Custodial symmetry and the Higgs sector

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Outline

Introduction: custodial symmetry in the Standard Model

Violating custodial symmetry? Models with triplets

Georgi-Machacek model

Loop-induced custodial symmetry violation and its consequences

Conclusions and outlook

Introduction: the Standard Model

Weinberg 1967

The electroweak part of the Standard Model is an SU(2)×U(1) gauge theory:

- Isospin SU$(2)_L$ gauge bosons W_μ^a, $a = 1, 2, 3$

- Hypercharge U$(1)_Y$ gauge boson B_μ

- Chiral fermions, left-handed transform as doublets under SU$(2)_L$, right-handed as singlets, hypercharge quantum numbers assigned according to electric charge $Q = T^3 + Y$.

Gauge invariance requires that the gauge bosons are massless.

To account for massive $W^\pm$ and Z, incorporate the Higgs mechanism of spontaneous symmetry breaking.

Minimal nontrivial representation of the Higgs field (Lorentz scalar) is a complex SU$(2)_L$ doublet with hypercharge $Y = 1/2$:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

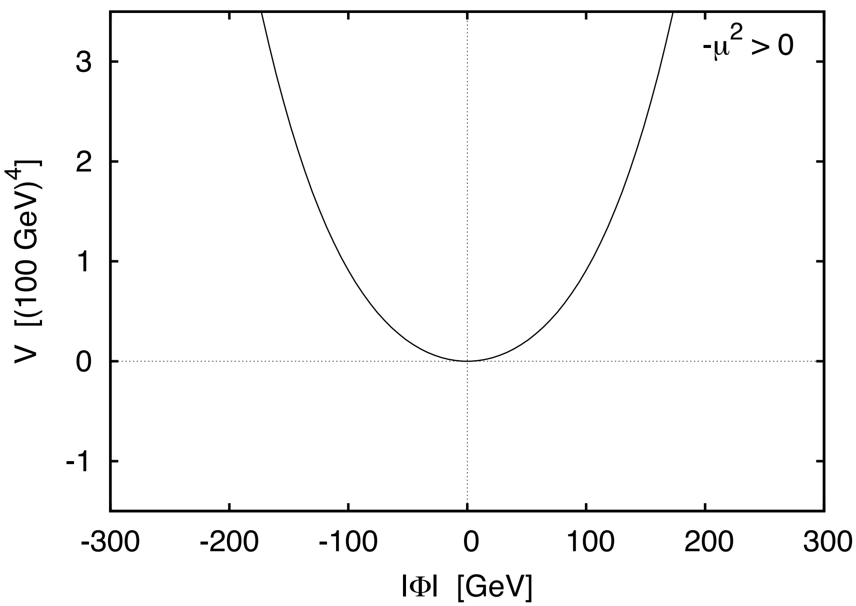
The most general gauge-invariant potential for this field (the so-called Higgs potential) is

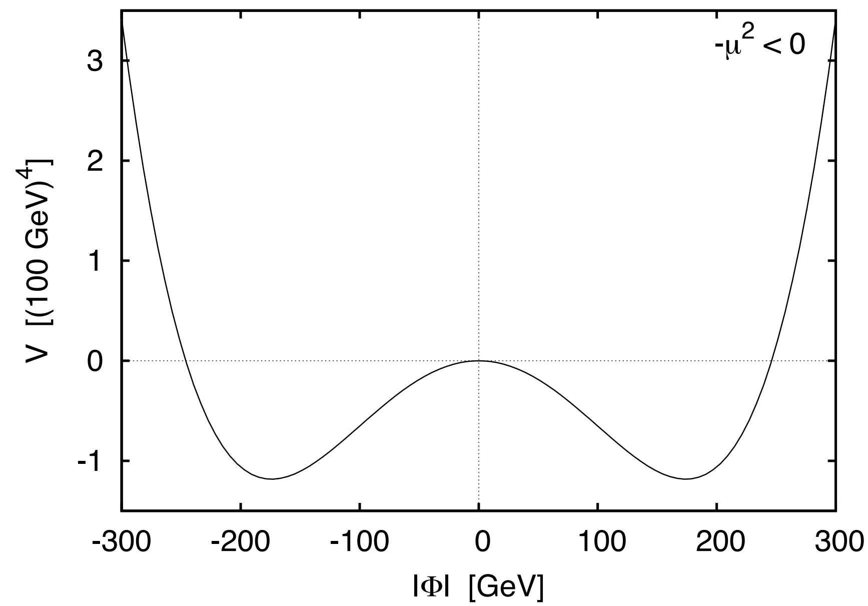
$$\begin{aligned} V &= -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \\ &= -\frac{\mu^2}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) + \frac{\lambda}{4}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2)^2 \end{aligned}$$

Clearly this potential is invariant under more than just SU$(2)_L \times$U$(1)_Y$: there is a global SO(4) symmetry (homomorphic to SU(2)$\times$SU(2)) under which $(\phi_1, \phi_2, \phi_3, \phi_4)$ transforms as a vector.

 Weinberg 1967

Spontaneous symmetry breaking: coefficient of $\Phi^\dagger\Phi$ is negative





Vacuum: $(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) \equiv v^2 = \mu^2/\lambda$

Vacuum value of $(\phi_1, \phi_2, \phi_3, \phi_4)$ must choose a direction:
Breaks three SO(4) rotations, preserves the remaining three.
$\cong$ Breaks SU(2)×SU(2) down to diagonal SU(2) subgroup.

This is the custodial SU(2).

Another way to see this: rewrite Φ as a "bidoublet":

$$\overline{\Phi} = \begin{pmatrix} \phi^{0*} & \phi^{+} \\ -\phi^{+*} & \phi^{0} \end{pmatrix}$$

- Second column is the original Φ.
- First column is the conjugate doublet $\tilde{\Phi} \equiv i\sigma^2\Phi^*$ (also transforms as a doublet because SU(2) is pseudo-real).

$$V = -\frac{\mu^2}{2}\mathrm{Tr}(\overline{\Phi}^\dagger\overline{\Phi}) + \frac{\lambda}{4}[\mathrm{Tr}(\overline{\Phi}^\dagger\overline{\Phi})]^2$$

V is invariant under $SU(2)_L \times SU(2)_R$ transformations:

$$\overline{\Phi} \to \exp(i\theta_L^a\tau^a)\overline{\Phi}\exp(-i\theta_R^b\tau^b)$$

Vacuum preserves diagonal subgroup $\theta_L^a = \theta_R^a$: (custodial SU(2))

$$\langle\overline{\Phi}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \propto I_{2\times 2}$$

These are global symmetries. Match them back to the gauge symmetries? SU(2)$_L$×SU(2)$_R$ $\leftarrow$? $\rightarrow$ SU(2)$_L$×U(1)$_Y$

- Global SU(2)$_L$ is the gauged SU(2)$_L$.

- The T^3 generator of global SU(2)$_R$ is the hypercharge U(1)$_Y$ generator.

- The T^3 generator of the custodial SU(2) is the electric charge operator (unbroken).

- Gauging only the one (hypercharge) generator of SU(2)$_R$ breaks the global symmetry without promoting it to a full SU(2) gauge symmetry. $\rightarrow$ hypercharge is going to cause some trouble down the line....

Gauge boson masses in the SM come from the gauge-covariant derivative terms in the Lagrangian acting upon the Higgs field's vacuum expectation value. ($Y = 1/2$, $\tau^a = \sigma^a/2$)

$$\mathcal{L} \supset (\mathcal{D}_\mu \Phi)^\dagger (\mathcal{D}^\mu \Phi), \qquad \mathcal{D}_\mu = \partial_\mu - ig' Y B_\mu - ig\tau^a W^a_\mu$$

Gauge boson mass terms generated:
write in matrix form in basis (W^1, W^2, W^3, B):

$$M^2 = \frac{v^2}{4} \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}$$

- $W^\pm_\mu = (W^1_\mu \mp i W^2_\mu)/\sqrt{2}$ have the same mass $M_W = gv/2$ and do not mix with anything else (charge is conserved).
- W^3_μ and B_μ mix by $\theta_W = \tan^{-1}(g'/g)$ to produce the massive Z with $M_Z = \sqrt{g^2 + g'^2} v/2$ and the massless photon.

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Gauge boson mass terms generated:
write in matrix form in basis (W^1, W^2, W^3, B):

$$M^2 = \frac{v^2}{4} \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}$$

The custodial symmetry manifests here in the limit $g' \to 0$ as an invariance under SU(2) rotations among (W^1, W^2, W^3).

Consequence with $g' \neq 0$ is that $\rho_0 \equiv M_W^2 / M_Z^2 \cos^2\theta_W = 1$.
Experiment: $\rho_0 = 1.00039 \pm 0.00019$ (PDG 2018).

Higgs bidoublet is $2 \otimes 2$ under SU(2)$_L \times$SU(2)$_R$:

$$\overline{\Phi} = \begin{pmatrix} \phi^{0*} & \phi^{+} \\ -\phi^{+*} & \phi^{0} \end{pmatrix}$$

Breaking SU(2)$_L \times$SU(2)$_R \to$ SU(2)$_{\text{custodial}}$ $\Rightarrow 2 \otimes 2 \to 3 \oplus 1$.
- Custodial triplet $(\phi^{+}, \sqrt{2}\text{Im}\phi^{0}, \phi^{+*})$ are the (eaten) Goldstone bosons.
- Custodial singlet $\sqrt{2}\text{Re}\phi^{0} = h$ is the (physical) Higgs boson.

Higgs couplings to $W^{+}W^{-}$ and ZZ have a characteristic pattern:

$$hW^{+}_{\mu}W^{-}_{\nu}: \quad 2i\frac{M_W^2}{v}g_{\mu\nu}$$

$$hZ_{\mu}Z_{\nu}: \quad 2i\frac{M_Z^2}{v}g_{\mu\nu}$$

Experiment: $\lambda_{WZ} \equiv (g_{hWW}/M_W^2)/(g_{hZZ}/M_Z^2) = 0.88^{+0.10}_{-0.09}$
(ATLAS + CMS 2016).

Models without custodial symmetry?

To get an appreciation of the importance of custodial symmetry, let's look at some ways of breaking the SM gauge symmetry that do not preserve it.

Example 1: Real triplet with $Y = 0$.

$$\Xi = \begin{pmatrix} \xi^+ \\ \xi^0 \\ -\xi^{+*} \end{pmatrix}, \qquad \langle \Xi \rangle = \begin{pmatrix} 0 \\ v_\xi \\ 0 \end{pmatrix}$$

Gauge boson mass matrix generated:

$$M^2_\Xi = v_\xi^2 \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Real triplet generates a mass for W, but no mass for Z!

see also Georgi & Glashow 1972

Combine with a doublet: θ_W stays the same, but now M_W gets an extra contribution. $\rho_0 \equiv M_W^2/M_Z^2 \cos^2\theta_W > 1$.

Models without custodial symmetry?

To get an appreciation of the importance of custodial symmetry, let's look at some ways of breaking the SM gauge symmetry that do not preserve it.

Example 2: Complex triplet with $Y = 1$.

$$X = \begin{pmatrix} \chi^{++} \\ \chi^{+} \\ \chi^{0} \end{pmatrix}, \qquad \langle X \rangle = \begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix}$$

Gauge boson mass matrix generated:

$$M_X^2 = v_\chi^2 \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & 2g^2 & -2gg' \\ 0 & 0 & -2gg' & 2g'^2 \end{pmatrix}$$

Complex triplet generates $\sqrt{2}$ more mass for Z than for W!

Combine with a doublet: θ_W stays the same, but now M_Z gets more contribution. $\rho_0 \equiv M_W^2 / M_Z^2 \cos^2\theta_W < 1$.

Models without custodial symmetry?

What if we combine the real triplet and the complex triplet?
(At least one doublet is needed to generate the fermion masses.)

$$M_W^2 = \frac{g^2}{4}(v_\phi^2 + 4v_\xi^2 + 4v_\chi^2), \qquad M_Z^2 = \frac{g^2 + g'^2}{4}(v_\phi^2 + 8v_\chi^2)$$

so (using $g^2 + g'^2 = g^2/\cos^2\theta_W$),

$$\rho_0 = \frac{v_\phi^2 + 4v_\xi^2 + 4v_\chi^2}{v_\phi^2 + 8v_\chi^2}$$

If we just fine-tune $v_\xi = v_\chi$ then we are in good shape!

But that is ugly, since the fine-tuning has to be pretty extreme.
Experiment: $\rho_0 = 1.00039 \pm 0.00019$ (PDG 2018).

Instead, let's construct a model including both of the triplets with custodial symmetry re-imposed! Georgi & Machacek 1985

Georgi-Machacek model

Georgi & Machacek 1985; Chanowitz & Golden 1985

SM Higgs bidoublet + the two triplets in a bitriplet:

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \qquad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

Impose a global SU(2)$_L \times$SU(2)$_R$ and write down the scalar potential (this is not the most general gauge invariant potential):

$$\begin{aligned} V(\Phi, X) &= \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 \\ &+\lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X) + \lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) \\ &+\lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b) \\ &-M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(UXU^\dagger)_{ab} - M_2\mathrm{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab} \end{aligned}$$

9 parameters, 2 fixed by G_F and m_h $\to$ 7 free parameters. Aoki & Kanemura, 0712.4053

Chiang & Yagyu, 1211.2658; Chiang, Kuo & Yagyu, 1307.7526

Hartling, Kumar & HEL, 1404.2640

Spontaneous symmetry breaking can be achieved preserving the custodial SU(2) $\to \langle X \rangle = v_\chi \times I_{3\times3}$, so $v_\xi = v_\chi$ naturally!

Georgi-Machacek model

Georgi & Machacek 1985; Chanowitz & Golden 1985

SM Higgs bidoublet + the two triplets in a bitriplet:

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \qquad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

Physical spectrum controlled by transformation under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_{\mathrm{custodial}}$:

Bidoublet: $2 \otimes 2 \to 1 \oplus 3$

Bitriplet: $3 \otimes 3 \to 1 \oplus 3 \oplus 5$

- Two custodial singlets mix $\to h^0$, H^0 m_h, m_H, angle α
  Usually identify $h^0 = h(125)$ $\lambda_{WZ} = 1$

- Two custodial triplets mix $\to (H_3^+, H_3^0, H_3^-)$ m_3 + Goldstones
  Phenomenology very similar to $H^\pm, A^0$ in 2HDM Type I, $\tan\beta \to \cot\theta_H$

- Custodial fiveplet $(H_5^{++}, H_5^+, H_5^0, H_5^-, H_5^{--})$ m_5
  Fermiophobic; H_5VV couplings $\propto s_H \equiv \sqrt{8} v_\chi / v_{\mathrm{SM}}$
  $\lambda_{WZ} = -1/2$ for H_5^0

  $s_H^2 \equiv$ exotic fraction of M_W^2, M_Z^2

Georgi-Machacek model

Georgi & Machacek 1985; Chanowitz & Golden 1985

SM Higgs bidoublet + the two triplets in a bitriplet:

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix} \qquad X = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}$$

Why add SU(2)-triplet scalars?

- They show up in some composite-Higgs models.

- Complex triplet $(\chi^{++}, \chi^+, \chi^0)$ generates Majorana neutrino masses: "type-II seesaw". But can do that with tiny vev, no need for custodial symmetry.

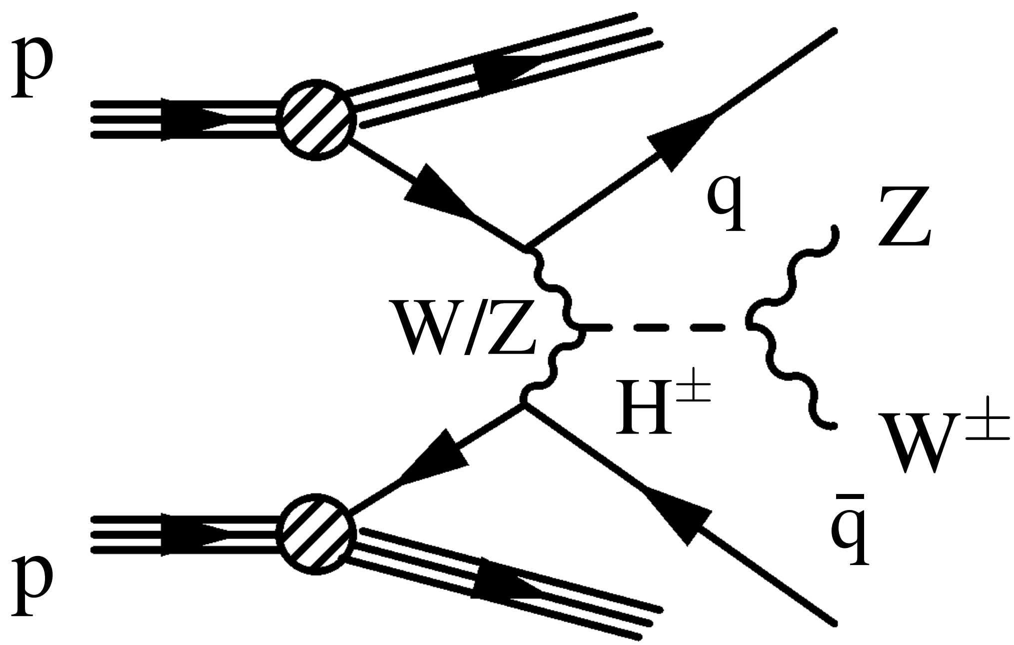
- Other than that, no particular "problem-solving" reason.

But, Georgi-Machacek model provides a phenomenological prototype for ALL "exotic" scalar sector extensions engineered to preserve $\rho_{\text{tree}} = 1$. ⇒ Generic search with LHC data.

Smoking-gun processes involve $(H_5^{++}, H_5^{+}, H_5^{0}, H_5^{-}, H_5^{--})$:

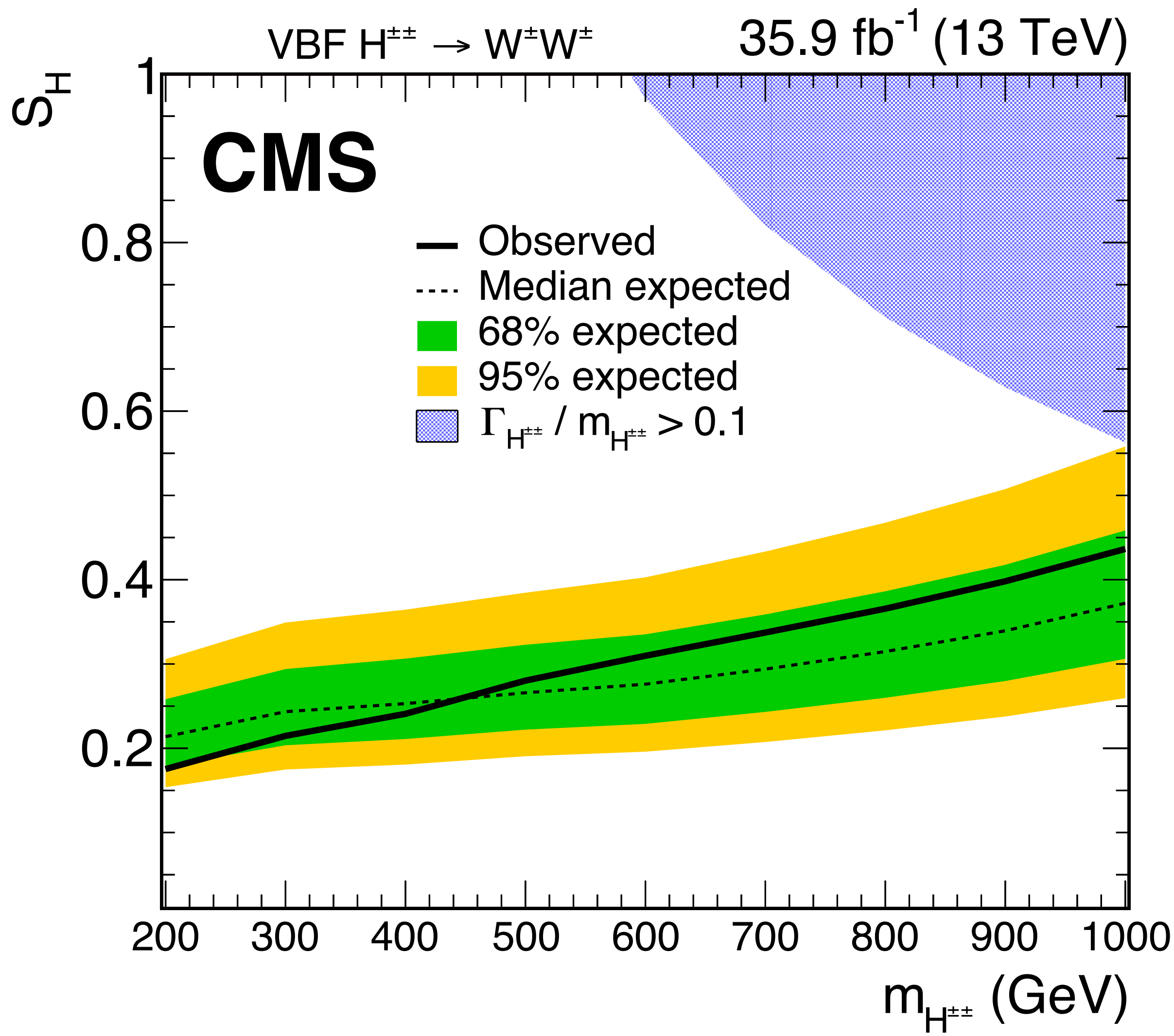
VBF $\to H_5^{\pm\pm} \to W^{\pm}W^{\pm}$ — VBF + like-sign dileptons + MET

VBF $\to H_5^{\pm} \to W^{\pm}Z$ — VBF + $qq\ell\ell$; VBF + 3ℓ + MET



Cross section $\propto s_H^2 \equiv$ fraction of M_W^2, M_Z^2 due to exotic scalars.

Most stringent constraint: VBF $\to H_5^{\pm\pm} \to W^{\pm}W^{\pm}$ CMS, arXiv:1709.05822



Also ATLAS + CMS searches for VBF $H_5^{\pm} \to W^{\pm}Z$

For $m_{H^{++}} > 1000$ GeV, theory upper bound on s_H from unitarity of quartic couplings takes over $\Rightarrow s_H \leq 0.5$ at $m_{H^{++}} = 1000$ GeV.

Cross section $\propto s_H^2 \equiv$ fraction of M_W^2, M_Z^2 due to exotic scalars

Probed by direct searches in GM model: $\sim 4\% - 20\%$

These are global symmetries. Match them back to the gauge symmetries? $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \leftarrow ? \rightarrow \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

- Global $\mathrm{SU}(2)_L$ is the gauged $\mathrm{SU}(2)_L$.

- The T^3 generator of global $\mathrm{SU}(2)_R$ is the hypercharge $\mathrm{U}(1)_Y$ generator.

- The T^3 generator of the custodial SU(2) is the electric charge operator (unbroken).

- Gauging only the one (hypercharge) generator of $\mathrm{SU}(2)_R$ breaks the global symmetry without promoting it to a full SU(2) gauge symmetry. $\rightarrow$ hypercharge is going to cause some trouble down the line....

Custodial symmetry violation in the GM model: a long history

Gunion, Vega & Wudka 1991 showed that computing the T parameter in the GM model yields infinity due to an uncancelled UV divergence caused by hypercharge violating the custodial symmetry at 1-loop. Full gauge-invariant but SU(2)$_L\times$SU(2)$_R$-violating scalar potential yields the needed counterterm.

Englert, Re & Spannowsky 1302.6505 applied S, T parameter constraints by subtracting a counterterm for T. (just divergence?)

Chiang, Kuo & Yagyu 1804.02633 calculated 1-loop renormalized predictions for h couplings in GM model and used measured T parameter as input to fix the relevant custodial-symmetry-violating counterterm.

Blasi, De Curtis & Yagyu 1704.08512 computed the RGEs and studied custodial violation from running up from custodial-symmetric theory at the weak scale. (RGEs independently calculated by us.)

Our implementation

B. Keeshan, HEL & T. Pilkington 1807.11511

- Assume custodial symmetry at some high scale Λ.

(accidental $SU(2)_L \times SU(2)_R$ coming from UV completion e.g. composite Higgs)

- Run down to weak scale $\Rightarrow$ custodial violation generated.

(1-loop RGEs, tree-level matching $\equiv$ leading log approximation)

(Have to do some iteration to get correct low-scale G_F, m_h, m_t.)

- Use measured value of ρ_0 to put an upper bound on scale Λ.

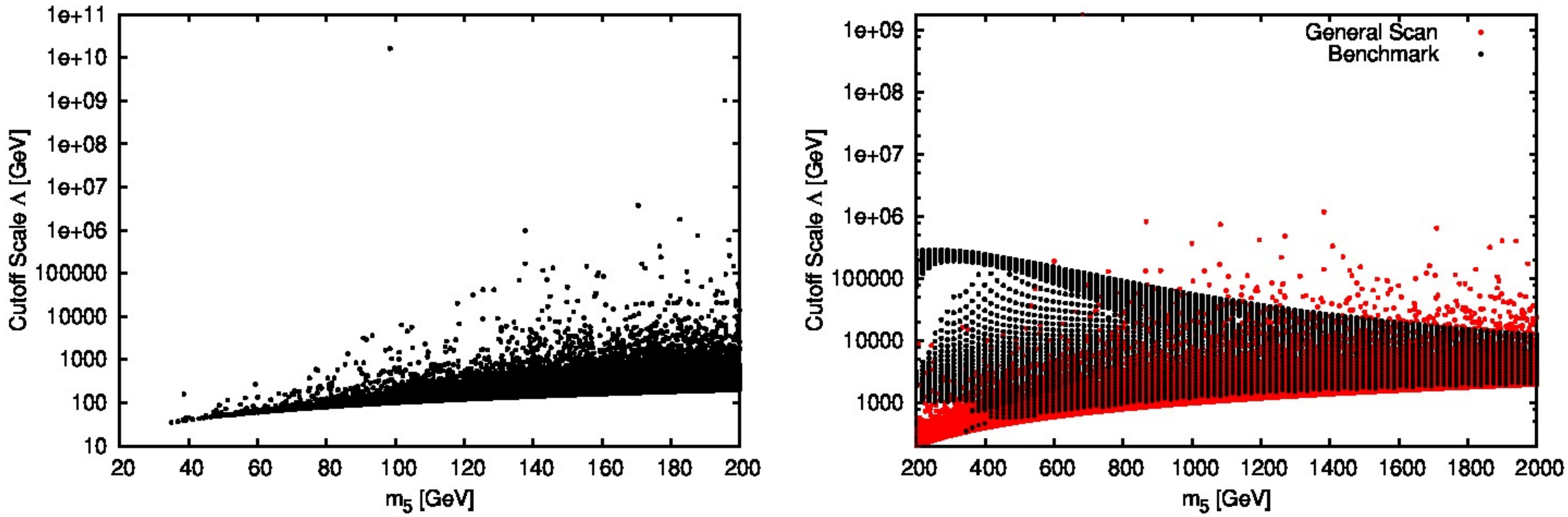
(Also require perturbative unitarity constraint on quartic couplings.)

- Subject to ρ_0 constraint (and perturbativity at Λ), quantify maximum allowed custodial symmetry violation and its phenomenological consequences.

Used a combination of benchmark plane and general parameter scans to study effects over the GM model parameter space.

Results: maximum cutoff scale Λ

B. Keeshan, HEL & T. Pilkington 1807.11511 + revisions in preparation



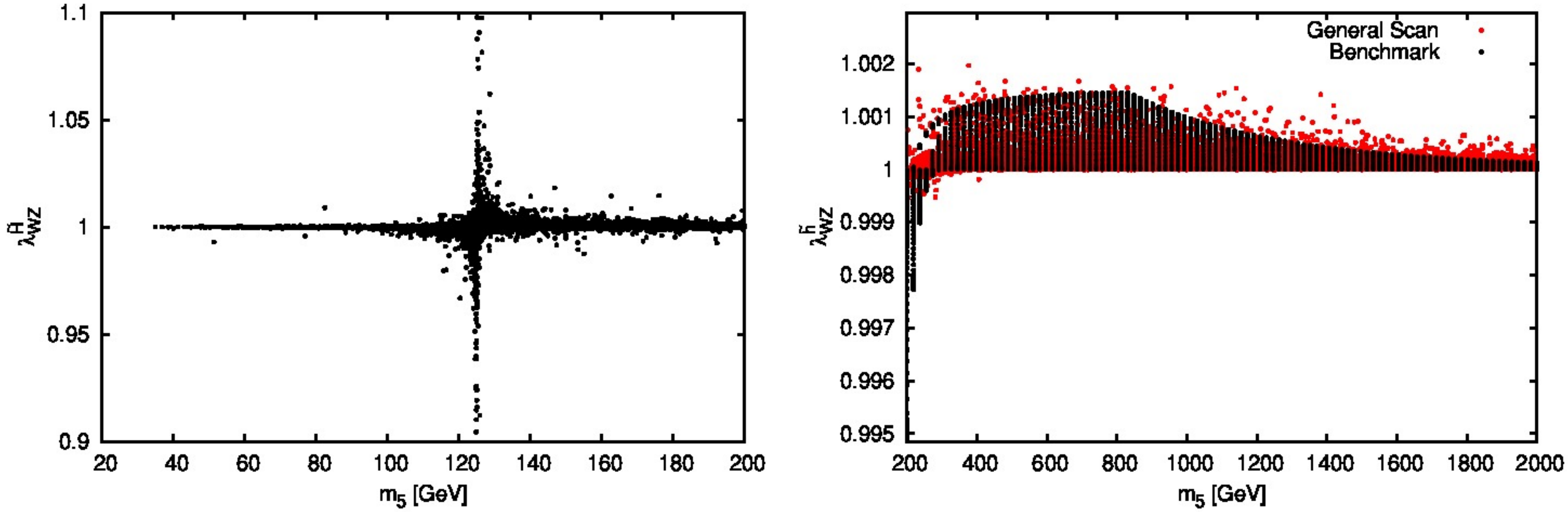
UV completion generally must appear below 10s to 100s of TeV.

Not too far away! Hierarchy problem is only "little".

But also not right on top of our heads: generally high enough to be able to ignore loop effects or dimension-6 operators induced by the UV completion.

Results: $\lambda_{WZ} \equiv hWW/hZZ$ normalized to SM

B. Keeshan, HEL & T. Pilkington 1807.11511 + revisions in preparation



Deviation from SM prediction ($\lambda^h_{WZ} = 1$) below percent-level except for resonant mixing between h and H_5^0 at $m_5 \sim 125$ GeV.

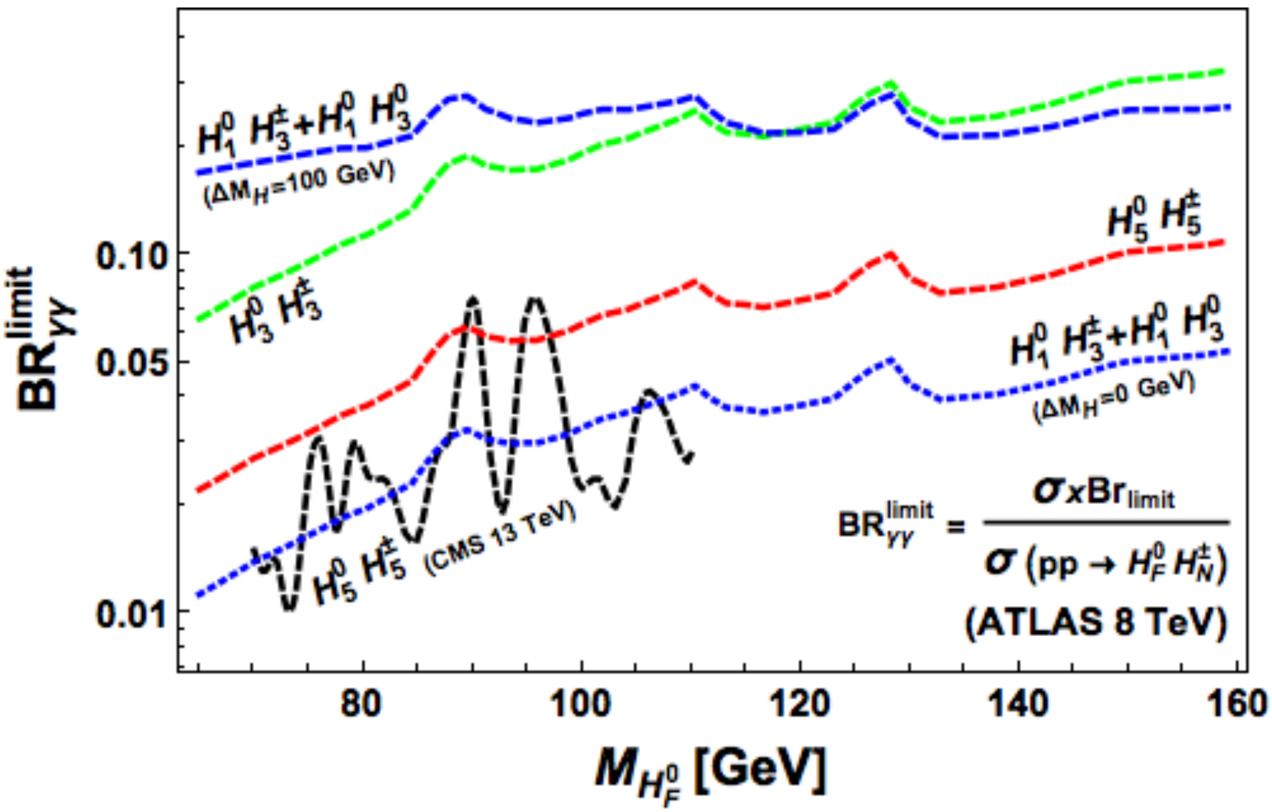
Current LHC precision: $\lambda^h_{WZ} = 0.88^{+0.10}_{-0.09}$ ATLAS + CMS Run 1, 1606.02266

Future: HL-LHC few % / ILC ~ 0.5% / FCC-ee ~ 0.2%

Results: custodial-violating mixing of Higgs states

At tree level, H_5^0 is fermiophobic due to custodial symmetry:
$H_5^0 \to \gamma\gamma$ gives a powerful search channel at low mass!

Drell-Yan $pp \to H_5^0 H_5^\pm$ depends only on m_5 and gauge couplings:



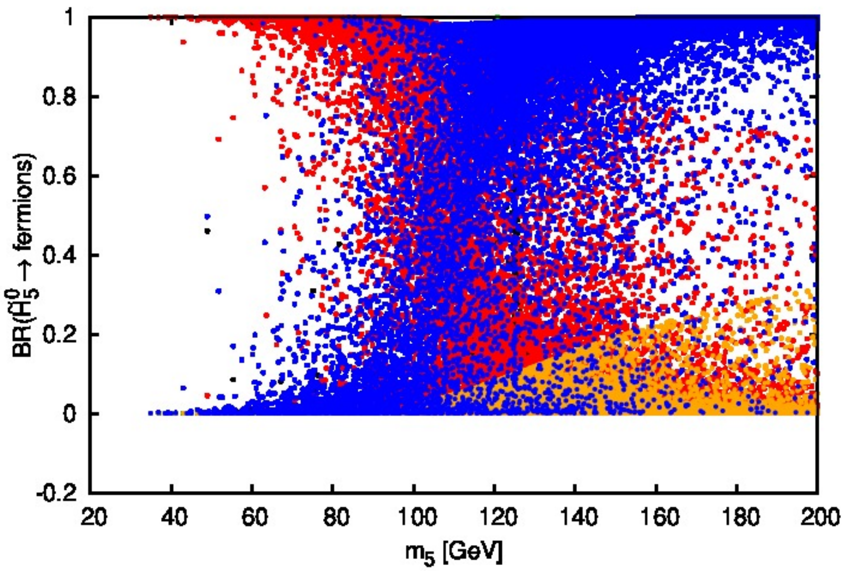
Vega, Vega-Morales & Xie, 1805.01970

Results: custodial-violating mixing of Higgs states

B. Keeshan, HEL & T. Pilkington 1807.11511 + revisions in preparation

Custodial symmetry violation mixes doublet into H_5^0, can induce fermionic decays that might compete with $\gamma\gamma$.

The effect is generally very small unless H_5^0–h mixing is resonant.



$H_5^0 \to \gamma\gamma$

$H_5^0 \to WW + ZZ$

$H_5^0 \to Z\gamma$

$H_5^0 \to f\bar{f}$ (few points at $m_5 \sim 125$ GeV)

$H_5^0 \to H_3 H_3$ (few points above 180 GeV)

$H_5^0 \to \gamma\gamma$ still strongly constraining for masses below ~ 110 GeV.

Conclusions and outlook

Custodial symmetry is accidental in the Standard Model!

Generally has to be built in to BSM models to avoid stringent constraints on the ρ_0 parameter.

Can build "exotic" extended Higgs sectors with custodial symmetry, but hypercharge interactions violate it at 1-loop level.

Fortunately the effect is fairly small!

Quantified explicitly in Georgi-Machacek model, prototype for LHC searches for "exotic" extended Higgs sectors:

- UV completion generally lies below 10s to 100s of TeV

  forced by perturbative unitarity + measured ρ parameter

- Custodial-violating effects are generally small

  assumption of custodial-symmetric GM is good for LHC searches

BACKUP SLIDES

Introduction

Can we constrain the possibility that "exotic" Higgs fields (isospin $> 1/2$) contribute to electroweak symmetry breaking?

Generically this is very strongly constrained by the ρ parameter:

$$\rho \equiv \frac{\text{weak neutral current}}{\text{weak charged current}} = \frac{(g^2 + g'^2)/M_Z^2}{g^2/M_W^2} = \frac{v_\phi^2 + a\langle X^0\rangle^2}{v_\phi^2 + b\langle X^0\rangle^2}$$

$$\begin{aligned} a &= 4\left[T(T+1) - Y^2\right] c \\ b &= 8Y^2 \end{aligned}$$

$Q = T^3 + Y$; SM doublet: $Y = 1/2$

Expt: $\rho = 1.00037 \pm 0.00023$ (2016 PDG)

Need to do some model-building; otherwise $v_{\text{exotic}} \ll v_{\text{doublet}}$.

There are only two known approaches:

1) Use the septet $(T, Y) = (3, 2)$: $\rho = 1$ by accident!
Doublet $\left(\frac{1}{2}, \frac{1}{2}\right)$ + septet $(3, 2)$: Scalar septet model

Hisano & Tsumura, 1301.6455; Kanemura, Kikuchi & Yagyu, 1301.7303

2) Use global $SU(2)_L \times SU(2)_R$ imposed on the scalar potential
Global $SU(2)_L \times SU(2)_R \to$ custodial SU(2) ensures tree-level $\rho = 1$
Doublet + triplets $(1, 0) + (1, 1)$: Georgi-Machacek model

Georgi & Machacek 1985; Chanowitz & Golden 1985

Doublet + quartets $\left(\frac{3}{2}, \frac{1}{2}\right) + \left(\frac{3}{2}, \frac{3}{2}\right)$:
Doublet + quintets $(2, 0) + (2, 1) + (2, 2)$:
Doublet + sextets $\left(\frac{5}{2}, \frac{1}{2}\right) + \left(\frac{5}{2}, \frac{3}{2}\right) + \left(\frac{5}{2}, \frac{5}{2}\right)$:
Generalized Georgi-Machacek models

Galison 1984; Robinett 1985; HEL 1999; Chang et al 2012; HEL & Rentala 2015

Larger than sextets $\to$ too many large multiplets, violates perturbativity

Can also have duplications, combinations $\to$ ignore that here.

Both approaches have theoretical "issues":

1) Can't give the septet a vev through spontaneous breaking without generating a physical massless Goldstone boson.

Have to couple it to the SM doublet through a dimension-7 $X\Phi^*\Phi^5$ term Hisano & Tsumura 2013

Need the UV completion to be nearby!

2) Global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is broken by gauging hypercharge.

Gunion, Vega & Wudka 1991

Special relations among params of *full* gauge-invariant scalar potential can only hold at one energy scale: violated by running due to hypercharge. Garcia-Pepin, Gori, Quiros, Vega, Vega-Morales, Yu 2014

Need the UV completion to be nearby!

This talk: quantify (2) in the Georgi-Machacek model.

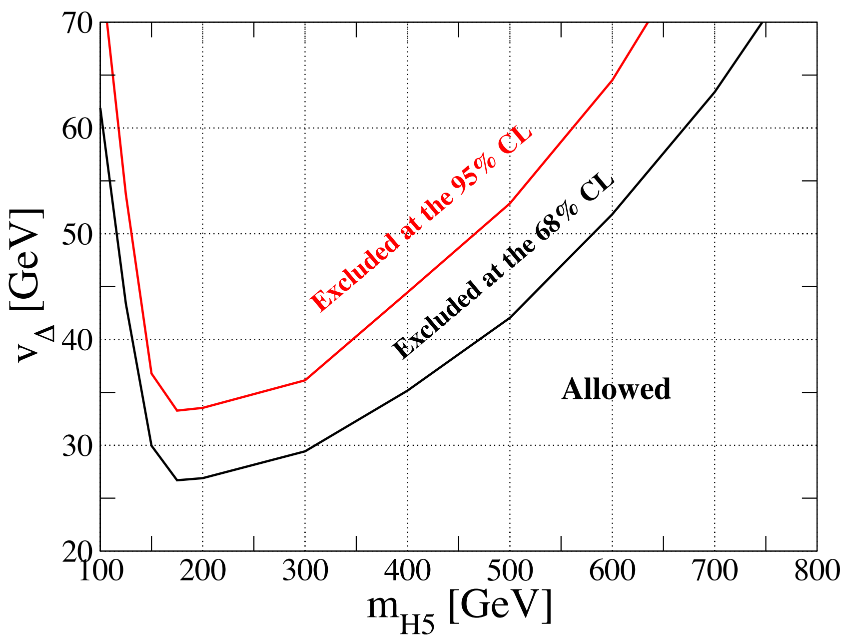
Searches

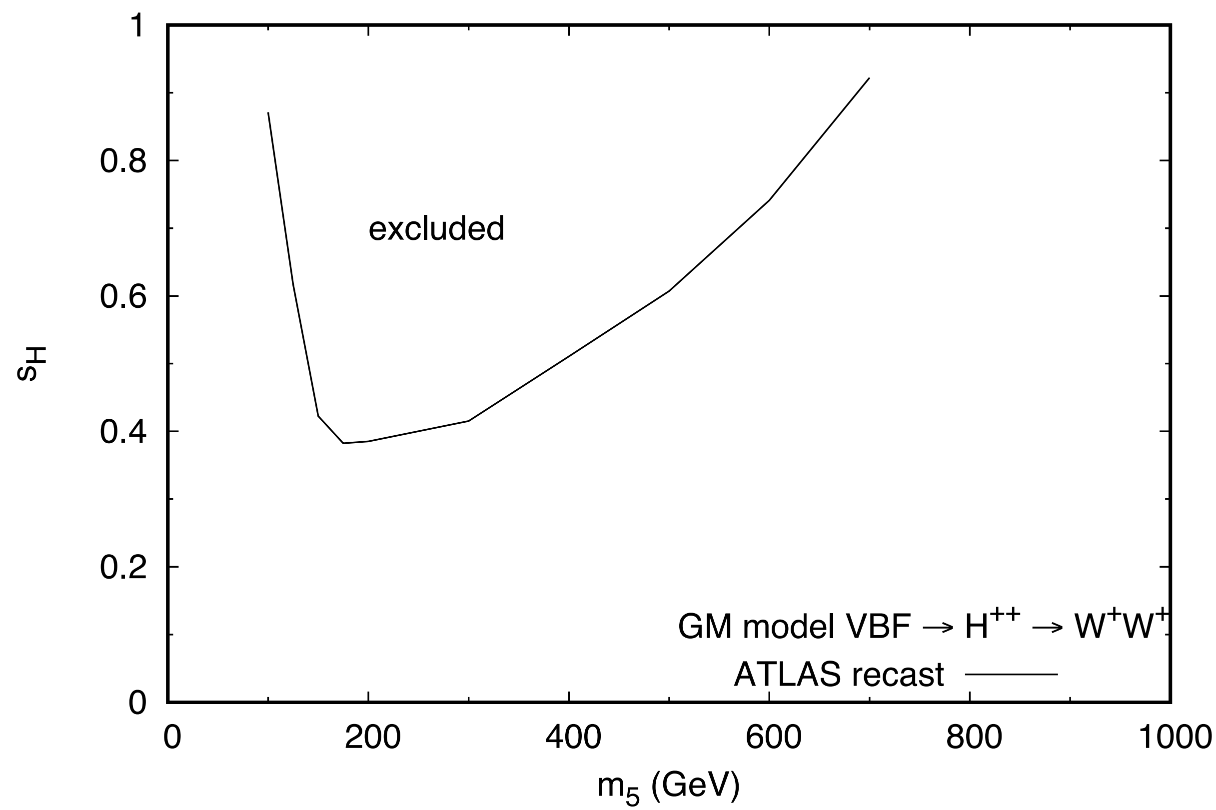
SM VBF $\to W^\pm W^\pm \to \ell^\pm \ell^\pm$ + MET cross section measurement

ATLAS Run 1 1405.6241, PRL 2014

Recast to constrain VBF $\to H_5^{\pm\pm} \to W^\pm W^\pm \to \ell^\pm \ell^\pm$ + MET

Chiang, Kanemura, Yagyu, 1407.5053

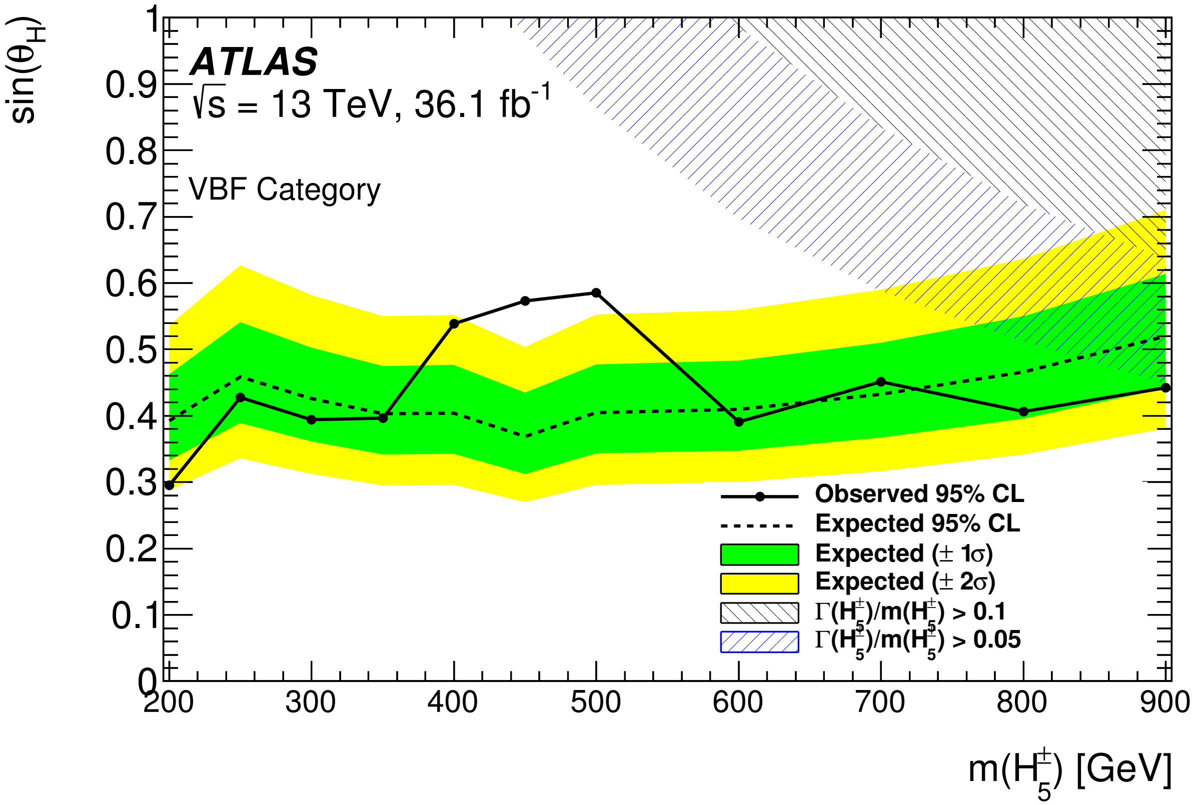




Searches

VBF $H_5^\pm \to W^\pm Z \to \ell^\pm \ell^+ \ell^- +$ MET (ATLAS Run 2)

ATLAS 1806.01532



Stronger upper bound on s_H for $m_5 \in (700, 900)$ GeV compared to $H_5^{\pm\pm}$

Full gauge-invariant potential:

$$
\begin{aligned}
V(\phi,\chi,\xi) \;=\; & \tilde\mu_2^2 \phi^\dagger\phi + \tilde\mu_3'^2 \chi^\dagger\chi + \frac{\tilde\mu_3^2}{2}\xi^\dagger\xi \\
& + \tilde\lambda_1 (\phi^\dagger\phi)^2 + \tilde\lambda_2 |\tilde\chi^\dagger\chi|^2 + \tilde\lambda_3 (\phi^\dagger\tau^a\phi)(\chi^\dagger t^a\chi) \\
& + \left[\tilde\lambda_4 (\tilde\phi^\dagger\tau^a\phi)(\chi^\dagger t^a\xi) + \text{h.c.}\right] + \tilde\lambda_5 (\phi^\dagger\phi)(\chi^\dagger\chi) \\
& + \tilde\lambda_6 (\phi^\dagger\phi)(\xi^\dagger\xi) + \tilde\lambda_7 (\chi^\dagger\chi)^2 + \tilde\lambda_8 (\xi^\dagger\xi)^2 \\
& + \tilde\lambda_9 |\chi^\dagger\xi|^2 + \tilde\lambda_{10} (\chi^\dagger\chi)(\xi^\dagger\xi) \\
& - \frac{1}{2}\left[\tilde M_1' \phi^\dagger\Delta_2\tilde\phi + \text{h.c.}\right] + \frac{\tilde M_1}{\sqrt{2}}\phi^\dagger\Delta_0\phi - 6\tilde M_2 \chi^\dagger\overline{\Delta}_0\chi
\end{aligned}
$$

where

$$
\begin{aligned}
\Delta_2 &\equiv \sqrt{2}\tau^a U_{ai}\chi_i = \begin{pmatrix} \chi^+/\sqrt{2} & -\chi^{++} \\ \chi^0 & -\chi^+/\sqrt{2} \end{pmatrix}, \\
\Delta_0 &\equiv \sqrt{2}\tau^a U_{ai}\xi_i = \begin{pmatrix} \xi^0/\sqrt{2} & -\xi^+ \\ -\xi^{+*} & -\xi^0/\sqrt{2} \end{pmatrix}, \\
\overline{\Delta}_0 &\equiv -t^a U_{ai}\xi_i = \begin{pmatrix} -\xi^0 & \xi^+ & 0 \\ \xi^{+*} & 0 & \xi^+ \\ 0 & \xi^{+*} & \xi^0 \end{pmatrix}.
\end{aligned}
$$

Minimize potential, compute mass matrices, etc.

16 Lagrangian parameters compared to 9 in original GM model: Matching gauge-invariant potential to original GM model yields

$$
\begin{aligned}
\tilde{\mu}_2^2 &= \mu_2^2 & \tilde{\lambda}_6 &= 2\lambda_2 \\
\tilde{\mu}_3'^2 &= \mu_3^2 & \tilde{\lambda}_7 &= 2\lambda_3 + 4\lambda_4 \\
\tilde{\mu}_3^2 &= \mu_3^2 & \tilde{\lambda}_8 &= \lambda_3 + \lambda_4 \\
\tilde{\lambda}_1 &= 4\lambda_1 & \tilde{\lambda}_9 &= 4\lambda_3 \\
\tilde{\lambda}_2 &= 2\lambda_3 & \tilde{\lambda}_{10} &= 4\lambda_4 \\
\tilde{\lambda}_3 &= -2\lambda_5 & \tilde{M}_1' &= M_1 \\
\tilde{\lambda}_4 &= -\sqrt{2}\lambda_5 & \tilde{M}_1 &= M_1 \\
\tilde{\lambda}_5 &= 4\lambda_2 & \tilde{M}_2 &= M_2
\end{aligned}
$$

RGEs with $g' = 0$ preserve these relations.

Keeping $g' \neq 0$ violates these relations and introduces custodial symmetry violation through the RGE running.

Our implementation

Details of the benchmark:

- Start with a benchmark scenario at the weak scale* (for concreteness, and to get G_F, m_h close to their correct values)
* "weak scale" $= m_5$

H5plane benchmark (introduced by HXSWG for H_5 LHC searches)

Fixed Parameters	Variable Parameters	Dependent Parameters
$G_F = 1.1663787 \times 10^{-5}$ GeV^{-2}	$m_5 \in [200, 3000]$ GeV	$\lambda_2 = 0.4 m_5/(1000$ GeV$)$
$m_h = 125$ GeV	$s_H \in (0, 1)$	$M_1 = \sqrt{2} s_H (m_5^2 + v^2)/v$
$\lambda_3 = -0.1$		$M_2 = M_1/6$
$\lambda_4 = 0.2$		

- Run up with $g' = 0$ (custodial symmetric!) to some scale Λ; check perturbativity of quartic couplings (avoid Landau pole)
$\Rightarrow$ upper bound on Λ to avoid perturbativity violation

Our implementation

Details (continued):

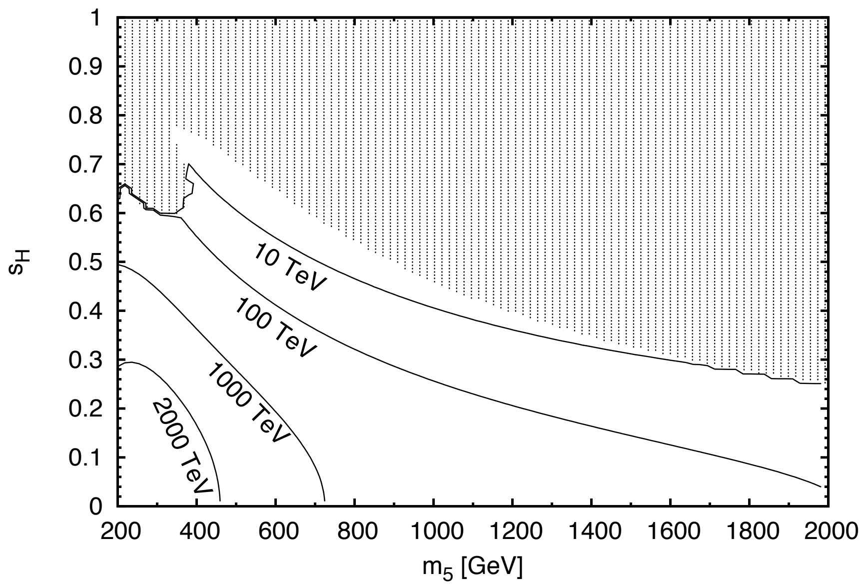
- Run back down with $g' \neq 0$ to get the custodial-violating Lagrangian parameters at the weak scale

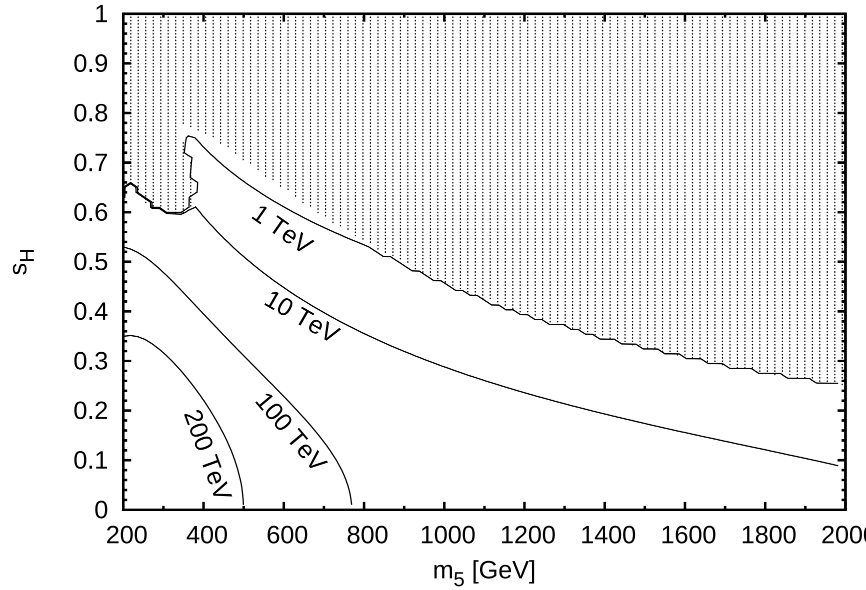
- Compute vevs $\to G_F$ and mass matrices $\to m_h$; adjust original weak-scale inputs and iterate until these match experiment in custodial violating theory

- Compute ρ; adjust upper bound on Λ if necessary
$\rho = 1.00037 \pm 0.00023$ (2016 PDG) [require within $\pm 2\sigma$]

- Compute weak-scale predictions for custodial-violating observables (λ^h_{WZ}, mass splittings, mixings)

Results (within H5plane benchmark): cutoff scale



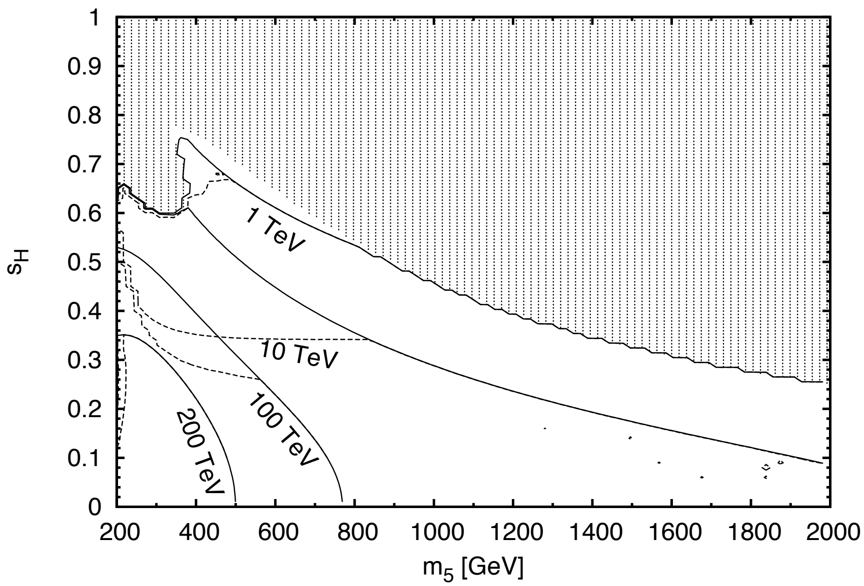


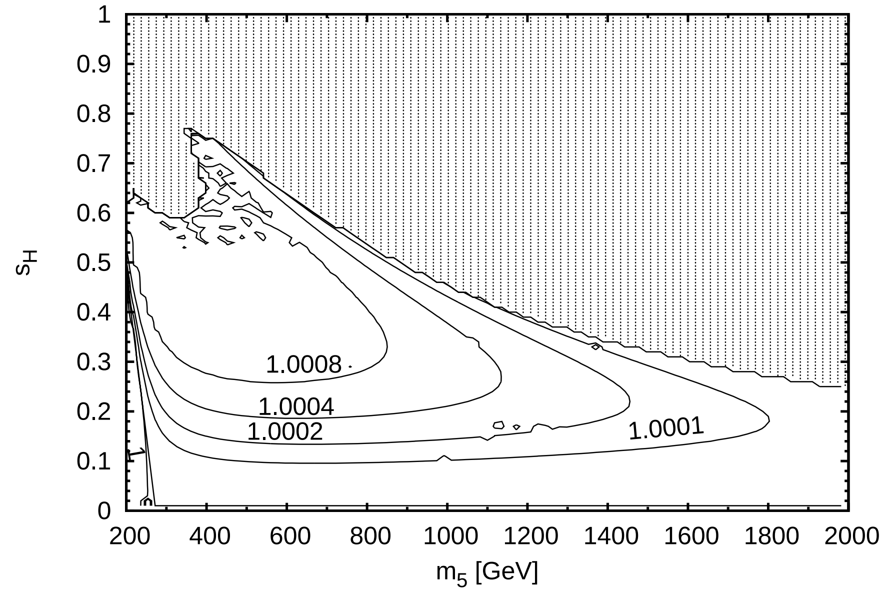
Left: Scale of Landau pole

Right: Highest scale at which perturbative unitarity constraints on custodial-symmetric λ_i remain satisfied

UV completion must appear below 10s to 100s of TeV

Results (within H5plane benchmark): ρ parameter





Left: Maximum cutoff scale including ρ parameter constraint (dashed) + perturbative unitarity (solid)

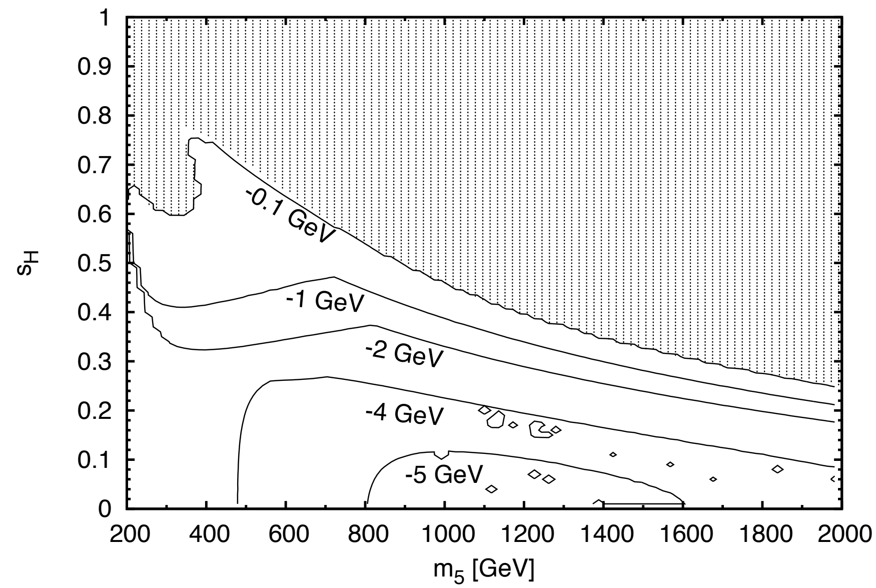
Right: Weak-scale value of ρ, for Λ as large as possible

ρ_0 samples full 2σ allowed range
$\Delta\rho_0$ is positive in most of H5plane benchmark parameter space

Results (within H5plane benchmark): mass splittings

Plot: $m_{H_3^\pm} - m_{H_3^0}$
for Λ as large as possible

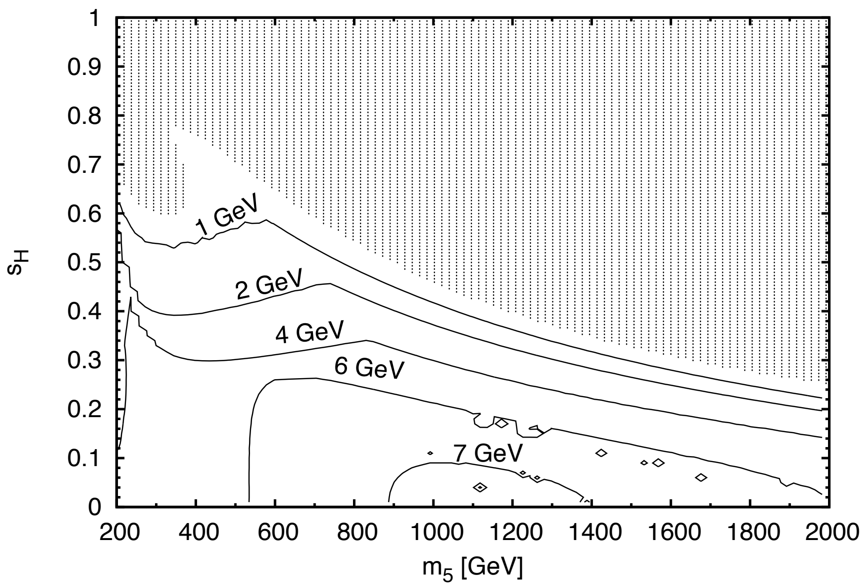
(negative values: $H_3^\pm$ is lighter)

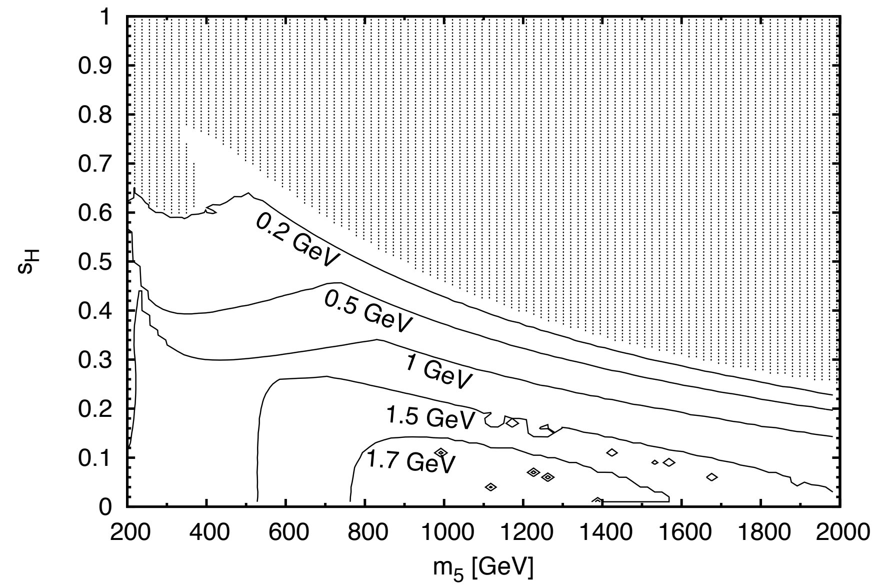


Custodial-violating mass splitting of $H_3^{0,\pm}$ is at most 5.3 GeV.
$m_{H_3^0} > m_{H_3^\pm}$ everywhere in H5plane benchmark.

Measurement prospects: $H_3^0 \to b\bar{b}, t\bar{t}$; $H_3^+ \to t\bar{b}$
Couplings as in Type-I 2HDM: down-type decays not enhanced
Mass splitting too small to detect at LHC

Results (within H5plane benchmark): mass splittings





Left: $m_{H_5^{\pm\pm}} - m_{H_5^0}$
for Λ as large as possible

Right: $m_{H_5^{\pm}} - m_{H_5^0}$

Custodial-violating mass splitting of $H_5^{0,\pm,\pm\pm}$ is at most 7.2 GeV.
$m_{H_5^{\pm\pm}} > m_{H_5^{\pm}} > m_{H_5^0}$ everywhere in H5plane benchmark.

Decays are to VV – similar challenges to detect small mass splittings at LHC.