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A connection between linearized Gauss–Bonnet gravity and classical electrodynamics

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Outline

- Motivation: Why we are doing this work
- Methodology: What we are doing
- Results: Benefits of this approach

Motivation

- Classical electrodynamics has special property of **complete gauge invariance**
- Complete gauge invariance of electrodynamics, **each of the following is independently invariant**: Lagrangian density, equation of motion, energy-momentum tensor, angular momentum tensor, dilatation tensor and conformal tensor
- **Other “gauge invariant” models don’t have precisely this property** with respect to every component of the model

Example: Electrodynamics vs. Spin-2

	Electromagnetism	**Spin-2**
Lagrangian		
Equation of motion		
Energy-momentum tensor		
Angular momentum tensor		

$\mathcal{L}$

E^A

$T^{\mu\nu}$

$M^{\lambda\mu\nu}$

Questions

- Do other models exist with the property of complete gauge invariance?
- How do we find these models?

Methodology

- Axiomatic approach to physics
- Set of rules/ a procedure that can be used to uniquely determine physical model(s)

Axiomatic Approach to Physics: One to One

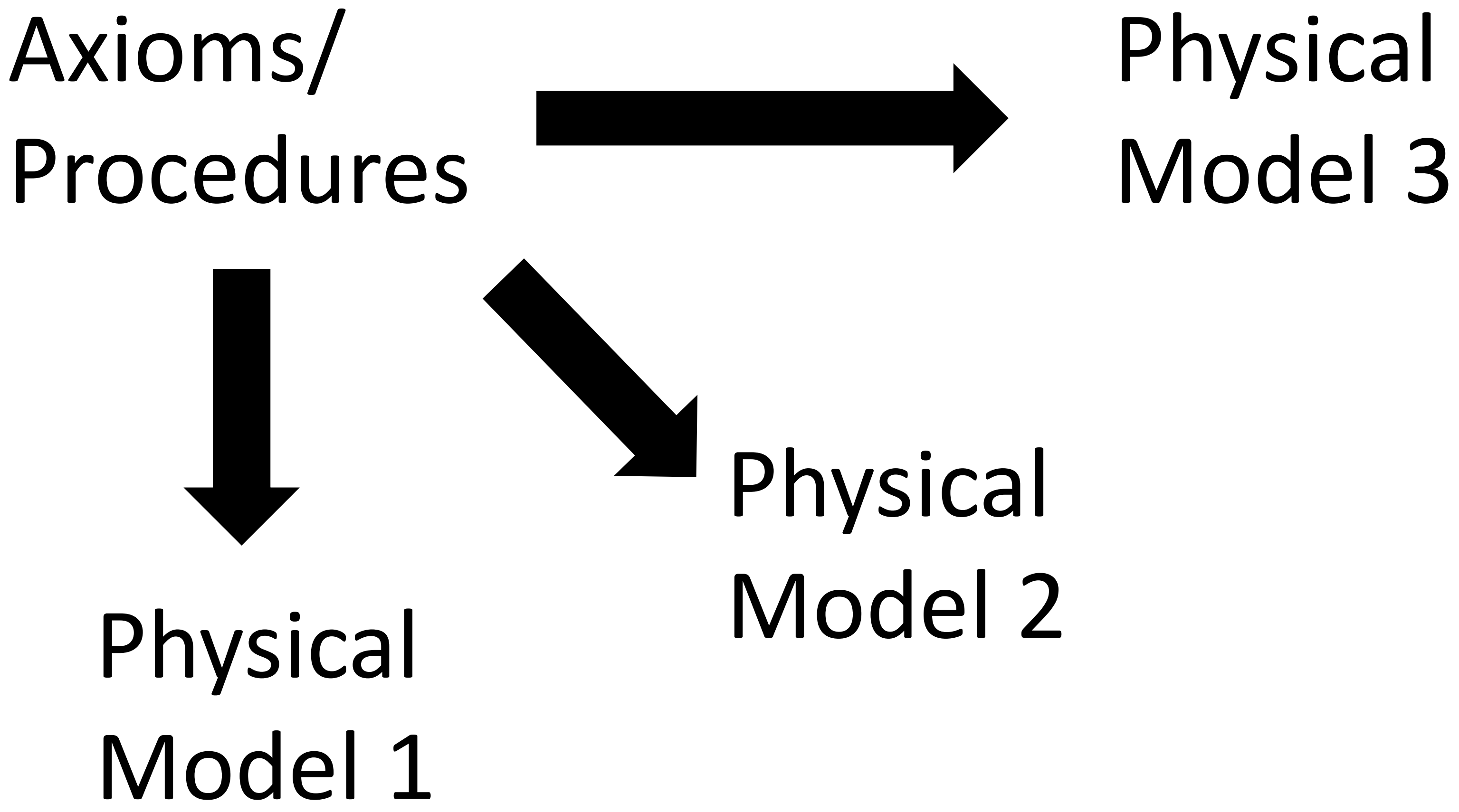
Axioms/
Procedures

→

Physical
Model

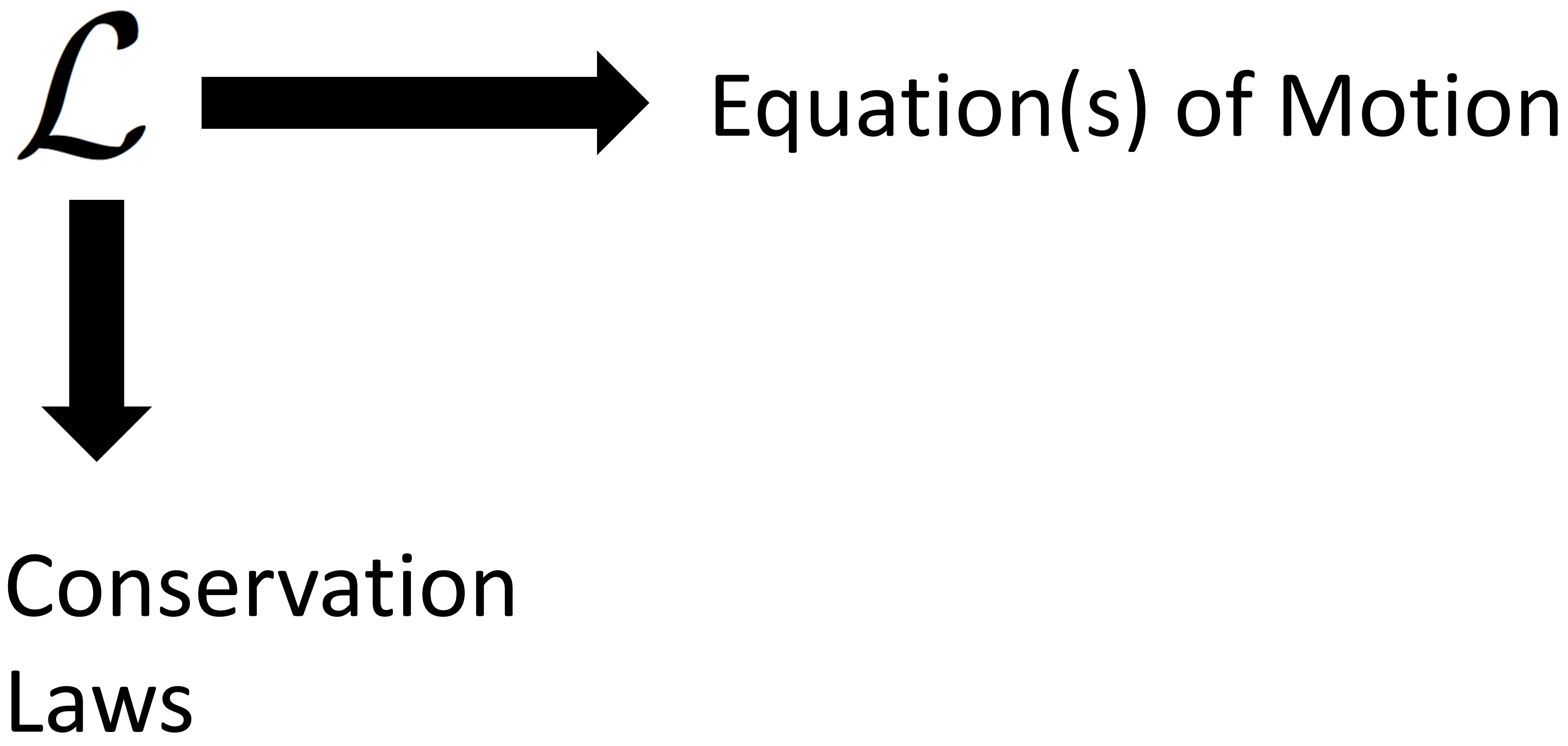
i.e. Mueller(2011), set of rules to uniquely recover Quantum Mechanics

Axiomatic Approach to Physics: One to Many



our goal

Noether's Theorem

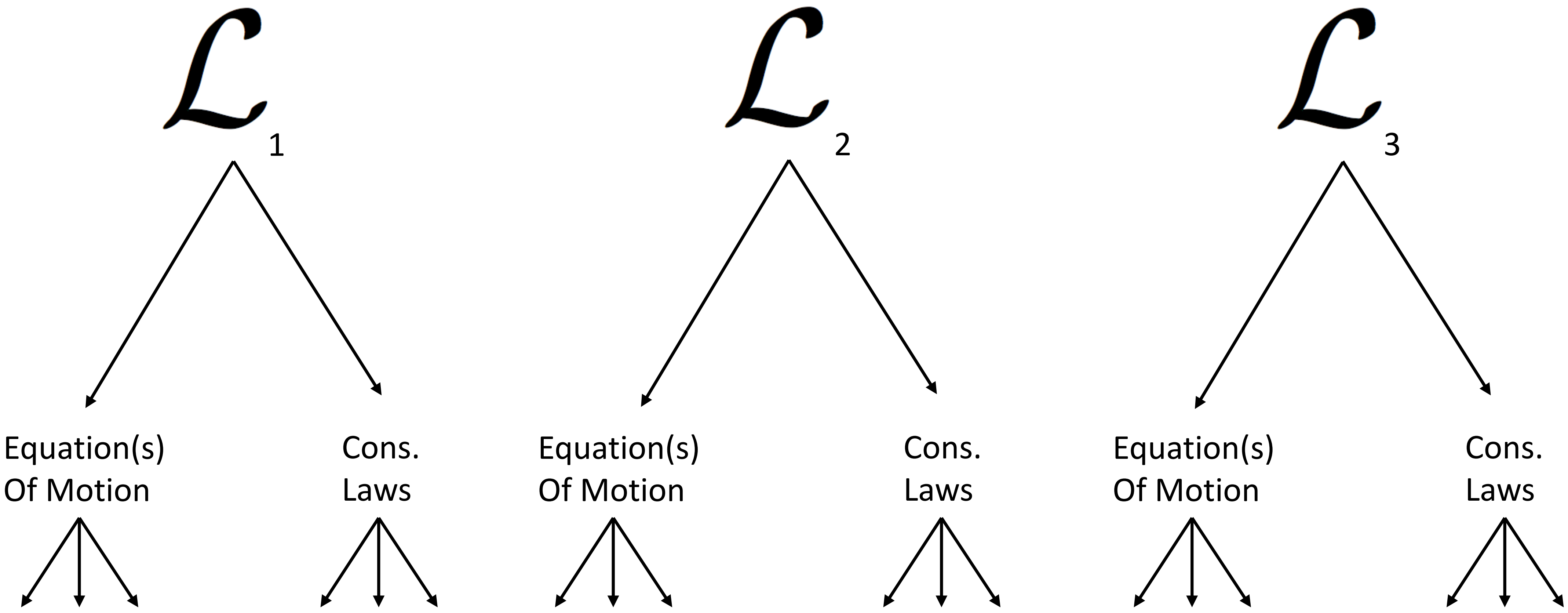


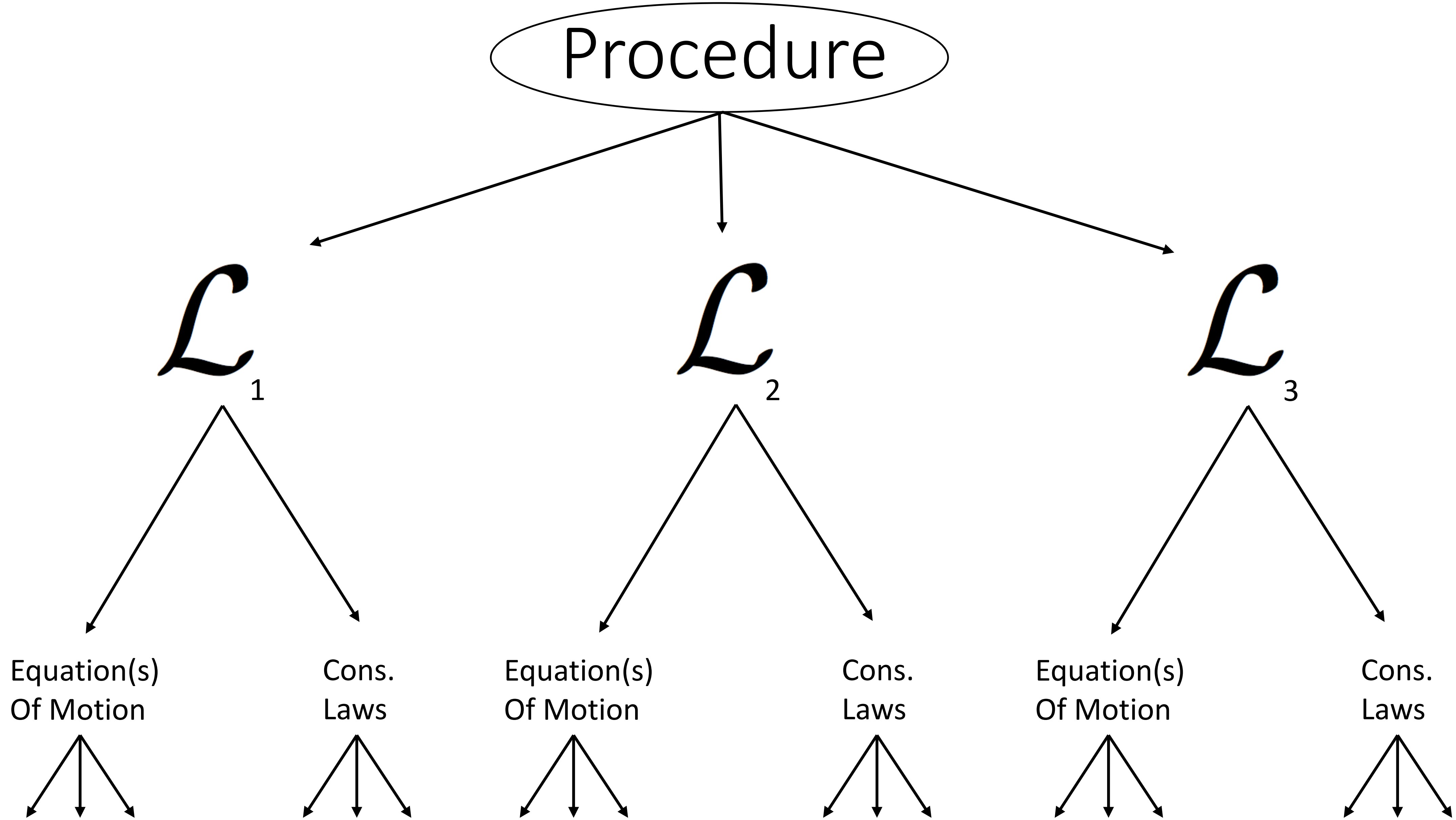
Noether's Theorem: Electrodynamics

$\mathcal{L} \longrightarrow$ Maxwell's Equations

$\Big\downarrow$

Poynting's Theorem
Maxwell Stress Tensor, etc.


$\mathcal{L}_1$
$\mathcal{L}_2$
$\mathcal{L}_3$
Equation(s) Of Motion
Cons. Laws
Equation(s) Of Motion
Cons. Laws
Equation(s) Of Motion
Cons. Laws


Procedure
$\mathcal{L}_1$
$\mathcal{L}_2$
$\mathcal{L}_3$
Equation(s) Of Motion
Cons. Laws
Equation(s) Of Motion
Cons. Laws
Equation(s) Of Motion
Cons. Laws

Basic Idea

- Define criteria that can uniquely determine known Lagrangian densities from a clearly defined procedure
- Criteria such as complete gauge invariance, number of dimensions, conformal invariance are example criteria

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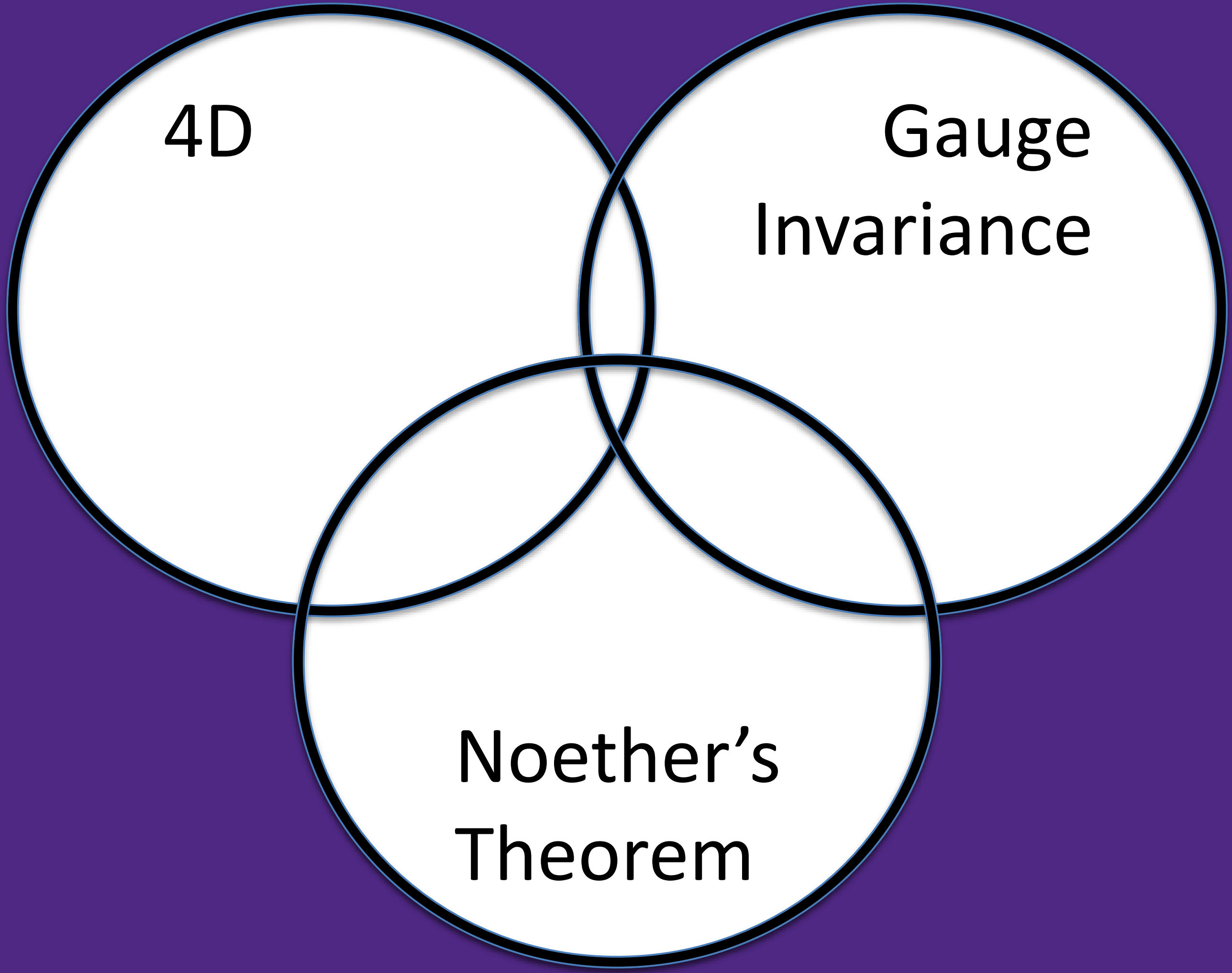

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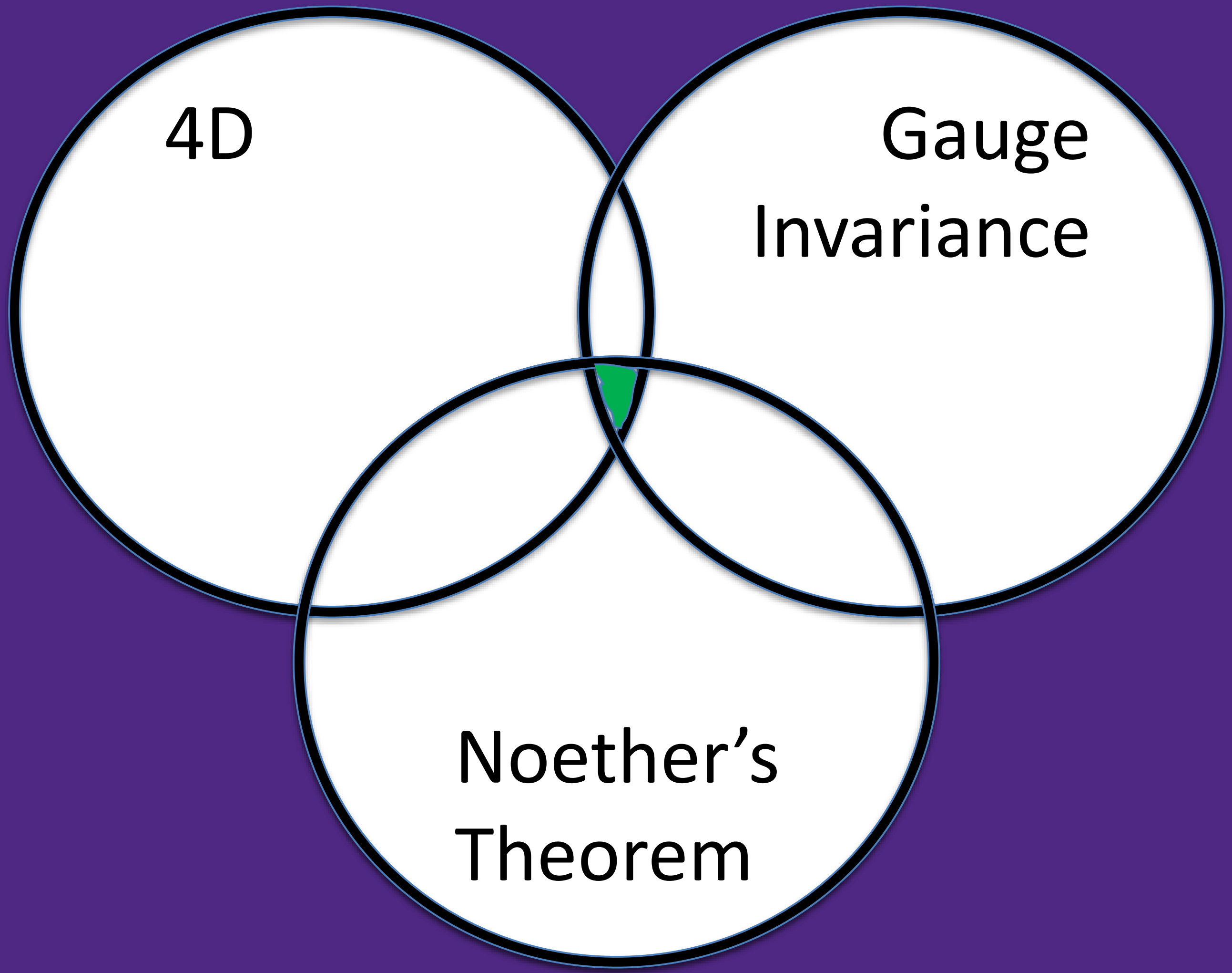

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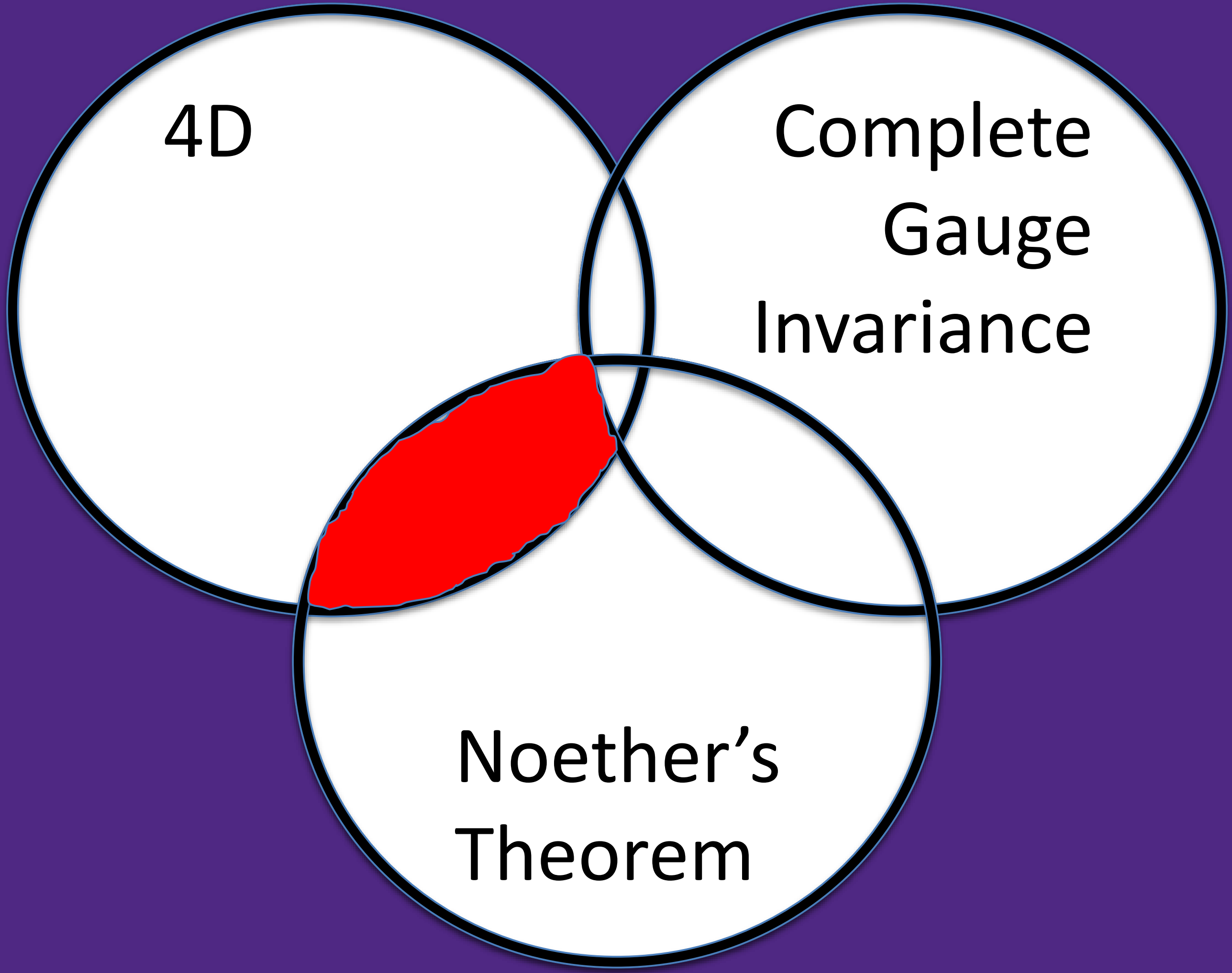

Example: Classical Electrodynamics

- Developed a procedure to derive explicitly gauge invariant Lagrangian densities from completely general scalars

$$\mathcal{L} = a\partial_\mu A_\nu \partial^\mu A^\nu + b\partial_\mu A^\mu \partial_\nu A^\nu + c\partial_\mu A_\nu \partial^\nu A^\mu$$

$$\Downarrow$$

$$A'_\mu = A_\mu + \partial_\mu \phi$$

$$\Downarrow$$

$$\begin{aligned}\mathcal{L} = a(&\partial_\mu A_\nu \partial^\mu A^\nu + \partial_\mu A_\nu \partial^\mu \partial^\nu \phi + \partial_\mu \partial_\nu \phi \partial^\mu A^\nu + \partial_\mu \partial_\nu \phi \partial^\mu \partial^\nu \phi) \\ &+ b(\partial_\mu A^\mu \partial_\nu A^\nu + \partial_\mu A^\mu \partial_\nu \partial^\nu \phi + \partial_\mu \partial^\mu \phi \partial_\nu A^\nu + \partial_\mu \partial^\mu \phi \partial_\nu \partial^\nu \phi) \\ &+ c(\partial_\mu A_\nu \partial^\nu A^\mu + \partial_\mu A_\nu \partial^\nu \partial^\mu \phi + \partial_\mu \partial_\nu \phi \partial^\nu A^\mu + \partial_\mu \partial_\nu \phi \partial^\nu \partial^\mu \phi)\end{aligned}$$

Example: Classical Electrodynamics

- This procedure led to the classical electrodynamics model,

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad\qquad E^{\rho} = \partial_{\mu} F^{\mu\rho}$$

$$T^{\alpha\omega} \;=\; F^{\alpha\gamma} F^{\omega}{}_{\gamma} - \frac{1}{4} \eta^{\alpha\omega} F_{\mu\nu} F^{\mu\nu}$$

Higher Order, Second Rank Tensor Potential

- First order quadratic terms do not yield a complete gauge invariant model, however in the second order,

$$\mathcal{L} = C_1\partial_\mu\partial^\mu h^\nu_\nu\partial_\alpha\partial^\alpha h^\beta_\beta + C_2\partial_\mu\partial^\mu h_{\alpha\beta}\partial_\nu\partial^\nu h^{\alpha\beta} + C_3\partial_\mu\partial_\nu h^{\mu\nu}\partial_\alpha\partial^\alpha h^\beta_\beta$$

$$+C_4\partial_\mu\partial_\nu h^\alpha_\alpha\partial_\beta\partial^\beta h^{\mu\nu} + C_5\partial_\mu\partial_\nu h^\nu_\beta\partial_\alpha\partial^\alpha h^{\mu\beta} + C_6\partial_\mu\partial_\nu h^\alpha_\alpha\partial^\mu\partial^\nu h^\beta_\beta + C_7\partial_\mu\partial_\nu h^\alpha_\alpha\partial^\mu\partial_\beta h^{\nu\beta}$$

$$+C_8\partial_\mu\partial_\nu h^{\mu\nu}\partial_\alpha\partial_\beta h^{\alpha\beta} + C_9\partial_\mu\partial_\nu h^{\nu\beta}\partial^\mu\partial_\alpha h^\alpha_\beta + C_{10}\partial_\mu\partial_\nu h^\nu_\beta\partial^\beta\partial_\alpha h^{\mu\alpha}$$

$$+C_{11}\partial_\mu\partial_\nu h_{\alpha\beta}\partial^\mu\partial^\nu h^{\alpha\beta} + C_{12}\partial_\mu\partial_\nu h_{\alpha\beta}\partial^\mu\partial^\alpha h^{\nu\beta} + C_{13}\partial_\mu\partial_\nu h_{\alpha\beta}\partial^\alpha\partial^\beta h^{\mu\nu}.$$

- We have a completely gauge invariant model under the spin-2 gauge transformation, $$h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu\xi_\nu + \partial_\nu\xi_\mu$$

Higher Order, Second Rank Tensor Potential

- System decouples into 3 independently gauge invariant terms,

$$\mathcal{L} = C_{11}(\partial_\mu \partial_\nu h_{\alpha\beta} \partial^\mu \partial^\nu h^{\alpha\beta} - 2\partial_\mu \partial_\nu h_{\alpha\beta} \partial^\mu \partial^\alpha h^{\nu\beta} + \partial_\mu \partial_\nu h_{\alpha\beta} \partial^\alpha \partial^\beta h^{\mu\nu})$$

$$+C_2(\partial_\mu \partial^\mu h_{\alpha\beta} \partial_\nu \partial^\nu h^{\alpha\beta} + 2\partial_\mu \partial_\nu h^\alpha_\alpha \partial_\beta \partial^\beta h^{\mu\nu} - 4\partial_\mu \partial_\nu h^\nu_\beta \partial_\alpha \partial^\alpha h^{\mu\beta}$$

$$+\partial_\mu \partial_\nu h^\alpha_\alpha \partial^\mu \partial^\nu h^\beta_\beta - 4\partial_\mu \partial_\nu h^\alpha_\alpha \partial^\mu \partial_\beta h^{\nu\beta} + 2\partial_\mu \partial_\nu h^{\nu\beta} \partial^\mu \partial_\alpha h^\alpha_\beta + 2\partial_\mu \partial_\nu h^\nu_\beta \partial^\beta \partial_\alpha h^{\mu\alpha})$$

$$+C_1(\partial_\mu \partial^\mu h^\nu_\nu \partial_\alpha \partial^\alpha h^\beta_\beta - 2\partial_\mu \partial_\nu h^{\mu\nu} \partial_\alpha \partial^\alpha h^\beta_\beta + \partial_\mu \partial_\nu h^{\mu\nu} \partial_\alpha \partial_\beta h^{\alpha\beta}).$$

- After factoring these are contractions of the linearized Riemann tensor, Ricci tensor and Ricci scalar,

$$\mathcal{L} = \tilde{a} R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + \tilde{b} R_{\mu\nu} R^{\mu\nu} + \tilde{c} R^2,$$

Linearized Gauss-Bonnet Gravity

- Repeating the procedure for this higher order combination has a unique solution of the linearized Gauss-Bonnet model, with 0 contribution to the EOM,

$$\mathcal{L} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$T^{\omega\nu} = -R^{\omega\rho\lambda\sigma}R^{\nu}{}_{\rho\lambda\sigma} + 2R_{\rho\sigma}R^{\omega\rho\nu\sigma} + 2R^{\omega\lambda}R^{\nu}{}_{\lambda} - RR^{\nu\omega}$$
$$+ \frac{1}{4}\eta^{\omega\nu}(R_{\mu\lambda\alpha\beta}R^{\mu\lambda\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2).$$

Conclusions

- Axiomatic approach to physics was used to derive classical electrodynamics and linearized Gauss-Bonnet gravity from a common set of axioms
- Additional results, such as the general class of spin-2 Lagrangians from which the Fierz-Pauli action can be obtain, are in the paper
- Possible fundamentally important characteristic of complete gauge invariance is worth exploring further