On the quantum origin of a small positive cosmological constant

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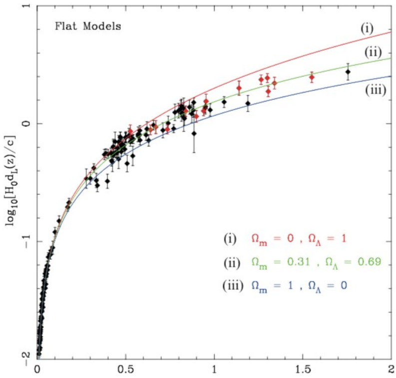
Overview

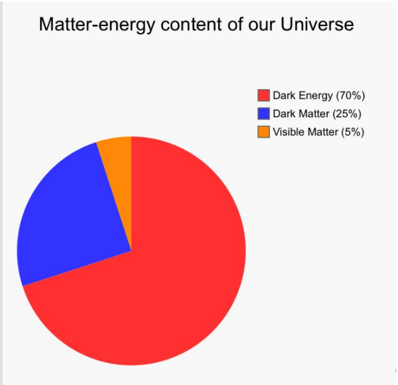
Dark Matter, Dark Energy

Luminosity distance $d_L(\Omega_\Lambda, \Omega_M, z) = \frac{(1+z)}{H_0} \int_0^z \frac{dz}{\sqrt{\underbrace{\Omega_\Lambda}_{0.7} + \underbrace{\Omega_M}_{0.3}(1+z)^3}}$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{crit}} \ , \quad \Omega_M = \frac{\rho_M}{\rho_{crit}}$$



(Pic: Copeland et al, hep-th/0603057)



Questions

- What constitutes Dark Matter?
- What constitutes Dark Energy/Λ?
- Why is Λ positive?
- Why is Λ tiny, about $10^{-123}\ell_{Pl}^{-2}$ where ℓ_{Pl} is the Planck length?
  $(\rho = \int_0^{k_{max}} dk\, k^2 \sqrt{k^2+m^2} \approx k_{max}^4 > 10^{50}\, \rho_\Lambda)$
- Currently $\rho_{DM} \approx \underbrace{\rho_\Lambda}_{\frac{\Lambda c^2}{8\pi G}} \approx \underbrace{\rho_{crit}}_{\frac{3H_0^2}{8\pi G}} \approx 10^{-26}\, kg\, m^{-3}$

  Why? The 'coincidence problem'

Bose-Einstein Condensate as Dark Matter

- Cold
- Dark
- Light bosons as DM $\Rightarrow$ no small scale structure
- Macroscopic *Quantum state*
- BEC $\Rightarrow$ DE ($\approx$ DM) via its (repulsive) Quantum Potential
- Few assumptions and free parameters

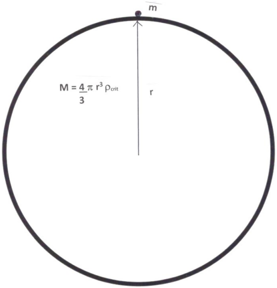
Critical temperature, universe temperature, boson mass

$$T_c(a) = \frac{\hbar c}{k_B}\left(\frac{(N/V)\,\pi^2}{\eta\zeta(3)}\right)^{1/3} = \frac{\hbar c}{k_B}\left(\frac{(0.25\,\rho_{crit}/ma^3)\,\pi^2}{\eta\zeta(3)}\right)^{1/3} = \frac{4.9}{m^{1/3}\ a}\ K$$

$$T(a) = \frac{3.7}{a}\ K$$

$T(a) < T_c(a) \ \rightarrow \quad m < 6\ eV/c^2 \ \Rightarrow$ BEC forms in the early universe

Dark Energy from BEC



Classical, Newtonian

$R_{space} = 0$, $R_{spacetime} = 10^{-123}\,\ell_{Pl}^{-2}$, will quantize, GR $\rightarrow$ same results

$$m\ddot{r} = -\frac{GMm}{r^2} = -\frac{G(\frac{4}{3}\pi r^3 \rho_{crit})m}{r^2} \quad , \quad r = r_0\, a(t) \ , \quad a = \text{scale factor}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_{crit} = -\omega^2 \qquad \text{Raychaudhuri Equation}$$

BEC in a harmonic trap for $t \ll H_0^{-1}$ (14 Gyr)

Quantum Potential

Quantize

$$-\frac{\hbar^2}{2m}\nabla^2\Psi + V\,\Psi = i\hbar\frac{\partial\Psi}{\partial t}$$

$$\Psi(\vec{r},t) = \mathcal{R}\,e^{i\frac{S}{\hbar}}, \quad \mathcal{R}, S = \text{Real} \;, \mathcal{R}^2 = |\Psi|^2 = \rho,\; \vec{v} = \frac{\vec{\nabla}S}{m}$$

Real and imaginary parts

$$\frac{\partial\rho}{\partial t} + \nabla.(\rho\vec{v}) = 0,$$

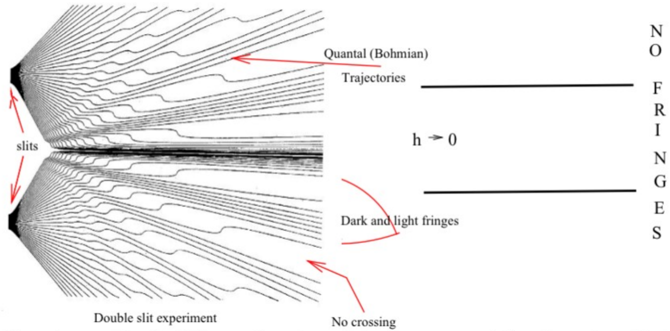
$$m\frac{d\vec{v}}{dt} = -\vec{\nabla}V + \frac{\hbar^2}{2m}\vec{\nabla}\left(\frac{1}{\mathcal{R}}\nabla^2\mathcal{R}\right)$$
$$= -\vec{\nabla}\left(V + V_Q\right)$$

$$V_Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\mathcal{R}}{\mathcal{R}}$$

$V_Q = -V + const.$ if $\Psi =$ stationary state

Quantum Potential - Example

Double slit experiment



(C. Phillipides, C. Dewdney, B. J. Hiley, *Quantum Interference and the Quantum Potenti*
il nuovo cimento B, **52**, no.1 (1979) 15-28)

Quantum Potential $\rightarrow$ Λ

$$m\frac{d\vec{v}}{dt} = -\vec{\nabla}\left(V + V_Q\right) \ , \ V_Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\mathcal{R}}{\mathcal{R}} \ , \ \vec{v} = \frac{d\vec{r}}{dt} \ , r = r_0 \, a(t)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + \frac{\hbar^2}{6m^2}\nabla^2\left(\frac{\nabla^2\mathcal{R}}{\mathcal{R}}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda}{3} \quad \text{(compare)}$$

$$\Lambda = \Lambda_Q = \frac{\hbar^2}{2m^2c^2}\nabla^2\left(\frac{\nabla^2\mathcal{R}}{\mathcal{R}}\right) = -\frac{1}{mc^2}\nabla^2 V_Q$$

Quantum Potential of BEC

Quantize

BEC wavefunction $$\Psi = \underbrace{R(a)}_{\text{DM decay}} \times \underbrace{e^{-r^2/\sigma^2}}_{\text{HO GS}} ,$$

$$R(a) = \frac{R_0}{a^{3/2}} \ , \ \rho_{DM} = |\Psi|^2 \propto \frac{1}{a^3} \ , \ \int dV |\Psi|^2 = N, \ \sigma^2 = \frac{2\hbar}{m\omega}$$

$$V_Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\Psi}{\Psi} = \frac{3}{2}\ \hbar\omega - \frac{1}{2}m\omega^2 r^2$$ *(Inverted HO!)*

$$\frac{\ddot{a}}{a} = -\frac{4\pi G\rho_{crit}}{3} + \frac{\Lambda_Q}{3}$$

$$\Lambda_Q = -\frac{1}{mc^2}\nabla^2 V_Q = \frac{3H_0^2}{2} = 4\pi G\rho_{crit} =$$ constant!

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} = \frac{\rho_{crit}}{2} \approx \rho_{DM}, \ \ p_\Lambda = -\frac{\Lambda c^2}{8\pi G} = -\rho_\Lambda$$

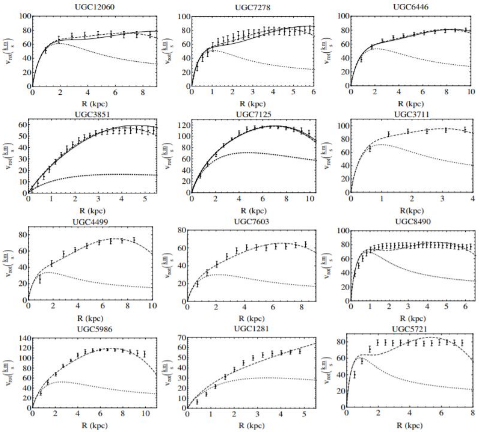
$$= -\frac{4\pi G}{3}\left(\rho_{DM} + \rho_\Lambda + 3p_\Lambda\right) = 0$$ *Einstein's static Universe!*

S.Das, R. K. Bhaduri, Class. Quant. Grav. **32** 105003 (2015) and arXiv:1812.07647

Results for $m = 10^{-32}\ eV/c^2$

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = \left[-\frac{\hbar^2}{2m}\Delta + mV_{ext}(\mathbf{r}) + \lambda\rho(\mathbf{r},t)\right]\psi(\mathbf{r},t)\,,$$

$$V_{ext} = U_Y(\mathbf{r}) = -\int \frac{G\rho_{BEC}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} e^{-\frac{|\mathbf{r}-\mathbf{r}'|}{R_g}}\, d^3\mathbf{r}'\,.$$



E. Kun, Z. Keresztes, S. Das, L.A. Gergely, Symmetry, **10**, 520 (2018), arXiv:1905.04336

Summary

- What constitutes DM? *BEC*
- What constitutes DE? *Quantum potential of the BEC*
- Why is Λ positive? *Because* $V_Q = -V > 0$
- Why is $\rho_{DM} \approx \rho_\Lambda \approx \rho_{crit}$? *Because* $|V_Q| = |V|$

Remarks

- We get $\rho_\Lambda = \not{3}\ \rho_{DM}$, because $\rho_{DM} \propto 1/a^3, \rho_\Lambda \propto$ constant and all bosons $\neq$ GS
- Newtonian $\rightarrow$ Relativistic (Straightforward)
- *i.e.* Raychaudhuri Equation $\rightarrow$ Raychaudhuri Equation $+V_Q$. Results don't change
- Prediction: ultralight bosons of $m < 6eV/c^2$ including the 'preferred' value of $m = 10^{-22}\ eV/c^2$ (prevents small scale structure)
- What are these bosons? gravitons, axions, ...
- Better fitting with galaxy data
- Prediction: Λ has changed in the far past and will change in the future