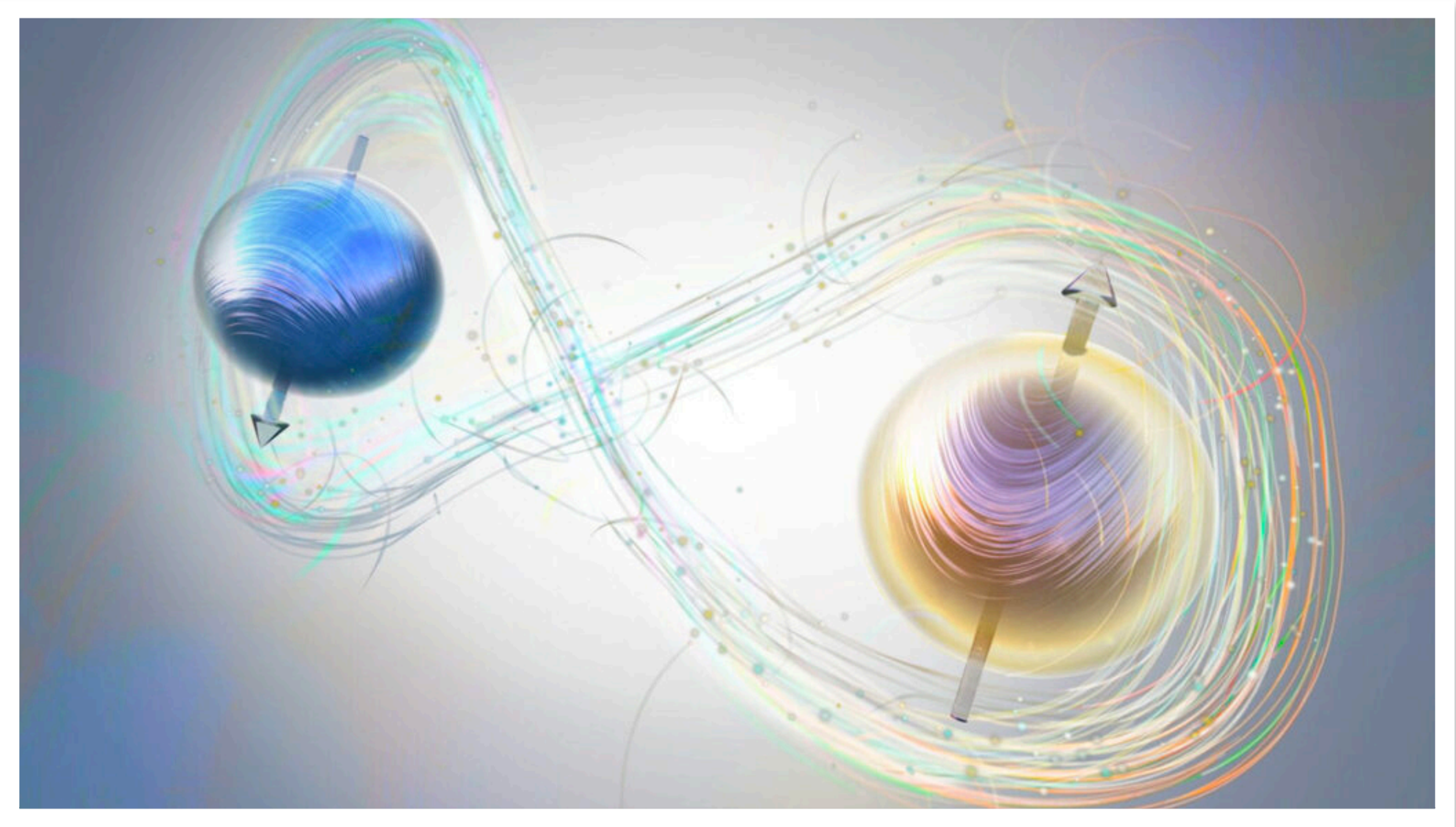
Entanglement and Bell inequalities with boosted top quarks

Physics Potential of Future Colliders (TRIUMF), Sep 19, 2024

Dorival Gonçalves



Dong, DG, Kong, Navarro '23
Dong, DG, Kong, Larkoski, Navarro '24
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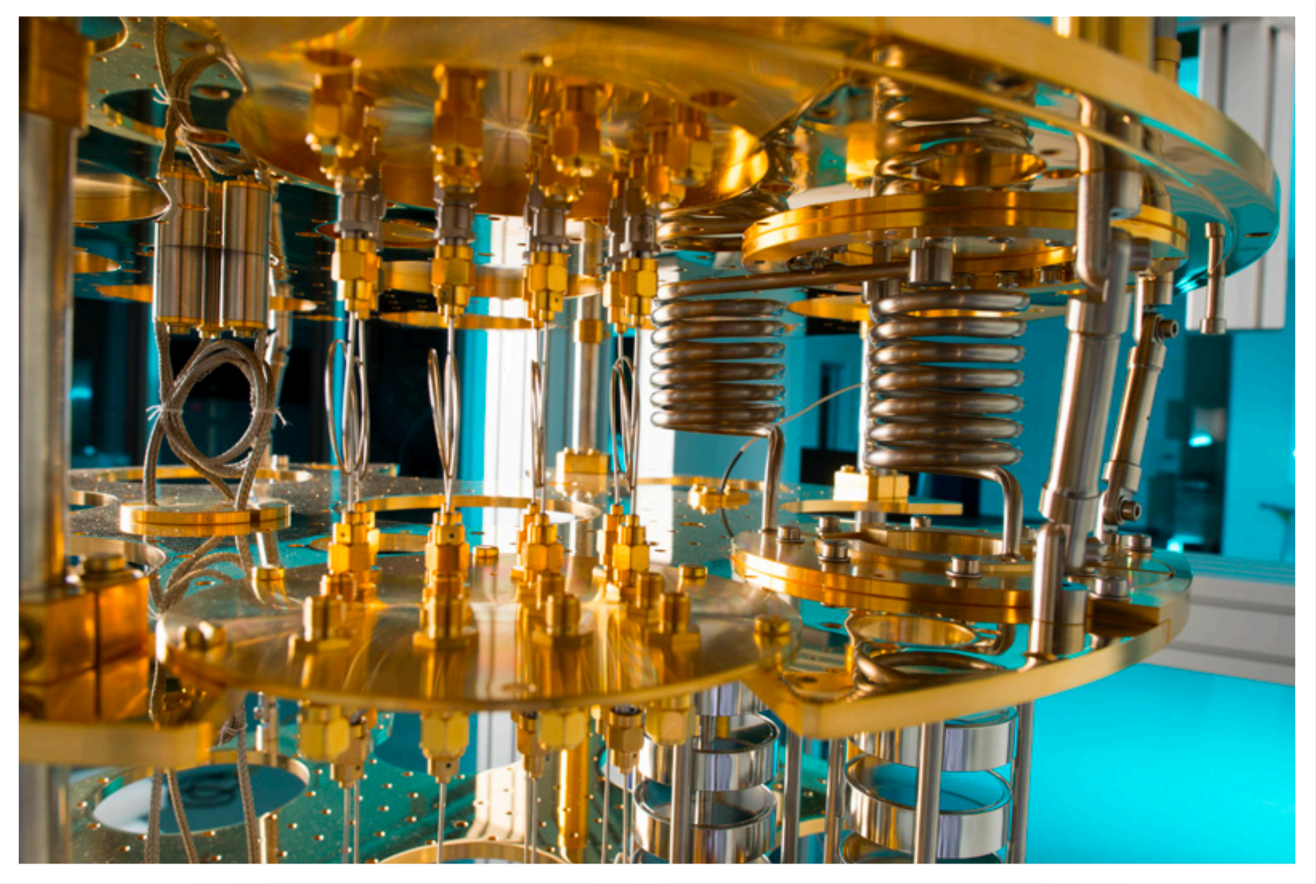
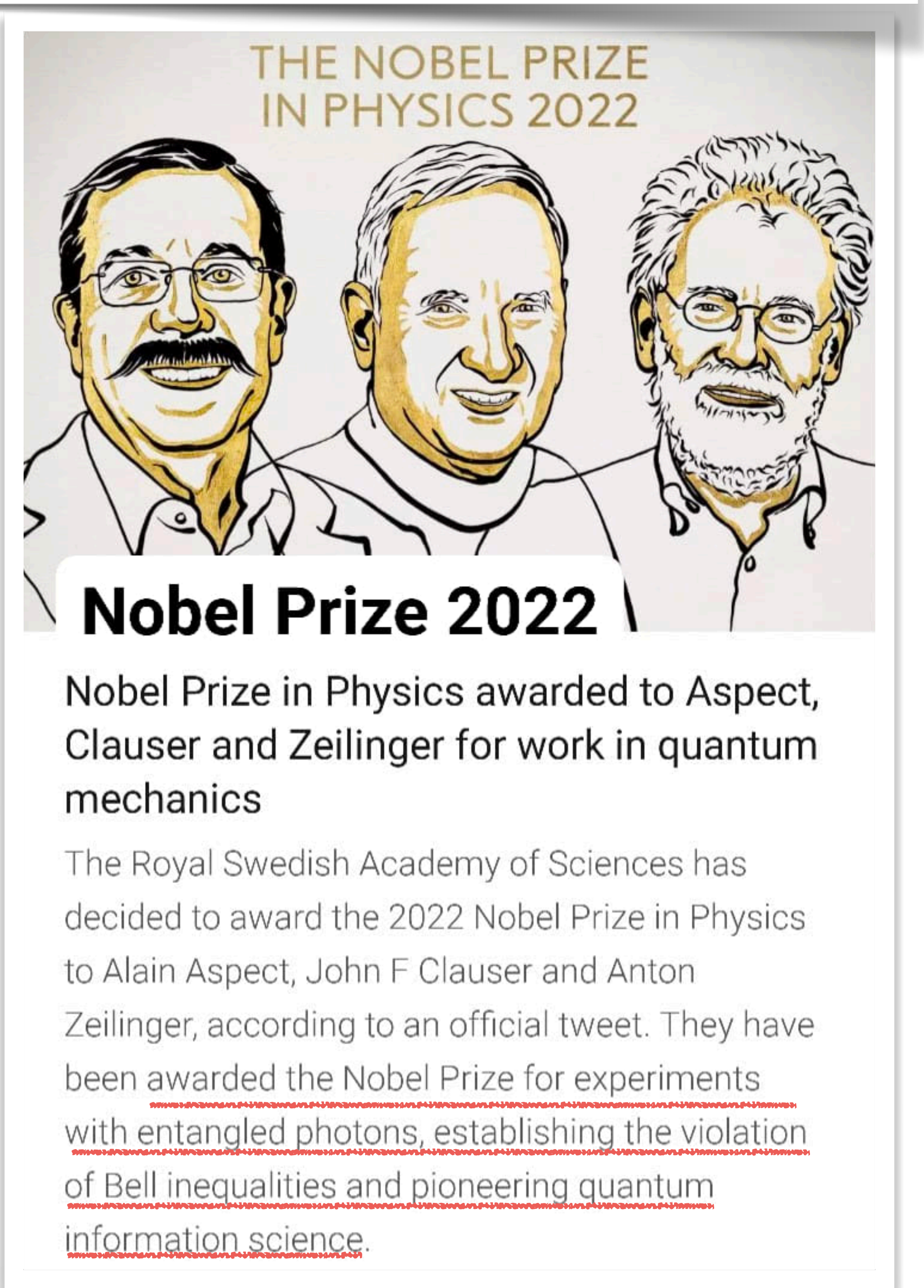
Entanglement and Bell Inequalities with Boosted Top Quarks





Nobel Prize 2022

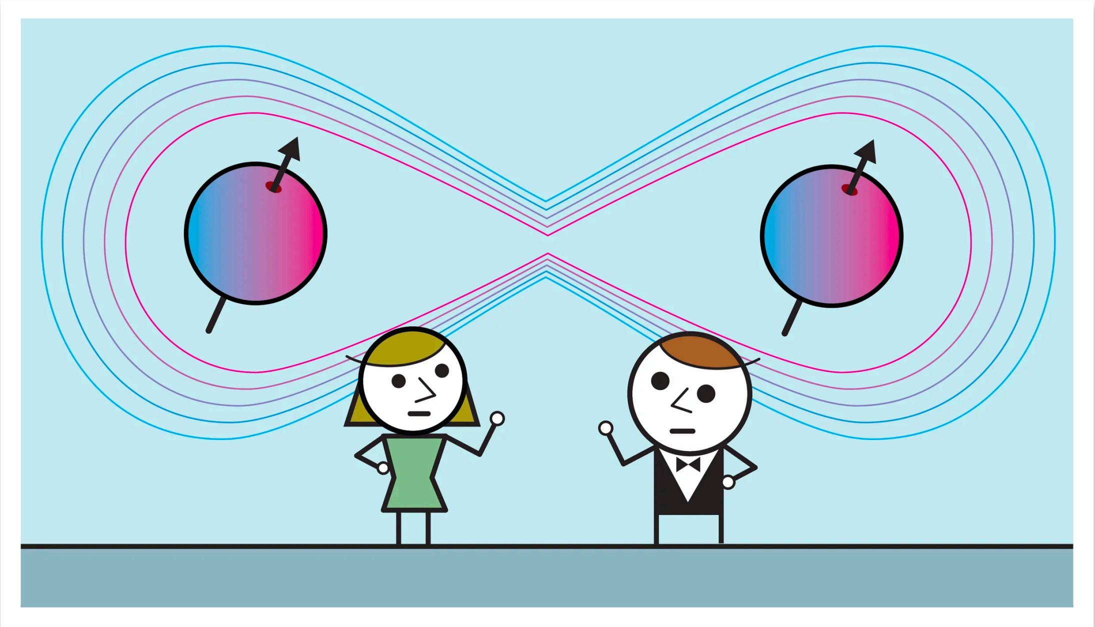
Nobel Prize in Physics awarded to Aspect, Clauser and Zeilinger for work in quantum mechanics

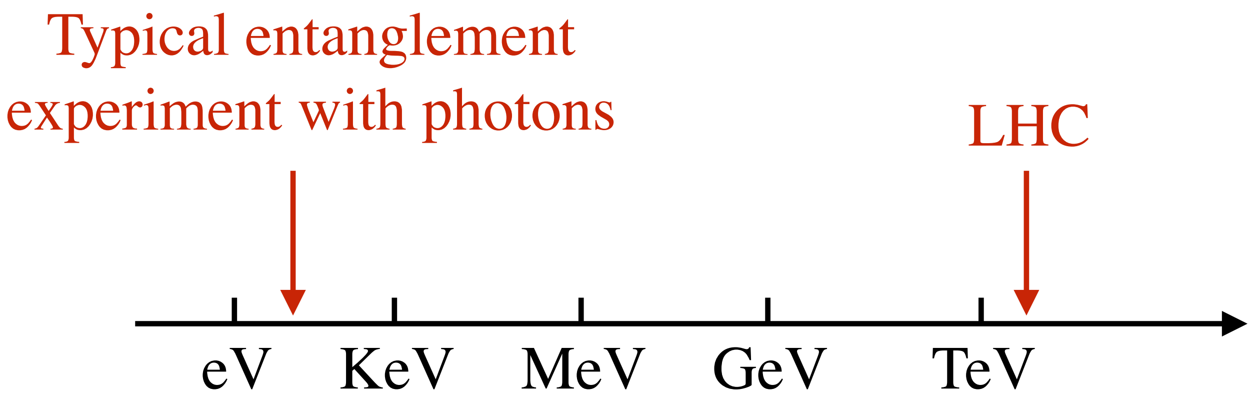
The Royal Swedish Academy of Sciences has decided to award the 2022 Nobel Prize in Physics to Alain Aspect, John F Clauser and Anton Zeilinger, according to an official tweet. They have been awarded the Nobel Prize for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science.

- QI theory provides a set of tools designed to probe the inner behavior of QM. While these phenomena have been widely tested at low energies, their study at higher energy scales is still largely unexplored

Entanglement and Bell Inequalities with Boosted Top Quarks

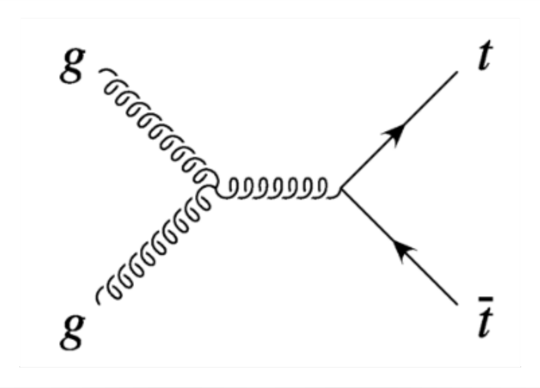
- LHC can provide a unique environment to study entanglement and violation of Bell's inequalities at the highest energy available to date





- Top quark pair production is an optimal candidate for these studies

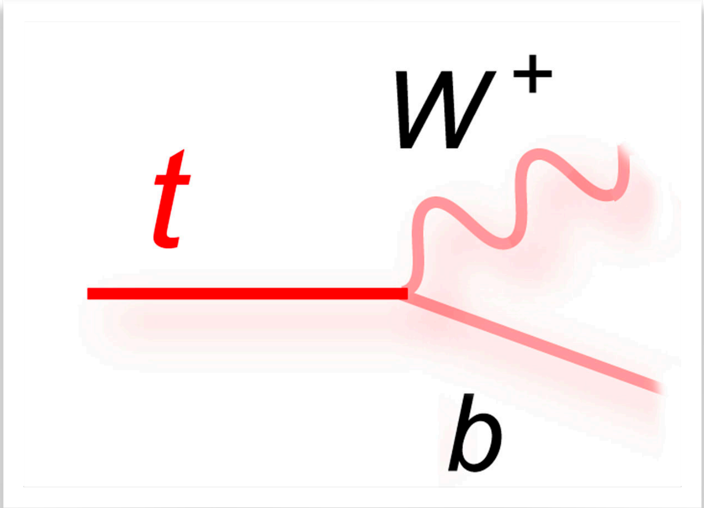
Afik, Nova '20; Fabbrichesi, Floreanini, Panizzo '21; Severi, Boschi, Maltoni, Sioli '21
Saavedra, Casas '22; Severi, Vryonidou 22
Dong, **DG**, Kong, Navarro '23
Han, Low, Wu '23
ATLAS Nature vol 633, 542-547 (2024)
CMS 2406.03976
CMS 2409.11067



Top quark as a one qubit system

- As a spin-1/2 particle, the most general spin density matrix for the top quark is

$$\rho = \frac{\mathbb{I}+B_i\sigma_i}{2}$$



- Characterized by three parameters: $B_i = \langle\sigma_i\rangle = tr(\sigma_i\rho)$

→ However, the top quark decays. In general, spin information could be lost by hadronization or spin decorrelation effects

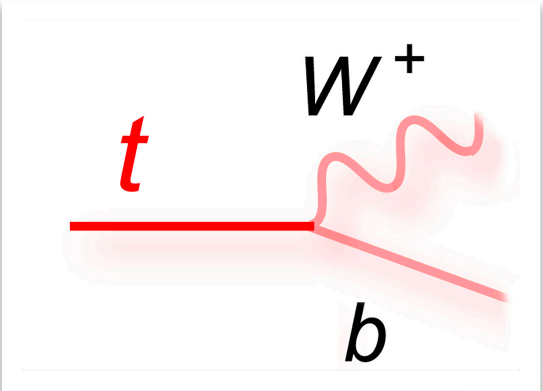
Top Quark is Unique

- Decays before it hadronizes or its spin flips

$$\tau_{top} \approx 5 \times 10^{-25} s$$

$$\tau_{had} \sim 1/\Lambda_{QCD} \sim 10^{-24} s$$

$$\tau_{flip} \sim m_t/\Lambda_{QCD}^2 \sim 10^{-21} s$$



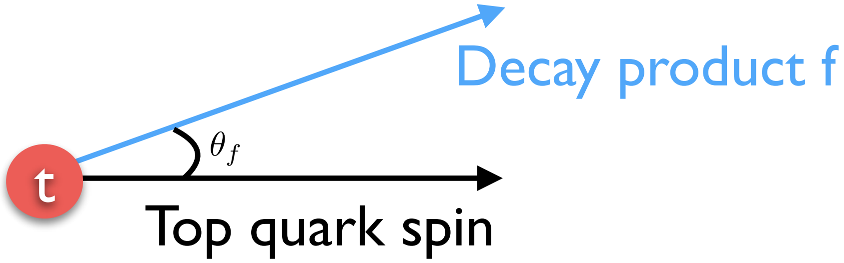
Bottom quark is several orders of magnitude behind $\tau_b \approx 10^{-12} s$

- Top polarization directly observable via angular distributions of its decay products

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \omega_f \cos\theta_f)$$

	$l^+, \bar{d}$	b	$\bar{\nu}, u$
ω_f	1	-0.4	-0.3

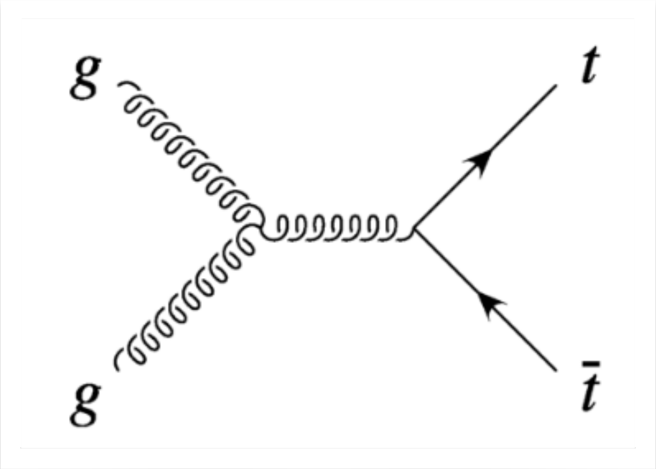
Spin analyzing power: maximum for charged leptons



Top quark pair production as a two qubit system

- The most general two-qubit system can be represented by

$$\rho = \frac{\mathbb{I}\otimes\mathbb{I}+\left(B_i\sigma_i\otimes\mathbb{I}+\bar{B}_i\mathbb{I}\otimes\sigma_i\right)+C_{ij}\sigma_i\otimes\sigma_j}{4}$$



- Characterized by 15 parameters parameters: B_i, $\bar{B}_i$, and C_{ij}

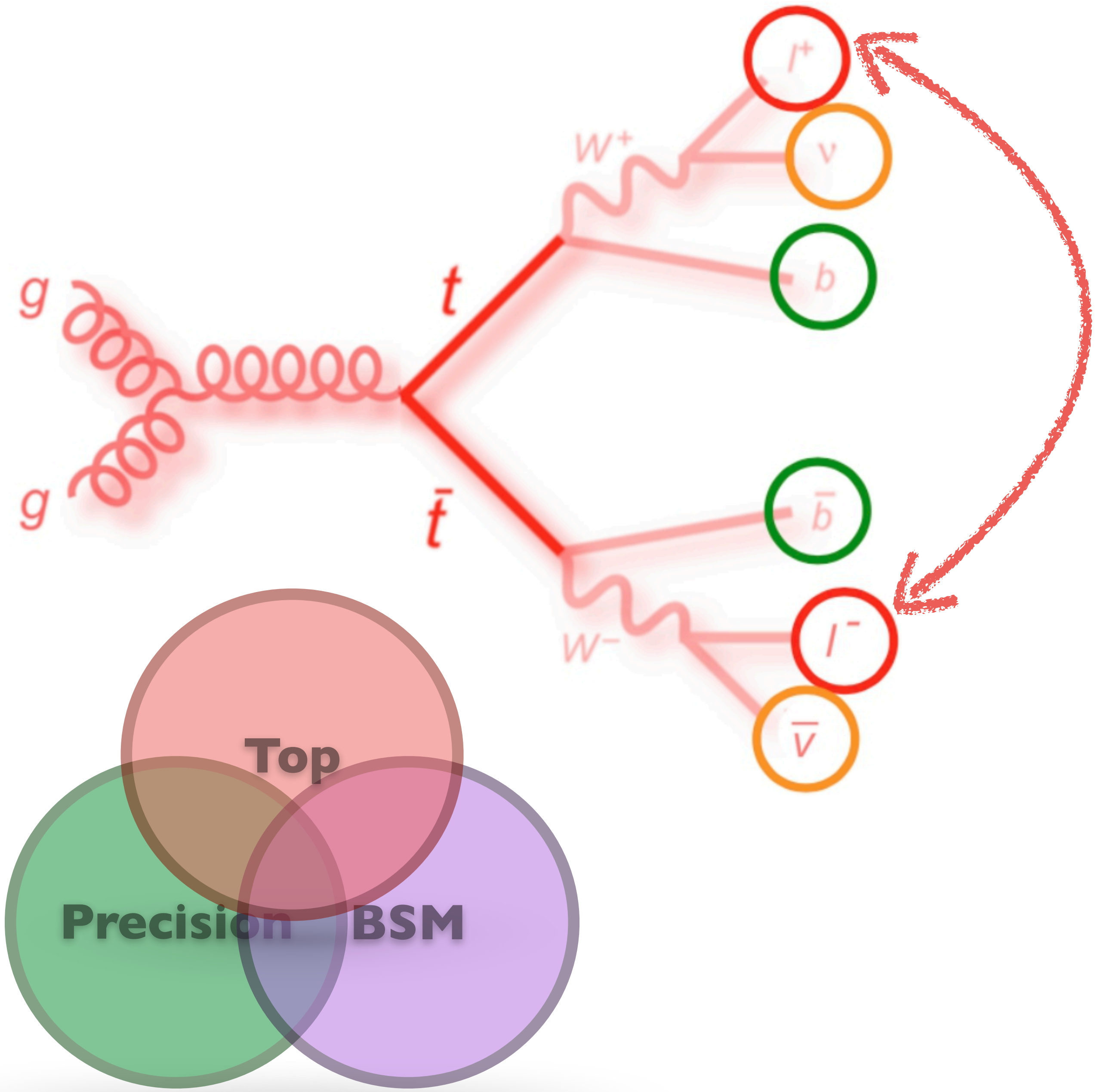
$$\begin{tikzcd}
B_i = \langle\sigma_i\otimes\mathbb{I}\rangle \arrow[dr] & \\
 & \text{Polarizations} \\
\bar{B}_i = \langle\mathbb{I}\otimes\sigma_i\rangle \arrow[ur] & \\
C_{ij} = \langle\sigma_i\otimes\sigma_j\rangle \arrow[r] & \text{Spin correlations}
\end{tikzcd}$$

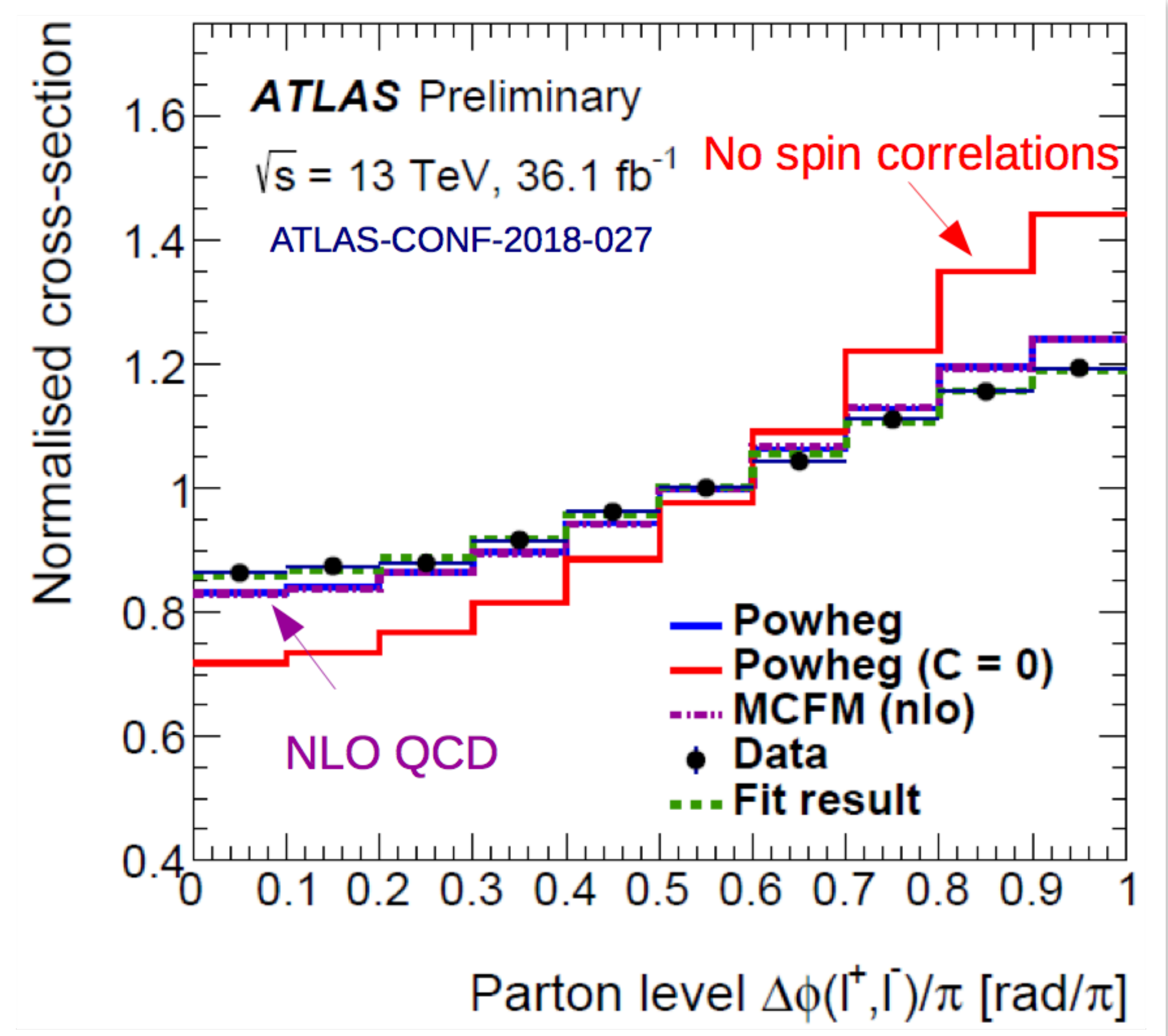
- P and CP invariance under $t\bar{t}$ production → $B_i = \bar{B}_i = 0$ and $C_{ij} = C_{ji}$
- Things further simplify in the helicity basis: only non-vanishing parameters are the diagonal terms C_{ii} and one off-diagonal term $C_{12} \simeq C_{21}$

Bernreuther, Heisler, Si '15
Frederix, Tsinikos, Vitos '21

Top quark pair production as a two qubit system

- We are probing the spin correlations of top and anti-top since the Tevatron era and continue to study them at the LHC in different forms: relevant precision observable





Parke, Mahlon '95

Top quark pair production as a two qubit system

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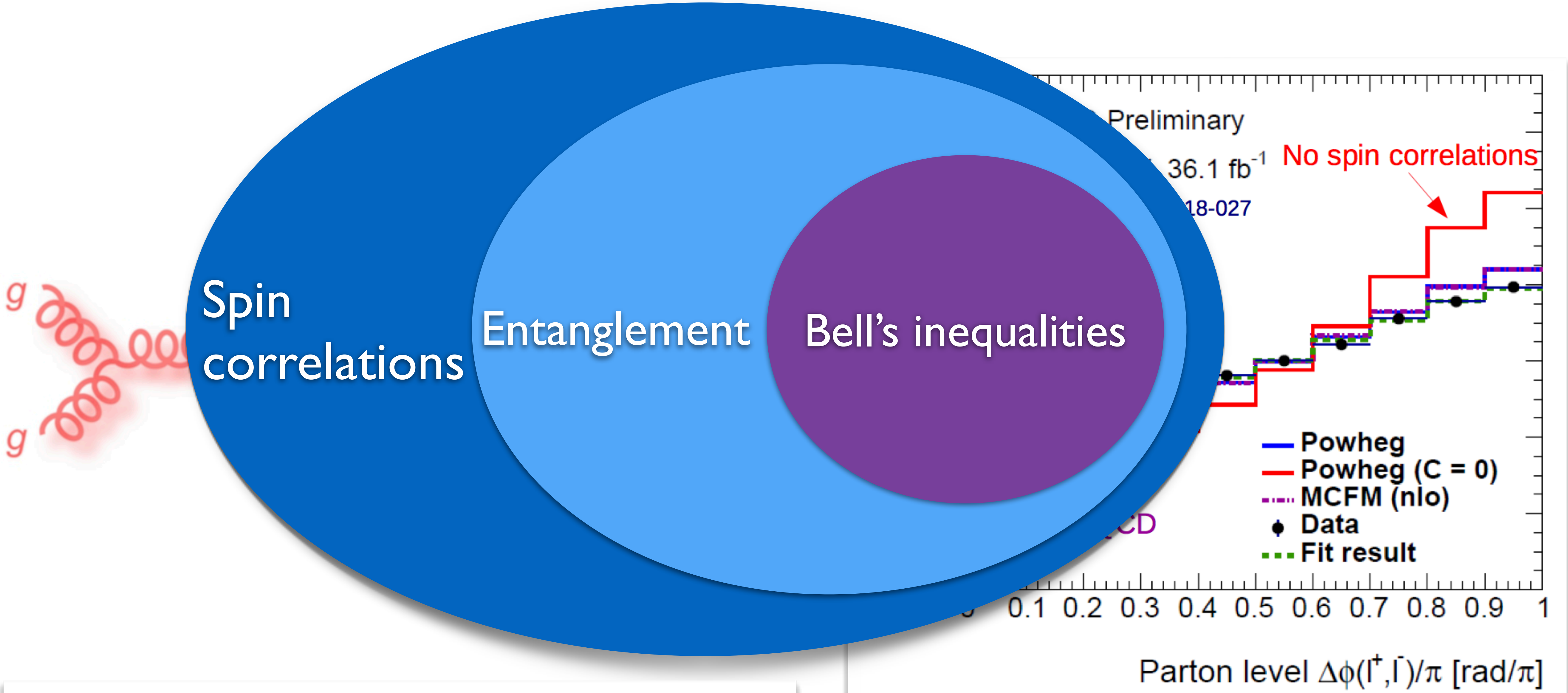


$$\rho = \frac{\mathbb{I}\otimes\mathbb{I} + \left(B_i\sigma_i\otimes\mathbb{I} + \bar{B}_i\mathbb{I}\otimes\sigma_i\right) + C_{ij}\sigma_i\otimes\sigma_j}{4}$$

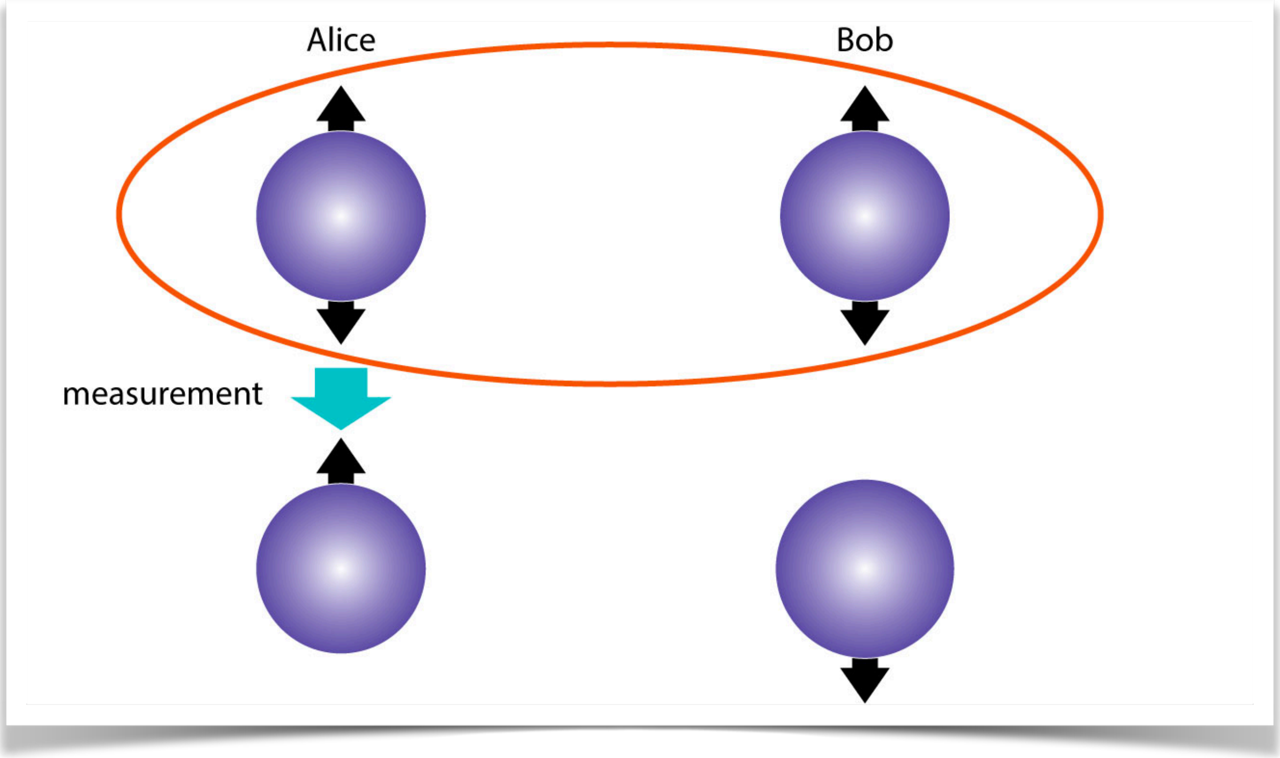
Parke, Mahlon '95

Quantum Entanglement

- A quantum state of two subsystems A and B is separable when its density matrix ρ can be expressed as a convex sum

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$$

→ If the state is not separable, it is named *entangled*



Measurement in one subsystem immediately affect the other, even if they are causally disconnected

Quantum Entanglement

- A quantum state of two subsystems A and B is separable when its density matrix ρ can be expressed as a convex sum

$$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$$

→ If the state is not separable, it is named *entangled*

- The Peres-Horodecki criterion provides a necessary and sufficient condition for entanglement in two-qubit systems:

Peres '96; Horodecki '97

Take the transpose of indices associated only to Bob (or Alice)

$$\rho^{T2} = \sum_i p_i \rho_A^i \otimes (\rho_B^i)^T$$

For a separable system, ρ^{T2} results in a non-negative operator

→ If ρ^{T2} displays at least one negative eigenvalue, the system is entangled

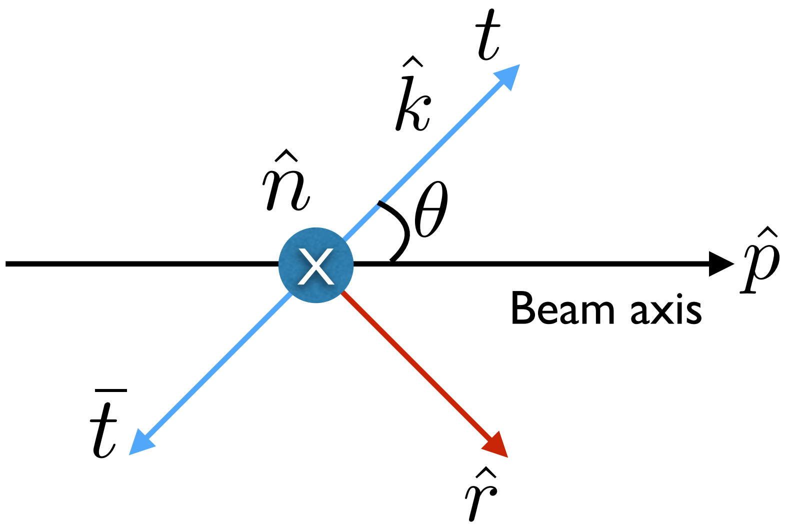
Quantum Entanglement

- Helicity basis

$$\hat{k} = \text{top quark direction}$$

$$\hat{r} = \text{sign}(\cos\theta)(\hat{p} - \cos\theta\hat{k})/\sin\theta$$

$$\hat{n} = \hat{k} \times \hat{r}$$



$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_a^i\, d\cos\bar{\theta}_b^j} = \frac{1}{4}\left(1 + \beta_a\beta_b C_{ij}\cos\theta_a^i\cos\bar{\theta}_b^j\right)$$

- Entanglement conditions:

$$|C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$

$$|C_{kk} - C_{rr}| + C_{nn} - 1 > 0$$

Afik, Nova '20
Severi, Boschi, Maltoni, Sioli '21
Saavedra, Casas '22

Quantum Entanglement

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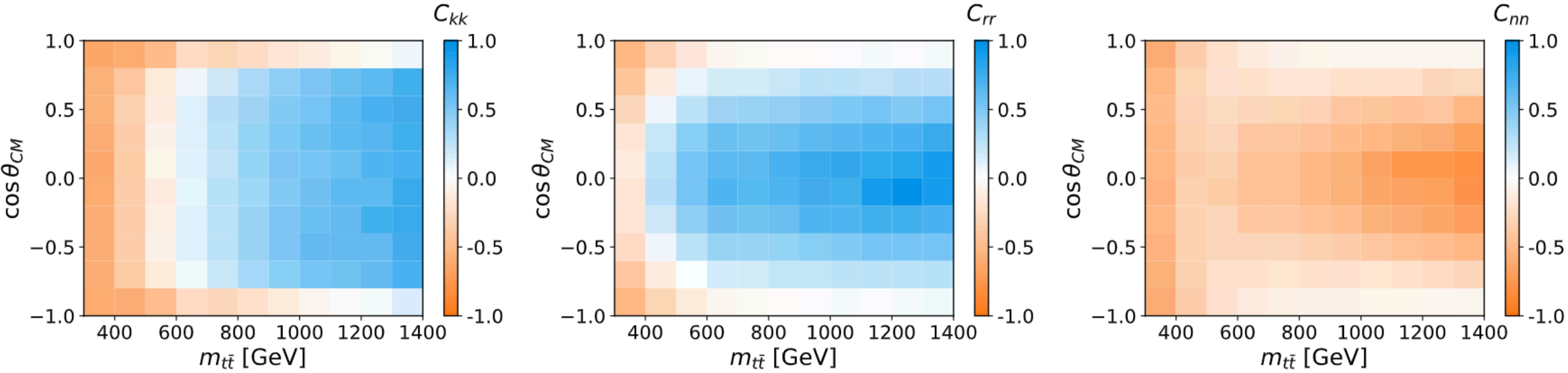
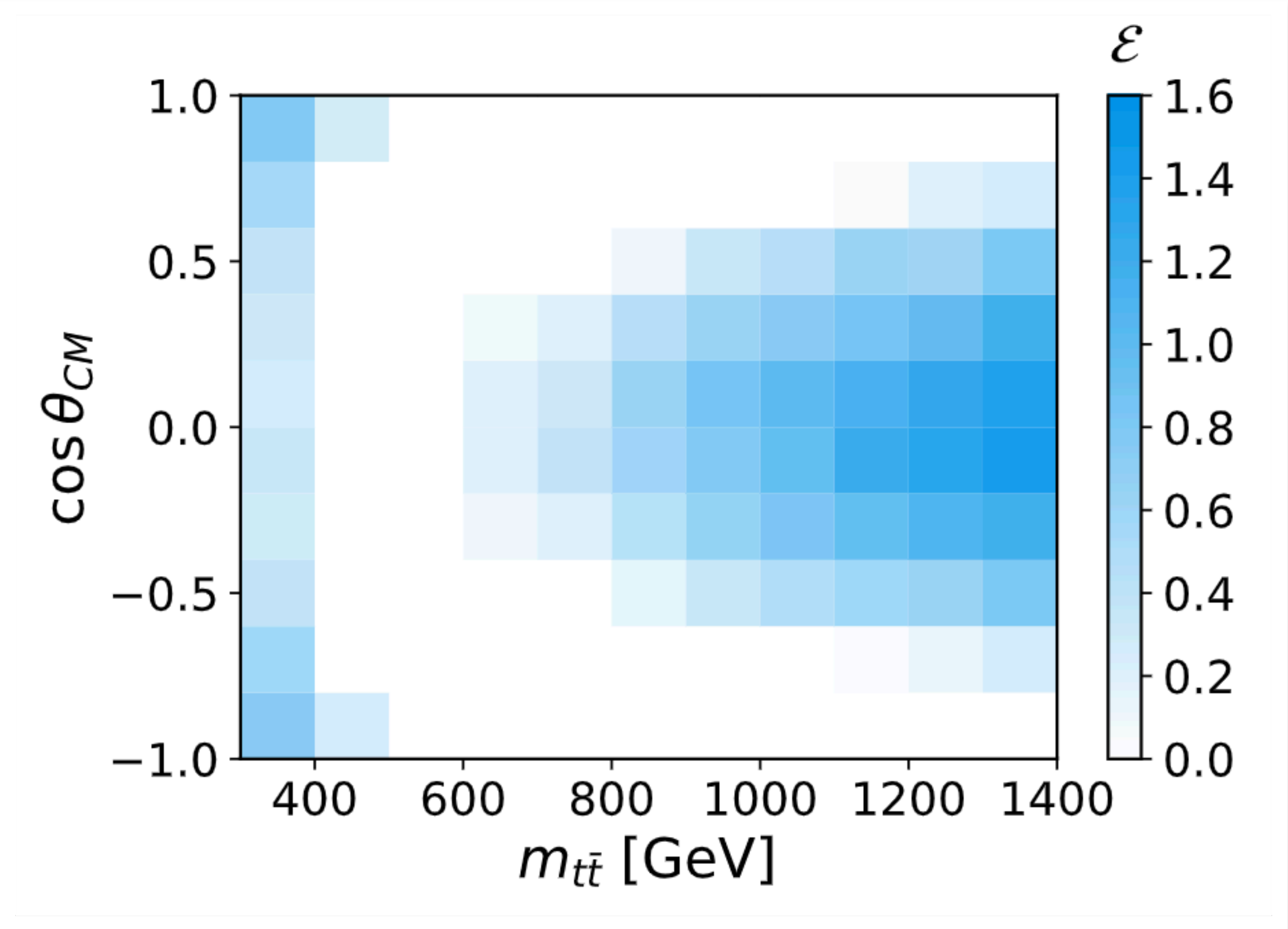


$$\boxed{\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0}$$

Dong, DG, Kong, Navarro '23

Quantum Entanglement

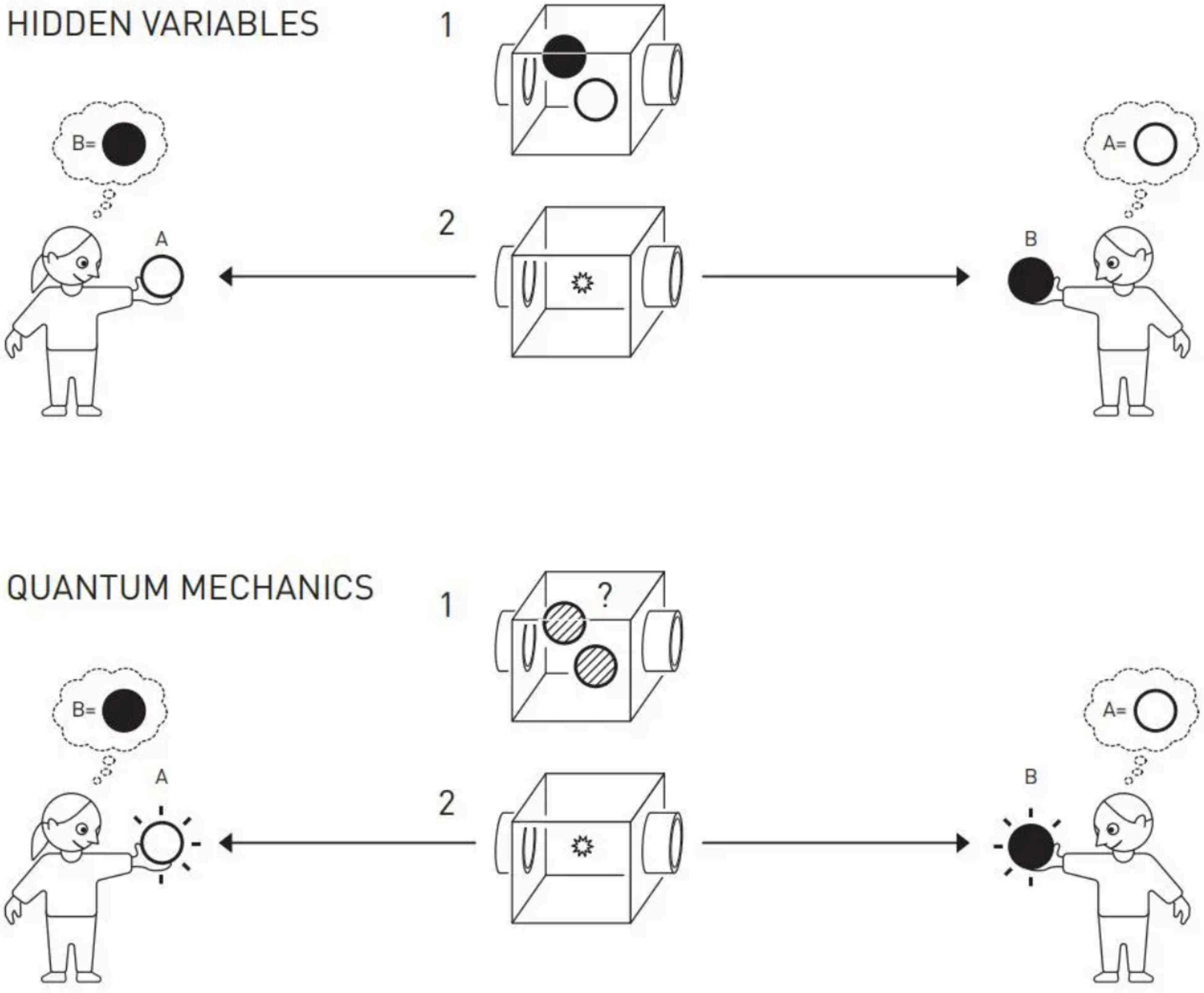
$$\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$$



Dong, DG, Kong, Navarro ’23

- Top pair displays entanglement in two disconnected regimes: **threshold** and **boosted** regions

Bell's Inequalities



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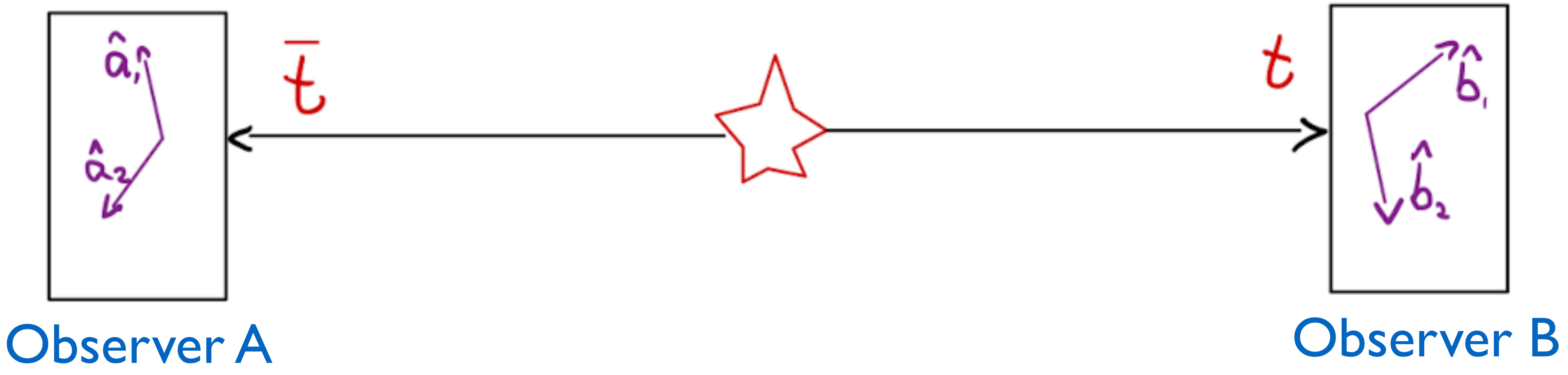
Bell's Inequalities

- Violation of Bell-type inequalities demonstrates that there is no hidden variable theory capable of encoding the generated entanglement. QM cannot be explained by classical laws
- Bell's inequality can be distilled in a simpler form: CHSH inequality

Clauser, Horne, Shimony, Holt '69

$$|\langle A_1 B_1\rangle + \langle A_2 B_1\rangle + \langle A_1 B_2\rangle - \langle A_2 B_2\rangle| \leq 2$$



Bell's Inequalities

- Bell/CHSH inequalities:

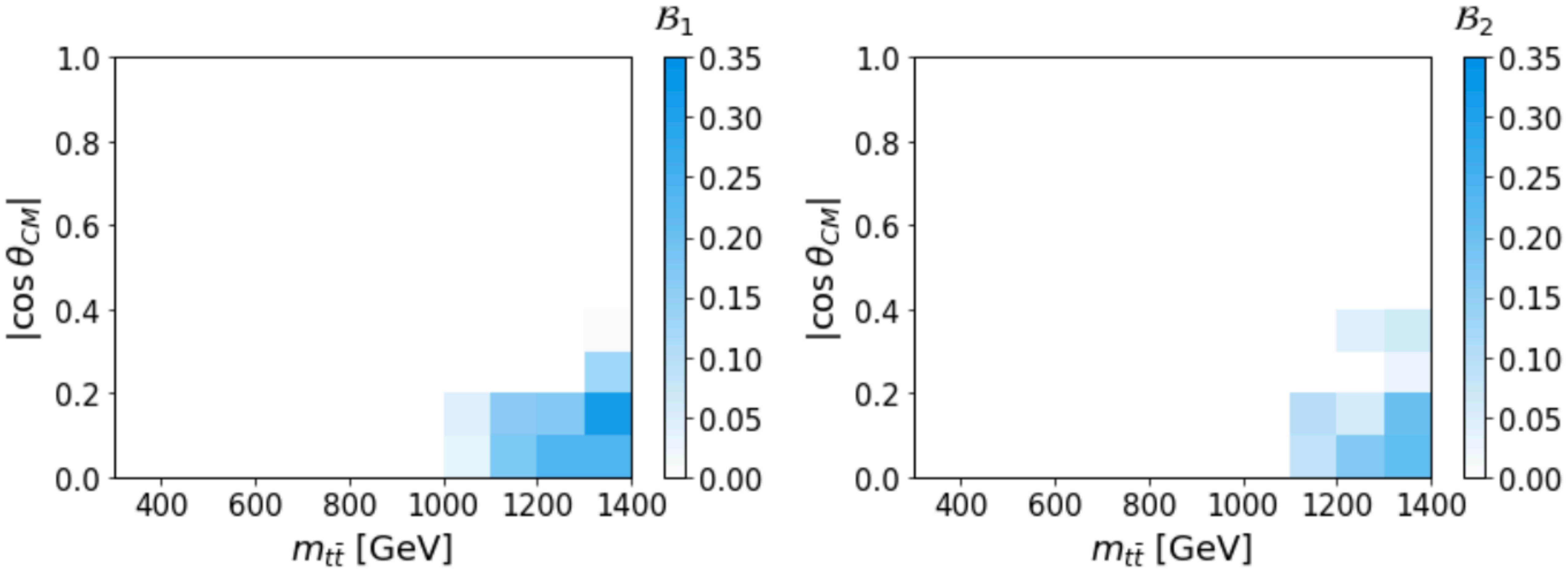
$$\mathcal{B}_1 \equiv |C_{rr} - C_{nn}| - \sqrt{2} > 0$$

$$\mathcal{B}_2 \equiv |C_{kk} + C_{rr}| - \sqrt{2} > 0,$$

Afik, Nova '20

Severi, Boschi, Maltoni, Sioli '21

Saavedra, Casas '22

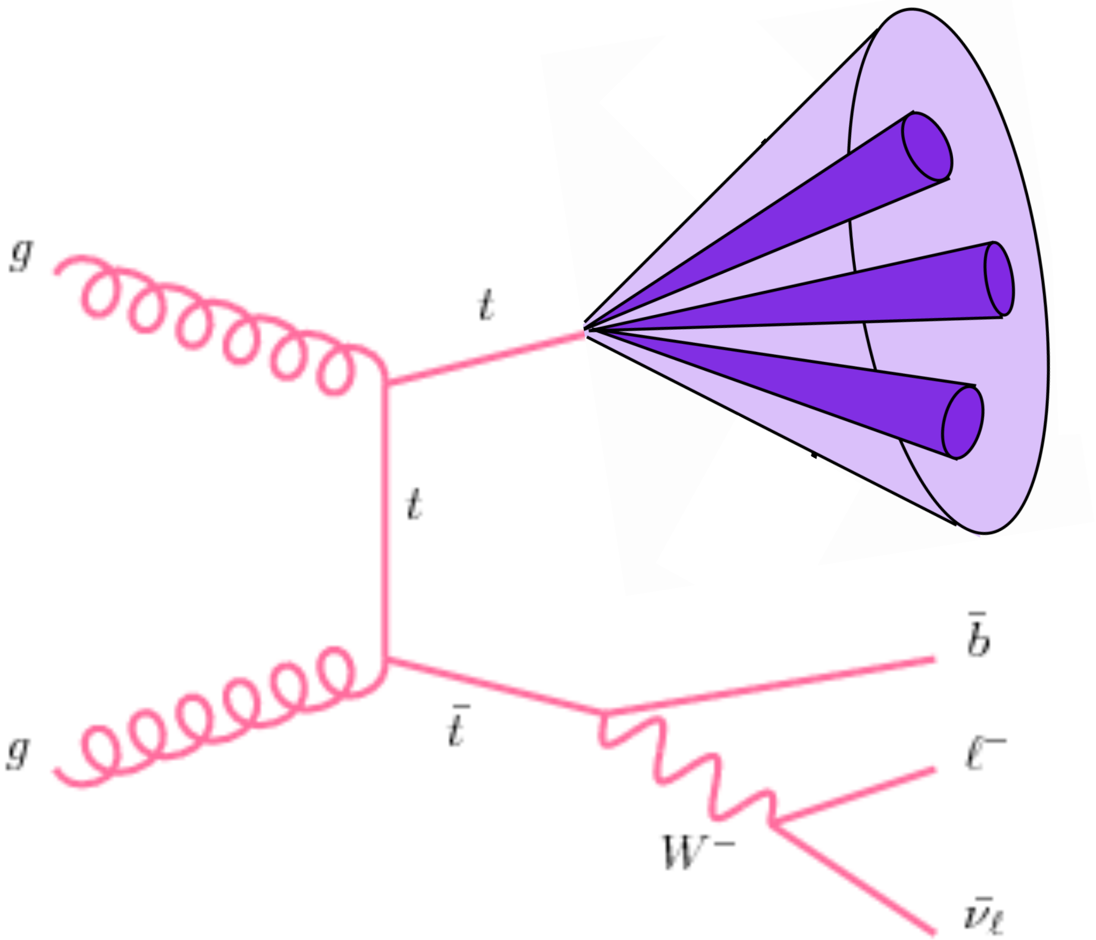


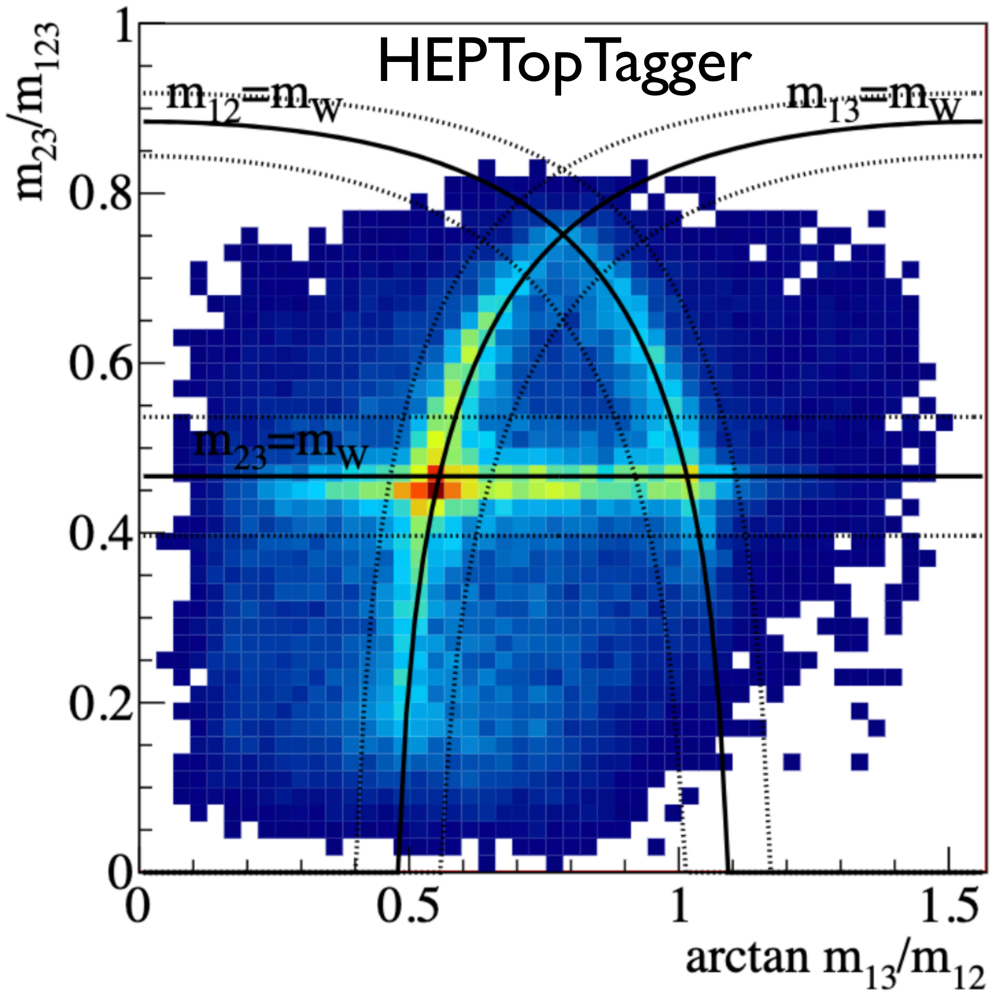
Dong, DG, Kong, Navarro '23

→ Bell/CHSH violation studies well match **boosted** top pair searches

Analysis

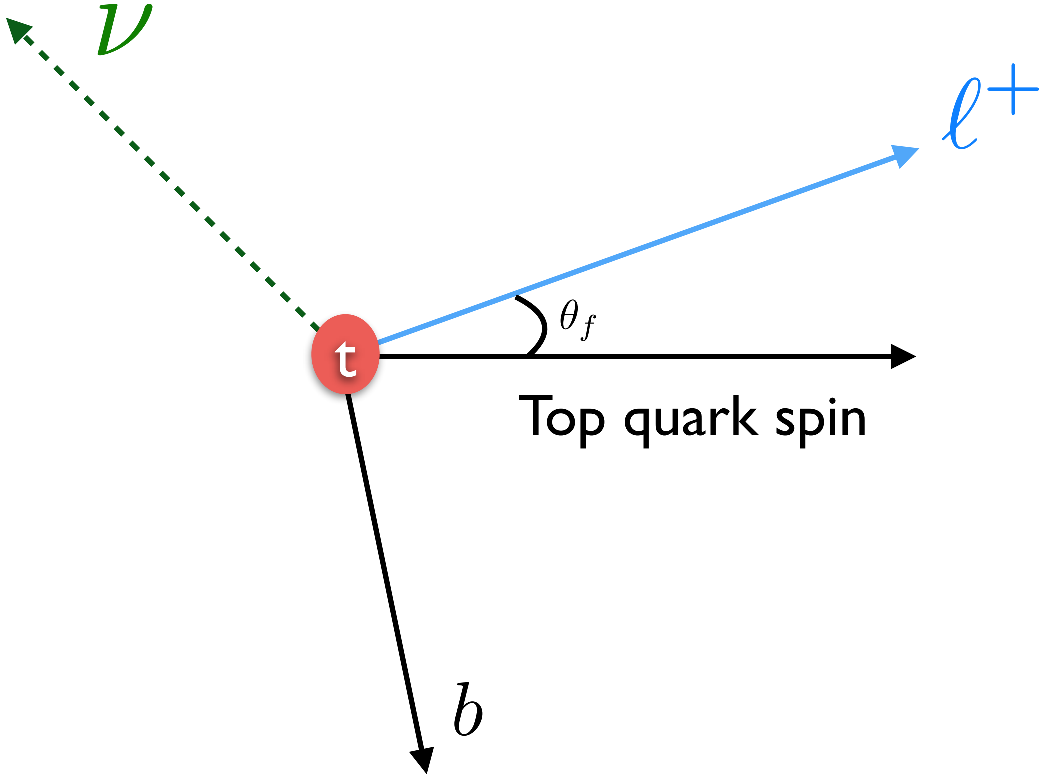
- Semi-leptonic top pair
  - High event rate: ~6 times higher than dileptonic case
  - Easier reconstruction
  - Boosted top tagging aid subject and light quark matching
  - Proxy for down-quark: optimal hadronic polarimeter





Hadronic top quark polarimetry

- In the high-energy regime, boosted techniques can be employed to tag the hadronic top and efficiently identify a spin analyzer



$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 + \mathbf{1.0}\,\cos\theta_f)$$

Hadronic top quark polarimetry

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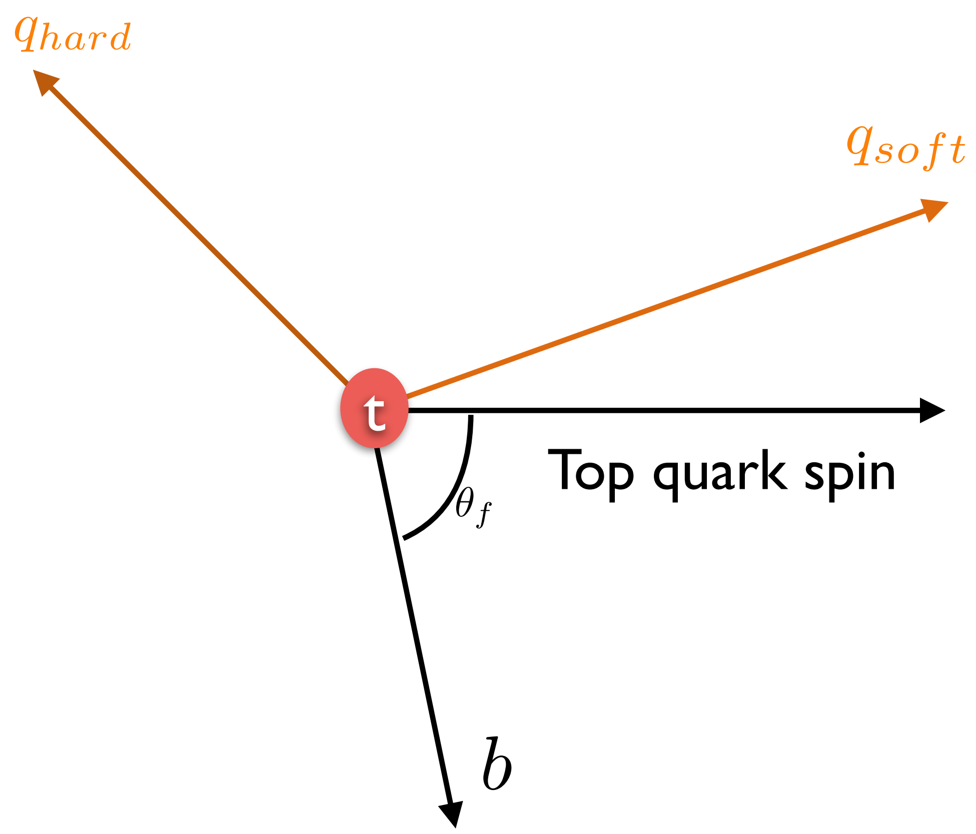

$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1 \;\; \mathbf{-0.4} \;\; \cos\theta_f)$$

Hadronic top quark polarimetry

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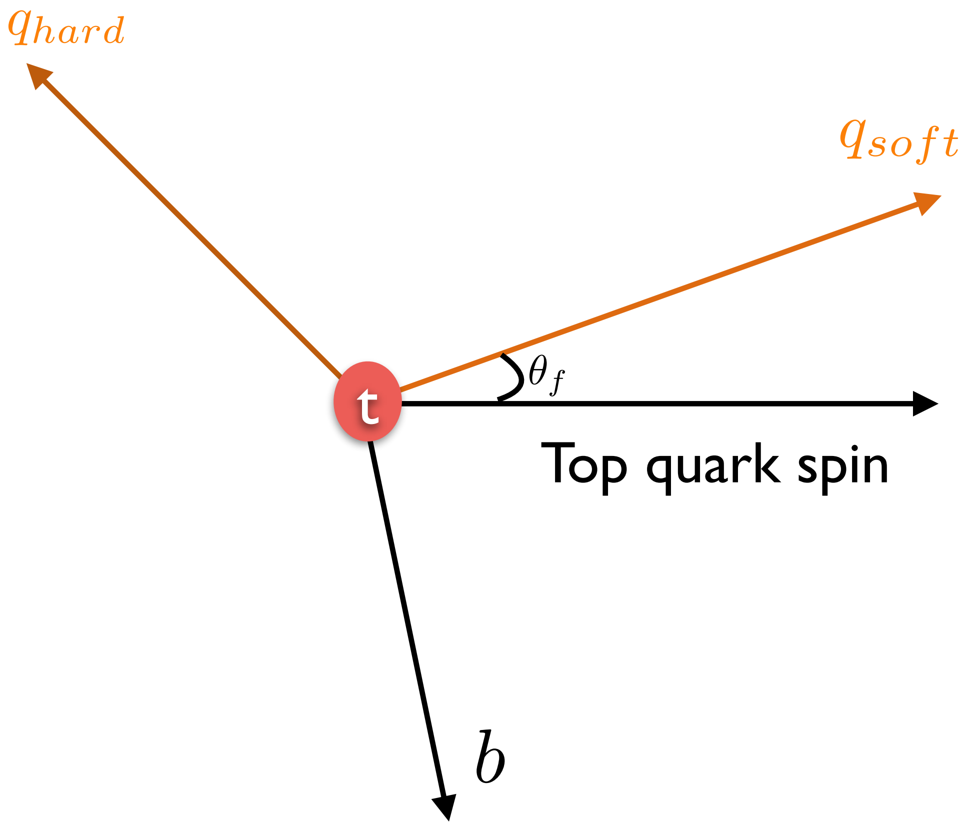


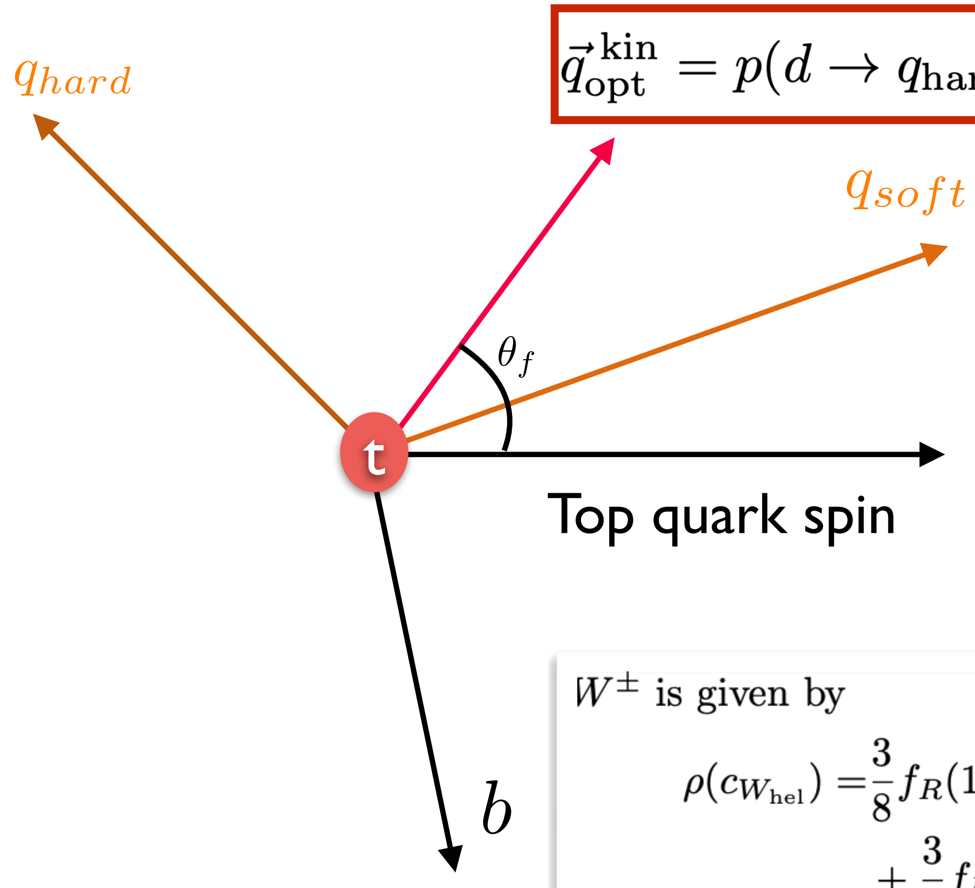
$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1+\mathbf{0.5}\,\cos\theta_f)$$

Jezabek ‘94

Hadronic top quark polarimetry

- In the high-energy regime, boosted techniques can be employed to tag the hadronic top and efficiently identify a spin analyzer

$$\boxed{\vec{q}^{\,\text{kin}}_{\text{opt}} = p(d\to q_{\text{hard}}|c_W)\,\hat{q}_{\text{hard}} + p(d\to q_{\text{soft}}|c_W)\,\hat{q}_{\text{soft}}}$$



$$\frac{1}{\Gamma_f}\frac{d\Gamma_f}{d\cos\theta_f} = \frac{1}{2}(1+\mathbf{0.64}\cos\theta_f)$$

$W^\pm$ is given by

$$\rho(c_{W_{\text{hel}}}) = \frac{3}{8}f_R(1\pm c_{W_{\text{hel}}})^2 + \frac{3}{4}f_0(1-c^2_{W_{\text{hel}}}) + \frac{3}{8}f_L(1\mp c_{W_{\text{hel}}})^2.$$

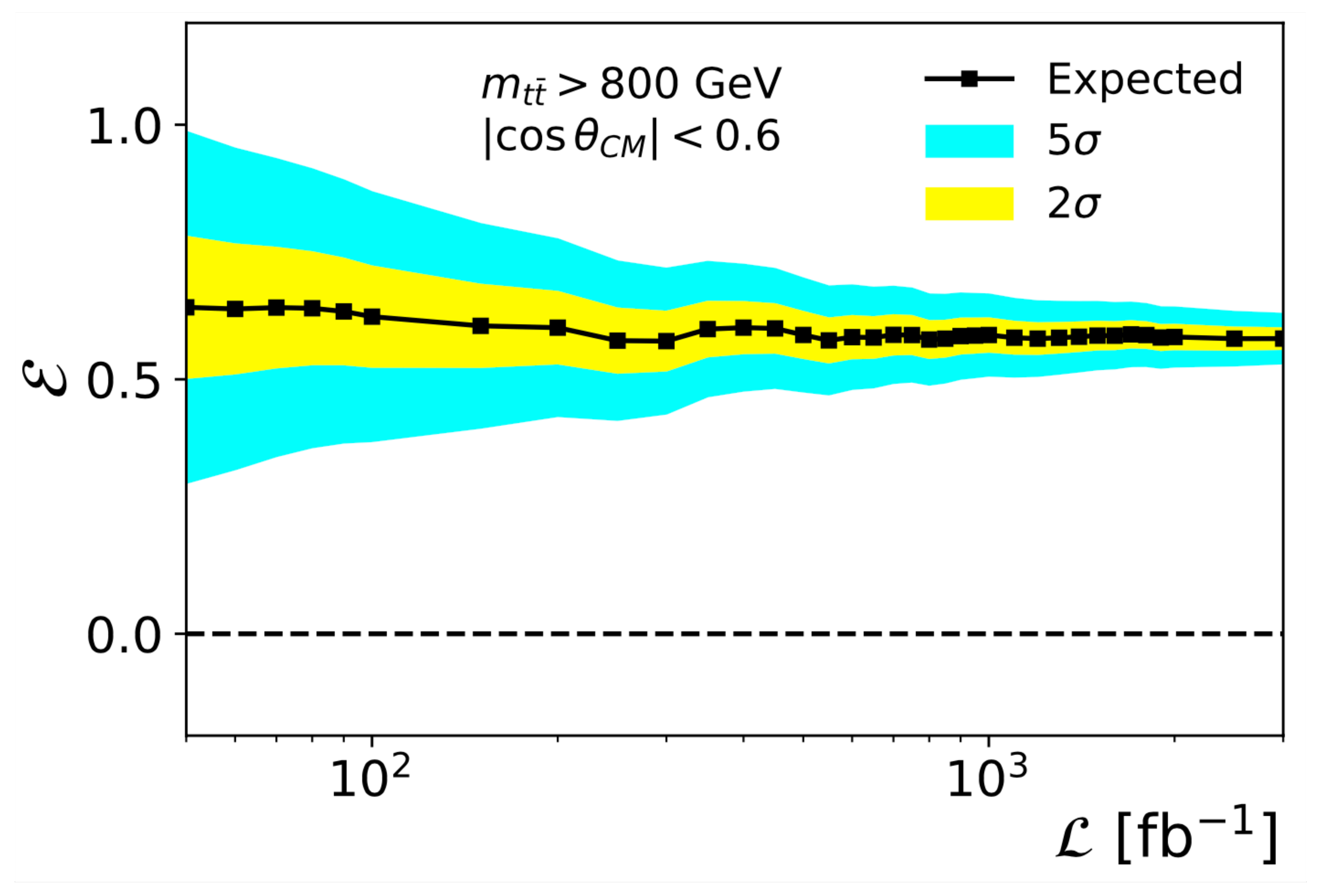
$$p(d\to q_{\text{hard}}) = \frac{\rho(|c_{W_{\text{hel}}}|)}{\rho(|c_{W_{\text{hel}}}|)+\rho(-|c_{W_{\text{hel}}}|)}$$

$$p(d\to q_{\text{soft}}) = \frac{\rho(-|c_{W_{\text{hel}}}|)}{\rho(|c_{W_{\text{hel}}}|)+\rho(-|c_{W_{\text{hel}}}|)}$$

Tweedie ‘14

LHC Projections

- **Entanglement:** $\mathcal{E} \equiv |C_{kk} + C_{rr}| - C_{nn} - 1 > 0$



Dong, DG, Kong, Navarro ’23

See also Han, Low and Wu ’23

LHC Projections

- **Bell/CHSH inequalities:** $\mathcal{B}_1 \equiv |C_{rr} - C_{nn}| - \sqrt{2} > 0$

$$\mathcal{B}_1 \equiv |C_{rr} - C_{nn}| - \sqrt{2} > 0$$

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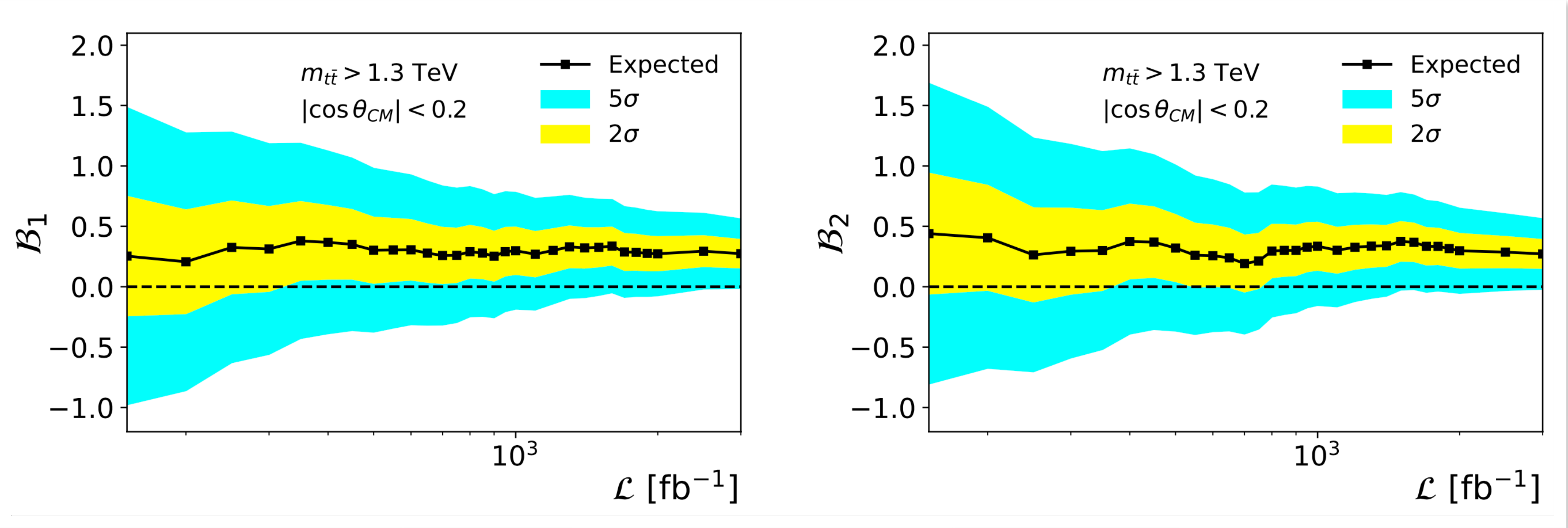

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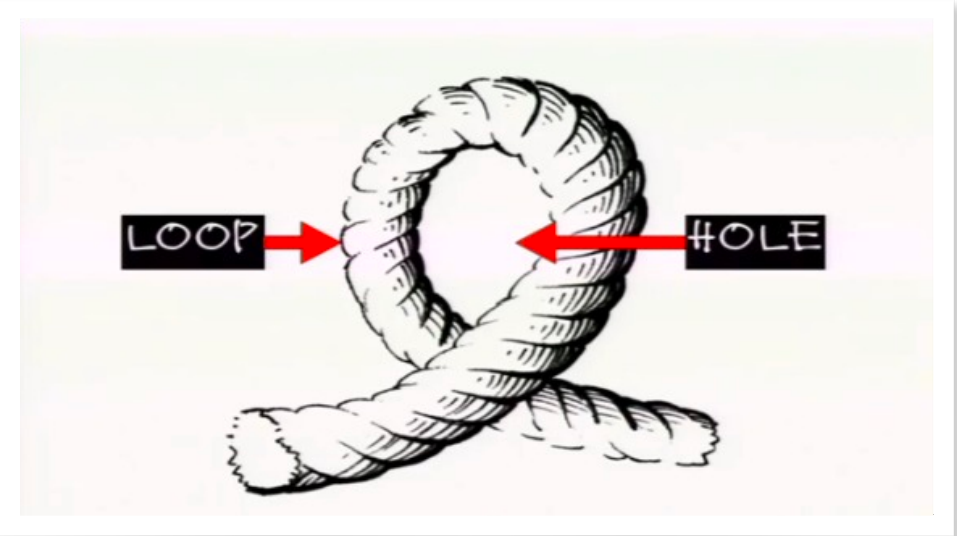
CHSH loopholes in top pair production

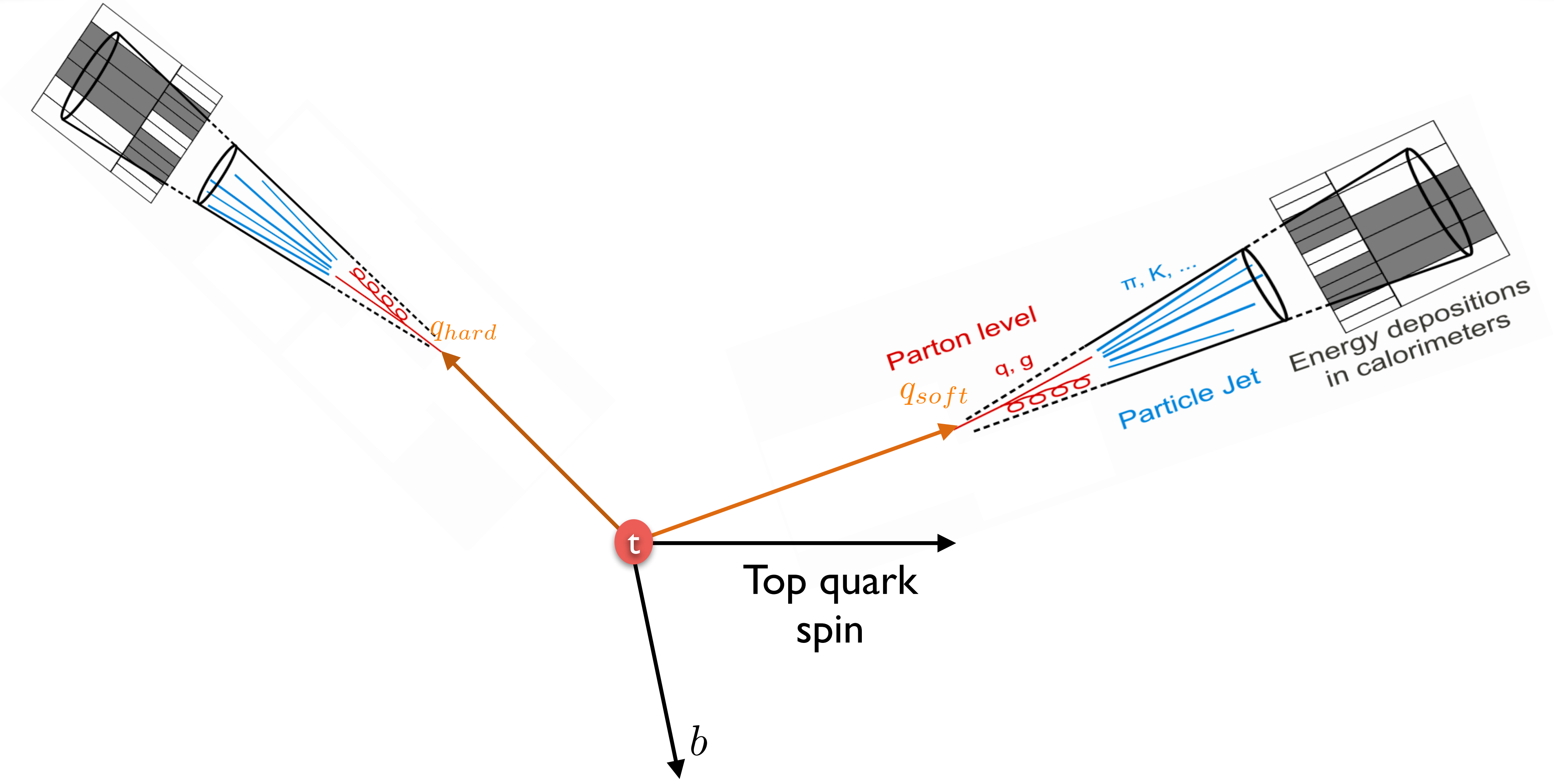
Measurement of loophole free Bell violation is more delicate than probing entanglement



- **Free-will loophole**: There is no external intervention in choosing measurements. ✗
  However, directions of final state leptons and q_{opt} are randomized ✓
- **Causally disconnected** at the boosted regime ✓ (statistical level) ✗
- **Detection loophole**: We examine a subset of events, potentially biasing results ✗

→ It is not surprising that the LHC can only probe weak violation of Bell's inequalities. LHC wasn't designed for this. In fact, it took decades to prepare loophole-free setups, with the first measurement in 2015-2018, leading to the 2022 Nobel Prize

Hadronic Top Quark Polarimetry with ParticleNet



$$\vec{q}_{\rm opt} = \; p(d \to q_{\rm hard}|c_W, \{\mathcal{O}\})\,\hat{q}_{\rm hard} + p(d \to q_{\rm soft}|c_W, \{\mathcal{O}\})\,\hat{q}_{\rm soft}$$

Dong, DG, Kong, Larkoski, Navarro ’24

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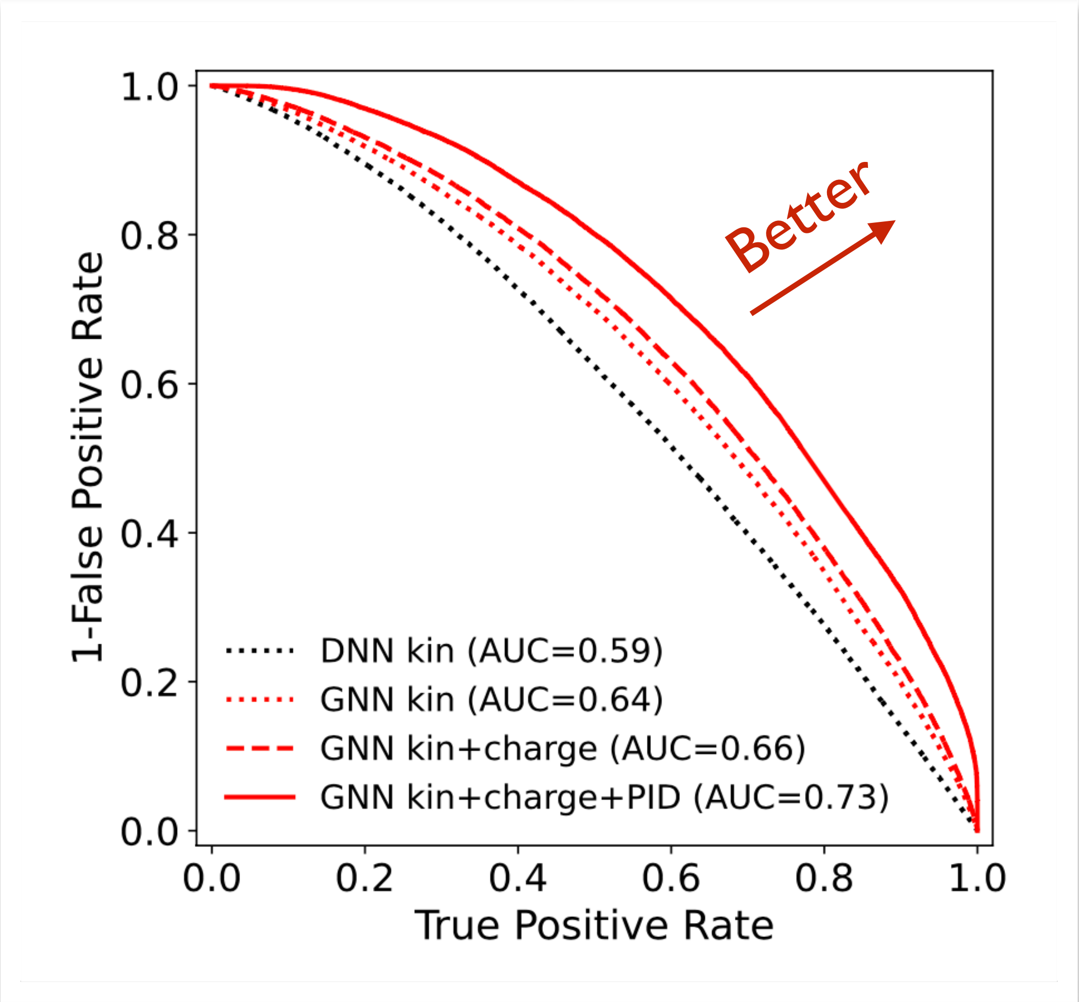
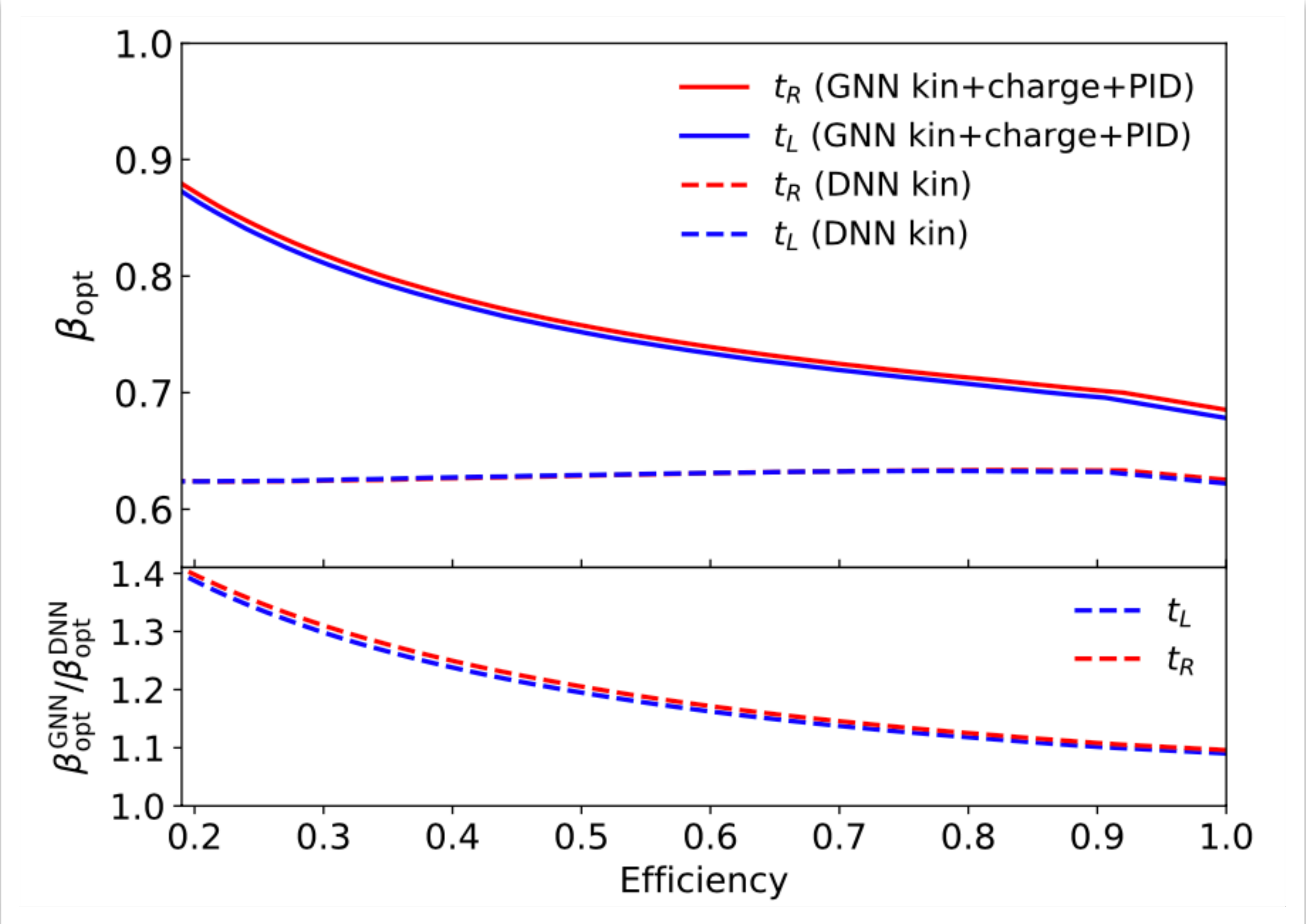
Hadronic Top Quark Polarimetry with ParticleNet





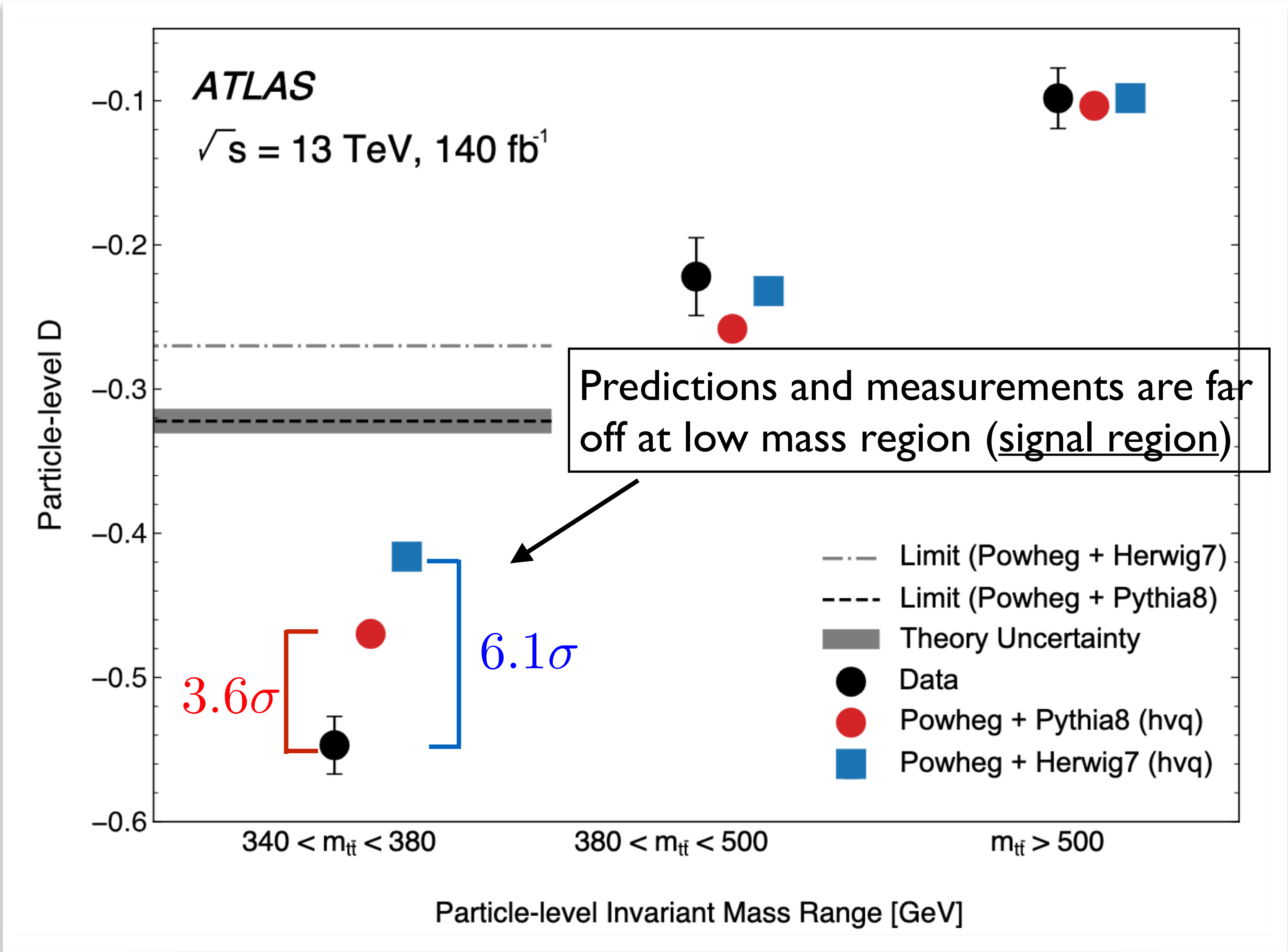
- This can significantly boost spin correlations, entanglement, Bell inequalities, new physics searches…

Dong, DG, Kong, Larkoski, Navarro '24

Dong, DG, Kong, Larkoski, Navarro '24

We are just at the beginning

- Entanglement stress test our current understanding
  - Theoretical modeling
  - Experimental uncertainties



Predictions and measurements are far off at low mass region (<u>signal region</u>)

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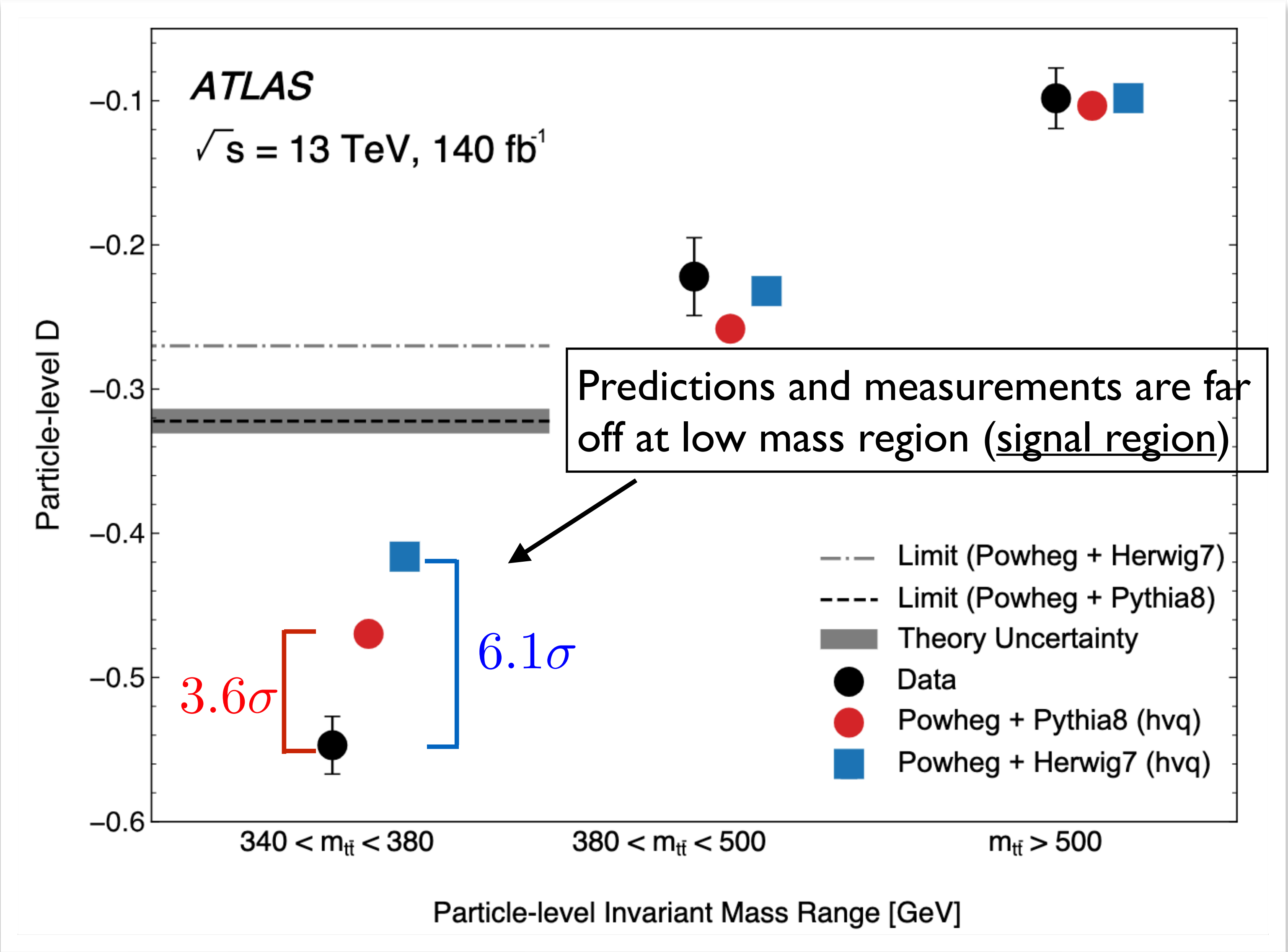
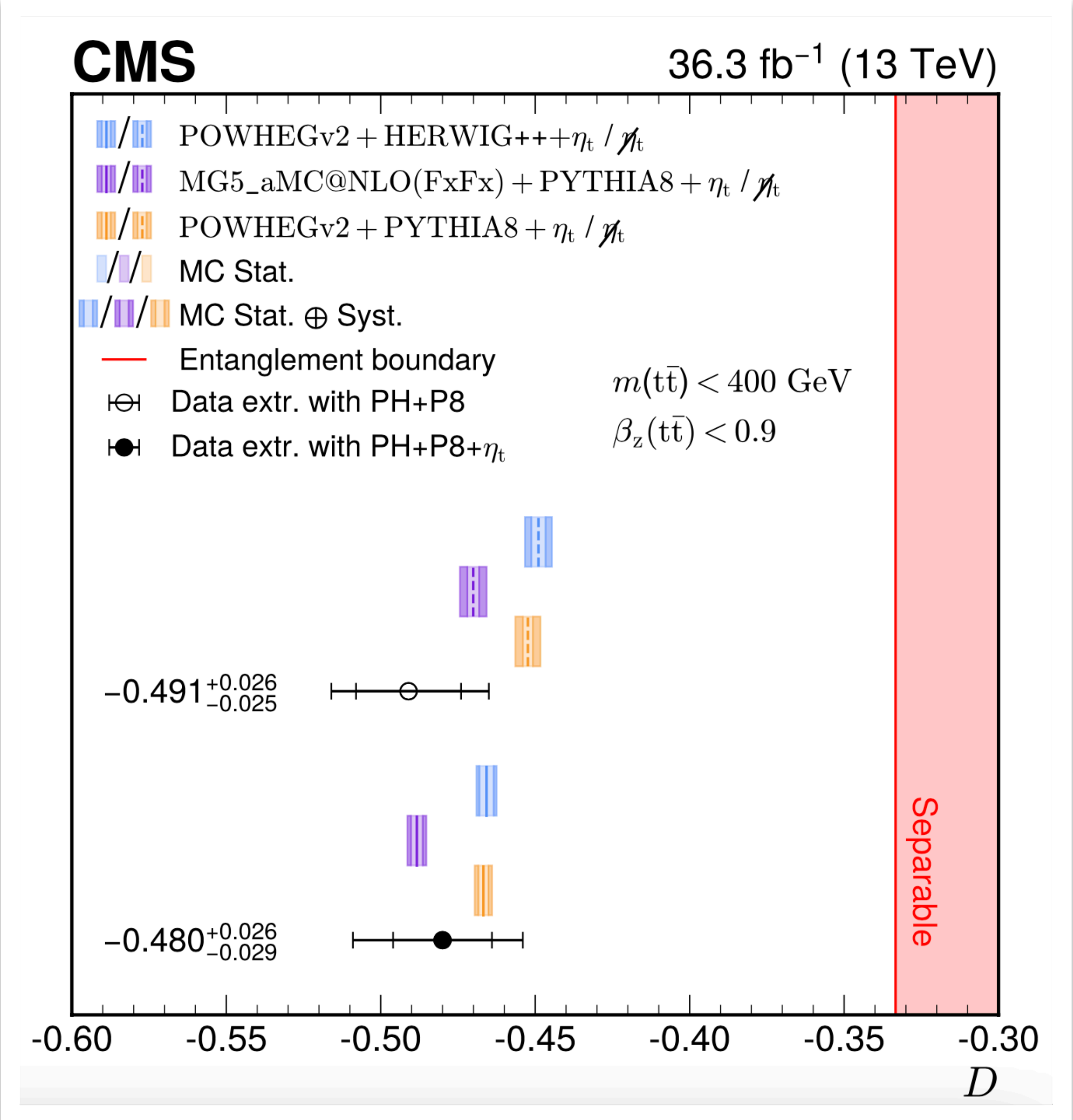
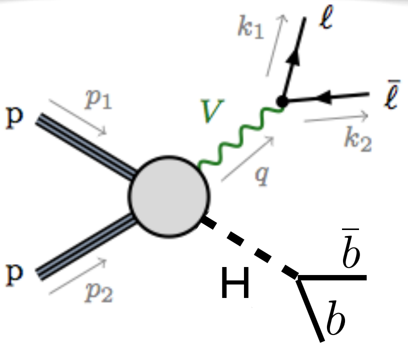


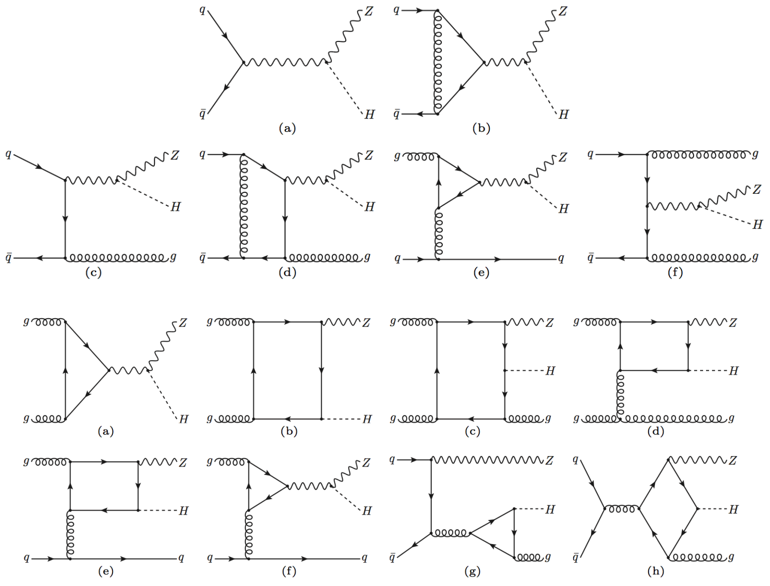
Predictions and measurements are far off at low mass region (signal region)



Paths towards precision



Higher order calculations



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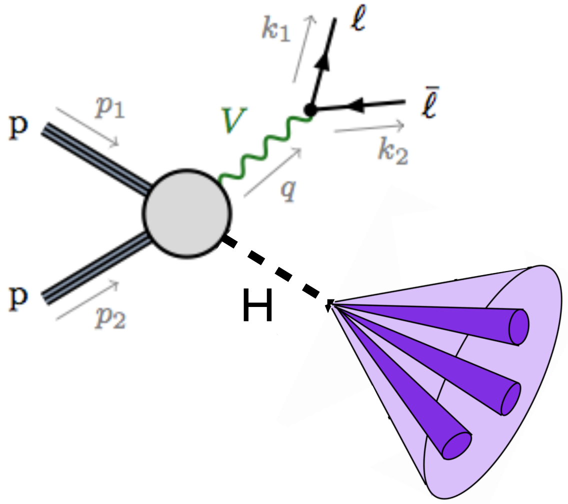
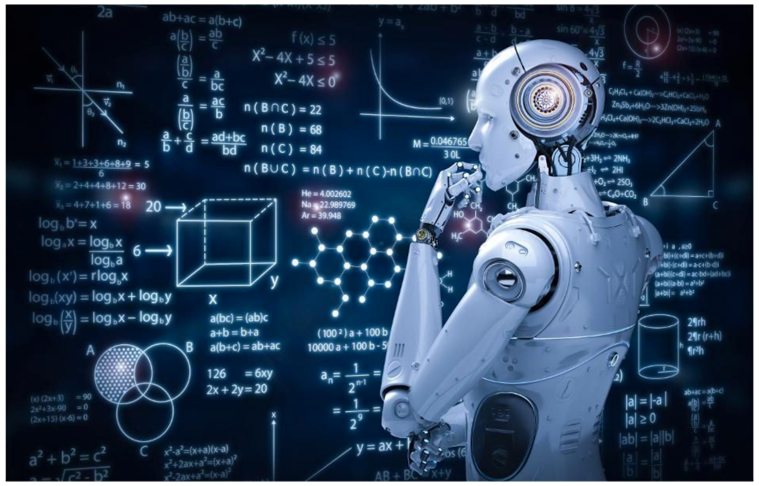
QCD corrections are dynamic and fundamental for robust predictions

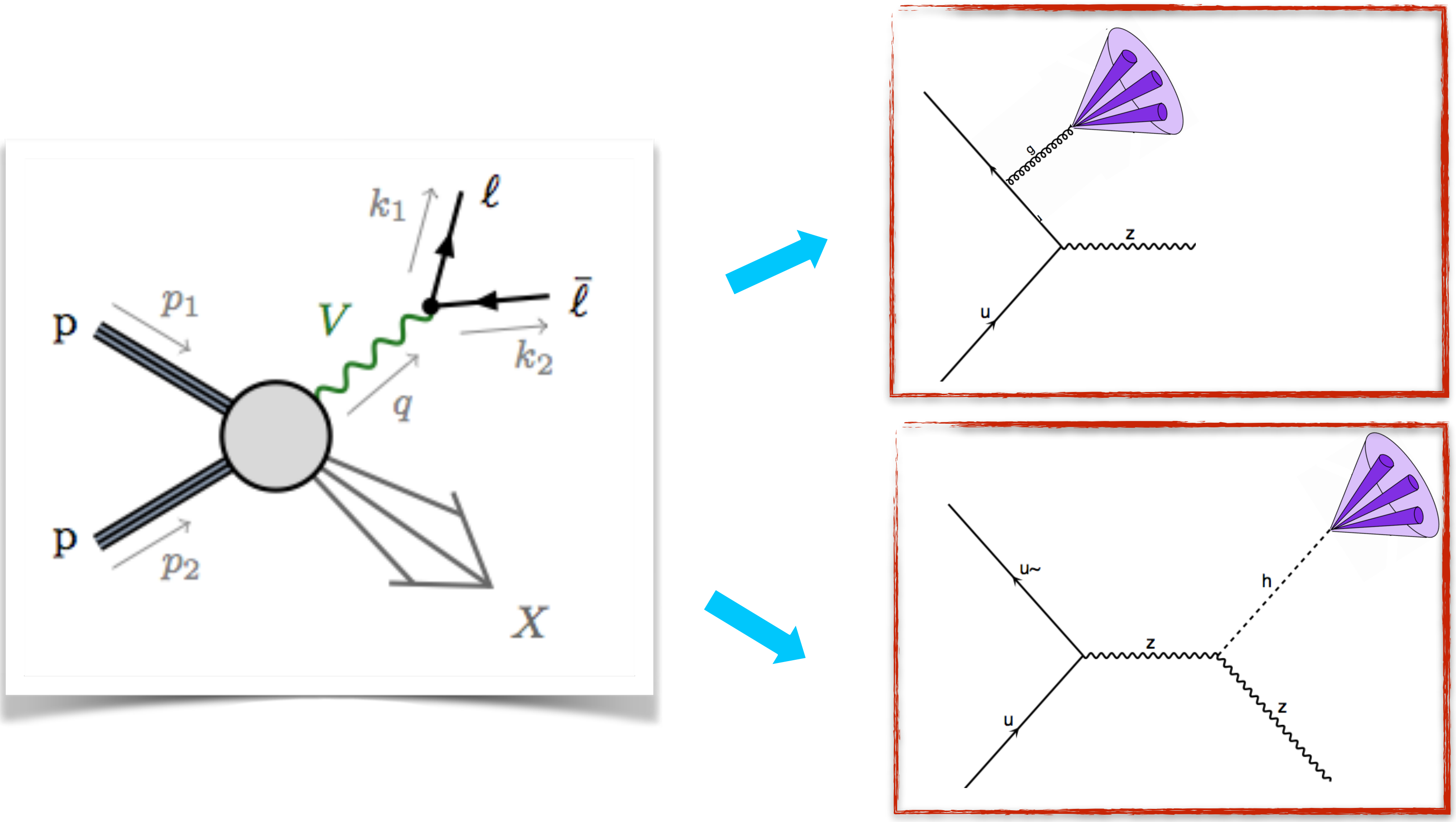
Efficient analysis technics





Role of the Z spin density in the H→bb measurement

- Z spin density displays a rich amount of information (8 multipoles coef.) and is not systematically limited

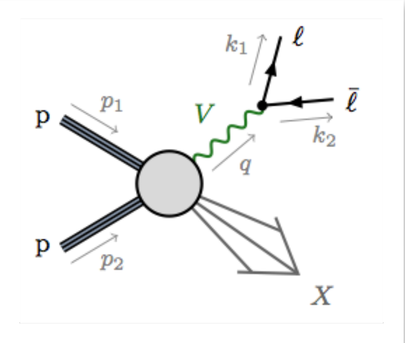


Can we significantly improve this search maximally exploring the Z spin density?

DG, Nakamura (2018)

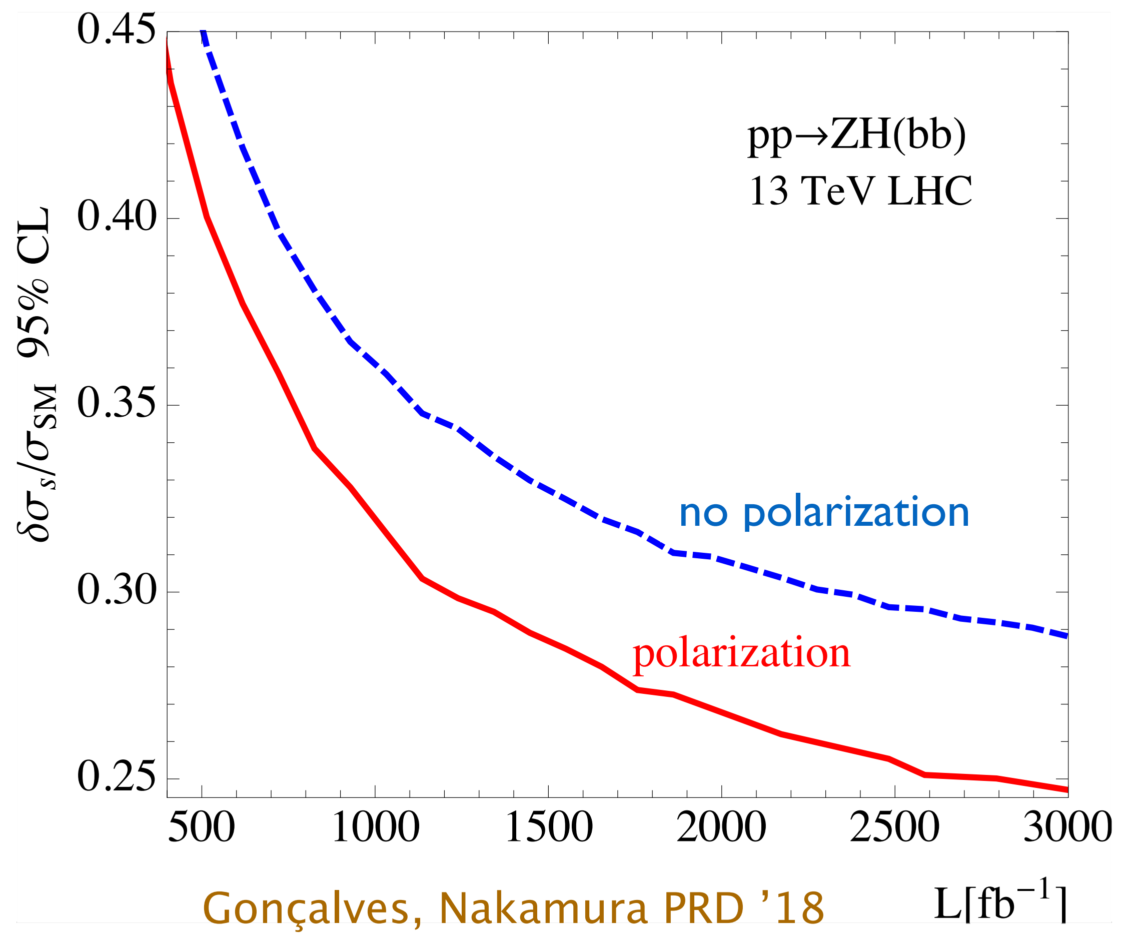
Role of the Z spin density in the H→bb measurement

- Z polarization displays a rich amount of information (8 multipoles coef.) and is not systematically limited



$$\ell^-(\ell^+):\ \frac{m_{\ell\ell}}{2}\left(1,\ \pm\sin\theta\cos\phi,\ \pm\sin\theta\sin\phi,\ \pm\cos\theta\right)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta d\phi} = 1+\cos^2\theta + A_1(1-3\cos^2\theta) + A_2\sin 2\theta\cos\phi + A_3\sin^2\theta\cos 2\phi$$
$$+ A_4\cos\theta + A_5\sin\theta\cos\phi + A_6\sin\theta\sin\phi + A_7\sin 2\theta\sin\phi + A_8\sin^2\theta\sin 2\phi$$



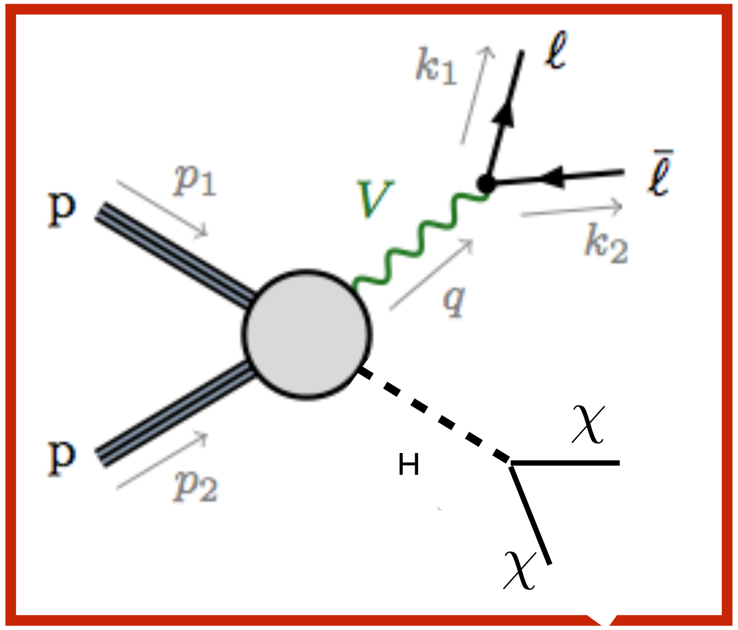
Gonçalves, Nakamura PRD '18

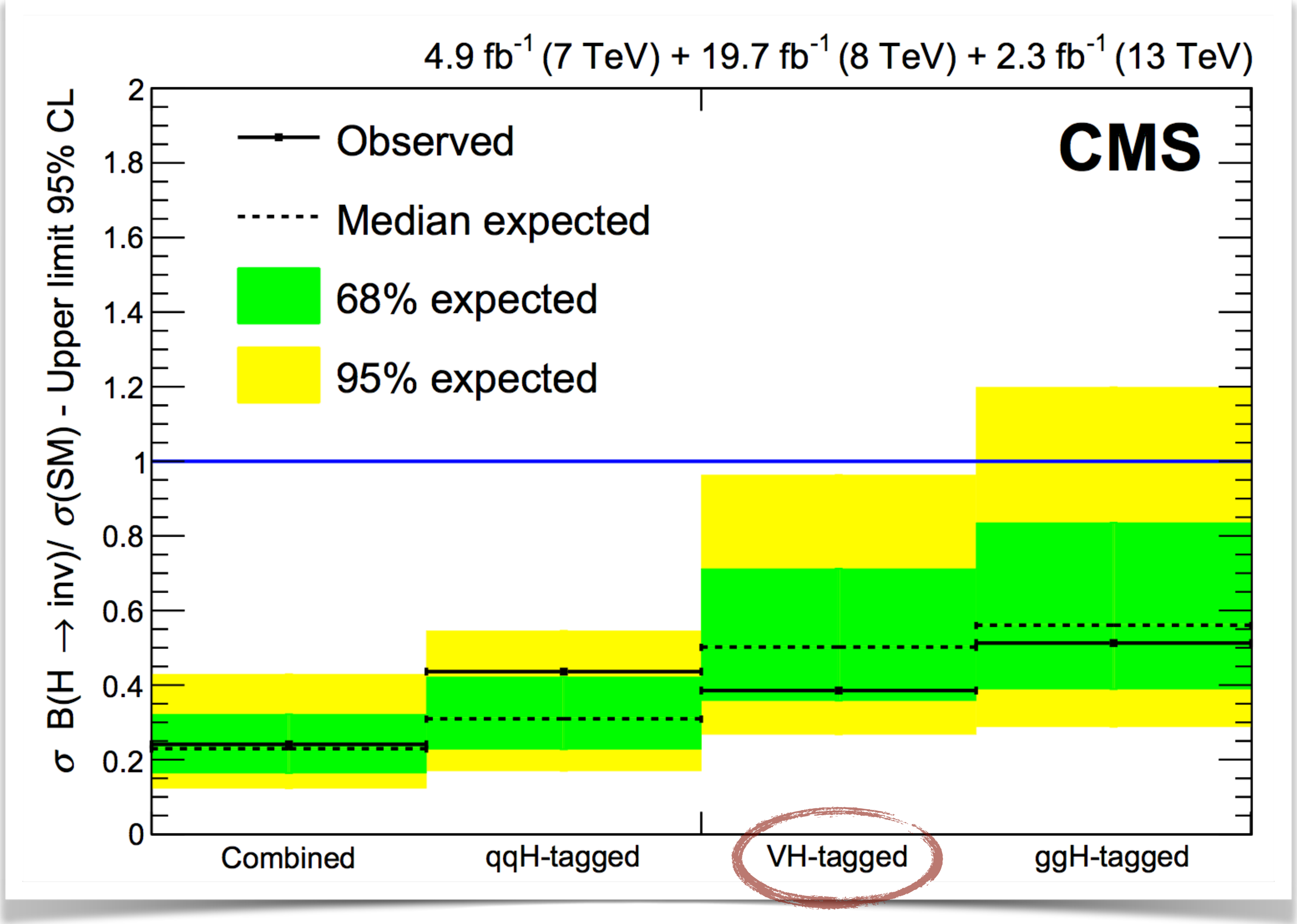
→ Lepton angular distribution works as an analyzer, probing underlying production dynamics encoded in the A_i coefficients: ZH(bb) vs Zbb

→ It relies only on lepton reconstruction with small experimental uncertainties. It can be promptly included in the ATLAS and CMS studies

Role of the Z spin density in the H→inv measurement

- It directly generalizes to other relevant searches such as Z(ll)H(inv):
  Does not depend on the Higgs decay, only on the Z polarization





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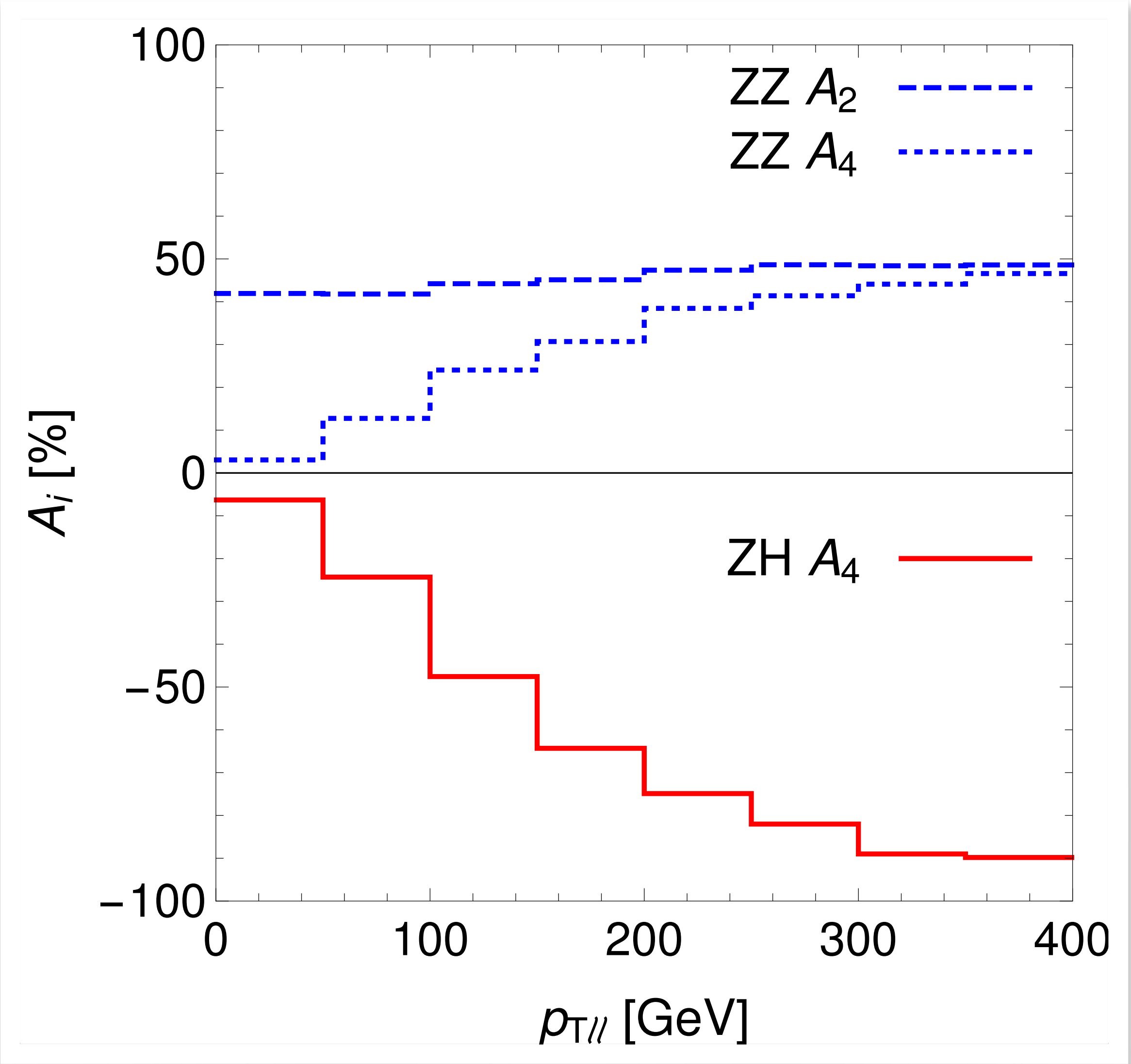
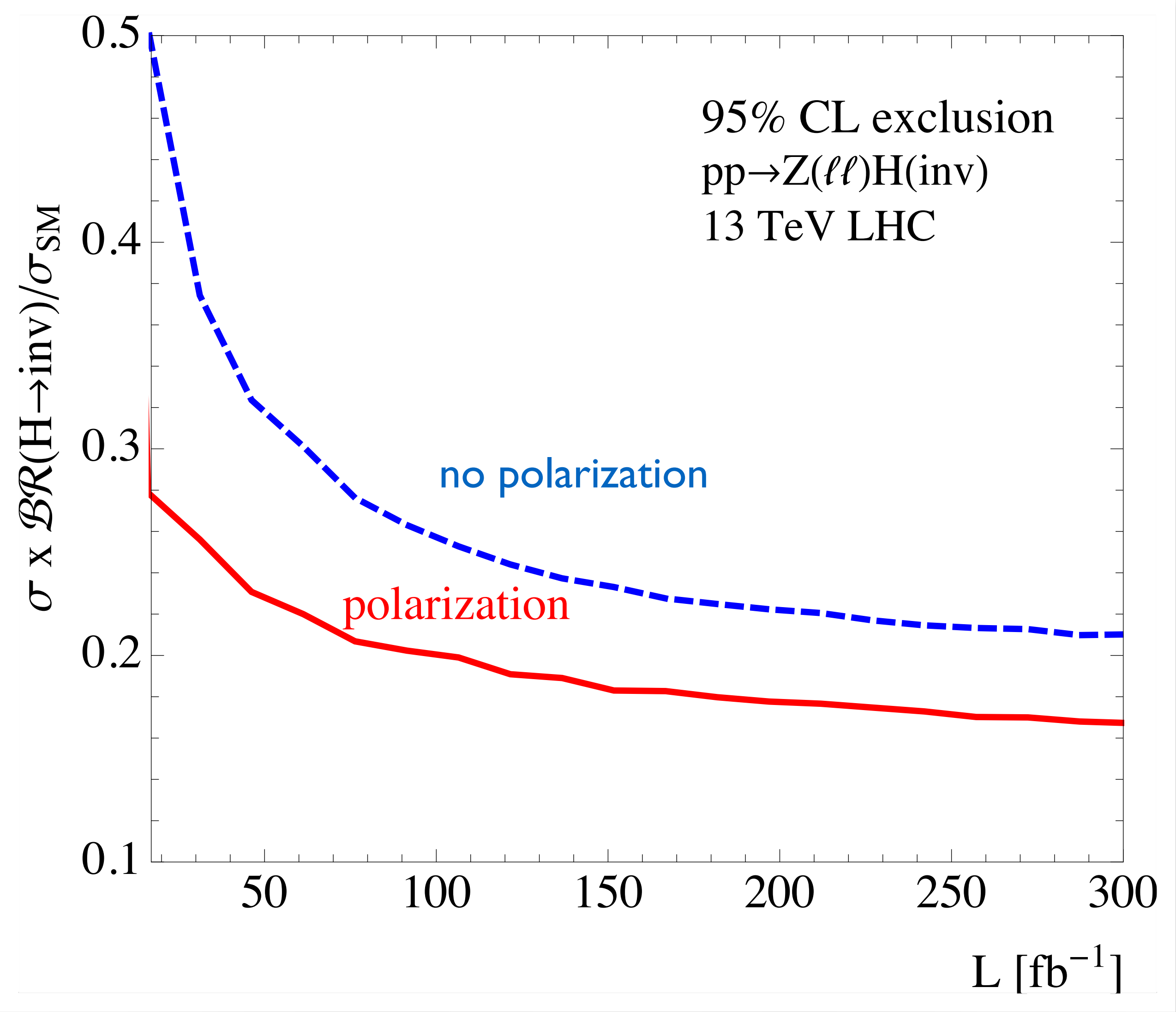




DG, Nakamura '19

Summary

- LHC provides a unique opportunity to study quantum correlations, such as entanglement and violation of Bell inequalities, at high energy scales: quantum tomography

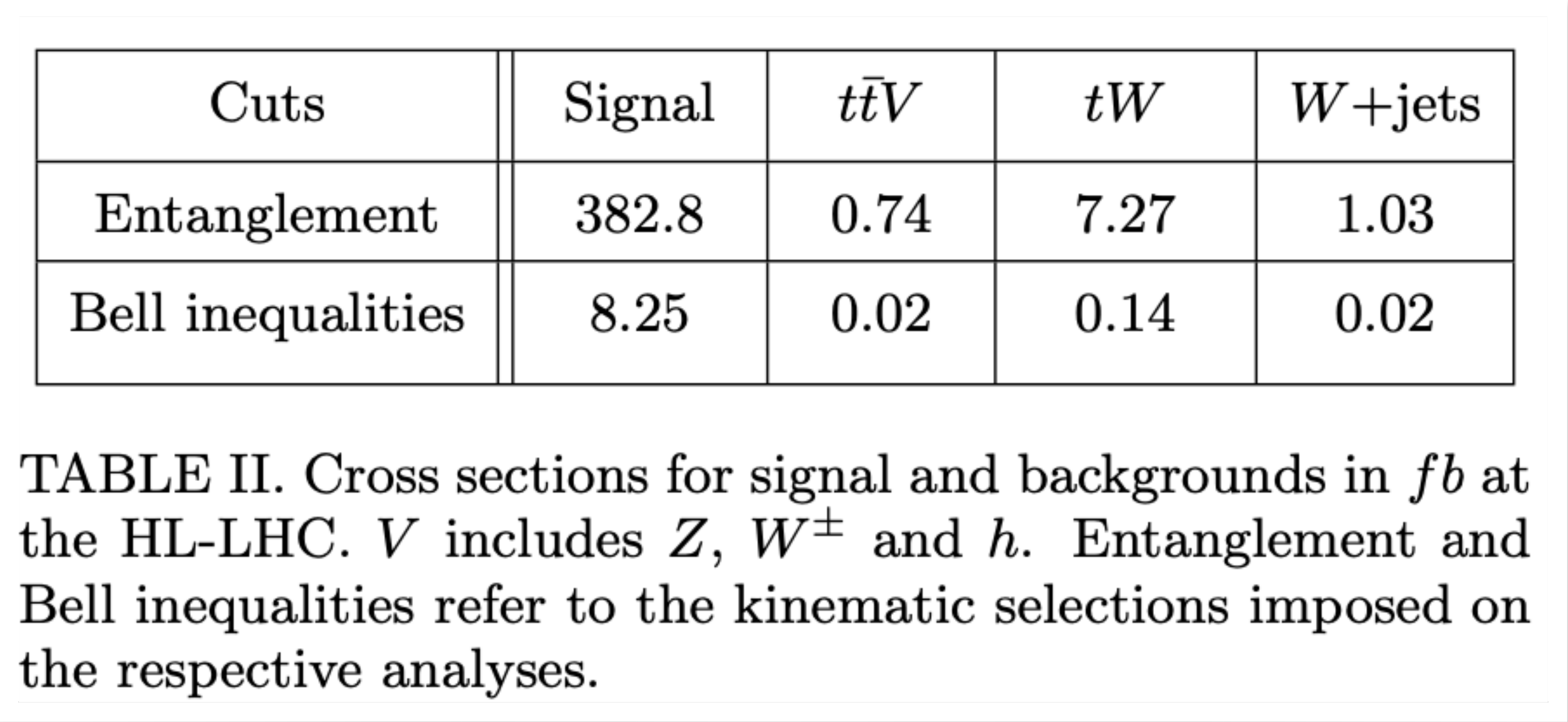
  Ex: Top quark pair Dong, DG, Kong, Navarro '23

- Optimal hadronic polarimeter Dong, DG, Kong, Larkoski, Navarro '24
  - Crucial to access the higher event rate from hadronic final states Dong, DG, Kong, Larkoski, Navarro '24
  - Can significantly boost spin correlations, entanglement, Bell inequalities, new physics searches…

- We are just at the beginning:
  - Brand new experimental results for entanglement
  - Bell/CHSH violation may be probed at 4-5 sigma level at the HL-LHC
  - Many new channels can be analogously studied in the SM and beyond: $t\bar{t},\ \tau\tau,\ ZZ,\ WW...$

U.S. DEPARTMENT OF ENERGY

Backgrounds



Cuts	Signal	$t\bar{t}V$	tW	W+jets
Entanglement	382.8	0.74	7.27	1.03
Bell inequalities	8.25	0.02	0.14	0.02

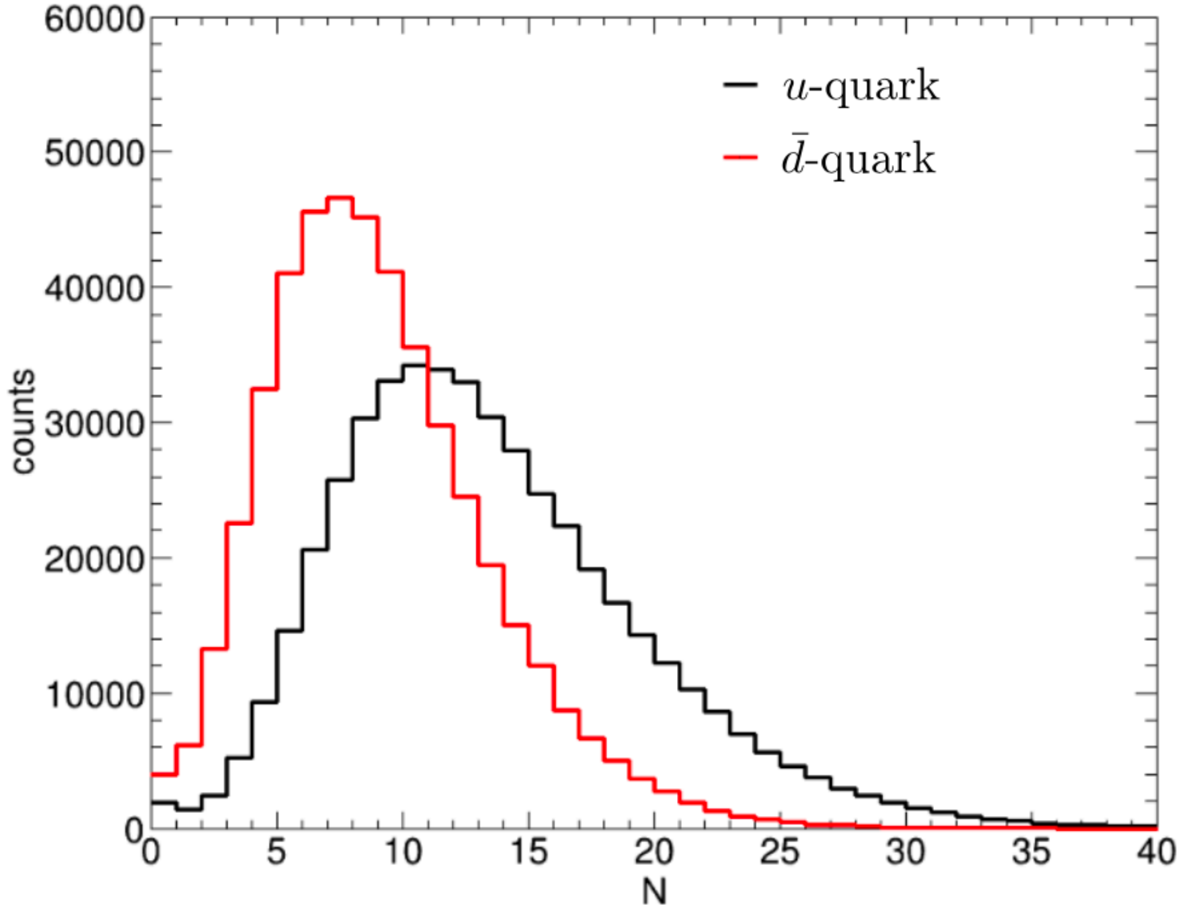
TABLE II. Cross sections for signal and backgrounds in fb at the HL-LHC. V includes Z, $W^{\pm}$ and h. Entanglement and Bell inequalities refer to the kinematic selections imposed on the respective analyses.

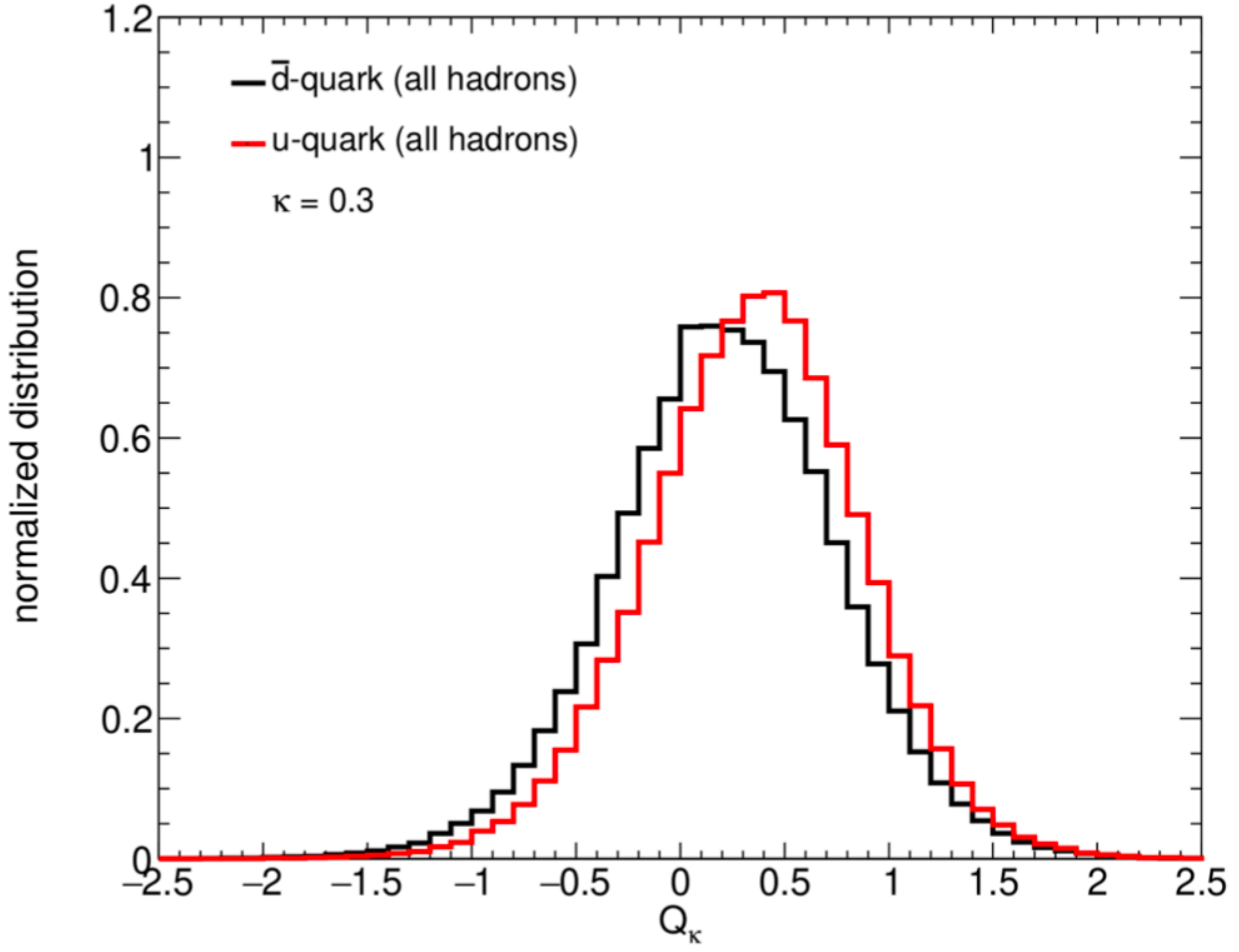
Dong, DG, Kong, Navarro '23

Multiplicity and Jet Charge

$$Q_\kappa = \sum_i Q_i\, z_i^\kappa = \sum_i Q_i \left(\frac{E_i}{E}\right)^\kappa$$

Field, Feynman 1978

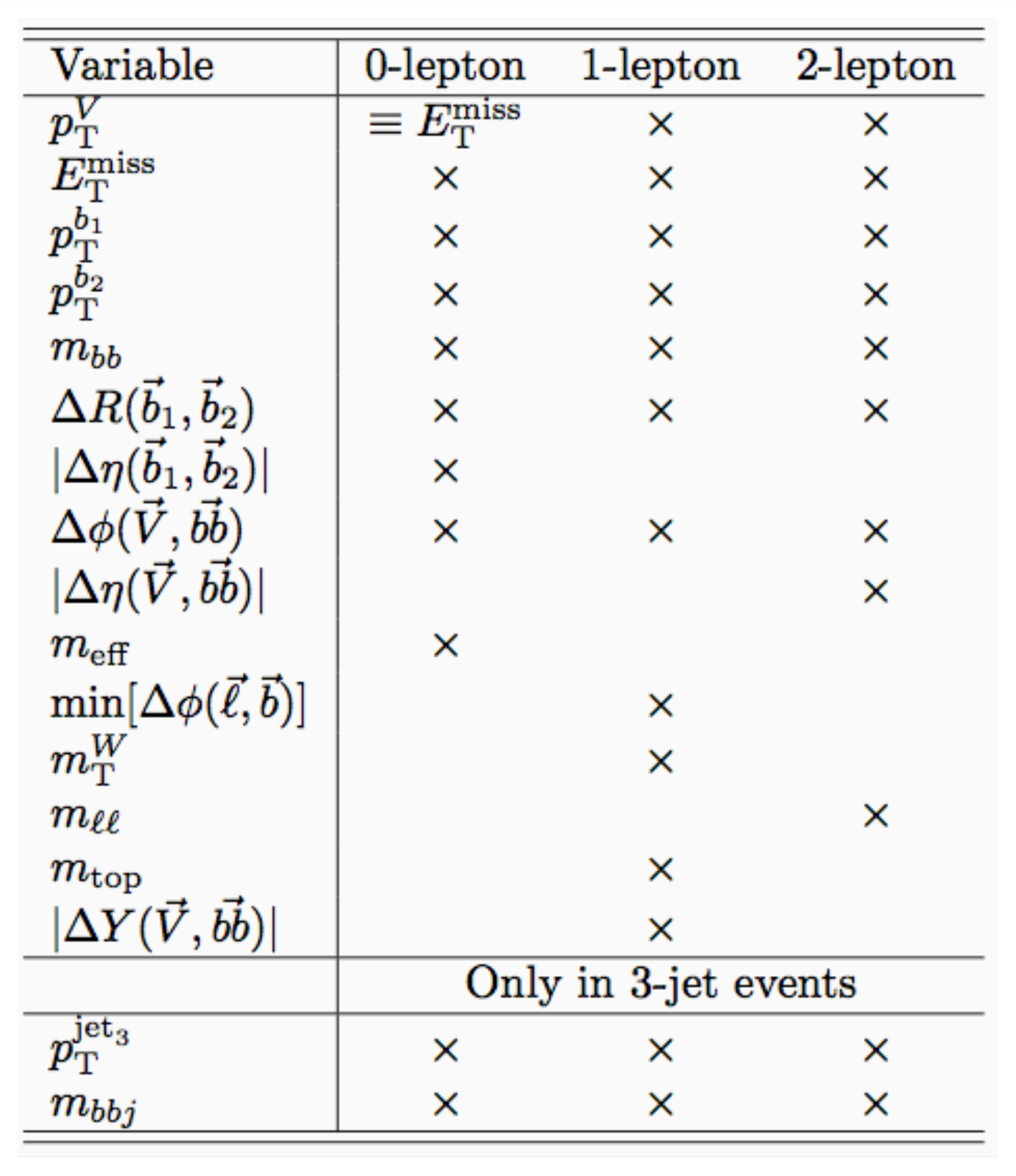




Role of the Z spin density in the H→bb measurement

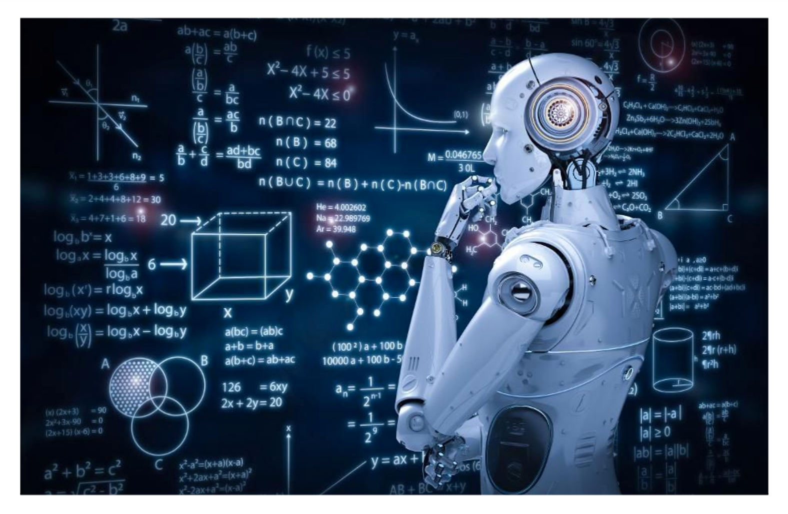
- While the spin density matrix is used in tt studies, it is typically disregarded when studying other channels at the LHC. Here, I will highlight its relevance for VH(bb) studies

- ATLAS & CMS limits use standard resolved jet studies, suppressing the backgrounds with MVA



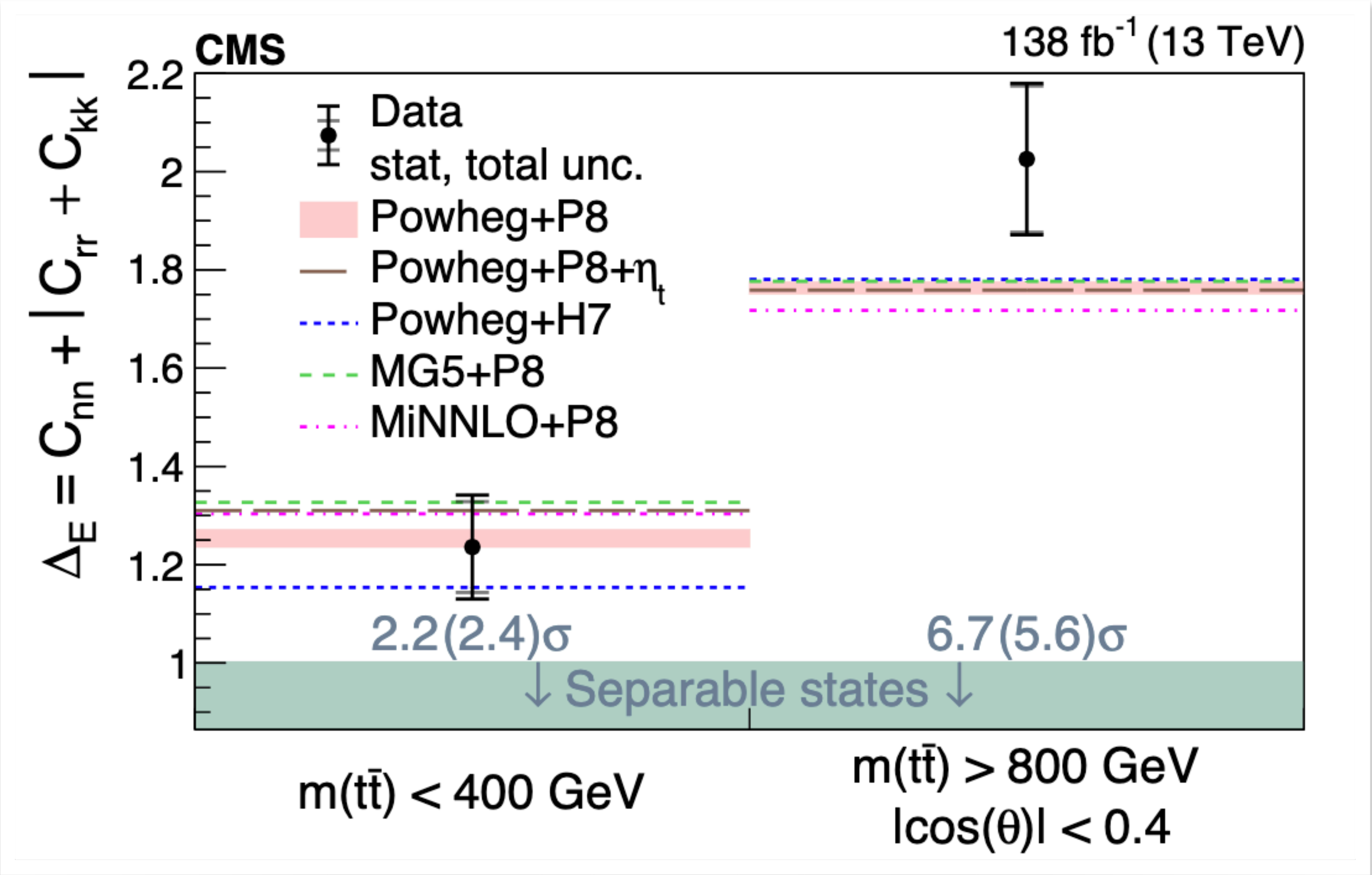
Variable	0-lepton	1-lepton	2-lepton
p_T^V	$\equiv E_T^{\text{miss}}$	×	×
E_T^{miss}	×	×	×
$p_T^{b_1}$	×	×	×
$p_T^{b_2}$	×	×	×
m_{bb}	×	×	×
$\Delta R(\vec{b}_1, \vec{b}_2)$	×	×	×
$\lvert\Delta\eta(\vec{b}_1, \vec{b}_2)\rvert$	×		
$\Delta\phi(\vec{V}, \vec{bb})$	×	×	×
$\lvert\Delta\eta(\vec{V}, \vec{bb})\rvert$			×
m_{eff}	×		
$\min[\Delta\phi(\vec{\ell}, \vec{b})]$		×	
m_T^W		×	
$m_{\ell\ell}$			×
m_{top}		×	
$\lvert\Delta Y(\vec{V}, \vec{bb})\rvert$		×	
	Only in 3-jet events		
$p_T^{\text{jet}_3}$	×	×	×
m_{bbj}	×	×	×

ATLAS



Z spin density matrix is disregarded

We are just at the beginning



Hadronic Top Quark Polarimetry with ParticleNet

Variable	Definition
$\Delta\eta_t$	difference in pseudorapidity between the particle and the top jet axis
$\Delta\phi_t$	difference in azimuthal angle between the particle and the top jet axis
$\Delta\eta_j$	difference in pseudorapidity between the particle and the subjet axis
$\Delta\phi_j$	difference in azimuthal angle between the particle and the subjet axis
$\log\, p_T$	logarithm of the particle's p_T
$\log\, E$	logarithm of the particle's Energy
q	electric charge of the particle
isElectron	if the particle is an electron
isMuon	if the particle is a muon
isPhoton	if the particle is a photon
isChargedHadron	if the particle is a charged hadron
isNeutralHadron	if the particle is a neutral hadron