Near-extremal black holes and Jackiw-Teitelboim gravity

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Introduction

- The Jackiw-Teitelboim model is a model of gravity coupled to a scalar field, called the dilaton, in 2D.
  – Jackiw 1985, Teitelboim 1983
- It has gained a lot of attention recently due to its connection with the Sachdev-Ye-Kitaev model of fermions in 1D.
  – Sachdev & Ye 1993, Kitaev 2015
- Both the models exhibit an identical pattern of symmetry breaking, and the associated dynamics is governed by a Schwarzian action.
- This gives tantalizing hints towards a possible duality between the two models.
  – Maldacena & Stanford 2016

- AdS_2 spacetime (with a varying dilaton) arises as the solution in the JT model.
  – Almheiri & Polchinski 2014; Maldacena, Stanford and Yang 2016
- AdS_2 spacetime is also known to arise as the geometry in the near-horizon region of near-extremal black holes.
- It would be interesting to know how well does the JT model capture the physics of near-extremal black holes.
- As we will illustrate, the thermodynamics and the low-energy behaviour of near-extremal black holes is well captured by the JT model.
- For concreteness, we will work with the magnetically charged near-extremal Reissner-Nordström black hole in asymptotically AdS_4 spacetime.

The Reissner-Nordström black hole

- The RN black hole is a spherically symmetric charged black hole solution to the Einstein-Maxwell system,

$$S = \frac{1}{16\pi G} \int d^4x\sqrt{-g}\,(R - 2\Lambda) - \frac{1}{4G} \int d^4x\sqrt{-g}\, F_{\mu\nu}F^{\mu\nu}$$

- The magnetically charged asymptotically AdS_4 black hole solution to the above action is

$$ds^2 = -a(r)^2\, dt^2 + \frac{1}{a(r)^2}\, dr^2 + b(r)^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2),$$

$$a(r)^2 = 1 - \frac{2GM}{r} + \frac{4\pi Q^2}{r^2} + \frac{r^2}{L^2}, \quad b(r) = r, \quad F_{\theta\varphi} = Q\sin\theta.$$

- The CC is related to the AdS radius via $\Lambda = -\frac{3}{L^2}$.

Extremal limit

- The black hole has two horizons $r_\pm$.
- In the extremal limit the two horizons coalesce $r_\pm = r_h$.
- The mass and charge at extremality are

$$M_{ext} = \frac{r_h}{G}\left(1 + \frac{2r_h^2}{L^2}\right), \quad Q_{ext}^2 = \frac{r_h^2}{4\pi}\left(1 + \frac{3r_h^2}{L^2}\right)$$

- To the leading order in $\frac{r-r_h}{r_h} \ll 1$ the near-horizon metric is

$$ds^2 = -\frac{(r-r_h)^2}{L_2^2}dt^2 + \frac{L_2^2}{(r-r_h)^2}dr^2 + r_h^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)$$

- This has the form of $AdS_2 \times S^2$, with

$$L_2 \approx \frac{L}{\sqrt{6}}, \quad R_{S^2} = r_h.$$

Near-extremal limit and thermodynamics

- In the near-extremal scenario, the two horizons are located at $r_{\pm} = r_h \pm \delta r_h$, with $\frac{\delta r_h}{r_h} \ll 1$.
- δr_h measures the splitting of the two horizons near extremality.
- The temperature is proportional to δr_h, $T \equiv \frac{1}{\beta} \sim \frac{\delta r_h}{L^2}$
- By computing the Euclidean onshell action with appropriate counter terms, one can compute the entropy of the black hole, which meets the Bekenstein entropy formula,

$$\mathcal{S} = \frac{\pi r_+^2}{G}$$

- The near-extremal free energy is then given by

$$\boxed{\beta \mathcal{F} = \beta M - \mathcal{S} \approx \beta M_{ext} - \beta \delta M - \frac{\pi r_h^2}{G}}$$

Response to a scalar: 4-point function

- The next thing we want to compute is the response of the system to a scalar, which is free except gravitational interactions

$$S = \frac{1}{2}\int d^4x\sqrt{g}\,\left[(\partial\sigma)^2 + m^2\sigma^2\right].$$

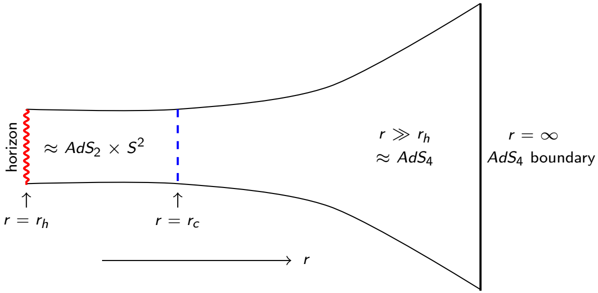
- The bulk scalar is dual to a scalar operator in the boundary theory, and we will be interested in the four point function of this operator at low-energies.
- For simplicity, we will assume spherical symmetry.
- As we will see, the non-trivial contribution at low energy comes from the near-horizon region of the geometry.

- Let's make the notion of low-energy precise.
- Consider the near-horizon $AdS_2 \times S^2$ region of the black hole. Construct a screen at $r = r_c$ in this region which acts like a boundary for this region, so

$$\begin{aligned} \text{Near-horizon limit:} \quad & \tfrac{r_c - r_h}{r_h} \ll 1, \\ \text{Near AdS}_2 \text{ boundary:} \quad & \tfrac{r_c - r_h}{L_2} \gg 1, \end{aligned}$$

- For consistency we work with large black holes, $r_h \gg L, L_2$
- σ can be mode expanded as $\sigma(t, r) = \int d\omega\, e^{i\omega t} \sigma(\omega, r)$
- $\sigma(\omega, r)$ satisfies the equation of motion

$$\frac{1}{r^2} \partial_r \left(r^2 a^2 \partial_r \sigma \right) - \left(\frac{\omega^2}{a^2} + m^2 \right) \sigma = 0.$$



- By low energy, we mean frequencies that satisfy $\frac{\omega}{a} \ll m$ for $r > r_c$.
- In the region $r_c < r < \infty$ the e.o.m. of σ then implies a solution of the factorized form,

$$\sigma \sim \hat{\sigma}(t) f(r)$$

with $f(r)$ being a power law.

- We now compute the four point function.
- The scalar stress tensor sources fluctuations in the metric,

$$ds^2 = a^2(r)\,(1 + h_{tt})\,dt^2 \;+\; \frac{1}{a^2(r)}\,dr^2 \;+\; b^2(r)\,(1 + h_{\theta\theta})\,d\Omega_2^2,$$

  where we have chosen the gauge $h_{rr} = h_{tr} = 0$.
- The interaction term is given by

$$\mathcal{I} = \frac{1}{4}\int d^4x\sqrt{g}\;\delta g^{\mu\nu}\;T_{\mu\nu}$$

- Using the e.o.m. of metric fluctuations and the conservation of the stress tensor, this can be written as

$$\mathcal{I} = -8\pi^2 G\int dt\,dr\,\left(\frac{2a^2b^3}{b'}\;T_{rr}\frac{1}{\partial_t}T_{tr} - a^2b^2\left(1+\frac{2a'b}{b'a}\right)\,T_{tr}\frac{1}{\partial_t^2}T_{tr}\right)$$

- Interestingly, in the low-energy limit, due to the factorized form of σ the region $r_c < r < \infty$ only gives rise to a contact term in the onshell action!
- The non-trivial contribution comes only from the near-horizon region $r_h < r < r_c$
- The term $\frac{2a'b}{ab'} \gg 1$ in this region.
- Using the coordinate $z = \frac{L_2^2}{r - r_h}$ the onshell action becomes

$$\boxed{\mathcal{I} \simeq 16\pi^2 G \, \frac{r_h^3}{L_2^2} \int dt \int_{\delta_c}^{\infty} dz \, z \left(T_{tz} \frac{1}{\partial_t^2} T_{tz} - z \, T_{tz} \frac{1}{\partial_t} T_{zz} \right)}$$

with $\delta_c = \frac{L_2^2}{r_c - r_h}$.

Jackiw-Teitelboim gravity

- The JT model is a 2D model of dilaton gravity,

$$S_{JT} = -\frac{r_h^2}{4G}\left(\int d^2x\sqrt{g}\,R + 2\int_{bdy}\sqrt{\gamma}\,K\right) - \frac{r_h^2}{2G}\left(\int d^2x\sqrt{g}\,\phi\,(R-\Lambda_2) + 2\int_{bdy}\sqrt{\gamma}\,\phi K\right).$$

- The first term is topological.
- The e.o.m. of the dilaton sets the background to be AdS_2,

$$ds^2 = \frac{L_2^2}{z^2}(dt^2 + dz^2), \quad \text{with } \Lambda_2 = -\frac{2}{L_2^2}.$$

- The non-trivial dynamics thus arises from the boundary term.

- Euclidean AdS_2 is like a disk.
- Let the boundary be located at $z = \delta$, with $\delta \to 0$.
- Small fluctuations of the boundary corresponding to time reparametrizations change it to $z(1 - \epsilon(t)) = \delta$.
- The coordinate transformation

$$t = \hat{t} + \epsilon(\hat{t}) - \frac{\hat{z}^2 \epsilon''(\hat{t})}{2}, \quad z = \hat{z}(1 + \epsilon'(\hat{t})).$$

  puts the boundary at $\hat{z} = \delta$, and changes the metric to

$$ds^2 = \frac{L_2^2}{\hat{z}^2}(1 + h_{tt})\, d\hat{t}^2 + \frac{L_2^2}{\hat{z}^2}\, d\hat{z}^2, \quad \text{with } h_{tt} = -\epsilon'''(\hat{t})\hat{z}^2$$

- The fluctuations are now parametrized by h_{tt}.

- The metric equation of motion from the JT action gives the solution for the dilaton to be $\phi = \frac{L_2^2}{r_h z}$.
- To compute the action for the modes $\epsilon(t)$ we substitute the solutions for ϕ and the metric in the action. This gives

$$S = -\frac{r_h L_2^2}{G} \int_{bdy} \epsilon'''(t).$$

- A more careful analysis keeping higher orders in ϵ yields

$$S = -\frac{r_h L_2^2}{G} \int_{bdy} \text{Sch}[\epsilon(t)],$$

where for $t \to t + \epsilon(t) \equiv f(t)$, $\text{Sch}[\epsilon(t)] = -\frac{1}{2}\frac{(f'')^2}{(f')^2} + \left(\frac{f''}{f'}\right)'$.

JT thermodynamics

- Let us now look at the thermodynamics in the JT model.
- The black hole metric is

$$ds^2 = \left(\frac{(r-r_h)^2}{L_2^2} - \frac{2G\delta M}{r_h}\right) dt^2 + \frac{dr^2}{\left(\frac{(r-r_h)^2}{L_2^2} - \frac{2G\delta M}{r_h}\right)}$$

- Onshell the topological term reproduces the correct extremal entropy, $\pi r_h^2/G$.
- The complete onshell action gives the free energy

$$\boxed{\beta\mathcal{F} = -\beta\delta M - \frac{\pi r_h^2}{G}}$$

which is in agreement with the near-extremal result (up to the extremal piece)!

4-point function in JT gravity

- We now compute the response to a scalar in the JT model.
- The scalar action is

$$S_\sigma = 2\pi r_h^2 \int d^2x \sqrt{g}\, \left((\partial\sigma)^2 + m^2\sigma^2\right)$$

- The scalar does not couple to the dilaton. Thus the dilaton e.o.m. still sets the background to be AdS_2.
- However there is a non-trivial coupling between the scalar and the boundary fluctuations,

$$S_\sigma = 4\pi r_h^2 \int dt\, \left(\epsilon'(t) z T_{zz} + \epsilon(t) T_{tz}\right).$$

- The action for $\epsilon(t)$ is the Schwarzian action described earlier.

- Integrating out $\epsilon(t)$ from the combined action and performing simplifications using the stress tensor conservation etc. gives the onshell action

$$\boxed{S_{OS} = \frac{16\pi^2 G r_h^3}{L_2^2} \int d^2x\, z\, T_{tz}\, \frac{1}{\partial_t^2}\,(T_{tz} - z\partial_t T_{zz})}$$

- This matches with the low-energy limit of the near-extremal computation!
- JT gravity thus provides a good description of the thermodynamics and low-energy dynamics for near-extremal RN black holes.

Dimensional reduction from 4D to 2D

- Before we conclude, let's see why JT is able to capture the low-energy dynamics so well.
- We perform dimensional reduction of the 4D theory assuming spherical symmetry.
- Take the 4D action

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{\hat{g}}\left(\hat{R}-2\hat{\Lambda}\right)-\frac{1}{8\pi G}\int d^3x\sqrt{\hat{\gamma}}K^{(3)}+\frac{1}{4G}\int d^4x\sqrt{\hat{g}}F^2$$

- We reduce it to 2D by taking the metric ansatz

$$ds^2 = g_{\alpha\beta}(t,r)\,dx^\alpha dx^\beta + \Phi^2(t,r)\,d\Omega_2^2$$

- We also need a Weyl rescaling $g_{\alpha\beta} \to \frac{r_h}{\Phi}\, g_{\alpha\beta}$.

- We restrict the action to the near-horizon region. For this, we insert

$$\Phi = r_h(1+\phi)$$

and expand up to quadratic order in ϕ.
- The resulting action is

$$S = -\frac{r_h^2}{4G}\left(\int d^2x\sqrt{g}\,R + 2\int_{bdy}\sqrt{\gamma}\,K\right) - \frac{r_h^2}{2G}\int d^2x\,\sqrt{g}\,\phi\,(R-\Lambda_2)$$
$$+\frac{3r_h^2\,\kappa}{G\,L_2^2}\int d^2x\,\sqrt{g}\,\phi^2 - \frac{r_h^2}{G}\int_{bdy}\sqrt{\gamma}\,\phi K - \frac{r_h^2}{2G}\int_{bdy}\sqrt{\gamma}\,\phi^2 K.$$

- This has additional terms on top of JT.

- The e.o.m. of ϕ implies that the geometry departs from AdS_2 at same order as ϕ,

$$R = \Lambda_2 + \mathcal{O}(\phi)$$

- However, the additional bulk and boundary terms present contain onshell an extra factor of $\frac{L_2}{r_h}$, and are therefore suppressed compared to the terms linear in ϕ.
- This explains why JT captures the near-horizon low energy dynamics to the leading order in $\frac{L}{r_h}$ so well.

Concluding comments

- It would be interesting to know how universally does JT capture the low energy dynamics of other near-extremal black holes.
- JT works well even when:
  – Departures from spherical symmetry are included
  – The matter coupled to the black hole is also charged
  - - - Moitra, Trivedi & Vishal 2018; Sachdev 2019
  – Rotating Kerr black holes in 4D and 5D
  - - - Moitra, Sake, Trivedi & Vishal 2019
- Currently investigating rotating BTZ black holes.
  - - - Kundu, Shukla & Vishal (*work in progress*)