On Negative Mass

On negative mass

Jonathan Belletête, M.B. Paranjape. Apr 4, 2013. 6 pp.
Int.J.Mod.Phys. D22 (2013) 1341017
DOI: 10.1142/S0218271813410174
e-Print: **arXiv:1304.1566 [gr-qc]**

Negative mass bubbles in de Sitter space-time

Saoussen Mbarek, M.B. Paranjape. Jul 6, 2014. 5 pp.
Phys.Rev. D90 (2014) no.10, 101502
e-Print: **arXiv:1407.1457 [gr-qc]**

Stable negative mass bubbles in de Sitter space-time

Matthew Johnson, Antoine Savard, Natalia Tapia-Arellano, M.B. Paranjape. in preparation

Jonathan Belletête, Matthew Johnson, Saoussen Mbarek, Antoine Savard, Natalia Tapia-Arellano and M. B. Paranjape

Groupe de physique des particules,
Département de physique, Université de Montréal,
Montréal, Québec, Canada

- The Schwarzschild solution of the vacuum Einstein equations contains one parameter, the mass.
- The mass can be any value, positive or negative.

$$G_{\mu\nu}[g_{\sigma\rho}(t, r, \theta, \varphi; M)] = 0$$

$$d\tau^2 = (1 - 2M/r)dt^2 - dr^2/(1 - 2M/r) - r^2 d\Omega^2$$

- The metric is singular at $r = 0$
- For positive mass there is a singularity at $r = 2M$ however this can be removed with an appropriate change of coordinates. $r = 2M$ defines the event horizon, and we are content to observe that the real singularity at the origin is hidden behind this horizon.
- As the role of the time and radial coordinates exchange at the event horizon, the singularity at the origin occurs on a future space-like hypersurface which is impossible to avoid, a circumstance that does not seem particularly pleasant

- For negative mass, there is no event horizon and the singularity at the origin is naked, it occurs at a distinct spatial position.
- But we deal with this kind of singularity all the time, and compared to the murderous, inevitable future singularity of positive mass, they are benign.
- For example, from far away, a proton say, appears as a singular point charge, however, on closer inspection, we find that the charge is spread out over a tiny volume.
- Can we not imagine the introduction of a smooth distribution of negative matter to give the same kind of resolution for a negative mass gravitational singularity?

- Negative mass particles, if they exists are strange beasts.
- A negative mass particle creates a negative gravitational field, however, because of the equivalence principle, it is attracted to positive mass particles.
- The force is exerted on a negative mass particle $-m$ by a positive mass particle M, towards the positive mass particle is:

$$F = \frac{GM(-m)}{r^2}$$

- But the acceleration felt by the negative mass particle is:

$$\dot{p} = (-m)a_{-} = F = \frac{GM(-m)}{r^2}$$

- Obviously the negative mass cancels from both sides of the equation giving:

$$a_- = \frac{GM}{r^2}$$

- On the other hand, the acceleration felt by the positive mass particle is obtained through:

$$\dot{p} = (M)a_+ = F_+ = \frac{G(-m)M}{r^2}$$

- That is: $$a_+ = \frac{G(-m)}{r^2}$$

- which is clearly negative or repulsive.

- This scenario was first expressed by Luttinger in 1951 GRFEC, “On Negative Mass in the Theory of Gravitation”.
- Subsequently, Bondi made a detailed analysis in the context of general relativity,

REVIEWS OF MODERN PHYSICS VOLUME 29, NUMBER 3 JULY, 1957

Negative Mass in General Relativity

H. BONDI

King's College, University of London, London, England

- He considered the situation only from the point of view of the exterior spacetime, the negative mass was considered to be inside a compact region that was not probed.

- Bondi's analysis is still not complete, as it does not address what is the source of the negative mass
- Indeed, in asymptotically Minkowski spacetime, there cannot exist negative mass that comes from physically reasonable sources, due to the positive energy theorem.
- Physically reasonable mass would satisfy the dominant energy condition.

$$T^{0\nu} u_\nu \geq 0 \qquad\qquad T^{\mu\nu} u_\nu T_{\mu\alpha} u^\alpha \geq 0$$

- for any timeline vector u

Dominant Energy Condition

- The dominant energy condition is a very reasonable, local constraint to impose on any physical energy-momentum distribution.
- It enforces that no observer will see energy-momentum is moving faster than the speed of light.
- Consider a spherically symmetric distribution of matter and the ensuing metric in Schwarzschild coordinates

$$d\tau^2 = (1 - 2M(r)/r)dt^2 - dr^2/(1 - 2M(r)/r) - r^2 d\Omega^2$$

- Here, $M(r)$ is the mass inside a sphere of coordinate radius r .
- Inserting this metric into the Einstein tensor gives the energy-momentum tensor that would create such a metric. It is easily found that:

$$T_0^0 = T_1^1 = \frac{2M'(r)}{r^2} \qquad T_2^2 = T_3^3 = \frac{M''(r)}{r}$$

- The dominant energy condition imposes:

$$T^{0\nu}u_\nu \geq 0 \qquad T^{\mu\nu}u_\nu T_{\mu\alpha}u^\alpha \geq 0$$

- For any time-like or light-like vector u

- This yields the conditions:

$$\frac{d}{dr}\left(\frac{M'(r)}{r^2}\right) \leq 0 \qquad \frac{d}{dr}\left(M'(r)r^2\right) \geq 0$$

Positive Energy Theorem

by Schoen and Yau, and Witten

- The positive energy theorem states that any space-time that is asymptotically flat and contains energy-momentum that everywhere satisfies the dominant energy condition, will necessarily have a positive ADM mass.
- In our specialized spherically symmetric geometry, this requires that:

$$M(r) \to M \text{ and } M \geq 0$$

- Thus we cannot have asymptotic flatness and

$$M(r) \to -M \text{ and } M \geq 0$$

- The positive energy theorem tells us that if the dominant energy condition were satisfied, the mass parameter would necessarily have to be positive.
- But we can imagine eschewing the positive energy theorem by dropping the constraint of asymptotic flatness. The inflationary phase of the universe or even in principle the present, accelerating universe are both asymptotically de Sitter universes which are not asymptotically flat.

- There is a more general exact solution of the Einstein equations with cosmological constant

$$d\tau^2 = \left(1 - \frac{(\Lambda/3)r^3 - 2GM}{r}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{(\Lambda/3)r^3 - 2GM}{r}\right)} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2$$

$$G_{\mu\nu}[g_{\lambda\rho}] = \Lambda g_{\mu\nu} \qquad T_{\mu\nu} = (\Lambda/8\pi G)g_{\mu\nu}$$

- containing two free parameters Λ and M which can be positive or negative.
- We ask the question: Is there a deformation of the positive Λ but negative M metric that satisfies the dominant energy condition everywhere?

- With Jonathan Belletête we were able to find exactly such a deformation.

On negative mass
Jonathan Belletête, M.B. Paranjape. Apr 4, 2013. 6 pp.
Int.J.Mod.Phys. D22 (2013) 1341017
DOI: 10.1142/S0218271813410174
e-Print: **arXiv:1304.1566** **[gr-qc]**

- It is just a mathematical deformation, there is no analysis of stability.
- The added energy-momentum is not seen as any kind of known energy-momentum

Adding perfect fluid energy momentum

Negative mass bubbles in de Sitter space-time
Saoussen Mbarek, M.B. Paranjape. Jul 6, 2014. 5 pp.
Phys.Rev. D90 (2014) no.10, 101502
e-Print: arXiv:1407.1457 **[gr-qc]**

- The previously obtained deformation was simply a mathematical deformation which satisfied the imposed constraints.
- We asked if there some actual physical energy-momentum which can provide the necessary deformation.
- We were successful with the energy-momentum of a perfect fluid.

- The energy momentum of a perfect fluid is given by:

$$T_{\mu\nu} = -pg_{\mu\nu} + (p+\rho)U_\mu U_\nu$$

- with metric of the form:

$$ds^2 = B(r)dt^2 - A(r)dr^2 - r^2 d\theta^2 - r^2 sin^2\theta d\phi^2$$

- We parametrise:

$$A = \left(1 - \frac{2m(r)}{r}\right)^{-1}$$

- The dominant energy condition demands simply:

$$\rho \geq 0$$

$$\rho \geq |p|$$

- while the Einstein equations are:

$$\frac{B''}{2A} - \frac{B'}{4A}\left[\frac{B'}{B} + \frac{A'}{A}\right] + \frac{B'}{rA} = 4\pi(3p + \rho)B$$

$$-\frac{B''}{2B} + \frac{B'}{4B}\left[\frac{B'}{B} + \frac{A'}{A}\right] + \frac{A'}{rA} = 4\pi(p - \rho)A$$

$$1 - \frac{r}{2A}\left[\frac{B'}{B} - \frac{A'}{A}\right] - \frac{1}{A} = 4\pi(p - \rho)r^2$$

- Normally the Einstein equations are augmented by an equation of state that links the pressure to the density, yielding four equations in the four functions $A(r)$ $B(r)$ $p(r)$ and $\rho(r)$.
- However we will take a different approach. Choosing:

$$m(r) = \begin{cases} 0 & \text{if } r < x \\ a(r-x)^3 & \text{if } x < r < y \\ \frac{\lambda r^3}{6} - M & \text{if } r > y \end{cases}$$

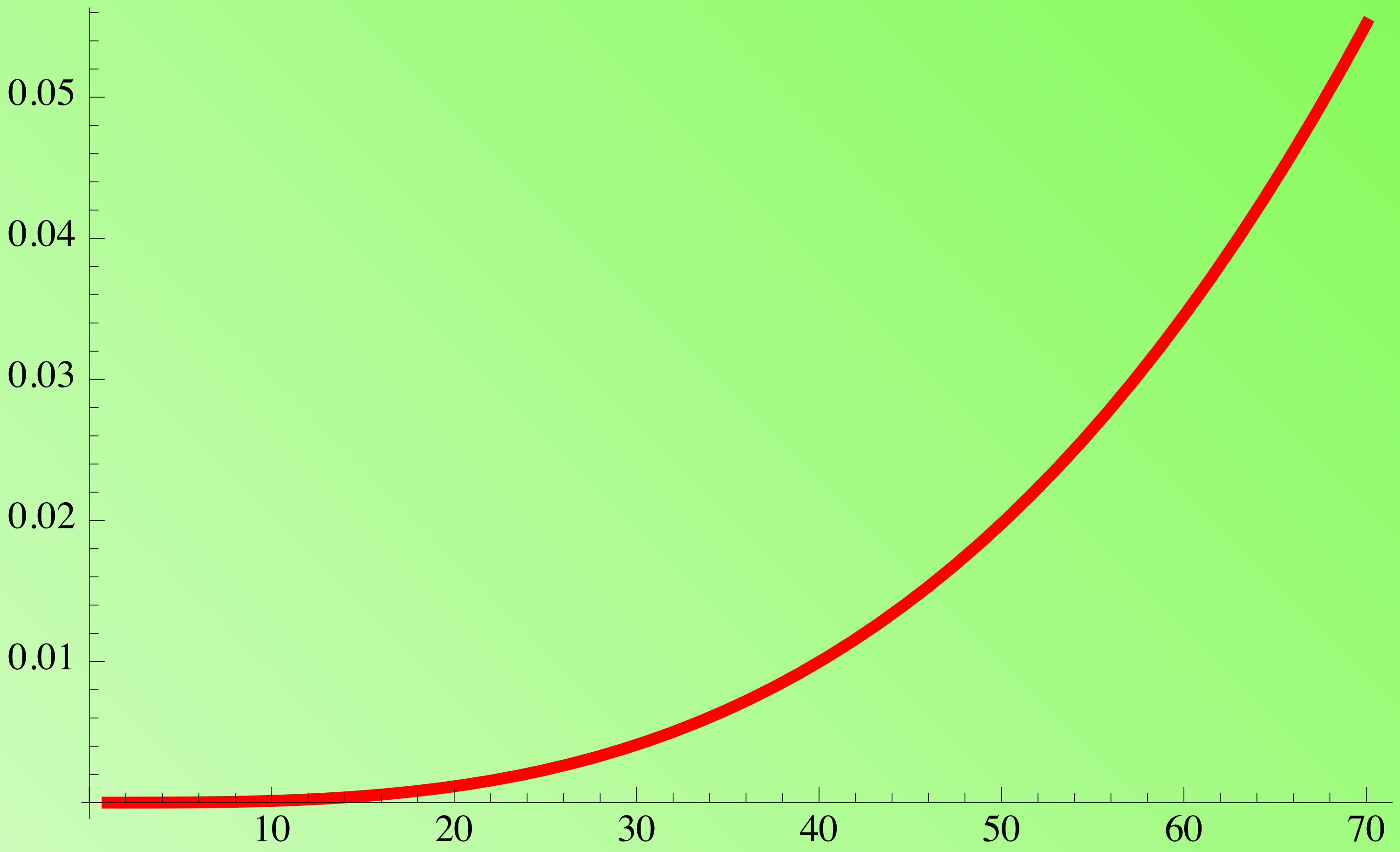
- This fixes $A(r)$ and the equations then uniquely give us the remaining variables.
- In this way, the required equation of state is uncovered rather than imposed.
- The Einstein equations become:

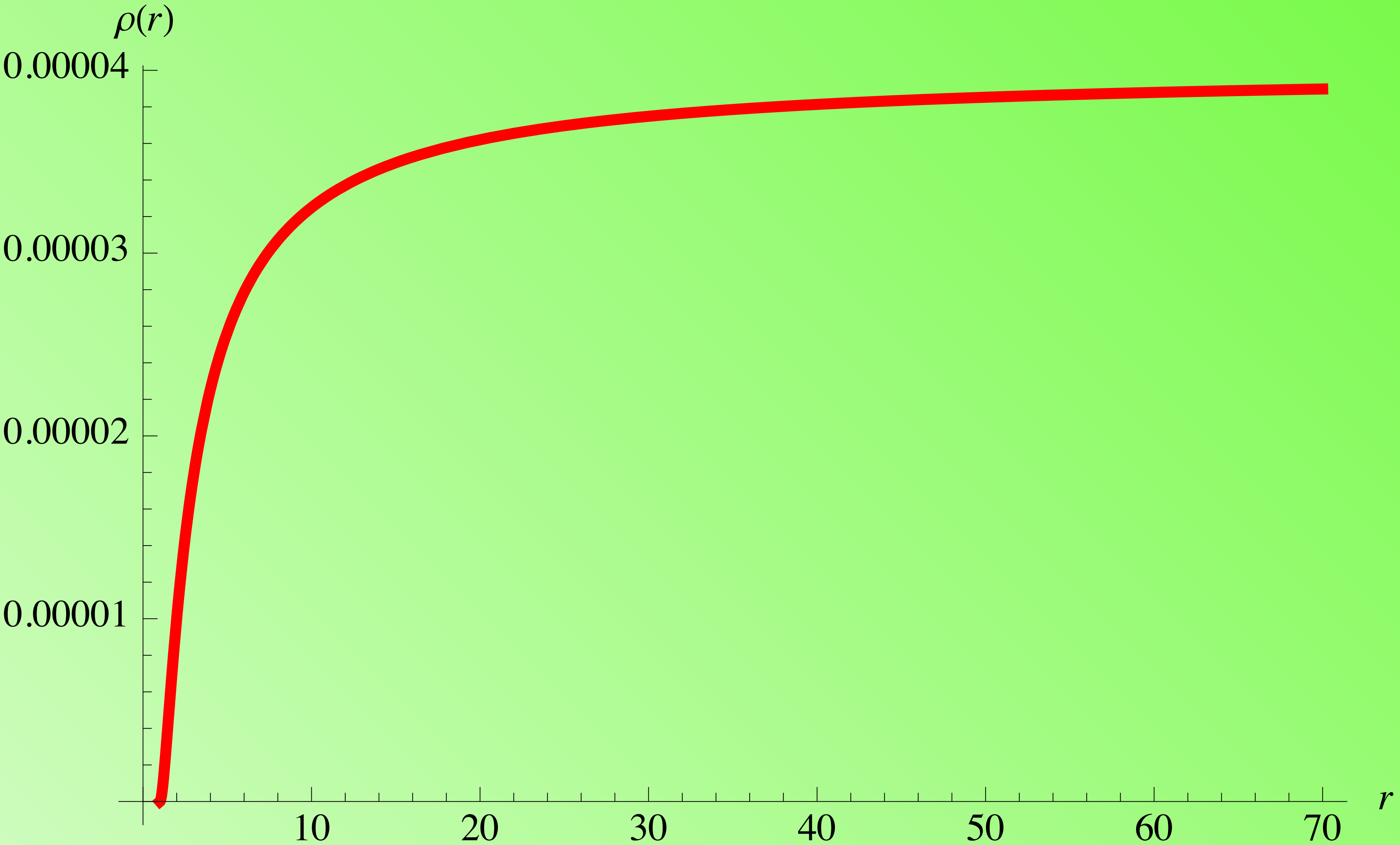
$$\rho(r) = \frac{1}{8\pi}\left(\frac{m'(r)}{r^2}\right)$$

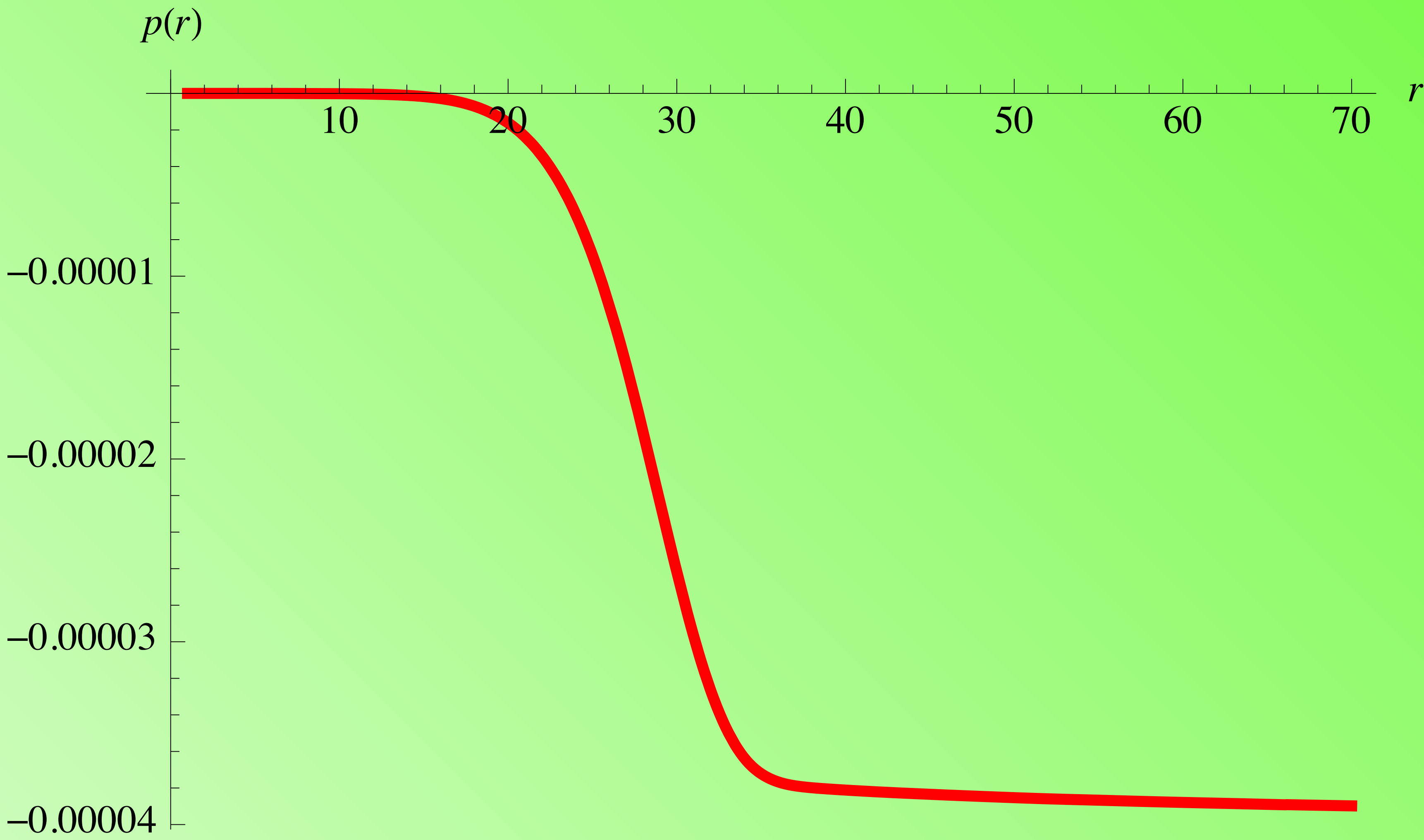
$$p'(r) = \frac{4\pi}{2m(r)-r}p^2(r) + \frac{m(r)+rm'(r)}{r\left(2m(r)-r\right)}p(r) + \\ + \frac{m'(r)m(r)}{4\pi r^3\left(2m(r)-r\right)}$$

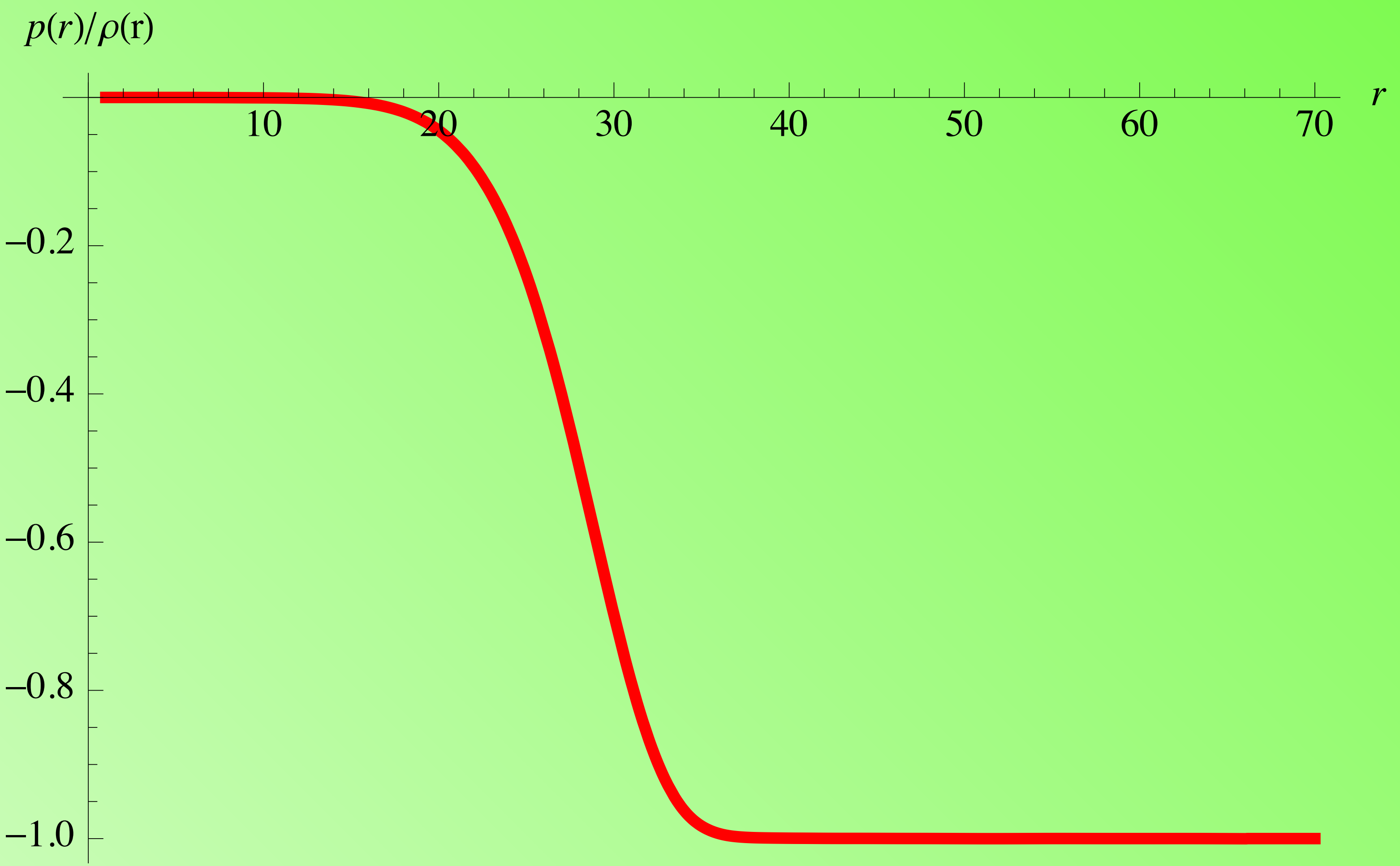
$$(\ln AB)' = 8\pi G\left(\rho(r)+p(r)\right)rA$$

- We integrate these equations numerically for some uninspired values of the parameters and find: $x = .1 \; G = 1, \; \Lambda = .001, \; M = .01, y \approx 245$

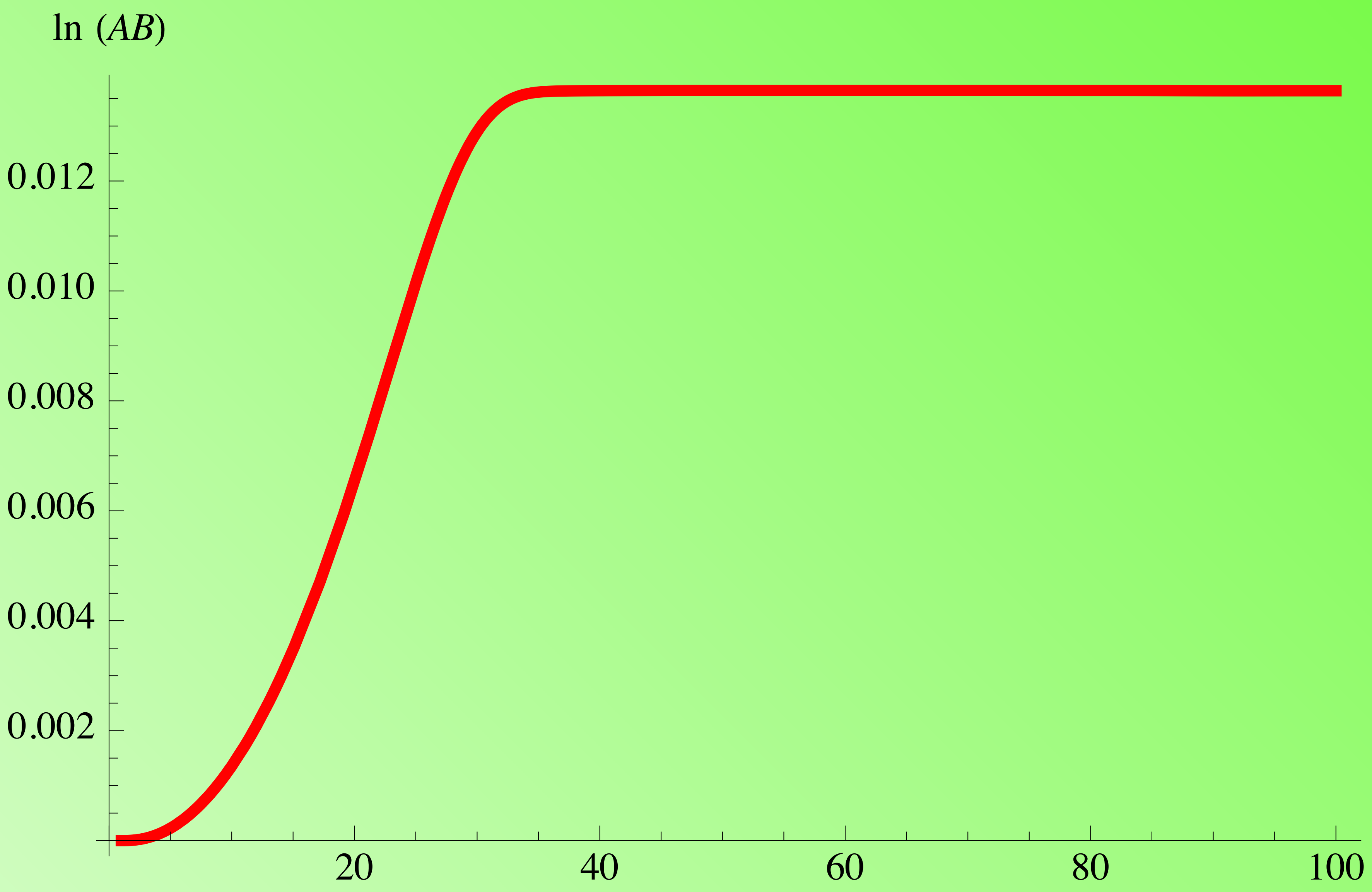



ρ(r)
0.00004
0.00003
0.00002
0.00001
10
20
30
40
50
60
70
r


p(r)
r
10
20
30
40
50
60
70
−0.00001
−0.00002
−0.00003
−0.00004



$p(r)/\rho(\mathrm{r})$
r
10
20
30
40
50
60
70
−0.2
−0.4
−0.6
−0.8
−1.0



ln (*AB*)
0.012
0.010
0.008
0.006
0.004
0.002
20
40
60
80
100
r

Stable negative mass solutions

- It would still be desirable to find dynamically stable solutions that give rise to negative mass.
- Ideally this would be obtained from some kind of non-standard matter which obeys an equation of state, but satisfies the dominant energy condition.
- We have not been able to find such an equation of state.
- However, we have been able to find stable, thin wall solutions. These correspond to spacetimes where and exterior negative mass Schwarzschild-de Sitter spacetime is separated from an interior spacetime by a thin wall. The Israel junction conditions are imposed at the wall. The source is inside the wall, and is imposed to satisfy the dominant energy condition everywhere.

Thin wall bubbles

- The Israel junction conditions guarantee the conservation of energy and momentum across a boundary.

- The induced metric is

$$h_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b = -\mathrm{d}\tau^2 + a(\tau)^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

- For a spherical wall at

$$r = a(t)$$

- The wall stress energy tensor is

$$S_{\hat{a}\hat{b}} = \mathrm{diag}(\sigma, -\vartheta, -\vartheta)$$

- and one gets

$$[[\sqrt{1 - 2m(a)/a}\, a^{-1}]] = -4\pi\sigma$$

- For the dynamic case, a slightly more complicated analysis yields

$$\sigma = -\frac{1}{4\pi a}[[\sqrt{1 - 2m(a)/a + \dot{a}^2}]]$$

- which can be rewritten as

$$\sqrt{1 - 2m_+(a)/a + \dot{a}^2} = \sqrt{1 - 2m_-(a)/a + \dot{a}^2} - 4\pi\sigma(a)a$$

- after squaring twice

$$m_s{}^2(1 + \dot{a}^2) + 4m_+m_- = \left[\frac{m_s^2}{2a} + (m_+ + m_-)\right]^2$$

- where the mass of the thin shell is defined as

$$m_s = 4\pi\sigma a^2$$

- This equation has the form of an energy equation for a non-relativistic particle

$$\frac{1}{2}\dot{a}^2 + V(a) = E$$

$$V(a) = \frac{1}{2}\left\{1 + \frac{4m_+(a)m_-(a)}{m_s^2(a)} - \left[\frac{m_s(a)}{2a} + \frac{(m_+(a) + m_-(a))}{m_s(a)}\right]^2\right\}$$

- however with vanishing "energy": $E = 0$
- There will be stable, static solutions if we can find a radius a_0 where

$$V(a_0) = 0, \qquad V'(a_0) = 0, \qquad V''(a_0) > 0$$

- We take the exterior mass function given by the negative mass Schwarzschild-de Sitter metric: $m_+(r) = -2M + \frac{\Lambda r^3}{3}$
- Solving for $m_-(r)$ we have

$$m_- = -8\pi^2\sigma^2 r^3 + -M + \frac{\Lambda r^3}{6} + 4\pi\sigma r^2 \sqrt{1 - 2V - \frac{2}{r}\left(-M + \frac{\Lambda r^3}{6}\right)}$$

- Now the aim is to pick the potential so that it has an exact, double zero, which fixes the radius of the bubble, but so that the interior mass satisfies the dominant energy condition.

- Defining $-2V = \frac{1+\tilde{V}}{\sigma^2 r^4}$ and rescaling the constants
- which gives

$$m_- = M\left(-1 + \tfrac{1}{2}\left(\Lambda - \sigma^2\right) r^3 + \sqrt{1 + \tilde{V} + \sigma^2 r^4 + 2\sigma^2 r^3 - \Lambda \sigma^2 r^6}\right)$$

- and writing

$$(1+U)^2 = 1 + \tilde{V} + \sigma^2 r^4 + 2\sigma^2 r^3 - \Lambda \sigma^2 r^6$$

- we have $m_- = M\left(\tfrac{1}{2}\left(\Lambda - \sigma^2\right) r^3 + U\right)$
- the dominant energy condition requires

$$\frac{d}{dr}\left(\frac{m'_-(r)}{r^2}\right) \leq 0 \qquad \frac{d}{dr}\left(m'_-(r) r^2\right) \geq 0$$

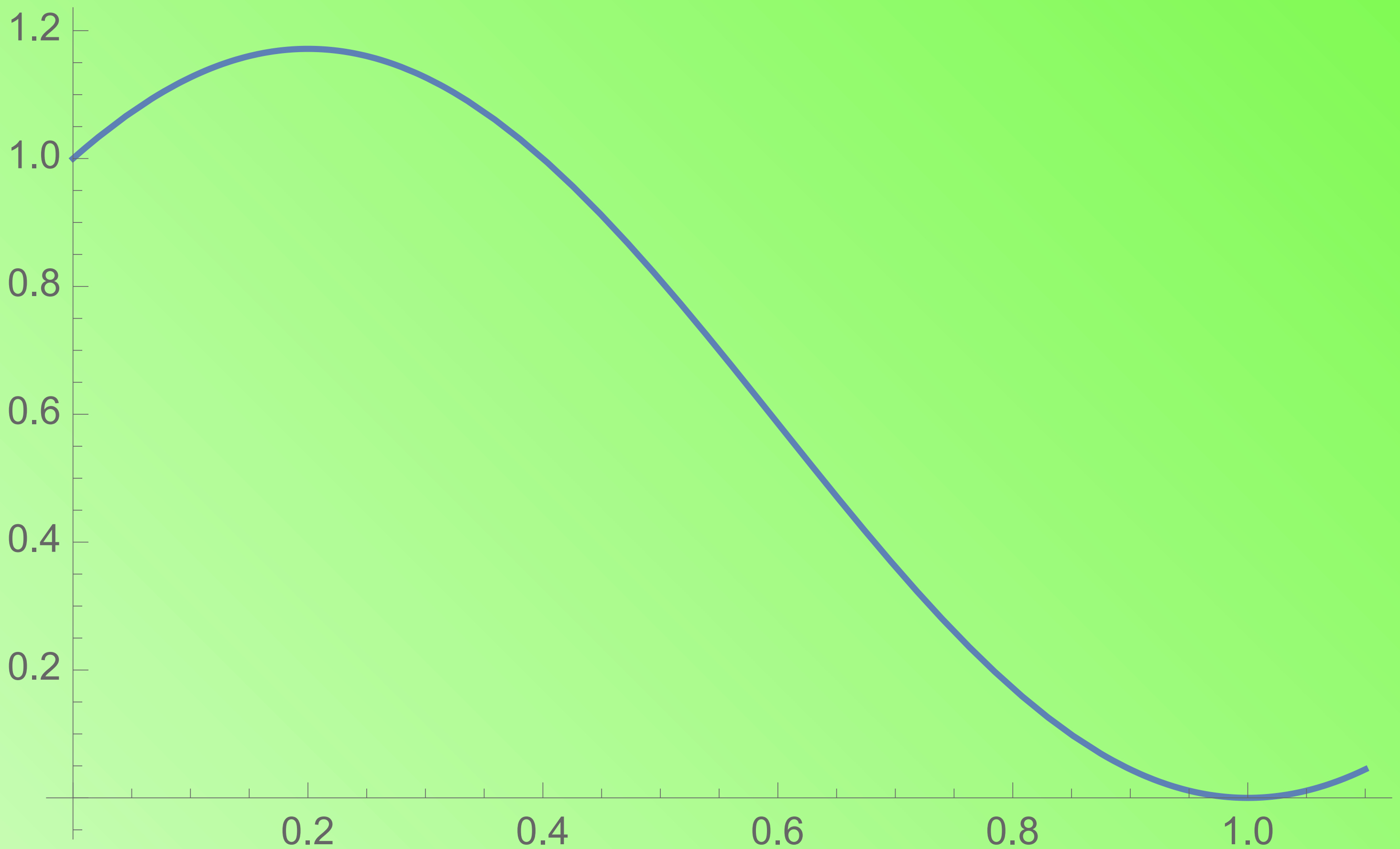
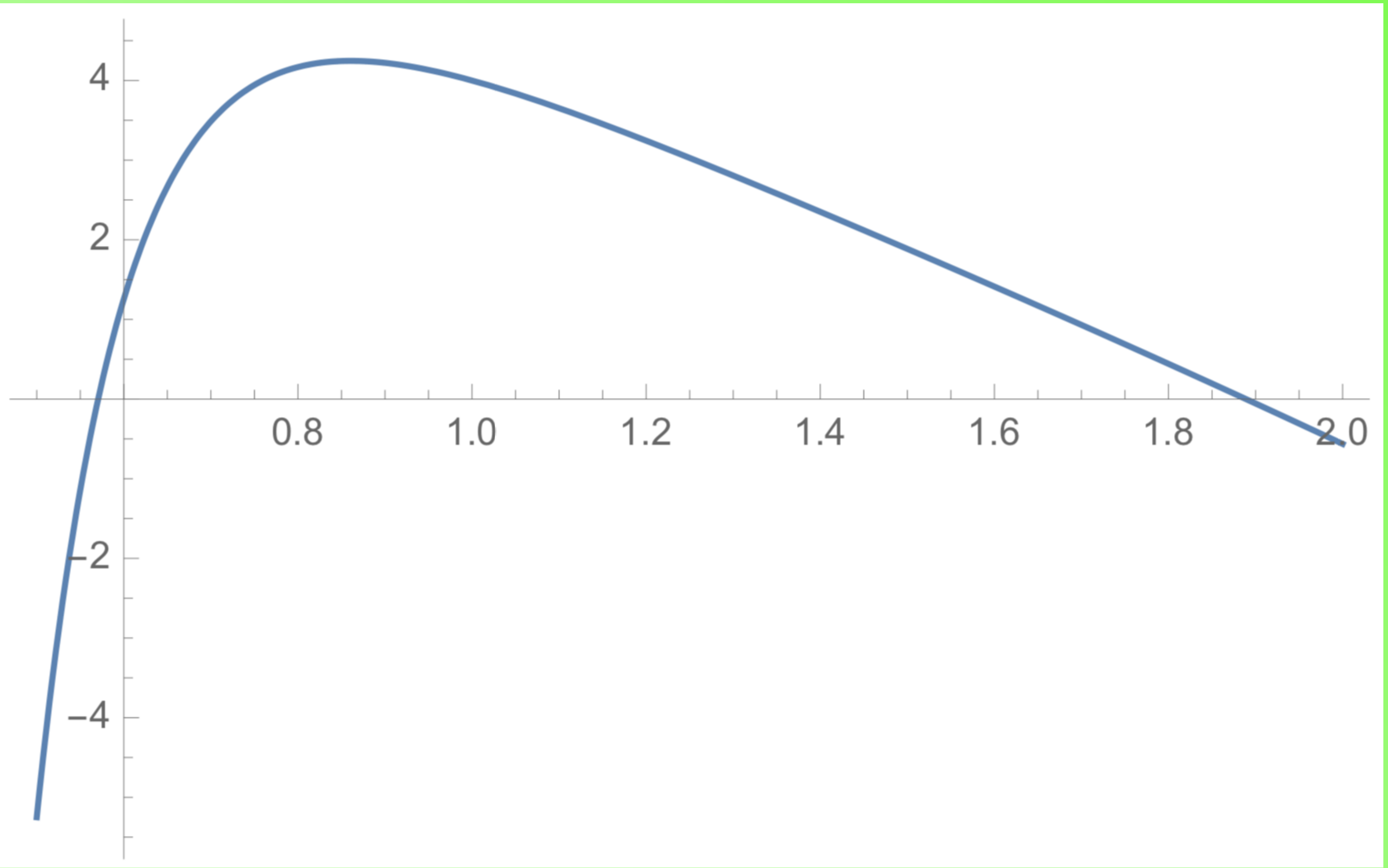
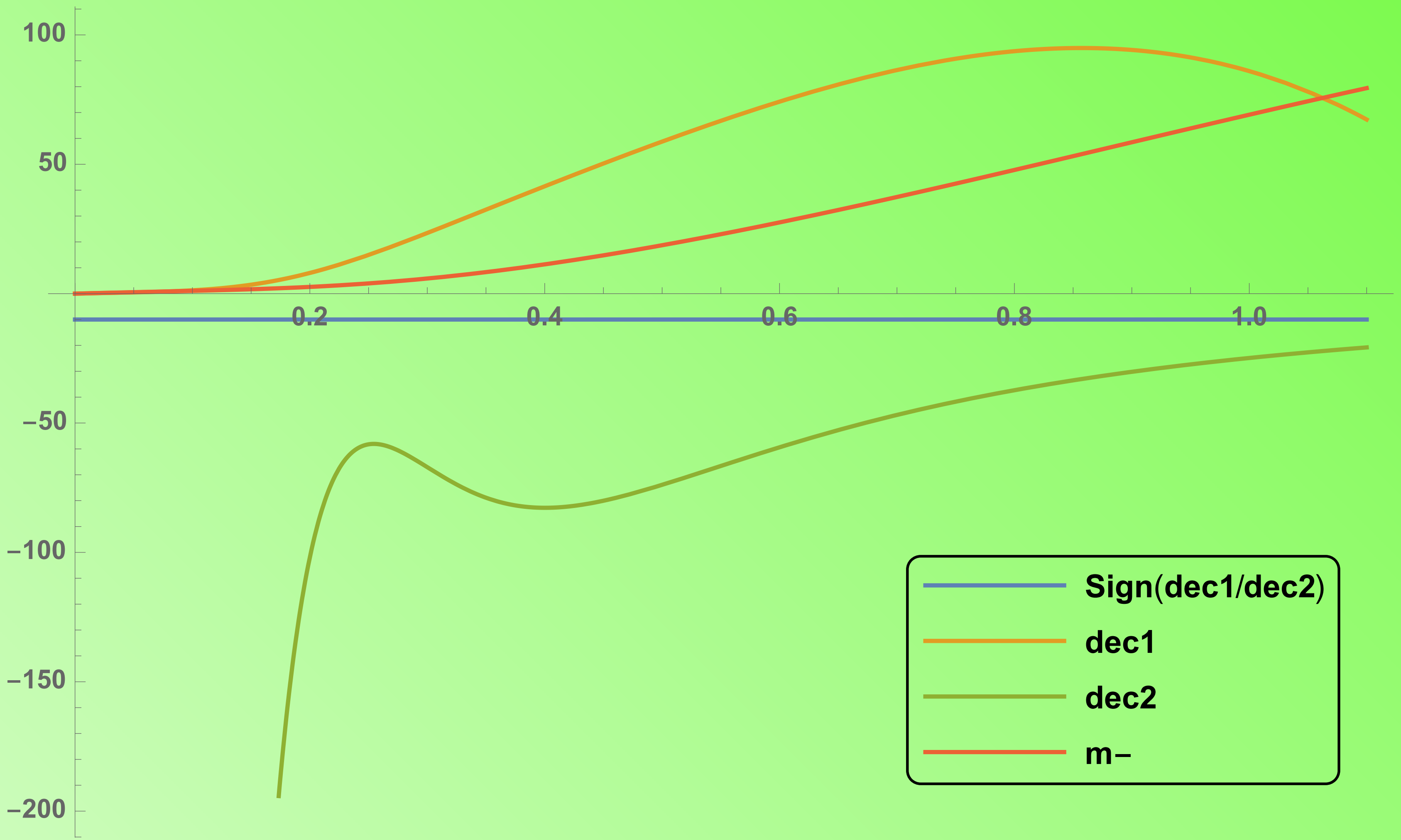


FIG. 2: (color online) The potential $1 + \tilde{V}(r)$.

- Potential for generic values of the parameters.



- Numerical experimentation gives the solution



Conclusions and Speculations

- We have shown that negative mass configurations which everywhere satisfy the dominant energy condition, can exist within non-asymptotically flat space-times.
- This could have important consequences for the early universe, where the inflationary phase corresponds to a de Sitter universe.
- Pair production of positive and negative mass pairs would give rise to a strange gravitational plasma.
- The negative mass particles would chase after the positive mass particles and in principle always exit any Hubble volume. However, for an infinite universe, there would always be such pairs entering the Hubble volume from outside, so the entire system would be stable.

- A gas/plasma of positive and negative mass particles would have strange damping properties.
- It could damp out gravitational waves!
- We speculate that such a plasma might screen gravitational waves rendering the initial singularity always hidden behind an opaque curtain.

Real negative mass as opposed to relative negative mass

- There has been criticism that the mass we are discussing is somehow not real.
- The reason is that a trivial analysis indicates that for even spin messenger fields, like charges attract (while for odd spin, they repel)
- This is based on the propagator of the messenger field and the force between like charges

- thus the potential between sources is given by

$$\begin{aligned} V &\sim \int d^3x d^3y J(x) J(y) \langle \phi(x) \phi(y) \rangle \\ &\sim \int d^3x d^3y J_\mu(x) J_\nu(y) \langle A^\mu(x) A^\nu(y) \rangle \\ &\sim \int d^3x d^3y T^{\mu\nu}(x) T^{\sigma\tau}(y) \langle h_{\mu\nu}(x) h_{\sigma\tau}(y) \rangle \end{aligned}$$

- However, for negative gravitational charges we must take $T^{\mu\nu}(x) \to -T^{\mu\nu}(x)$
- which grossly violates the dominant energy condition.

Cosmological constant

- We need to find a mechanism for the creation of positive-negative mass pairs (since the overall mass is conserved).
- There are exact solutions which correspond to black holes separated by struts, Weyl type metrics. A configuration of two black holes on either side of a negative mass solution would require no struts, or two negative mass solutions on either side of a black hole if M=1/4 (-m) or vice versa. Euclideanization of these solutions might give the appropriate Euclidean instanton.

- The pair creation would of course be controlled by the cosmological constant, and if there is any kind of Lenz's law behaviour, pair creation would tend to reduce the cosmological constant.
- Thus pairs would be created until the cosmological constant is essentially rendered zero, after which pairs can no longer be created.

Idle Speculation and Conclusions

- A negative-positive mass pair would accelerate away, the negative mass particle chasing the positive mass one. Latching on to a system like this would provide a kind of "warp drive" for fuel-less transport.
- Creating a negative mass particle is an exothermic process. One could mine unlimited energy by creating such particles, and then sending them away.
- Negative mass particles would make the ultimate armour piercing ordnance. The denser the material of the armour, the more the negative mass particle is attracted to it, and would destroy it upon contact. See:
- "Negative mass"

Richard T Hammond. Aug 6, 2013. 7 pp.
e-Print: **arXiv:1308.2683** **[gr-qc] |** **PDF**