Quantum tasks in holography

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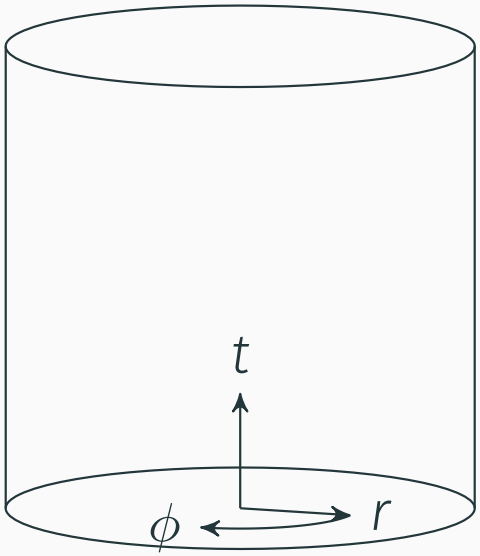
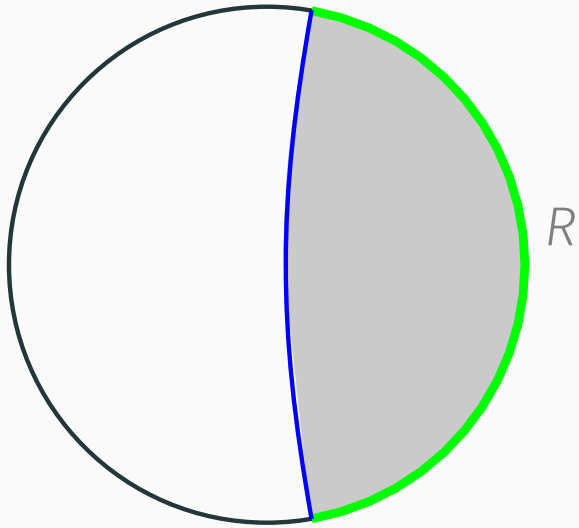
AdS/CFT

- AdS/CFT is a duality between string theory in an asymptotically AdS spacetime and a CFT living on the boundary of that spacetime.
- In an appropriate limit ($N \gg 1$), the bulk is described by QFT in a curved background.



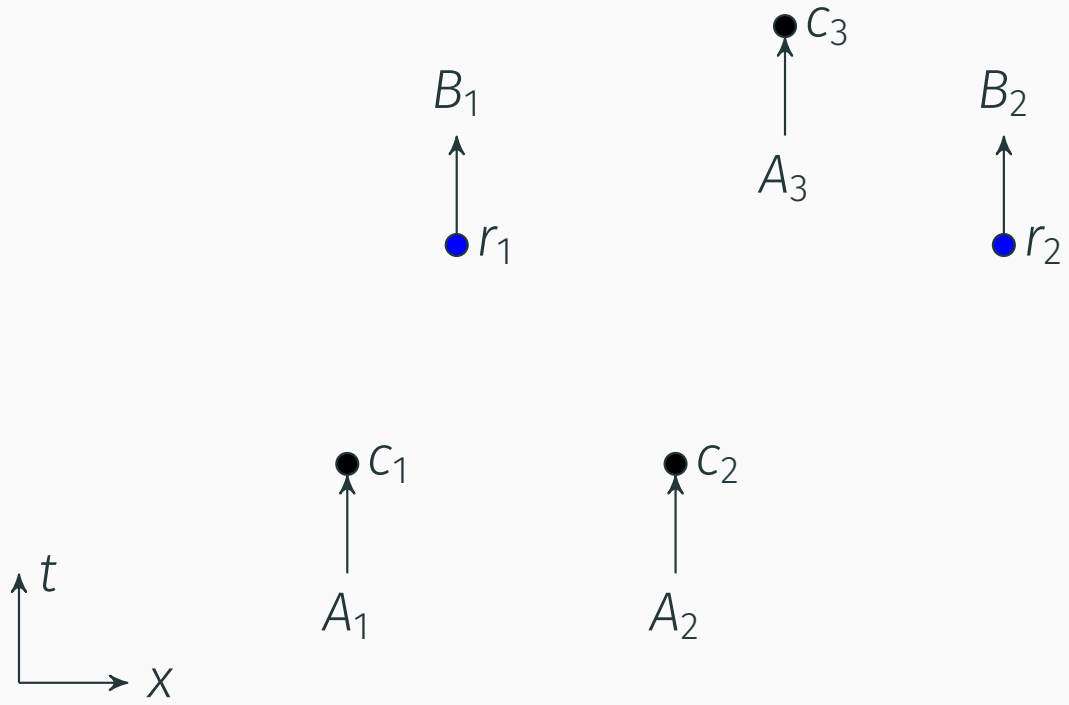
(a) $Z_{CFT}[J] = Z_{ADS}[\phi \to J] \approx e^{-S[\phi \to J]}$



(b) $S(R) = \frac{A[\chi]}{4G} + S_{bulk}$

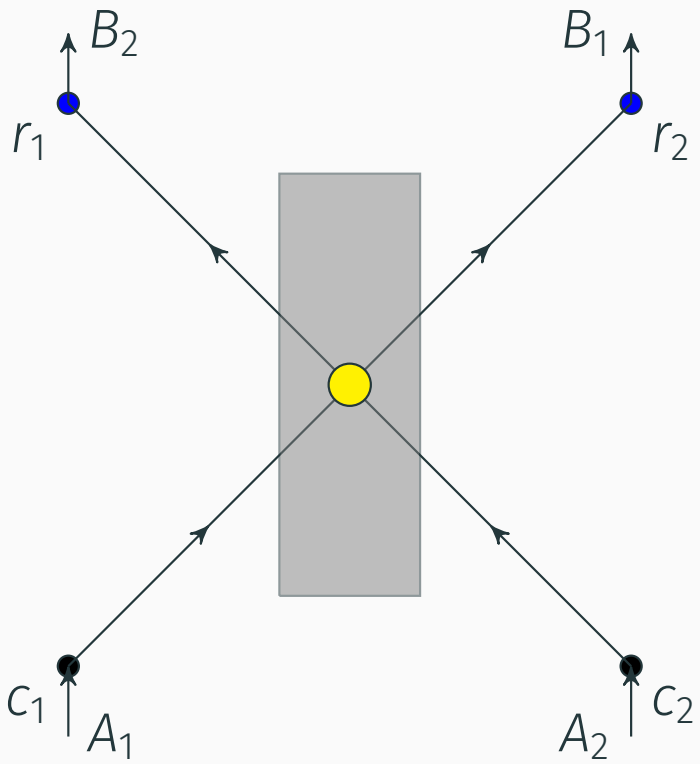
Quantum tasks

- To study the holographic principle, we will find it useful to introduce **quantum tasks**

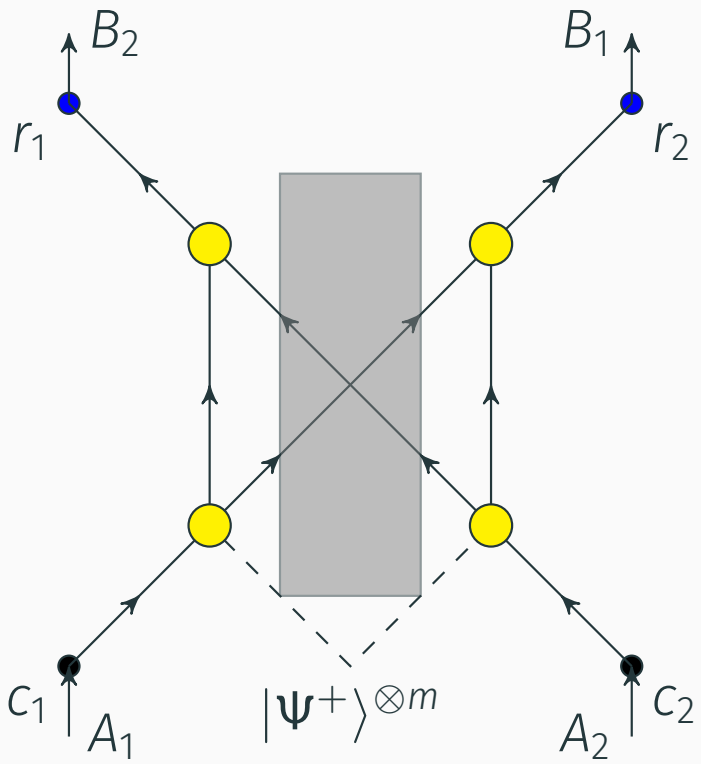


- Quantum tasks are distributed quantum computations, with inputs received and outputs given at various spacetime locations.

Position based cryptography



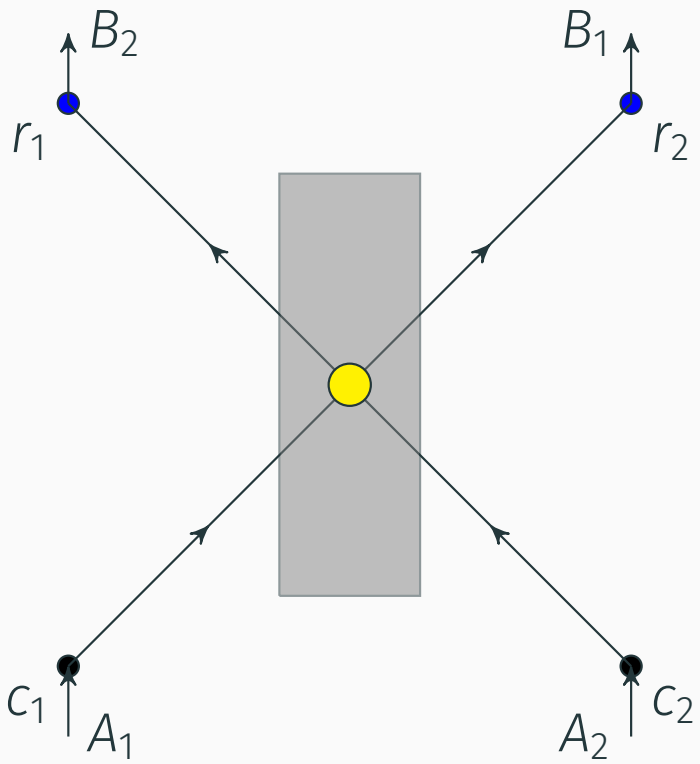
(a)



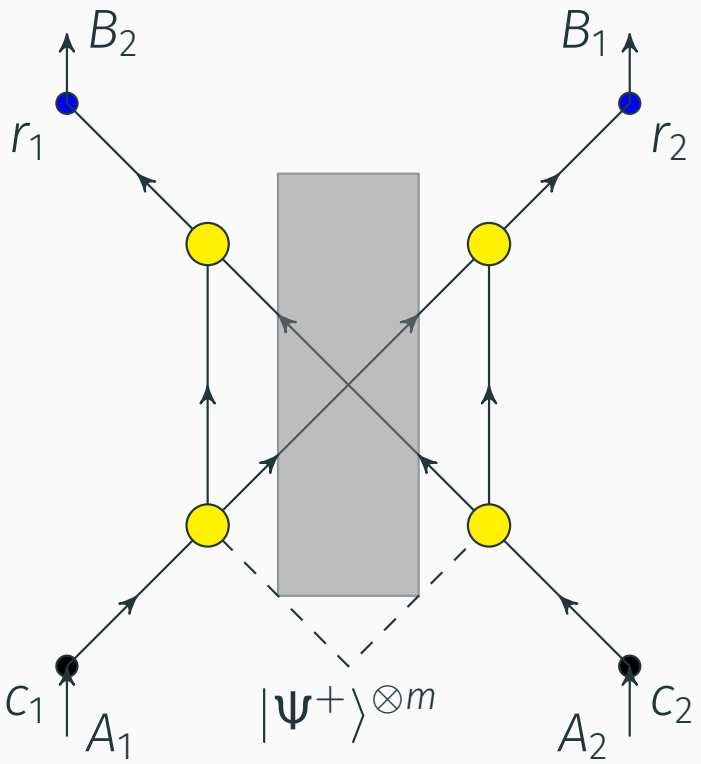
(b)

- In the cryptographic task of **position verification**, it has been understood that access to the grey region can be replaced with entanglement across the region, and only with entanglement.
- This is reminiscent of AdS/CFT...

Position based cryptography



(a)



(b)

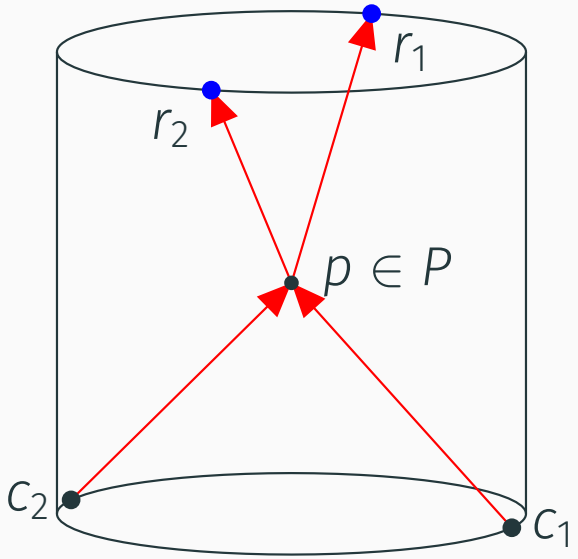
For some tasks, we can show,

$$I(C_1 : C_2) \geq \beta\, n \tag{1}$$

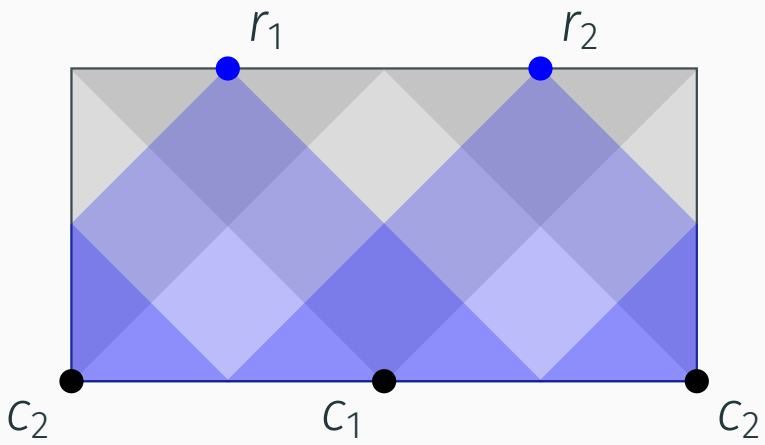
$n = \log\dim A_1 = \log\dim A_2$ is number of qubits in the input.

Some AdS geometry

Consider the central region $P = J^+(c_1) \cap J^+(c_2) \cap J^-(r_1) \cap J^-(r_2)$



(a) Bulk: P is non-empty.



(b) Boundary: P is empty.

- In bulk, can do task $n \approx O(1/G)$ times.
- In boundary, must accomplish same task, but using entanglement, with $I(C_1 : C_2) \geq \beta n$

Some AdS geometry

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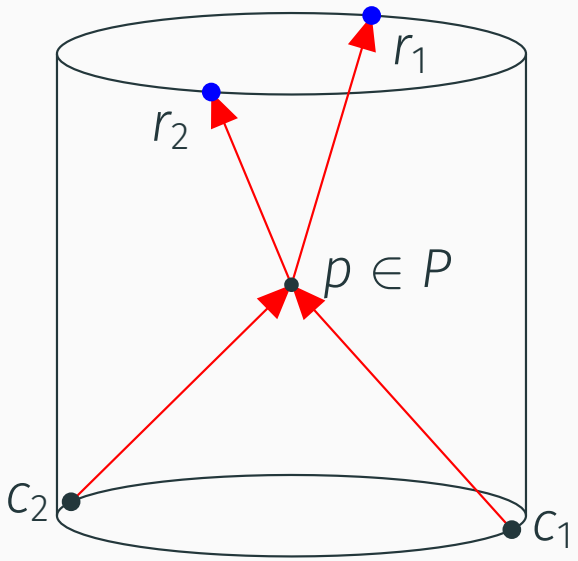

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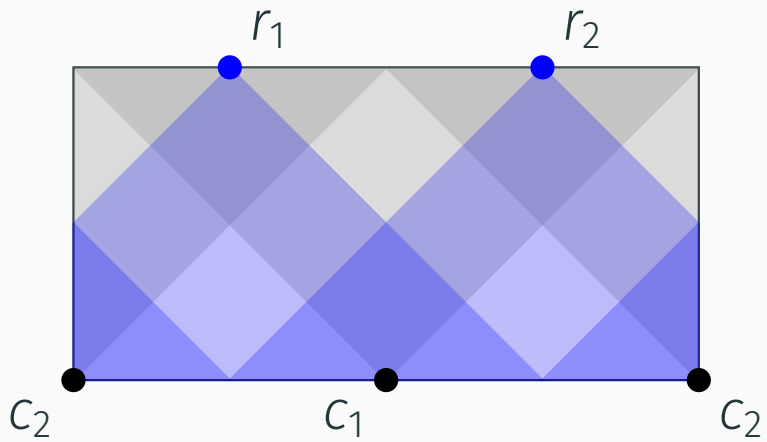


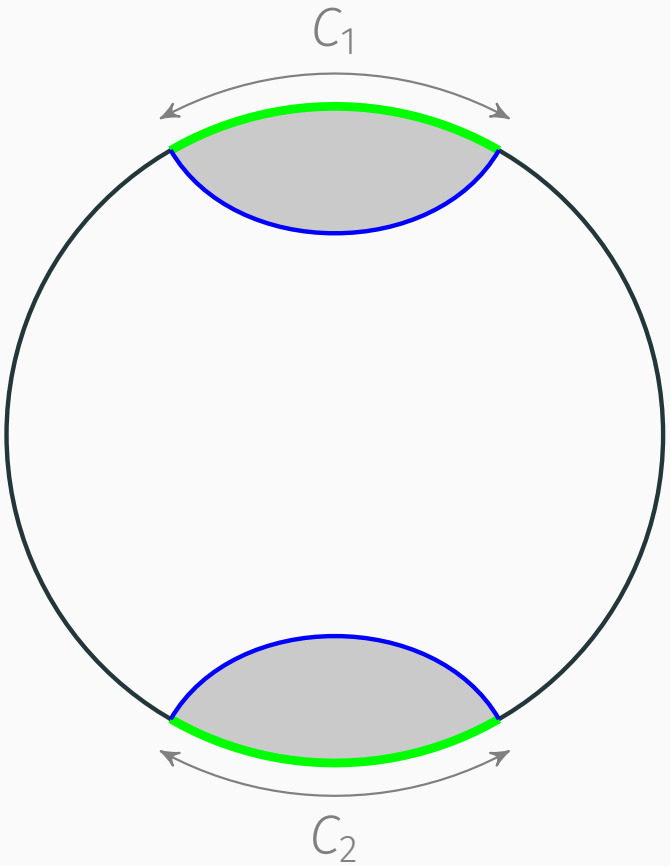
(b) Boundary: P is empty.

Entanglement-causal structure theorem

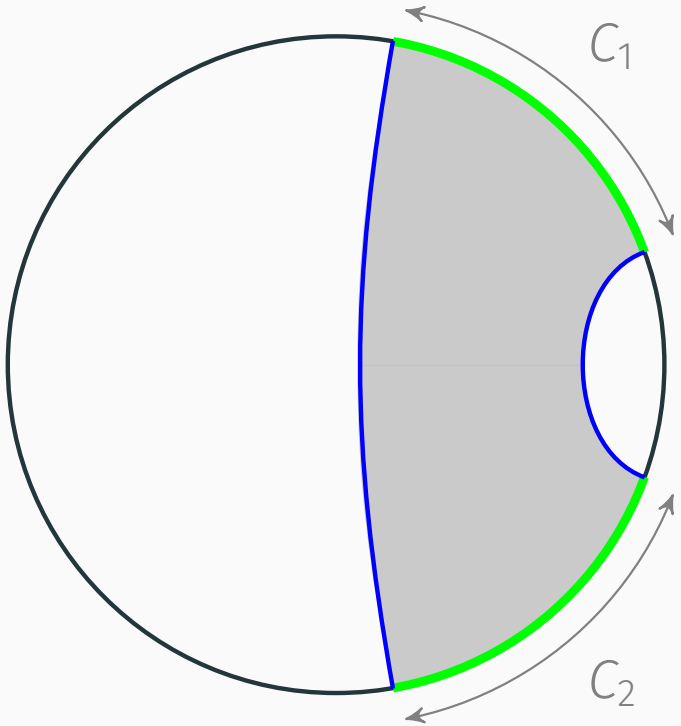
$P \neq \emptyset$ in the bulk implies $I(C_1 : C_2) = O(1/G)$.

Minimal surfaces and mutual information

The mutual information undergoes a phase transition:



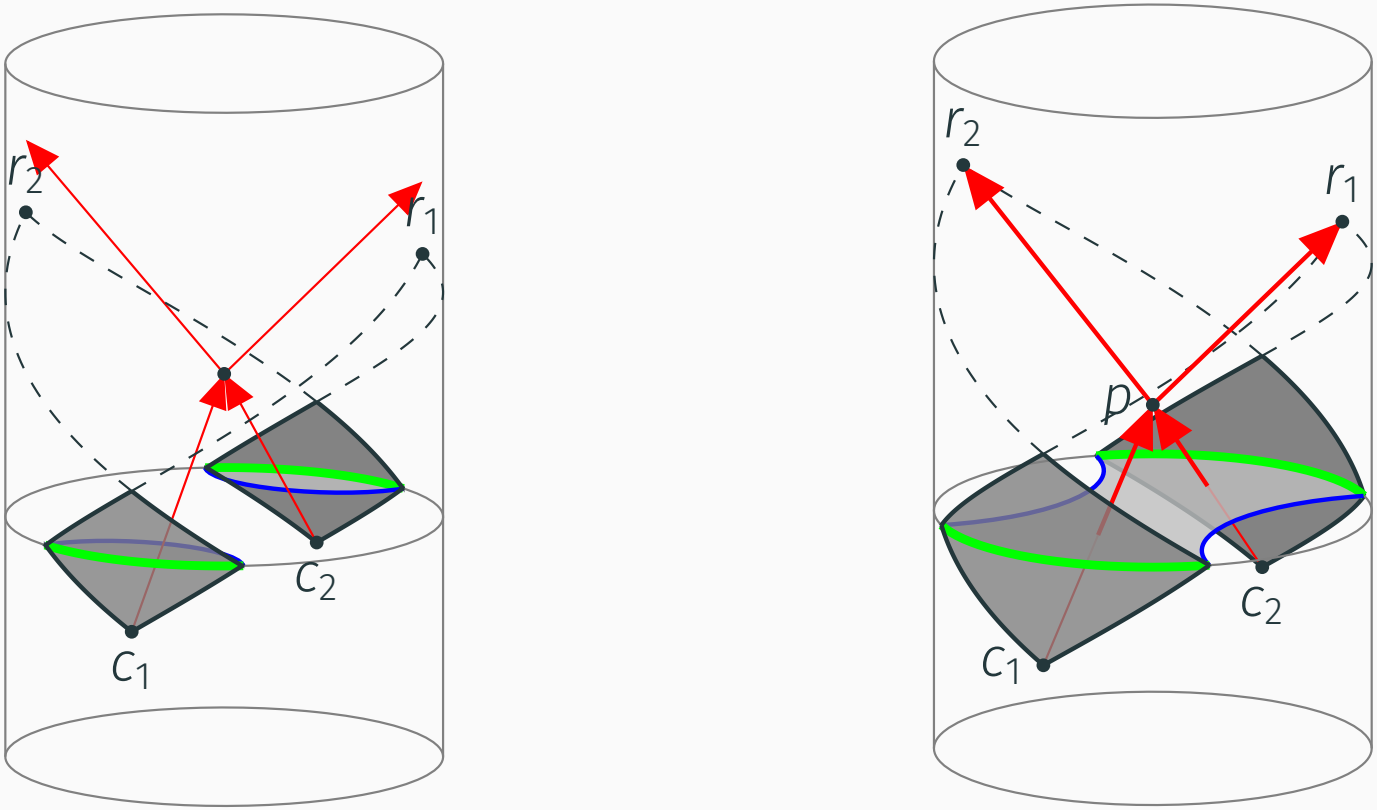
(a) $I(C_1 : C_2) = O(1)$



(b) $I(C_1 : C_2) = O(1/G)$

The blue line is the minimal surface enclosing $C_1 \cup C_2$.

Entanglement and causal structure: A geometric result



Minimal surface-causal structure theorem

If the bulk central region P is non-empty, then the minimal surfaces take on the connected configuration.

Conclusions

Conclusions:

- The cryptographic theorems on position verification give a novel connection between entanglement and causal structure in AdS/CFT.
- Via the Ryu-Takayanagi formula, this allows us to relate minimal surfaces to light cone structure.

Acknowledgements

Thank you!

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