Reduced Quantum Dynamics from Observables

arXiv:190x.xxxxx w/ Oleg Kabernik (UBC) and Ashmeet Singh (Caltech)

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Prior Work:
1801.09770 (OK)
1801.10168 (JP+AS)

Disclaimer

- I work on quantum gravity, but this talk will not (explicitly) be about quantum gravity.
- I care about this problem mainly because I'm interested in the (approximate) emergence of spacetime from a (more) fundamental Hilbert-space description.
  - (Including, but not limited to, a holographic description.)
- But (I hope) the results will be interesting more generally.

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- Arbitrary macrostates = collections of/distributions over microstates
- *Good* macrostates = possible to measure macroscopically, approximately preserved under time evolution (macrostates evolve to macrostates)
  - States with definite values of thermodynamic/hydrodynamic properties, planets/stars, …

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- "Dynamics"
  - Decoherence program: classical states are preserved under the action of the environment
- "Observables"
  - Reduced state preserves information about *specified set of observables* – more general than tensor factor

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- We have a more general method ("generalized bipartition tables") of generating reduced states than the partial-trace map
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- We have a more general method ("generalized bipartition tables") of generating reduced states than the partial-trace map
  - i.e. we can generate information-preserving maps from the space of density matrices in L(H) to space of density matrices in L(H'), where H' is not necessarily a factor of H
- We have an (explicit, polynomial-time) algorithm for going from the subalgebra generated by a set of observables to a generalized BPT which preserves expectation values of this subalgebra
  - Block-diagonal BPTs—c.f. explicit construction of block decomposition of a Hilbert space given a vN algebra
  - Involves graph-theoretic reduction of a special set of projectors

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- The standard (Zurekian) decoherence analysis can be applied to these new reduced states.
- Variation over possible sets of observables with the dynamics fixed yields the classical observables.

Wait, I only have 15 minutes?

- I'll try to define generalized BPTs and sketch the algorithm for reducing a state given a subalgebra of observables.
- Ask me about the rest later!

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  - Preserves expectation values
- (pure/mixed) state + particular choice of partial-trace map respecting factorization of Hilbert space → reduced density matrix
- "Bipartition table": tool to visualize the relevant bipartition for the partial-trace map
  - Essentially, arrangement of basis states into a grid

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- Bipartition operators for each pair of columns

$$S_{kl} := \sum_{i=1\ldots d_A} |a_i, b_k\rangle \langle a_i, b_l| = I \otimes |b_k\rangle \langle b_l|$$

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- Coarse-graining: e.g. particle on a lattice
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  - Clear what new Hilbert space is, but not how to embed this in the old one
- Collective observables
  - E.g. multiple particles → effective particles

Generalized BPTs

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- Example: 2 spin ½ particles in total spin-z basis

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- Traces out multiplet degree of freedom, preserves information about total spin operators

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- A general non-rectangular BPT preserves information in span $\{S_{kl}\}$, which need not be an algebra (not closed under products)
- E.g. N spin ½ particles, N even:

				$0,0$				
				$\vdots$				
				$0,0$				
			$1,+1$	$1,0$	$1,-1$			
			$\vdots$	$\vdots$	$\vdots$			
			$1,+1$	$1,0$	$1,-1$			
		$2,+2$	$2,+1$	$2,0$	$2,-1$	$2,-2$		
		$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
$\frac{N}{2},+\frac{N}{2}$	$\cdots$	$\frac{N}{2},+2$	$\frac{N}{2},+1$	$\frac{N}{2},0$	$\frac{N}{2},-1$	$\frac{N}{2},-2$	$\cdots$	$\frac{N}{2},-\frac{N}{2}$

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- Any vN algebra has a decomposition into irreps

$$Alg\left(\mathcal{O}\right) \cong \bigoplus_{q=1\ldots s} I_{m_q} \otimes \mathcal{M}\left(n_q, \mathbb{C}\right) \qquad d = \sum_{q=1}^{s} m_q n_q$$

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- There is an associated decomposition of the Hilbert space, $\mathcal{H} = \bigoplus_{q=1\ldots s} \mathcal{N}_q \otimes \mathcal{M}_q.$

- That is, there is some basis for $\mathcal{H}$ where all elements of the algebra are block-diagonal:

$$A = \begin{pmatrix} \underbrace{\begin{matrix} A_1 & & \\ & \ddots & \\ & & A_1 \end{matrix}}_{m_1} & & & \\ & \underbrace{\begin{matrix} A_2 & & \\ & \ddots & \\ & & A_2 \end{matrix}}_{m_2} & & \\ & & \ddots & \\ & & & \underbrace{\begin{matrix} A_s & & \\ & \ddots & \\ & & A_s \end{matrix}}_{m_s} \end{pmatrix} = \begin{pmatrix} I_{m_1} \otimes A_1 & & & \\ & I_{m_2} \otimes A_2 & & \\ & & \ddots & \\ & & & I_{m_s} \otimes A_s \end{pmatrix}$$

$$A_q \in \mathcal{M}\left(n_q, \mathbb{C}\right)$$

- The decomposition can be described by a *block-diagonal* generalized BPT, with each block giving a product basis for a $\mathcal{N}_q \otimes \mathcal{M}_q$

e^q_{11}	e^q_{12}	e^q_{13}	e^q_{14}
e^q_{21}	e^q_{22}	e^q_{23}	e^q_{24}
$\vdots$	$\vdots$	$\vdots$	$\vdots$
e^q_{r1}	e^q_{r2}	e^q_{r3}	e^q_{r4}

$$\longrightarrow \qquad \mathcal{N}_q \otimes \mathcal{M}_q$$

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- The BPOs form a basis spanning $Alg(\mathcal{O})$, with a simple action under products $S^q_{kl} S^{q'}_{l'k'} = \delta_{ll'} \delta_{qq'} S^q_{kk'}$

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- "Scatter" products of projectors, i.e. decompose them into new projectors.

$$\begin{cases} \Pi^a\Pi^b\Pi^a = & \Pi_1^{ab} + \sum_k \lambda_k \Pi_k^a + 0\Pi_0^a \\ \Pi^b\Pi^a\Pi^b = & \Pi_1^{ab} + \sum_k \lambda_k \Pi_k^b + 0\Pi_0^b \end{cases} \qquad \begin{matrix} \Pi^a \\ \Pi^b \end{matrix} \overset{\text{scatter}}{\longmapsto} \begin{Bmatrix} \Pi_1^{ab}, \Pi_2^a \ldots \Pi_0^a \\ \Pi_1^{ab}, \Pi_2^b \ldots \Pi_0^b \end{Bmatrix}$$

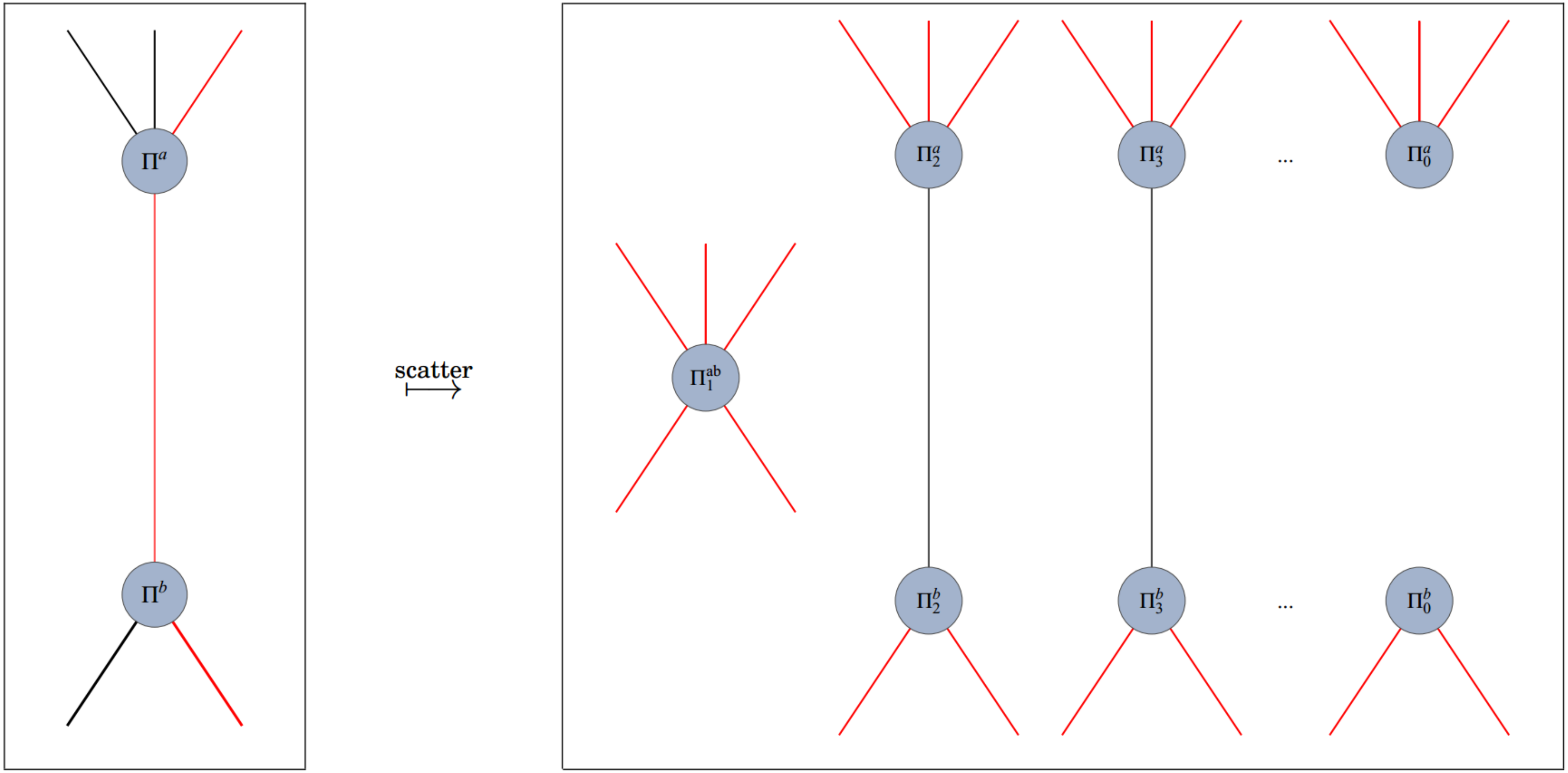
- Repeat process until all scattering is trivial (projectors reflecting or orthogonal)

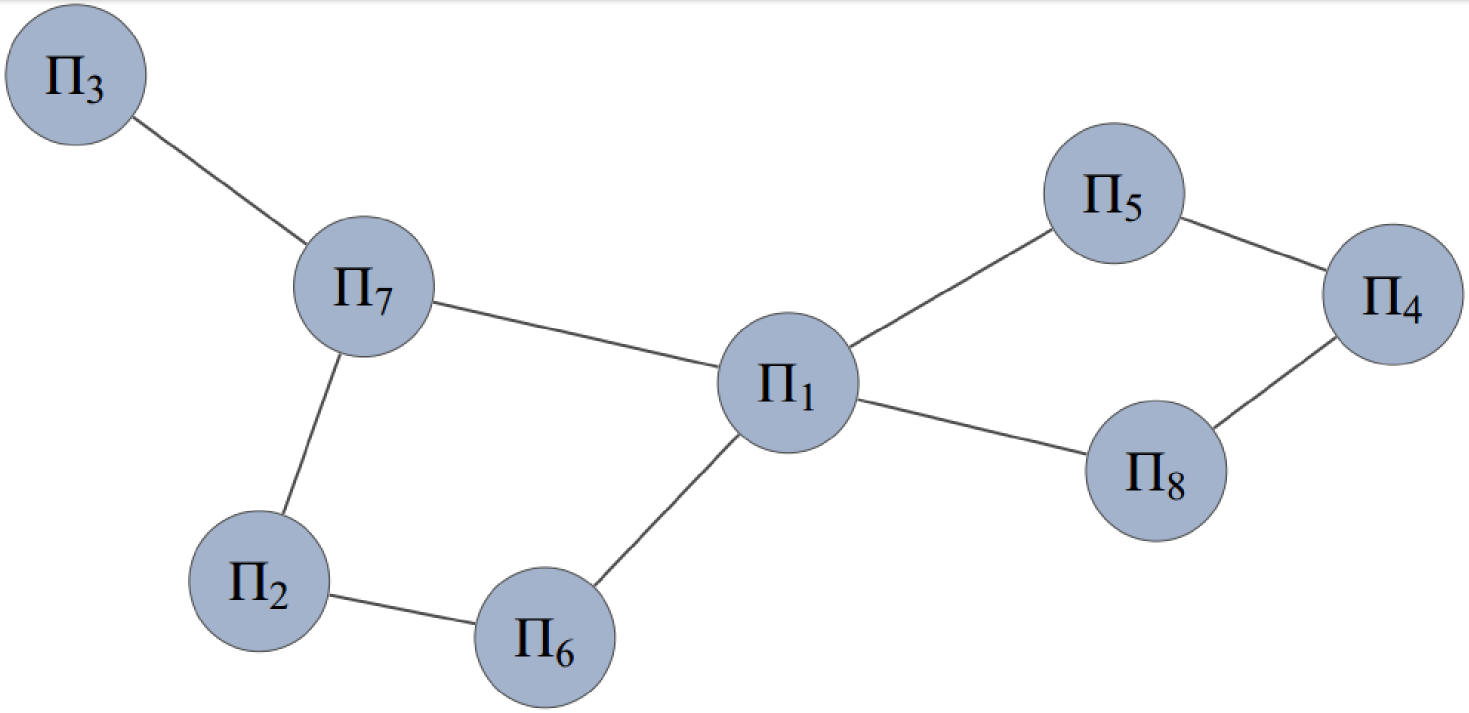
$$\begin{cases} \Pi^a \Pi^b \Pi^a = & \lambda \Pi^a \\ \Pi^b \Pi^a \Pi^b = & \lambda \Pi^b \end{cases}$$

$$\begin{matrix} \Pi^a \\ \Pi^b \end{matrix} \quad \xmapsto{\text{scatter}} \quad \begin{matrix} \Pi^a \\ \Pi^b \end{matrix}$$

$$\begin{cases} \Pi^a \Pi^b \Pi^a = & 0 \Pi^a \\ \Pi^b \Pi^a \Pi^b = & 0 \Pi^b \end{cases}$$

- Graph-theoretic interpretation




Π_3
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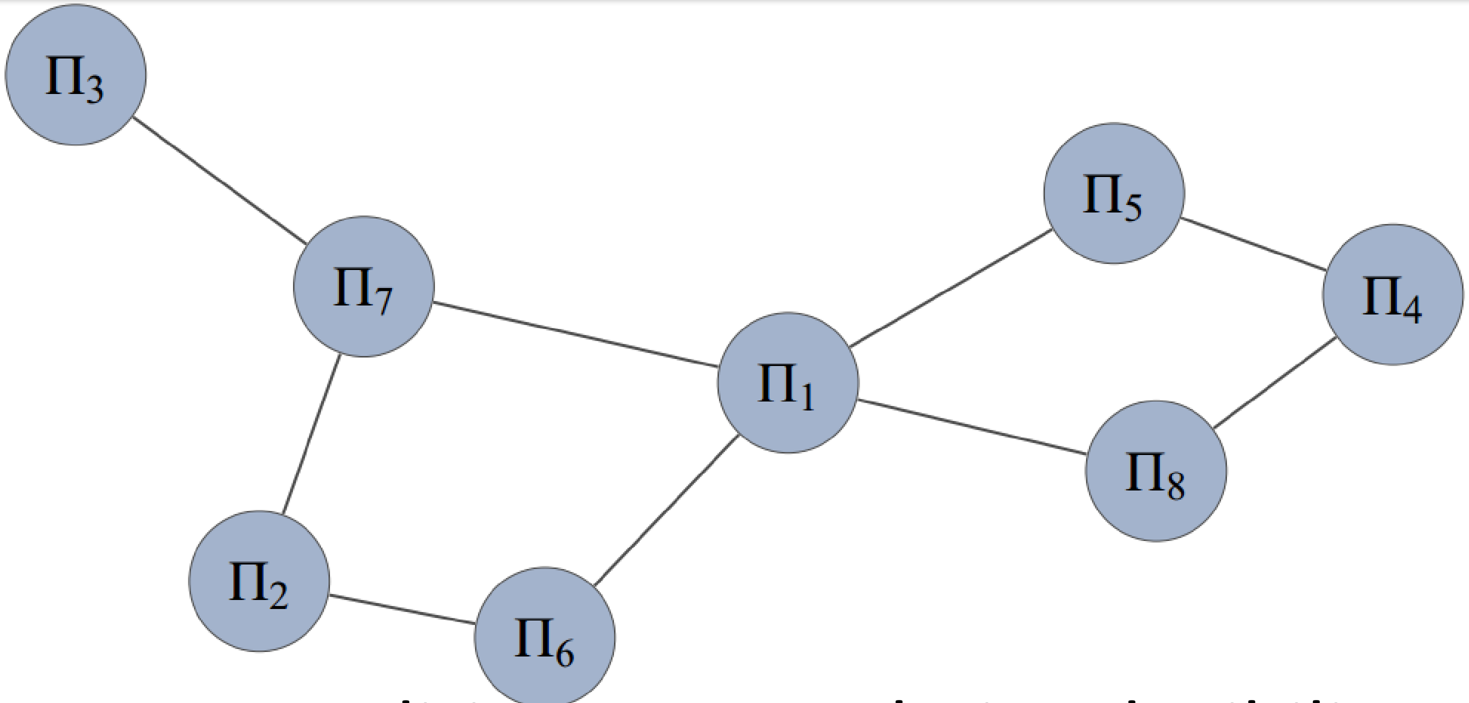
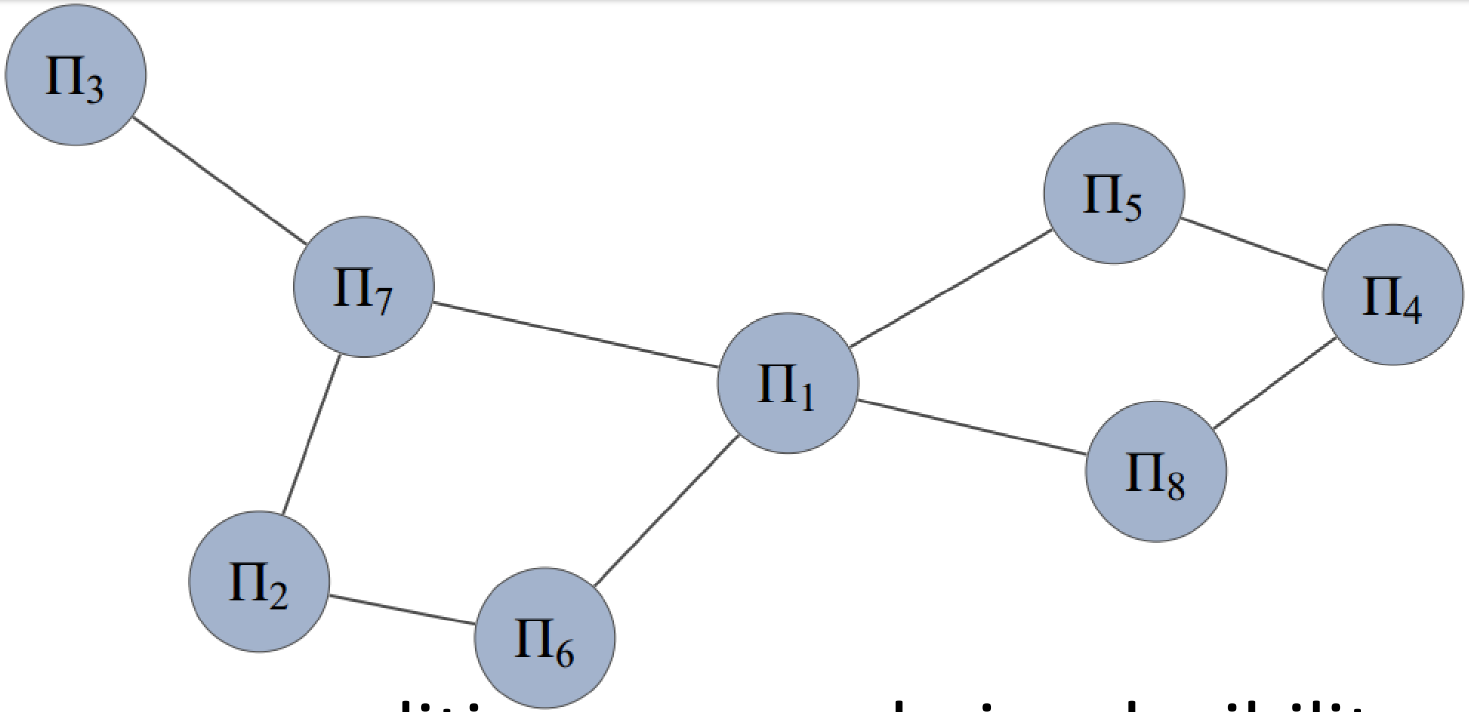
Π_8
Π_2
Π_6



- Impose conditions on graph: irreducibility and completeness (by adding more projectors if necessary)



- Impose conditions on graph: irreducibility and completeness (by adding more projectors if necessary)
- Construct BPT by traversing graph

Applications

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- Error correction

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- Error correction
- Bulk reconstruction

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- Bulk reconstruction
- Quantum gravity

Applications

- Error correction
- Bulk reconstruction
- Quantum gravity
- …more to come?

Thank you!

Generalized BPT example

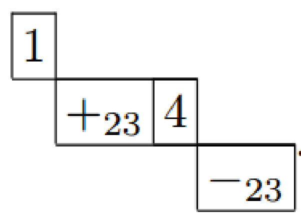
$$\{|1\rangle, |2\rangle, |3\rangle, |4\rangle\} \qquad |\pm_{ij}\rangle := \frac{|i\rangle \pm |j\rangle}{\sqrt{2}} \qquad |\phi_{234}\rangle = \frac{\sqrt{2}}{\sqrt{3}}|+_{23}\rangle + \frac{1}{\sqrt{3}}|4\rangle$$

$$\Pi_1^A = |1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3| \quad \Pi_2^A = |4\rangle\langle 4| \qquad \Pi^B = |\phi_{234}\rangle\langle\phi_{234}| + |-_{23}\rangle\langle -_{23}|$$

$$Q_{11} = \Pi_1^A \Pi^B \Pi_1^A = \Pi_1^A |\phi_{234}\rangle\langle\phi_{234}| \Pi_1^A + \Pi_1^A |-_{23}\rangle\langle -_{23}| \Pi_1^A = \frac{2}{3}|+_{23}\rangle\langle +_{23}| + |-_{23}\rangle\langle -_{23}|$$

$$Q_{22} = \Pi_2^A \Pi^B \Pi_2^A = \frac{1}{3}|4\rangle\langle 4|$$

$$Q_{12} = \Pi_1^A \Pi^B \Pi_2^A = \Pi_1^A |\phi_{234}\rangle\langle\phi_{234}| \Pi_2^A + \Pi_1^A |-_{23}\rangle\langle -_{23}| \Pi_2^A = \frac{\sqrt{2}}{3}|+_{23}\rangle\langle 4|.$$



Collective Observables

- Consider many particles, interacting in some potential (e.g. two bound particles). Say we can only track the center of mass + total momentum.
- We don't have access to measurements that distinguish individual particles. So the observable physics is invariant under permutations of the particles.
- The results BPT is block-diagonal in the irreps of the permutation group (e.g. symmetric vs antisymmetric under exchange of particles.)
- Ex 2: reproduces Clebsch-Gordon decomp for spins

Variational Approach

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- Optimize over possible BPTs

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- Change of basis states

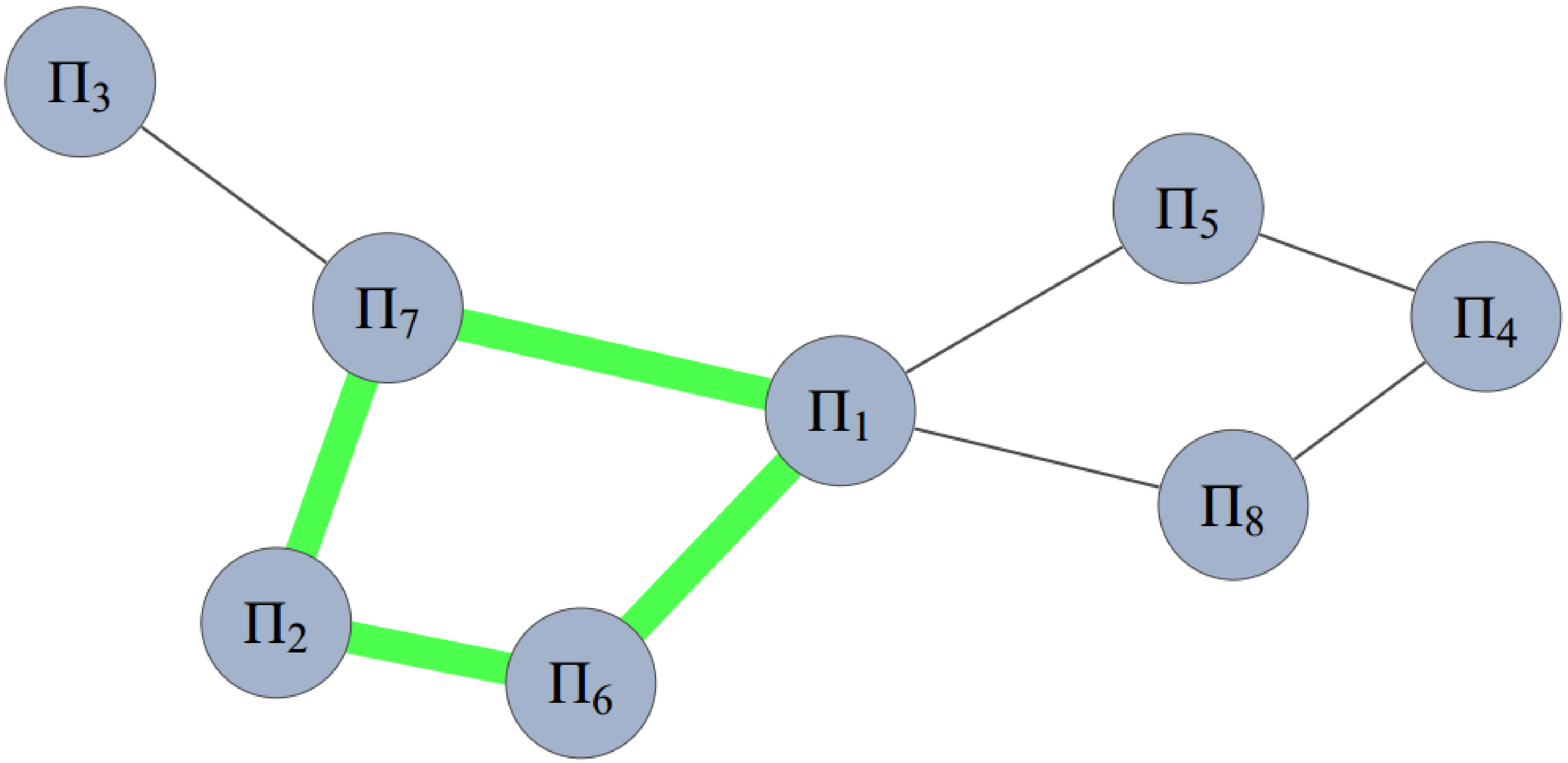
Variational Approach

- Optimize over possible BPTs
- Change of basis states
- Change of table arrangements

Decoherence

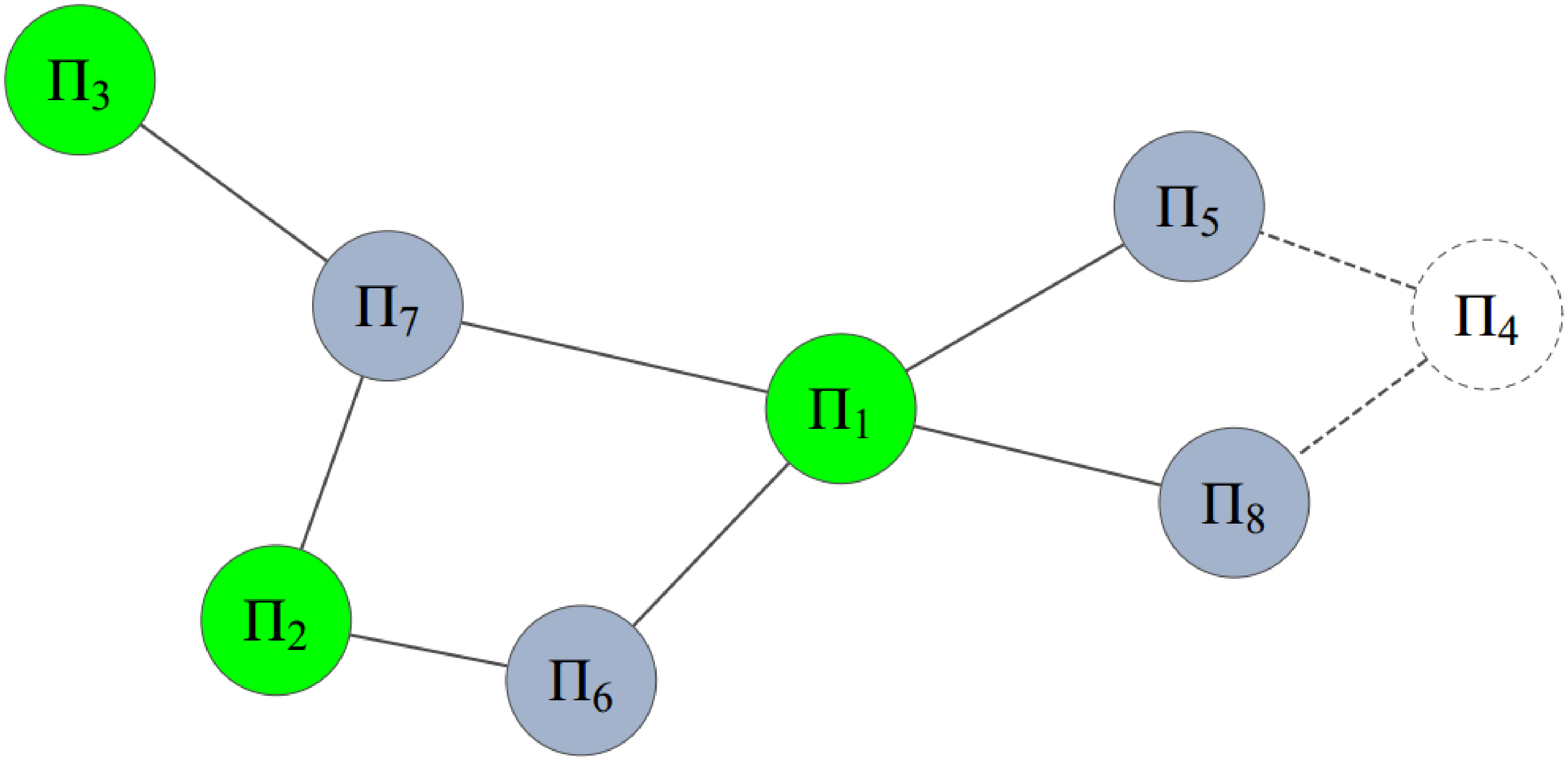
- Zurekian story
  - System-environment split
  - Pointer basis
  - Branching + classical states
  - Relies on existence of reduced density matrix + Hamiltonian

Irreducibility



$$\Pi_2\Pi_6\Pi_1\Pi_7\Pi_2 \propto S_{22} \stackrel{?}{=} \Pi_2$$

Completeness

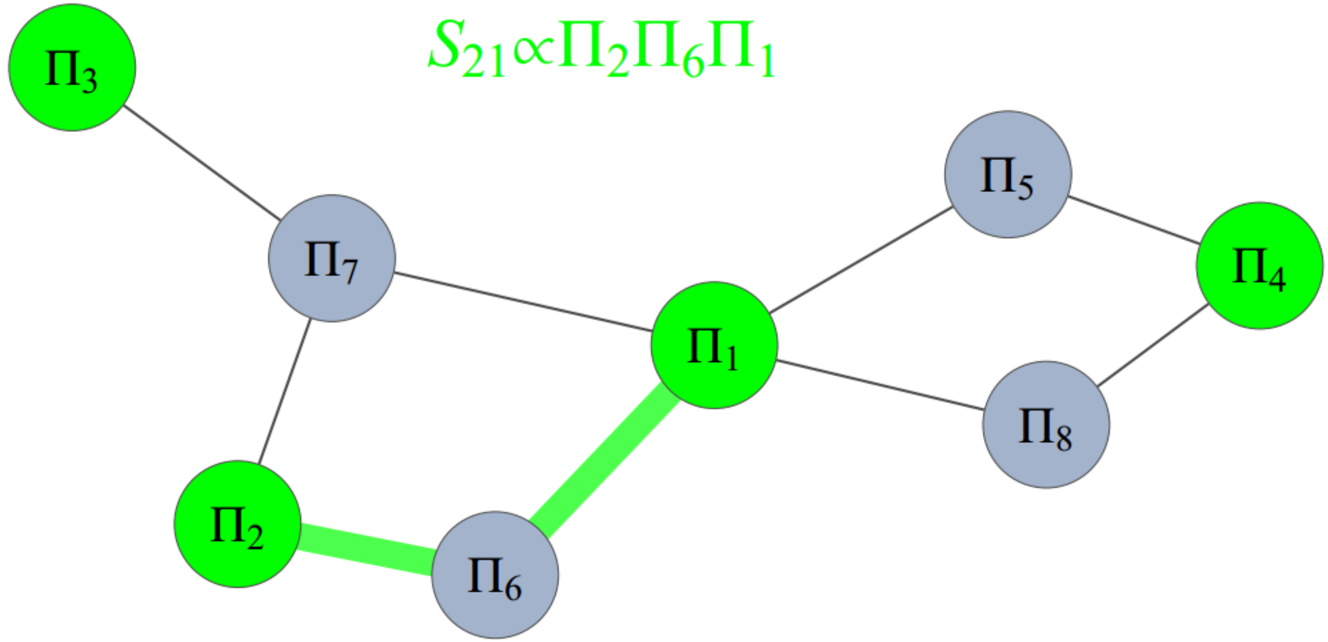


$$\Pi_1 + \Pi_2 + \Pi_3 \neq I_c$$

$$\Pi_4 \propto \Pi_4 \Pi_8 \Pi_4 = (I - \Pi_1 - \Pi_2 - \Pi_3)\, \Pi_8\, (I - \Pi_1 - \Pi_2 - \Pi_3)$$

Reconstruction

$$S_{21} \propto \Pi_2 \Pi_6 \Pi_1$$



Π_1	Π_2	Π_3	Π_4
e_{11}	e_{12}		
e_{21}	e_{22}		
$\vdots$	$\vdots$		
e_{r1}	e_{r2}		

$$e_{i2} = S_{21} e_{i1}$$