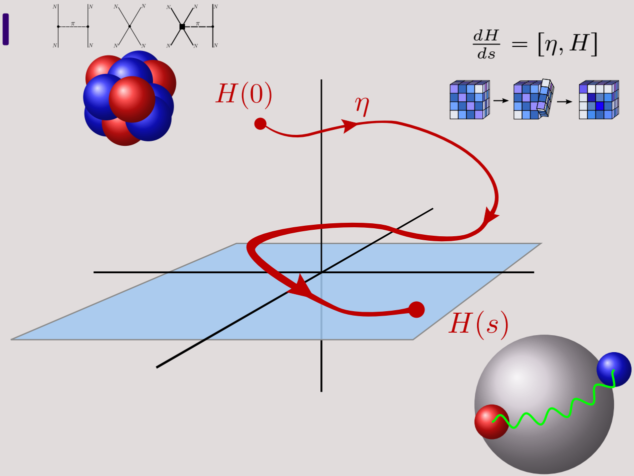
The Status of the Nuclear Shell Model

Still kicking after 70 years



Ragnar Stroberg

University of Washington

Theory Canada 14
University of British Columbia
Vancouver, BC
May 31, 2019

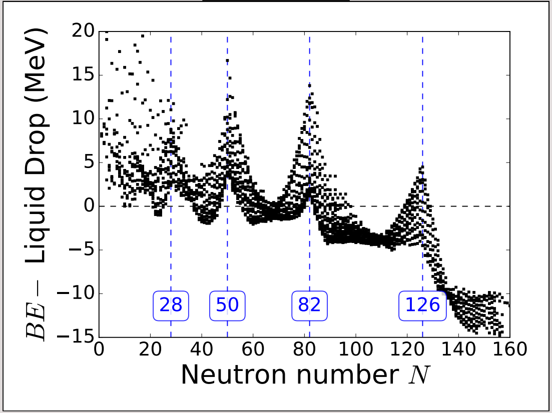
Background

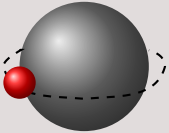
- The Mayer-Jensen shell model
- The interacting shell model
- Attempts to derive the residual interaction
- Phenomenology

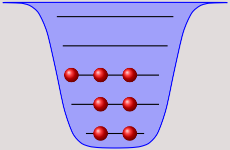
Recent progress

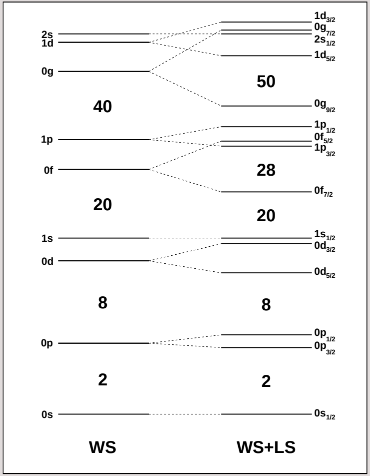
- Forces from chiral EFT (with consistent 3N)
- Similarity renormalization group
- Valence-space in-medium SRG
- Some results and future work

The nuclear shell model

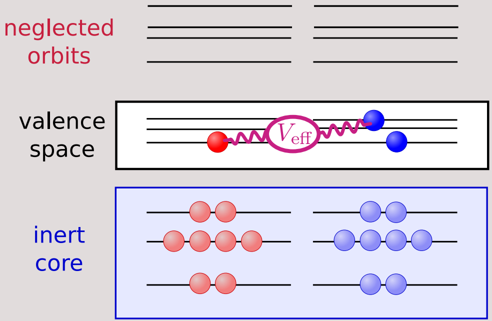








Mayer 1948; Haxel, Jensen, and Suess 1949



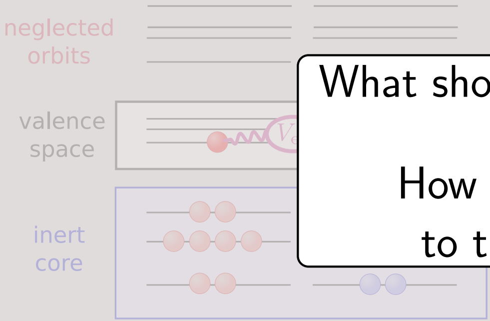
$$H_{SM} = E_{core} + \sum_i \epsilon_i \; a_i^\dagger a_i + \sum_{ijkl} V_{ijkl} \; a_i^\dagger a_j^\dagger a_l a_k$$

Diagonalize H_{SM} in space of all possible valence configurations.

^{56}Ni in fp shell:

- $\sim 10^9$ possible configurations
- $\gtrsim 10^{11}$ nonzero matrix elements

$$H_{SM} = E_{core} + \sum_i \epsilon_i \; a_i^\dagger a_i + \sum_{ijkl} V_{ijkl} \; a_i^\dagger a_j^\dagger a_l a_k$$



What should we use for V_{eff}?

How is H_{SM} related to the "bare" H?

...n space of all
...nfigurations.

- $\sim 10^9$ possible configurations
- $\gtrsim 10^{11}$ nonzero matrix elements

Effective interaction theory

Operator P projects from large Hilbert space to smaller, tractable space.
Complementary projector $Q = \mathbb{1} - P$

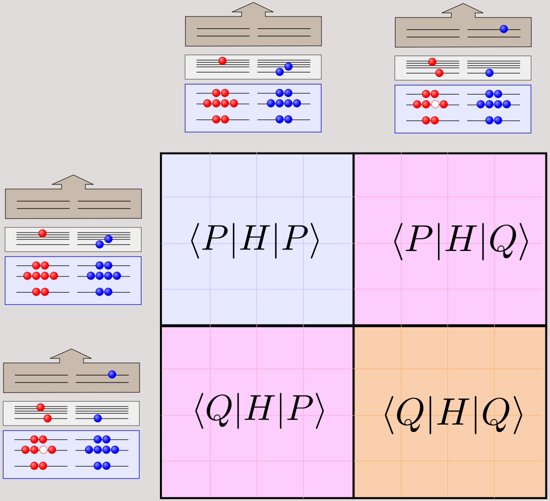
Basic idea:

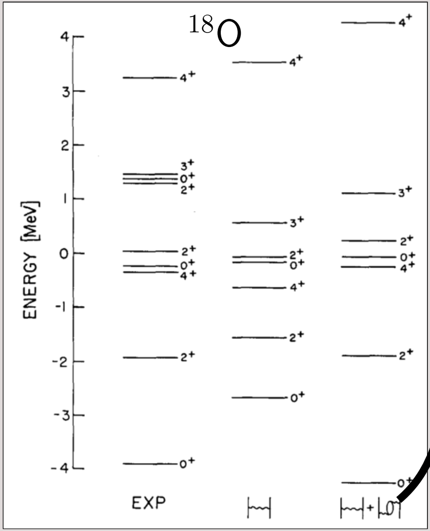
We want an *effective interaction* H_{eff} that acts only in the P space and reproduces the spectrum of H in the big space.

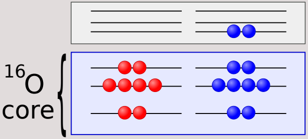
$$H|\Psi_i\rangle = E_i|\Psi_i\rangle$$

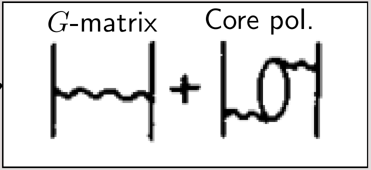
$$H_{\text{eff}}|\Phi_i\rangle = E_i|\Phi_i\rangle, \quad P|\Phi_i\rangle = |\Phi_i\rangle$$

So we require: $\quad QH_{\text{eff}}P = 0.$



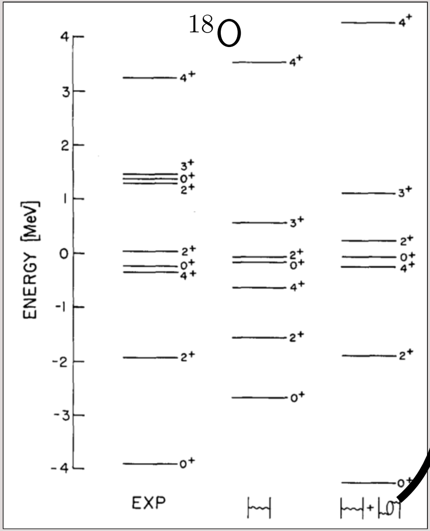


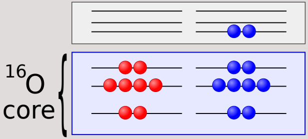


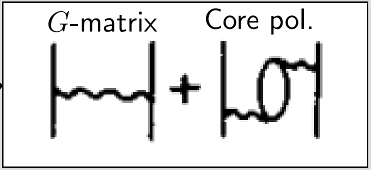


$$V_{\text{eff}} = V + VQ\sum_{n=1}^{\infty}\left(\frac{1}{E_0 - H_0}QV\right)^n$$

Bertsch 1965; Kuo and Brown 1966; Barrett and Kirson 1970; Schucan and Weidenmüller 1973; Vary, Sauer, and Wong 1973







Trouble:

- **3rd order not small, makes it worse**
- **Perturbation series diverges**
- **Neglected virtual states important**

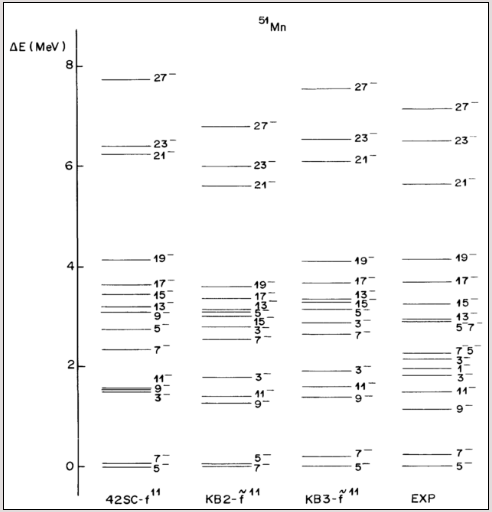
Bertsch 1965; Kuo and Brown 1966; Barrett and Kirson 1970; Schucan and Weidenmüller 1973; Vary, Sauer, and Wong 1973

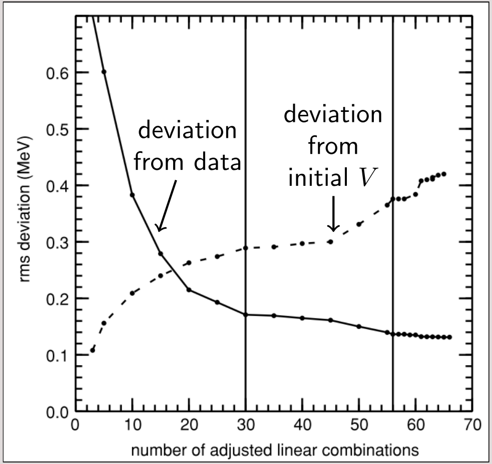
Shell model interaction

1. Take core energy from data
2. Fit single particle energies ϵ_i to core+1 system
3. Adjust "monopoles" of V_{ijkl} to data (or treat as free parameters)
4. Scale V_{ijkl} by $\approx A^{1/3}$
5. Use effective operators for electroweak observables, e.g. $e_p = 1.5, e_n = 0.5$

Talmi 1962; Chung 1976; Poves and Zuker 1981; Wildenthal 1984...

Recipe for success



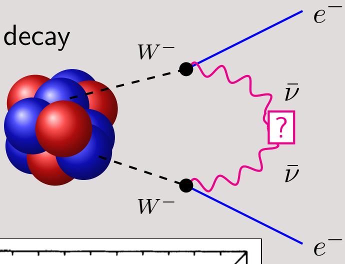


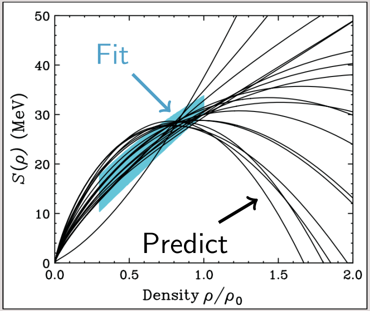
Poves and Zuker 1981; Brown et al. 2006

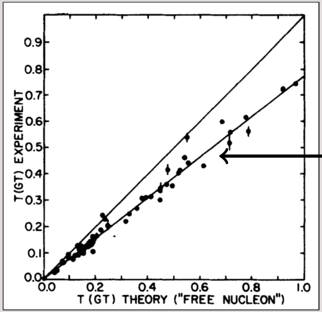
Downsides of the phenomenological approach

- Overfitting?
- Extrapolation?
- Lost connection to underlying interaction
- New valence space $\Rightarrow$ refit everything

$0\nu\beta\beta$ decay



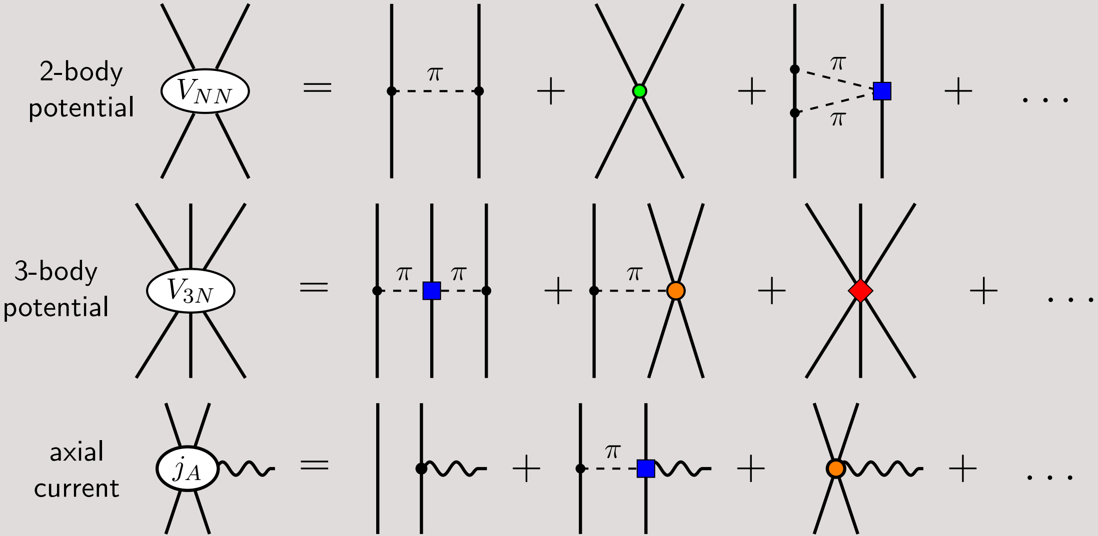




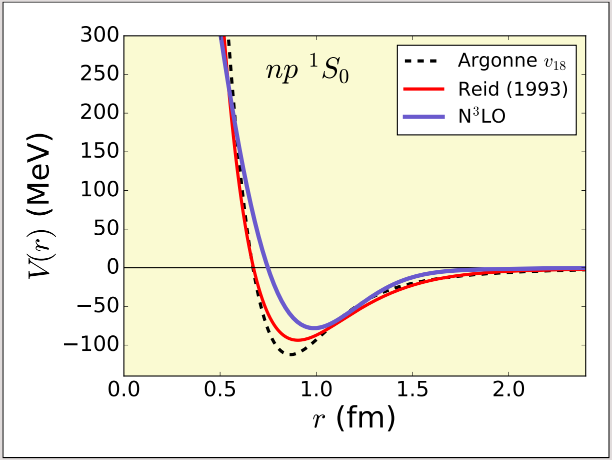
Bare Gamow-Teller operator needs to be "quenched" by factor ∼0.75

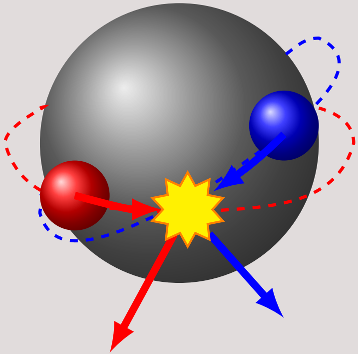
- Consistent NN and $3N$ potential, and electroweak operators from chiral EFT
- Similarity Renormalization Group
- Non-perturbative many-body methods

Chiral EFT



Weinberg 1990; Kolck 1993; Ordóñez, Ray, and Kolck 1994; Epelbaum et al. 2002; Entem and Machleidt 2002; Navrátil 2007...



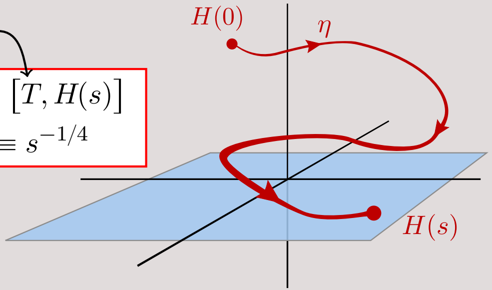


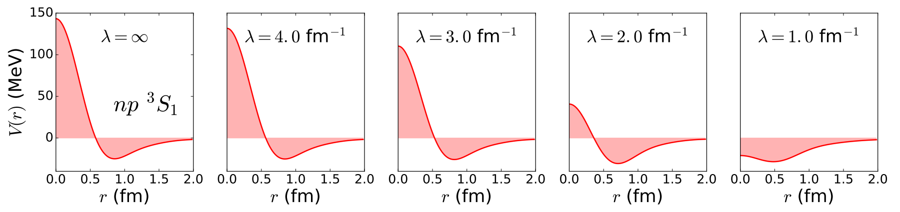
Short-range repulsion	$\longrightarrow$	Indpendent particles is a bad approx.

SRG

- $H(s) \equiv U(s) H U^\dagger(s)$
- $\frac{dU(s)}{ds} = \eta(s) U(s)$
- $\frac{dH(s)}{ds} = [\eta(s), H(s)]$

- $\eta(s) \equiv [T, H(s)]$ (kinetic energy)
- $\lambda_{SRG} \equiv s^{-1/4}$ (resolution scale)





Glazek and Wilson 1994; Wegner 1994; White 2002; Bogner, Furnstahl, and Perry 2007

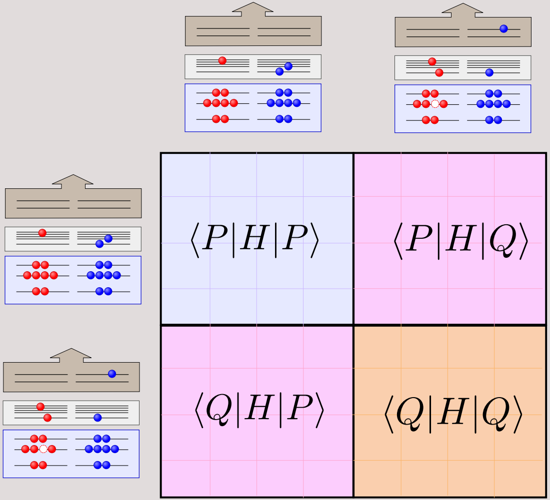
Using SRG to derive a shell model interaction

We want $QHP = 0$.

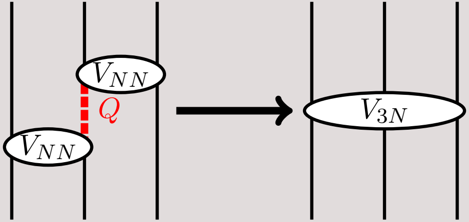
1. Partition $H = H^{\mathrm{d}} + H^{\mathrm{od}}$ with $QH^{d}P = 0$

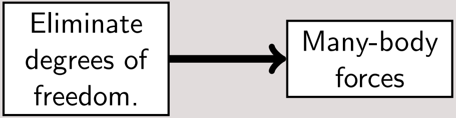
2. Define $\eta(s) = \frac{H^{\mathrm{od}}(s)}{\Delta}$

3. $H^{\mathrm{od}}(s) \to 0$, $QHP \to 0$ is a fixed point of the RG flow.

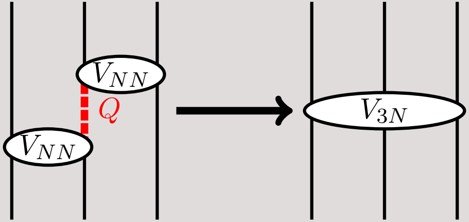


White 2002; Tsukiyama, Bogner, and Schwenk 2011; Bogner et al. 2014; Stroberg et al. 2019



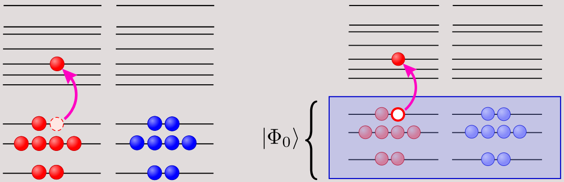


White 2002; Tsukiyama 2010



Eliminate degrees of freedom. → Many-body forces

Redefine the vacuum

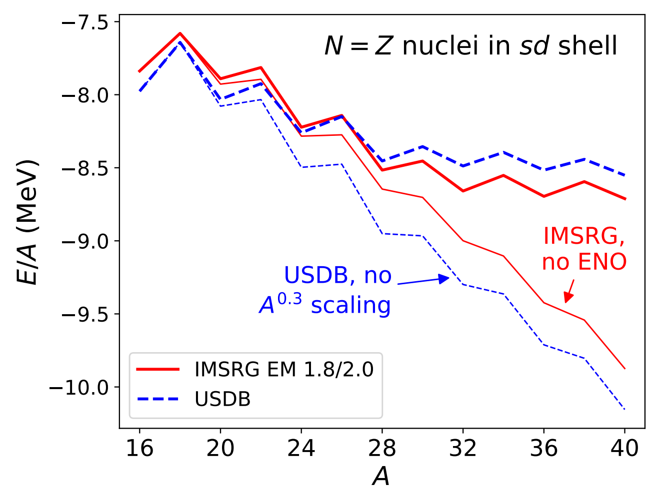


16 particles | 1 particle, 1 hole

Many-particle forces | Many-*quasi*particle forces

White 2002; Tsukiyama 2010

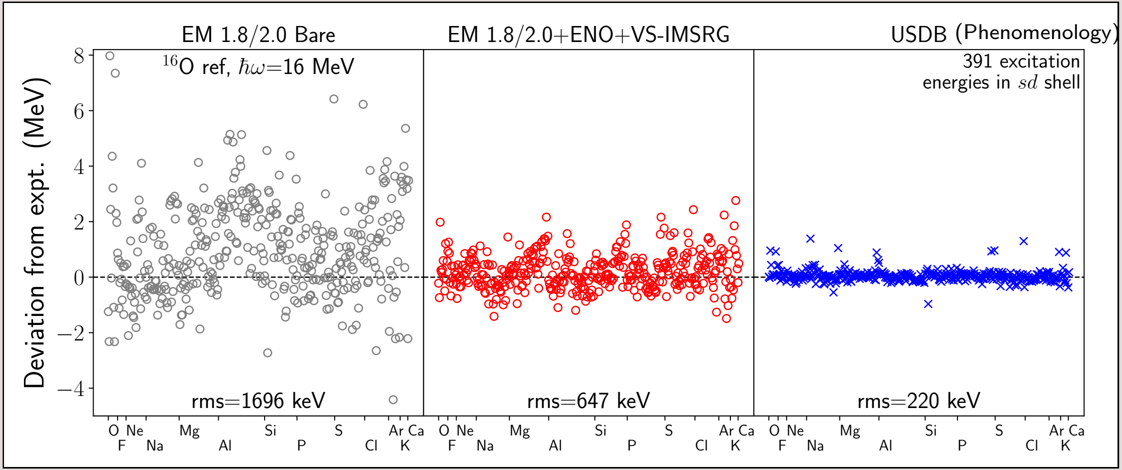
- Optimal reference depends on the nucleus being studied.
- Generate a different shell model interaction for *each nucleus*
- No loss of predictive power (no parameters to fit)



(USDB = phenomonology)

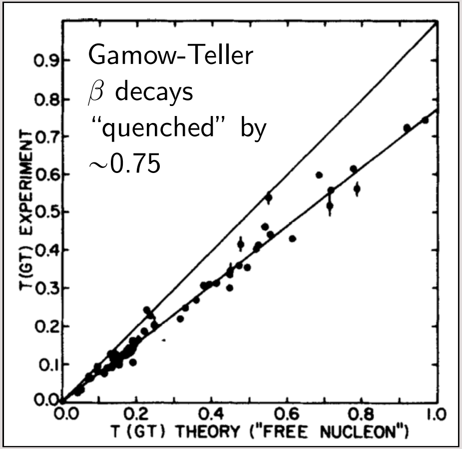
Stroberg et al. 2019

Deviation from experimental spectroscopy

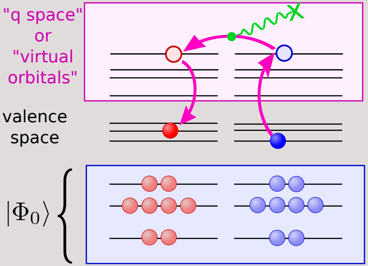


Stroberg et al. 2019

Quenching of Gamow-Teller β decays

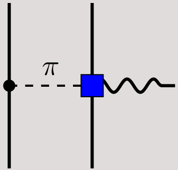


Correlations

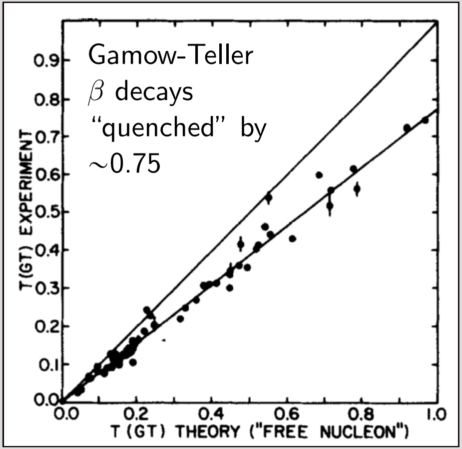


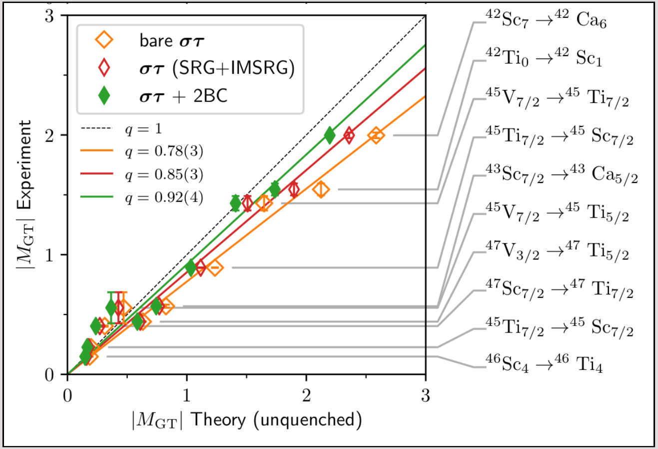
$$\mathcal{O}(s) = U(s)\mathcal{O}U^{\dagger}(s)$$

Currents



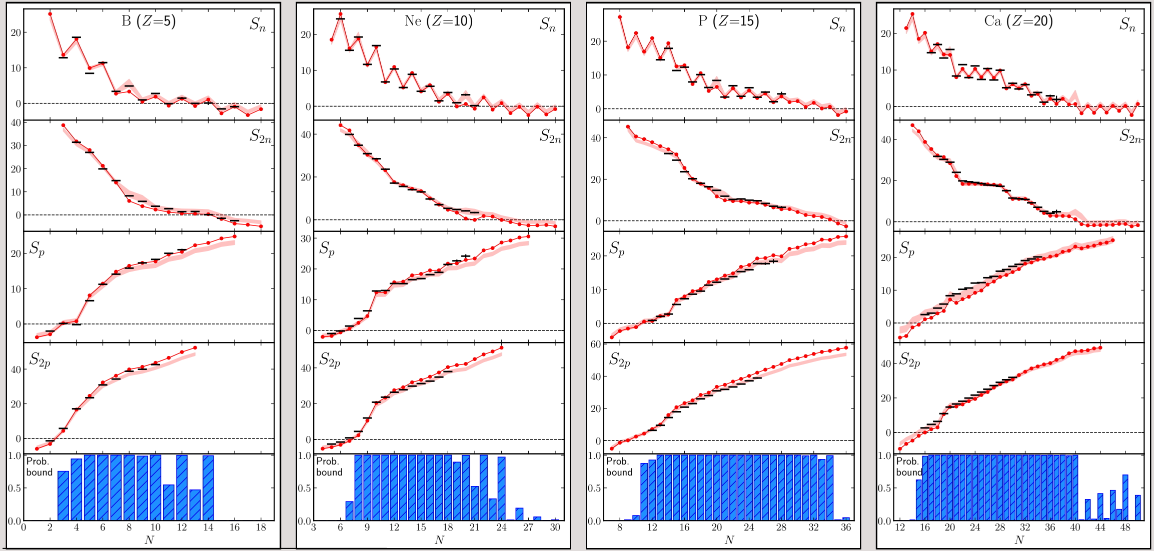
Gysbers et al. 2019



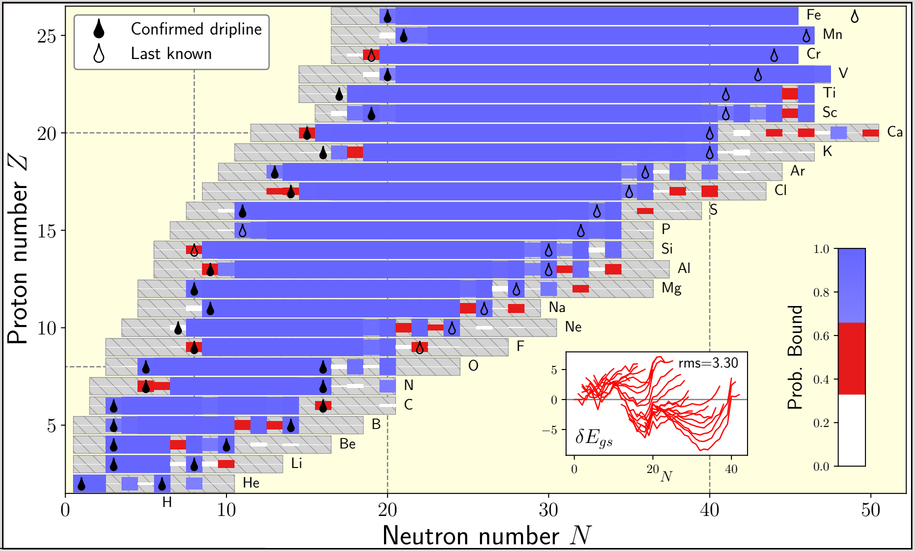


Gysbers et al. 2019

Predicting the driplines

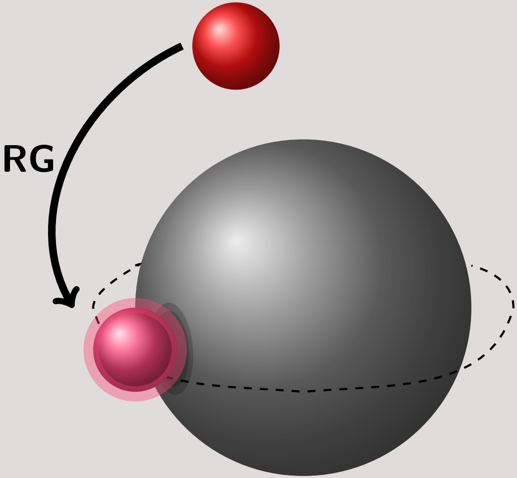


Holt et al. 2019

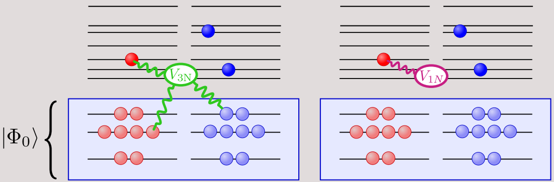


Holt et al. 2019

- RG connects free nucleons to shell model quasiparticles
- Old phenomenological adjustments can be traced to 3N forces
- Some indications of universality in shell model interactions
- Strategy: Short distance $\rightarrow$ RG flow, long distance $\rightarrow$ diagonalization in valence space
- Diagonalization is exact, so bigger valence space should be better (see talk by **T. Miyagi**)

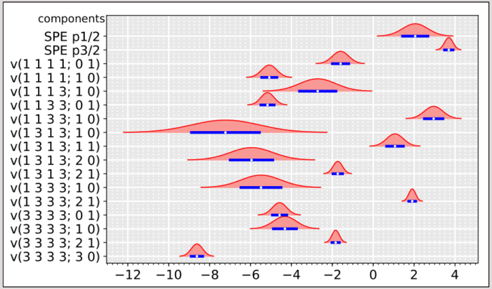


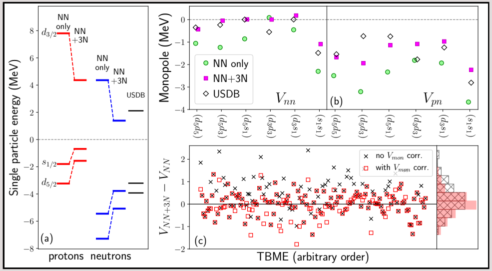
- RG connects free nucleons to shell model quasiparticles
- Old phenomenological adjustments can be traced to 3N forces
- Some indications of universality in shell model interactions
- Strategy: Short distance → RG flow, long distance → diagonalization in valence space
- Diagonalization is exact, so bigger valence space should be better (see talk by **T. Miyagi**)



3-body force　　effective 1-body force

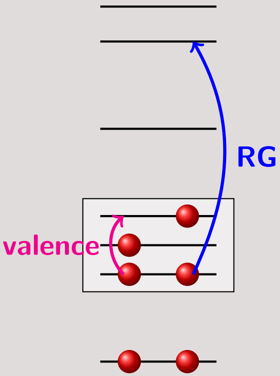
- RG connects free nucleons to shell model quasiparticles
- Old phenomenological adjustments can be traced to 3N forces
- Some indications of universality in shell model interactions
- Strategy: Short distance → RG flow, long distance → diagonalization in valence space
- Diagonalization is exact, so bigger valence space should be better (see talk by T. Miyagi)



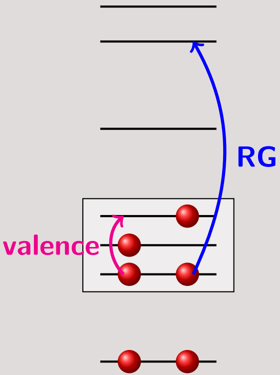


Final thoughts

- RG connects free nucleons to shell model quasiparticles
- Old phenomenological adjustments can be traced to 3N forces
- Some indications of universality in shell model interactions
- Strategy: Short distance → RG flow, long distance → diagonalization in valence space
- Diagonalization is exact, so bigger valence space should be better (see talk by **T. Miyagi**)



- RG connects free nucleons to shell model quasiparticles
- Old phenomenological adjustments can be traced to 3N forces
- Some indications of universality in shell model interactions
- Strategy: Short distance → RG flow, long distance → diagonalization in valence space
- Diagonalization is exact, so bigger valence space should be better
(see talk by **T. Miyagi**)



Summary

- The shell model is useful and successful because it picks the “right” d.o.f.
- Modern approaches can harness the virtues and insight of the old shell model, while avoiding some of the downsides
- Outstanding issues include: rigorous EFT power counting, quantification of many-body uncertainties, extended valence spaces, going beyond mass 100, the continuum . . .

Collaborators:

TRIUMF A. Calci, J. Holt, P. Navrátil, P Gysbers

ORNL/UT G. Hagen, G. Jansen, T. Morris, T. Papenbrock

NSCL/MSU S. Bogner, H. Hergert

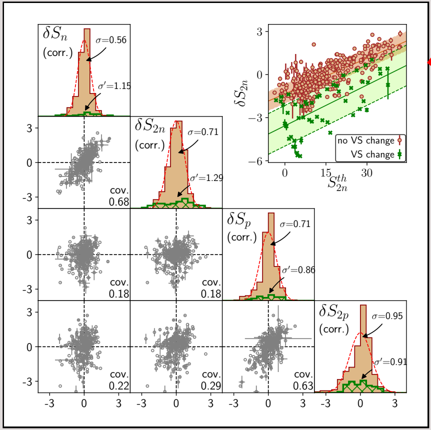
TU Darmstadt K. Hebeler, R. Roth, A. Schwenk

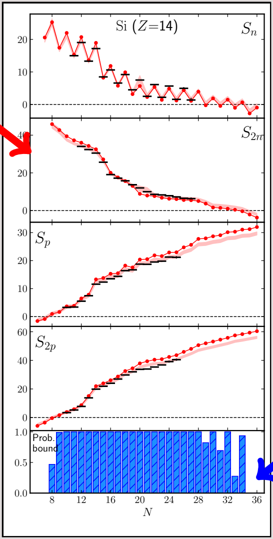
LLNL S. Quaglioni, K. Wendt

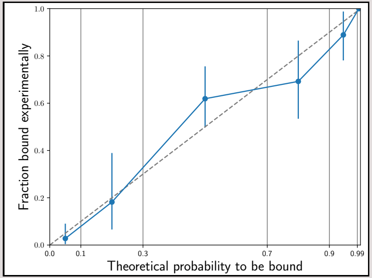
Mainz J. Simonis

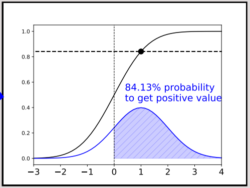
Additional figures

The “magic” EM1.8/2.0 interaction and the dripline





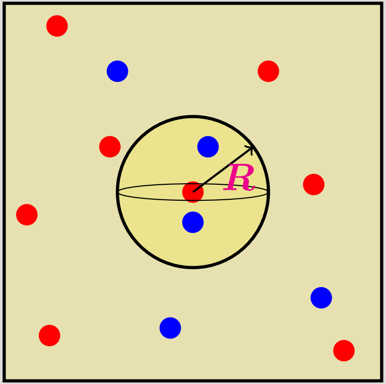


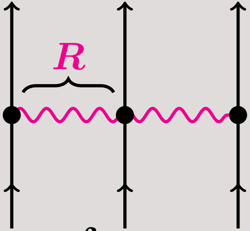


J.D. Holt, **SRS**, A. Schwenk, J. Simonis (in prep)

What's the small parameter?

Cluster hierarchy $H_{2b} > H_{3b} > H_{4b} \ldots$ justified if $\rho R^3 \ll 1$





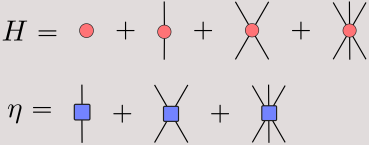
$$\rho \sim 0.16 \text{ fm}^{-3}$$
$$\Rightarrow R \ll 1.1 \text{ fm}$$

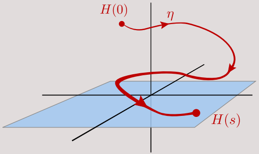
$$\langle H_{3b} \rangle \sim \int d^3r_1 d^3r_2 d^3r_3 \; |\psi_1|^2 |\psi_2|^2 \underbrace{|\psi_3|^2}_{\rho} \underbrace{V(r_{12}, r_{13})}_{\sim V_0 \theta(R-r)}$$

$$\sim \left(\tfrac{4\pi}{3} R^3 \rho\right)^2 V_0 \int d^3 r_1 |\psi_1|^2$$

Similarity Renormalization Group for the effective interaction

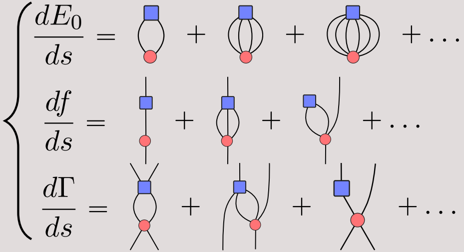
$$H = E_0 + \sum_{ij} f_{ij}\{a_i^\dagger a_j\} + \frac{1}{4}\sum_{ijkl}\Gamma_{ijkl}\{a_i^\dagger a_j^\dagger a_l a_k\} + \frac{1}{36}\sum_{\substack{ijk\\lmn}} W_{ijklmn}\{a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l\}$$

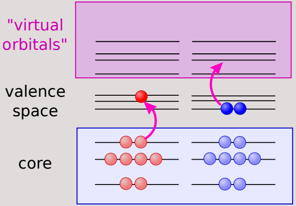




Flow equation:

$$\frac{dH}{ds} = [\eta(s), H(s)] \Rightarrow$$

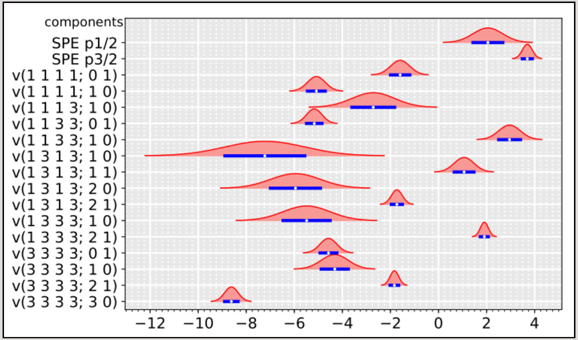


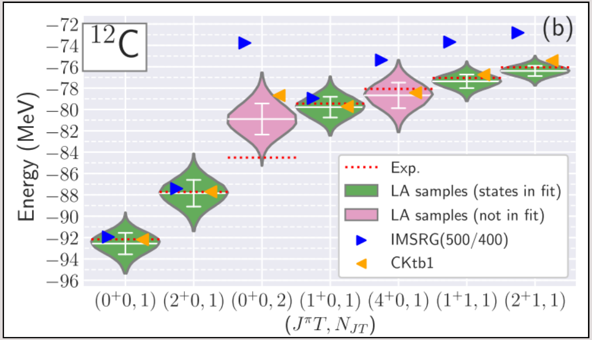


$$H^{\text{od}} \sim QHP \to 0$$

Glazek1993; Wegner 1994; Tsukiyama, Bogner, and Schwenk 2011; Bogner et al. 2014

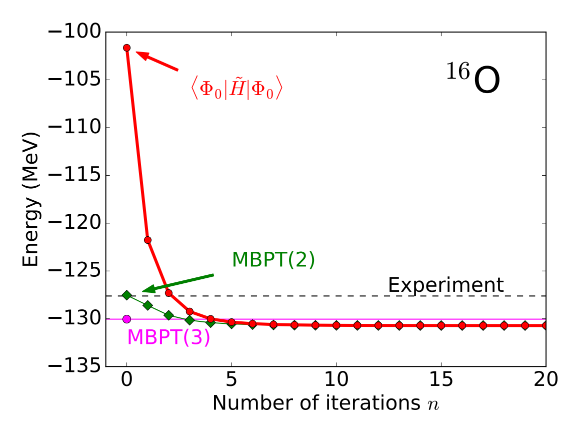
A new twist on phenomenology

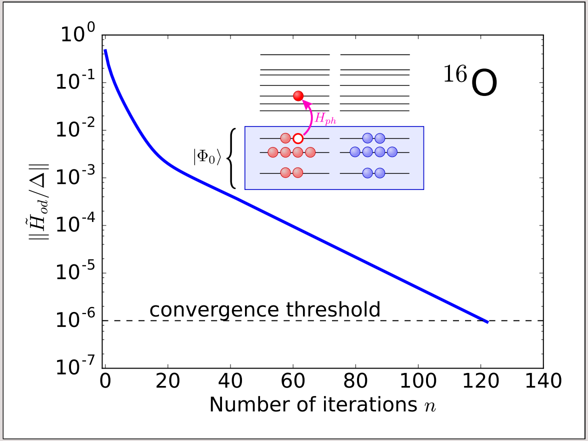




Yoshida et al. 2018

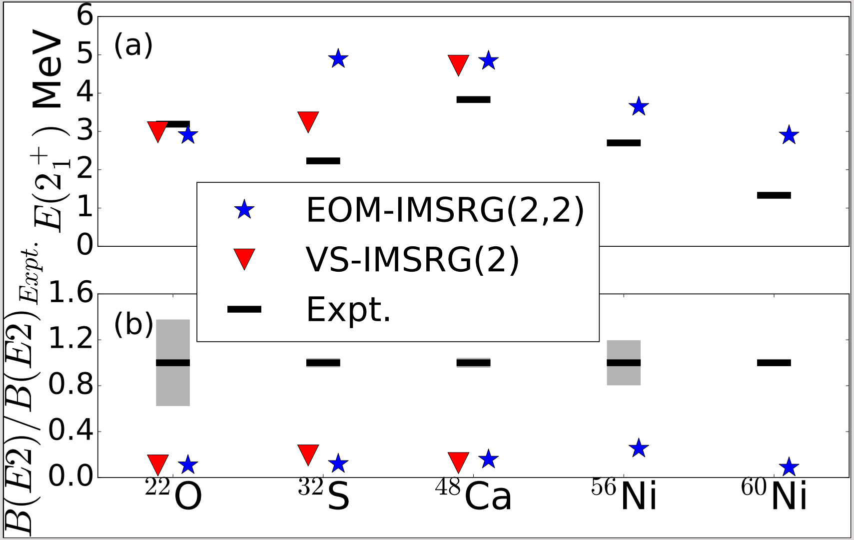
In-medium Similarity Renormalization Group





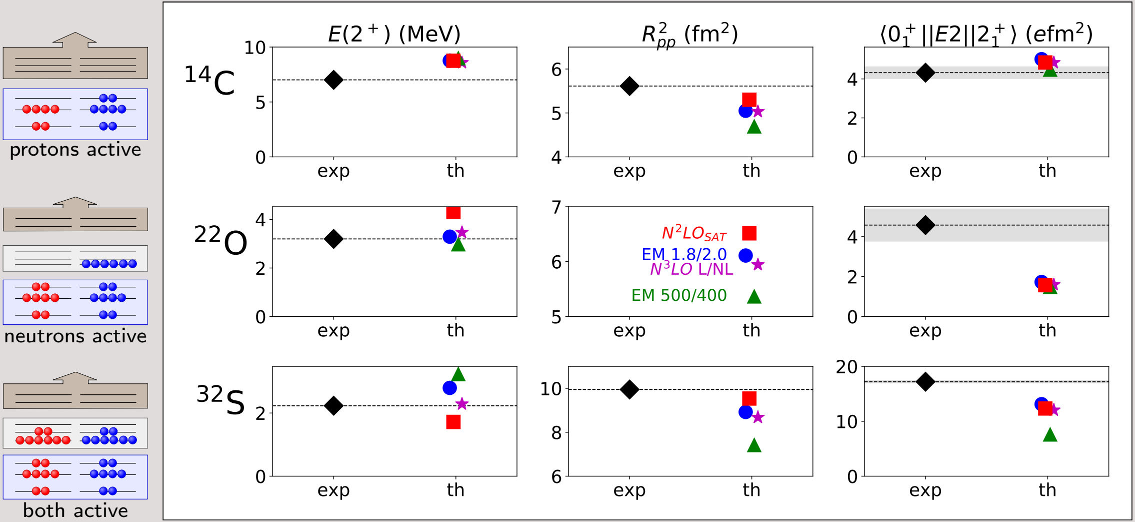
Tsukiyama 2010; Tsukiyama, Bogner, and Schwenk 2011; Hergert et al. 2013

$E2$ transitions—severe underprediction of strength

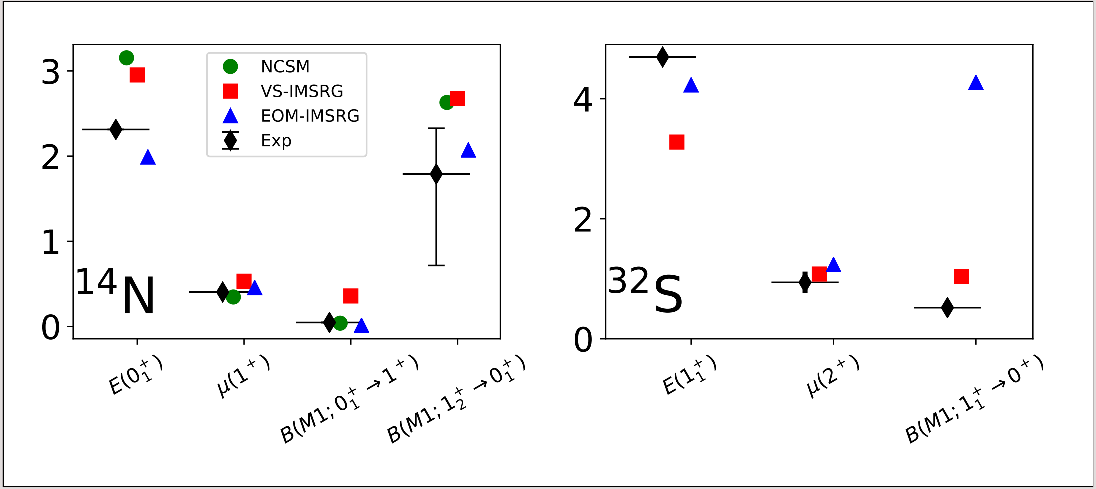


Parzuchowski, Morris, and Bogner 2017

Dependence on choice of interaction

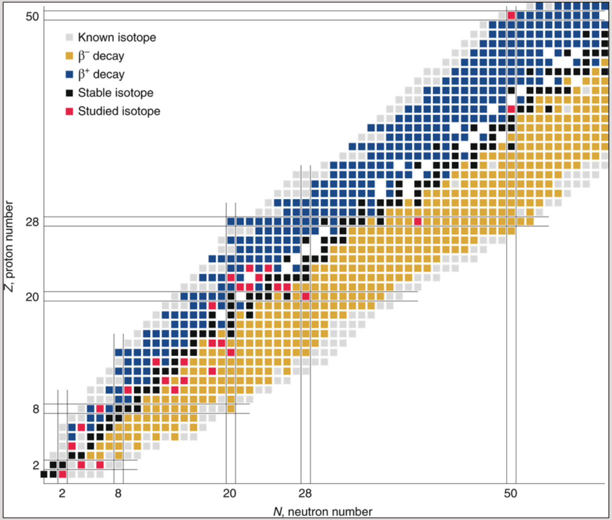


Entem and Machleidt 2003; Navrátil 2007; Gazit, Quaglioni, and Navrátil 2009; Ekström et al. 2015; Simonis et al. 2017



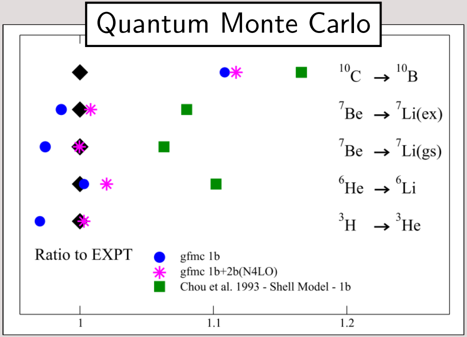
Parzuchowski, Morris, and Bogner 2017

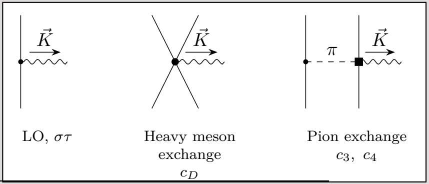
Currents for Gamow-Teller decays



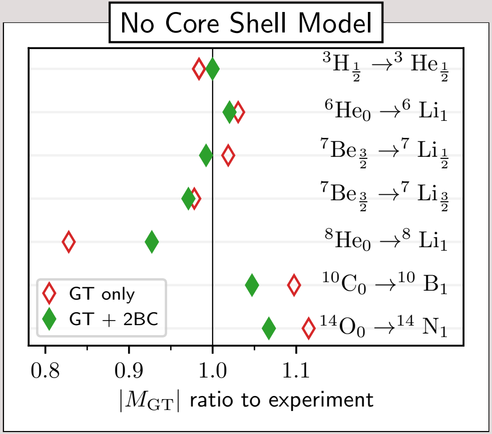
Rios 2019

Quantum Monte Carlo



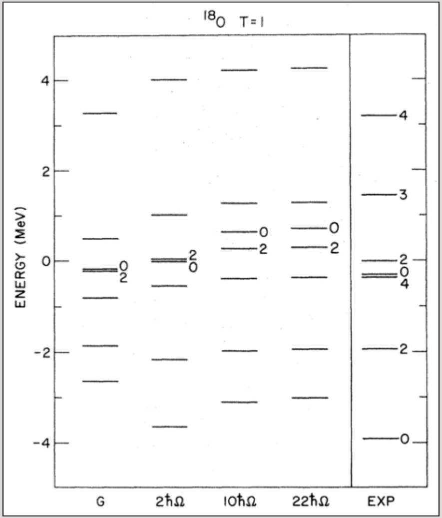


No Core Shell Model

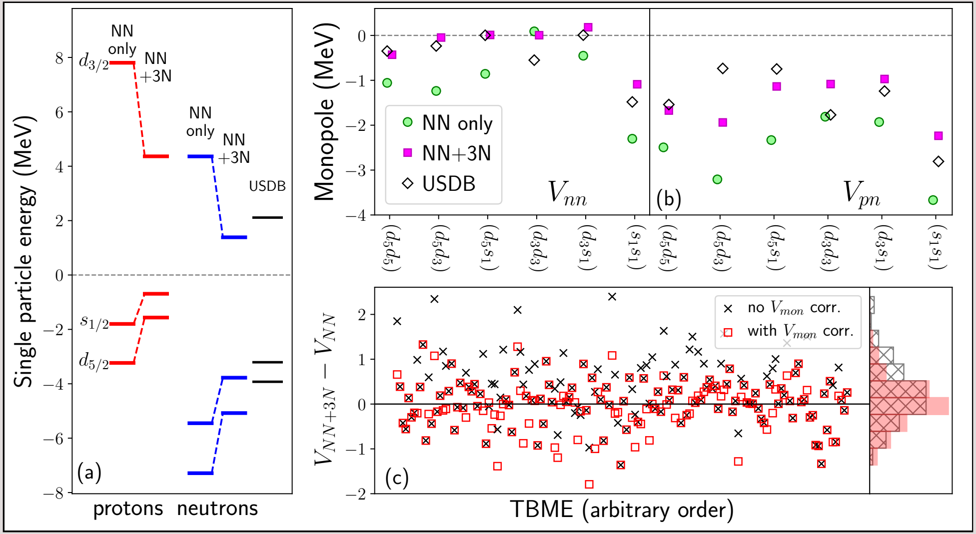


Pastore et al. 2017; Gysbers et al. 2019

Vary-Sauer-Wong (1973) convergence with intermediate states



Vary, Sauer, and Wong 1973



NN only
NN +3N
USDB
$d_{3/2}$
$s_{1/2}$
$d_{5/2}$
Single particle energy (MeV)
(a)
protons
neutrons
Monopole (MeV)
NN only
NN+3N
USDB
V_{nn}
(b)
V_{pn}
(d_5d_5)
(d_5d_3)
(d_5s_1)
(d_3d_3)
(d_3s_1)
(s_1s_1)
$V_{NN+3N} - V_{NN}$
no V_{mon} corr.
with V_{mon} corr.
(c)
TBME (arbitrary order)

Barrett, Bruce R. and Michael W. Kirson (1970). "Higher-order terms and the apparent non-convergence of the perturbation expansion for the effective interaction in finite nuclei". In: *Nucl. Phys. A* 148.1, pp. 145–180. ISSN: 03759474. DOI: 10.1016/0375-9474(70)90617-2. URL: http://linkinghub.elsevier.com/retrieve/pii/0375947470906172.

Bertsch, George F. (1965). "Role of core polarization in two-body interaction". In: *Nucl. Phys.* 74.1, pp. 234–240. ISSN: 00295582. DOI: 10.1016/0029-5582(65)90262-2.

Bogner, S. K., R. J. Furnstahl, and R. J. Perry (2007). "Similarity renormalization group for nucleon-nucleon interactions". In: *Phys. Rev. C* 75.6, p. 061001. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.75.061001. URL: http://link.aps.org/doi/10.1103/PhysRevC.75.061001.

Bogner, S. K. et al. (2014). "Nonperturbative shell-model interactions from the in-medium similarity renormalization group". In: *Phys. Rev. Lett.* 113.14, p. 142501. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.113.142501. URL: http://link.aps.org/doi/10.1103/PhysRevLett.113.142501.

Brown, B. A. et al. (2006). "Tensor interaction contributions to single-particle energies". In: *Phys. Rev. C* 74.6, p. 061303. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.74.061303. URL: http://link.aps.org/doi/10.1103/PhysRevC.74.061303.

Chung, Wilton (1976). "Empirical renormalization of shell-model hamiltonians and magnetic dipole moments of sd-shell nuclei". PhD. Michigan State University, p. 135.

Ekström, A. et al. (2015). "Accurate nuclear radii and binding energies from a chiral interaction". In: *Phys. Rev. C* 91.5, p. 051301. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.91.051301. URL: http://link.aps.org/doi/10.1103/PhysRevC.91.051301.

Entem, D. R. and R. Machleidt (2002). "Accurate nucleon nucleon potential based upon chiral perturbation theory". In: *Phys. Lett. B* 524.January, pp. 93–98.

— (2003). "Accurate charge-dependent nucleon-nucleon potential at fourth order of chiral perturbation theory". In: *Phys. Rev. C* 68.4, p. 041001. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.68.041001. URL: http://link.aps.org/doi/10.1103/PhysRevC.68.041001.

Epelbaum, E. et al. (2002). "Three-nucleon forces from chiral effective field theory". In: *Phys. Rev. C* 66.6, p. 064001. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.66.064001. URL: https://link.aps.org/doi/10.1103/PhysRevC.66.064001.

Gazit, Doron, Sofia Quaglioni, and Petr Navrátil (2009). "Three-Nucleon Low-Energy Constants from the Consistency of Interactions and Currents in Chiral Effective Field Theory". In: *Phys. Rev. Lett.* 103.10, p. 102502. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.103.102502. URL: http://link.aps.org/doi/10.1103/PhysRevLett.103.102502.

Glazek, Stanislaw D. and Kenneth G. Wilson (1994). "Perturbative renormalization group for Hamiltonians". In: *Phys. Rev. D* 49.8, pp. 4214–4218. ISSN: 0556-2821. DOI: 10.1103/PhysRevD.49.4214. URL: http://journals.aps.org.ezproxy.library.ubc.ca/prd/abstract/10.1103/PhysRevD.49.4214http://link.aps.org/doi/10.1103/PhysRevD.49.4214.

Gysbers, P et al. (2019). "Discrepancy between experimental and theoretical β-decay rates resolved from first principles". In: *Nat. Phys.* ISSN: 1745-2473. DOI: 10.1038/s41567-019-0450-7. arXiv: arXiv:1903.00047v1. URL: https://arxiv.org/pdf/1903.00047.pdfhttp://www.nature.com/articles/s41567-019-0450-7.

Haxel, O., J. H. D. Jensen, and H. E. Suess (1949). "On the "magic numbers" in nuclear structure". In: *Phys. Rev.* P. 1766. URL: http://link.aps.org/doi/10.1103/PhysRev.75.1766.2.

Hergert, H. et al. (2013). "In-medium similarity renormalization group with chiral two- plus three-nucleon interactions". In: *Phys. Rev. C* 87.3, p. 034307. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.87.034307. URL: http://link.aps.org/doi/10.1103/PhysRevC.87.034307.

Holt, J. D. et al. (2019). "Ab initio limits of atomic nuclei". In: arXiv: 1905.10475. URL: http://arxiv.org/abs/1905.10475.

Kolck, Ubirajara van (1993). "Applications of Effective Chiral Lagrangians to Nuclear Physics and Quark Models". PhD thesis, p. 142. URL: http://inspirehep.net/record/364521/files/van-kolck.pdf.

Kuo, T. T. S. and G. E. Brown (1966). "Structure of finite nuclei and the free nucleon-nucleon interaction". In: *Nucl. Phys.* 85.1, pp. 40–86. ISSN: 00295582. DOI: 10.1016/0029-5582(66)90131-3. URL: http://www.sciencedirect.com/science/article/pii/0029558266901313.

Mayer, M. G. (1948). "On closed shells in nuclei". In: *Phys. Rev.* 74.3, p. 235. URL: http://www.osti.gov/scitech/biblio/4424422.

Navrátil, P. (2007). "Local three-nucleon interaction from chiral effective field theory". In: *Few-Body Syst.* 41.3-4, pp. 117–140. ISSN: 0177-7963. DOI: 10.1007/s00601-007-0193-3. URL: http://link.springer.com/10.1007/s00601-007-0193-3.

Ordóñez, C., L. Ray, and U. van Kolck (1994). "Nucleon-nucleon potential from an effective chiral Lagrangian". In: *Phys. Rev. Lett.* 72.13, pp. 1982–1985. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.72.1982. URL: http://link.aps.org/doi/10.1103/PhysRevLett.72.1982.

Parzuchowski, N. M., T. D. Morris, and S. K. Bogner (2017). "Ab initio excited states from the in-medium similarity renormalization group". In: *Phys. Rev. C* 95.4, p. 044304. ISSN: 2469-9985. DOI: 10.1103/PhysRevC.95.044304. arXiv: 1705.05511. URL: http://arxiv.org/abs/1705.05511http://link.aps.org/doi/10.1103/PhysRevC.95.044304.

Pastore, S. et al. (2017). "Quantum Monte Carlo calculations of weak transitions in $A = 6$–10 nuclei". In: arXiv: 1709.03592. URL: http://arxiv.org/abs/1709.03592.

Poves, A. and A. Zuker (1981). "Theoretical spectroscopy and the fp shell". In: *Phys. Rep.* 70.4, pp. 235–314. URL: http://www.sciencedirect.com/science/article/pii/0370157381901538.

Rios, Arnau (2019). "Beta decay gets the ab initio treatment". In: *Nat. Phys.* ISSN: 1745-2473. DOI: 10.1038/s41567-019-0483-y. URL: http://www.nature.com/articles/s41567-019-0483-y.

Schucan, Thomas H and Hans A Weidenmüller (1973). "Perturbation theory for the effective interaction in nuclei". In: *Ann. Phys. (N. Y).* 76.2, pp. 483–509. ISSN: 00034916. DOI: 10.1016/0003-4916(73)90044-4. URL: http://linkinghub.elsevier.com/retrieve/pii/0003491673900444http://www.sciencedirect.com.ezproxy.library.ubc.ca/science/article/pii/0003491673900444.

Simonis, J. et al. (2017). "Saturation with chiral interactions and consequences for finite nuclei". In: *Phys. Rev. C* 96.1, p. 014303. ISSN: 2469-9985. DOI: 10.1103/PhysRevC.96.014303. arXiv: 1704.02915. URL: http://arxiv.org/abs/1704.02915http://link.aps.org/doi/10.1103/PhysRevC.96.014303.

Stroberg, S. Ragnar et al. (2019). "Non-Empirical Interactions for the Nuclear Shell Model: An Update". In: pp. 1–57. arXiv: 1902.06154. URL: http://arxiv.org/abs/1902.06154.

Talmi, I. (1962). "Effective Interactions and Coupling Schemes in Nuclei". In: *Rev. Mod. Phys.* 34.4, pp. 704–722. ISSN: 0034-6861. DOI: 10.1103/RevModPhys.34.704. URL: http://link.aps.org/doi/10.1103/RevModPhys.34.704.

Tsukiyama, K. (2010). "Similarity Renormalization Group for Nuclear Many-Body Systems". PhD thesis. The University of Tokyo.

Tsukiyama, K., S. K. Bogner, and A. Schwenk (2011). "In-Medium Similarity Renormalization Group For Nuclei". In: *Phys. Rev. Lett.* 106.22, p. 222502. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.106.222502. URL: http://link.aps.org/doi/10.1103/PhysRevLett.106.222502.

Vary, J. P., P. U. Sauer, and C. W. Wong (1973). "Convergence Rate of Intermediate-State Summations in the Effective Shell-Model Interaction". In: *Phys. Rev. C* 7.5, pp. 1776–1785. ISSN: 0556-2813. DOI: 10.1103/PhysRevC.7.1776. URL: https://link.aps.org/doi/10.1103/PhysRevC.7.1776.

Wegner, Franz (1994). "Flow-equations for Hamiltonians". In: *Ann. der Physic* 3, pp. 77–91.

Weinberg, Steven (1990). "Nuclear forces from chiral lagrangians". In: *Phys. Lett. B* 251.2, pp. 288–292. ISSN: 03702693. DOI: 10.1016/0370-2693(90)90938-3. URL: http://www.sciencedirect.com/science/article/pii/037026939090938З.

White, Steven R. (2002). "Numerical canonical transformation approach to quantum many-body problems". In: *J. Chem. Phys.* 117.16, p. 7472. ISSN: 00219606. DOI: 10.1063/1.1508370. URL: http://scitation.aip.org.proxy2.cl.msu.edu/content/aip/journal/jcp/117/16/10.1063/1.1508370.

Wildenthal, B. H. (1984). "Empirical strengths of spin operators in nuclei". In: *Prog. Part. Nucl. Phys.* 11, pp. 5–51. ISSN: 01466410. DOI: 10.1016/0146-6410(84)90011-5. URL: http://www.sciencedirect.com/science/article/pii/0146641084900115.

Yoshida, Sota et al. (2018). "Uncertainty Quantification in Nuclear Shell Model". In: pp. 1–6. arXiv: arXiv:1810.03263v1.