Nuclear reactions important for astrophysics from *ab initio* theory

Theory Canada 14

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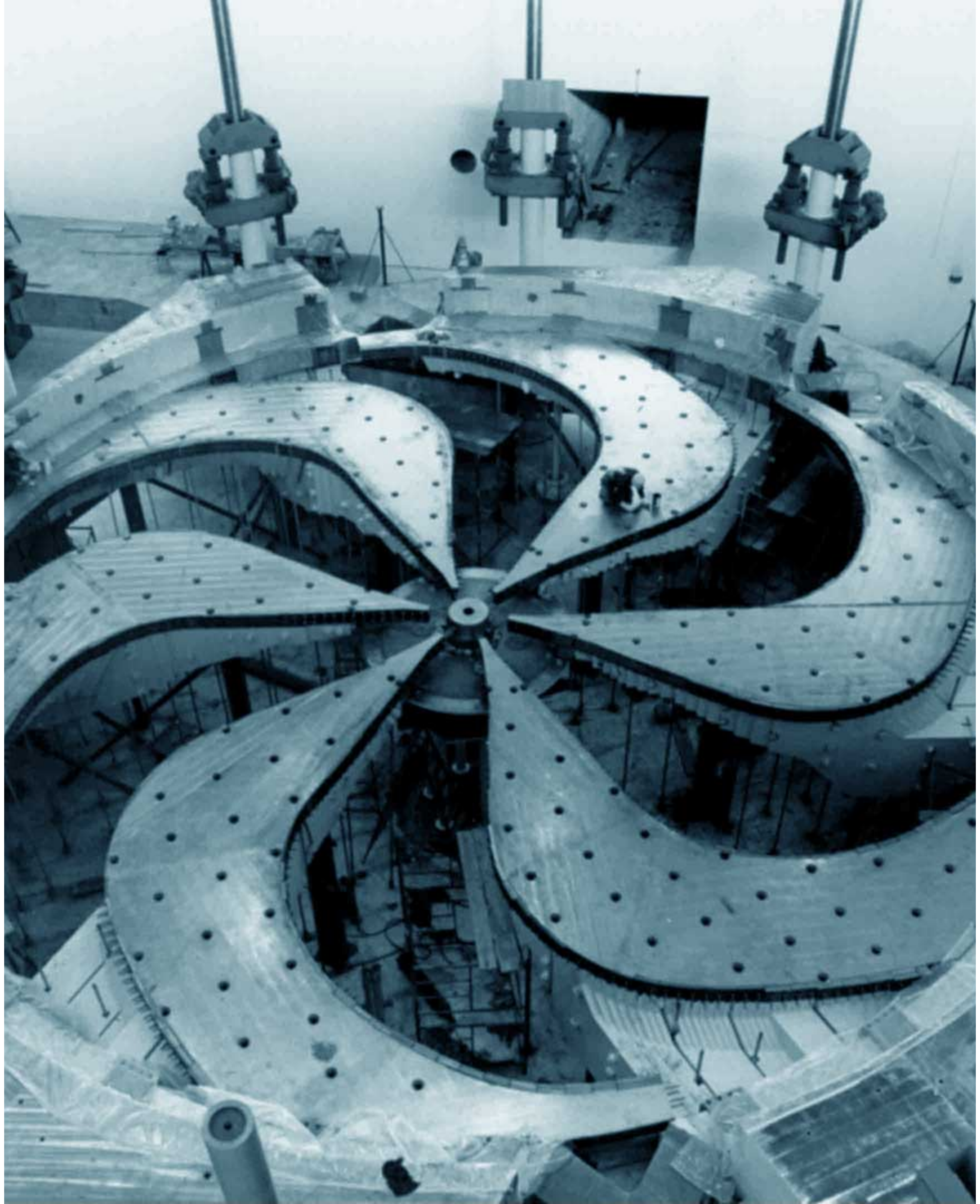
R. Roth (Darmstadt)

2019-05-30

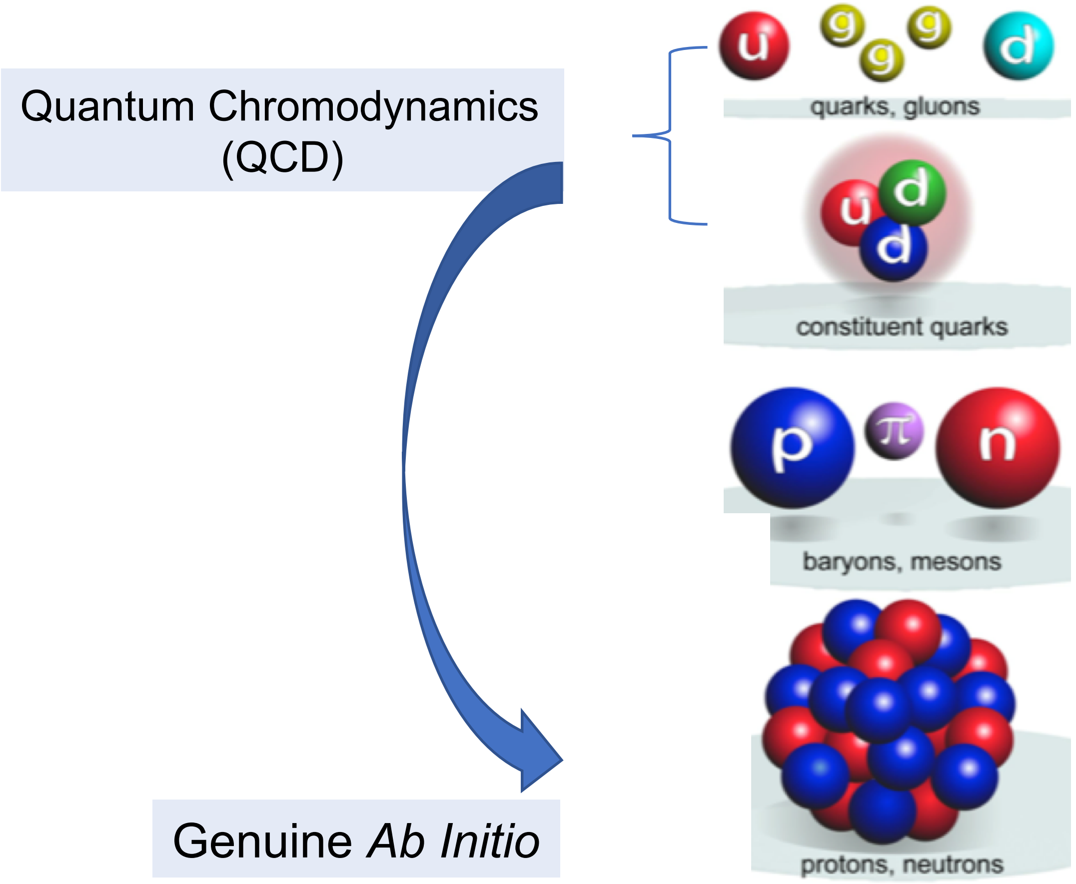


Discovery,
accelerated

Outline

- Introduction to *ab initio* No-Core Shell Model with Continuum (NCSMC)
- Polarized ^{3}H(d,n)^{4}He fusion
- ^{3}He(α,γ)^{7}Be & ^{3}H(α,γ)^{7}Li capture reactions
- ^{11}C(p, γ)^{12}N
- ^{14}C(n, γ)^{15}C

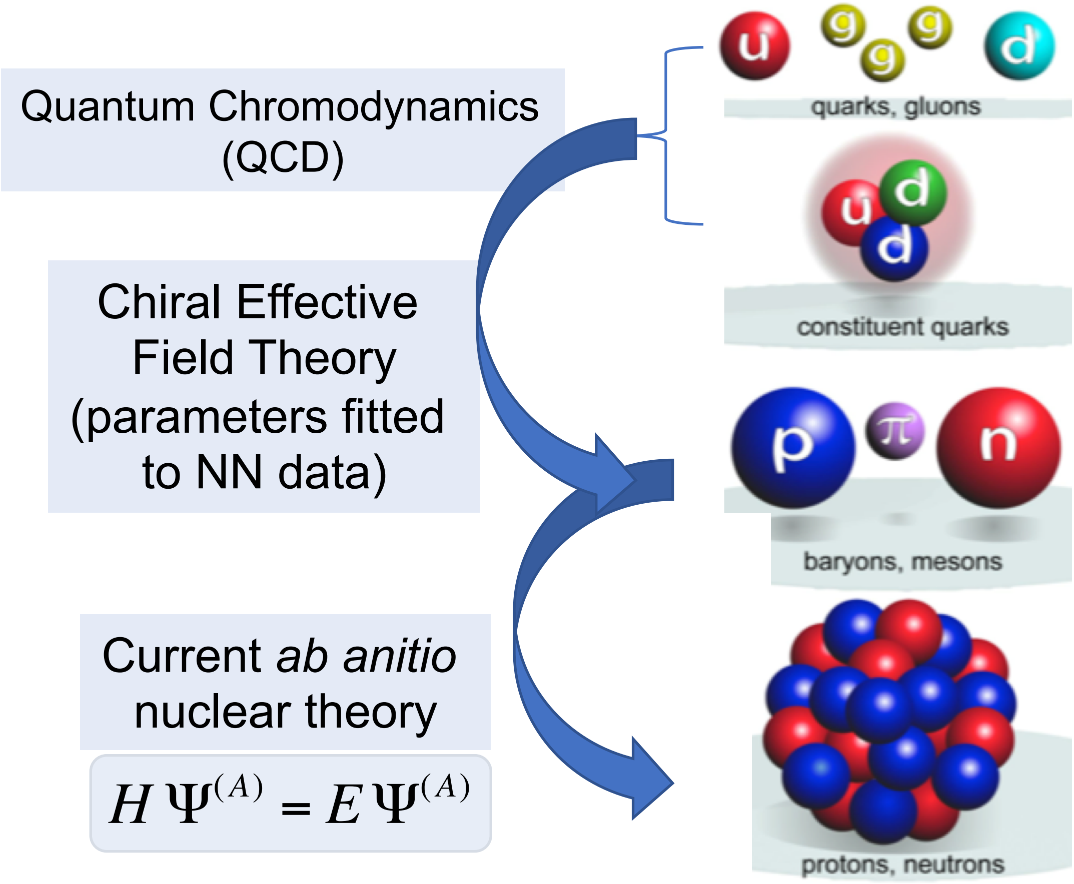
First principles or *ab initio* nuclear theory



Quantum Chromodynamics (QCD)

Genuine *Ab Initio*

First principles or *ab initio* nuclear theory – what we do at present



Quantum Chromodynamics (QCD)

Chiral Effective Field Theory (parameters fitted to NN data)

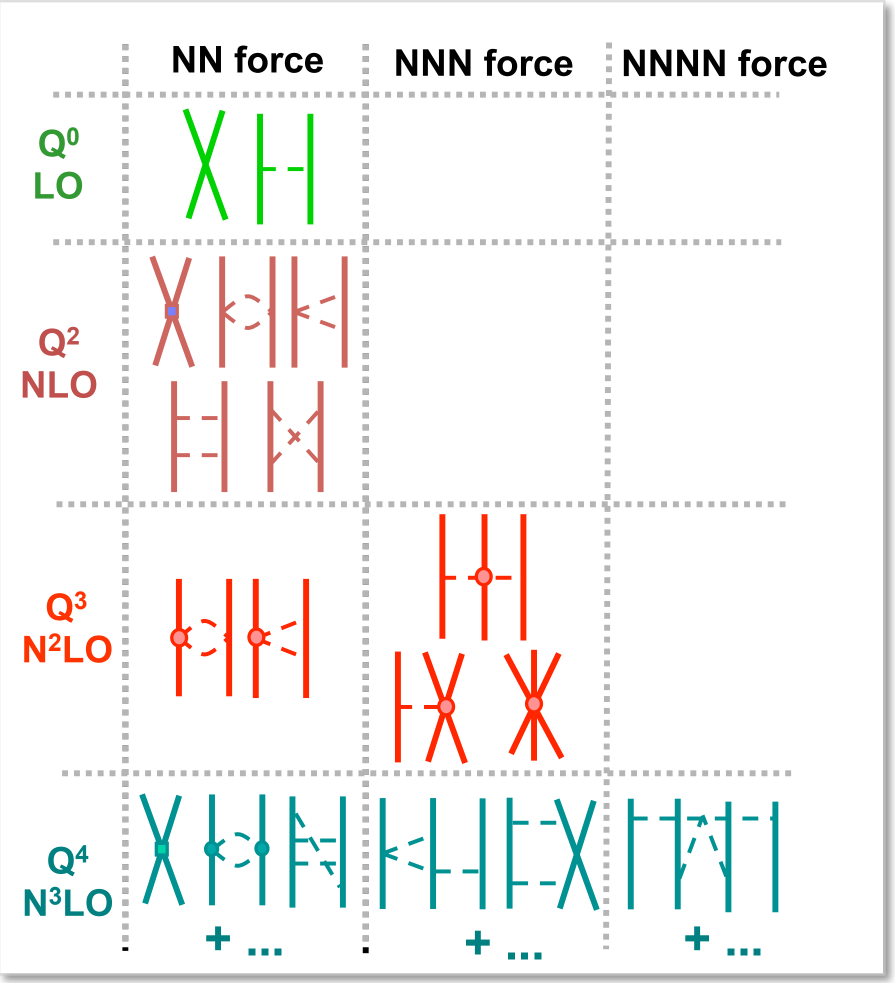
Current *ab anitio* nuclear theory

$$H\,\Psi^{(A)} = E\,\Psi^{(A)}$$

- *Ab initio*
- ✧ Degrees of freedom: Nucleons
- ✧ All nucleons are active
- ✧ Exact Pauli principle
- ✧ Realistic inter-nucleon interactions
  - ✧ Accurate description of NN (and 3N) data
- ✧ Controllable approximations

Chiral Effective Field Theory

- Inter-nucleon forces from chiral effective field theory
  - Based on the symmetries of QCD
    - Chiral symmetry of QCD ($m_u \approx m_d \approx 0$), spontaneously broken with pion as the Goldstone boson
    - Degrees of freedom: nucleons + pions
  - Systematic low-momentum expansion to a given order (Q/Λ_χ)
  - Hierarchy
  - Consistency
  - Low energy constants (LEC)
    - Fitted to data
    - Can be calculated by lattice QCD

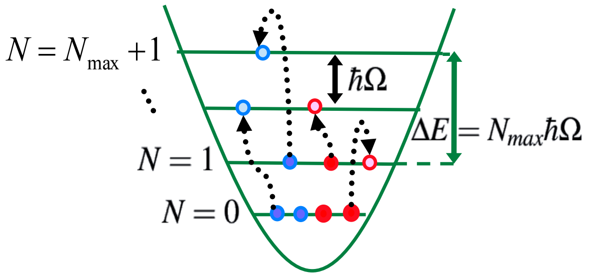


Λ_χ~1 GeV :
Chiral symmetry breaking scale

Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)

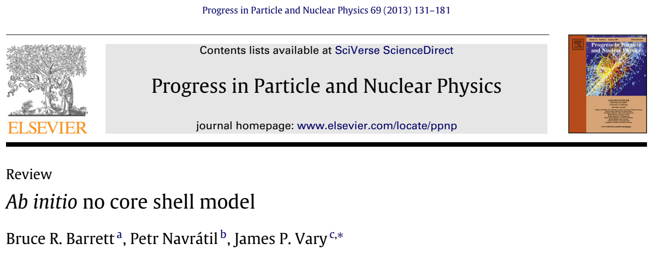
NCSM

- Basis expansion method
  - Harmonic oscillator (HO) basis truncated in a particular way ($N_{\max}$)
  - Why HO basis?
    - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 – ^{4}He, ^{16}O, ^{40}Ca)
    - Equivalent description in relative-coordinate and Slater determinant basis
- Short- and medium range correlations
- Bound-states, narrow resonances



$$(A) \quad \Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi^{HO}_{Ni}(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})$$

$$(A) \quad \Psi^A_{\mathrm{SD}} = \sum_{N=0}^{N_{\max}} \sum_j c^{\mathrm{SD}}_{Nj} \Phi^{HO}_{\mathrm{SD}\,Nj}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$



Progress in Particle and Nuclear Physics 69 (2013) 131–181

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Review

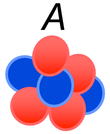
Ab initio no core shell model

Bruce R. Barrett[a], Petr Navrátil[b], James P. Vary[c,*]

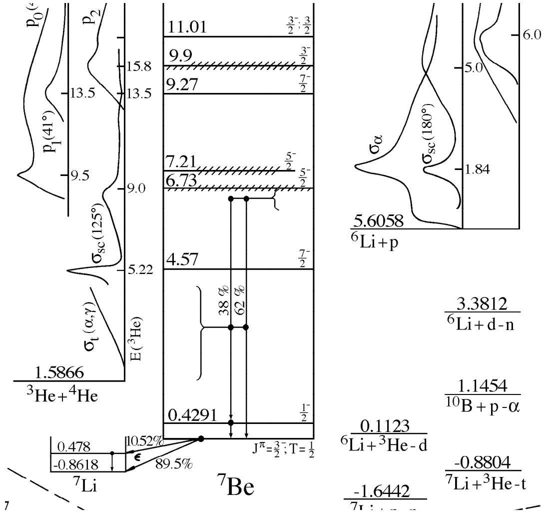
Extending no-core shell model beyond bound states

Include more many nucleon correlations…

NCSM →

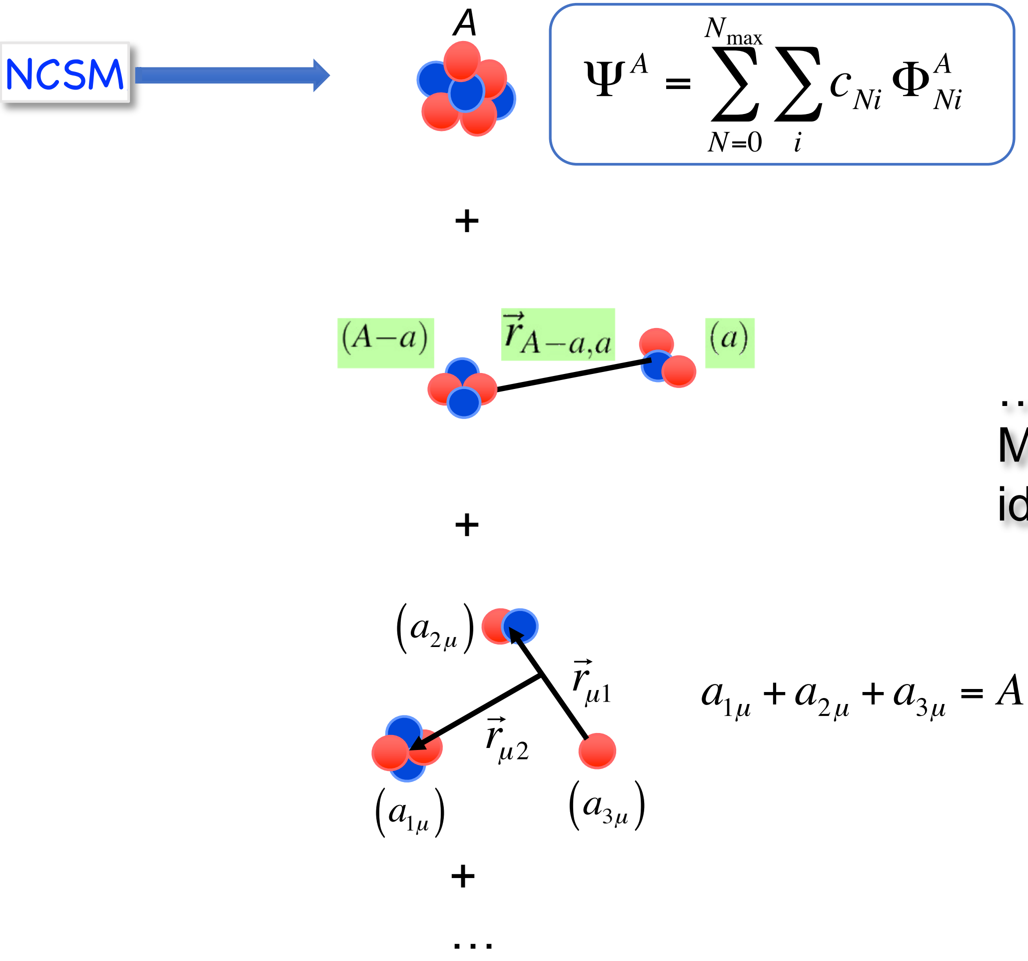


$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi^A_{Ni}$$



Extending no-core shell model beyond bound states

Include more many nucleon correlations…



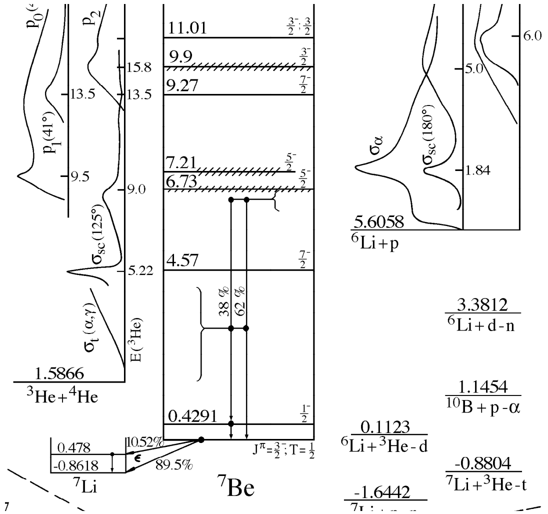
$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni}\,\Phi^A_{Ni}$$

$\vec{r}_{A-a,a}$

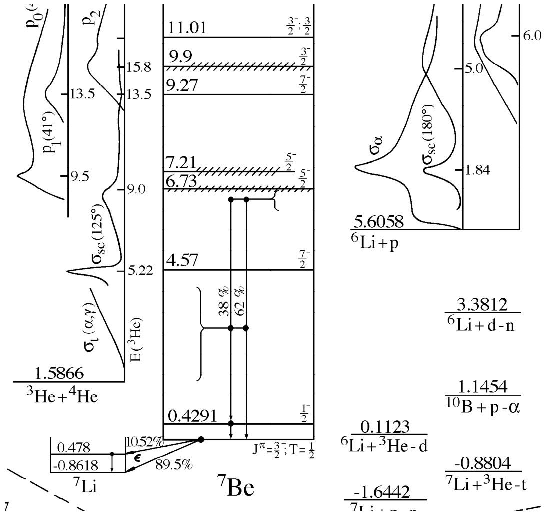
$(a_{2\mu})$ $\vec{r}_{\mu 1}$ $\vec{r}_{\mu 2}$ $(a_{1\mu})$ $(a_{3\mu})$

$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

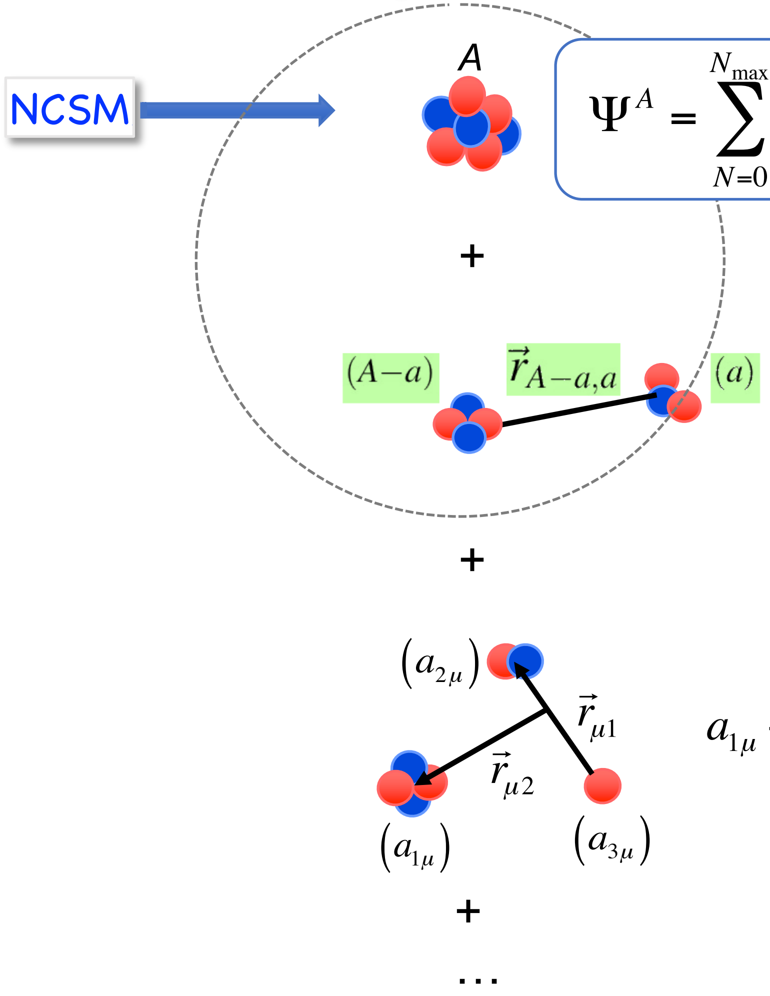
…using the Resonating Group Method (RGM) ideas



Extending no-core shell model beyond bound states



Include more many nucleon correlations…



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

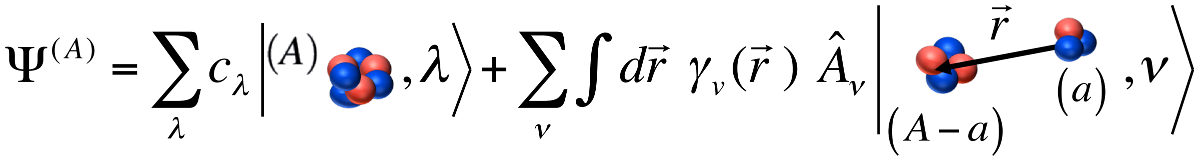
$$a_{1\mu} + a_{2\mu} + a_{3\mu} = A$$

…using the Resonating Group Method (RGM) ideas

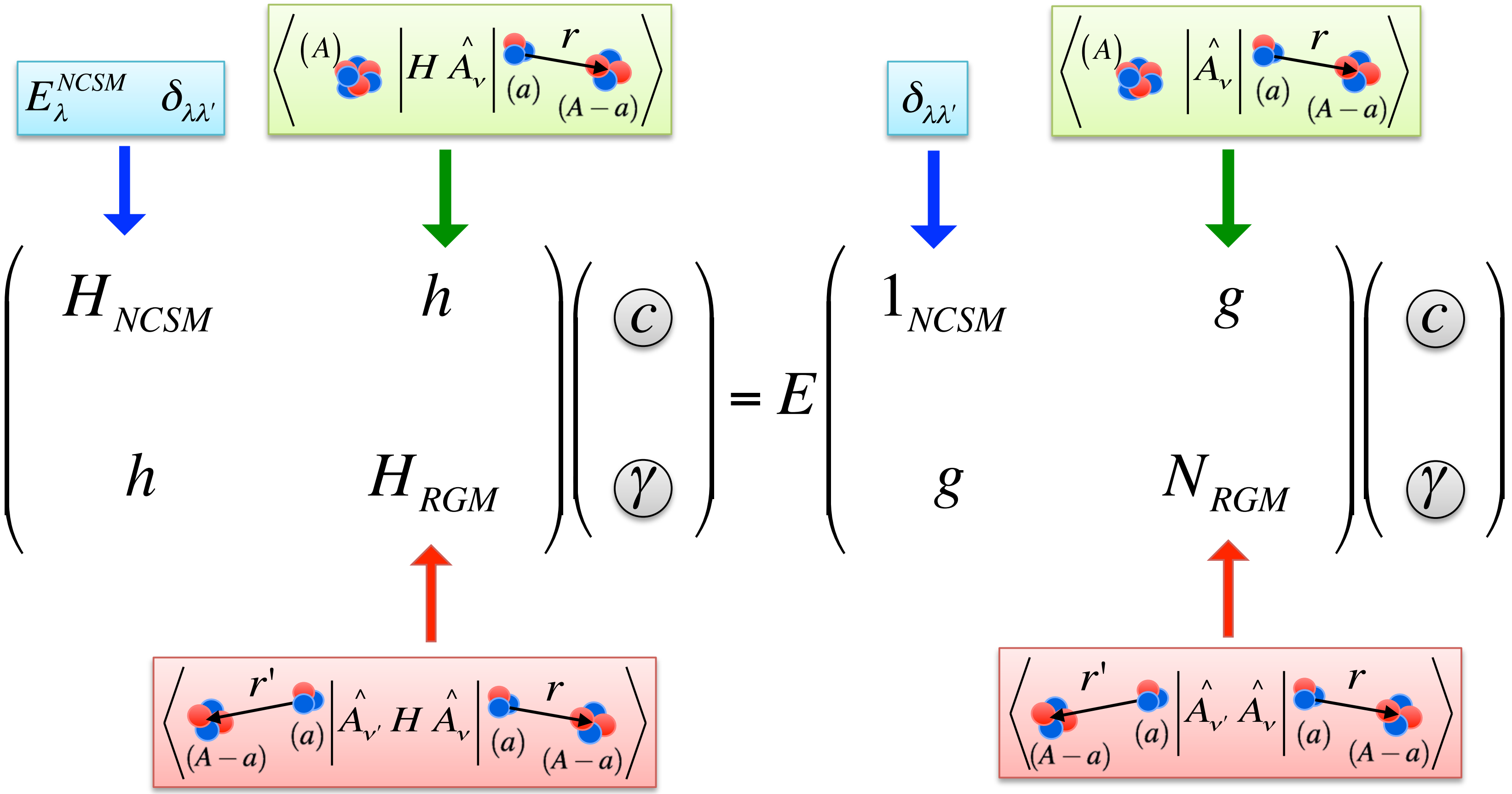
Coupled NCSMC equations

S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

$$H\,\Psi^{(A)} = E\,\Psi^{(A)}$$



$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left|{}^{(A)}, \lambda\right\rangle + \sum_{\nu} \int d\vec{r}\; \gamma_{\nu}(\vec{r})\; \hat{A}_{\nu} \left| \overset{\vec{r}}{(A-a)\quad(a)}, \nu \right\rangle$$



$$E_{\lambda}^{NCSM}\;\delta_{\lambda\lambda'}$$

$$\left\langle {}^{(A)} \right| H\,\hat{A}_{\nu} \left| \overset{r}{(a)\quad(A-a)} \right\rangle$$

$$\delta_{\lambda\lambda'}$$

$$\left\langle {}^{(A)} \right| \hat{A}_{\nu} \left| \overset{r}{(a)\quad(A-a)} \right\rangle$$

$$\begin{pmatrix} H_{NCSM} & h \\ h & H_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix} = E \begin{pmatrix} 1_{NCSM} & g \\ g & N_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix}$$

$$\left\langle \overset{r'}{(A-a)\quad(a)} \right| \hat{A}_{\nu'}\, H\, \hat{A}_{\nu} \left| \overset{r}{(a)\quad(A-a)} \right\rangle$$

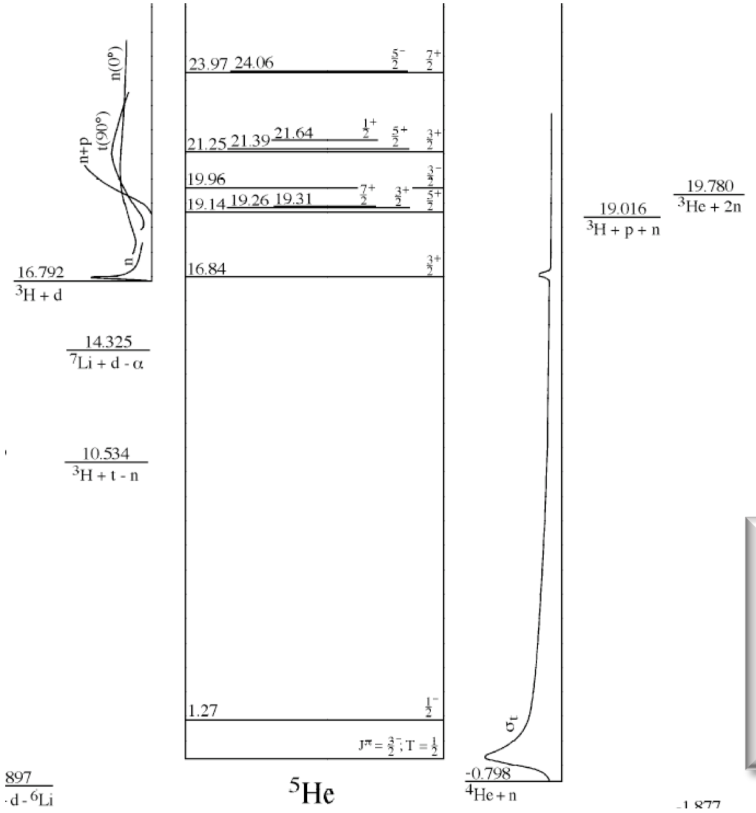
$$\left\langle \overset{r'}{(A-a)\quad(a)} \right| \hat{A}_{\nu'}\, \hat{A}_{\nu} \left| \overset{r}{(a)\quad(A-a)} \right\rangle$$

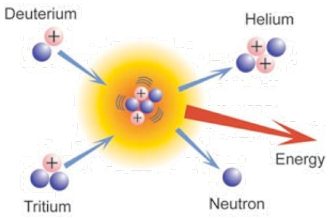
Solved by Microscopic R-matrix theory on a Lagrange mesh – efficient for coupled channels

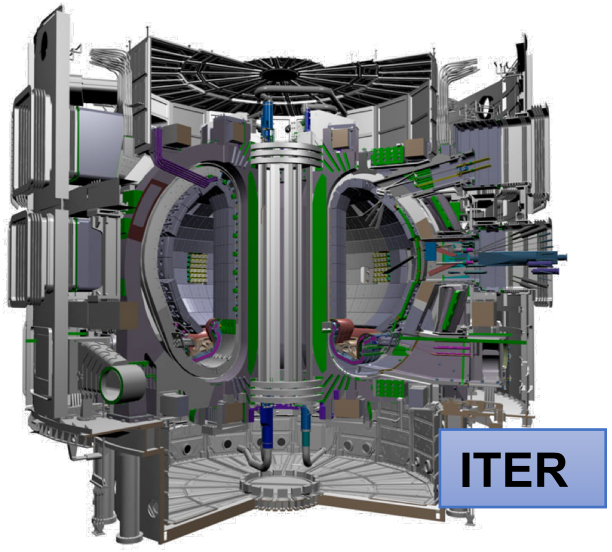
Deuterium-Tritium fusion

- The d+^{3}H$\rightarrow$$n$+^{4}He reaction
  - The most promising for the production of fusion energy in the near future
  - Used to achieve inertial-confinement (laser-induced) fusion at NIF, and magnetic-confinement fusion at ITER
  - With its mirror reaction, ^{3}He(d,p)^{4}He, important for Big Bang nucleosynthesis

NIF





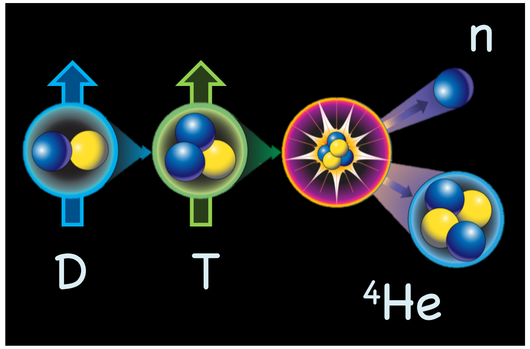


ITER

Resonance at E_{cm} =48 keV (E_d=105 keV) in the J=3/2^{+} channel
Cross section at the peak: 4.88 b

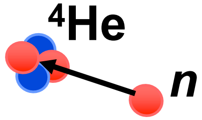
17.64 MeV energy released:
14.1 MeV neutron and 3.5 MeV alpha

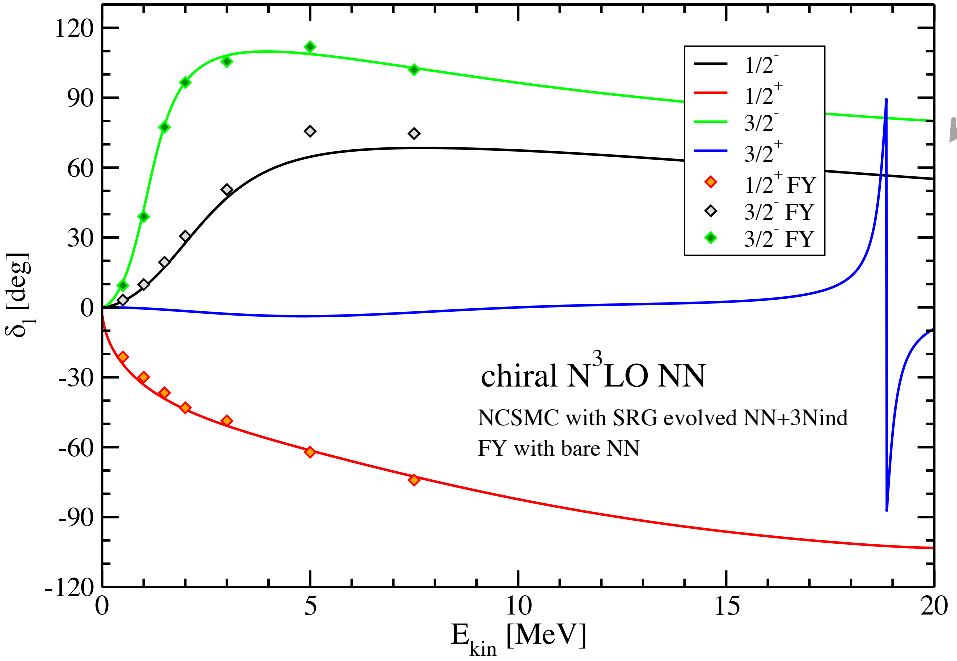
n-^{4}He scattering and ^{3}H+d fusion within NCSMC



^{4}He+n -> ^{3}H+d

n-^{4}He and *d*+^{3}H scattering phase-shifts



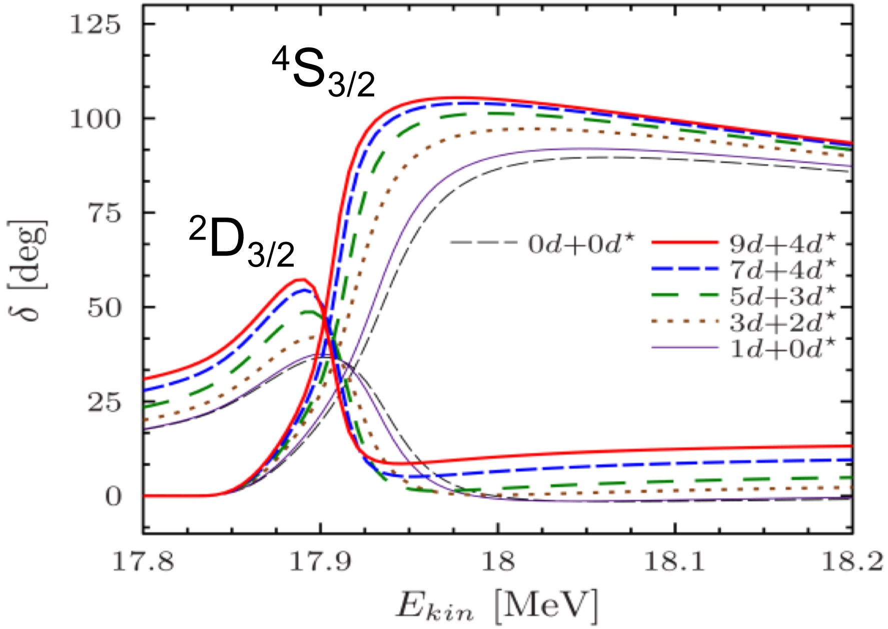


chiral N^{3}LO NN

NCSMC with SRG evolved NN+3Nind

FY with bare NN

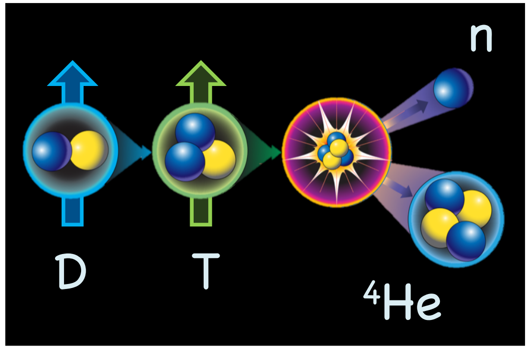
^{4}He+n



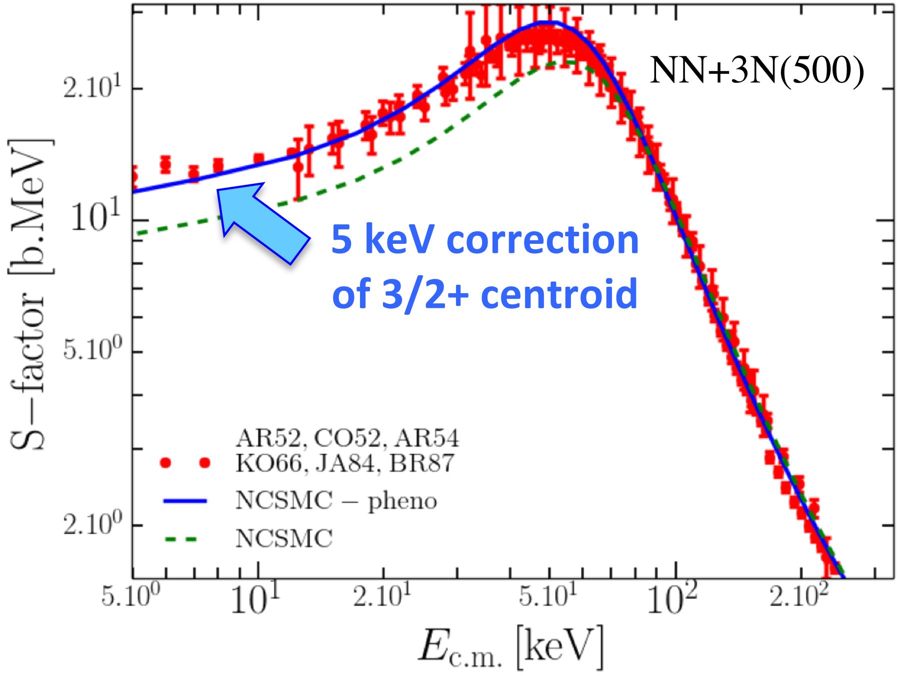
FY: Faddeev-Yakubovsky method - Rimantas Lazauskas

The *d*-^{3}H fusion takes place through a transition of *d*+^{3}H is *S*-wave to *n*+^{4}He in *D*-wave: Importance of the **tensor and 3N force**

^{3}H(d,n)^{4}He with chiral NN+3N(500) interaction



Astrophysical S-factor



5 keV correction of 3/2+ centroid

Assuming the fusion proceeds only in S-wave
with spins of D and T completely aligned:
Polarized cross section 50% higher than unpolarized

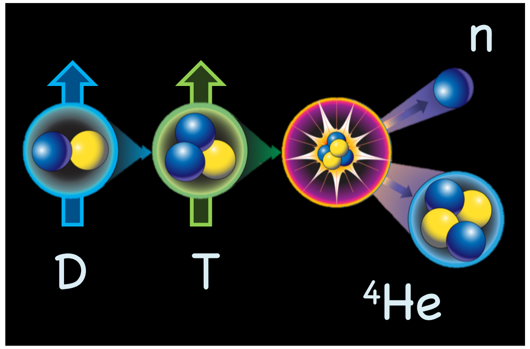
- While the DT fusion rate has been measured extensively, a fundamental understanding of the process is still missing
- Very little is known experimentally of how the polarization of the reactants' spins affects the reaction

$$\sigma_{unpol} = \sum_J \frac{2J+1}{(2I_D+1)(2I_T+1)}\sigma_J$$

$$\approx \frac{1}{3}\cancel{\sigma_{\frac{1}{2}}} + \frac{2}{3}\sigma_{\frac{3}{2}}$$

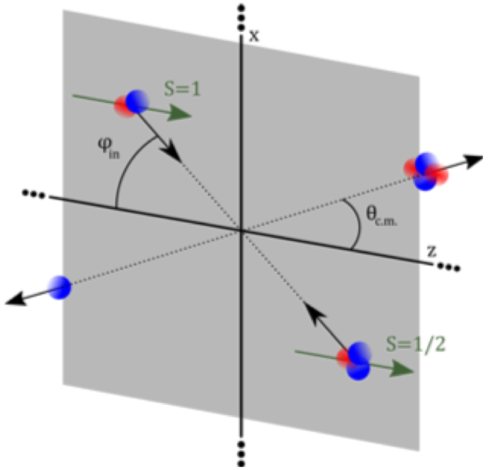
$$\sigma_{pol} \approx 1.5\,\sigma_{unpol}$$

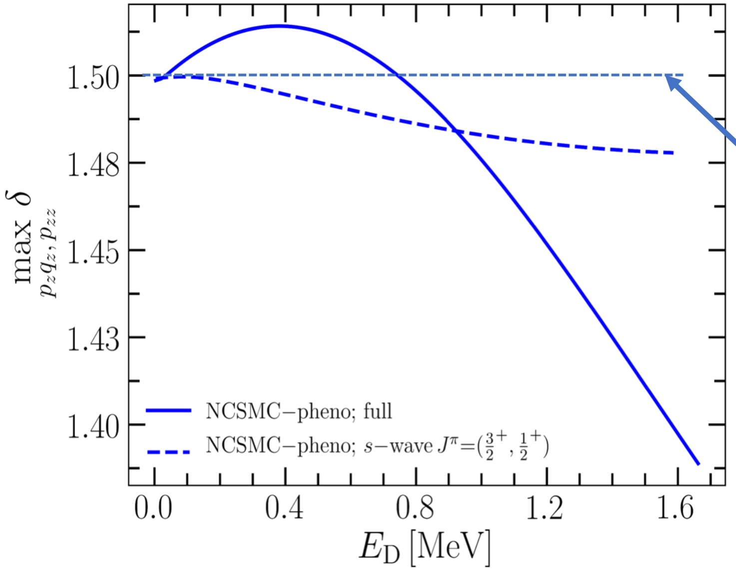
^{3}H(d,n)^{4}He with chiral NN+3N(500) interaction



Polarized fusion

$$\frac{\partial\sigma_{pol}}{\partial\Omega_{c.m.}}(\theta_{\text{c.m.}}) = \frac{\partial\sigma_{unpol}}{\partial\Omega_{c.m.}}(\theta_{\text{c.m.}})\left(1+\frac{1}{2}p_{zz}A_{zz}^{(b)}(\theta_{\text{c.m.}})+\frac{3}{2}p_z q_z C_{z,z}(\theta_{\text{c.m.}})\right)$$





$$\sigma_{unpol} = \sum_J \frac{2J+1}{(2I_D+1)(2I_T+1)}\sigma_J$$

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ARTICLE

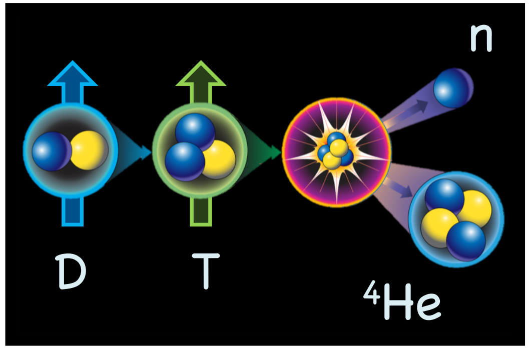
https://doi.org/10.1038/s41467-018-08052-6 OPEN

Ab initio predictions for polarized deuterium-tritium thermonuclear fusion

Guillaume Hupin[1,2,3], Sofia Quaglioni[3] & Petr Navrátil[4]

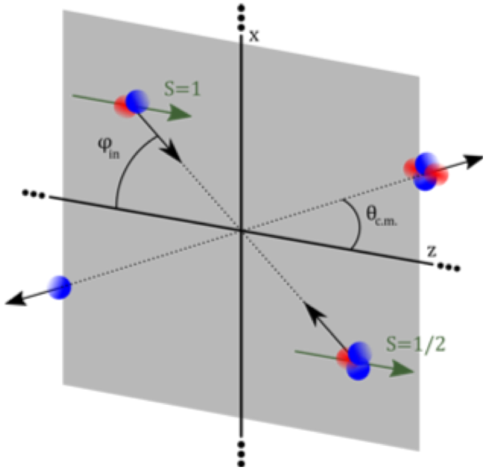
NCSMC calculation demonstrates impact of partial waves with $l > 0$ as well as the contribution of $l = 0$ $J^{\pi} = \frac{1}{2}^{+}$ channel

^{3}H(d,n)^{4}He with chiral NN+3N(500) interaction



Polarized fusion

$$\frac{\partial\sigma_{pol}}{\partial\Omega_{c.m.}}(\theta_{\text{c.m.}}) = \frac{\partial\sigma_{unpol}}{\partial\Omega_{c.m.}}(\theta_{\text{c.m.}})\left(1+\frac{1}{2}p_{zz}A^{(b)}_{zz}(\theta_{\text{c.m.}})+\frac{3}{2}p_z q_z C_{z,z}(\theta_{\text{c.m.}})\right)$$



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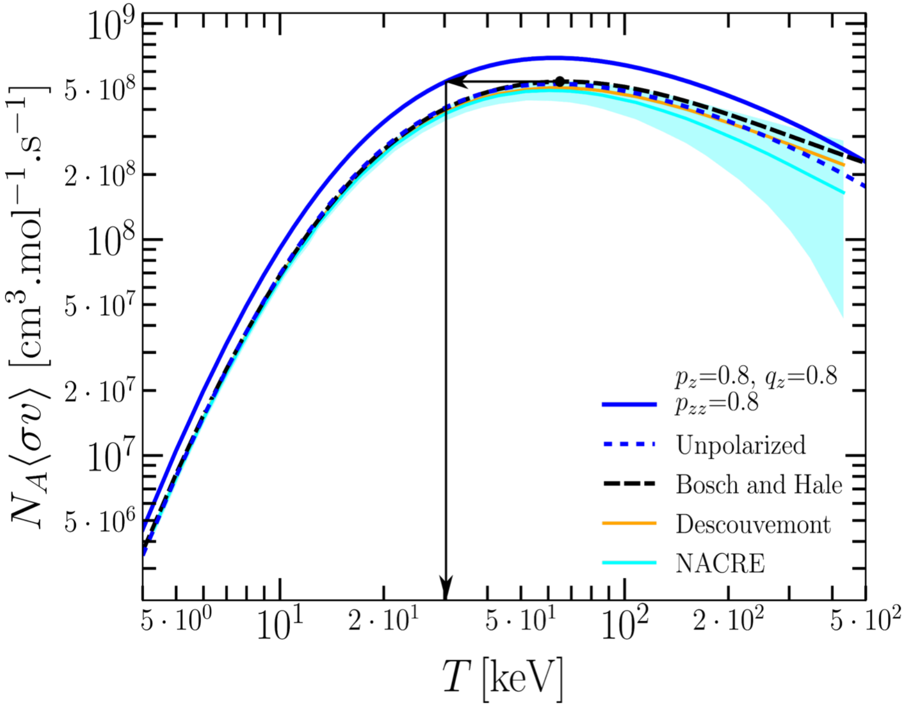
Ab initio predictions for polarized deuterium-tritium thermonuclear fusion

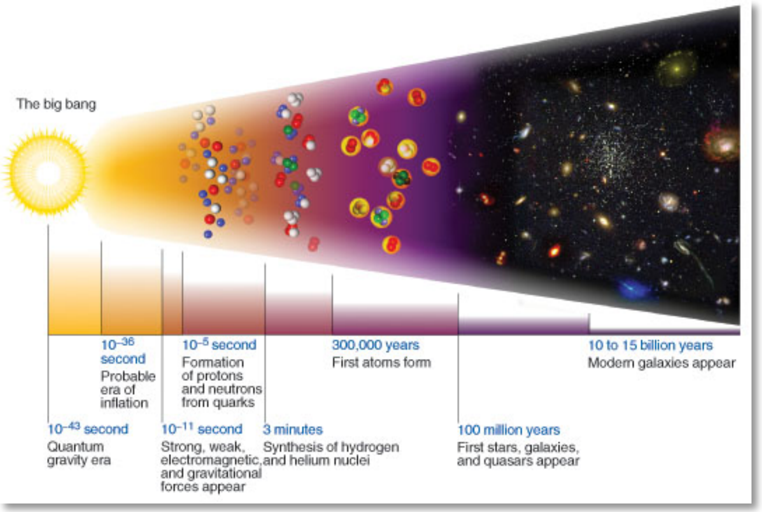
Guillaume Hupin[1,2,3], Sofia Quaglioni[3] & Petr Navrátil[4]

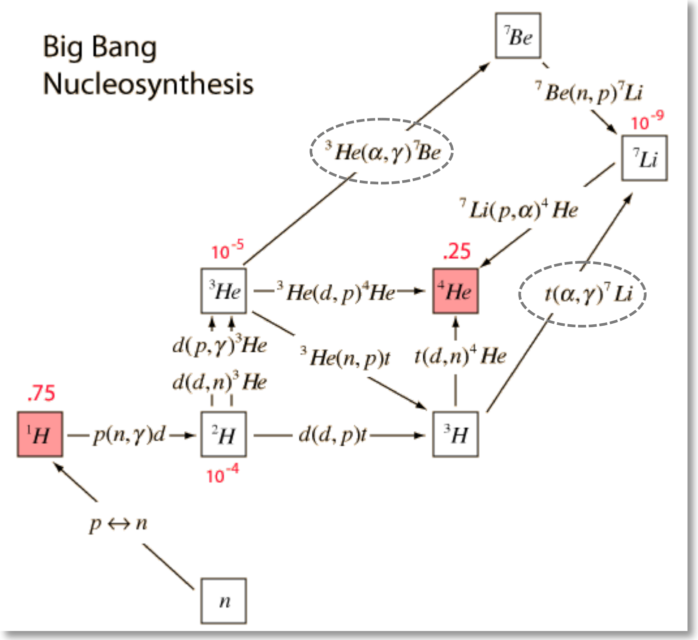


$$\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu(k_bT)^3}}\int_0^\infty S(E)\,exp\left(-\frac{E}{k_bT}-\sqrt{\frac{E_g}{E}}\right)dE,$$

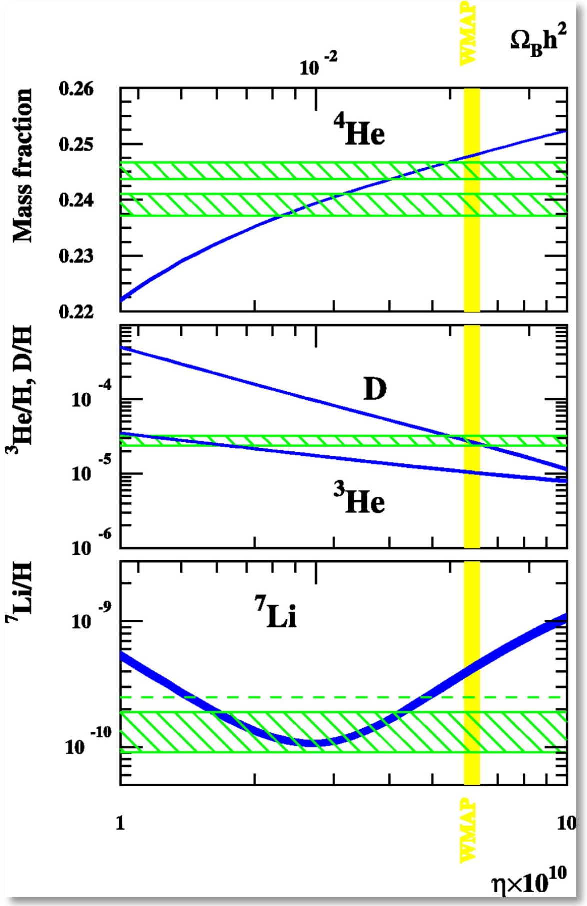
For a realistic 80% polarization, reaction rate increases by ~32% or the same rate at ~45% lower temperature

Big Bang nucleosynthesis



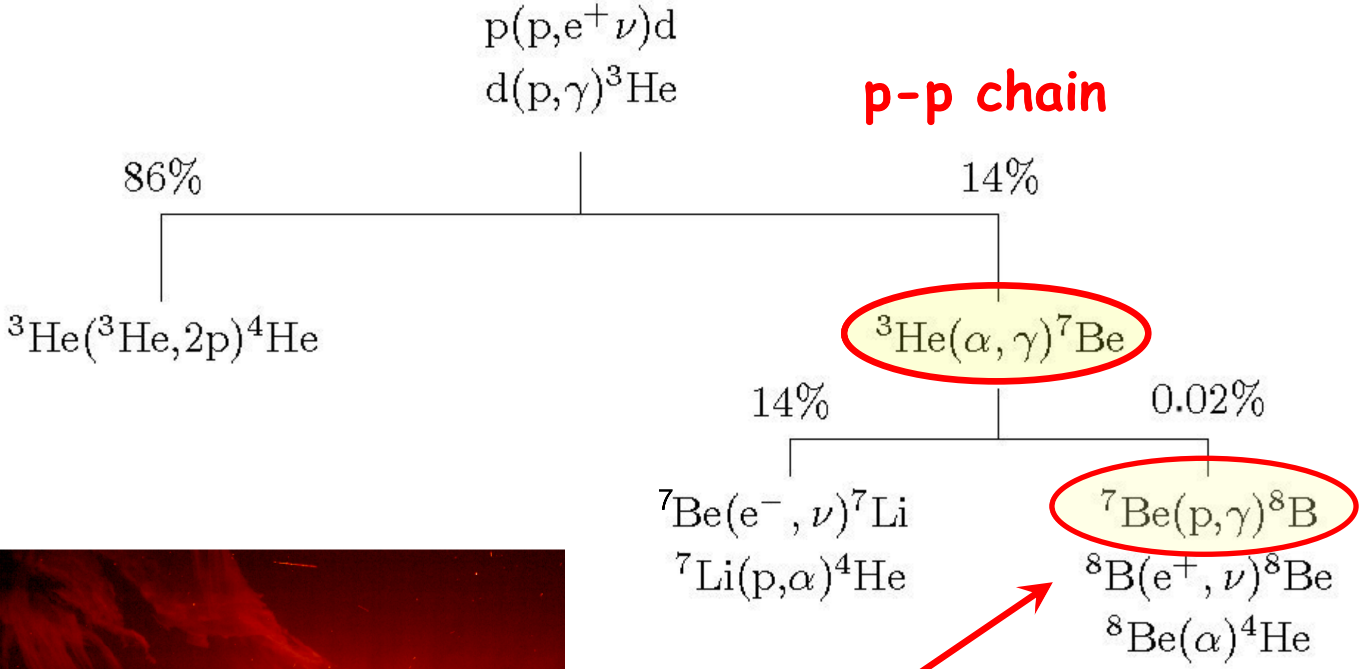


Key reactions



^{7}Li puzzle

Solar *p-p* chain



$p(p,e^+\nu)d$
$d(p,\gamma)^3He$

p-p chain

86%

14%

$^3He(^3He,2p)^4He$

$^3He(\alpha,\gamma)^7Be$

14%

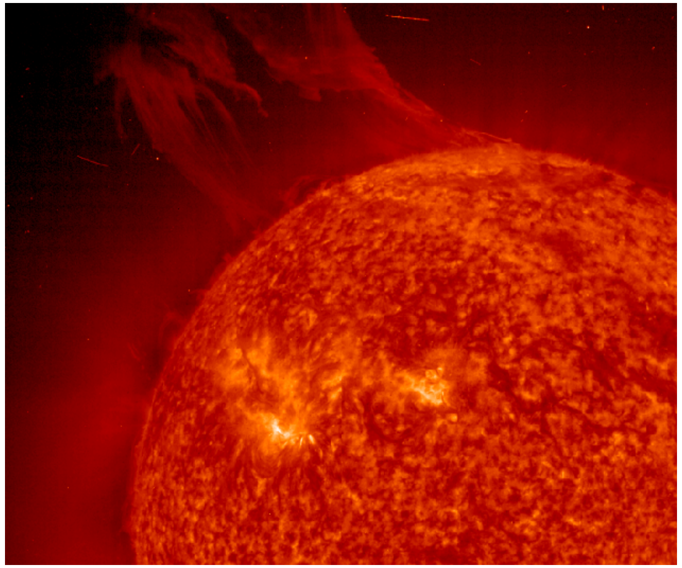
0.02%

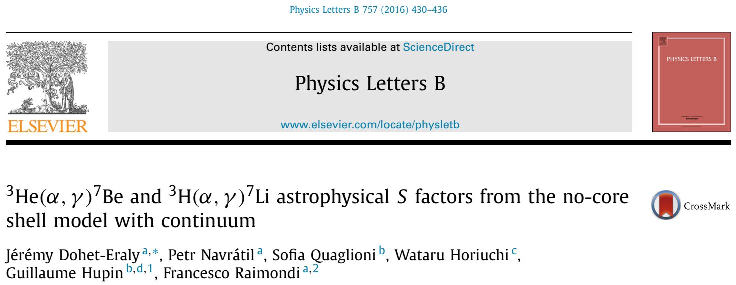
$^7Be(e^-,\nu)^7Li$
$^7Li(p,\alpha)^4He$

$^7Be(p,\gamma)^8B$
$^8B(e^+,\nu)^8Be$
$^8Be(\alpha)^4He$

Solar neutrinos

$E_\nu < 15$ MeV





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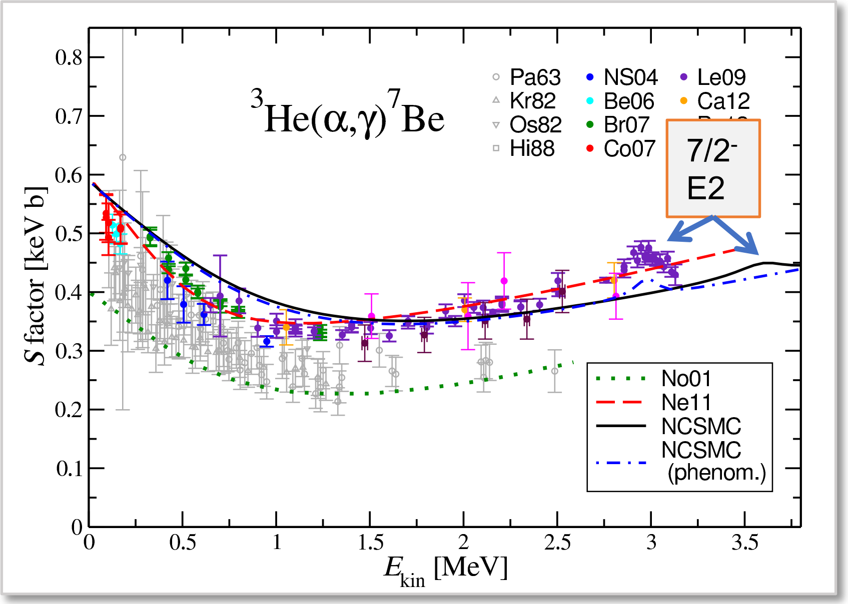
ELSEVIER

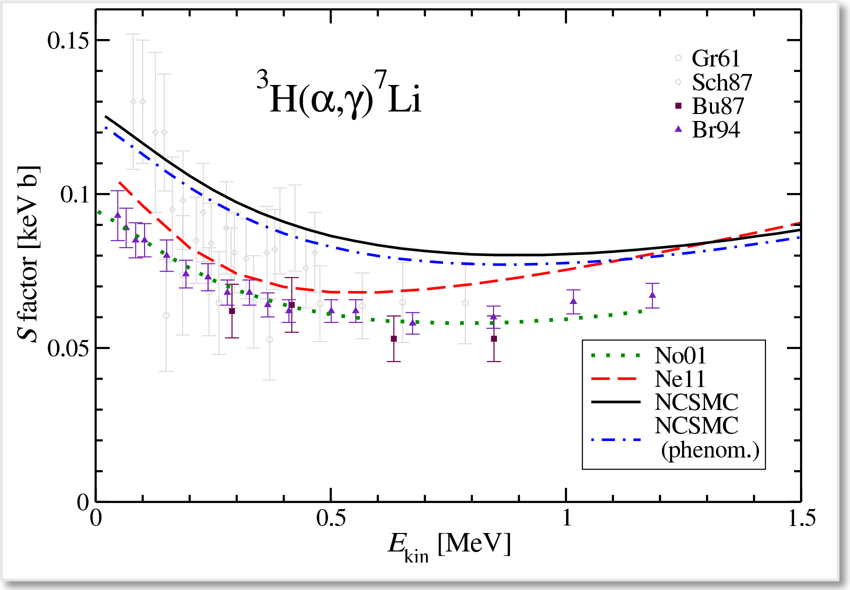
^{3}He$(\alpha,\gamma)^7$Be and ^{3}H$(\alpha,\gamma)^7$Li astrophysical S factors from the no-core shell model with continuum

Jérémy Dohet-Eraly [a,*], Petr Navrátil [a], Sofia Quaglioni [b], Wataru Horiuchi [c], Guillaume Hupin [b,d,1], Francesco Raimondi [a,2]

^{3}He-^{4}He and ^{3}H-^{4}He radiative capture

E1 radiative capture with small E2 contribution at 7/2$^-$ resonance





Theoretical calculations suggest that the most recent and precise ^{7}Be and ^{7}Li data are inconsistent

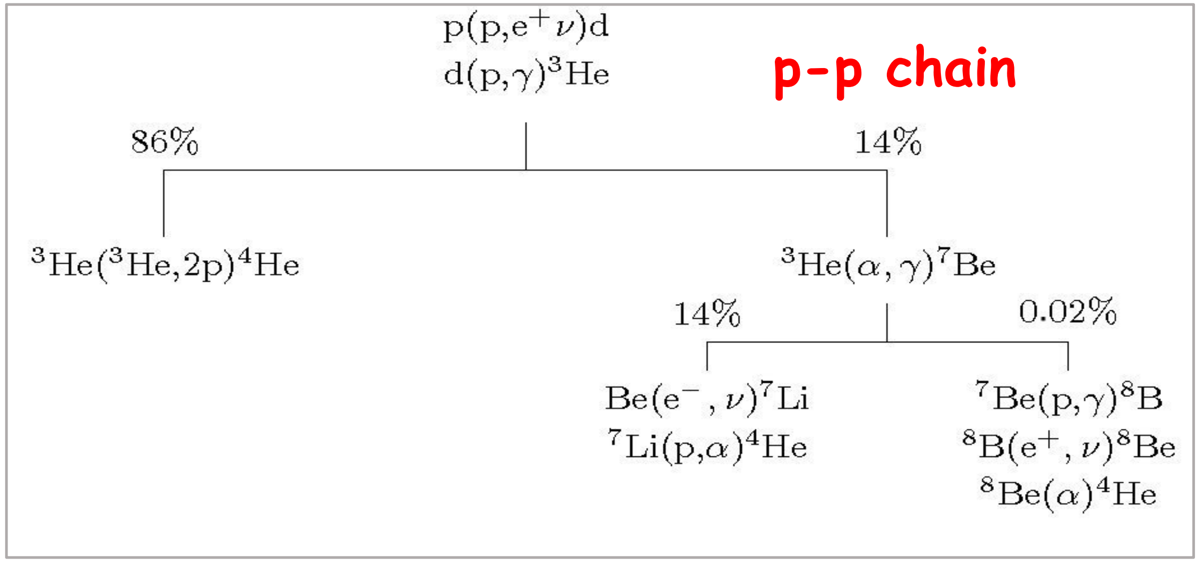
NCSMC calculations with chiral SRG-N^{3}LO *NN* potential (λ=2.15 fm^{-1})

^{3}He, ^{3}H, ^{4}He ground state, 8(π-) + 6(π+) eigenstates of ^{7}Be and ^{7}Li

Preliminary: N_{max}=12, hΩ=20 MeV

p+^{11}C scattering and ^{11}C(p,γ)^{12}N capture

- ^{11}C(p,γ)^{12}N capture relevant in hot *p-p* chain: Link between pp chain and the CNO cycle - bypass of slow triple alpha capture ^{4}He(αα,γ)^{12}C



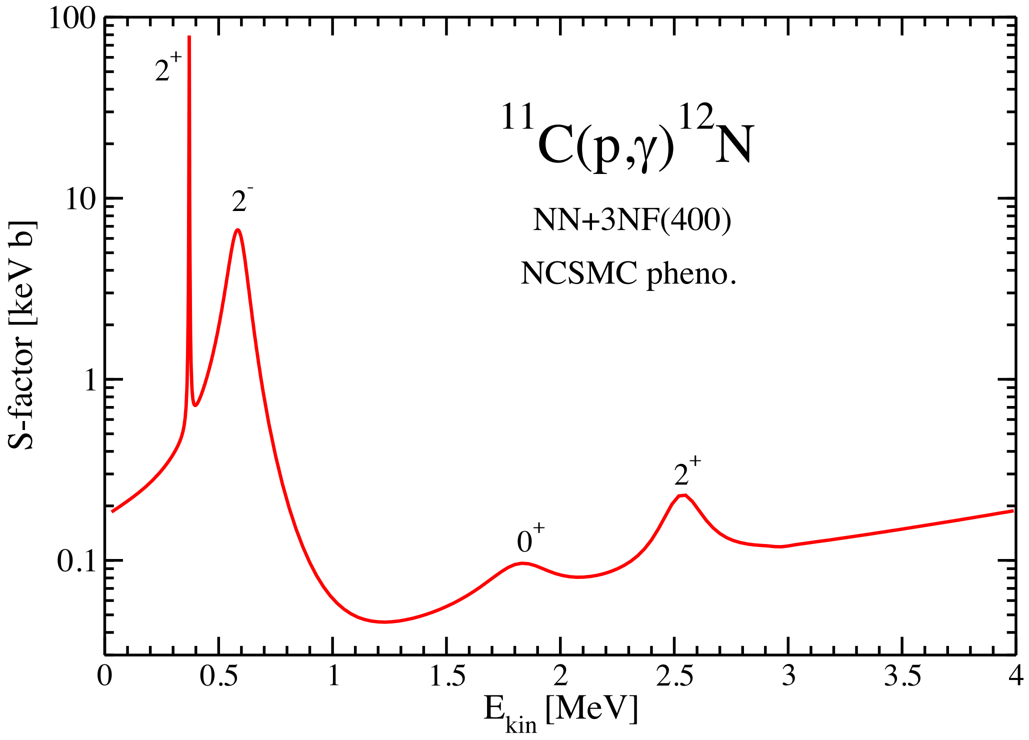
$$^{3}He(\alpha,\gamma)^{7}Be(\alpha,\gamma)^{11}C(p,\gamma)^{12}N(p,\gamma)^{13}O(\beta^{+},\nu)^{13}N(p,\gamma)^{14}O$$

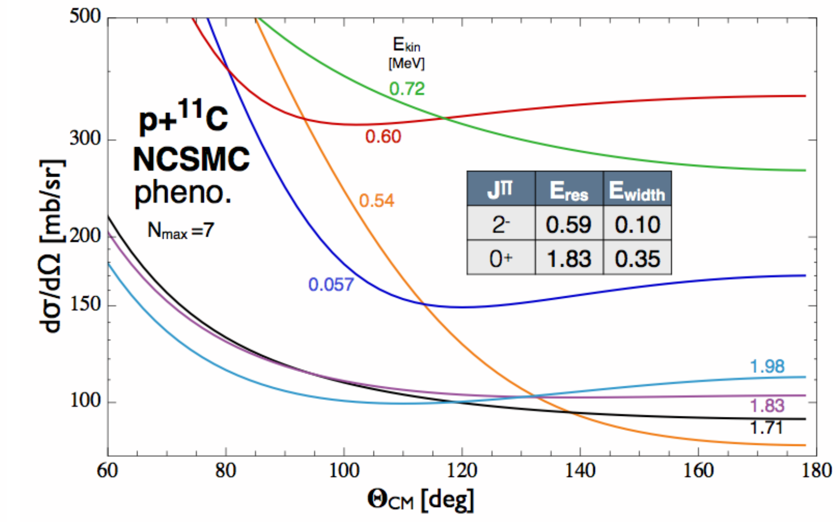
$$^{3}He(\alpha,\gamma)^{7}Be(\alpha,\gamma)^{11}C(p,\gamma)^{12}N(\beta^{+},\nu)^{12}C(p,\gamma)^{13}N(p,\gamma)^{14}O$$

$$^{11}\mathrm{C}(\beta^{+}\ \nu)^{11}\mathrm{B}(p,\alpha)^{8}\mathrm{Be}(^{4}\mathrm{He},{}^{4}\mathrm{He})$$

p+^{11}C scattering and ^{11}C(p,γ)^{12}N capture

- NCSMC calculations of ^{11}C(p,p) with chiral NN+3N under way
    - ^{11}C: 3/2$^-$, 1/2$^-$, 5/2$^-$, 3/2$^-$ NCSM eigenstates
    - ^{12}N: ≥6 π = +1 and ≥4 π = -1 NCSM eigenstates



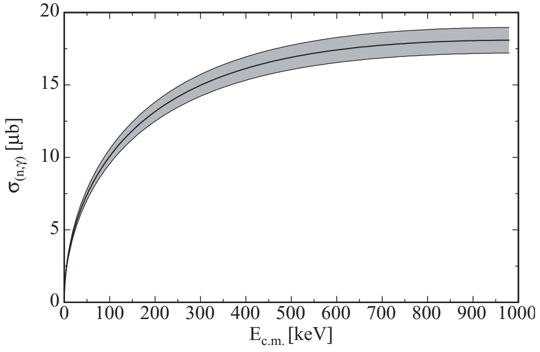


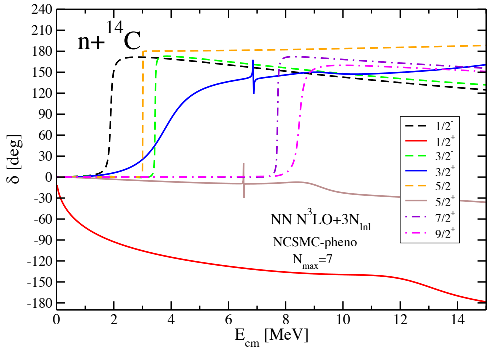
Jπ	E_{res}	E_{width}
2$^-$	0.59	0.10
0$^+$	1.83	0.35

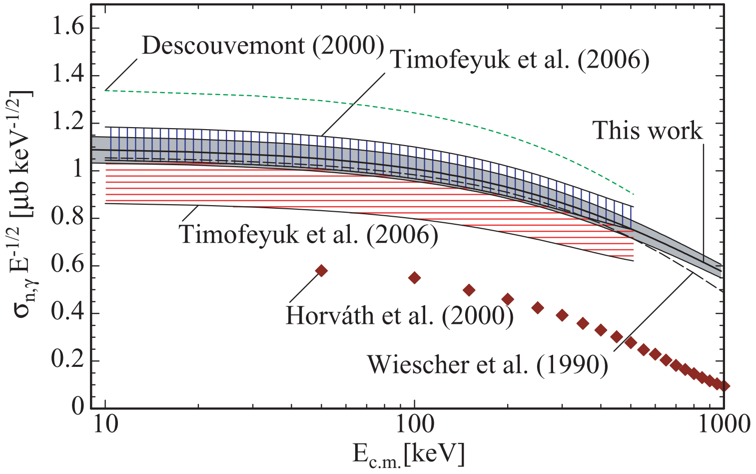
NCSMC calculations to be validated by TRIUMF scattering cross section measurement and applied to calculate the ^{11}C(p,γ)^{12}N capture

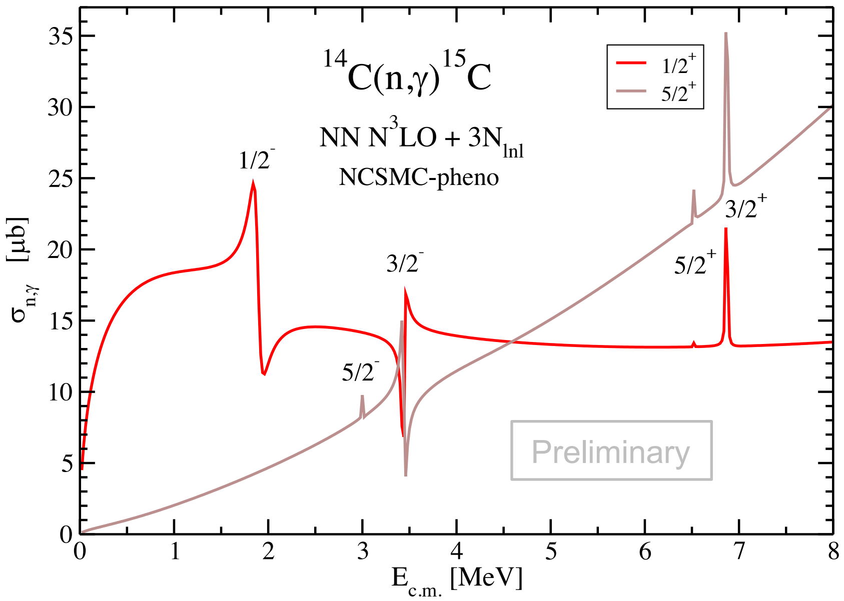
$^{14}C(n,\gamma)^{15}C$ capture cross section

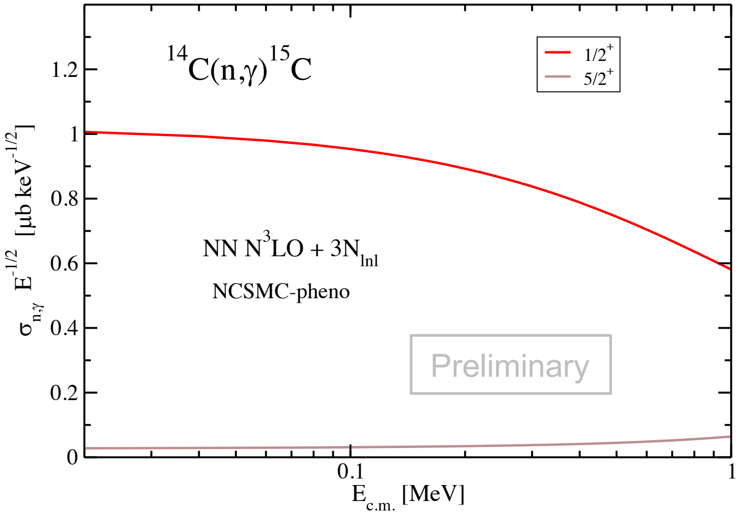
- Comparison to Karlsruhe experiment – Phys. Rev. C 77, 015804 (2008)











Relevant for

Inhomogeneous Big Bang models

Neutron induced CNO cycles

Neutrino driven wind models for the r-process

Validation of Coulomb dissociation method

Conclusions

- *Ab initio* calculations of nuclear structure and reactions becoming feasible beyond the lightest nuclei
  - Make connections between the low-energy QCD, many-body systems, and nuclear astrophysics

- Polarized DT fusion investigated within NCSMC
  - Sheds light on importance of *l*>0 partial waves

- Analysis of ^{3}He(α,γ)^{7}Be & ^{3}H(α,γ)^{7}Li capture reactions
  - Experimental data inconsistent

- NCSMC applied to predict for ^{11}C(p,γ)^{12}N capture
  - Support for upcoming TRIUMF ^{11}C+p scattering experiment

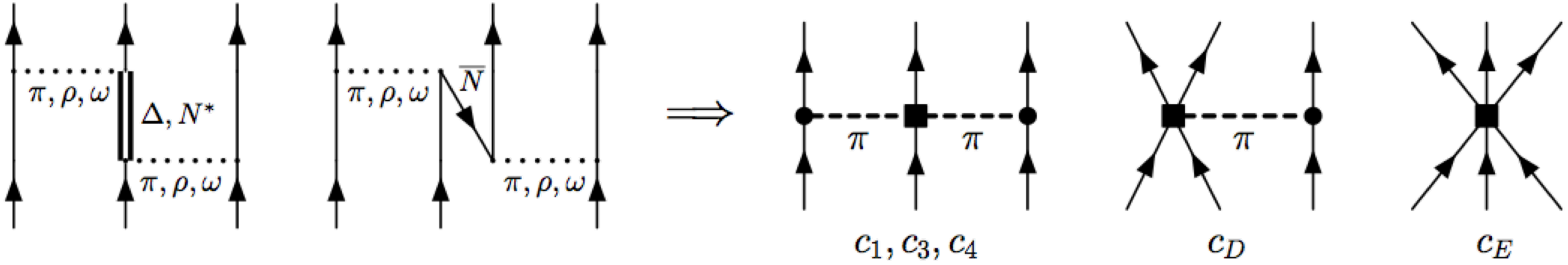
- NCSMC calculations of ^{15}C *sd*-shell halo nucleus in progress
  - 14C(n,γ)^{15}C capture cross section in agreement with experiment

Thank you!

Merci!

Discovery, accelerated

Why three-nucleon forces?



Eliminating degrees of freedom leads to three-body forces.

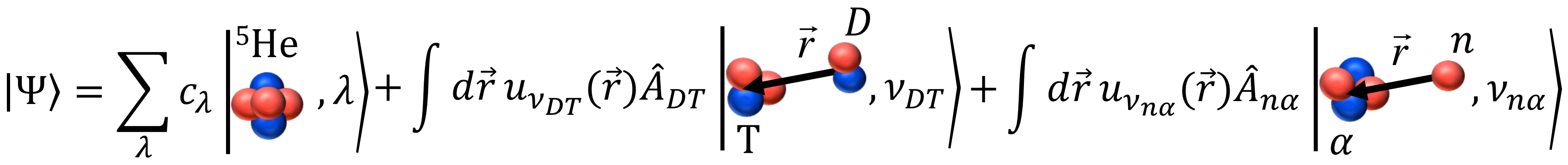
Two-pion exchange with **virtual Δ excitation** – Fujita & Miyazawa (1957)

- Leading three-nucleon force terms
  - Long-range two-pion exchange
  - Medium-range one-pion exchange + two-nucleon contact
  - Short range three-nucleon contact

The question is not: Do three-body forces enter the description?

The only question is: How large are three-body forces?

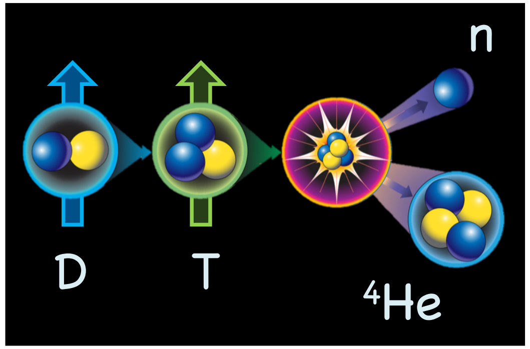
NCSMC calculation of the DT fusion



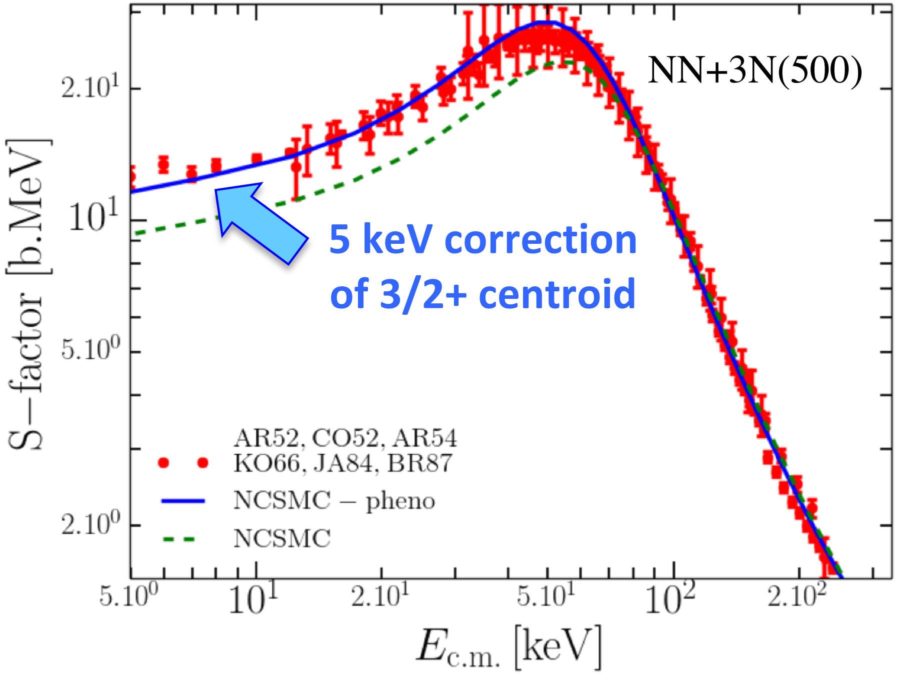
$$|\Psi\rangle = \sum_{\lambda} c_{\lambda} \left| {}^{5}\mathrm{He} , \lambda \right\rangle + \int d\vec{r}\, u_{\nu_{DT}}(\vec{r}) \hat{A}_{DT} \left| \mathrm{T} \; \vec{r} \; D , \nu_{DT} \right\rangle + \int d\vec{r}\, u_{\nu_{n\alpha}}(\vec{r}) \hat{A}_{n\alpha} \left| \alpha \; \vec{r} \; n , \nu_{n\alpha} \right\rangle$$

- 2x7 static ^{5}He eigenstates computed with the NCSM
- Continuous D-T(g.s.) cluster states (entrance channel)
  - Including positive-energy eigenstates of D to account for distortion
- Continuous n-^{4}He(g.s.) cluster states (exit channel)
- Chiral NN+3N(500) interaction

^{3}H(d,n)^{4}He with chiral NN+3N(500) interaction



Astrophysical S-factor



Astrophysical S-factor:
nuclear contribution

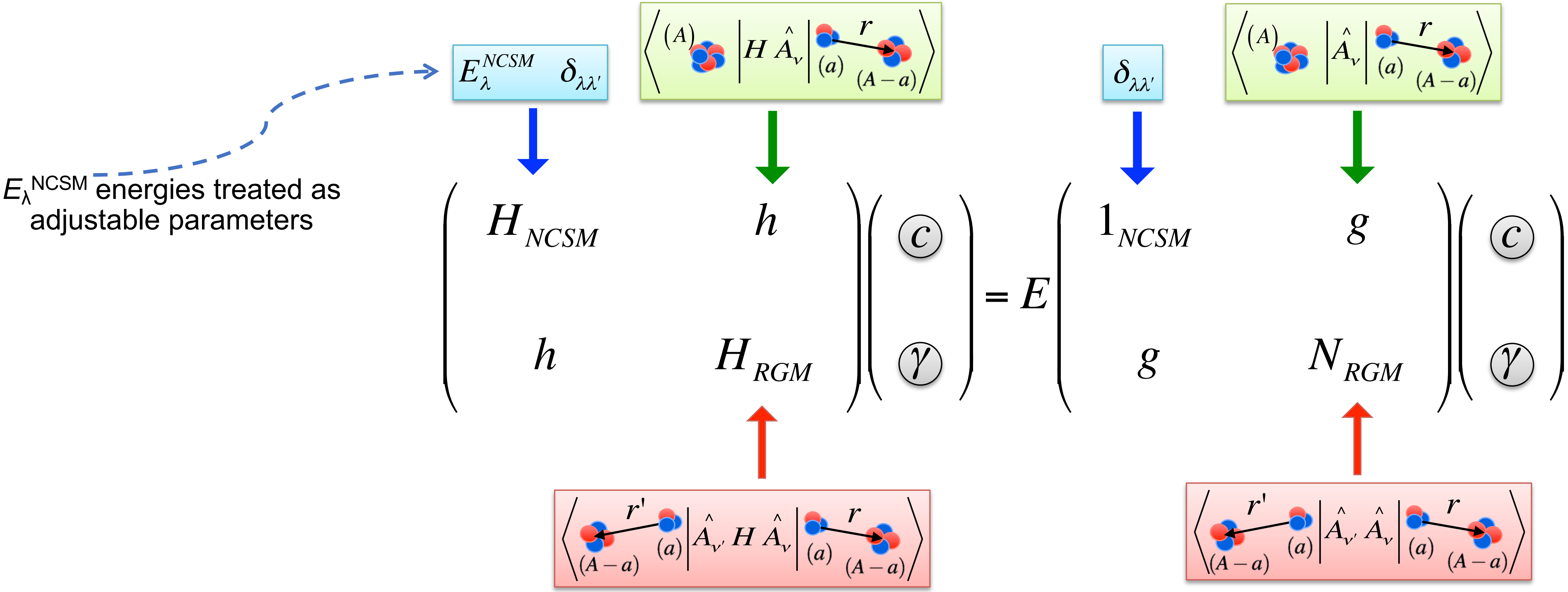
Fusion
cross section

$$\sigma(E) = \frac{\boxed{S(E)}}{E}\exp\left(-\frac{2\pi Z_1 Z_2 e^2}{\hbar\sqrt{2E/m}}\right)$$

'Coulomb'
Contribution
(tunneling)

NCSMC phenomenology

$$H\,\Psi^{(A)} = E\,\Psi^{(A)} \qquad \Psi^{(A)} = \sum_{\lambda} c_\lambda \left|{}^{(A)}, \lambda\right\rangle + \sum_{\nu} \int d\vec{r}\;\gamma_\nu(\vec{r})\;\hat{A}_\nu \left|\underset{(A-a)}{} \overset{\vec{r}}{\longleftarrow} (a), \nu\right\rangle$$

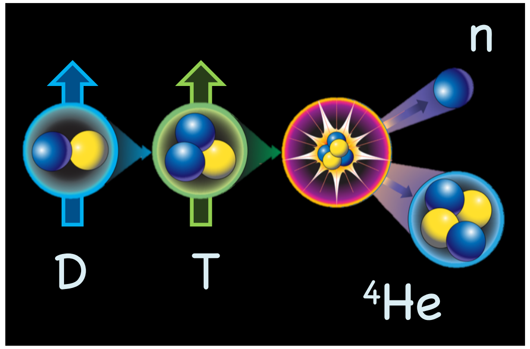


$$E_\lambda^{NCSM}\;\delta_{\lambda\lambda'} \qquad \left\langle {}^{(A)} \right| H\,\hat{A}_\nu \left| (a) \overset{r}{\longrightarrow} (A-a) \right\rangle \qquad \delta_{\lambda\lambda'} \qquad \left\langle {}^{(A)} \right| \hat{A}_\nu \left| (a) \overset{r}{\longrightarrow} (A-a) \right\rangle$$

$$\begin{pmatrix} H_{NCSM} & h \\ h & H_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix} = E \begin{pmatrix} 1_{NCSM} & g \\ g & N_{RGM} \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix}$$

$$\left\langle (A-a) \overset{r'}{\longleftarrow} (a) \right| \hat{A}_{\nu'}\,H\,\hat{A}_\nu \left| (a) \overset{r}{\longrightarrow} (A-a) \right\rangle \qquad \left\langle (A-a) \overset{r'}{\longleftarrow} (a) \right| \hat{A}_{\nu'}\,\hat{A}_\nu \left| (a) \overset{r}{\longrightarrow} (A-a) \right\rangle$$

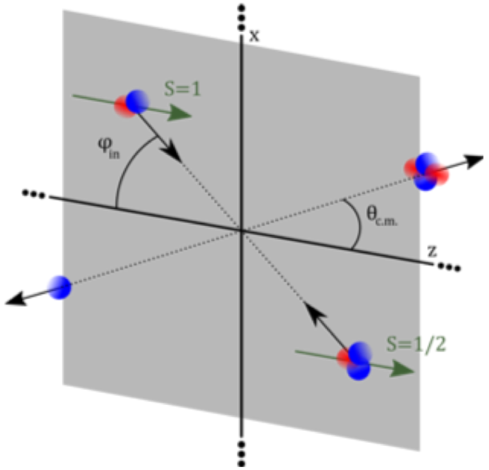
E_λ^{NCSM} energies treated as adjustable parameters



^{3}H(d,n)^{4}He with chiral NN+3N(500) interaction

Polarized fusion

$$\frac{\partial\sigma_{pol}}{\partial\Omega_{c.m.}}(\theta_{\text{c.m.}}) = \frac{\partial\sigma_{unpol}}{\partial\Omega_{c.m.}}(\theta_{\text{c.m.}})\left(1+\frac{1}{2}p_{zz}A_{zz}^{(b)}(\theta_{\text{c.m.}})+\frac{3}{2}p_z q_z C_{z,z}(\theta_{\text{c.m.}})\right)$$

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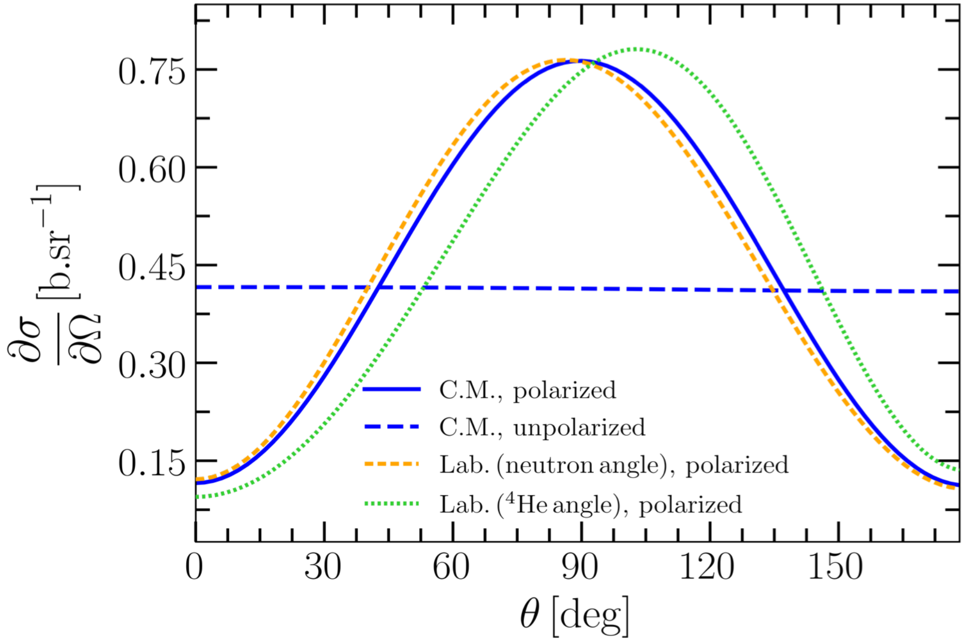
ARTICLE

https://doi.org/10.1038/s41467-018-08052-6 OPEN

Ab initio predictions for polarized deuterium-tritium thermonuclear fusion

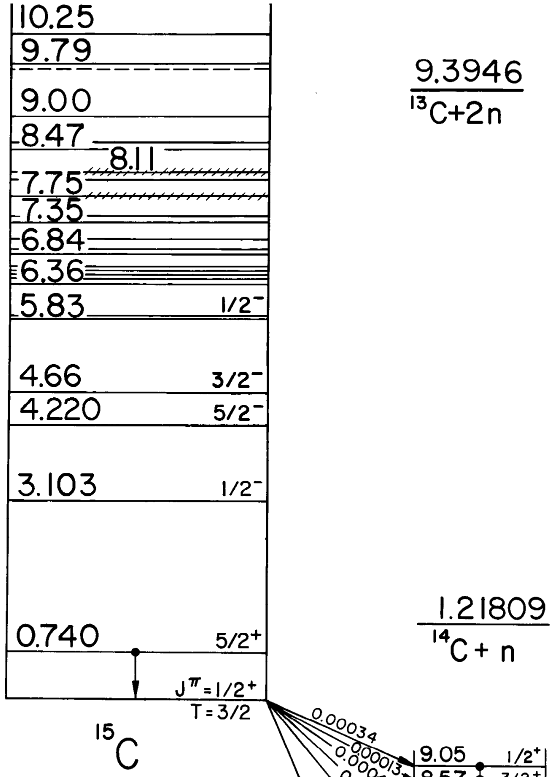
Guillaume Hupin[1,2,3], Sofia Quaglioni[3] & Petr Navrátil[4]



For a realistic 80% polarization, outgoing neutrons and alphas emitted dominantly in the perpendicular direction to the magnetic field

Halo *sd*-shell nucleus ^{15}C and its unbound mirror ^{15}F

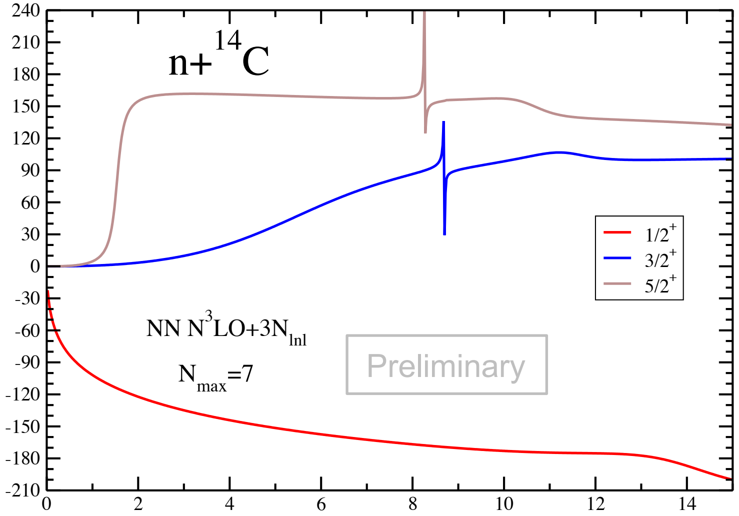
- Motivation:
  - Halo ½+ *S*-wave and $5/2^+$ *D*-wave bound states
  - ^{14}C(n,γ)^{15}C capture relevant for astrophysics
  - Unbound ^{15}F mirror – very narrow unnatural parity resonances embedded in continuum predicted in ^{14}O(p,p)^{14}O
    - measured at GANIL

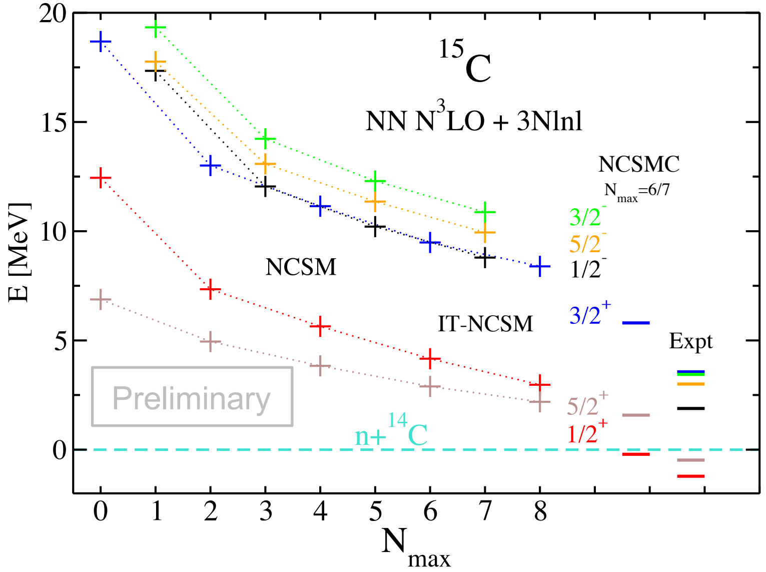


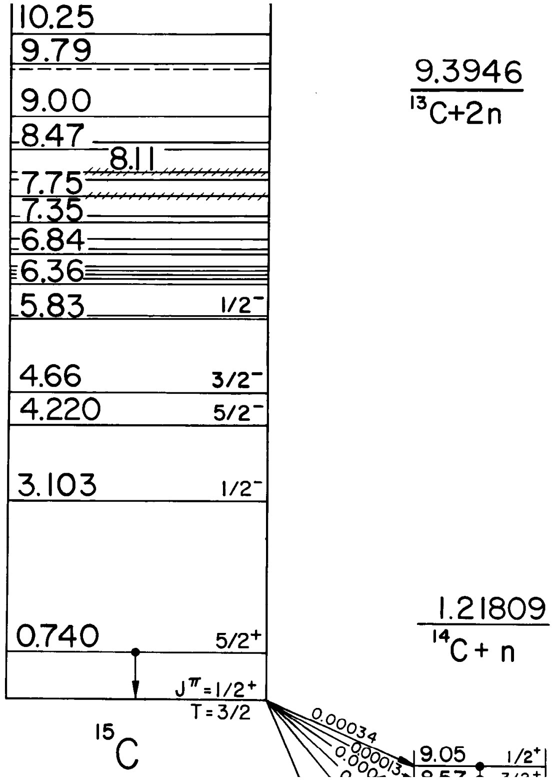
- Calculations in progress – all results preliminary
- NN chiral interaction – N^{3}LO Entem & Machleidt 2003, SRG evolved with λ = 2.0 fm^{-1}
- 3N chiral interaction – N^{2}LO with local/non-local regulator, SRG evolved with λ = 2.0 fm^{-1}

Halo *sd*-shell nucleus ^{15}C

- NCSMC
  - ^{14}C (^{14}O) 0^+ and 2^+ eigenstates
  - ^{15}C (^{15}F) lowest 7 positive and 3 negative parity eigenstates







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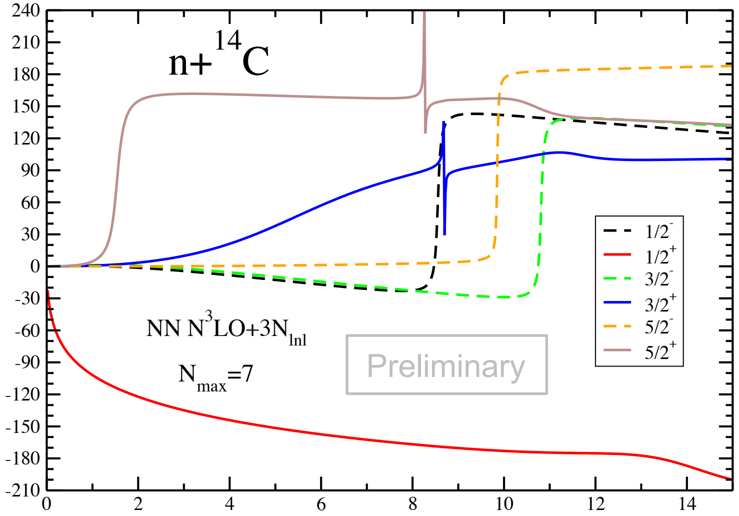

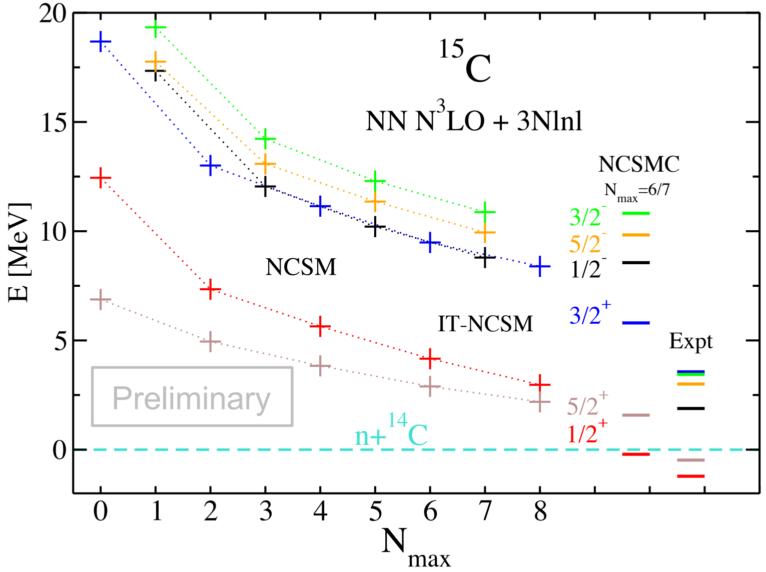

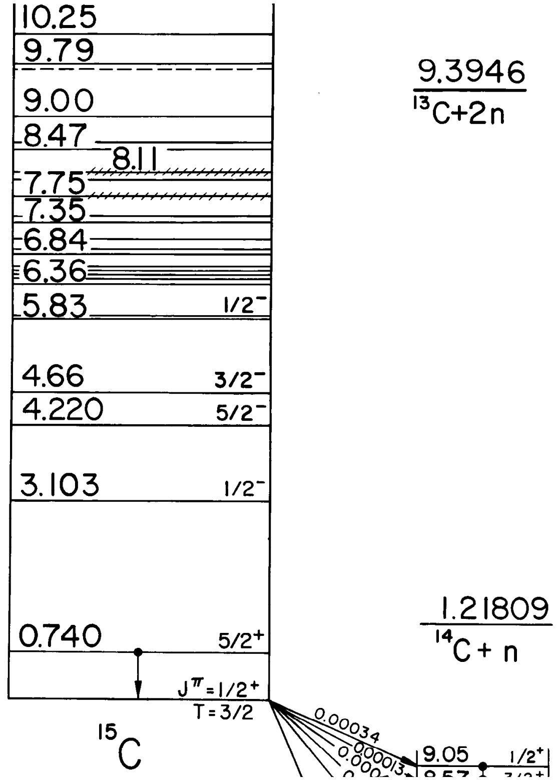

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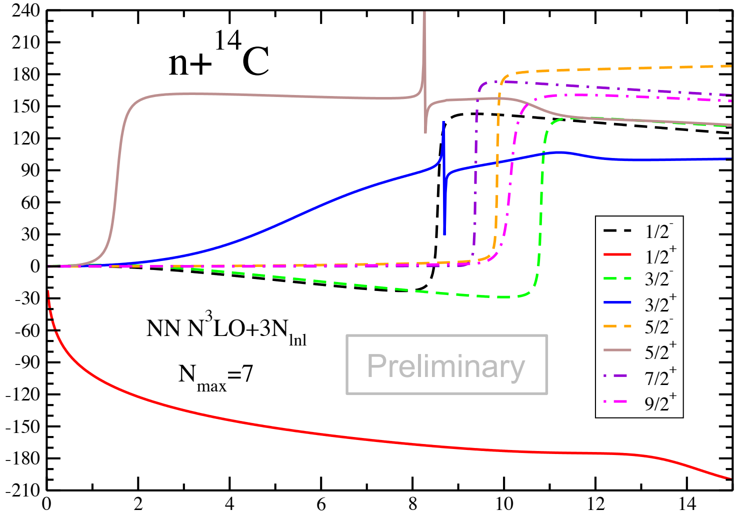

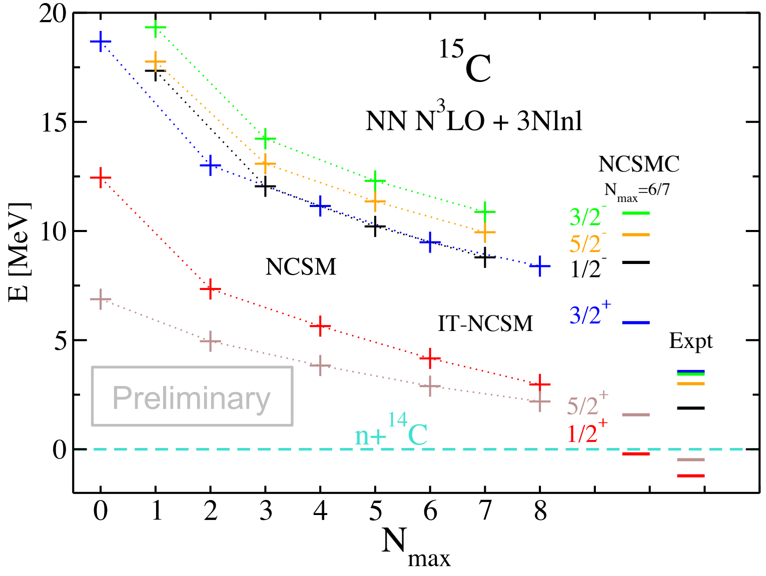

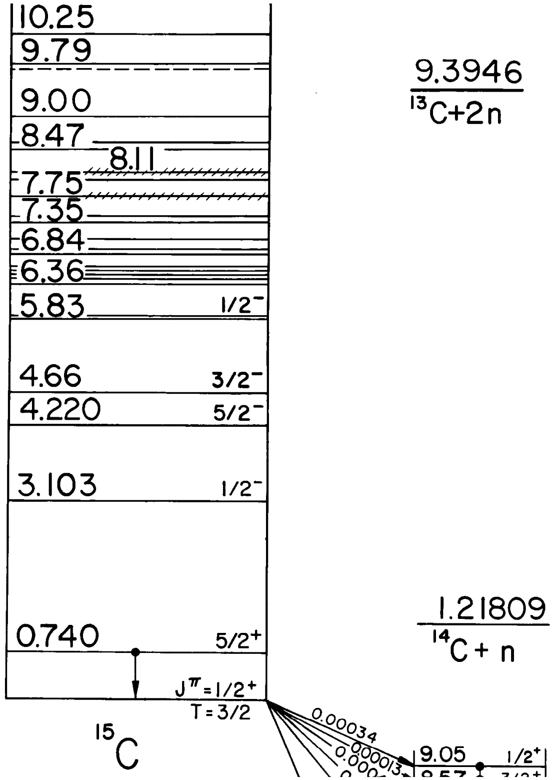

Halo *sd*-shell nucleus ^{15}C

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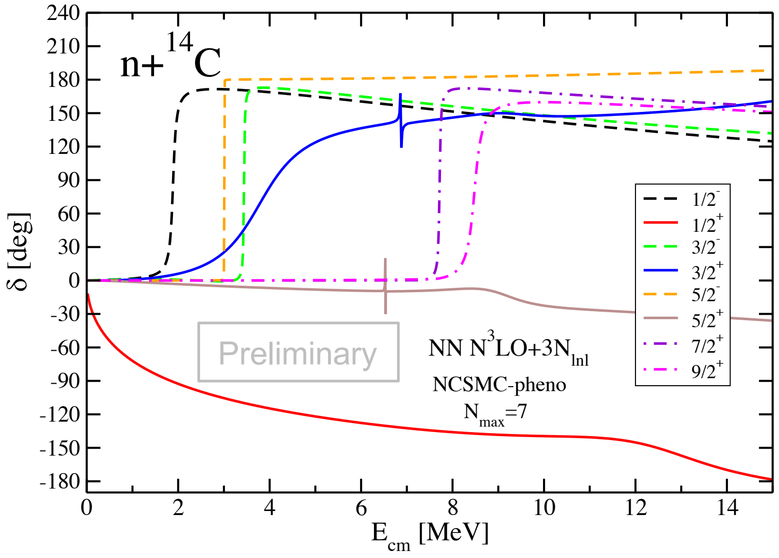
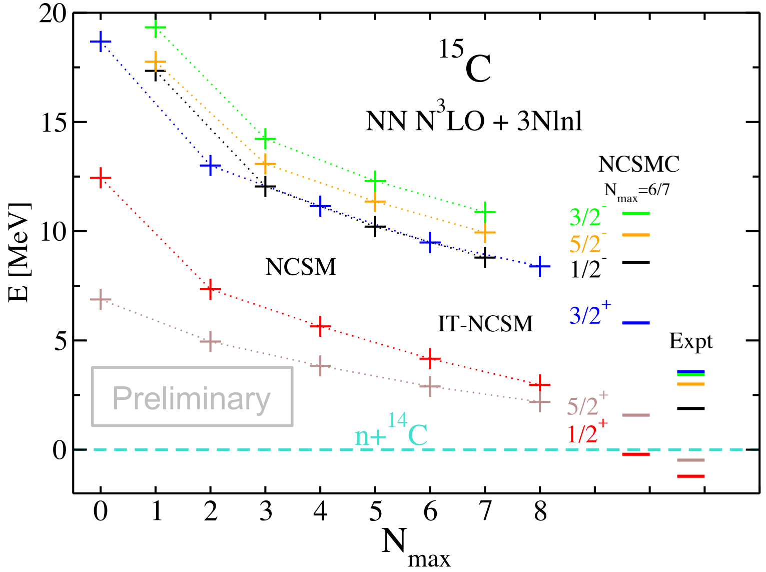
NCSMC-pheno





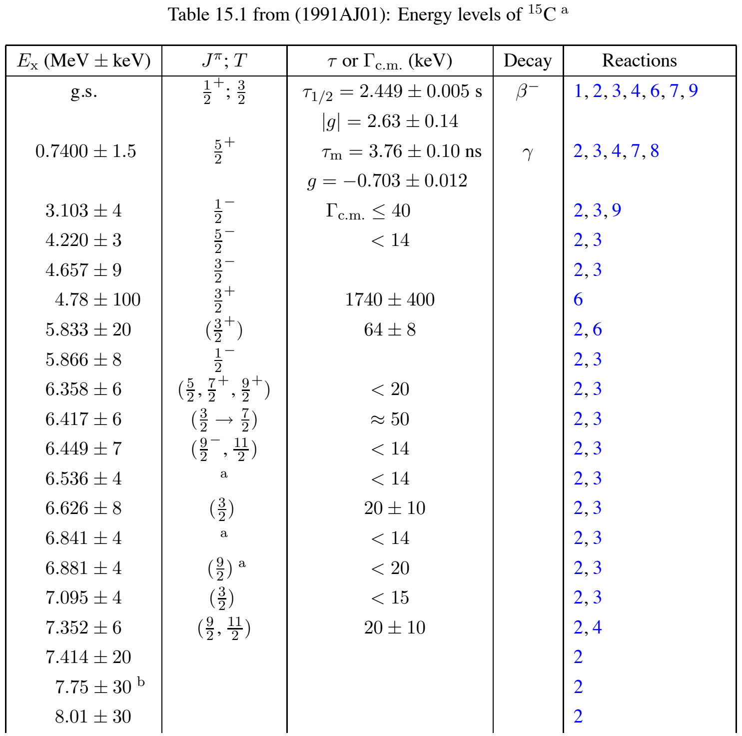


Table 15.1 from (1991AJ01): Energy levels of ^{15}C [a]

E_x (MeV ± keV)	$J^\pi; T$	τ or $\Gamma_{c.m.}$ (keV)	Decay	Reactions
g.s.	$\frac{1}{2}^+; \frac{3}{2}$	$\tau_{1/2} = 2.449 \pm 0.005$ s $\lvert g\rvert = 2.63 \pm 0.14$	β^-	1, 2, 3, 4, 6, 7, 9
0.7400 ± 1.5	$\frac{5}{2}^+$	$\tau_m = 3.76 \pm 0.10$ ns $g = -0.703 \pm 0.012$	γ	2, 3, 4, 7, 8
3.103 ± 4	$\frac{1}{2}^-$	$\Gamma_{c.m.} \leq 40$		2, 3, 9
4.220 ± 3	$\frac{5}{2}^-$	< 14		2, 3
4.657 ± 9	$\frac{3}{2}^-$			2, 3
4.78 ± 100	$\frac{3}{2}^+$	1740 ± 400		6
5.833 ± 20	$(\frac{3}{2}^+)$	64 ± 8		2, 6
5.866 ± 8	$\frac{1}{2}^-$			2, 3
6.358 ± 6	$(\frac{5}{2}, \frac{7}{2}^+, \frac{9}{2}^+)$	< 20		2, 3
6.417 ± 6	$(\frac{3}{2} \rightarrow \frac{7}{2})$	≈ 50		2, 3
6.449 ± 7	$(\frac{9}{2}^-, \frac{11}{2})$	< 14		2, 3
6.536 ± 4	[a]	< 14		2, 3
6.626 ± 8	$(\frac{3}{2})$	20 ± 10		2, 3
6.841 ± 4	[a]	< 14		2, 3
6.881 ± 4	$(\frac{9}{2})$ [a]	< 20		2, 3
7.095 ± 4	$(\frac{3}{2})$	< 15		2, 3
7.352 ± 6	$(\frac{9}{2}, \frac{11}{2})$	20 ± 10		2, 4
7.414 ± 20				2
7.75 ± 30 [b]				2
8.01 ± 30				2

^{15}C cluster form factors

- $1/2^+$ *S*-wave and $5/2^+$ *D*-wave ANCs
  - $C_{1/2+}$ = 1.282 fm$^{-1/2}$ - compare to Moschini & Capel inferred from transfer data: 1.26(2) fm$^{-1/2}$
  - $C_{5/2+}$ = 0.048 fm$^{-1/2}$ 0.056(1) fm$^{-1/2}$
  - Spectroscopic factors: 0.96 for $1/2^+$ and 0.90 for $5/2^+$ - experiments 0.95(5) and 0.69, resp.