Emergence of a Black Hole at the center of some Galaxies.

Réjean Plamondon

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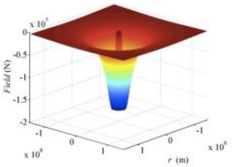
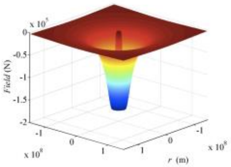


POLYTECHNIQUE MONTRÉAL

Topics

- **Emerging gravity: a recap.**
- **Symmetric space-time geometry.**
- **Axisymmetric Interpretation.**
- **Black hole emergence**



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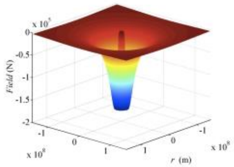
1-Einstein Gravitation Equation

$$G = KT$$

« Spacetime tells matter how to move; matter tells spacetime how to curve ».

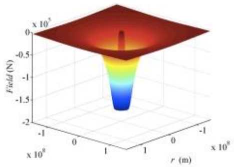
Wheeler, J.A., Ford, K.W., *Geons, Black Holes, and Quantum Foam: A Life in Physics*. W. W. Norton & Company, 2000.



2-Interdependence principle

Space-time curvature (S) and energy-momentum (E) are two inextricable descriptive approaches defining the physically observable probabilistic universe (U); they must be mutually exploited to describe any subset U_i of this universe. The probability of observing a subset (U_i) is:

$$P(U_i) = P(S_i, E_i) = P(S_i / E_i)P(E_i) = P(E_i / S_i)P(S_i)$$



A link with Einstein's law?

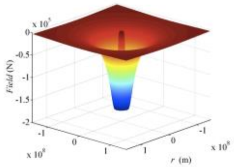
$$f(S_i/E_i) = \frac{f(E_i\ /\ S_i)\, f(S_i)}{f(E_i)} \Leftrightarrow G_{00} = KT_{00}$$

$$f(S_i/E_i) = k_1 tr G_{00}$$

$$\frac{f(S_i)}{f(E_i)} = k_2 tr T_{00}$$

$$G = \frac{k_2 k_3}{k_1} T_{00} f(E_i\ /\ S_i)$$

$$f(E_i\ /\ S_i) = ?$$



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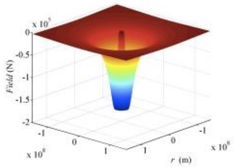
3-Modifying Einstein Equation

$$G = KT$$

A Bayesian Approach:

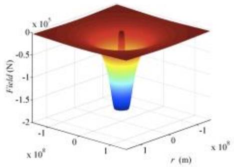
$$G_{00} = K \; T_{00} \times f(E_i / S_i)$$



4-A simple stochastic model of a galaxy formation…

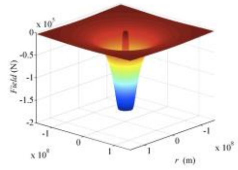
Assuming that in a far remote and isolated part of the Universe, a galaxy is slowly building up from the gradual agglomeration of stars.

Whatever the physical processes involved, these stars can be considered as random variables described by their own density functions and, from a probabilistic point of view, the whole process is equivalent to adding random variables, i.e. making the convolution of their corresponding probability density functions. .



5- Gaussian Convergence

- These densities are real, normalized, non-negative functions with a finite third moment and a scaled dispersion.
- The central limit theorem predicts that in a Euclidean flat spacetime, when the number of random stars is very large ($N \rightarrow \infty$), the ideal form of the global probability density will be a Gaussian multivariate.



6-Moving from flat to curved space

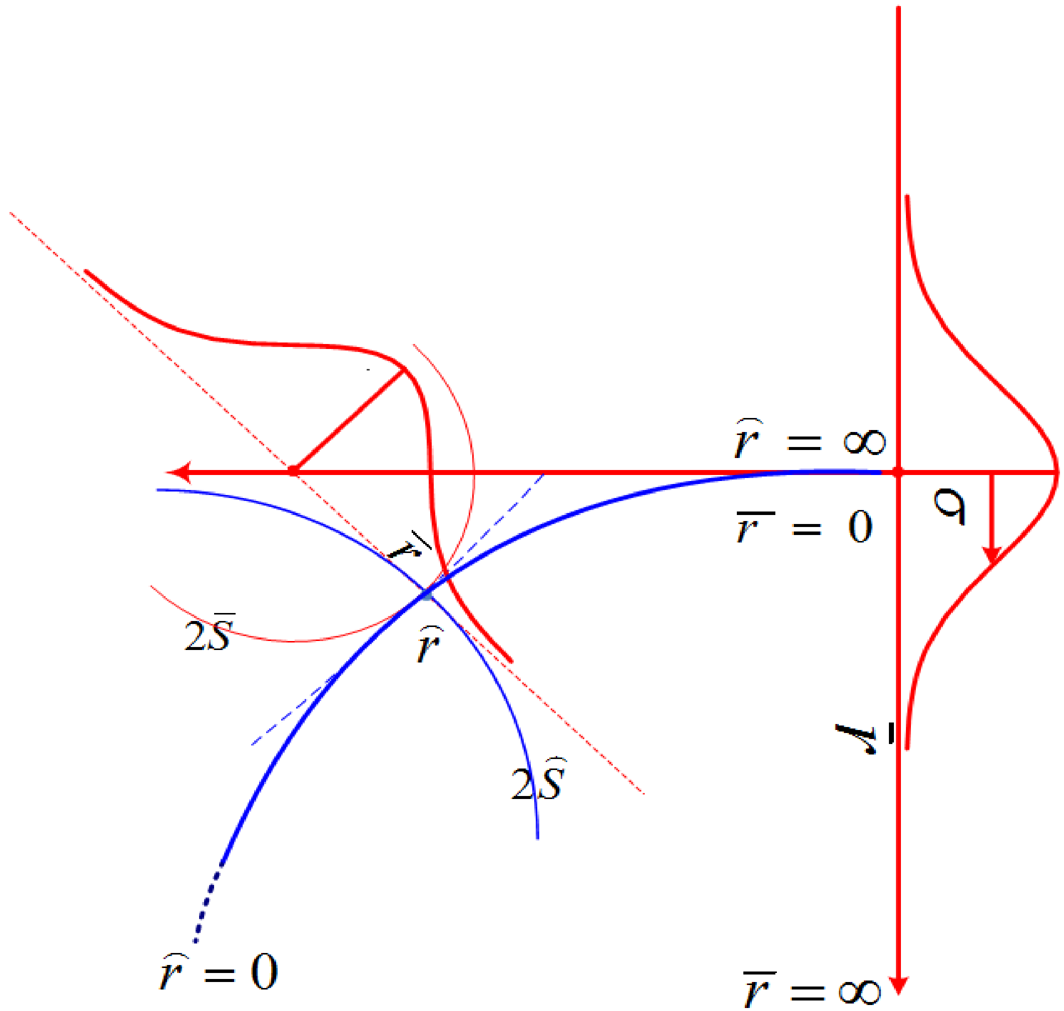
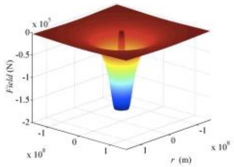


Figure 1

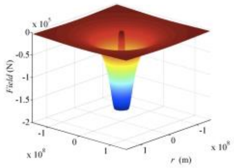


Curved vs flat space representations

$$\begin{bmatrix} \bar{r} = 0 \text{ at } \overset{\frown}{r} = \infty \\ \bar{r} = \infty \text{ at } \overset{\frown}{r} = 0 \end{bmatrix} \Rightarrow \bar{r} = \frac{s}{\overset{\frown}{r}}$$

$$\bar{r} = \sigma = \overset{\frown}{r} \Rightarrow s = \sigma^2$$

$$\frac{\bar{r}}{\sigma} = \frac{\sigma}{\overset{\frown}{r}}$$



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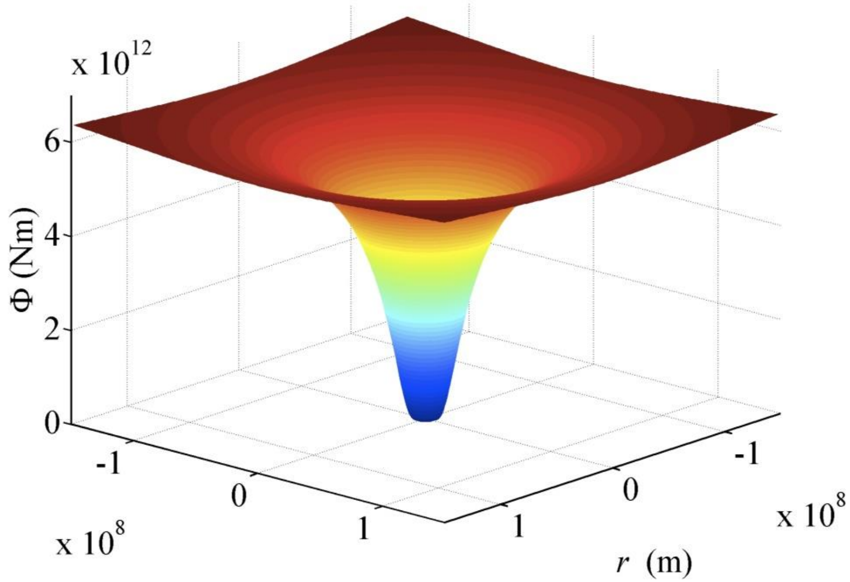
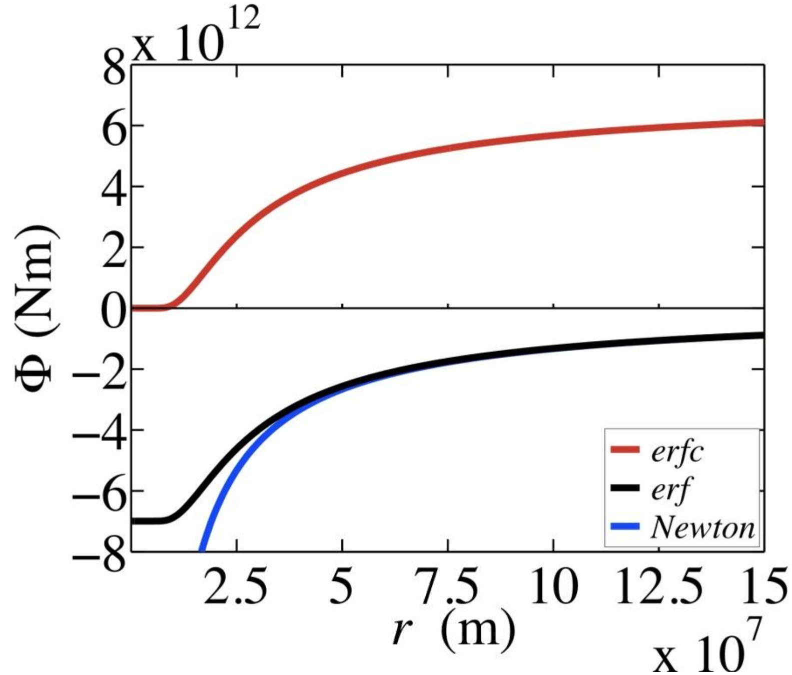
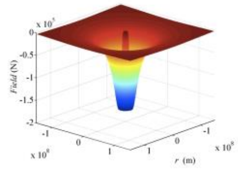
7-Emergence of Newton's law of gravitation: the potential

$$\Phi_{erfc}(r) = \frac{2KMc^4}{4\pi\sigma^3}\left(\frac{\sqrt{\pi}}{\sqrt{2}\sigma}\right) erfc\left(\frac{\sigma}{\sqrt{2}r}\right) = \mathit{\Phi}_{erfc}(r)$$







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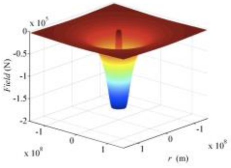


POLYTECHNIQUE MONTRÉAL

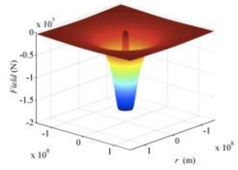
An Emergent Symmetric Metric

$$ds^2 = \left[1 + \frac{2K_\sigma}{c^2} erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right] c^2 dt^2$$

$$- \left[1 + \frac{2K_\sigma}{c^2} erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2$$

$$- r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

where: $K_\sigma = \sqrt{\pi} GM \; / \; \sigma\sqrt{2}$



Réjean Plamondon, Theory Canada 12, May 26, 2017, Toronto.

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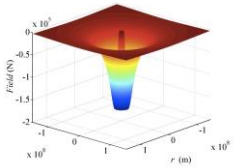
An Emergent Symmetric Metric

No Singularities,
(neither coordinate or intrinsic)

Two types of corrections
predicted by the *erfc* potential

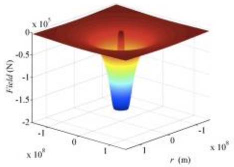
$$erfc(x) = 1 - erf(x)$$

$$erf(x)\big|_{x\to\infty} \cong 1/x$$



Potential explanations for:

1. Hubble's constant
2. Fly-by anomalies
3. Residual Pioneer's delay
4. Secular increase of the astronomical unit
5. Sun oblateness
6. …



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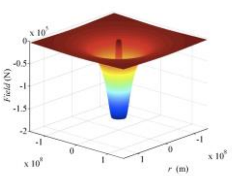
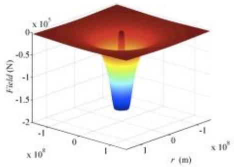


QUESTION

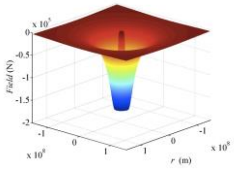
Can we convert this symmetric metric into into an asymetric one?



YES…here is how it goes:

- Algebrical Equivalence

$$ds^2 = \left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right] c_{th}^2 dt^2 + 2K_\sigma dt^2$$
$$+ \frac{2K_\sigma}{c_{th}^2}\left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} \times \left[1 + \frac{2K_\sigma}{c_{th}^2} erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2$$
$$- \left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$



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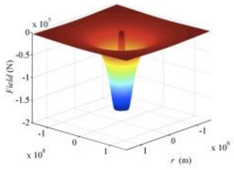
Defining a Rotation ω_{st} and a Space-time Expansion v_{st} Speed Components

$$\omega_{st} = \frac{d\phi}{dt}$$

$$v_{st} = \frac{dr}{dt}$$



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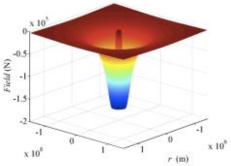
A New Axisymmetric Metric

$$ds^2 = \left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right] c_{th}^2 dt^2 + \frac{2K_\sigma}{\omega_{st}} d\phi dt$$

$$+ \frac{2K_\sigma v_{st}}{c_{th}^2}\left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} \times \left[1 + \frac{2K_\sigma}{c_{th}^2} erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} drdt$$

$$- \left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$



A New Axisymmetric Metric

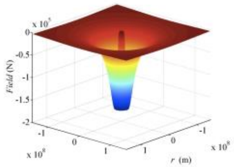
$$ds^2 = \left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right] c_{th}^2 dt^2 + \frac{2K_\sigma}{\omega_{st}} d\phi dt$$

$$+ \frac{2K_\sigma v_{st}}{c_{th}^2}\left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} \times \left[1 + \frac{2K_\sigma}{c_{th}^2} erfc\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} drdt$$

$$- \left[1 - \frac{2K_\sigma}{c_{th}^2} erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

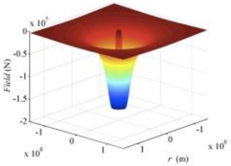
A general format describing a rotating and expanding manifold:

$$ds^2 = g_{00} dt^2 + 2g_{03} d\phi dt + 2g_{01} dr dt + g_{11} dr^2 + g_{22} d\theta^2 + g_{33} d\phi^2$$



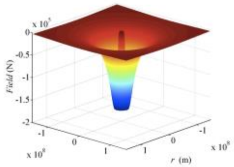
An interpretation…

In other words, the static energy associated with the offset of the *erfc* potential can be seen as the source of the body rotation and its corresponding spacetime expansion.



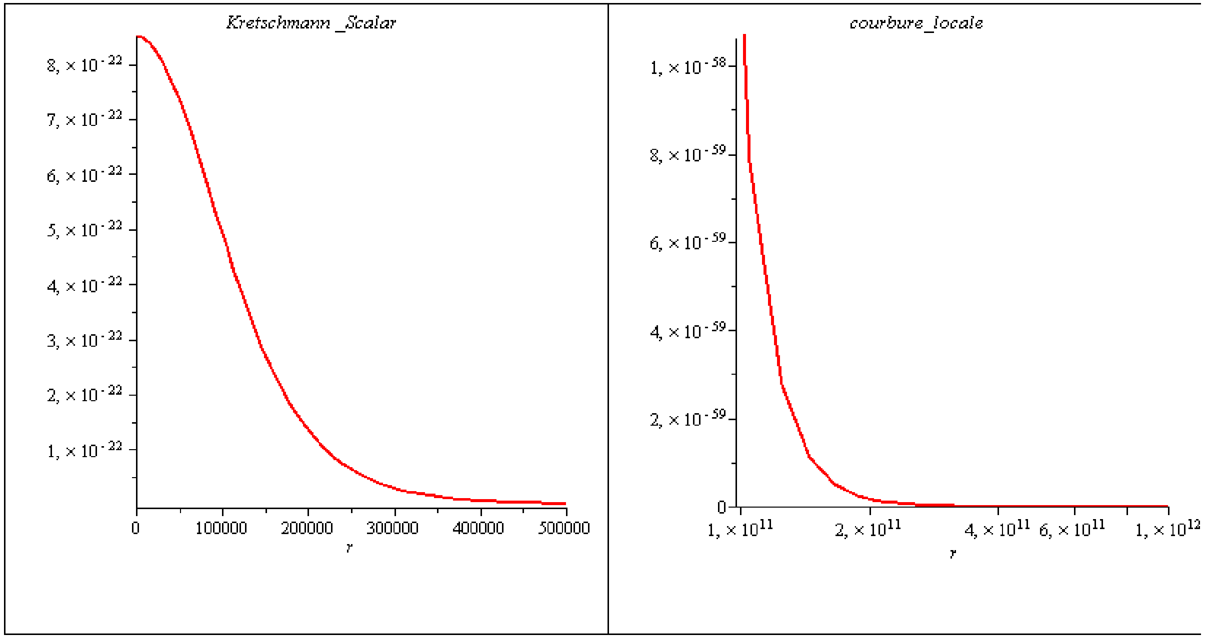
A complex axisymetric metrics

	Non zero components
Covariant metric components (6)	$g_{00}, g_{03}, g_{01}, g_{11}, g_{22,} g_{33}.$
Contravariant metric components (7)	$g^{00}, g^{01}, g^{03}, g^{11}, g^{13}, g^{22}, g^{33}.$
Christoffel symbols of the first kind (10)	$\Gamma_{00,1}, \Gamma_{01,0}, \Gamma_{11,0}, \Gamma_{11,1}, \Gamma_{12,2}, \Gamma_{13,3},$ $\Gamma_{22,1}, \Gamma_{23,3}, \Gamma_{33,1}, \Gamma_{33,2}.$
Christoffel symbols of the second kind (21)	$\Gamma^0_{00}, \Gamma^0_{01}, \Gamma^0_{11}, \Gamma^0_{13}, \Gamma^0_{22}, \Gamma^0_{23}, \Gamma^0_{33},$ $\Gamma^1_{00}, \Gamma^1_{01}, \Gamma^1_{11}, \Gamma^1_{13}, \Gamma^1_{22}, \Gamma^1_{23}, \Gamma^1_{33},$ $\Gamma^3_{00}, \Gamma^3_{01}, \Gamma^3_{11}, \Gamma^3_{13}, \Gamma^3_{22}, \Gamma^3_{23}, \Gamma^3_{33}.$
Riemann tensor covariant components (16)	$R_{0101}, R_{0103}, R_{0113}, R_{0202}, R_{0203}, R_{0212}, R_{0213},$ $R_{0303}, R_{0312}, R_{0313}, R_{1212}, R_{1213}, R_{1223}, R_{1313},$ R_{1323}, R_{2323}
Ricci tensor components (9)	$R_{00}, R_{01}, R_{03}, R_{11}, R_{12},$ $R_{13}, R_{22}, R_{23}, R_{33}.$
Einstein tensor components (9)	$G_{00}, G_{01}, G_{03}, G_{11}, G_{12},$ $G_{13}, G_{22}, G_{23}, G_{33}.$

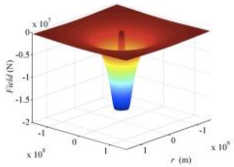


Kretschmann Scalar

- A typical example…



- No intrinsic singularity



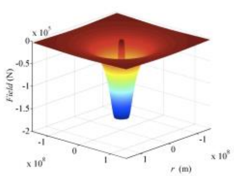
Under some specific conditions

- One coordinate singularity

$$\left[1-\frac{2K_\sigma}{c^2}erf\left(\frac{\sigma}{\sqrt{2}r}\right)\right]=0$$

$$erf\left(\frac{\sigma}{\sqrt{2}r_{\mathrm{coord.sing}}}\right)=\frac{c_{th}^2}{2K_\sigma}$$

where $K_\sigma = \sqrt{\pi}GM \;/\; \sigma\sqrt{2}$



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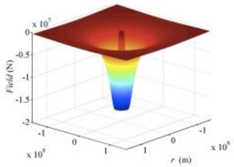
A Double Horizon

$$g^{11} = \frac{g_{00}g_{33} - g_{03}^2}{g_{00}g_{11}g_{33} - g_{03}^2 g_{11} - g_{01}^2 g_{33}} = 0$$

$$c_{th}^2 r^2 \sin^2\theta - 2K_\sigma erf\left(\frac{\sigma}{\sqrt{2}r}\right) r^2 \sin^2\theta + \frac{4K_\sigma^2}{\omega_{st}^2} = 0$$

$$r \cong \frac{K_\sigma \sigma}{\sqrt{2}c_{th}^2} \pm \frac{K_\sigma \sigma}{\sqrt{2}c_{th}^2}\left[1 - \frac{8c_{th}^2}{\omega_{st}^2}\right]^{1/2}$$



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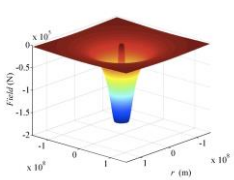
Gravitational collapse

A system described by *erfc* or an *erf* metric will hardly experience unbounded gravitational collapse. For a neutron star of mass M and of proper length σ, the extreme relativistic *N* electron gas will have a total energy of:

$$\frac{N^{4/3}(9\pi c^3\hbar^3 \;/\; 4)^{1/3}}{r} + \frac{\sqrt{\pi}GM^2}{\sigma\sqrt{2}}\,erfc\left(\frac{\sigma}{\sqrt{2}r}\right)$$

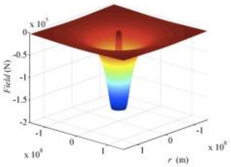
which will have a minimum as compared to the Newton potential.



No Stationnary Limit Surface
No ergosphere

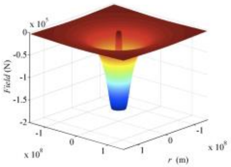
$$g_{01} \neq 0$$

No fixed spatial coordinate



Under some specific conditions

- No intrinsic singularity
- No gravitational collapse
- One coordinate singularity
- A double horizon
- No stationnary limit surface
- No ergosphere



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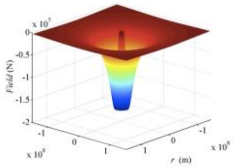
Complexity

The overall geometry is quite complex, with various zero crossings which points out the complexity of the spacetime associated to a rotating mass as seen from within the system.

An interesting model that worths pursuing its exploration…



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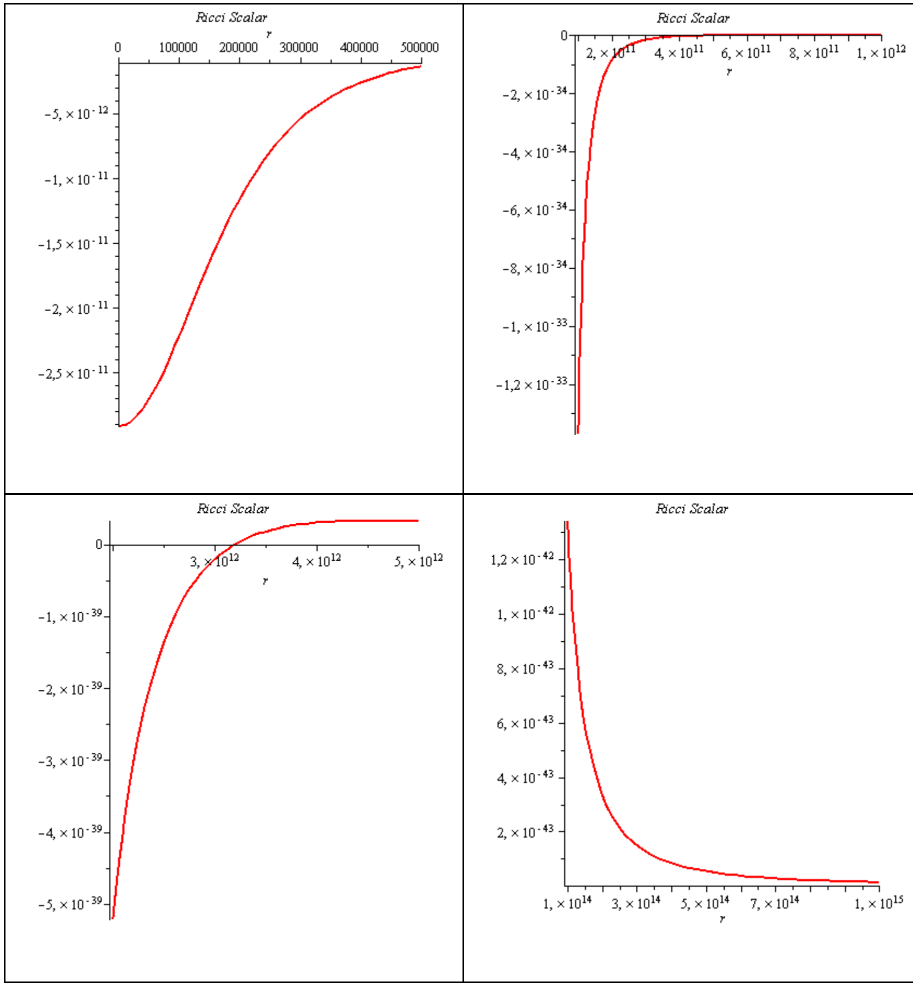
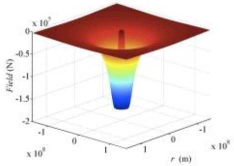
Its Corresponding Ricci Scalar…





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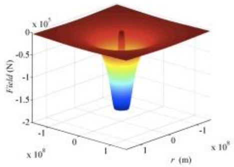
Food for thought

- According to this model, in some regions, the Galaxy would appear as surrounded by a repulsive ring when only the trace of the Ricci tensor is taken into account...



Emergence of a Black Hole at the center of some Galaxies.

See you

Tuesday, session T1-9 General Relativity II

10h00, room SSB 9242

Réjean Plamondon

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Questions?