Sending quantum information through a quantum field

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and
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Goal:

Understand (fundamentally) how quantum information is transmitted through free space

Why send quantum information through free space?

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- Crucial for quantum computing/communication/cryptography
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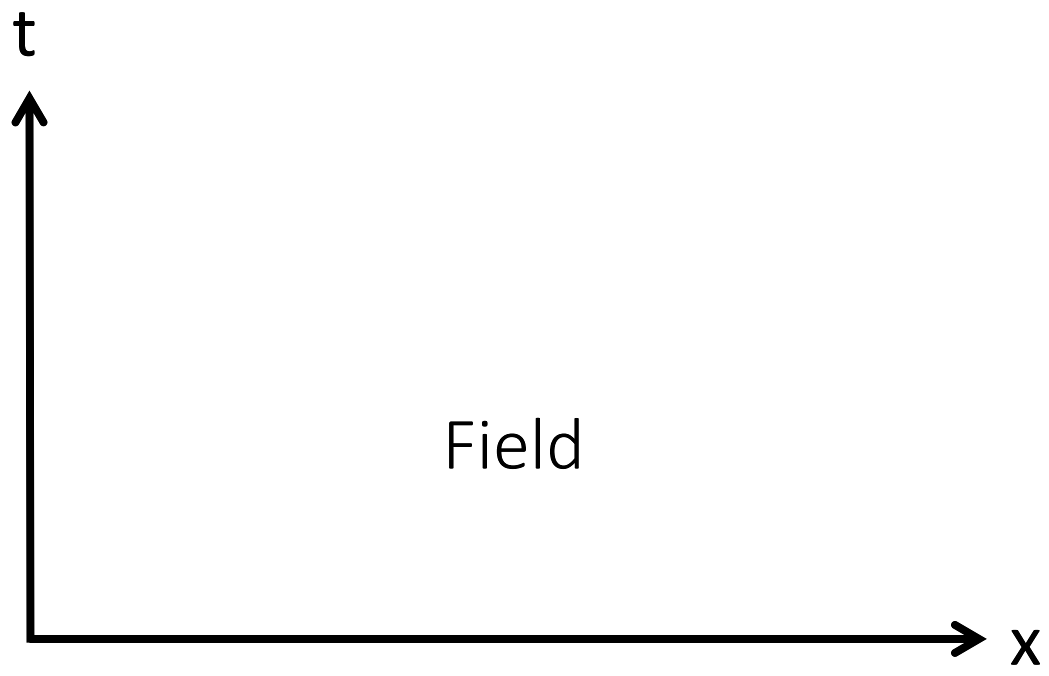
Why through free space?

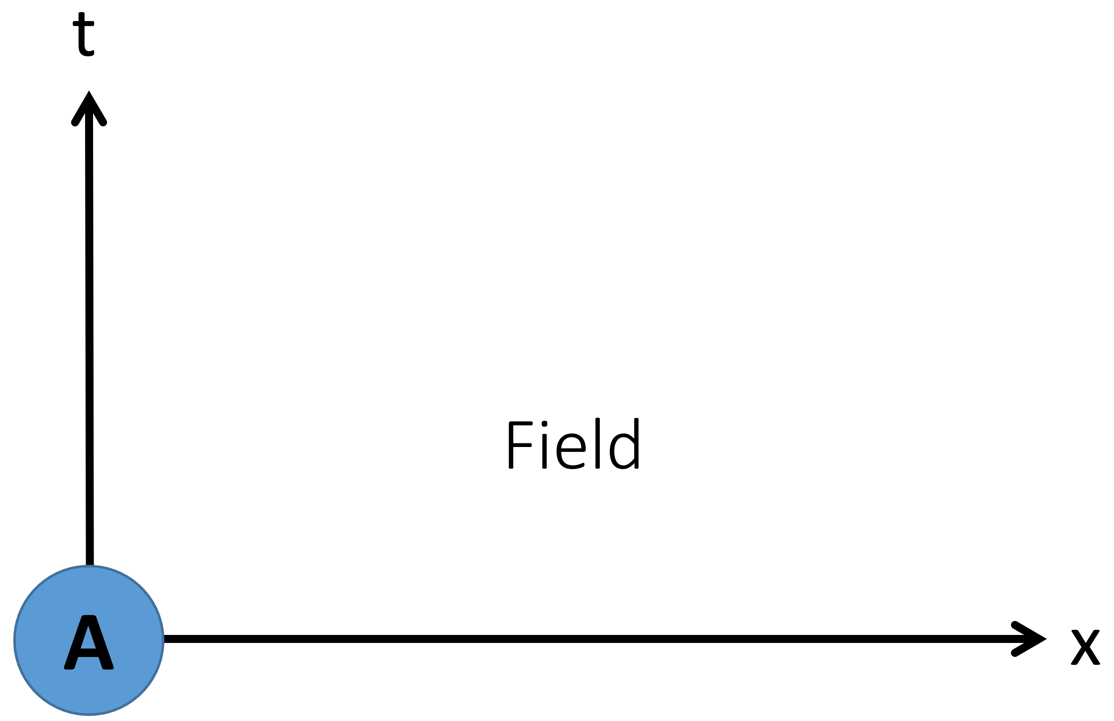
- Greater distances than with optical fibres (1000+ km)
- Develop "global quantum internet" [Kimble 2008]
- Probe gravitational-quantum interactions [Rideout *et al.* 2012]

Communication via relativistic quantum fields



Communication via relativistic quantum fields

1. Alice encodes message in field



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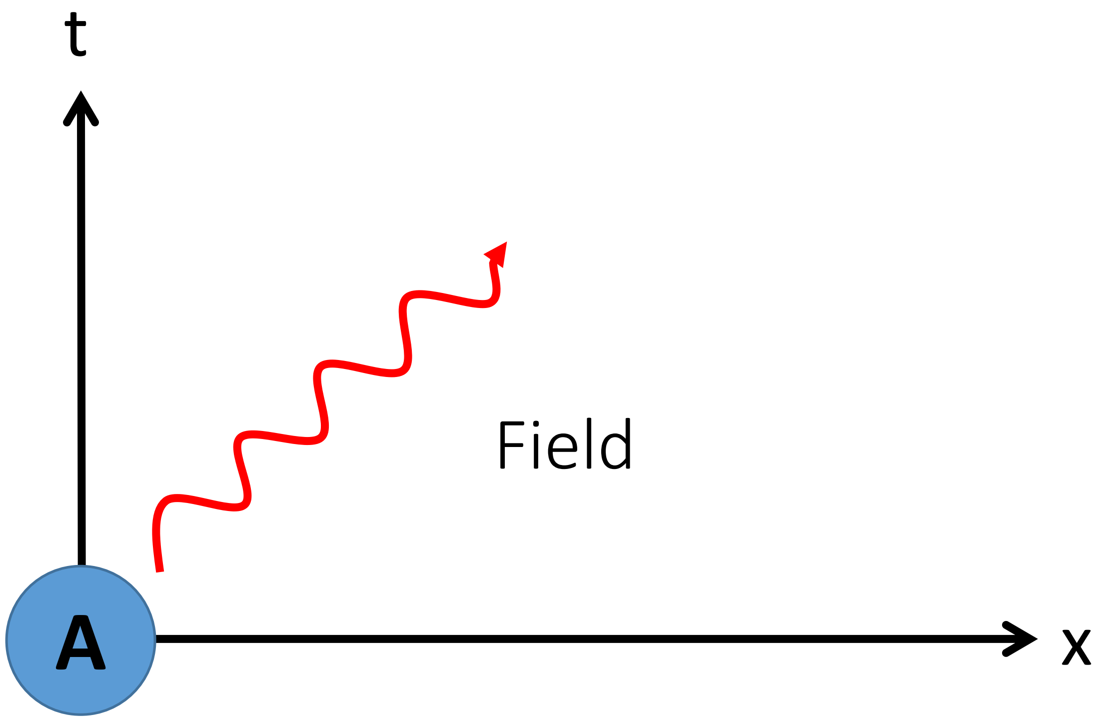

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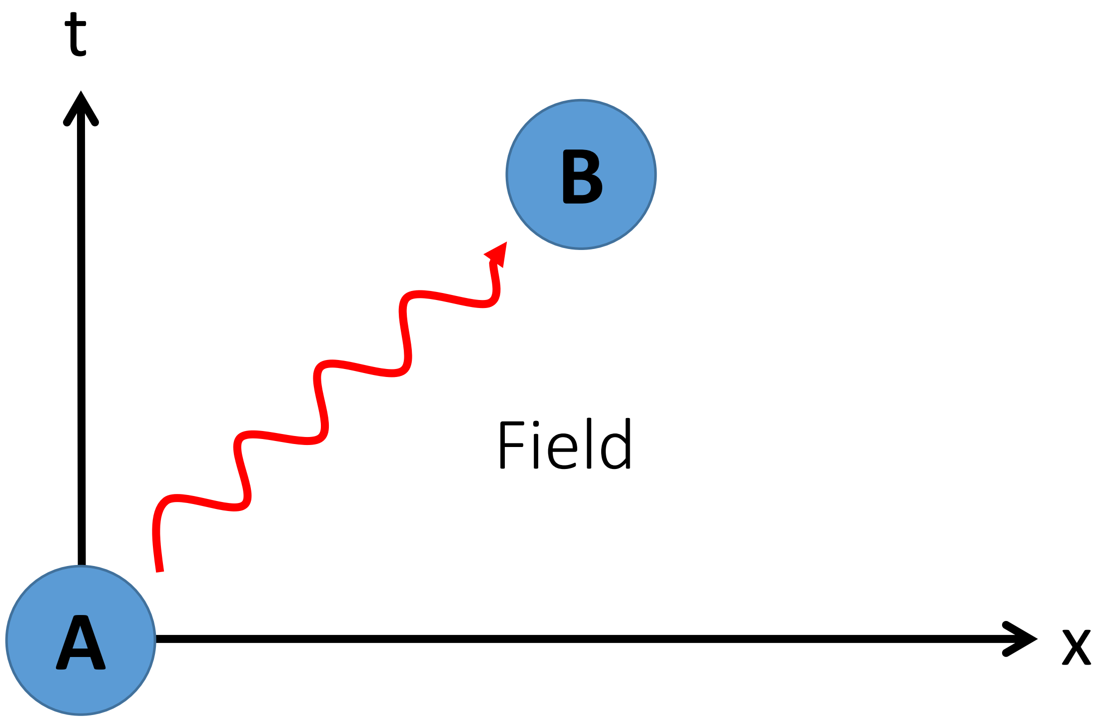

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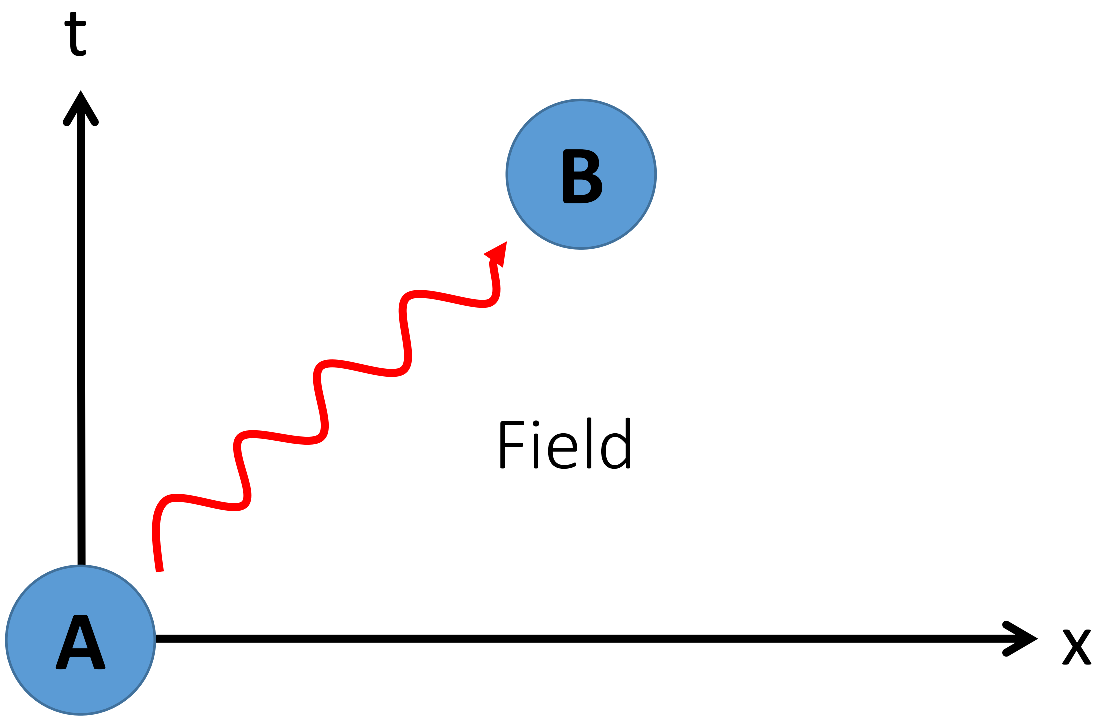

How do we model this?

Unruh-DeWitt model

- Model of light-matter interaction:
  - Light: massless scalar field $\phi(x,t) = \int \frac{d^d k}{\sqrt{(2\pi)^d 2|k|}} \left(a_k e^{-i(|k|t - k\cdot x)} + h.c.\right)$
  - Matter (Alice and Bob): 2-level quantum systems (qubits)
  - Interactions: Linear coupling, e.g.

$$H_I(t) = \lambda\, \chi(t)\sigma_x \otimes \int d^d x\, F(x)\phi(x,t)$$

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- Realistic model of atom-EM field interaction [Martín-Martínez 2013]
- Used to study Unruh/Hawking effects, probe spacetime entanglement structure, etc.

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quantum communication via quantum field

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But first:

Classical communication via quantum field

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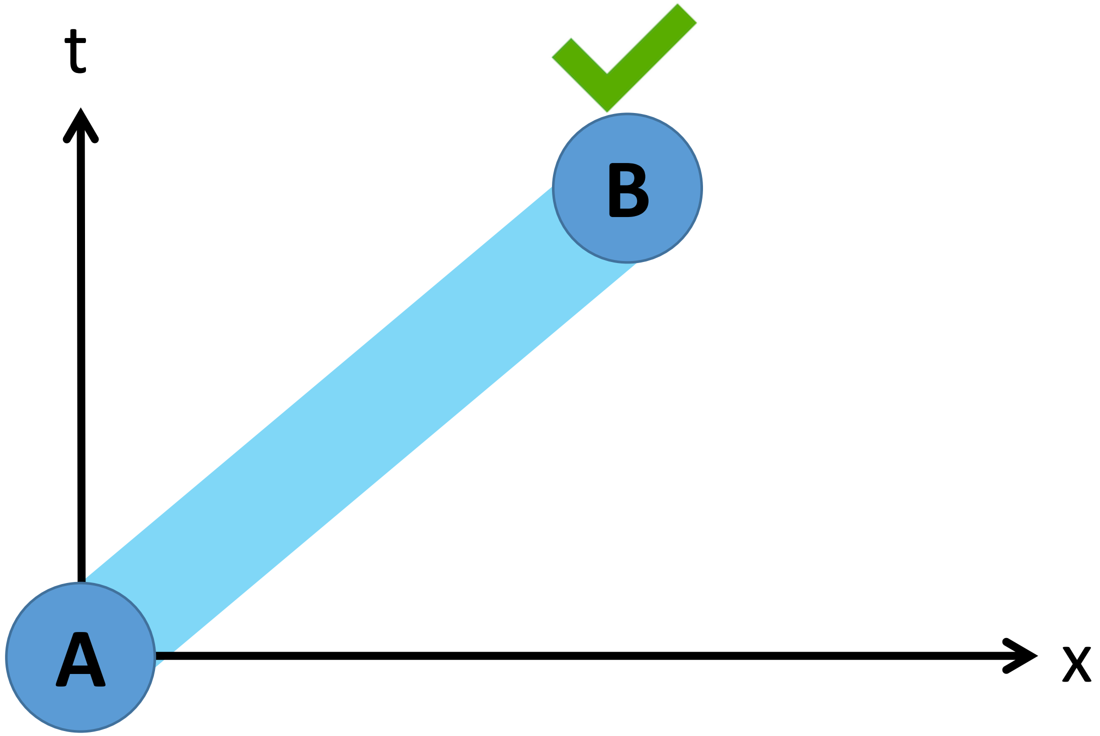
Does this *extremely simple* protocol work??

Does classical communication protocol work?

- If Bob is on Alice's light cone: YES



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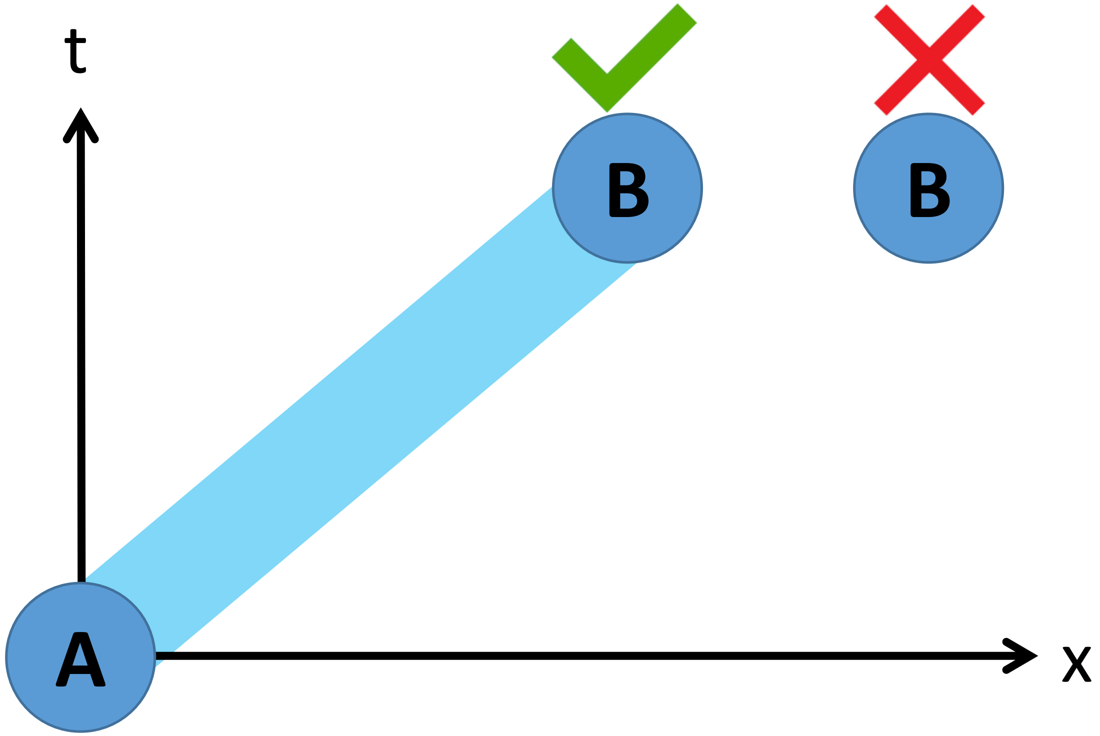

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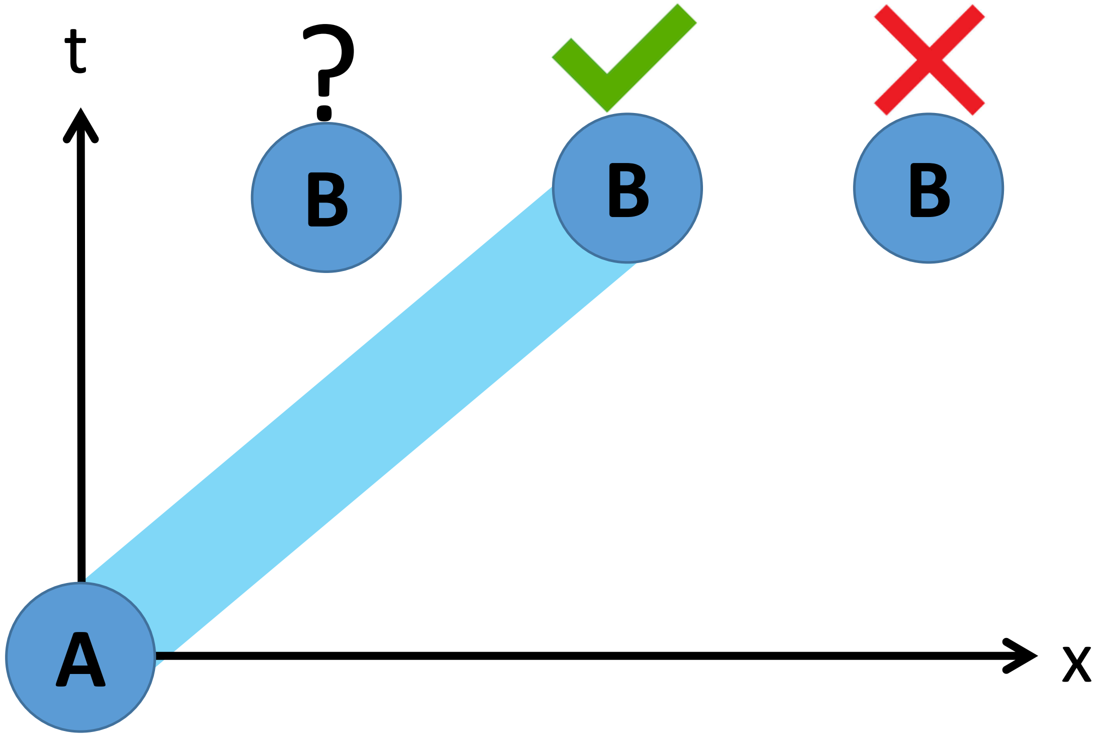

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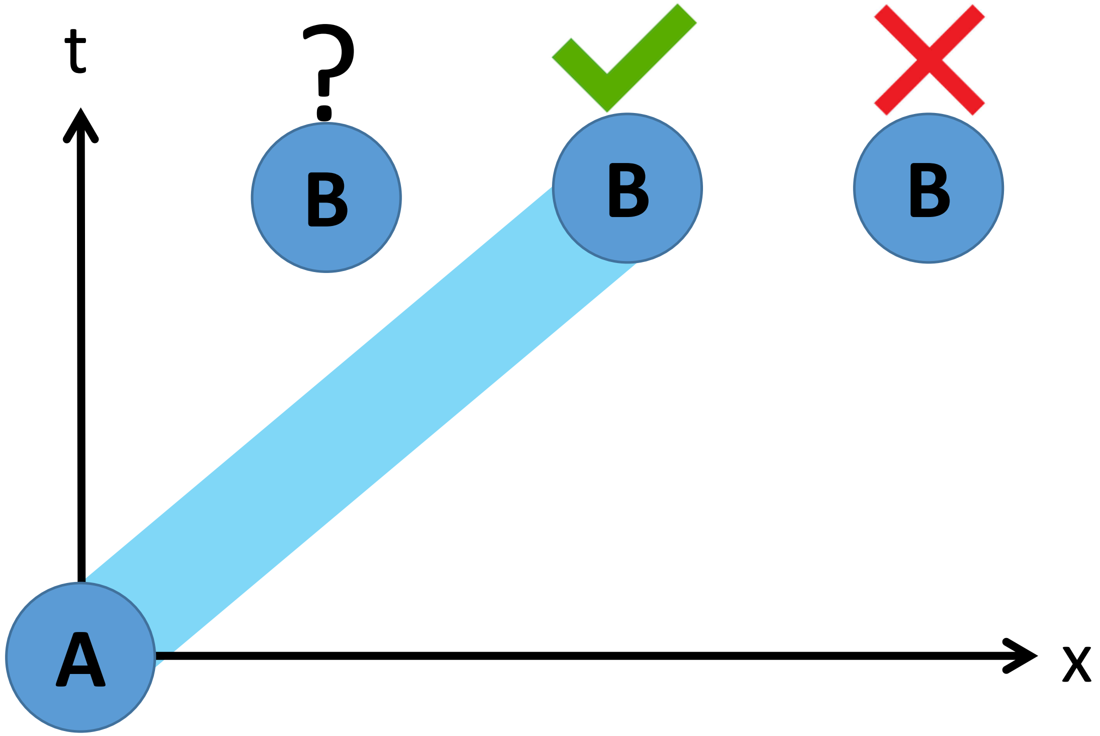

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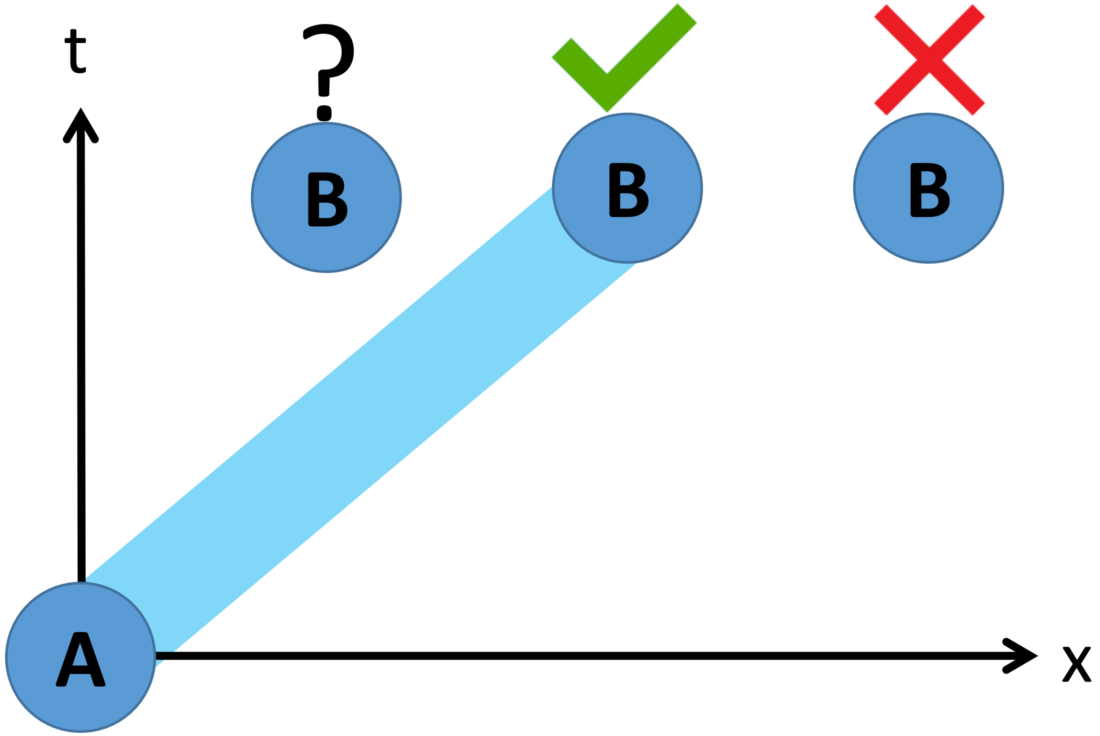

- What if Bob is inside light cone?
  - In (3+1)D flat space: NO (strong Huygens principle)
  - In (2+1)D flat space (and most other spacetimes): YES [Jonsson *et al.* 2015, P.S. *et al.* 2017]

Main message:

Constructing a classical channel via a quantum field is <u>easy</u>!

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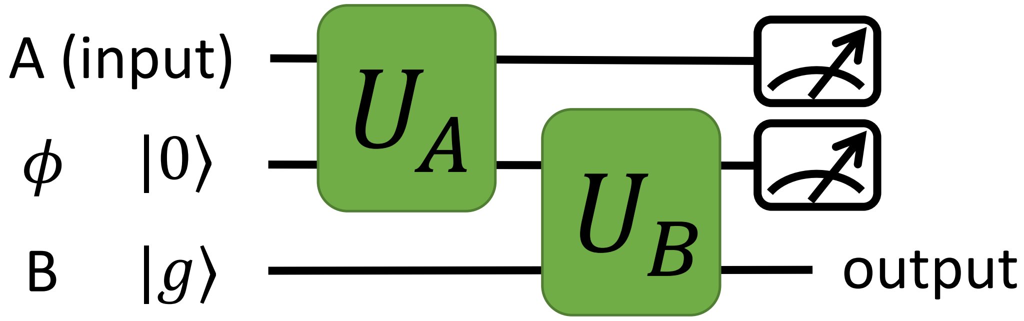
What about constructing a *quantum* channel?

Constructing a quantum channel

- Want a channel of the form:



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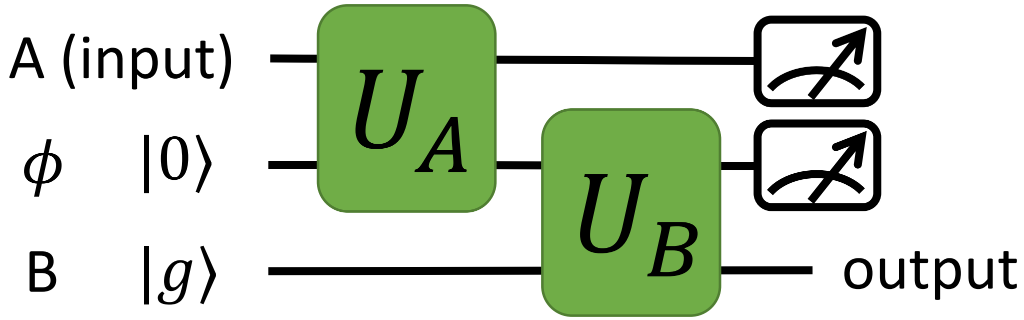

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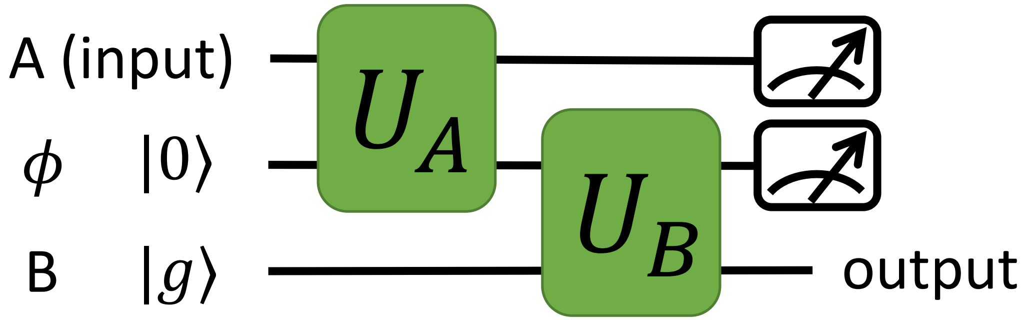

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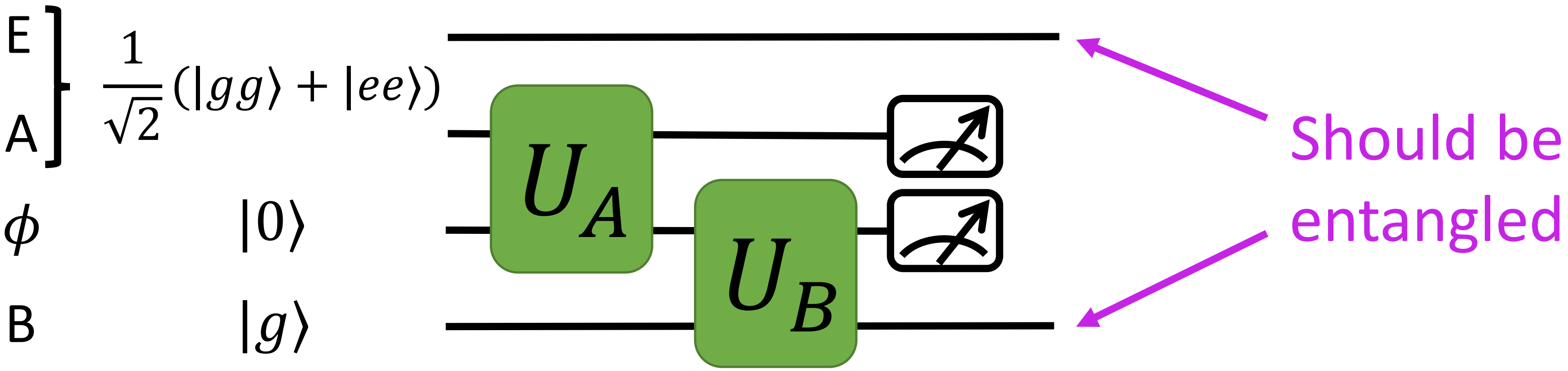
- Sending quantum information ⇒ sending entanglement



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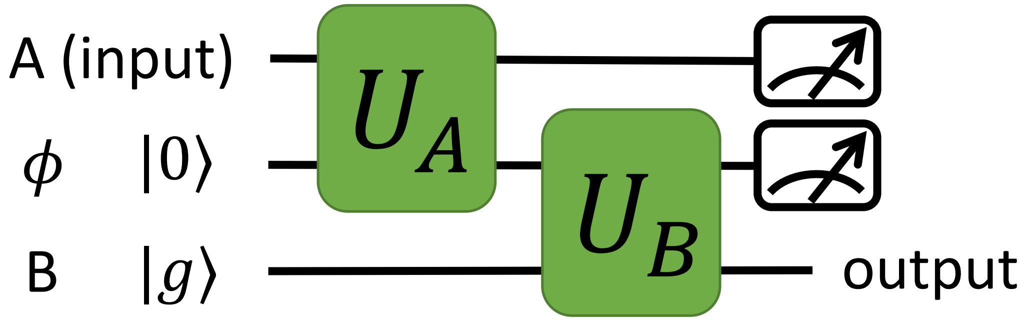

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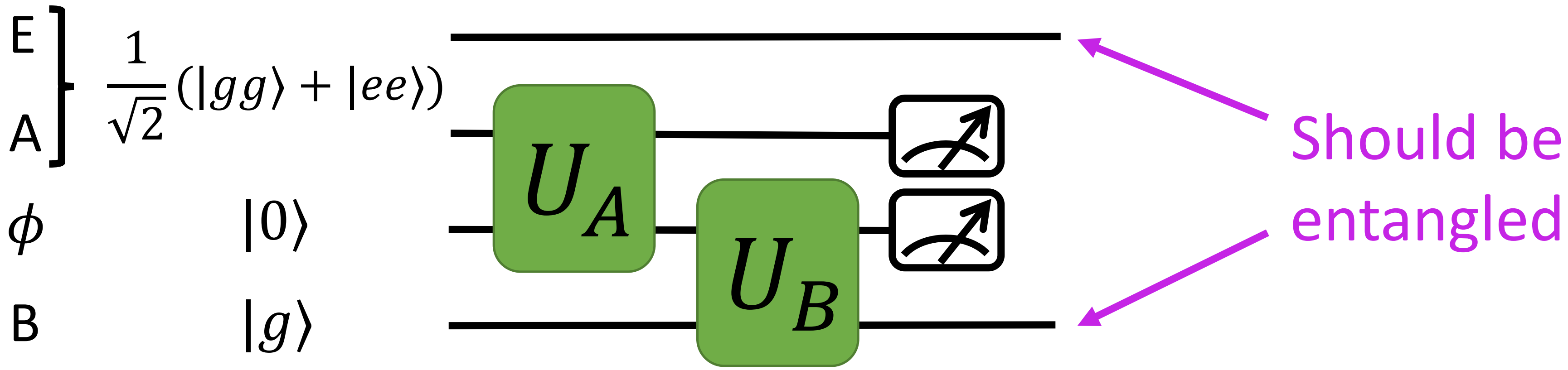

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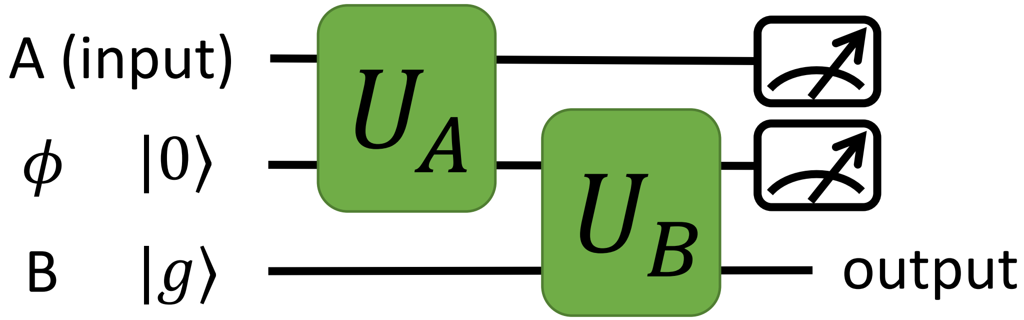

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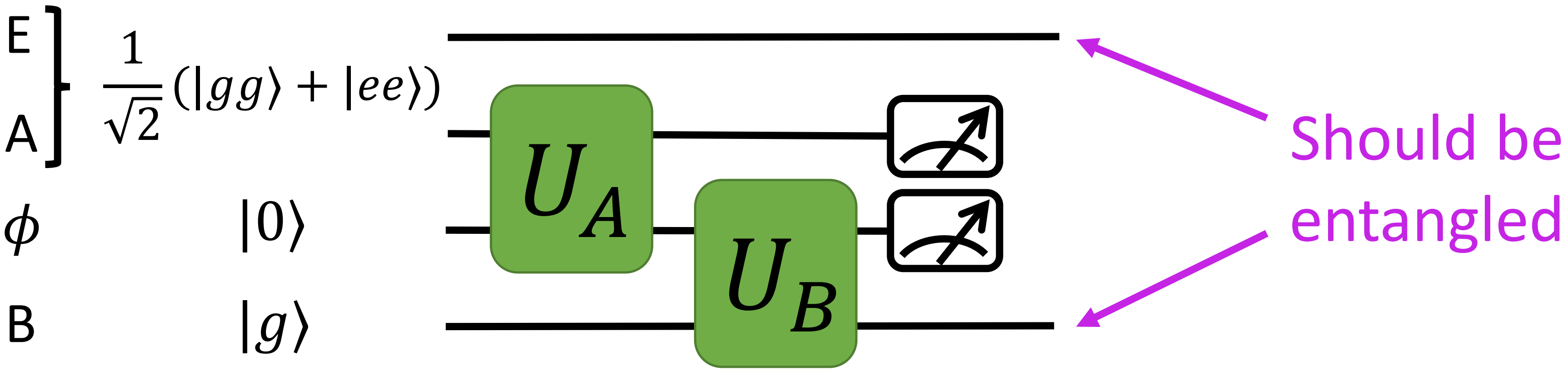
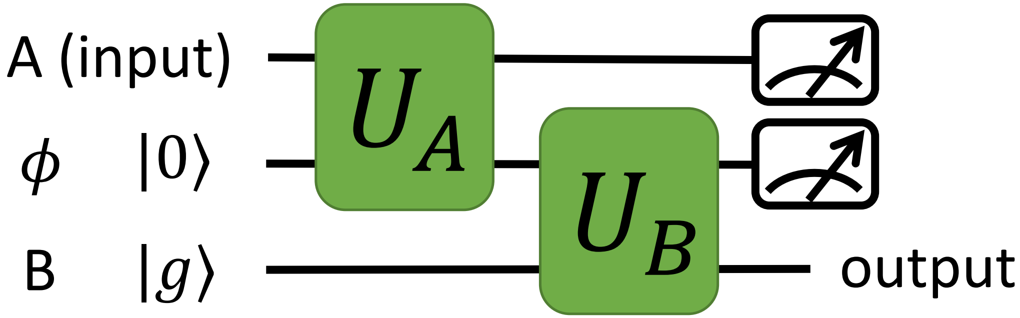


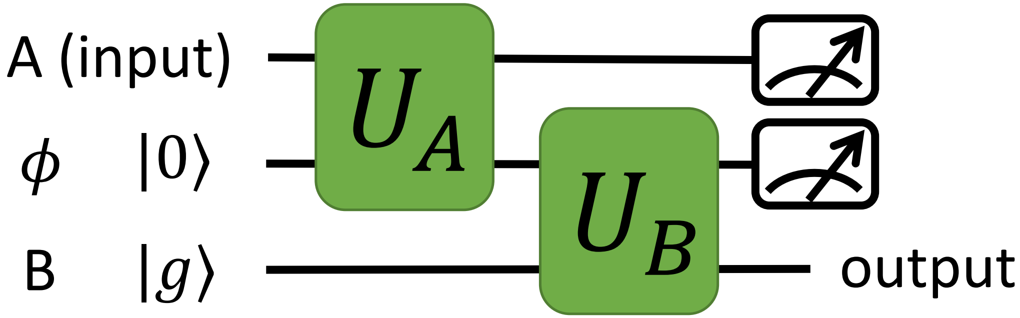
Quantum channel capacity Q $\geq S[\rho_B] - S[\rho_{EB}]$ coherent information

Constructing a quantum channel



- What are U_A and U_B?

Constructing a quantum channel



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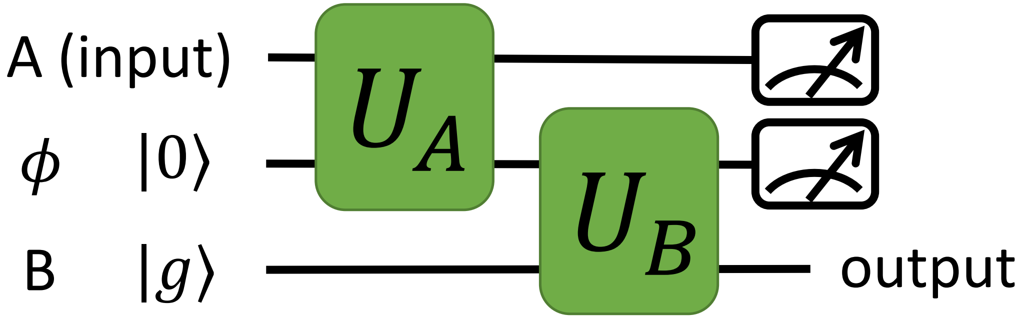

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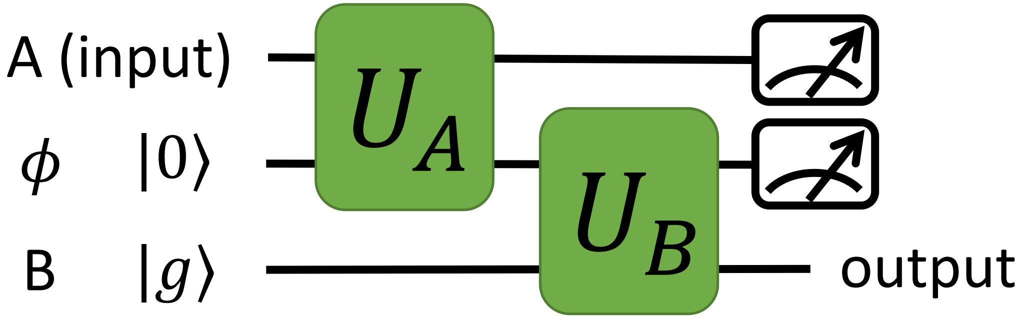

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- For general $\chi(t)$ must work perturbatively in small λ
- If $\chi(t) = \sum_i \delta(t - t_i)$ time ordering is trivial $\Rightarrow$ Can work non-perturbatively!

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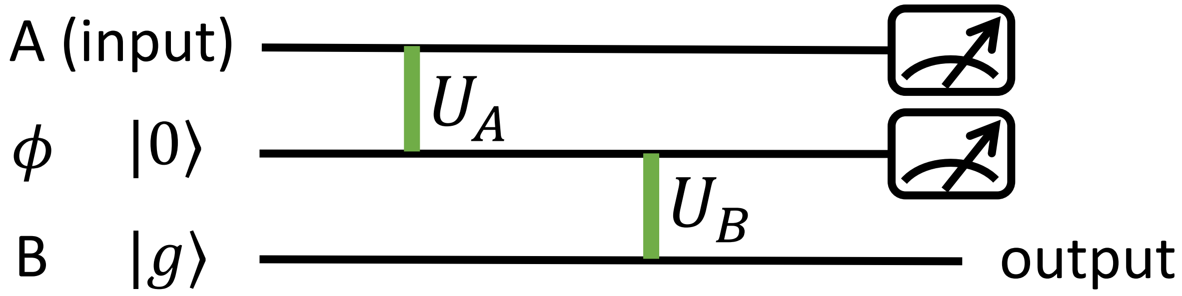
qubit field



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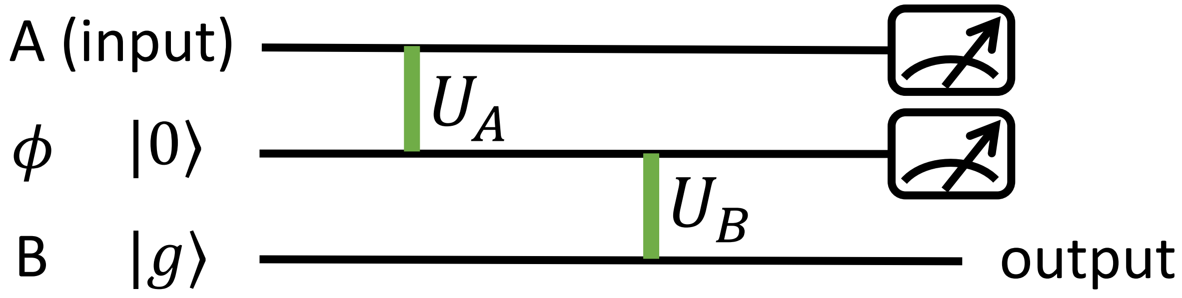

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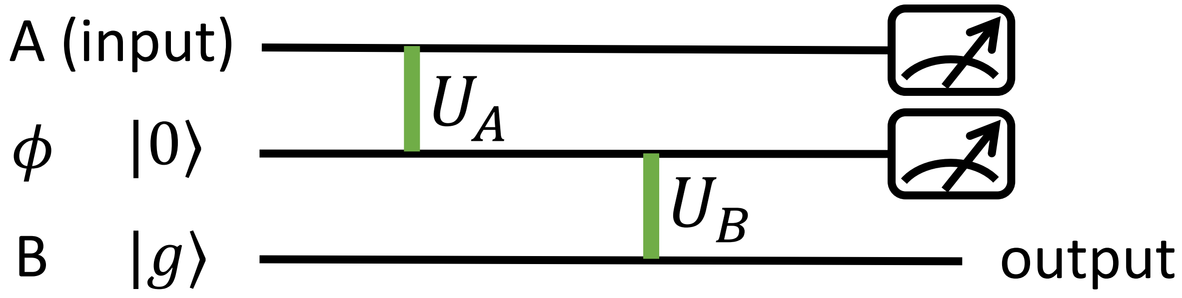
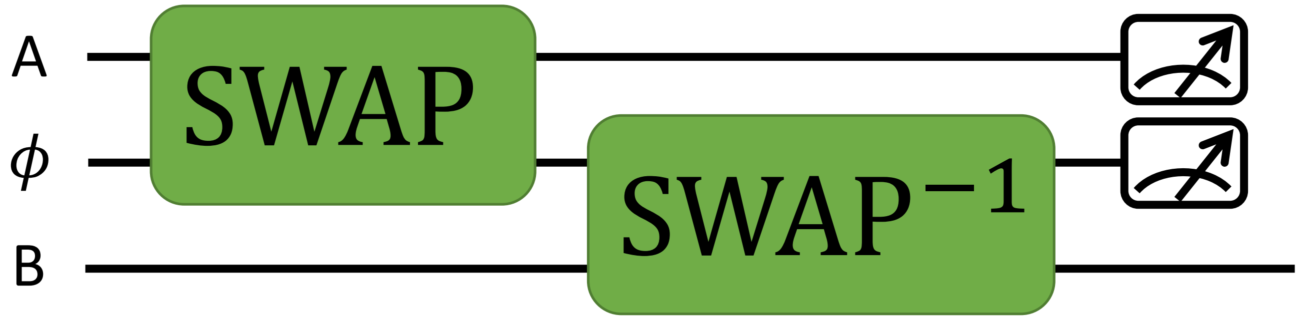


- However: $U = e^{iX\otimes Y}$ are entanglement-breaking [P.S.—Jonsson—Martín-Martínez 2018]
- Need more complicated couplings to transmit quantum information

Constructing a quantum channel

Inspiration: if ϕ were a qubit



Perfect quantum channel

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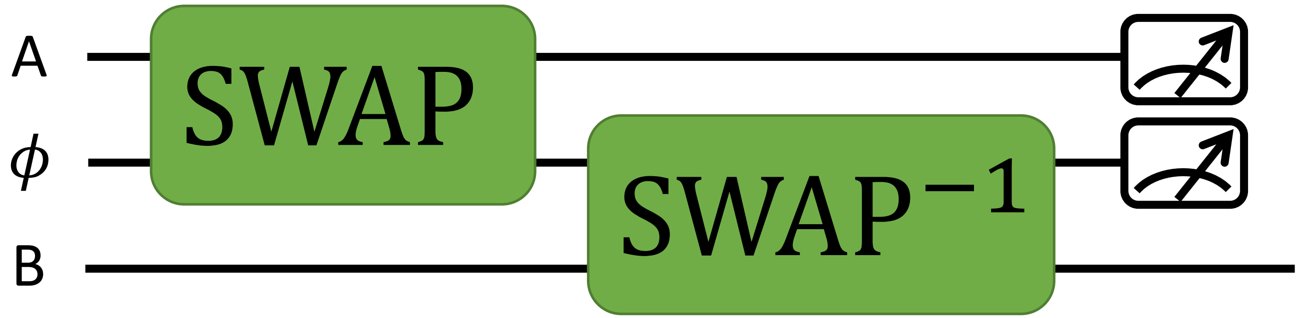

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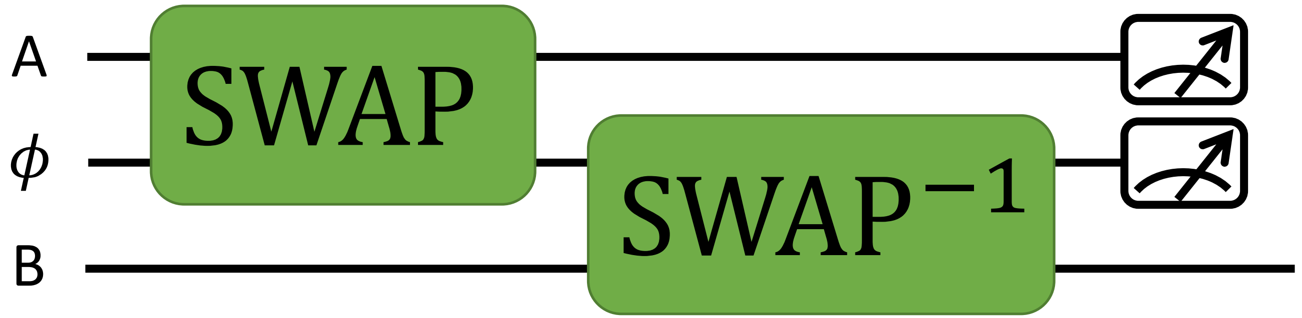

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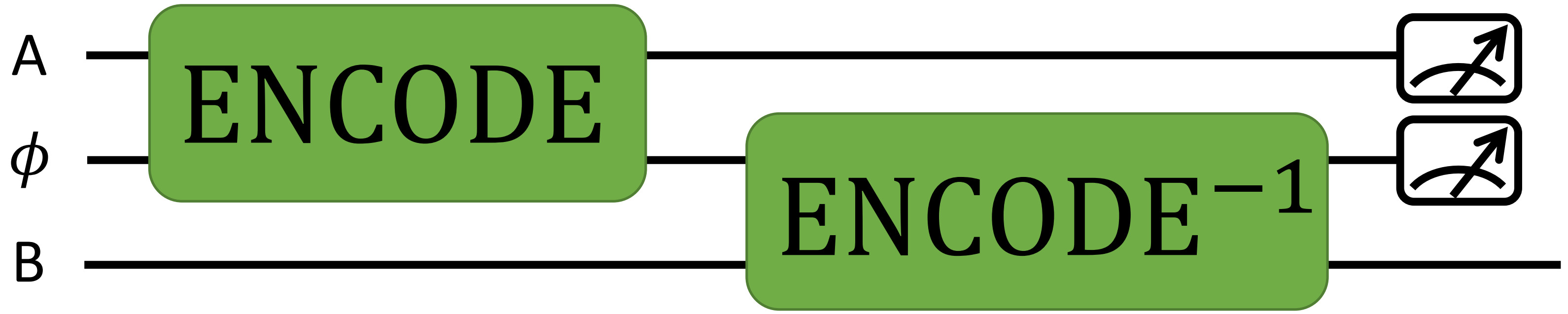

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(2 δ-couplings)

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For optimal encoding want $|+\alpha\rangle$ and $|-\alpha\rangle$ to be orthogonal: Need $\lambda_\phi \gg 1$

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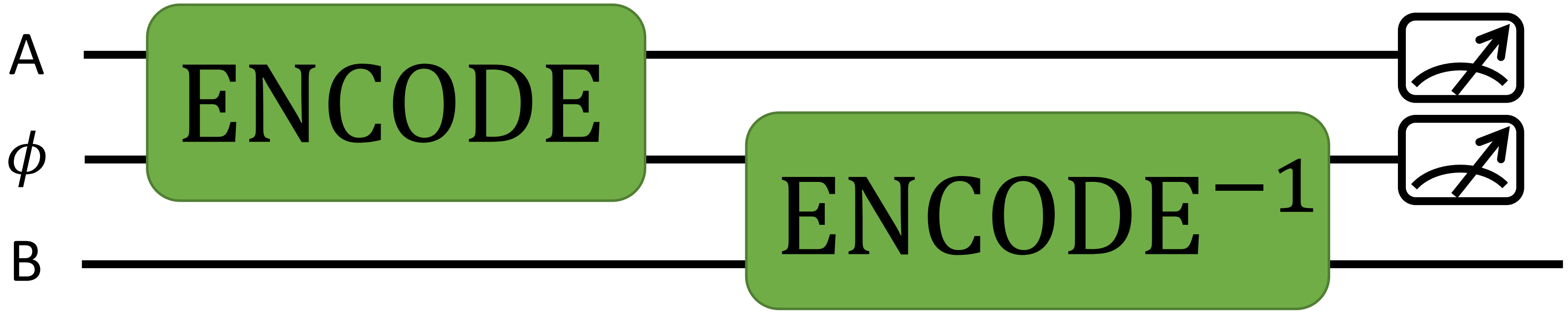

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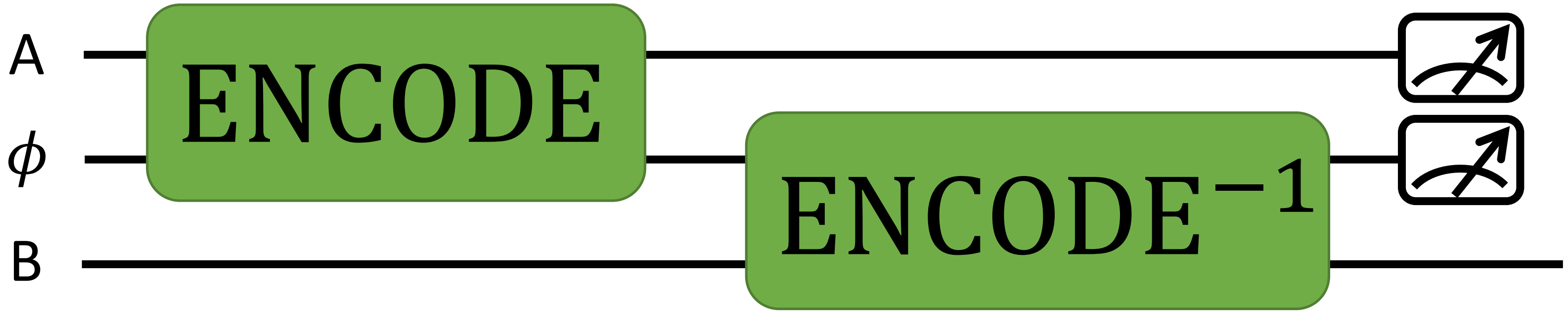

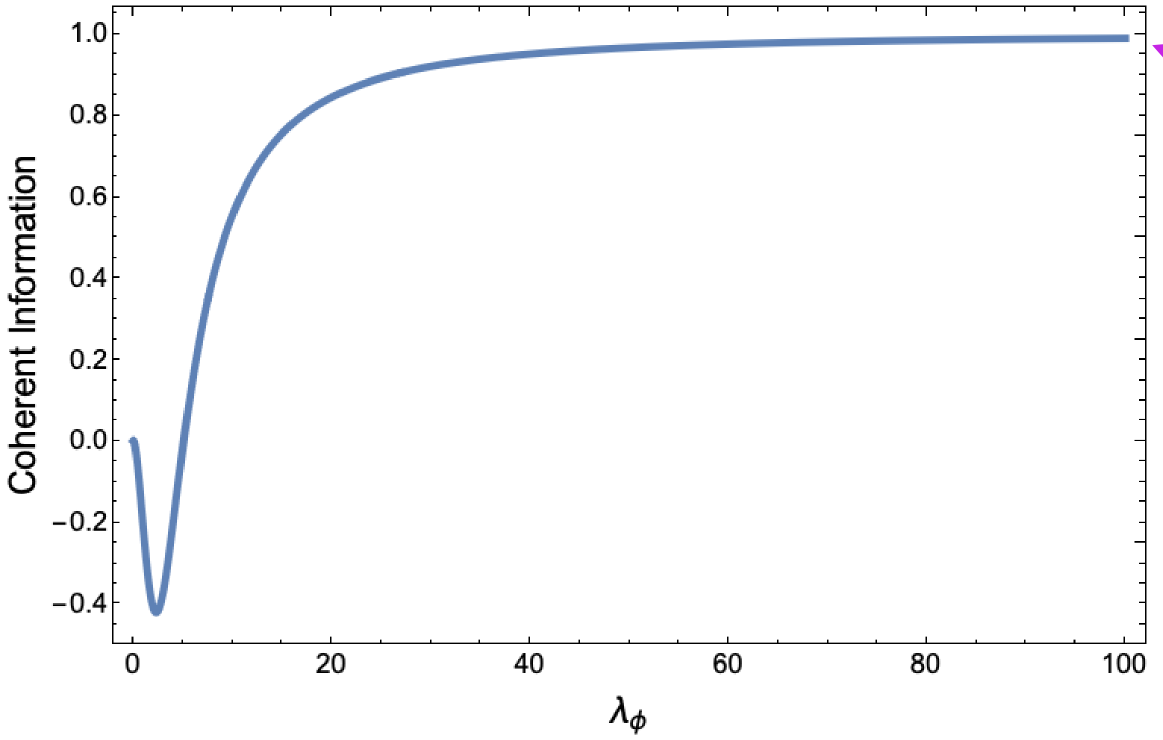

Perfect channel:
$C_I \approx 1$ for $\lambda_\phi \gg 1$

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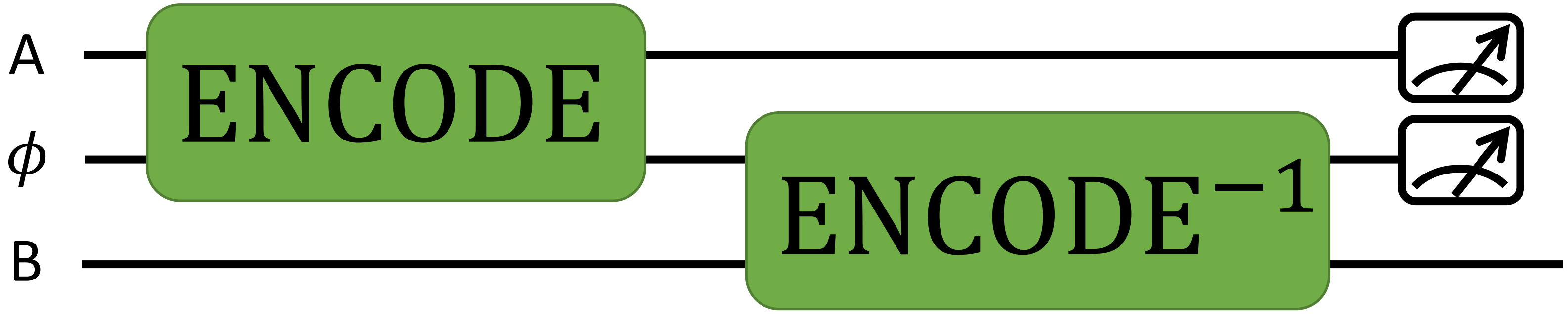

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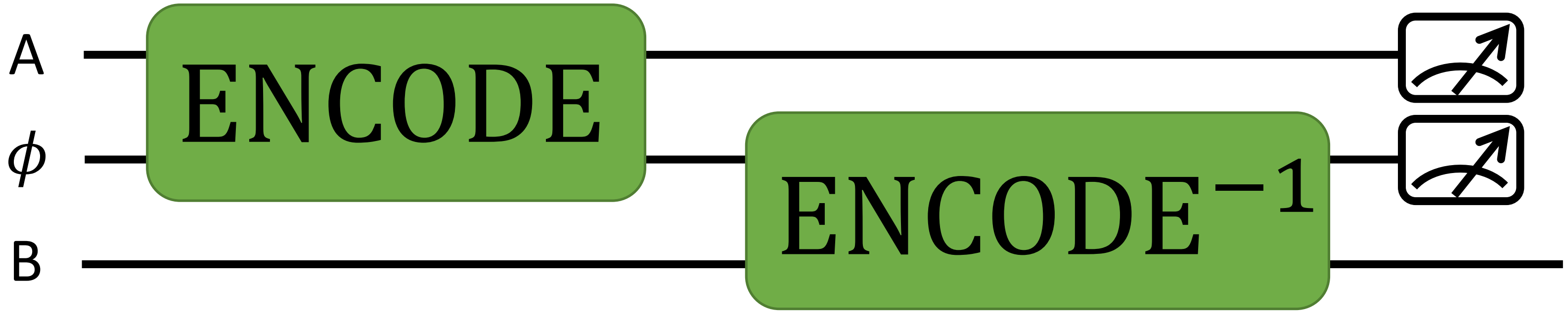

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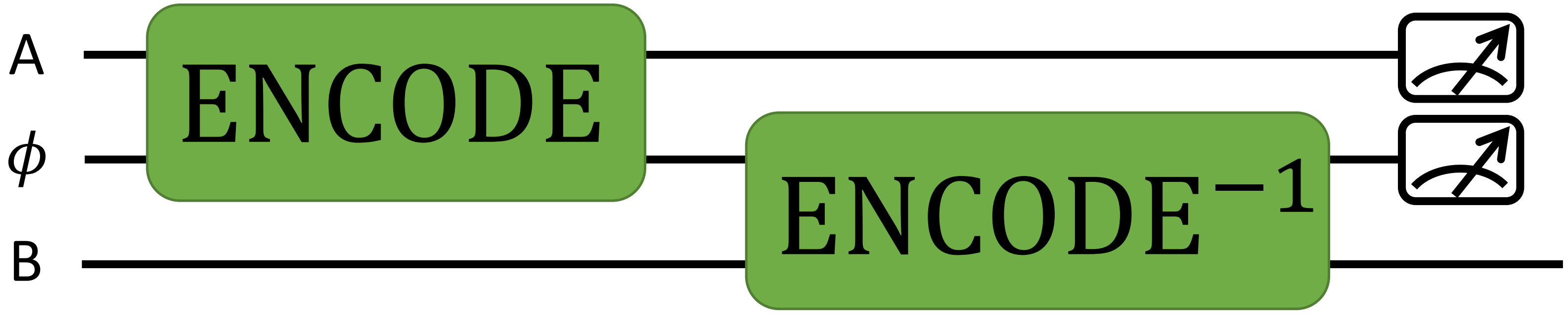

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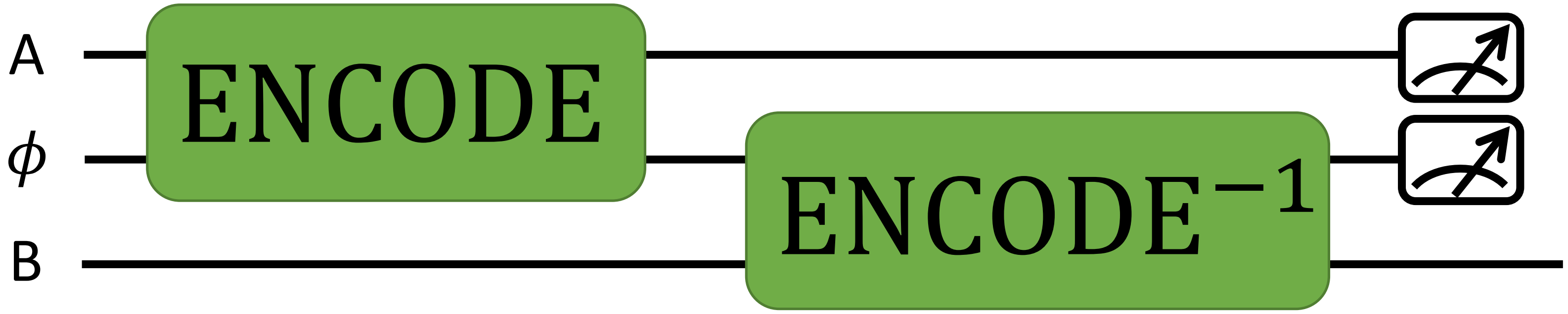

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field observables at $t = t_A$

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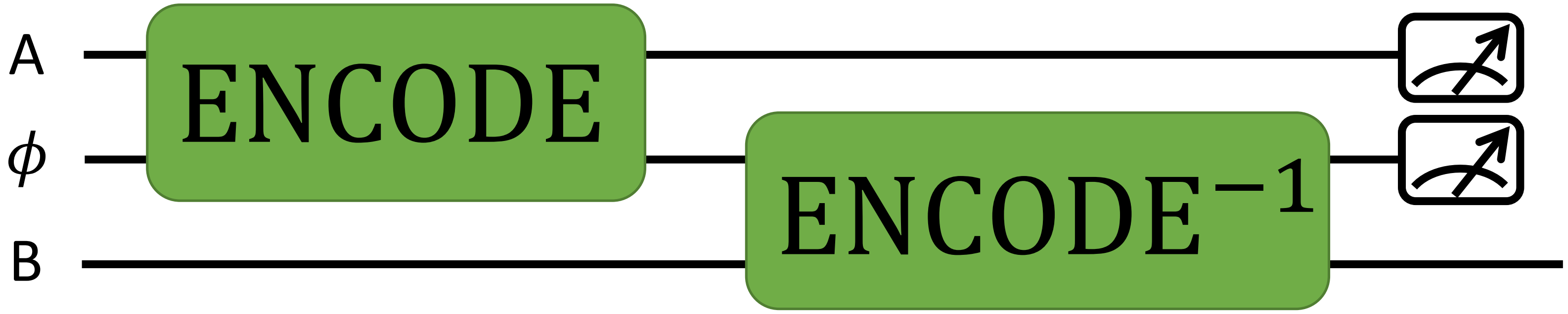

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Need to construct ENCODE^{-1} with observables at $t = t_B$

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Claim: For any free field in any spacetime dimension:

$$\phi[F](t_A) = \phi[F_1](t_B) + \pi[F_2](t_B)$$

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F_1 and F_2 are defined via Fourier transform:

$$\widetilde{F_1}(k) := \tilde{F}(k)\cos(\Delta|k|)\,,$$
$$\widetilde{F_2}(k) := \tilde{F}(k)\,\mathrm{sinc}(\Delta|k|)\,(-\Delta).$$

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F_1 and F_2 are defined via Fourier transform:

$$\widetilde{F_1}(k) := \tilde{F}(k)\cos(\Delta|k|)\,,$$
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Corollary: We can write $\boxed{\mathrm{ENCODE}^{-1}}$ using observables at $t = t_B$. Problem solved!

An algebraic QFT result

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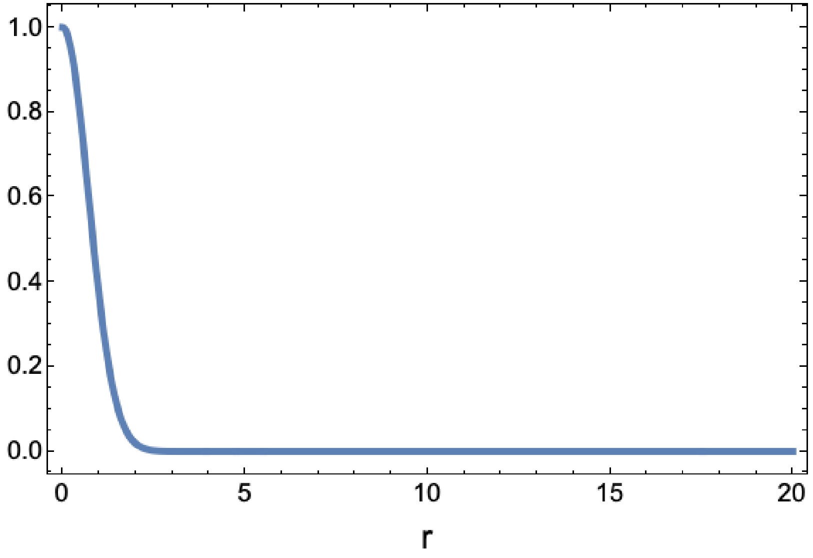
Where should Bob be in 3+1D?

Location of Alice at $t = t_A$



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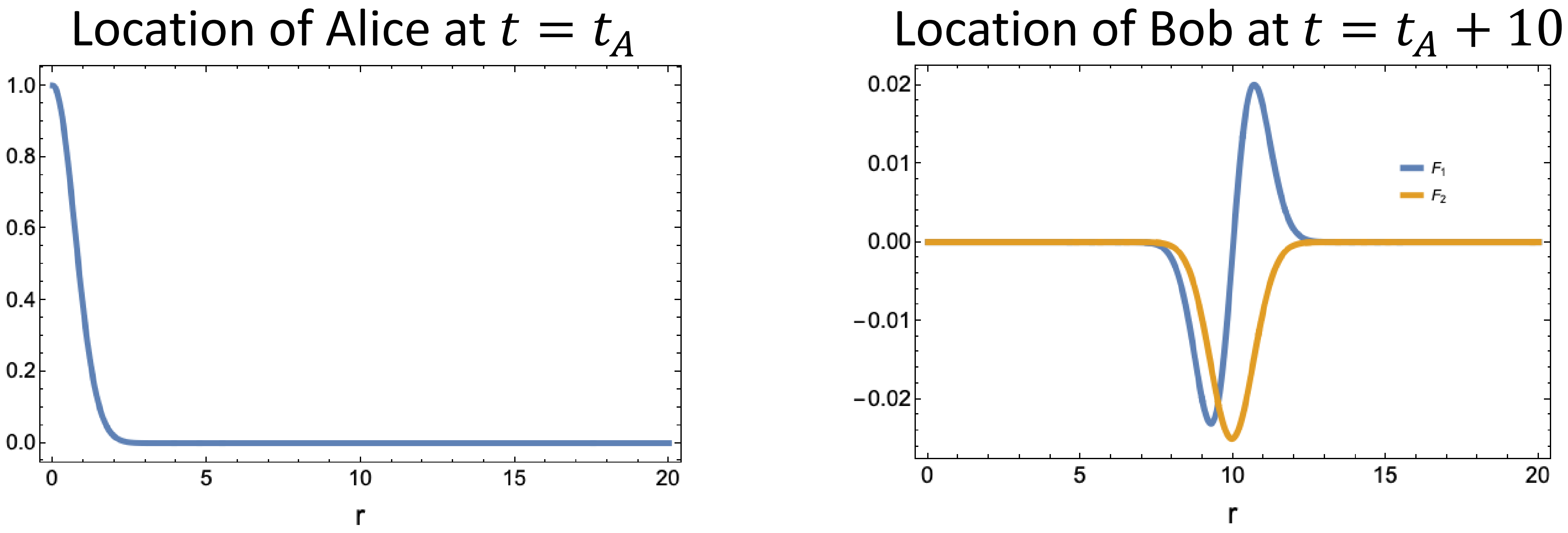

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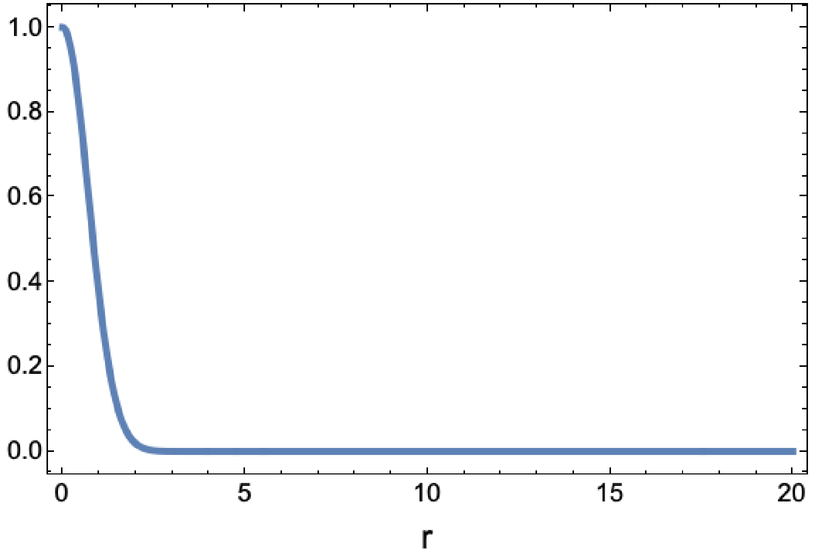
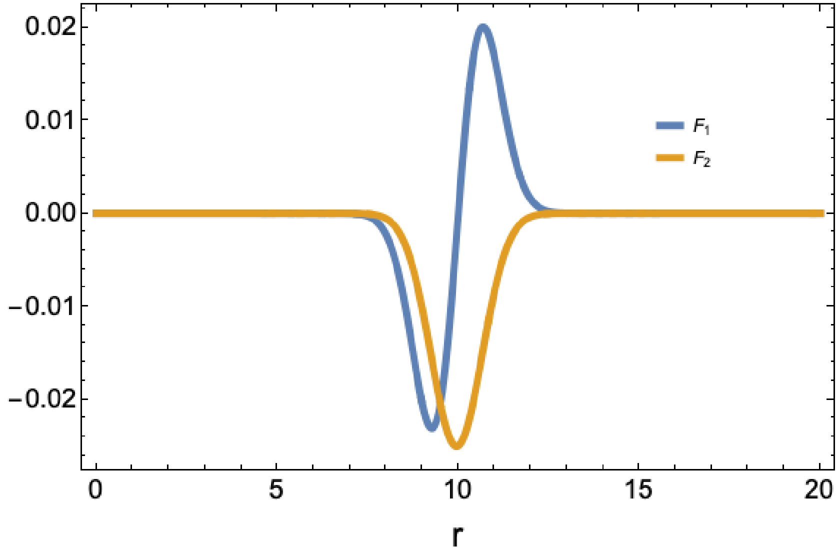
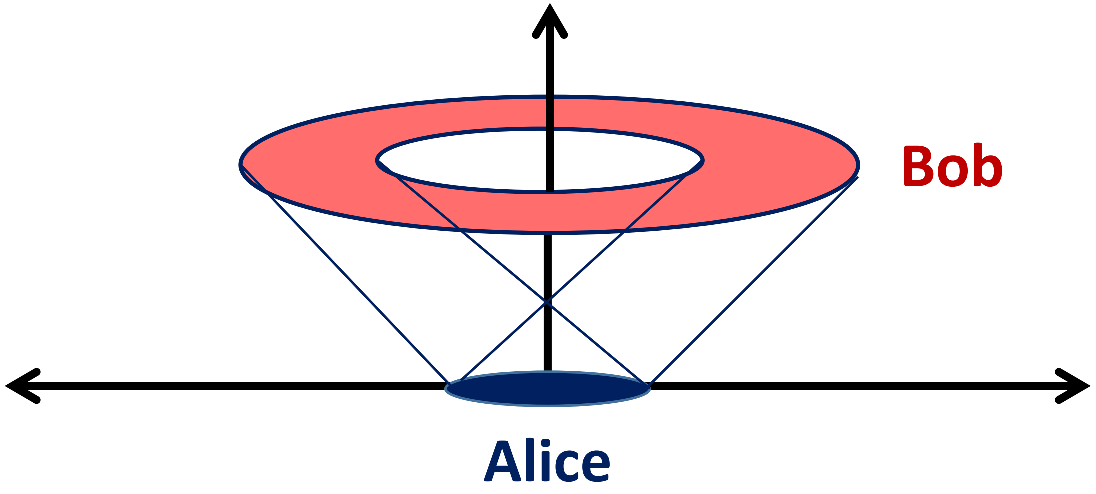


Location of Bob at $t = t_A + 10$



Quantum information propagates on light-cone (strong Huygens principle).



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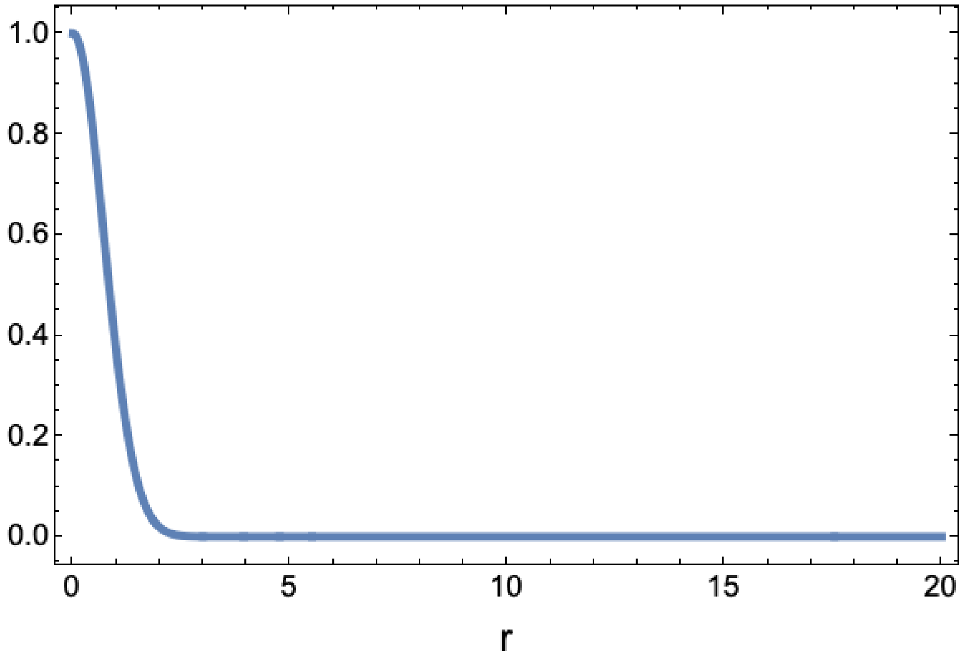

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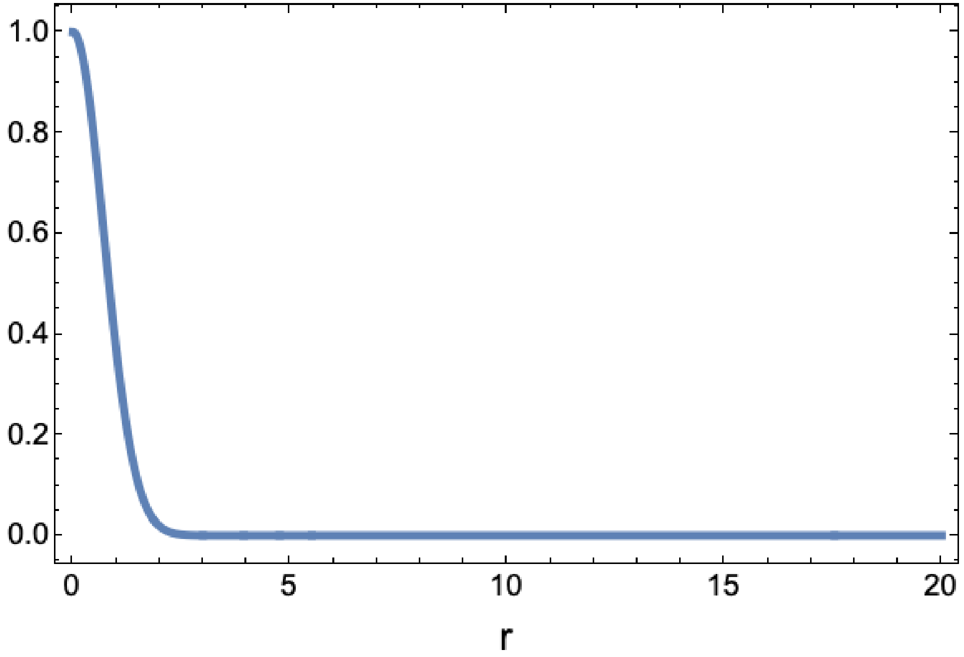

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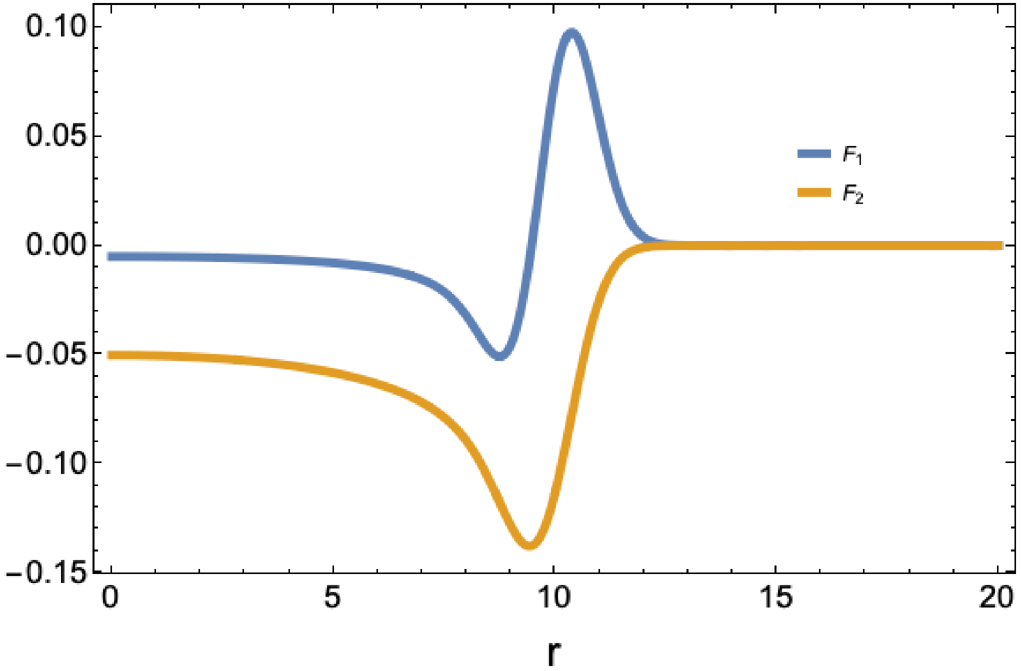

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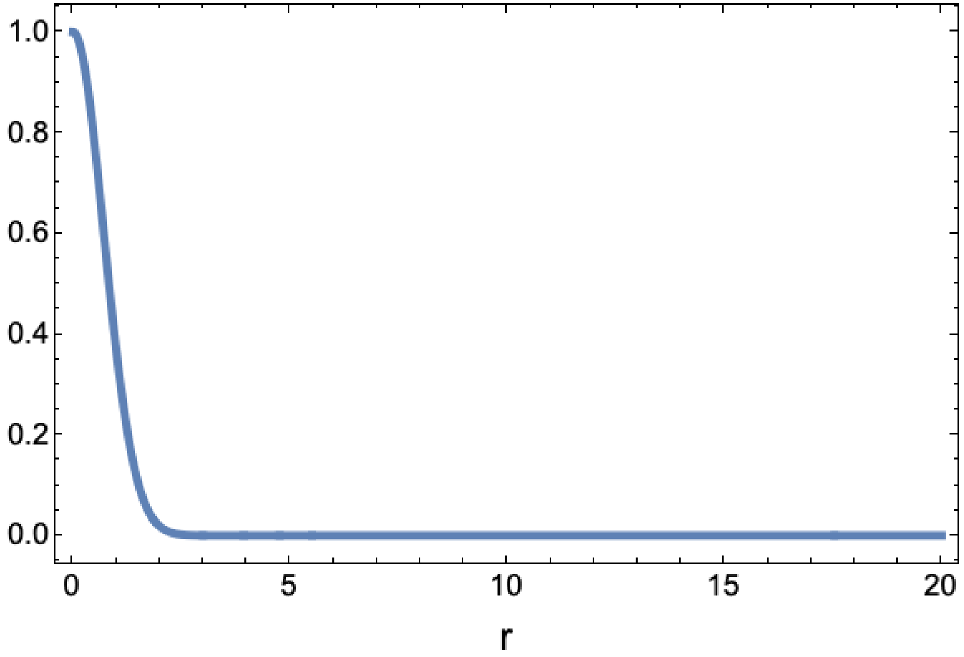

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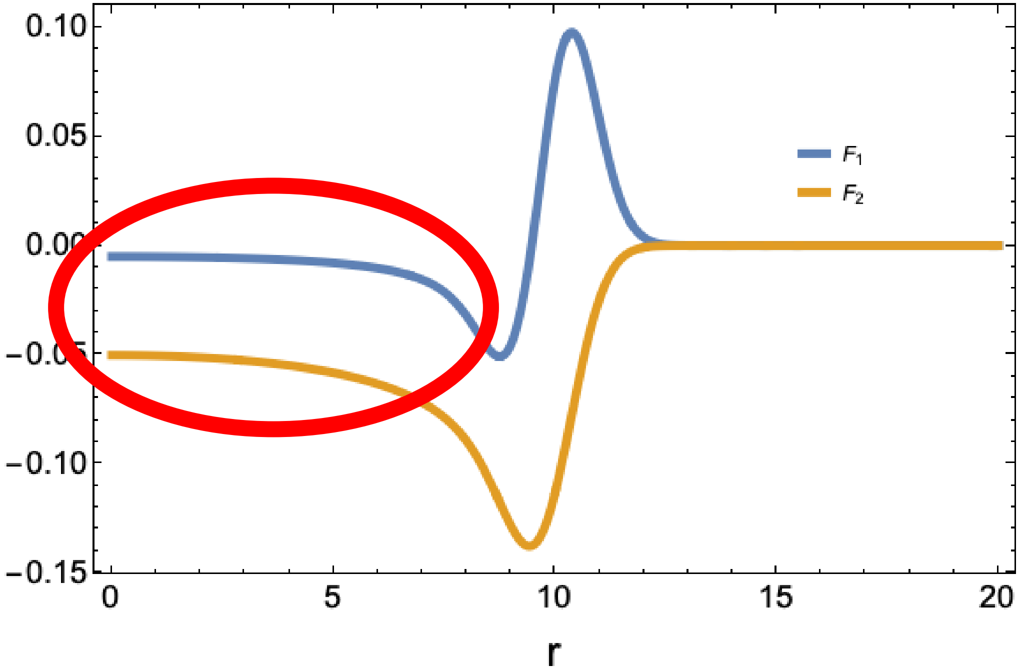

Quantum information leaks inside light-cone (strong Huygens violation).

Quantum information broadcasting?

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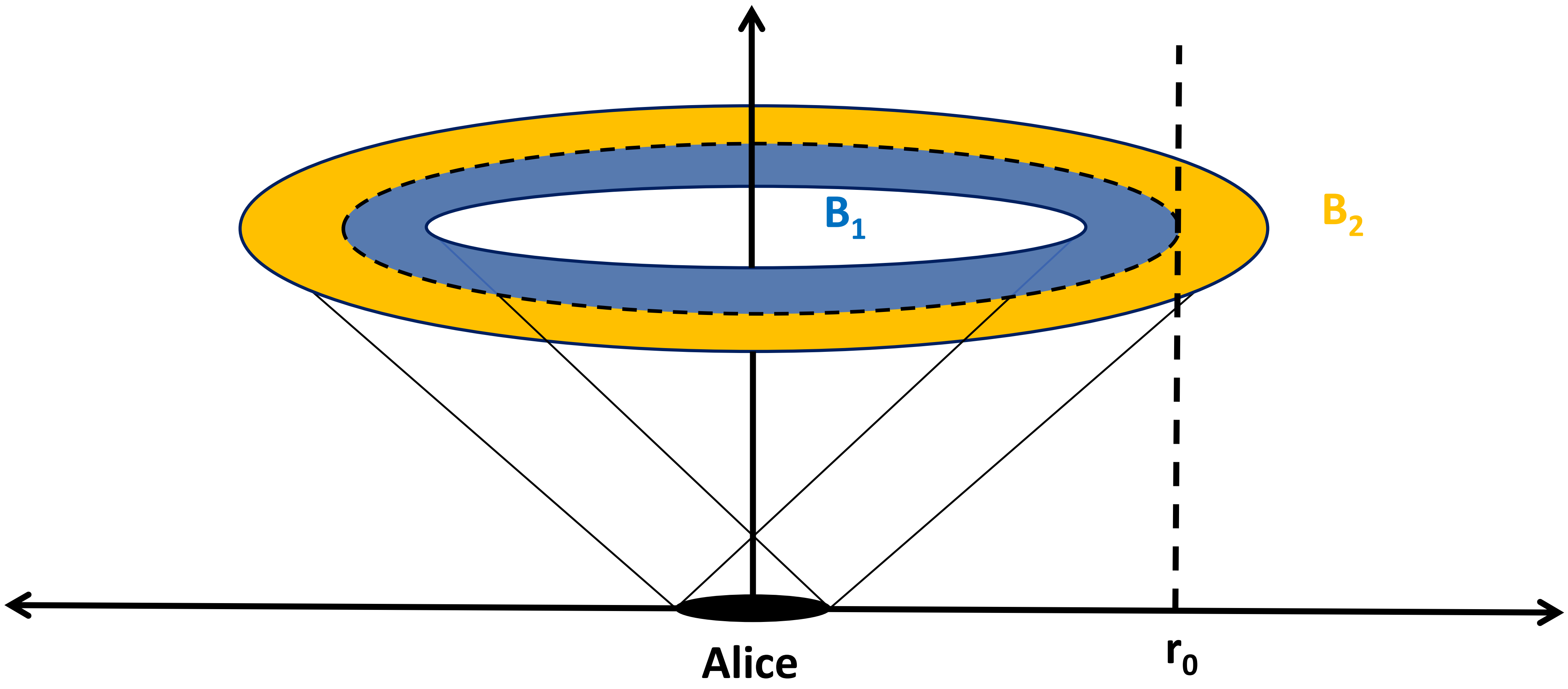
- Can Alice broadcast a *small* amount of quantum info to multiple Bobs?



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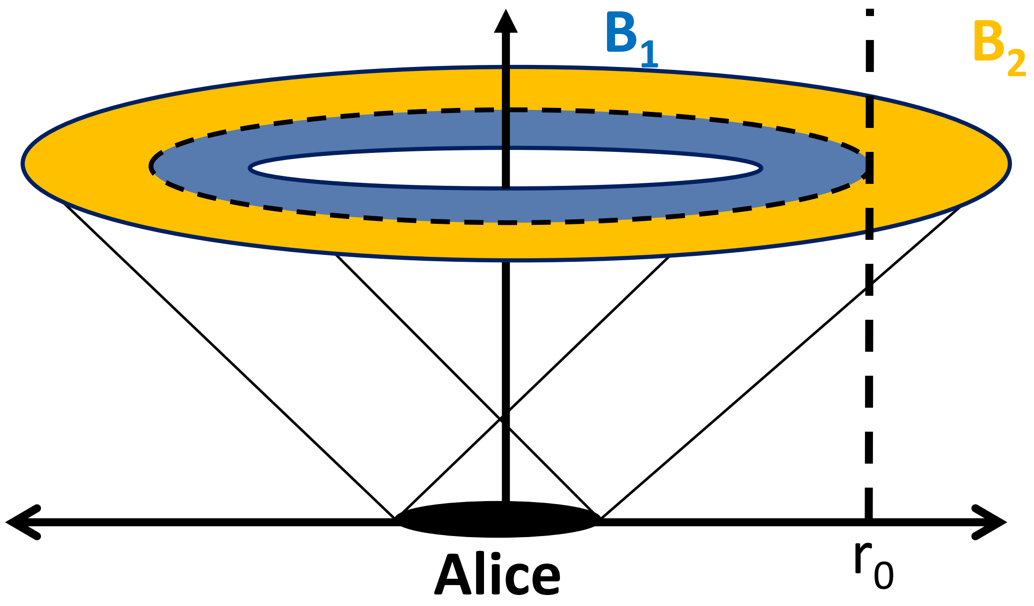
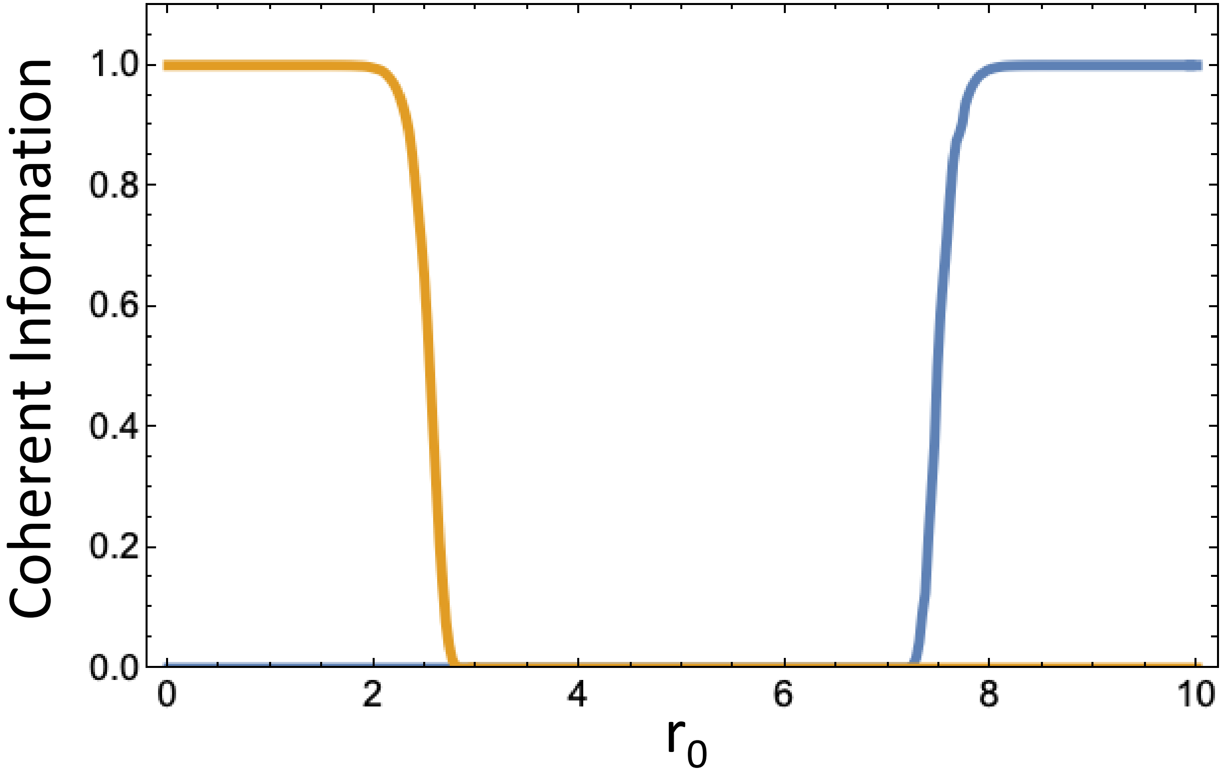
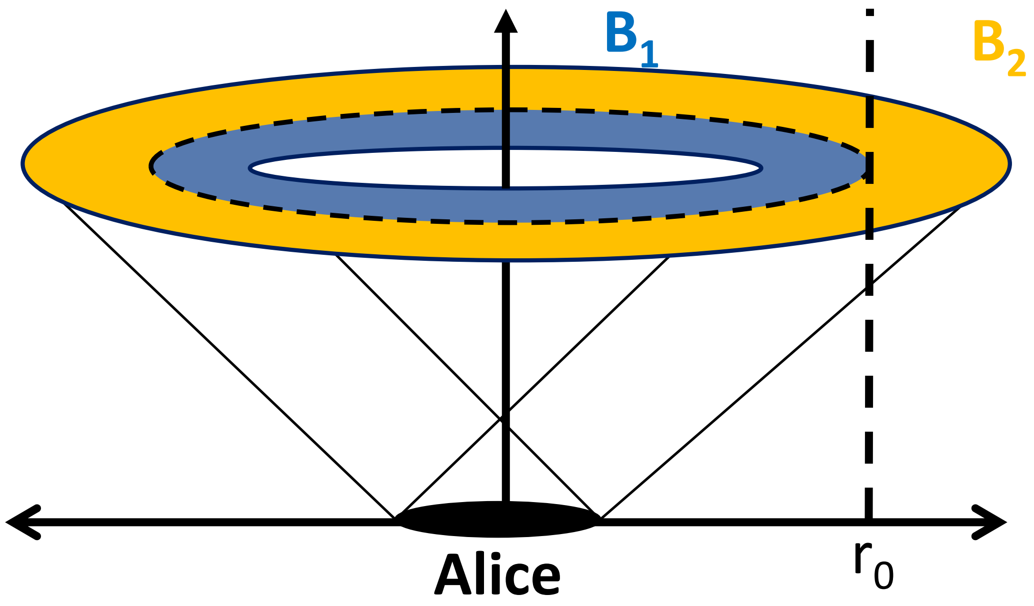
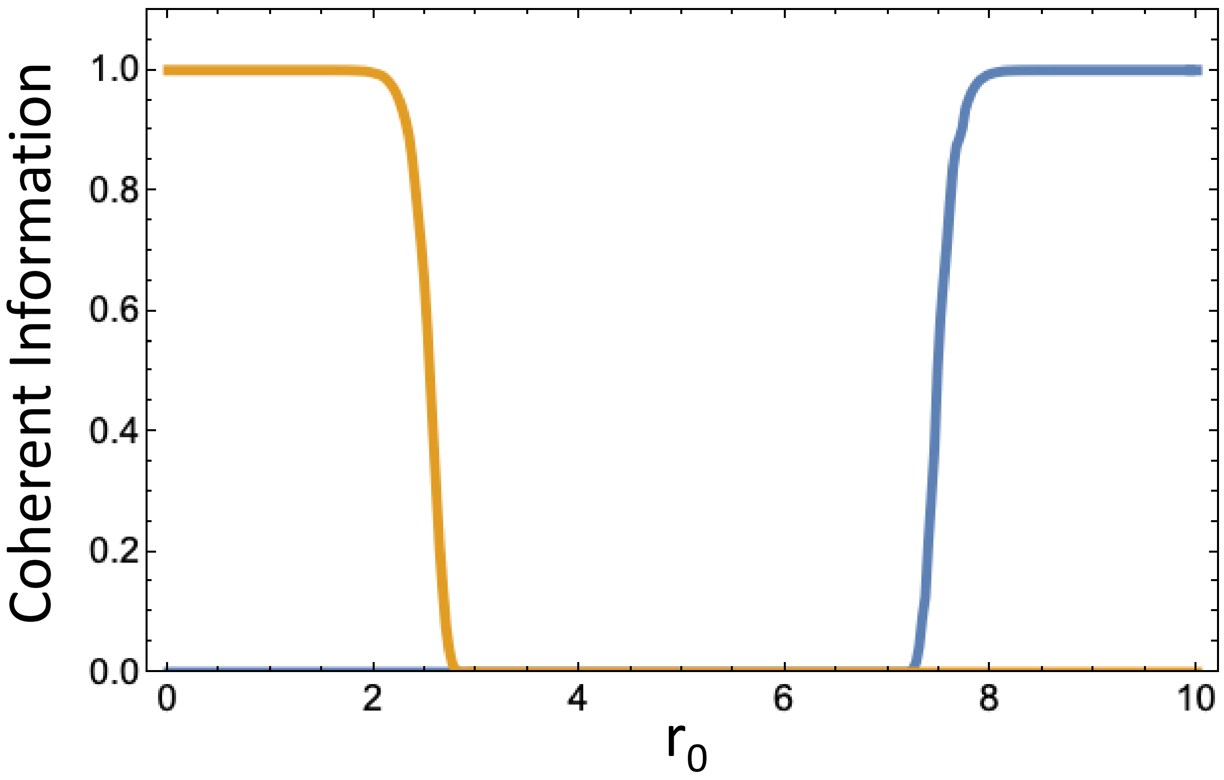




Quantum information broadcasting?





<u>Cannot</u> broadcast message to both Bobs simultaneously

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THANK YOU!