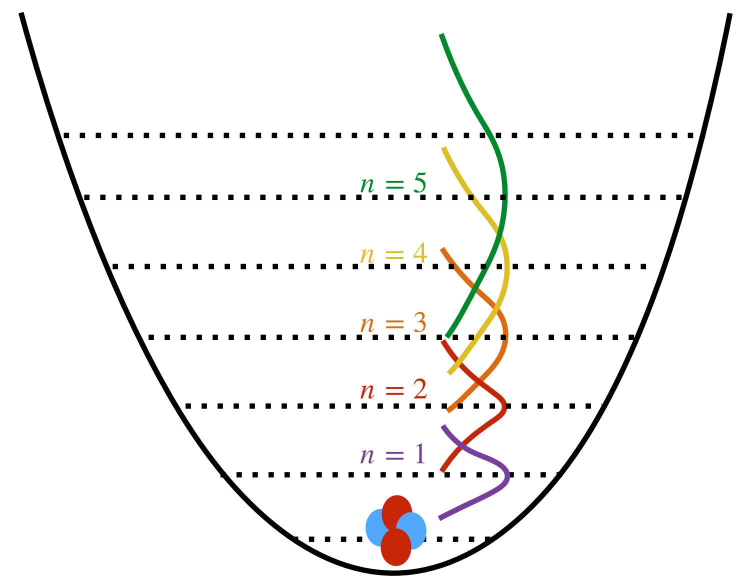
Campbell-Deem, Knapen, TL, Villarama 2205.02250

The spherical guinea pig

Incoherent approximation

Harmonic crystal

Isotropic crystal



Integrate over n-phonon phase space:

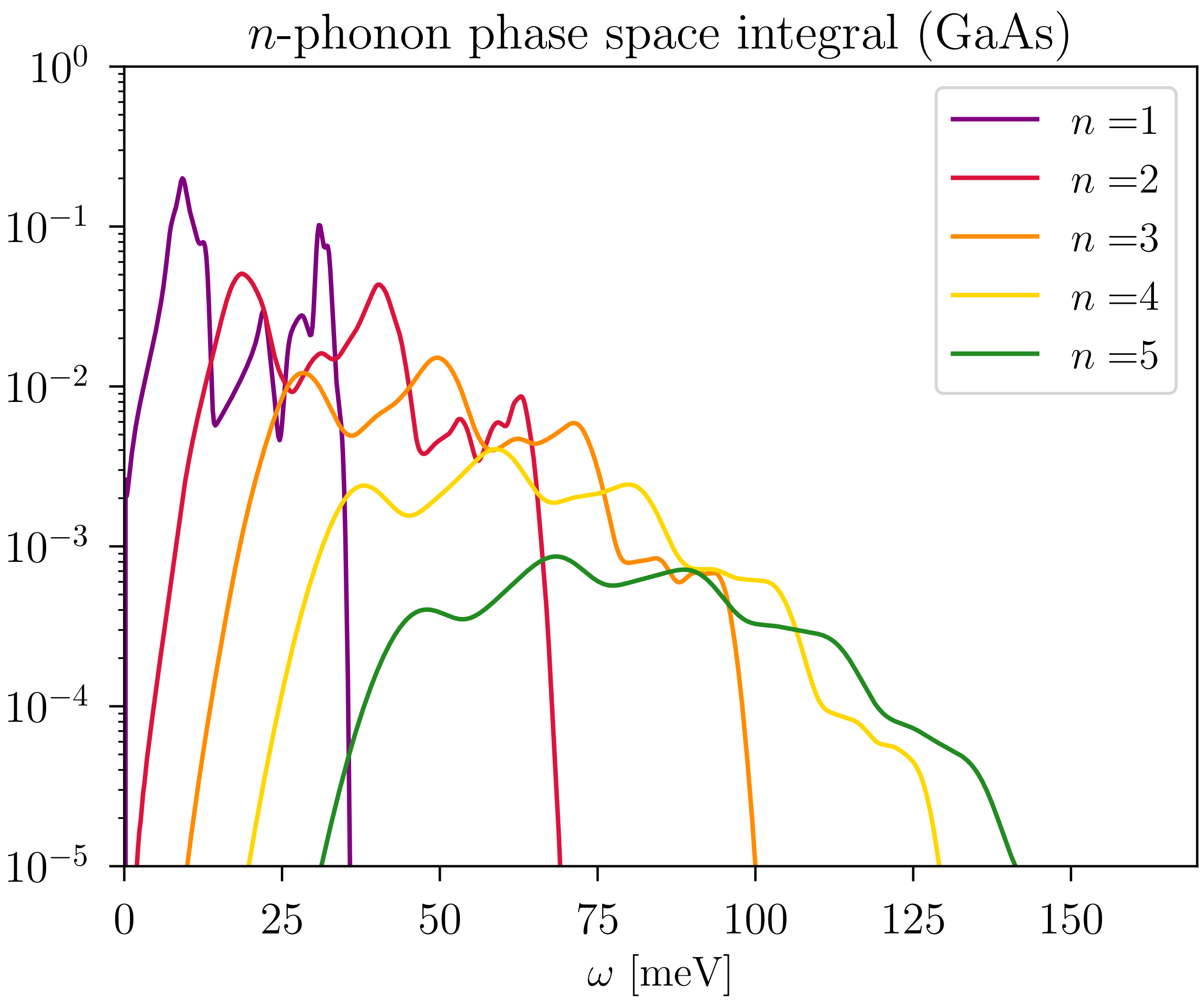
$$\sum_n \frac{1}{n!}\left(\frac{q^2}{2m_N}\right)^n \left(\prod_{i=1}^n \int d\omega_i \frac{D(\omega_i)}{\omega_i}\right)\delta\left(\sum_j \omega_j - \omega\right)$$

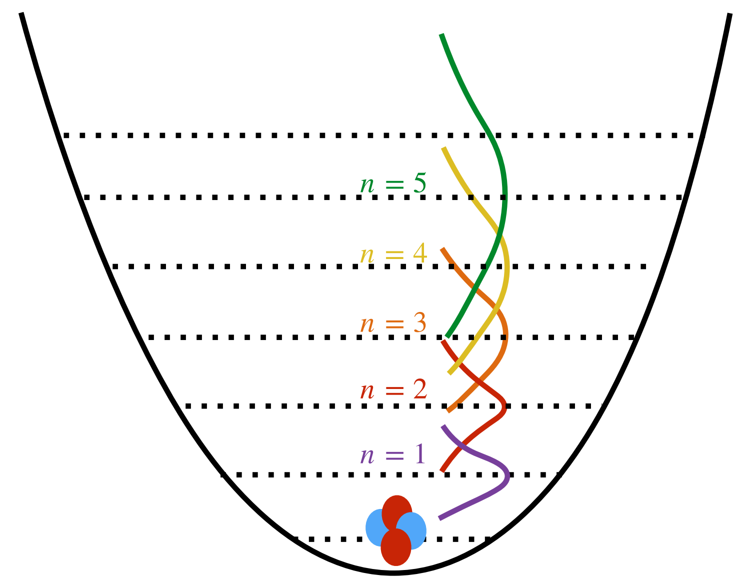
Public code: **DarkELF**

https://github.com/tongylin/DarkELF

Campbell-Deem, Knapen, TL, Villarama 2205.02250

The spherical guinea pig





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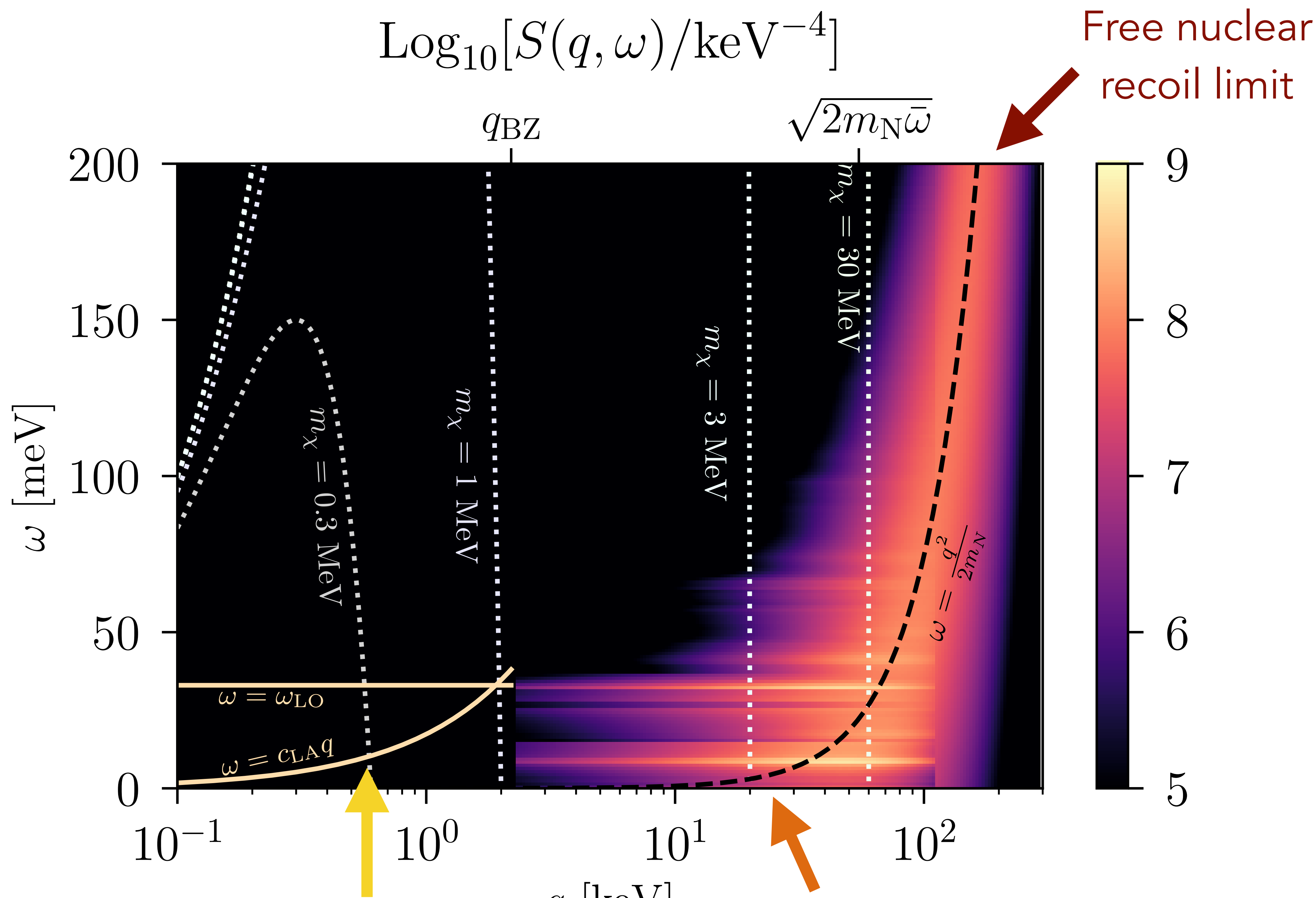
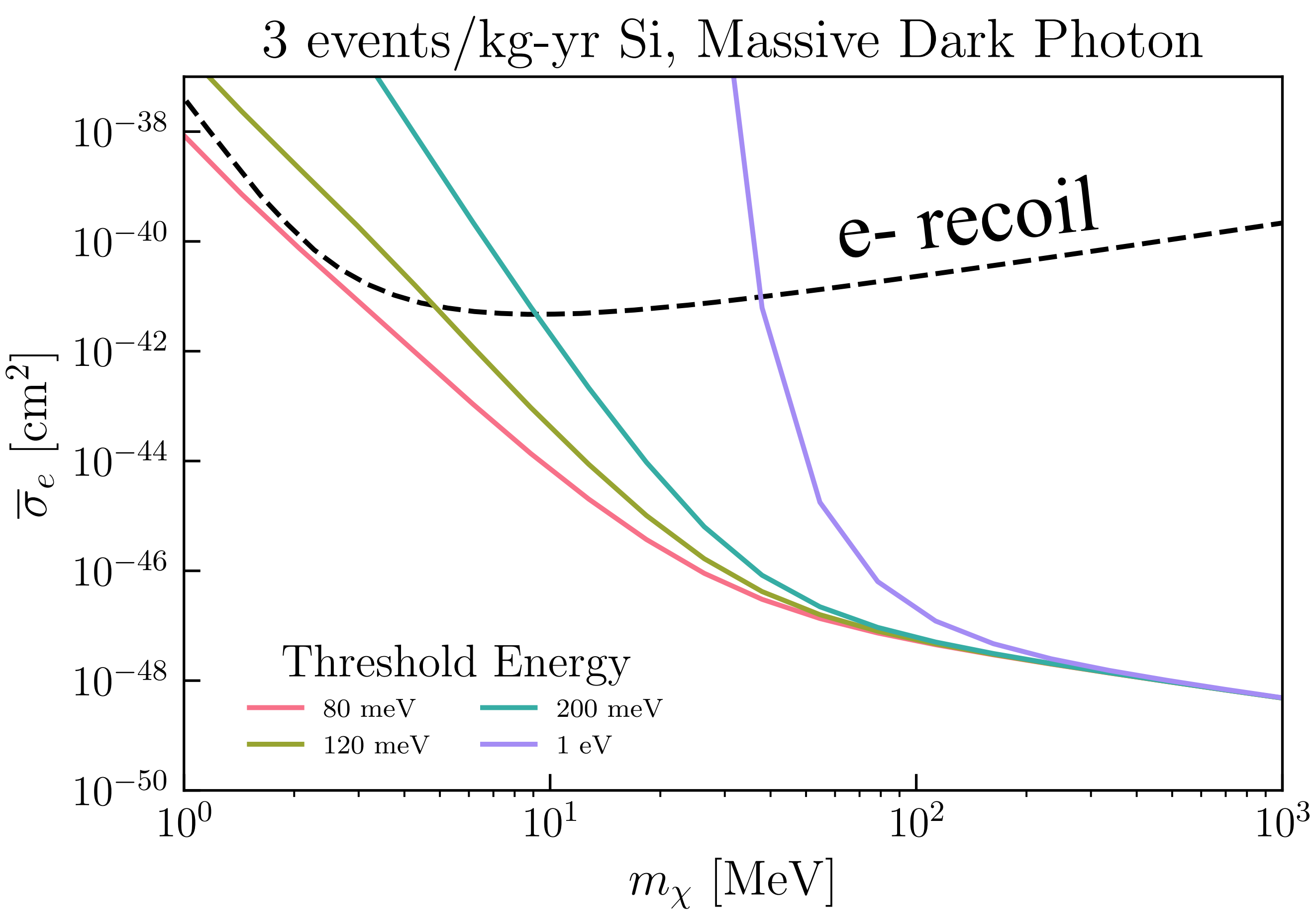
Campbell-Deem, Knapen, TL, Villarama 2205.02250

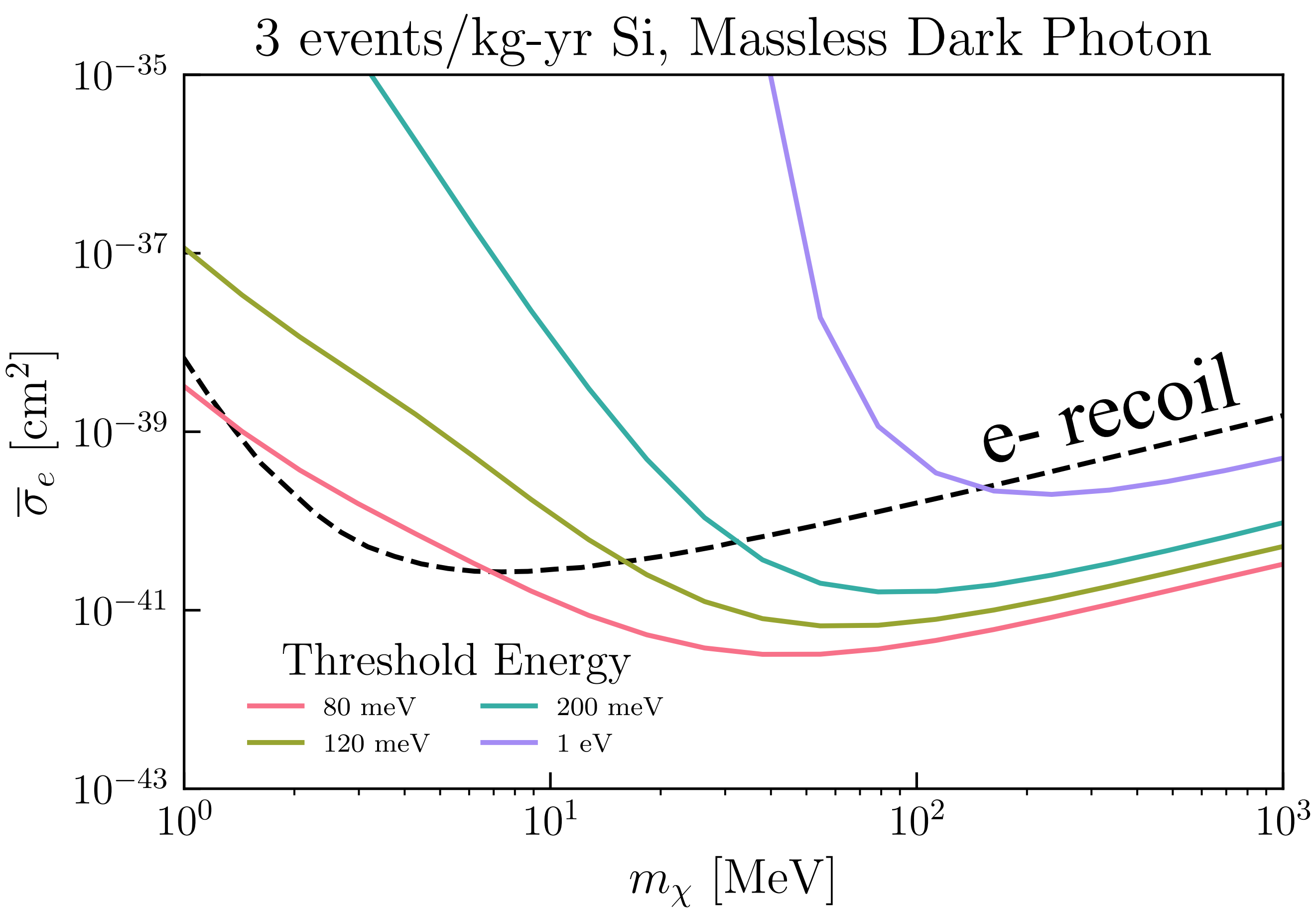
Dynamic structure factor



Campbell-Deem, Knapen, TL, Villarama 2205.02250

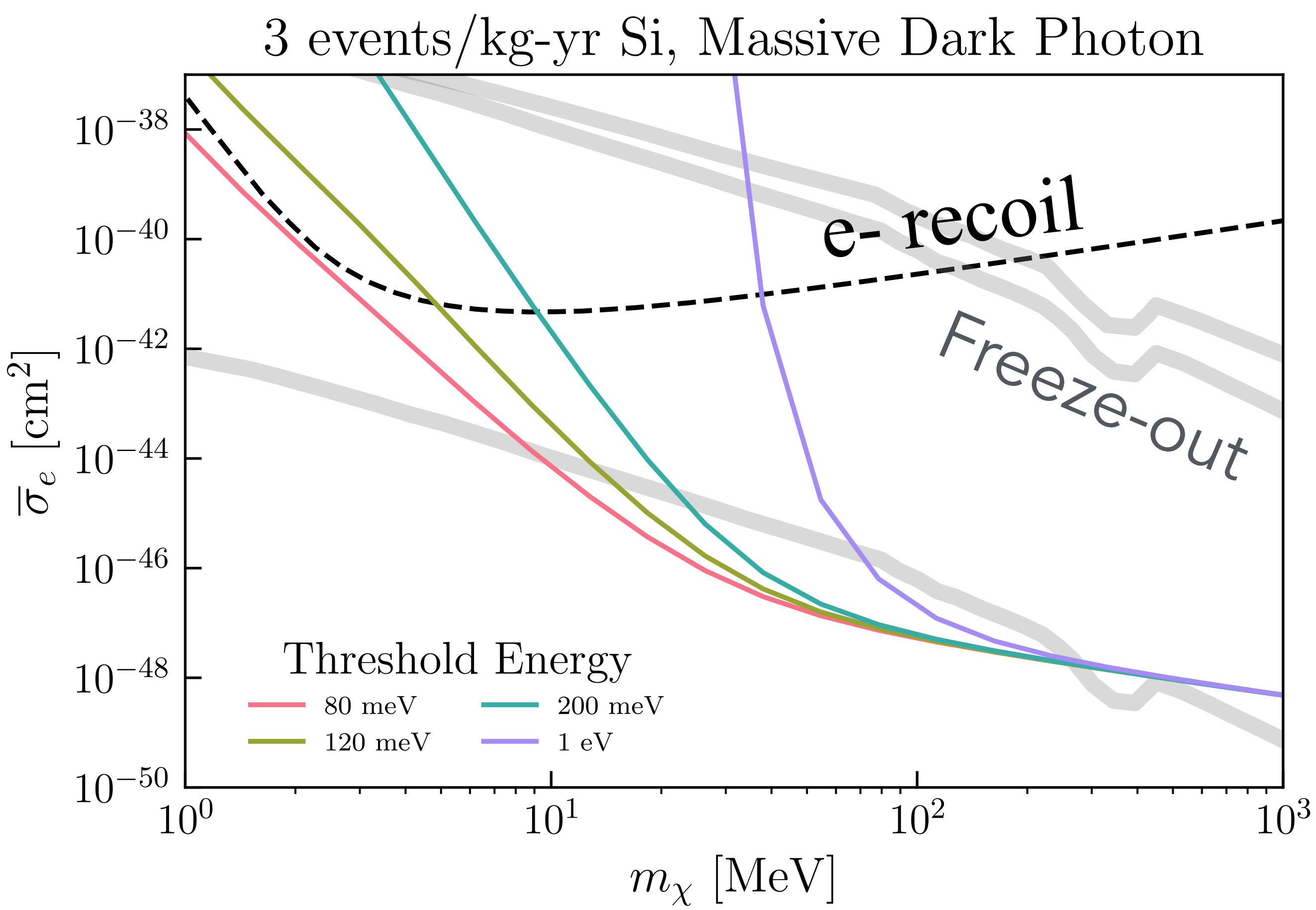
Dark photon mediator

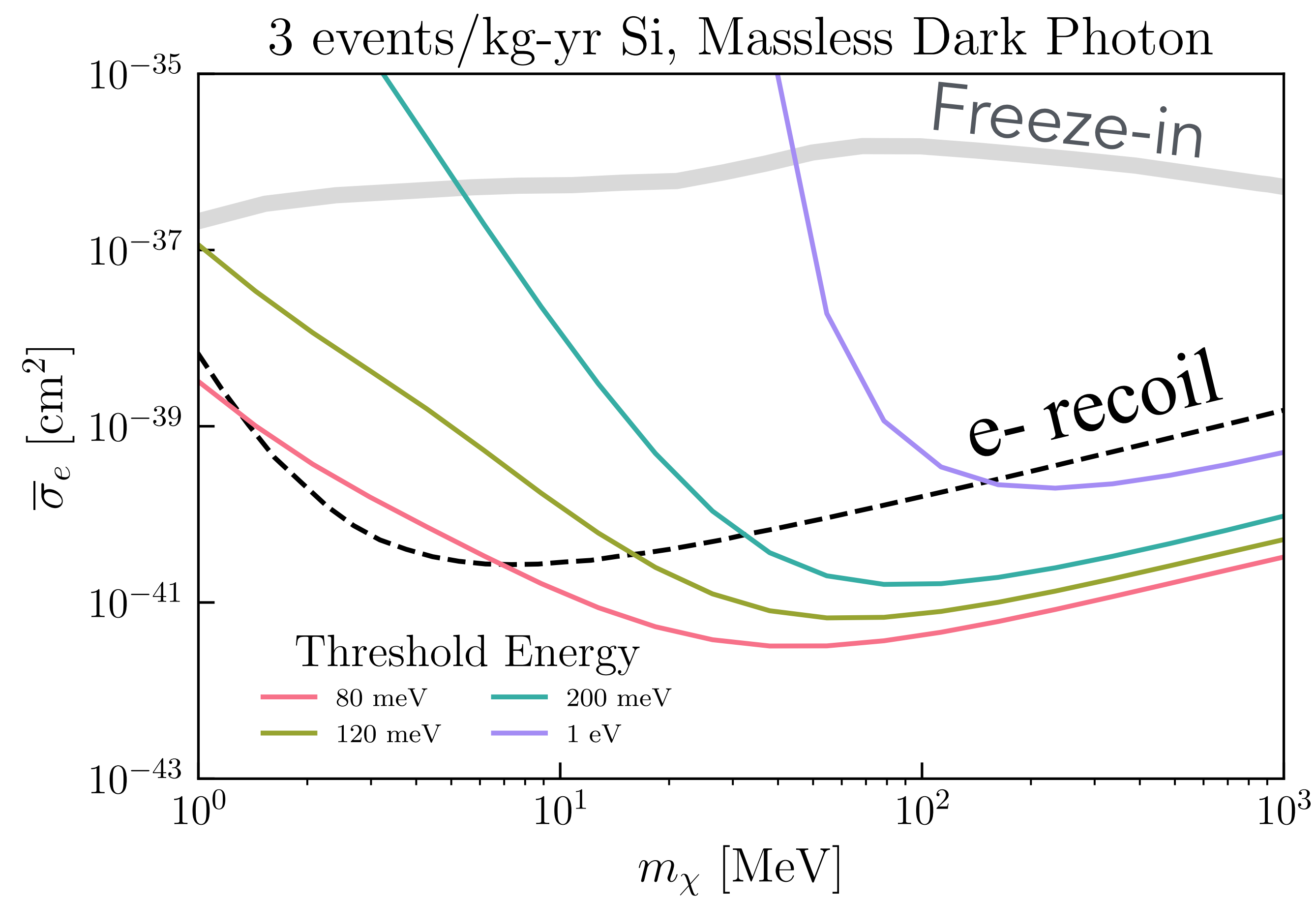




Campbell-Deem, Knapen, TL, Villarama 2205.02250
See also Kahn, Krnjaic, Mandava 2011.09477

Dark photon mediator





Campbell-Deem, Knapen, TL, Villarama 2205.02250
See also Kahn, Krnjaic, Mandava 2011.09477

Anharmonic crystals

$$V_{\text{lattice}} = \frac{1}{2} \sum_{\boldsymbol{\ell},d,\boldsymbol{\ell}',d'} \sum_{\alpha,\beta} k^{(2)}_{\alpha\beta}(\boldsymbol{\ell}d, \boldsymbol{\ell}'d')\ u_\alpha(\boldsymbol{\ell}d)\ u_\beta(\boldsymbol{\ell}'d')$$

Harmonic (2nd order force constants)

$$+ \frac{1}{3!} \sum_{\boldsymbol{\ell},d,\boldsymbol{\ell}',d',\boldsymbol{\ell}'',d''} \sum_{\alpha,\beta,\gamma} k^{(3)}_{\alpha\beta\gamma}(\boldsymbol{\ell}d, \boldsymbol{\ell}'d', \boldsymbol{\ell}''d'') \times u_\alpha(\boldsymbol{\ell}d)\ u_\beta(\boldsymbol{\ell}'d')\ u_\gamma(\boldsymbol{\ell}''d'') + \ldots$$

3rd order force constants

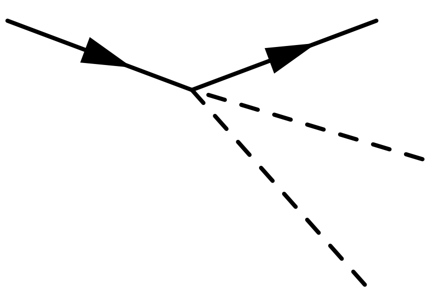
Anharmonicity related to phonon decay, thermal conductivity and expansion observed in crystals.

Anharmonic crystals

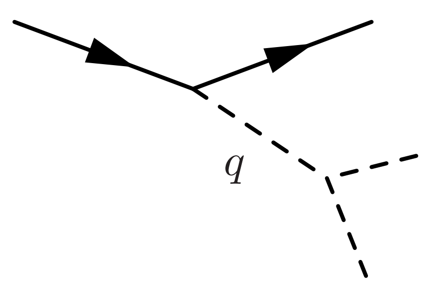
Momentum transfer

$$q \ll q_{\rm BZ} = \frac{2\pi}{a} \sim \mathrm{keV}$$

Harmonic



Anharmonic



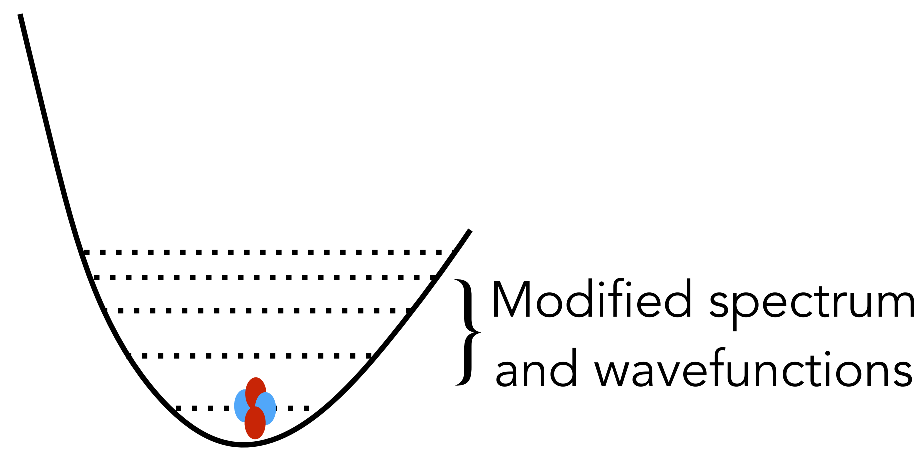
Use EFT of elastic waves

$$\delta H_{anh} \sim \int d^3\mathbf{r}\, \alpha(\nabla_i \mathbf{u}_j)^3$$

to calculate for acoustic phonons

Momentum transfer

$$q \gtrsim q_{\rm BZ} = \frac{2\pi}{a} \sim \mathrm{keV}$$



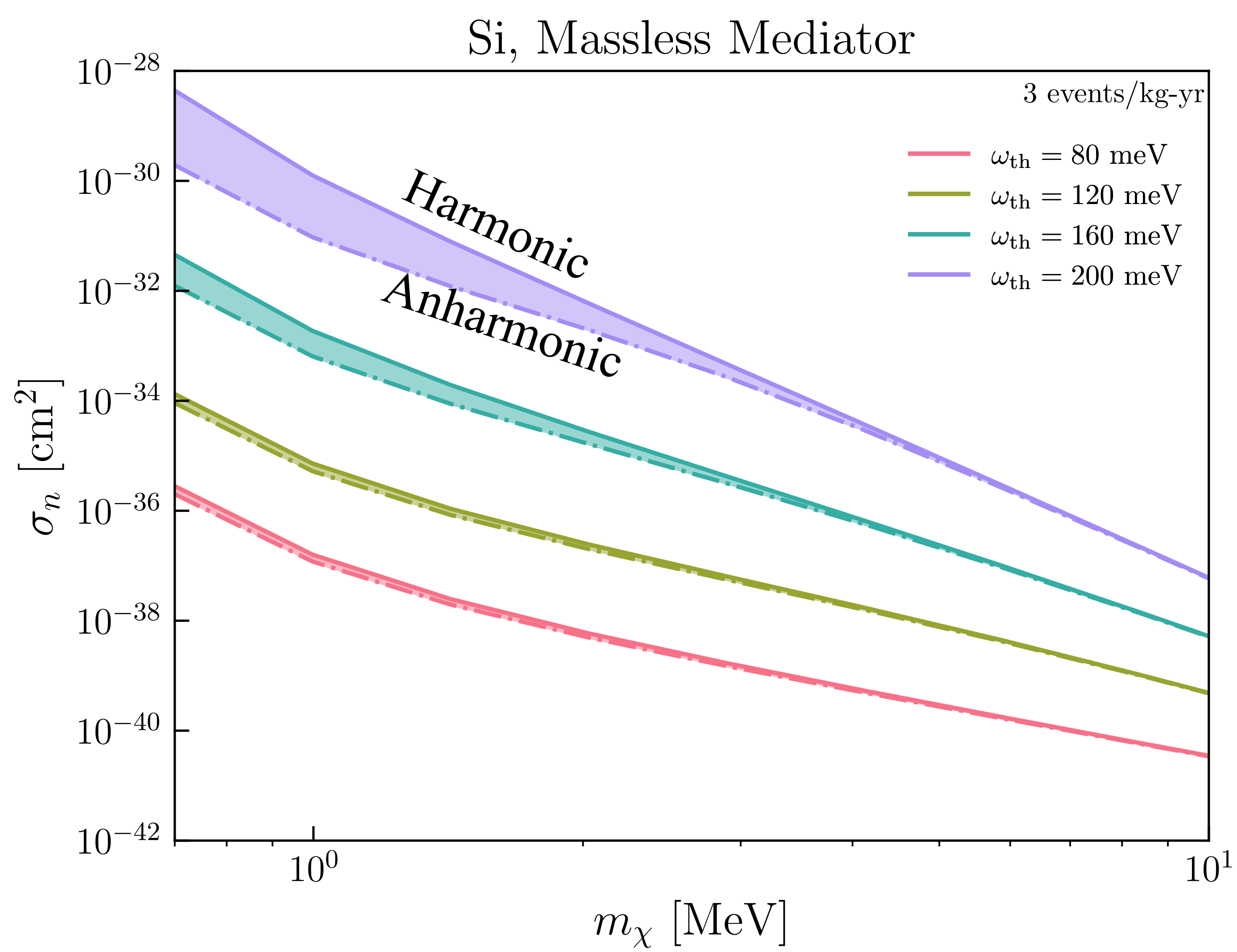
Toy model: scattering off a particle in a 1D anharmonic potential $V(x)$



Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482

TL, Shen, Sholapurkar, Villarama 2309.10839

Anharmonic interactions



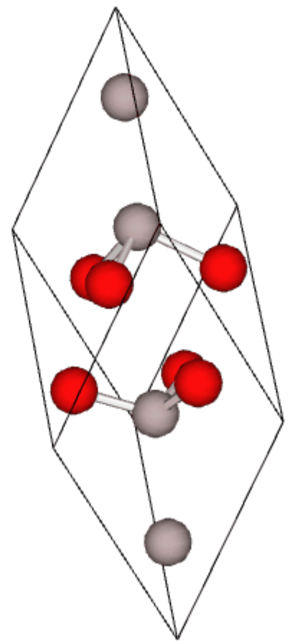
Takeaway: anharmonic effects are **limited** to DM masses $\lesssim$ MeV and high cross sections (typically not viable in realistic models).

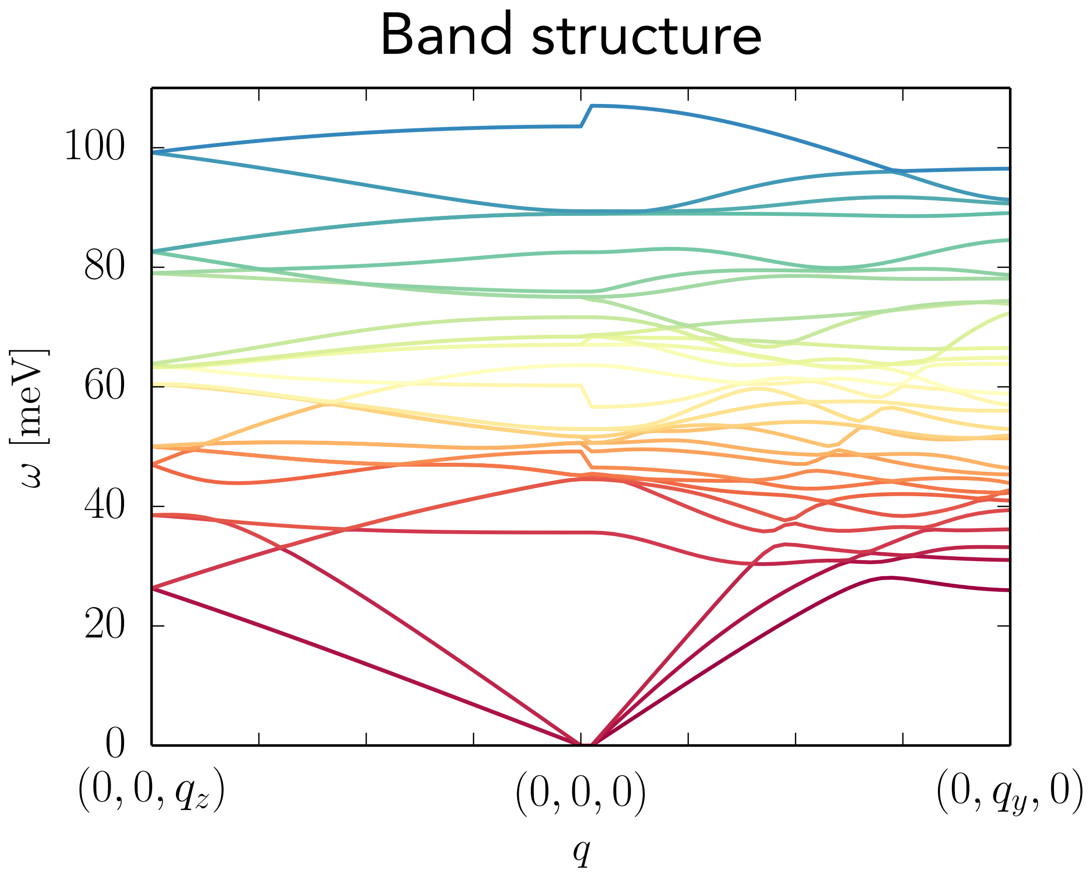
TL, Shen, Sholapurkar, Villarama 2309.10839

Anisotropic crystals

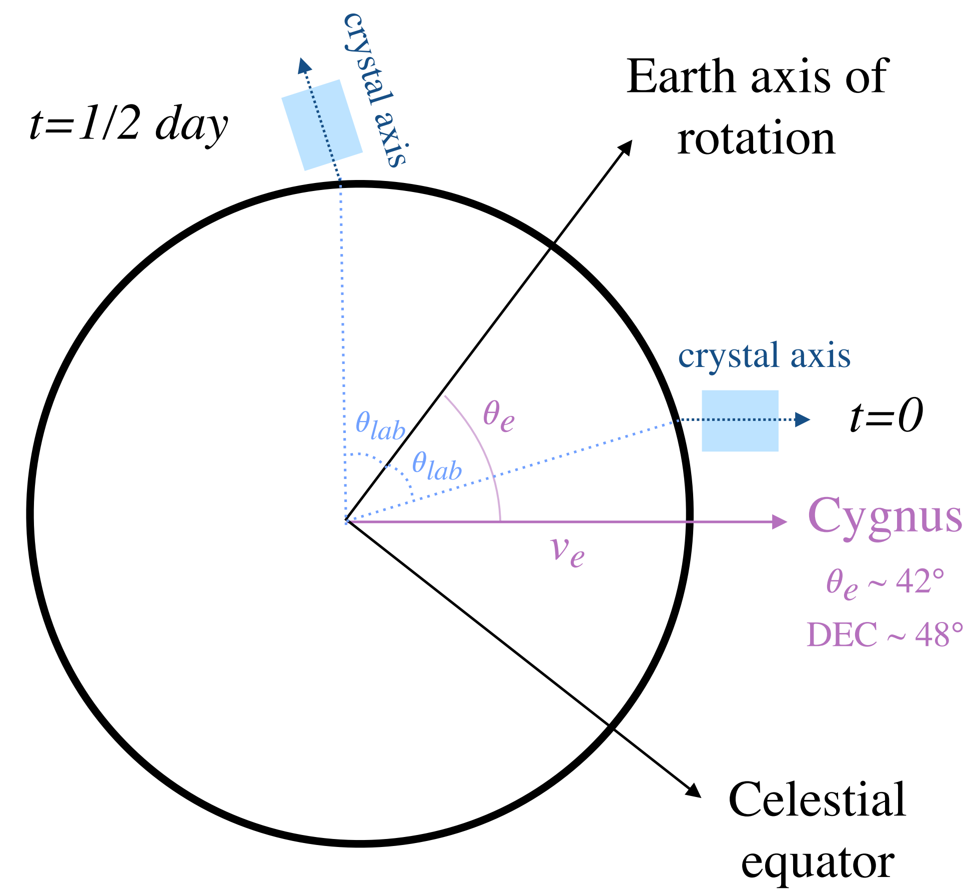
Al_2O_3

Sapphire; to be used in SPICE



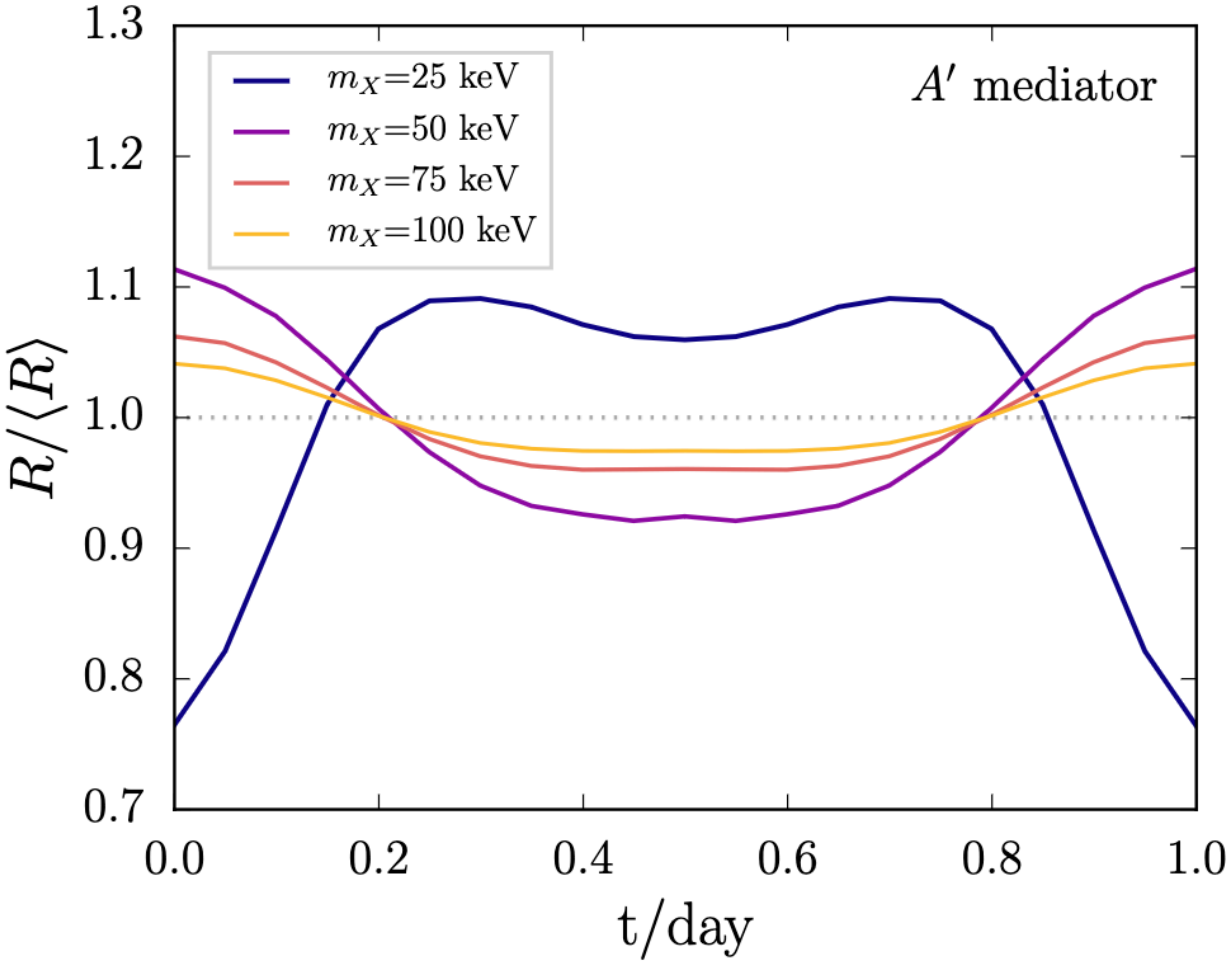


Phonon modes (DoS) depend on direction of momentum



Daily modulation in rate as Earth rotates

Daily modulation with single phonons



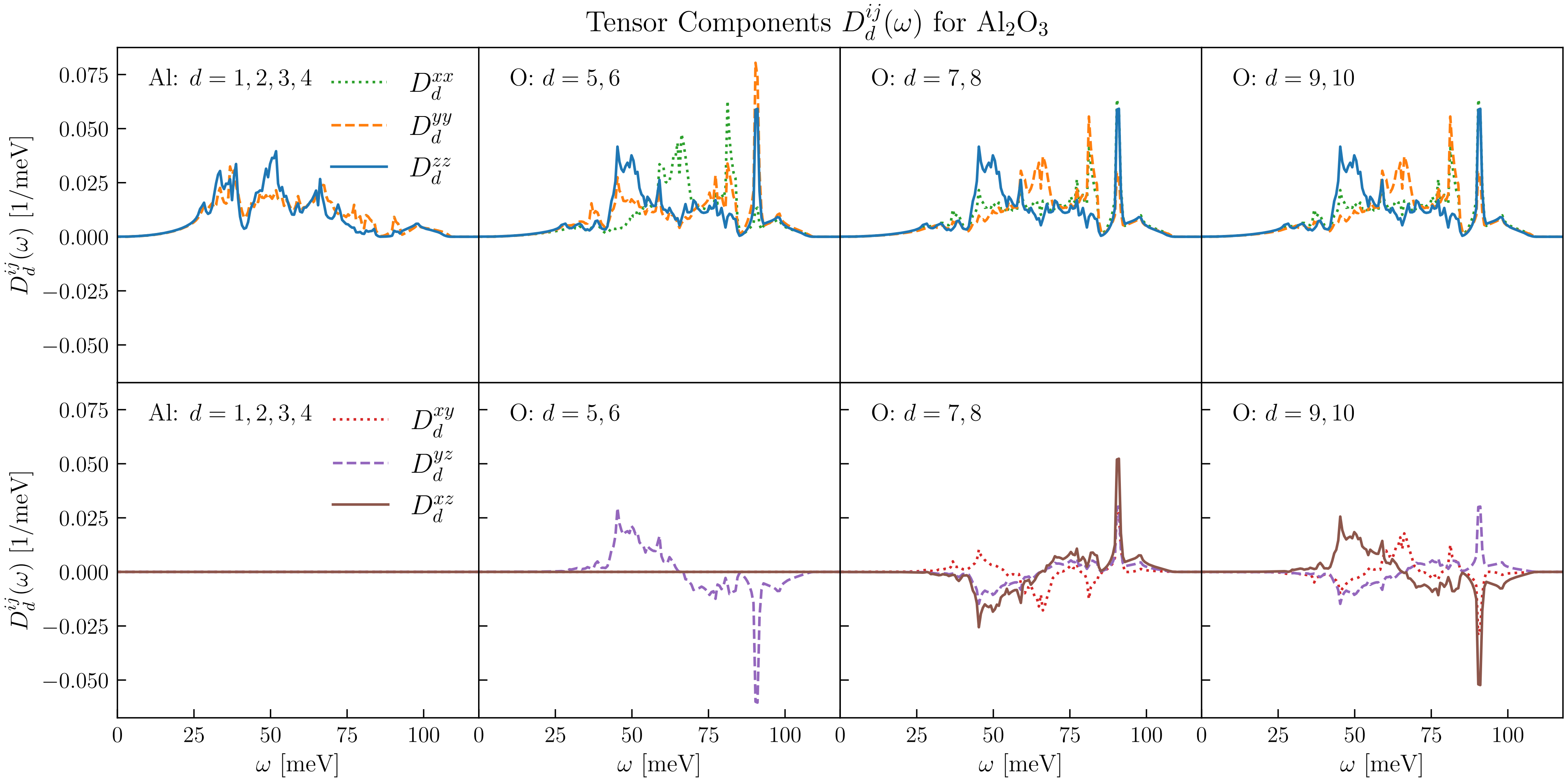
Griffin, Knapen, TL, Zurek 1807.10291
Griffin, Hochberg, Inzani, Kurninsky, TL, Yu 2008.08560
Coskuner, Trickle, Zhang, Zurek 2102.09567

Directional detection with multi phonons

With Connor Stratman, to appear

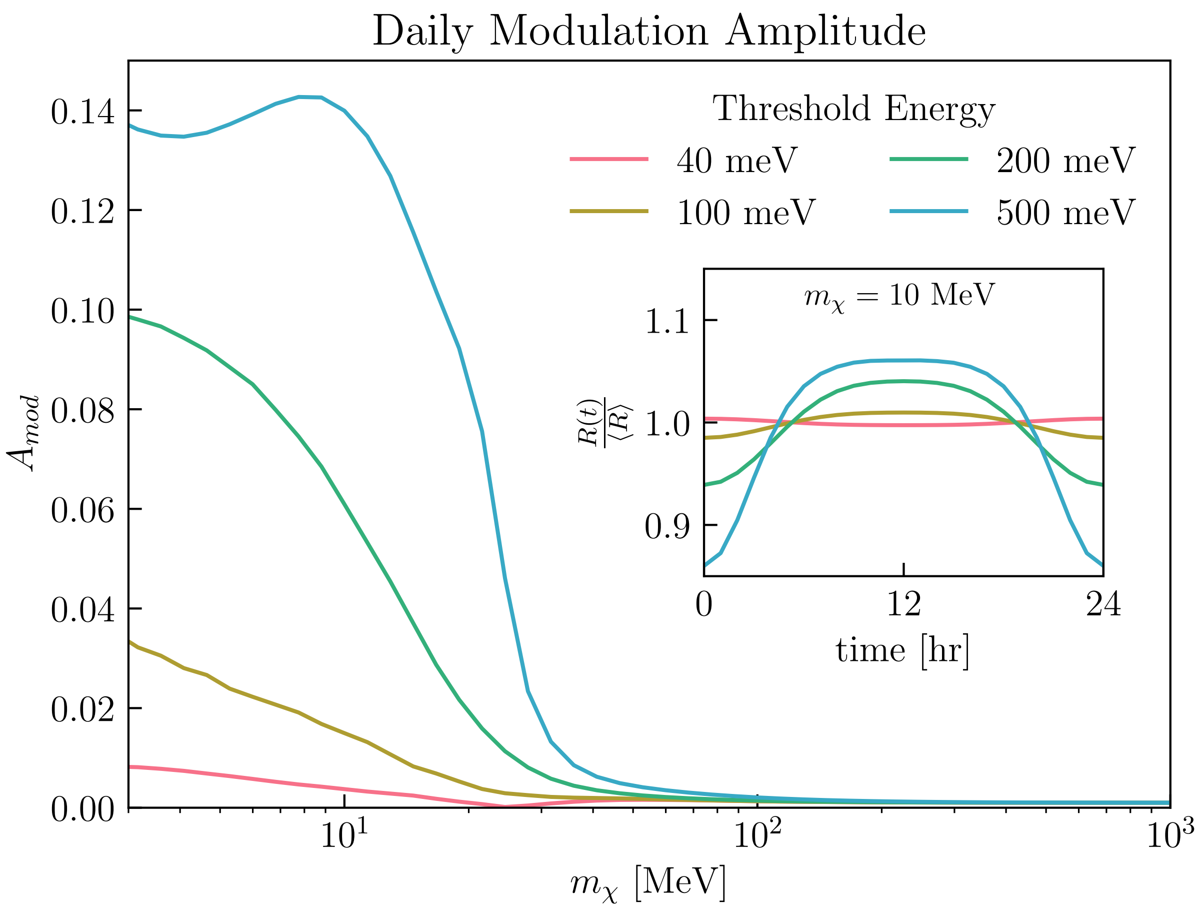
Density of states is now a tensor: $D(\omega) \rightarrow D^{ij}(\omega)$

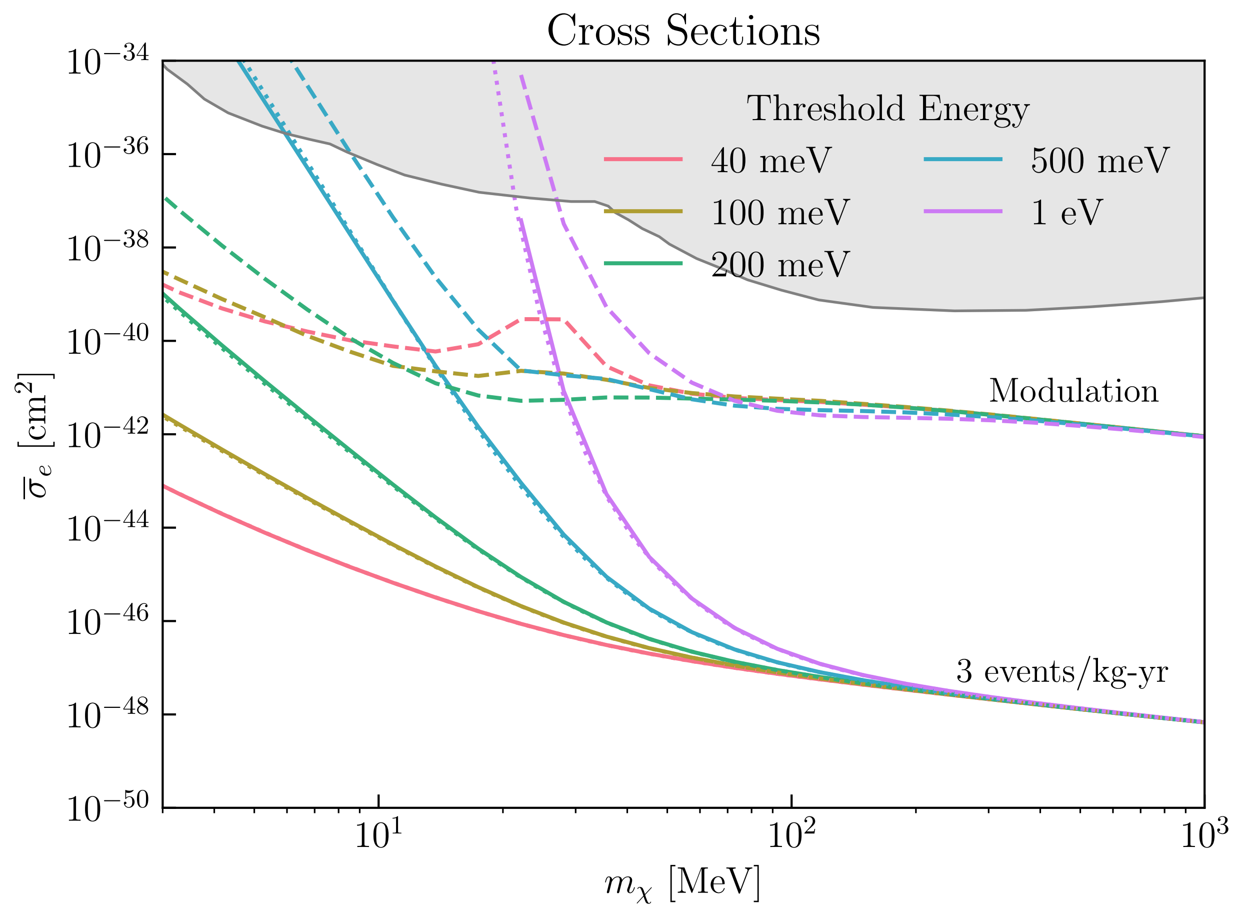
and scattering now depends on $\hat{q}_i D^{ij}(\omega) \hat{q}_j$



Daily modulation for multi phonons

Scattering in Al_2O_3 via massive dark photon mediator





Modulation calculation also to be made public in DarkELF (https://github.com/tongylin/DarkELF) when paper submitted.

Stratman, TL, to appear

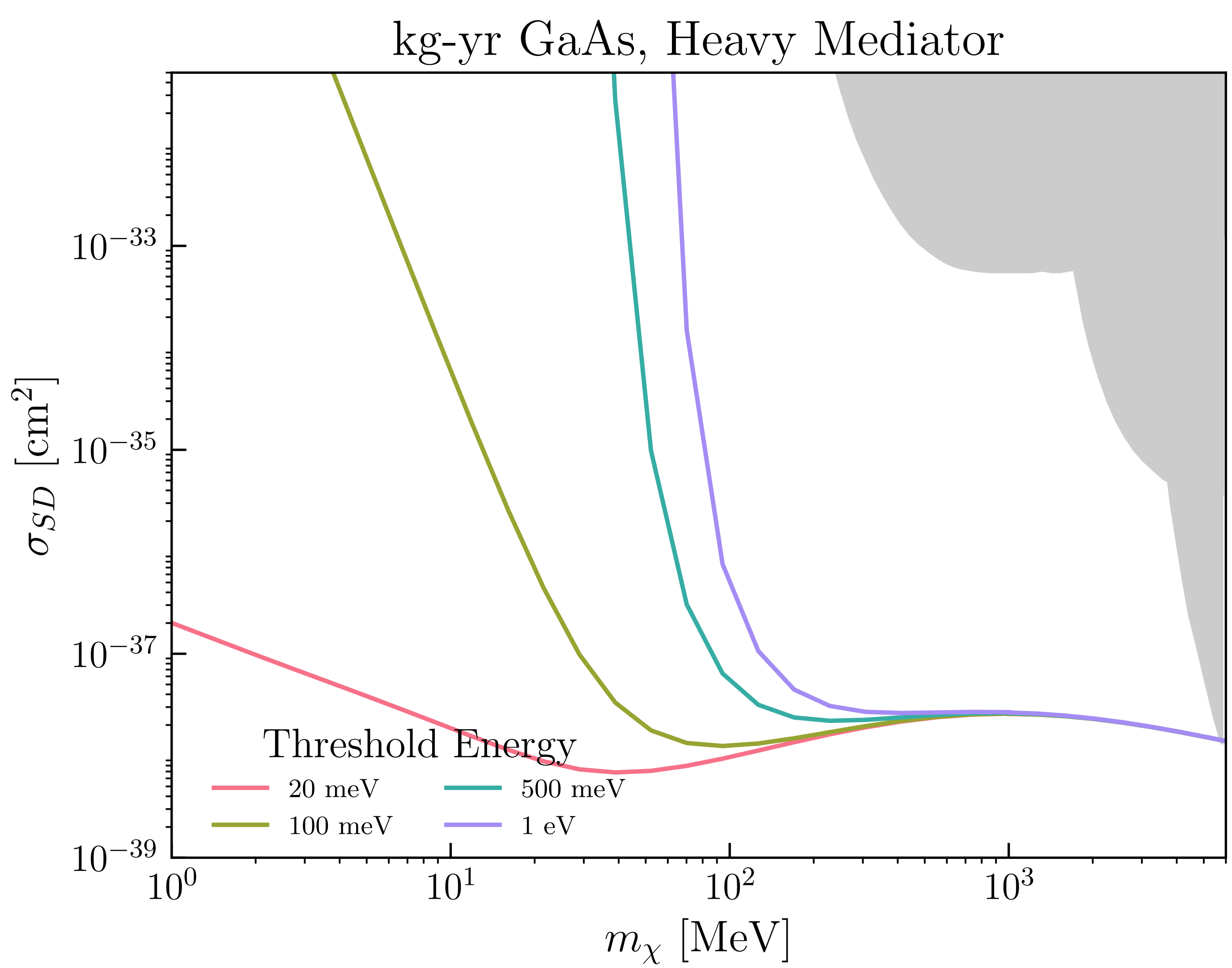
Spin-dependent scattering

With Pankaj Munbodh, Bethany Suter,
Stefania Gori, Simon Knapen (in progress)

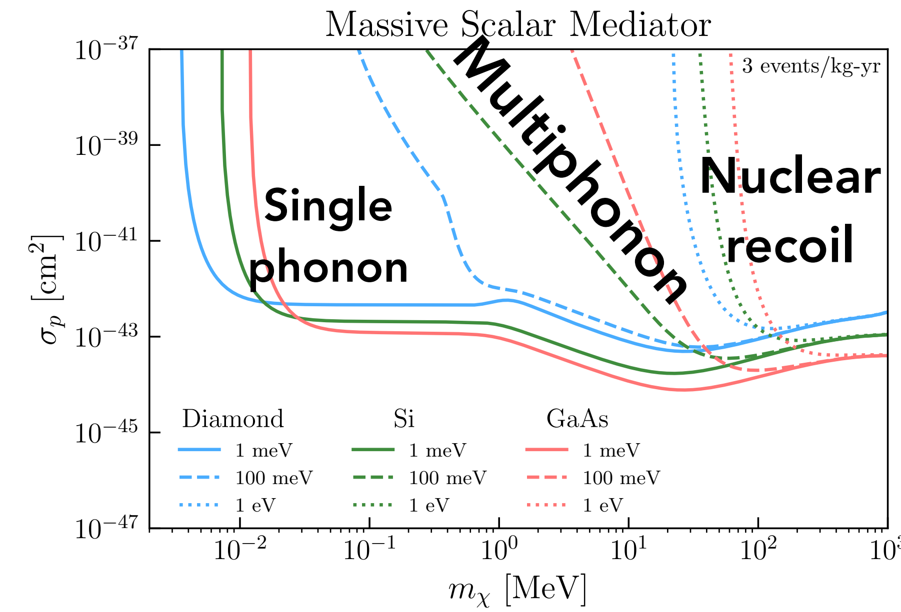
No long-range spin order in (typical) direct detection materials → spherical guinea pig approximation.

Also soon to appear in DarkELF!

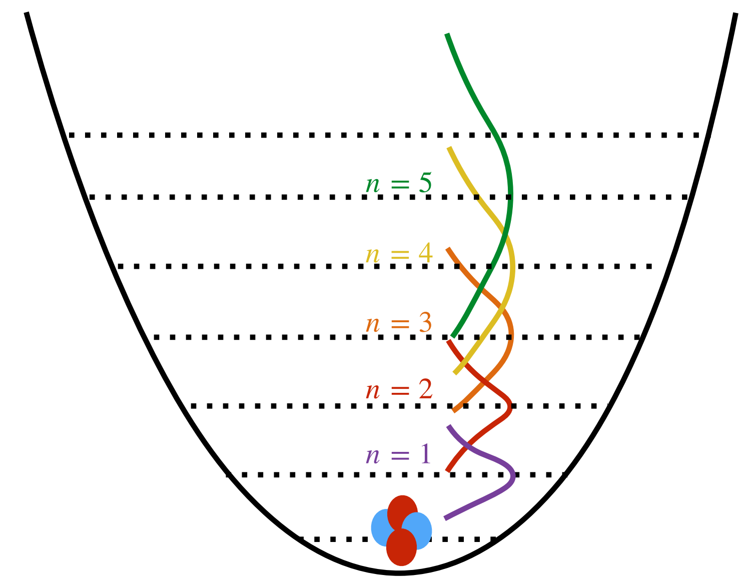
SD scattering off protons



Key takeaways



Multiphonons are the next step in the road towards single-phonon & MeV DM detection.



Full DFT multiphonon calculations are extremely daunting — we instead rely on a variety of approximations.

Theory progress: calculate anharmonic effects, anisotropies (daily modulation), spin-dependent scattering. Guinea pigs are actually pretty spherical.

Thanks!