Quantum chaos and effective field theory

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(+ work in progress with W. Reeves & M. Rozali)

Summary

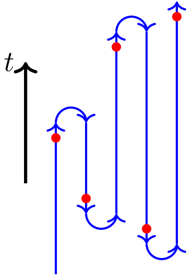
1. A class of "out-of-time-order" correlation functions in thermal systems exhibits a **hierarchy of timescales** $t_*^{(k)} \sim \log(N)$. These characterize aspects of quantum chaos.
2. To understand quantum aspects of black holes via AdS/CFT we are interested in **maximally chaotic** theories. The relevant physics is described by a novel "hydrodynamic" **effective field theory** of very few collective degrees of freedom.
   $\rightarrow$ new tool to study large-c CFTs

Motivation

- Usually QFT focuses on time-ordered (Feynman) path integrals
- QFT has a lot more correlation functions than the time-ordered ones:

$$\langle \widehat{\mathbb{O}}_1(t_1) \cdots \widehat{\mathbb{O}}_n(t_n) \rangle$$

$\longrightarrow$ $n!$ time orderings



- Seem to be very relevant for **black holes**, many body physics,...
  - **Dissipation, chaos, scrambling,...**
  - Generalized fluctuation relations
  - Usually about QI-theoretic ideas (entanglement, complexity, circuits...)

Schwinger, Keldysh; Feynman-Vernon, '60s

(Maldacena-)Shenker-Stanford '15

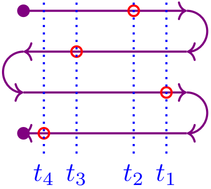
Roberts-Yoshida '16, Sekino-Susskind '08

Yunger Halpern '17,...

Out-of-time-order correlation functions

- Convenient way to represent n-point function with generic time ordering is the **k-OTO contour**

$$\langle \widehat{\mathbb{O}}_4(t_4)\widehat{\mathbb{O}}_1(t_1)\widehat{\mathbb{O}}_3(t_3)\widehat{\mathbb{O}}_2(t_2)\rangle \;=$$



  - Feynman (time-ordered) correlators:

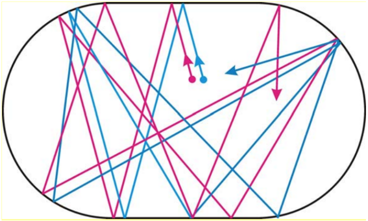
  - 'Schwinger-Keldysh' contour ($k = 1$):

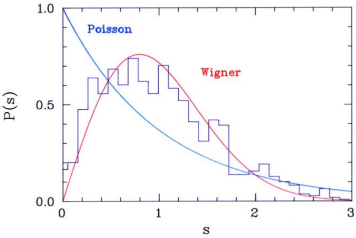
Review: classical chaos

- Early times: **classical Lyapunov exponents** quantify divergence of phase space trajectories

$$\{q(t), p\} \equiv \frac{\delta q(t)}{\delta q(0)} \sim e^{\lambda_L t}$$

- Late times: ergodicity, thermalization





Review: quantum chaos

(Maldacena–)Shenker–Stanford '13-'15

Leichenauer '14; Kitaev '15; ...

- Out-of-time-order correlators (OTOCs): quantify **early time quantum chaos**

$$\langle W(t)V(0)W(t)V(0)\rangle_\beta \sim a_0 - a_1\, e^{\lambda_L (t-t_*)}$$

  - Quantum Lyapunov exponent obeys fundamental bound:

$$\lambda_L \leq \frac{2\pi}{\beta}$$

  - scrambling time: $t_* \sim \frac{\beta}{2\pi}\log N$

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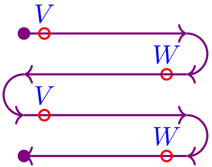
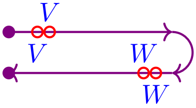
- Contrast this with TOCs:

$$\langle W(t)W(t)V(0)V(0)\rangle_\beta \sim \langle WW\rangle_\beta \,\langle VV\rangle_\beta + \mathcal{O}(e^{-t/t_{diss}})$$



vs.



Higher-point OTOCs

- The 4-point OTOC $\langle W(t)V(0)W(t)V(0)\rangle_\beta$ is **"2-OTO"**
- The space of n-point OTOCs is classified mathematically *[FH et al. '17]*

Q: What is the *physics* of higher-point OTOCs?

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Q: What is the *physics* of higher-point OTOCs?

- We studied a particular **"k-OTO" $2k$-point function** *[FH–Rozali '17 '18]*
  - Its characteristic thermalization time is

$$t_*^{(k)} \sim (k-1) \times t_*$$

Hierarchy of timescales in early-time quantum chaos associated with increasingly fine-grained probes of the thermal state

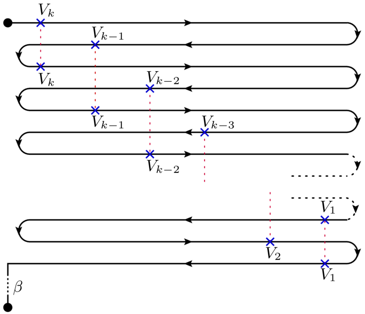
Higher-point OTOCs

FH–Rozali '17, '18

- Consider the following OTOC (assume $t_1 > t_2 > \ldots > t_k$):

$$F_{2k}(t_1, \ldots, t_k) = \frac{\langle V_1[V_2, V_1][V_3, V_2][V_4, V_3] \cdots [V_k, V_{k-1}]V_k \rangle_\beta^{\text{reg.}}}{\langle V_1 V_1 \rangle_\beta \cdots \langle V_k V_k \rangle_\beta}$$

- Dropping all commutators, the essential term is the following:



  - **"k-OTO"**, i.e., requires k switchbacks in time
  - **maximally "braided"** in imaginary time

Higher-point OTOCs

FH–Rozali '17, '18

$$F_{2k}(t_1, \dots, t_k) = \frac{\left\langle V_1 [V_2, V_1][V_3, V_2][V_4, V_3] \cdots [V_k, V_{k-1}] V_k \right\rangle_\beta^{\text{reg.}}}{\langle V_1 V_1 \rangle_\beta \cdots \langle V_k V_k \rangle_\beta}$$

- Claim:

$$F_{2k} \sim e^{\lambda_L (t-(k-1)t_*)} \qquad \text{with } t = t_1 - t_k\,, \quad t_* = \frac{2\pi}{\beta} \log N$$

  - Depends only on total "duration of experiment" $t = t_1 - t_k$
  - Characteristic time scale is $(k-1)t_*$
  - Sensitive to some more **fine-grained information about the state**

Effective field theory of chaos

- We computed F_{2k} in the **Schwarzian theory** (low energy SYK), and in **2d CFTs at large central charge**. Common features:
  - Maximally chaotic ($\lambda_L = \frac{2\pi}{\beta}$) and scrambling hierarchy ($(k-1)t_*$)

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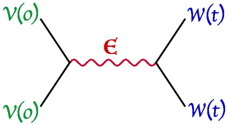
  - Lyapunov behavior of OTOC can be described using **effective field theory** of very few collective degrees of freedom $\epsilon_i(t,x)$

$$\begin{aligned}&\langle W(t,x_1)V(0,x_3)W(t,x_2)V(0,x_4)\rangle_\beta \\ &\quad\sim \langle \mathcal{D}_{x_1,x_2}[\epsilon_i(t)]\,\mathcal{D}_{x_3,x_4}[\epsilon_i(0)]\rangle \\ &\quad\sim \langle \epsilon_i(t)\epsilon_i(0)\rangle\end{aligned}$$



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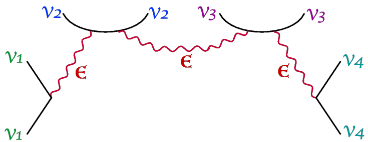
$$\frac{\left\langle V_1[V_2,V_1][V_3,V_2][V_4,V_3]\cdots[V_k,V_{k-1}]V_k\right\rangle_\beta^{\text{reg.}}}{\langle V_1V_1\rangle_\beta\cdots\langle V_kV_k\rangle_\beta}$$

$$\sim \langle\epsilon_i(t_1)\epsilon_i(t_2)\rangle\cdots\langle\epsilon_i(t_{k-1})\epsilon_i(t_k)\rangle$$



Effective field theory of chaos

- What is this "scramblon" mode $\epsilon_i(t,x)$ (in CFT)?
  - Goldstone mode of **spontaneously broken conformal symmetry**
  - Describes the physics of stress tensor exchanges
    - ⋆ Formally looks like a **"hydrodynamic" mode**
  - **Effective action** (can be derived from conformal symmetry):

Schwarzian (1d): $$S = \frac{N}{\mathcal{J}} \int d\tau \, (\partial_\tau^3 + \partial_\tau)\epsilon \;\; \partial_\tau \epsilon$$

Kitaev '15, Maldacena–Stanford '16, ...

CFT (2d): $$S = c \int d^2x \, (\partial_\tau^3 + \partial_\tau)\epsilon \;\; (\partial_\tau + i\partial_x)\epsilon + \text{anti-holo.}$$

FH–Rozali '18, Cotler–Jensen '18

CFT (d>2): (work in progress)

FH–Reeves–Rozali

  - Perturbative parameter: $\frac{1}{c}$
  - Has a propagator $\langle \epsilon\epsilon \rangle$ with exponentially growing terms

Effective field theory of large-c physics

- Use **EFT tools for universal aspects of large-c CFT**
  - Captures the **universal physics of energy conservation** at large c
    - ⋆ Easy calculation of ($2k$-point) OTOCs *[FH–Rozali '18]*
    - ⋆ "Boundary gravitons" in AdS/CFT *[Cotler–Jensen '18]*
    - ⋆ Explains "pole skipping" *[Blake–Lee–Liu '18] [Blake–Davison–Grozdanov–Liu '18]*
  - $\epsilon_i(x)$ naturally lives in **kinematic space** *[Czech et al. '16] [de Boer et al. '16]*
    - ⋆ New perspective on *kinematic space* and *shadow operator formalism*
    - ⋆ Novel tools for computing *conformal blocks* *[FH–Reeves–Rozali w.i.p.]*
    - ⋆ In particular: "gravity channels" of stress tensor exchanges
    - ⋆ Higher-point blocks relevant to AdS/CFT *[Anous–FH–Perlmutter w.i.p.]*
    - ⋆ $\frac{1}{c}$ corrections

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Further Details

Reparametrization modes in CFT$_2$

- Go to finite temperature: $(z, \bar{z}) \longrightarrow (e^{iz}, e^{-i\bar{z}})$ $\qquad (z \sim z + 2\pi)$
- Consider **holomorphic (and anti-holomorphic) reparametrizations:**

$$z \mapsto z + \epsilon(z, \bar{z}) \qquad \bar{z} \mapsto \bar{z} + \bar{\epsilon}(z, \bar{z})$$

$$S_{CFT} \mapsto S_{CFT} + \int d^2z \, \left\{ \bar{\partial}\epsilon(z, \bar{z})\, T(z) \, + \, \partial\bar{\epsilon}(z, \bar{z})\, \bar{T}(\bar{z}) \right\}$$

  - For conformal transformations ($\epsilon(z, \bar{z}) = \epsilon(z)$ and $\bar{\epsilon}(z, \bar{z}) = \bar{\epsilon}(\bar{z})$), this is a symmetry, generated by standard conserved currents:

$$\bar{\partial} J(z) = \partial \bar{J}(\bar{z}) = 0 \quad \text{with} \quad J = \epsilon(z) T(z)\,, \;\; \bar{J} = \bar{\epsilon}(\bar{z}) \bar{T}(\bar{z})$$

- Want to treat $(\epsilon, \bar{\epsilon})$ as soft modes associated with conformal symmetry breaking (c.f., *[Turiaci–Verlinde '16]*)

Reparametrization modes in CFT_2

- Next: Legendre transform, i.e., trade $(T, \bar{T})$ fluctuations due to sources $(\bar{\partial}\epsilon, \partial\bar{\epsilon})$ for fluctuations of $(\epsilon, \bar{\epsilon})$
  - Dynamics of $(\epsilon, \bar{\epsilon})$ encodes same physics as **stress tensor exchanges**
  - Holographically: gravitons
- Quadratic effective action for the "soft modes":

$$I_{quad} = -\frac{1}{2}\int d^2z_1 d^2z_2\; \bar{\partial}\epsilon(z_1,\bar{z}_1)\,\bar{\partial}\epsilon(z_2,\bar{z}_2)\;\langle T(z_1)T(z_2)\rangle + \text{(anti-holo.)}$$

- This is universal since $\langle T(z_1)T(z_2)\rangle$ is fixed by conformality
- Euclidean quadratic action reads ($z = \tau + i\sigma$):

$$I_{quad} = \frac{c\pi}{12}\int d\tau d\sigma\; (\partial_\tau + i\partial_\sigma)\epsilon\;(\partial_\tau^3 + \partial_\tau)\epsilon\; +\; \text{(anti-holo.)}$$

Pole skipping

- *[Blake–Lee–Liu '18]* and *[Blake–Davison–Grozdanov–Liu '18]* discussed **pole skipping**
  - Retarded energy-energy 2-point function has line of diffusion poles in complex ω-plane
  - However, there is **no pole at** $(\omega, k) = (i\lambda_L, \frac{i\lambda_L}{v_B})$
  - Proposed this as smoking gun of Lyapunov behavior of OTOCs
  - Starting point for "hydrodynamic" theory of an effective chaos mode

- Can see this explicitly in CFT_2: *[FH–Rozali '18]*

$$G^R_{\bar{T}\bar{T}}(\omega, k) = \frac{c\pi}{6}\frac{\omega(\omega^2+1)}{\omega - k}$$

  - Universal for any CFT_2
  - What are the precise assumptions in order to associate pole skipping with chaos?

Conformal Blocks

- *[Cotler–Jensen '18]* derived a non-linear version of our action
  - Chiral QFT of boundary gravitons in AdS$_3$ (reparametrization field on Diff(S^1))
  - Aka Alekseev-Shatashvili path integral quantization of Diff(S^1)/PSL(2,$\mathbb{R}$) coadjoint orbit of Virasoro
- Reproduced basic results about **vacuum block** (in "light-light" and "heavy-light" limits) from Feynman diagram calculations in this effective theory
- E.g., "light-light" vacuum block ($h_V, h_W \ll c$)

$$\langle V(z_1)V(z_2)W(z_3)W(z_4)\rangle$$

$$= \frac{1}{(z_{12})^{2h_V}(z_{34})^{2h_W}} \underbrace{\left[1 + \underbrace{\langle \mathcal{B}^{(1)}_{h_V}(z_1,z_2)\mathcal{B}^{(1)}_{h_W}(z_3,z_4)\rangle}_{\frac{2h_V h_W}{c}\, z^2\; {}_2F_1(2,2,4,z)} + \mathcal{O}\left(\frac{1}{c^2}\right)\right]}_{\exp\left(\frac{2h_V h_W}{c}\, z^2\; {}_2F_1(2,2,4,z)\right)(1+\mathcal{O}(1/c))}$$