Detecting μeV photons and meV phonons via inelastic charge tunnelling across Josephson junctions

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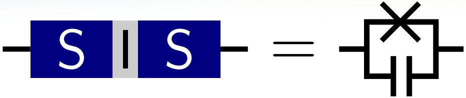
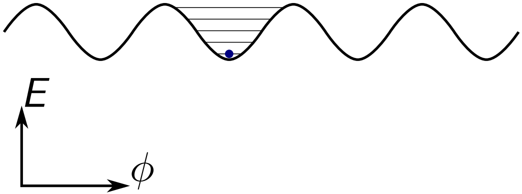
The Josephson junction in quantum circuits





- Josephson junction forms anharmonic oscillator ➔ qubit

$$H = -E_{\mathrm{J}}\cos(\phi)$$

$$V = \frac{\hbar}{2e}\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

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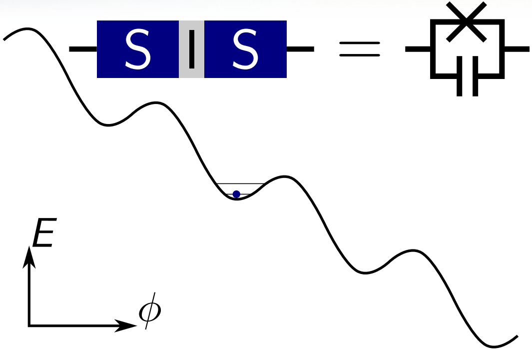

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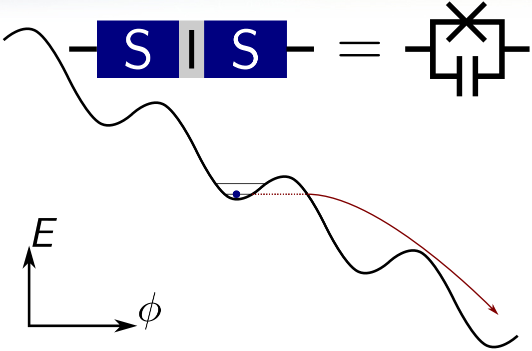


$$H = -E_J \cos(\phi)$$

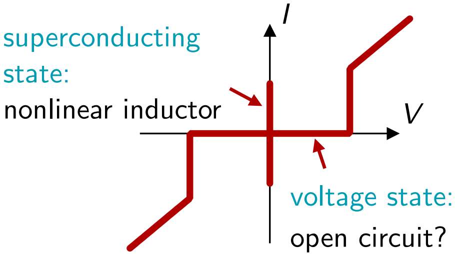
$$V = \frac{\hbar}{2e}\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

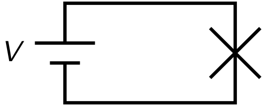
- Josephson junction forms anharmonic oscillator ➔ qubit
- DC current tilts potential
- current too high ➔ phase runs down potential
  - $V > 0$
  - energy gets dissipated somewhere
  - qubit is gone

Stay below the critical current!

Voltage state of the Josephson junction: Semi-classical view

Josephson junction in the voltage state is also dissipationless!





$$I = I_C \sin(\omega_J t)$$

$$\omega_J = \frac{2eV}{\hbar}, \quad I_C = \frac{2eE_J}{\hbar}$$

AC current but no DC current

Holst *et al.*, Phys. Rev. Lett. **73**, 3455 (1994)

Ingold, Nazarov, in Single Charge Tunnelling, cond-mat/0508728 (1992)

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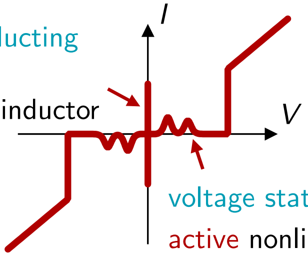
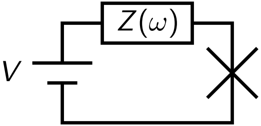
superconducting state:
nonlinear inductor



voltage state:
active nonlinear drive



$$I = I_C \sin(\omega_J t)$$

$$\omega_J = \frac{2eV}{\hbar}, \quad I_C = \frac{2eE_J}{\hbar}$$

Dissipated power

$$P = \mathrm{Re}\, Z(\omega_J) \frac{I_C^2}{2}$$

Power is drawn from bias:

$$I = \frac{P}{V} = \frac{2e}{\hbar} \frac{\mathrm{Re}\, Z(\omega_J)}{\omega_J} \frac{I_C^2}{2}$$

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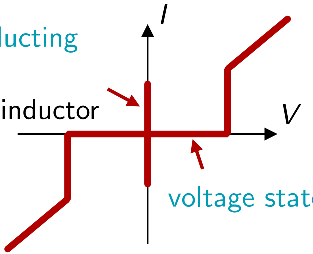
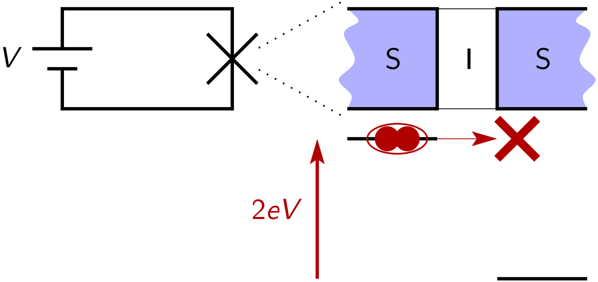


voltage state:



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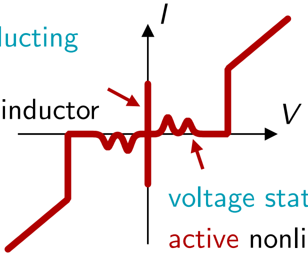

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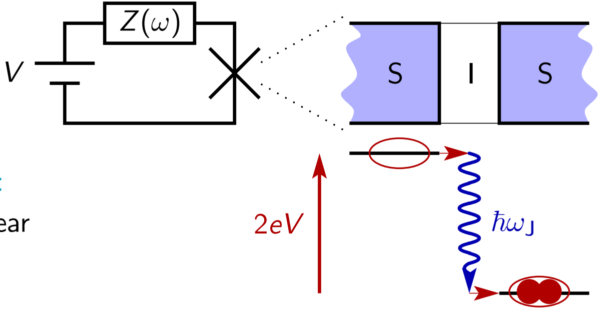

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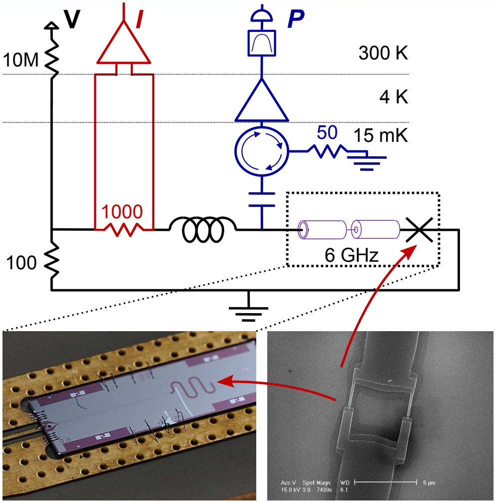
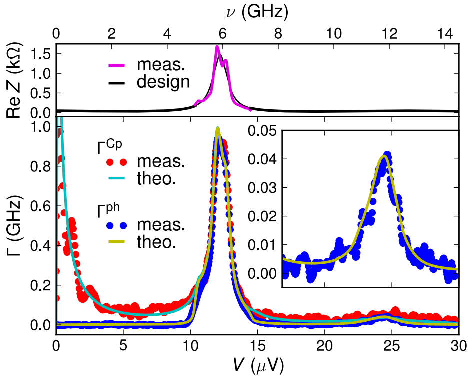
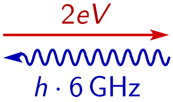
Bright side of inelastic Cooper-pair tunnelling







Hofheinz *et al.*, Phys. Rev. Lett. **106**, 217005 (2011)

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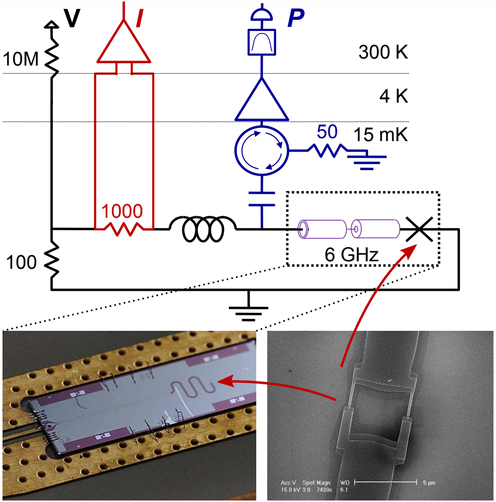
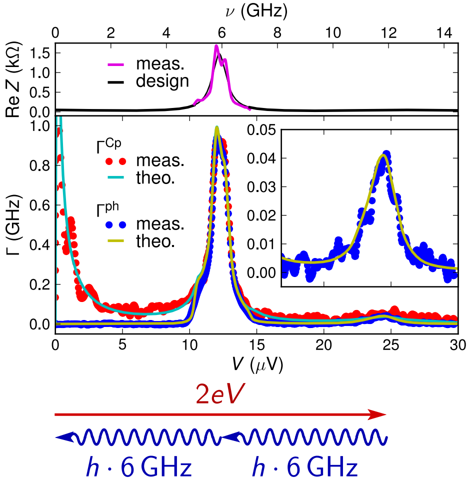
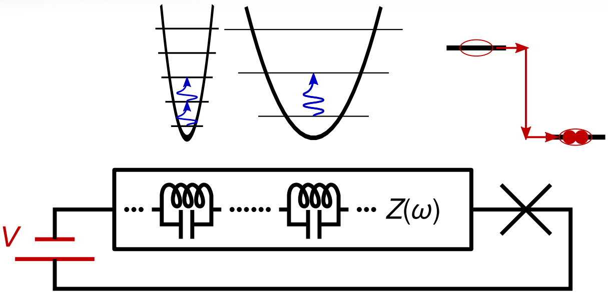




Hofheinz *et al.*, Phys. Rev. Lett. **106**, 217005 (2011)

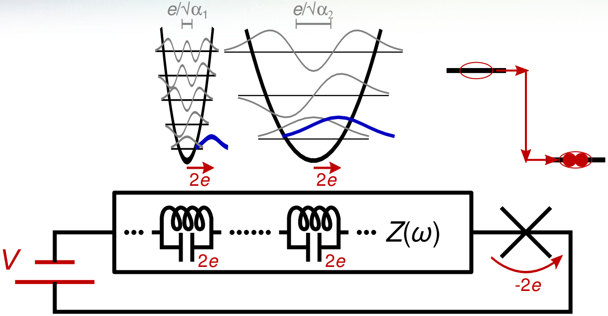
Inelastic Cooper pair tunnelling: Nonlinearity depends on impedance



$$\Gamma \propto E_J^2 \delta(2eV - n_1\hbar\omega_1 - n_2\hbar\omega_2 \ldots)$$

- One or several modes can absorb $2eV$ as photons

Inelastic Cooper pair tunnelling: Nonlinearity depends on impedance



$$M_n^{(k)} = \left| \langle n | e^{i\sqrt{\alpha_k}(a+a^\dagger)} | 0 \rangle \right|^2 = \frac{\alpha_k^n e^{-\alpha_k}}{n!}$$

$$\alpha_k = \pi \frac{4e^2}{h} Z_k$$

$\sqrt{\alpha_k}$: 0-point phase fluctuations

$$\Gamma \propto E_J^2 M_{n_1}^{(1)} M_{n_2}^{(2)} \cdots \delta(2eV - n_1\hbar\omega_1 - n_2\hbar\omega_2 \ldots)$$

- One or several modes can absorb $2eV$ as photons
- Z_k determine **how** $2eV$ is split up into photons

$Z(\omega)$ can be engineered, V controlled ➜ very versatile

Josephson photonics

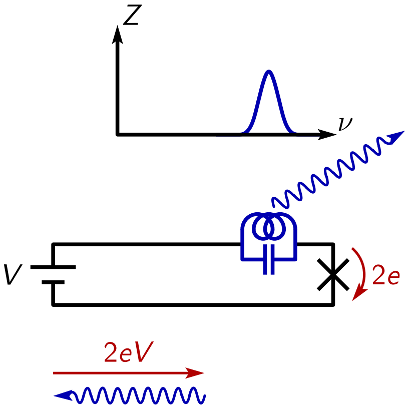
Engineering $Z(\nu)$ ➡ Full toolbox for wideband quantum microwave devices

Sources

- **Coherent**
- Single photons
- Entangled photons

Measurement

- Amplifiers
- Frequency shifters
- Photomultipliers



Holst *et al.*,
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Hofheinz *et al.*,
Phys. Rev. Lett. **106**, 217005 (2011)

Gramich *et al.*,
Phys. Rev. Lett. **111** 247002 (2013)

Chen *et al.*,
Phys. Rev. B **90**, 020506(R) (2014)

Cassidy *et al.*,
Science **355** 939 (2017)

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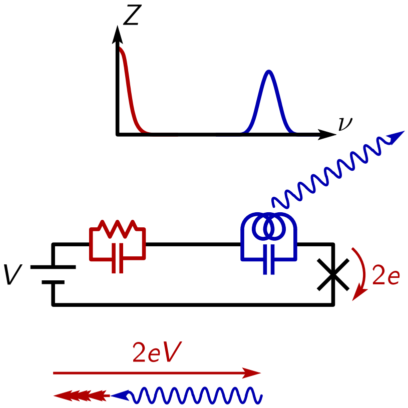

Leppäkangas *et al.*, Phys. Rev. Lett. **115** 027004 (2015)

Armour *et al.*, Phys. Rev. B **91** 184508 (2015)

Dambach *et al.*, Phys. Rev. B **92** 054508 (2015)

Souquet *et al.*, Phys. Rev. A **93** 060301 (2016)

Grimm *et al.*, Phys. Rev. X **9** 021016 (2019)

Rolland *et al.*, Phys. Rev. Lett. **122** 186804 (2019)

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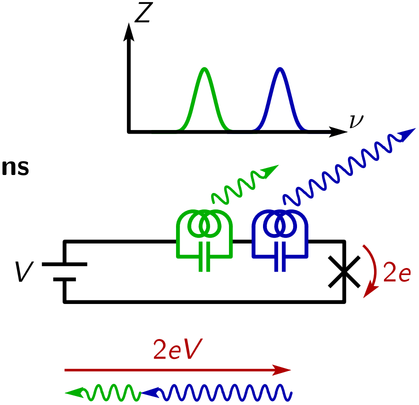

Padurariu *et al.*,
prb **86** 054514 (2012)

Leppäkangas *et al.*,
Phys. Rev. Lett. **110** 267004 (2013)

Trif *et al.*,
Phys. Rev. B **92** 014503 (2015)

Westig *et al.*,
Phys. Rev. Lett. **119** 137001 (2017)
Wood *et al.*,
Phys. Rev. B **104** 155424 (2021)

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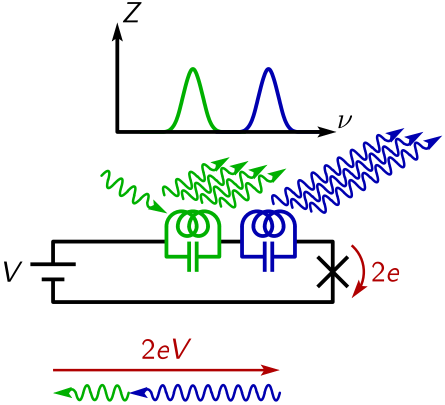

Safi *et al.*,
Phys. Rev. B **84** 205129 (2011)

Lähteenmäki *et al.*,
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Jebari *et al.*,
Nat. Electron. **1** 223 (2018)

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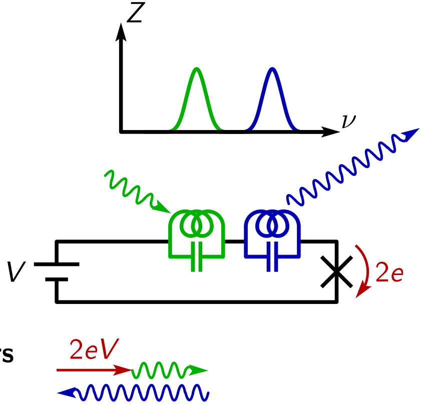

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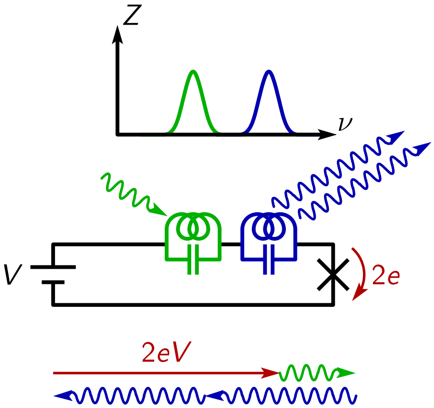
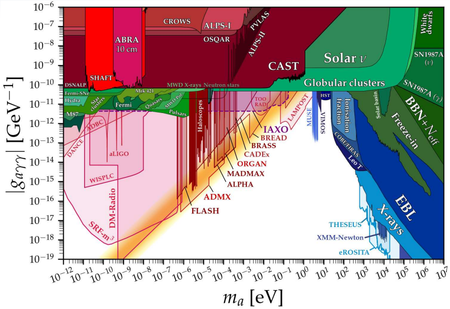


Leppäkangas *et al.*,
Phys. Rev. A **97** 013855 (2018)

Albert *et al.*,
Phys. Rev. X **14** 011011 (2024)

Quantum measurement devices for THz blind spot



Ciaran O'Hare, cajohare.github.io/AxionLimits

Gap frequencies $2\Delta/h$

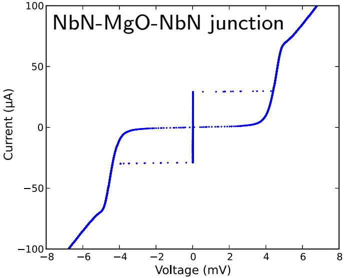
Al: 90 GHz

Nb: 700 GHz

NbN: 1.2 THz

Josephson photonics at high frequency

- No microwave pump needed
- Josephson inductance cancels
- Frequency only limited by gap



Grimm *et al.*, Supercond. Sci. Technol. **30** 105002 (2017)

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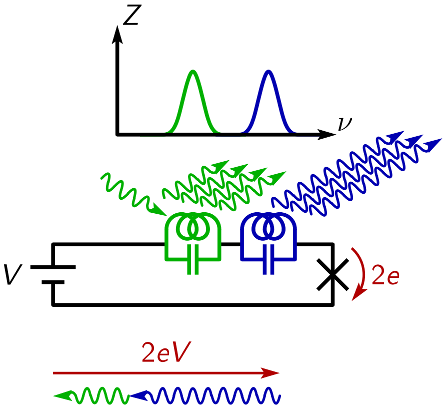

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Lähteenmäki *et al.*,
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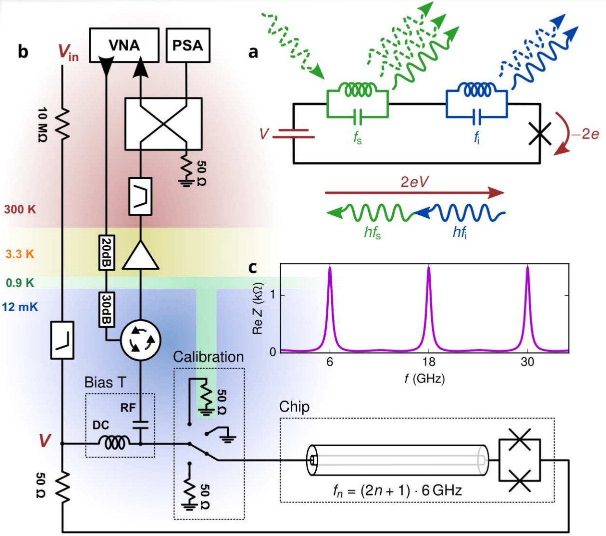
Jebari *et al.*,
Nat. Electron. **1** 223 (2018)

Weak nonlinearity: Amplification



- send signal to one of the modes
- chose any mode as idler
- bias at the sum of the two modes

→ quantum limited amplification?

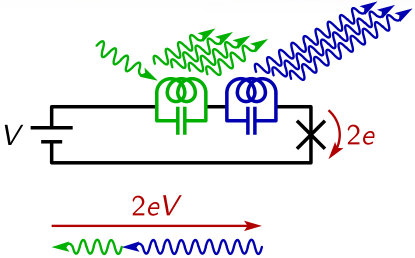
The Inelastic Cooper pair tunneling amplifier (ICTA)

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b - E_J \cos(\phi)$$

with

$$\phi = \omega_J t + \varphi_a \left(a^\dagger + a\right) + \varphi_b \left(b^\dagger + b\right)$$

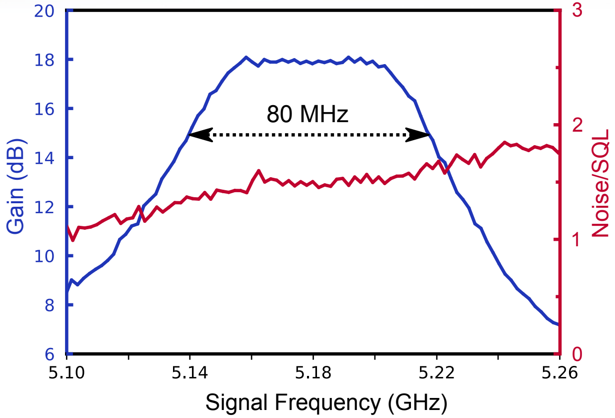
$$\varphi_i = \sqrt{\pi \frac{4e^2}{h} Z_i}$$



- Suppose small fields $\varphi_a a$, $\varphi_b b \ll 1$ ➔ expand to second order in $\varphi_a a$ and $\varphi_b b$
- Suppose $\frac{\varphi_a E_J}{2\hbar}$, $\frac{\varphi_b E_J}{2\hbar} \ll |\omega_a - \omega_b|$ ➔ RWA at $\omega_J = \omega_a + \omega_b$

$$E_J \cos(\phi) = \frac{E_J}{2}\left(e^{i\phi} + \text{h.c.}\right) \approx \frac{E_J^* \varphi_a \varphi_b}{2}\left(e^{i\omega_J t} a^\dagger b^\dagger + \text{h.c.}\right)$$

Inelastic Cooper-pair Tunneling Amplifier



$P_{\text{in}}^{-1\,\text{dB}} \approx -122\,\text{dBm} \text{ @ } G = 18\,\text{dB}$

Salha Jebari

Florian Blanchet

Ulrich Martel

Naveen Nehra

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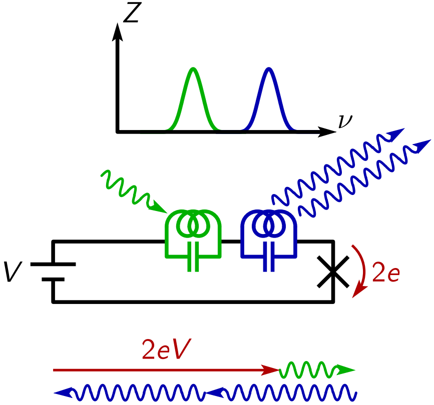

Leppäkangas *et al.*,
Phys. Rev. A **97** 013855 (2018)

Albert *et al.*,
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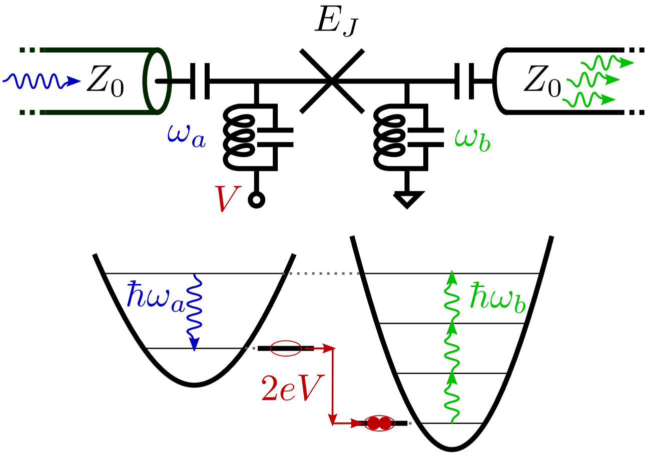
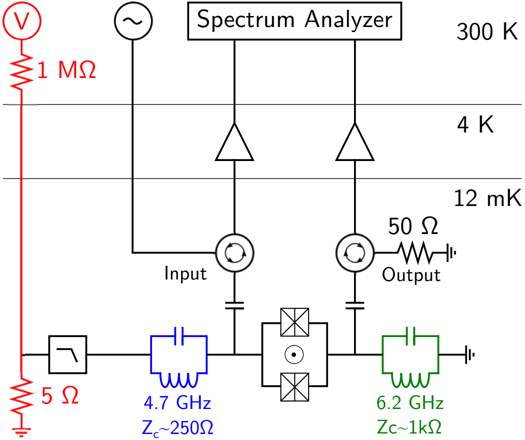
Strong nonlinearity: Photomultiplication



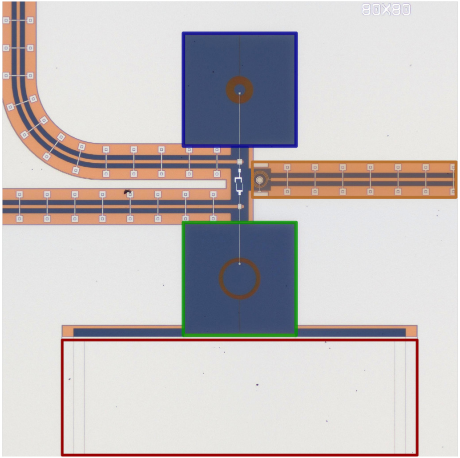
- spontaneous tunneling forbidden
- incident photon provides energy complement
- ➜ tunneling creates several photons in other mode
- process involving ≥ 3 photons
- need $Z_{\text{out}} \sim 2\,\text{k}\Omega$
- adjust E_J to cancel reflection

Leppäkangas *et al.*, Phys. Rev. A **97** 013855 (2018)

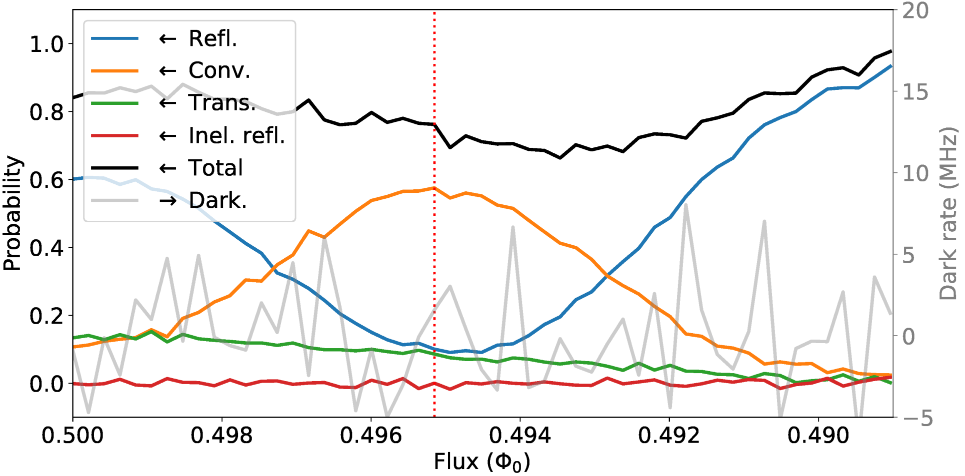
Device



Albert *et al.*, Phys. Rev. X **14** 011011 (2024)

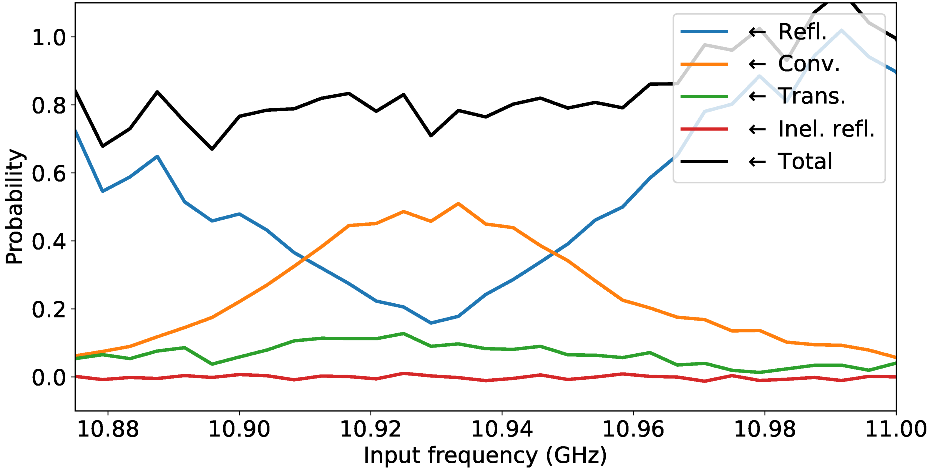


Conversion 1 ➡ 3



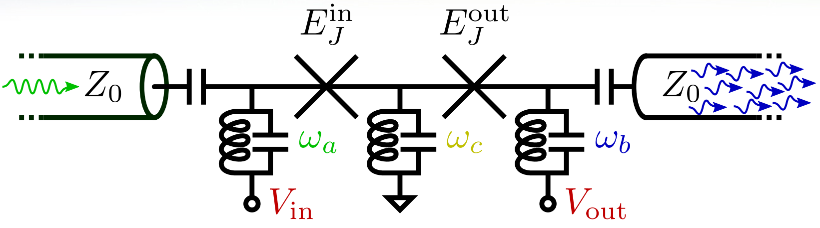
Input frequency	Input power	Bias $2eV/h$	Efficiency	Dark rate
10.933 GHz	−120 dBm	4.6 GHz	59 %	200 kHz

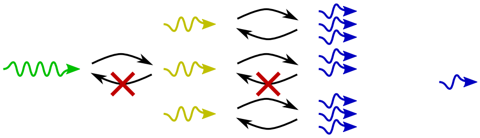
Bandwidth



Input power	Bias $2eV/h$	Flux	Efficiency	Bandwidth
-120 dBm	4.6 GHz	$0.497\,\phi_0$	0.59 %	50 MHz

Cascaded photomultipliers ➡ single photon detector





Leppäkangas *et al.*, Phys. Rev. A **97** 013855 (2018)

- photon is either fully converted or reflected
- impedance matching by tuning one Josephson energy
- need 2 to 3 stages followed by quantum limited amplifier
- number resolving, no dead time

Photomultiplier

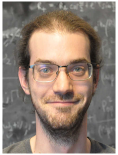
Where we are at:

- linear to a few photons
- 0.6 quantum efficiency
- dark rate $\sim 200\,\text{kHz}$
- bandwidth $\sim 50\,\text{MHz}$
- for single photon detector:
  - cascade 2 or 3 stages
  - follow by linear amplifier
  - follow by threshold detector
  - expect dark count rate $<$ dark rate

Juha Leppäkangas

Romain Albert

Joël Griesmar

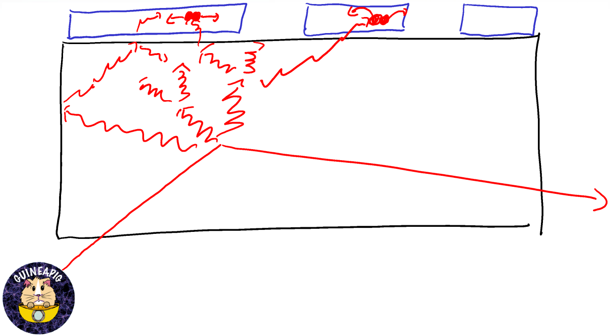


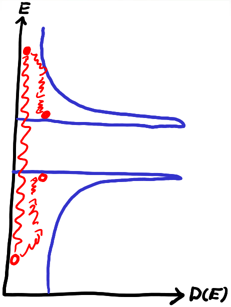
Nicolas Bourlet

Leppäkangas *et al.*, Phys. Rev. A **97** 013855 (2018)

Albert *et al.*, Phys. Rev. X **14** 011011 (2024)

Extending to phonons

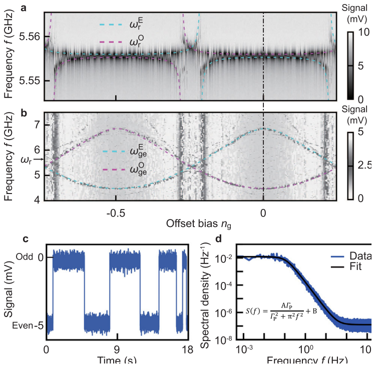




- Spherical guineapig scatters in substrate
- Coupling of substrate phonons to superconducting film?

- Phonons break Cooper pairs
- Quasi particles relax to quasi-thermal state.

Quasi-particle tunneling across voltage biased junction



Pan *et al.*, Nat. Comm. **13** 7196 (2022)

So far

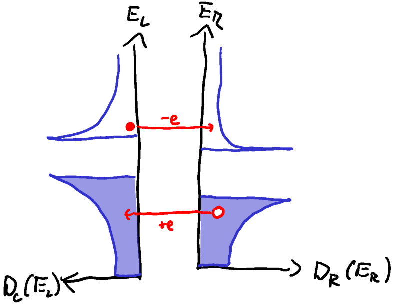
- Measurement of large QP numbers (KIT, SNSPD, ...)
  ➔ Poor energy resolution
- Parity / number fluctuations due to diffusion through junction
  ➔ Assessing total energy difficult

Good energy resolution with

- Junction with preferred tunneling direction
- Way to count number of tunnelled QPs

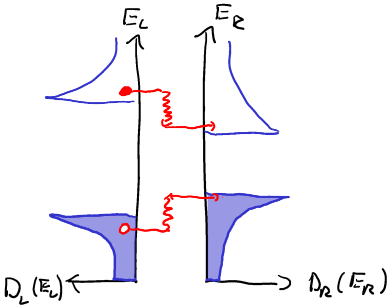
Tunneling with preferred direction

Voltage-biased junction



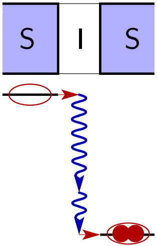
- Current $\propto$ QP density
- Readout with charge sensor
- Background: Inelastic CP tunneling

Unbiased hetero-junction



- Photon flux $\propto$ QP density
- Readout with photomultiplier
- Need $\hbar\omega_P \approx \Delta_L - \Delta_R$

Superconducting charge tunneling devices



Josephson photonics

- no microwave pump needed
- quantum limited amplification
- photon number amplification
- not limited by plasma frequency
- expect photon detection up to $\sim$ meV

Jebari *et al.*, Nat. Electron. **1** 223 (2018)

Albert *et al.*, Phys. Rev. X **14** 011011 (2024)

Quasiparticle tunneling

- energy funnel (relaxation to Δ)
- spatial funnel (bulk phonons absorbed in circuit)
- extends quantum circuits to direct detection $>$ meV?

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prompt

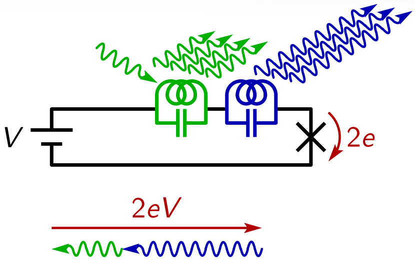
The Inelastic Cooper pair tunneling amplifier (ICTA)

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b - E_J \cos(\phi)$$

with

$$\phi = \omega_J t + \varphi_a \left(a^\dagger + a\right) + \varphi_b \left(b^\dagger + b\right)$$

$$\varphi_i = \sqrt{\pi \frac{4e^2}{h} Z_i}$$



- Suppose small fields $\varphi_a a$, $\varphi_b b \ll 1$ ➔ expand to second order in $\varphi_a a$ and $\varphi_b b$
- Suppose $\frac{\varphi_a E_J}{2\hbar}$, $\frac{\varphi_b E_J}{2\hbar} \ll |\omega_a - \omega_b|$ ➔ RWA at $\omega_J = \omega_a + \omega_b$

$$E_J \cos(\phi) = \frac{E_J}{2}\left(e^{i\phi} + \text{h.c.}\right) \approx \frac{E_J^* \varphi_a \varphi_b}{2}\left(e^{i\omega_J t} a^\dagger b^\dagger + \text{h.c.}\right)$$

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Usual parametric amplifier Hamiltonian

ICTA	↔	JPA
Josephson energy E_J	↔	pump power
0-point fluctuations φ_i	↔	participation ratio
voltage bias ω_J	↔	pump frequency

- Suppose small fields $\varphi_a a$, $\varphi_b b \ll 1$ → expand to second order in $\varphi_a a$ and $\varphi_b b$
- Suppose $\frac{\varphi_a E_J}{2\hbar}$, $\frac{\varphi_b E_J}{2\hbar} \ll |\omega_a - \omega_b|$ → RWA at $\omega_J = \omega_a + \omega_b$

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From here follow JPC derivation Abdo *et al.* Phys. Rev. B **87** 014508 (2013)

ICTA power handling

$$H = \hbar\omega_a a^\dagger a + \hbar\omega_b b^\dagger b - E_J \cos(\phi)$$

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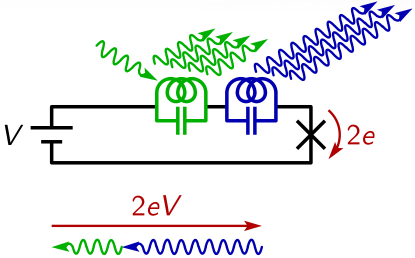

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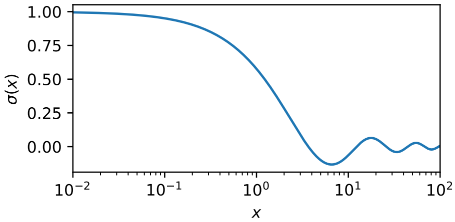
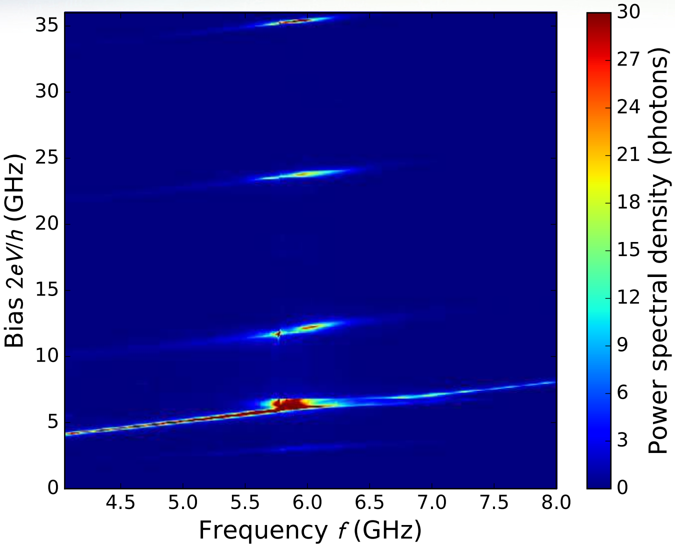


- ~~Suppose small fields $\varphi_a a$, $\varphi_b b \ll 1$ ➡ expand to second order in $\varphi_a a$ and $\varphi_b b$~~
- Suppose $\frac{\varphi_a E_J}{2\hbar}, \frac{\varphi_b E_J}{2\hbar} \ll |\omega_a - \omega_b|$ ➡ RWA at $\omega_J = \omega_a + \omega_b$

$$E_J \cos(\phi) = \frac{E_J}{2}\left(e^{i\phi} + \text{h.c.}\right) \approx \frac{E_J^* \varphi_a \varphi_b}{2}\left(e^{i\omega_J t} : a^\dagger b^\dagger \sigma\left(\varphi_a^2 a^\dagger a\right) \sigma\left(\varphi_b^2 b^\dagger b\right) : + \text{h.c.}\right)$$

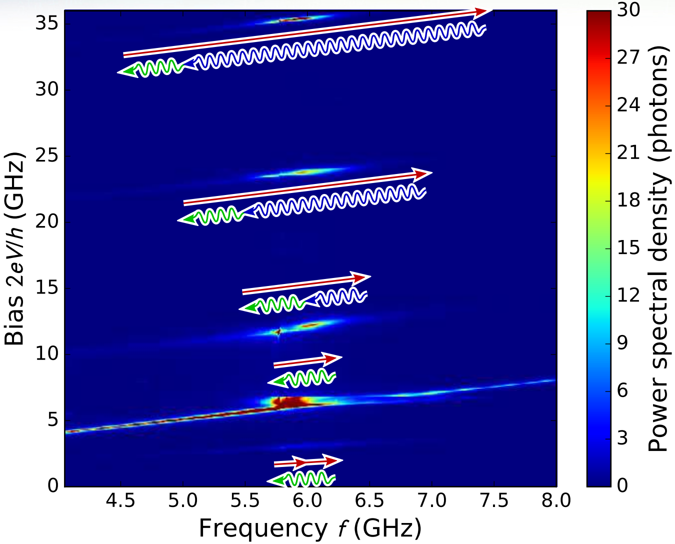
with $\sigma(x) = \frac{J_1(2\sqrt{x})}{\sqrt{x}}$

Power spectral density



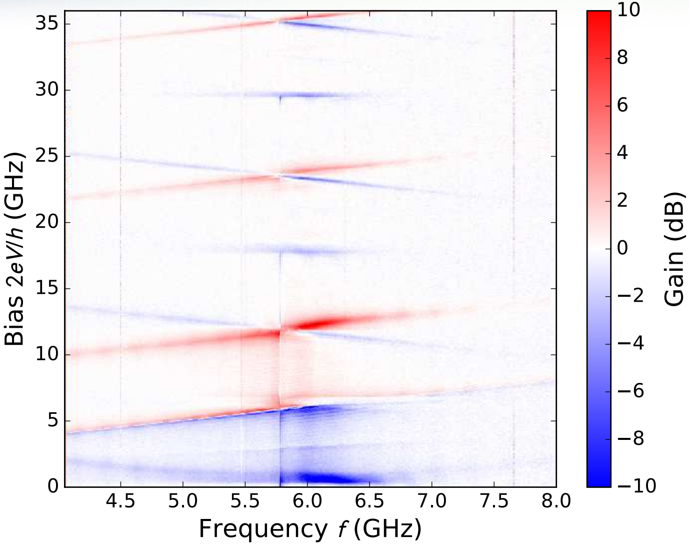
- Same sample as in the beginning
- Resolve photon emission rate in frequency

Power spectral density



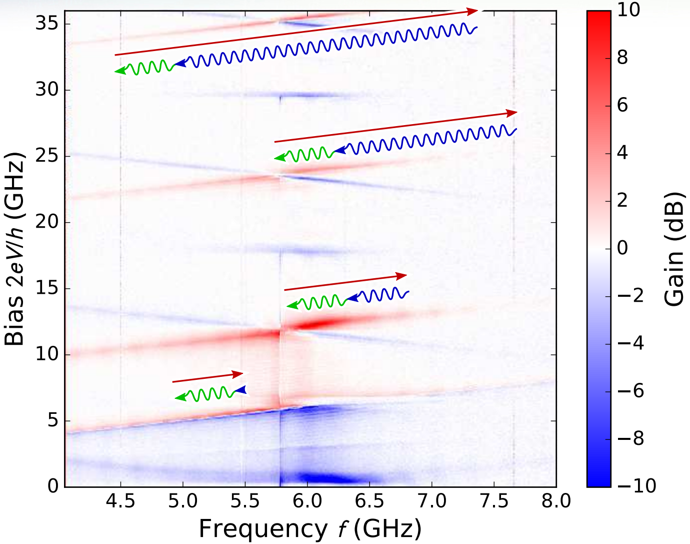
- Same sample as in the beginning
- Resolve photon emission rate in frequency
- Amplifier noise

Gain



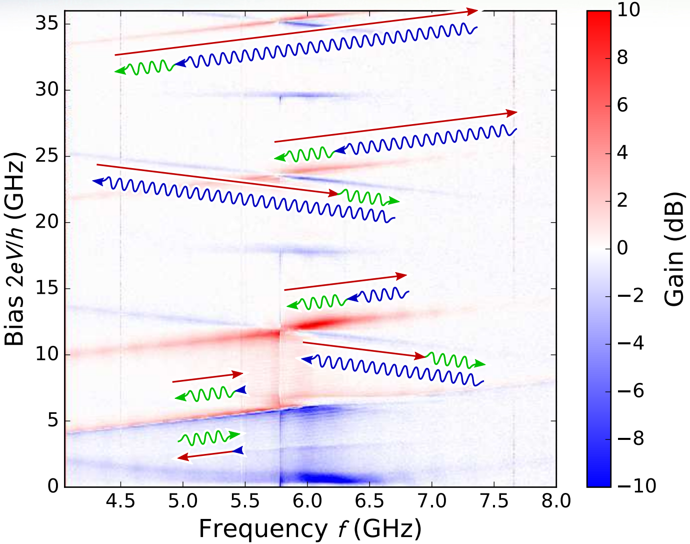
35
30
25
20
15
10
5
0
Bias 2eV/h (GHz)
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
Frequency f (GHz)
10
8
6
4
2
0
−2
−4
−6
−8
−10
Gain (dB)

Gain



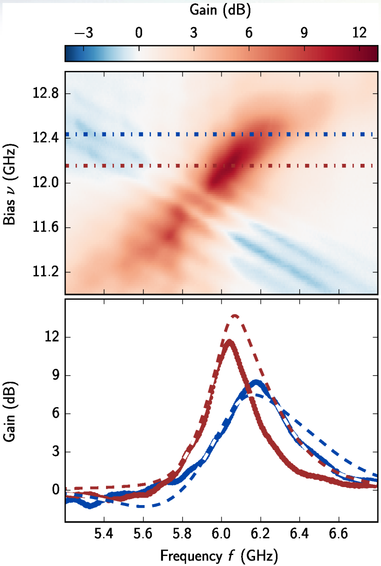
- Down conversion
- Gain

Gain



- Down conversion
- Gain
- Frequency conversion
- Loss

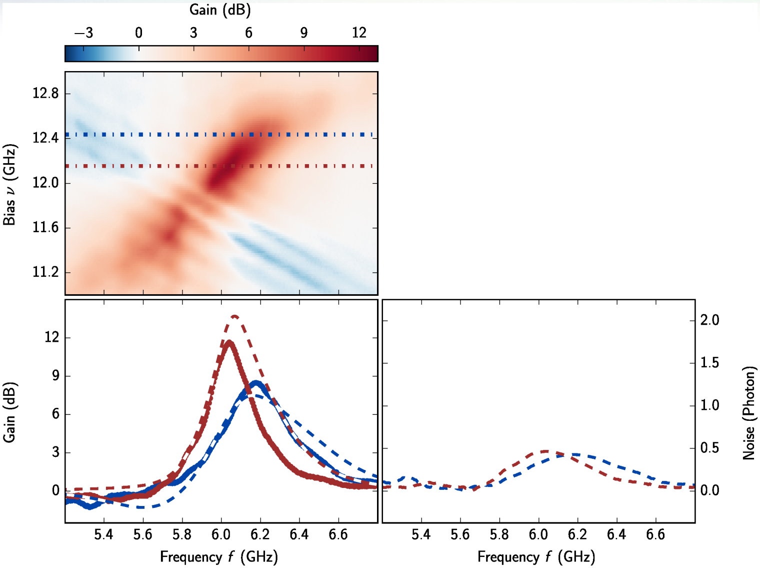
Amplification close to the quantum limit



- Gain > 10 dB for sample not designed as amplifier
- Qualitatively explained by $P(E)$ theory

Jebari *et al.*, Nat. Electron. **1** 223 (2018)

Amplification close to the quantum limit



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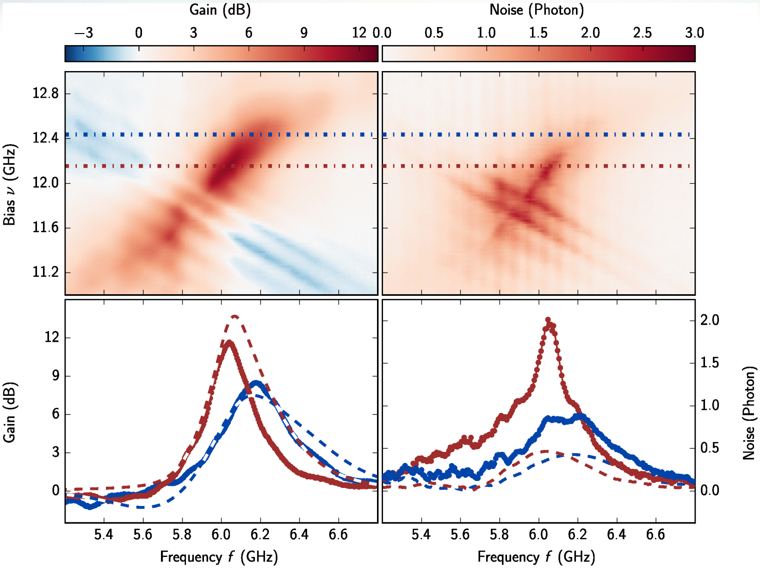

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- quantum limit $\frac{1}{2}|1 - G^{-1}|$
- Best noise $\sim 2 \times$ QL

Jebari *et al.*, Nat. Electron. **1** 223 (2018)

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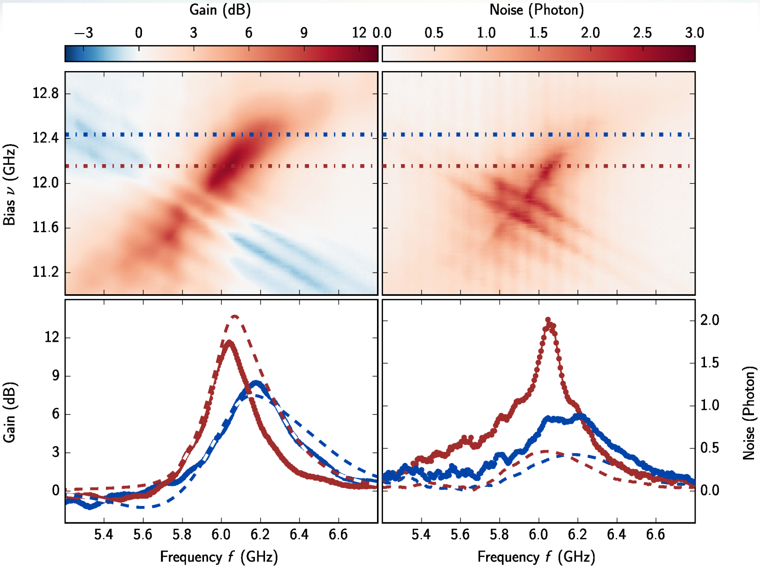

Limited performance

- Gain limited to $\sim 10\,\mathrm{dB}$
- Best noise $\sim 2 \times \mathrm{QL}$

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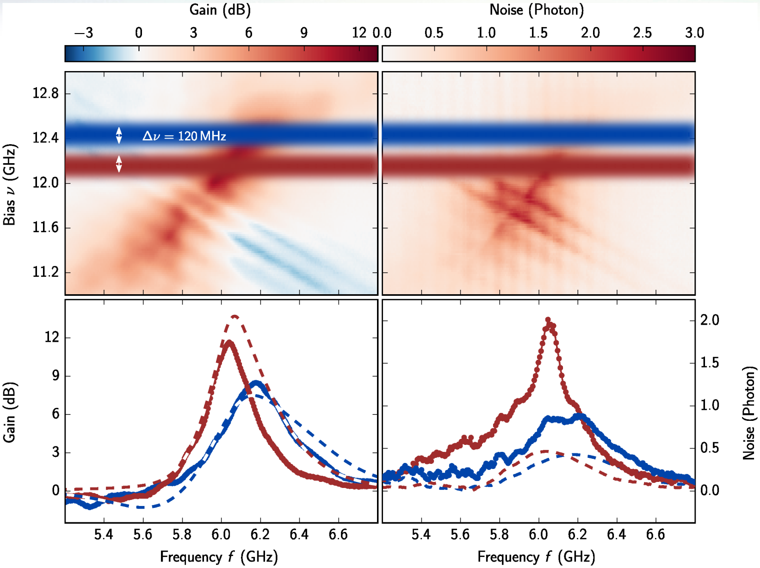
Amplification close to the quantum limit



Limited performance

- Gain limited to $\sim 10\,\text{dB}$
- Best noise $\sim 2 \times$ QL

Reason: Pump fluctuations $\Delta\nu$

- JPA: $\sim 1\,\mu\text{Hz}$
- ICTA: $\sim 100\,\text{MHz}$

Optimize:

- reduce voltage noise
- increase bandwidth

Jebari *et al.*, Nat. Electron. **1** 223 (2018)