# Electron scattering within ab-initio approaches

Alessandro Lovato

In collaboration with:
Stefano Gandolfi, Joe Carlson, Juan Nieves, Maria Piarulli, Steve Pieper, Noemi Rocco, and Rocco Schiavilla

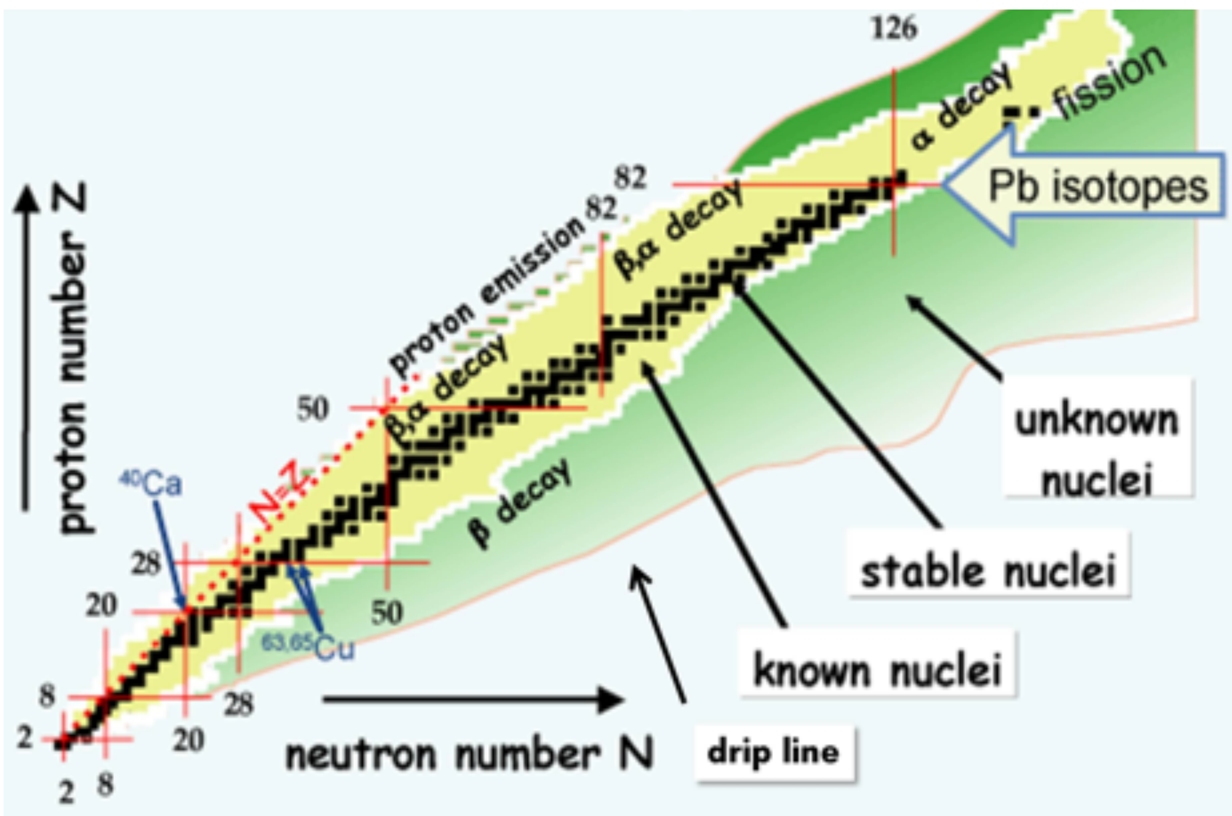
INFN Istituto Nazionale di Fisica Nucleare

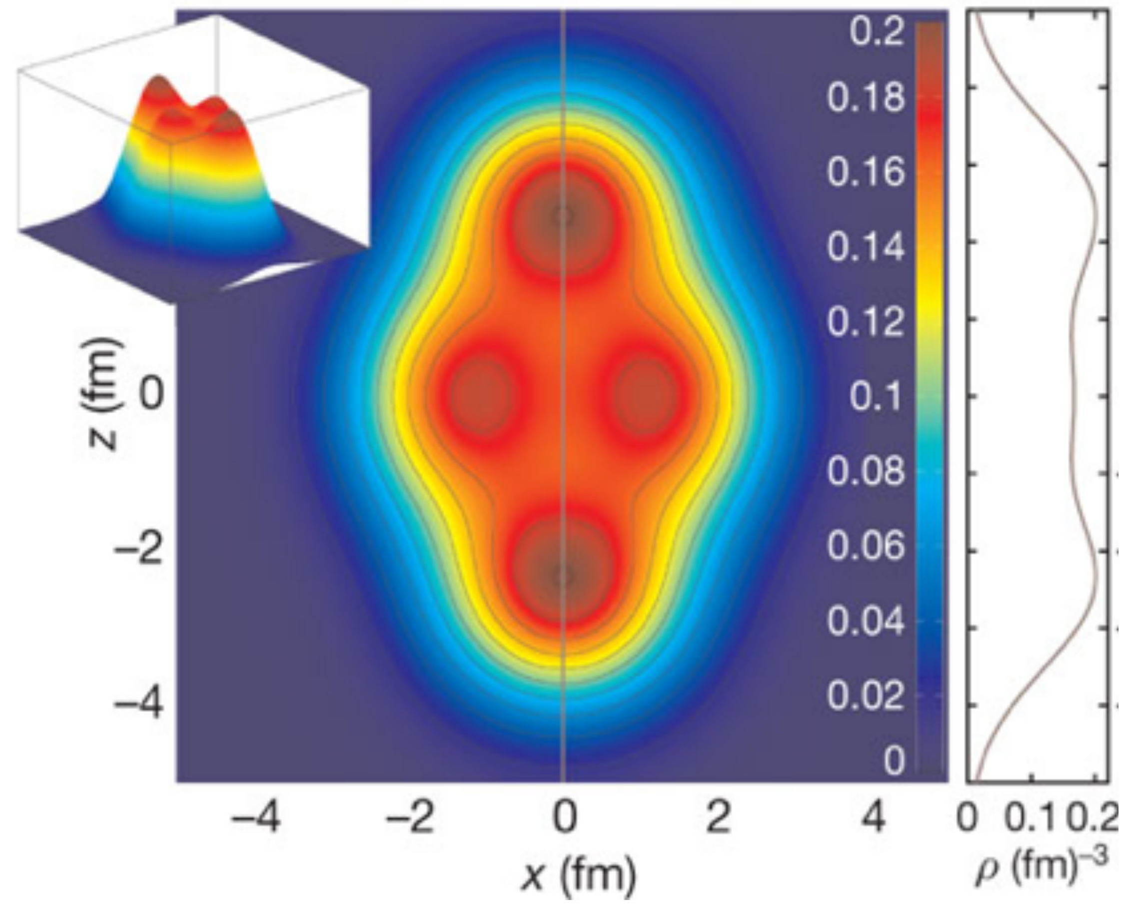
Argonne NATIONAL LABORATORY

NUCLEI Nuclear Computational Low-Energy Initiative

# Why nuclear Physics is (still) cool?

- Atomic nuclei are strongly interacting many-body systems exhibiting fascinating properties including: shell structure, pairing and superfluidity, deformation, and self-emerging clustering.



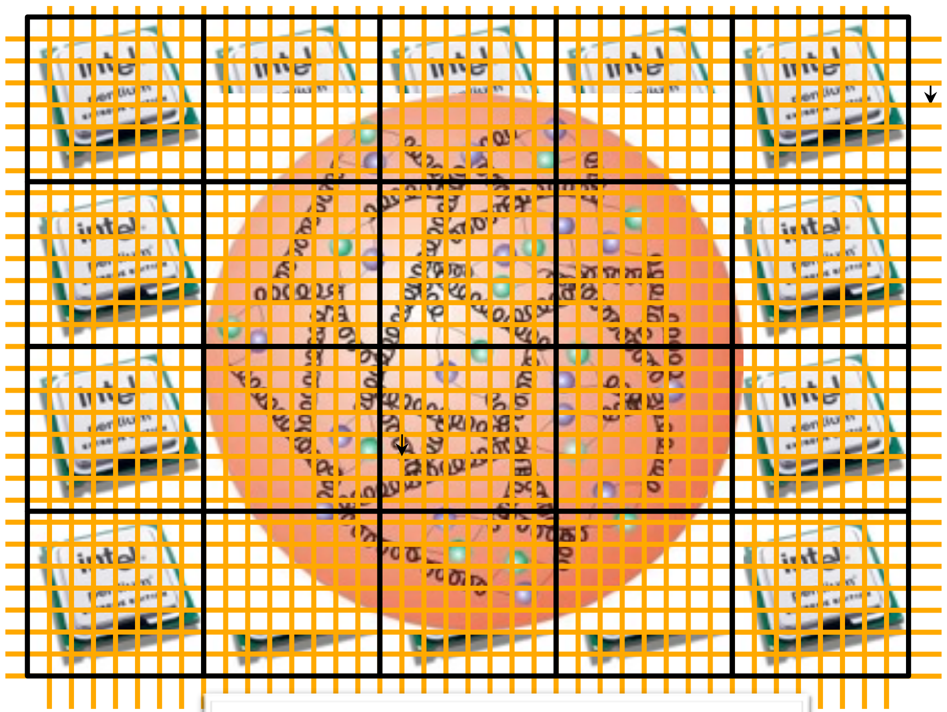

- Understanding their structure, reactions, and electroweak properties within a unified framework well-rooted in quantum chromodynamics has been a long-standing goal of nuclear physics.

# QCD and the nuclear Hamiltonian

- Owing to its non-abelian character, QCD is strongly nonperturbative at “large” distances.

- Lattice-QCD is the most reliable way of “solving” QCD in the low-energy regime, and it promises to provide a solid foundation for the structure of nuclei directly from QCD

- The applicability of Lattice-QCD is limited to few body systems, (A<4), and to a nuclear physics in which the pion mass must be kept much higher than the physical one.



Courtesy of M. Savage

- Quark and gluons do not exists in the physical spectrum as asymptotic states

- Effective theory: non relativistic nucleons interacting via instantaneous potentials

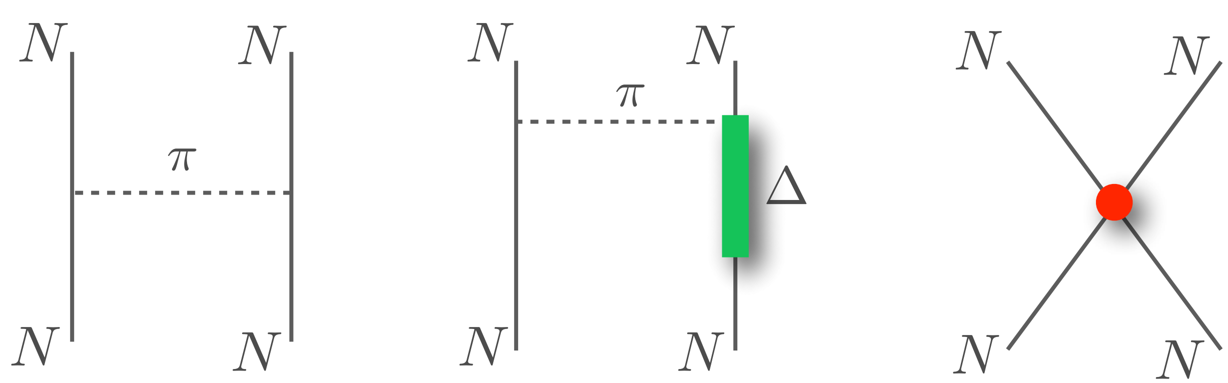
$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

# The nuclear Hamiltonian

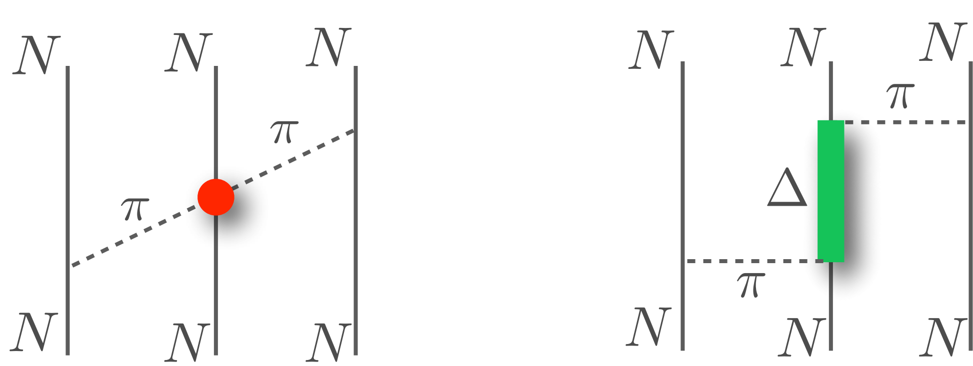
- Ab initio approaches are based on the non relativistic hamiltonian

$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

The Argonne $v_{18}$ is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database

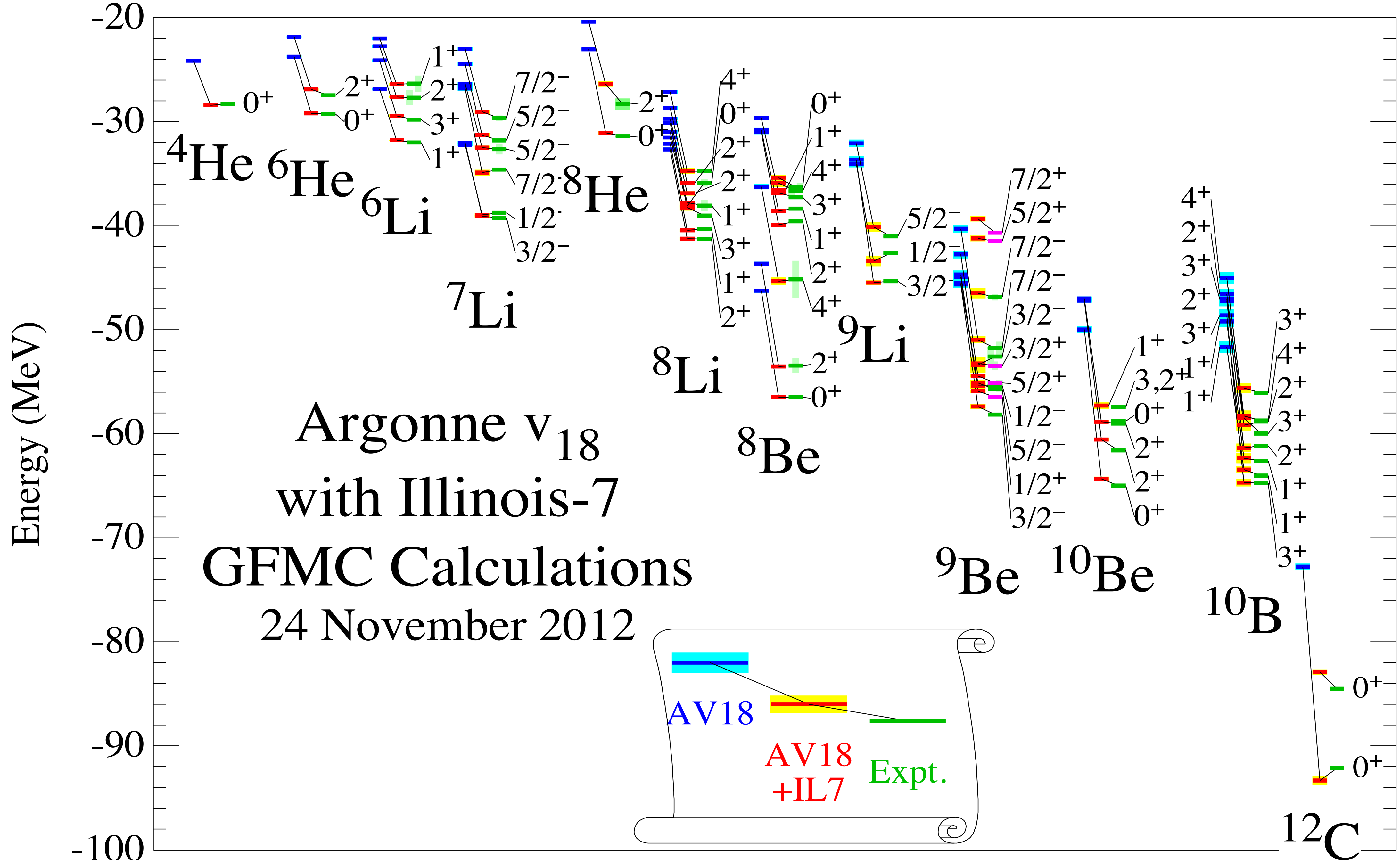



Three-nucleon interactions effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects



# Ab initio nuclear methods

Ab initio approaches accurately predict the energy spectrum of light nuclei

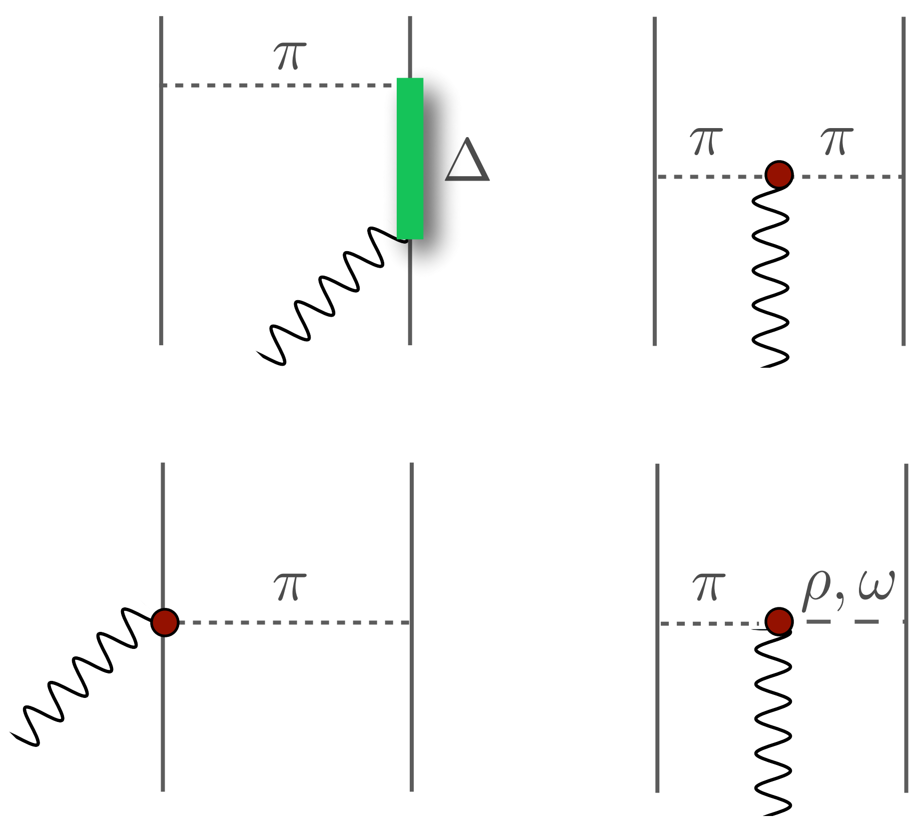


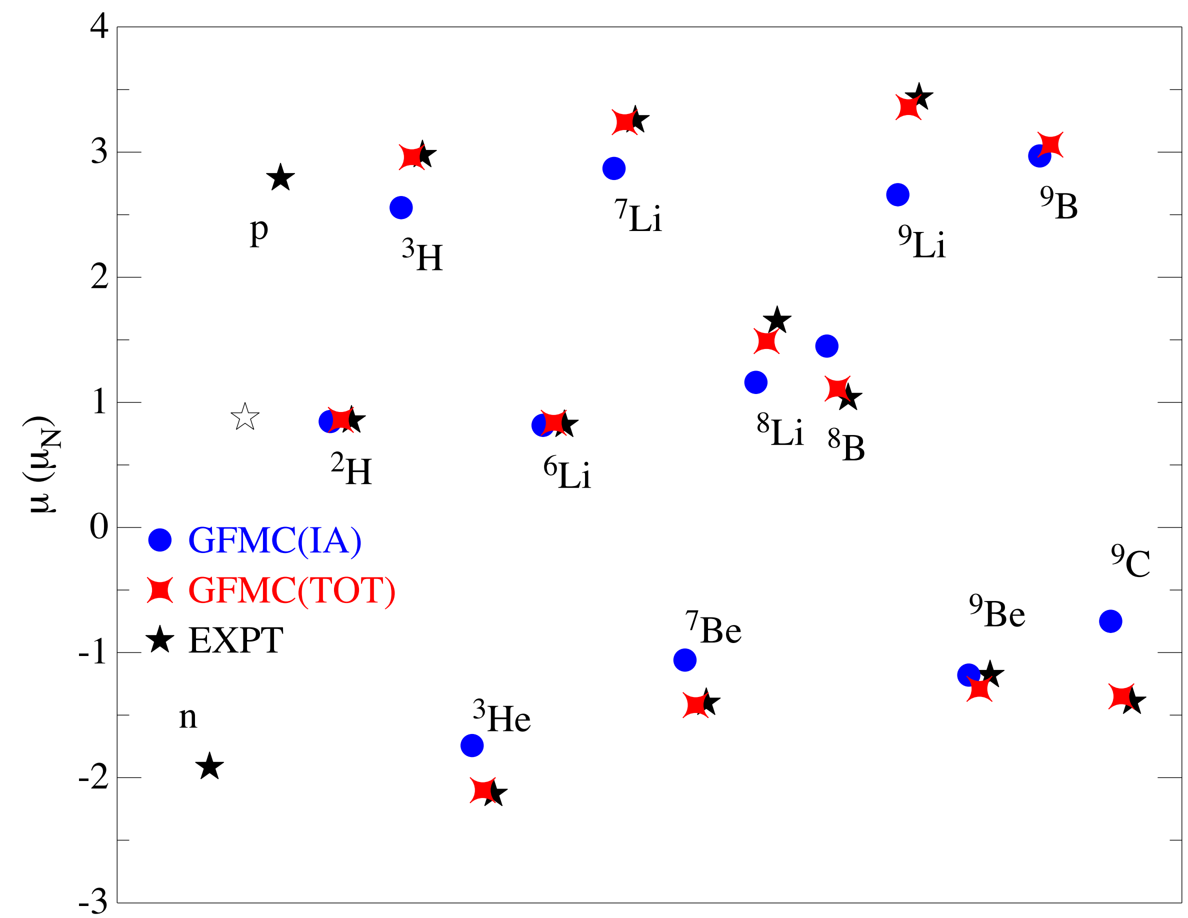
# Ab initio nuclear methods

The nuclear electromagnetic current is constrained by the Hamiltonian through the continuity equation

$$\nabla \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0$$

- The above equation implies that $\mathbf{J}_{\rm EM}$ involves two-nucleon contributions.
- They are essential for low-momentum and low-energy transfer transitions.



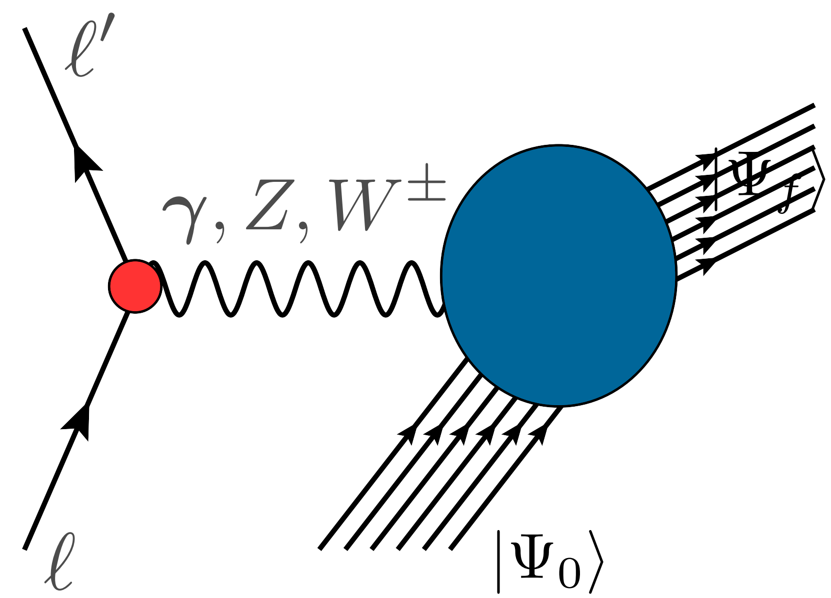




S. Pastore at al., PRC 87, 035503 (2013)

# Lepton-nucleus scattering

The inclusive cross section of the process in which a lepton scatters off a nucleus can be written in terms of five response functions

$$\frac{d\sigma}{dE_{\ell'} d\Omega_\ell} \propto [v_{00}R_{00} + v_{zz}R_{zz} - v_{0z}R_{0z} + v_{xx}R_{xx} \mp v_{xy}R_{xy}]$$

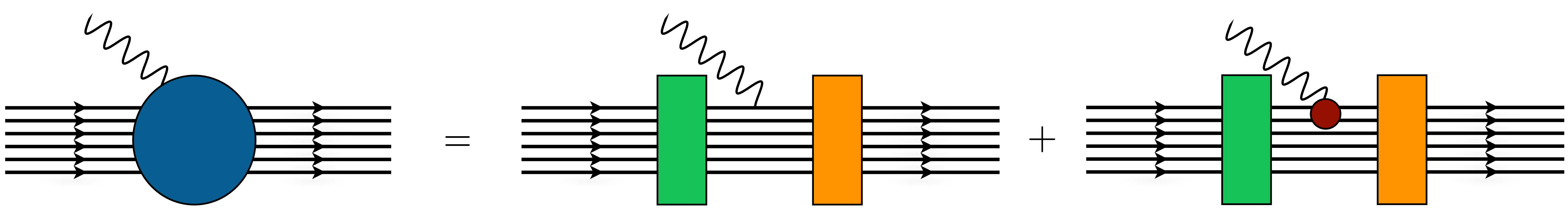



- In the electromagnetic case only the longitudinal and the transverse response functions contribute

- The response functions contain all the information on target structure and dynamics

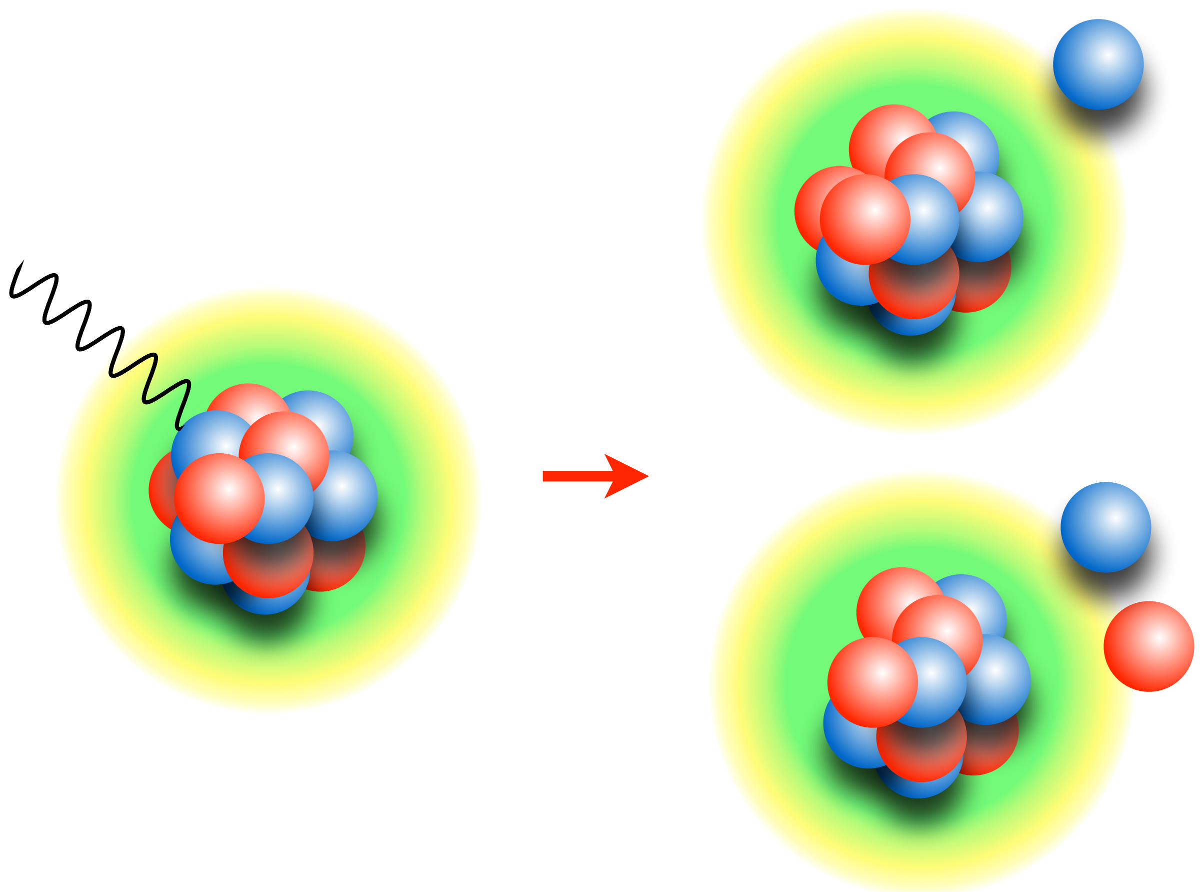
$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle\Psi_0|J_\alpha^\dagger(\mathbf{q})|\Psi_f\rangle\langle\Psi_f|J_\beta(\mathbf{q})|\Psi_0\rangle\delta(\omega - E_f + E_0)$$

- They account for initial state correlations, final state correlations and two-body currents



# Lepton-nucleus scattering

- At (very) large momentum transfer, scattering off a nuclear target reduces to the sum of scattering processes involving bound nucleons → short-range correlations.



$$|\Psi_f\rangle \simeq |p_1\rangle \otimes |\Psi_f\rangle_{A-1}$$

$$|\Psi_f\rangle \simeq |p_1, p_2\rangle \otimes |\Psi_f\rangle_{A-2}$$

- Relativistic effects play a major role and need to be accounted for along with nuclear correlations (Non trivial interplay between them)

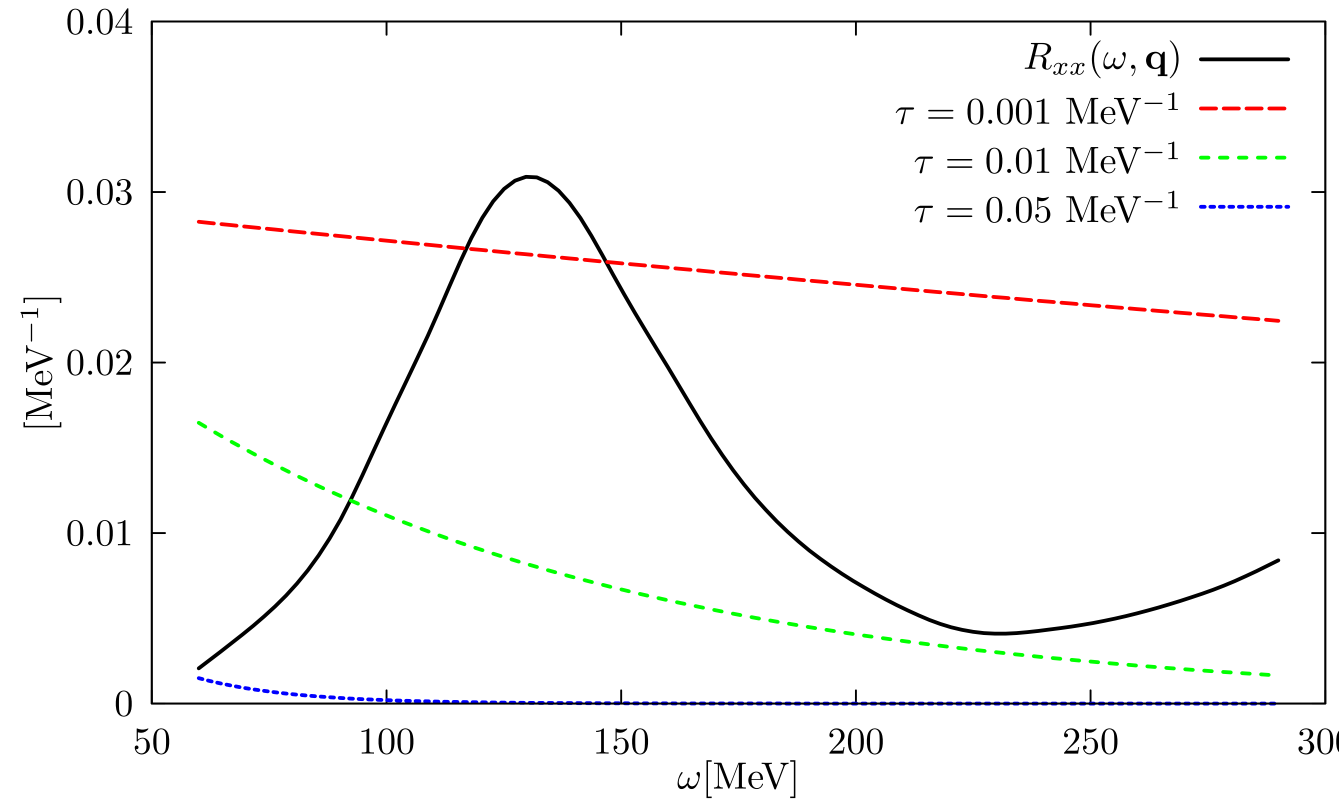
- Resonance production and deep inelastic scattering also need to be accounted for

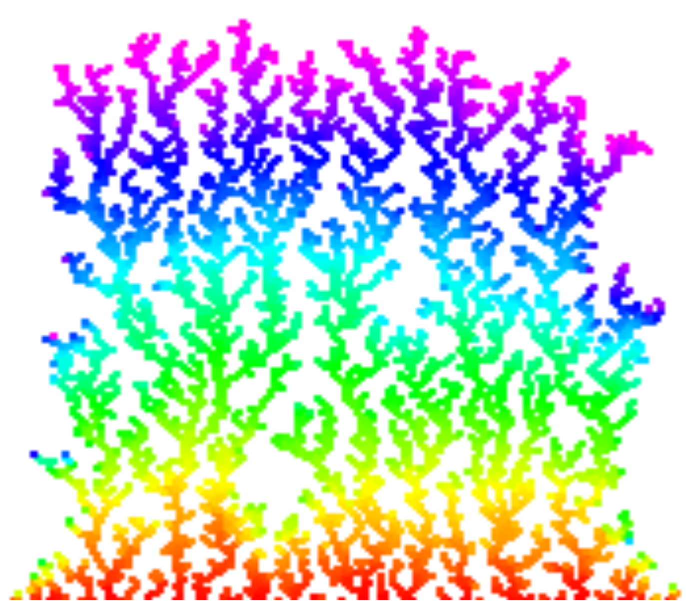
# Euclidean response function

Valuable information on the energy dependence of the response functions can be inferred from their Laplace transforms

$$E_{\alpha\beta}(\tau, \mathbf{q}) \equiv \int d\omega e^{-\omega\tau} R_{\alpha\beta}(\omega, \mathbf{q})$$

At finite imaginary time the contributions from large energy transfer are quickly suppressed



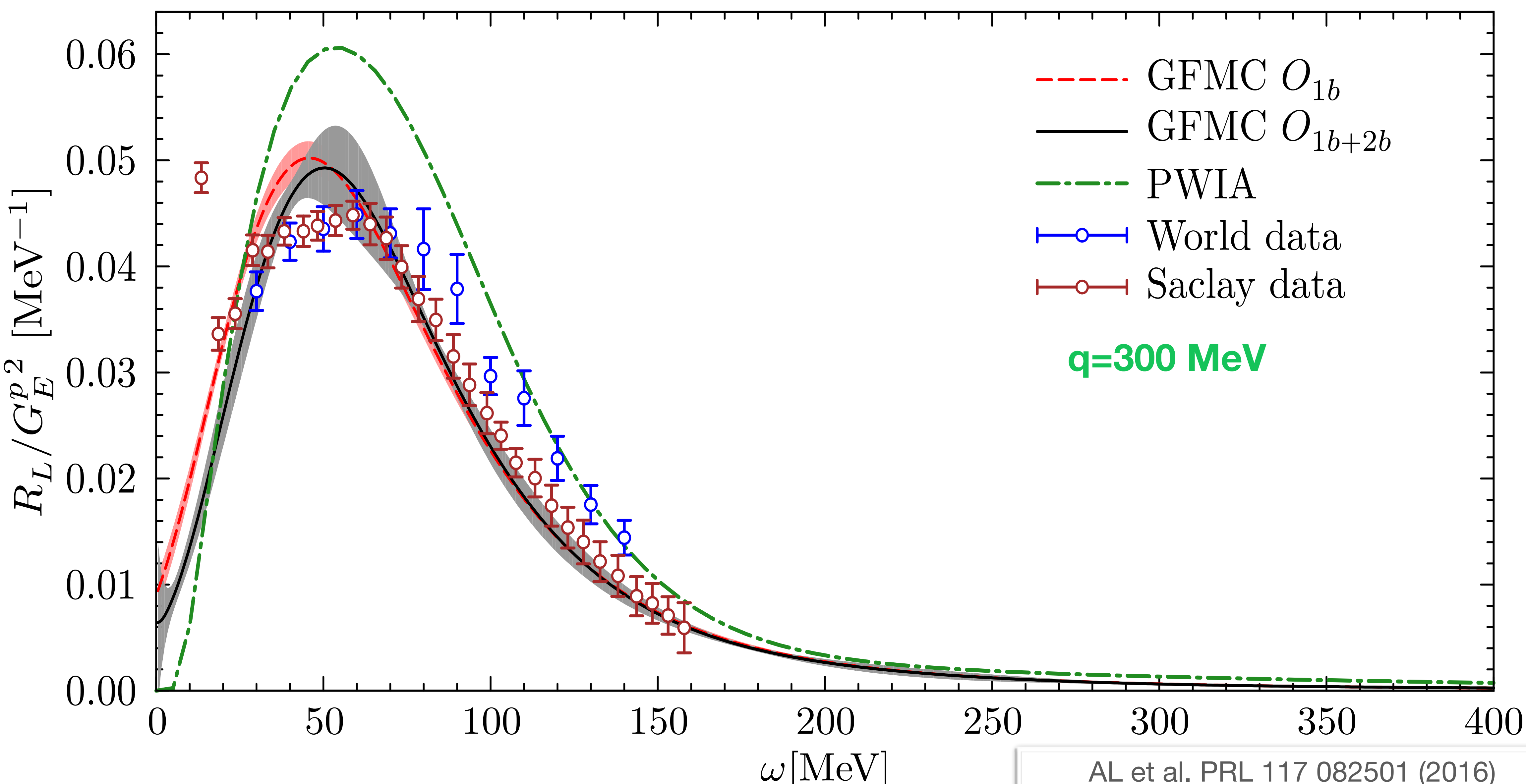
The system is first heated up by the transition operator.
Its cooling determines the Euclidean response of the system

$$E_{\alpha\beta}(\tau, \mathbf{q}) = \langle \Psi_0 | J^{\dagger}_{\alpha}(\mathbf{q}) e^{-(H-E_0)\tau} J_{\beta}(\mathbf{q}) | \Psi_0 \rangle$$

Same technique used in Lattice QCD, condensed matter physics…

# $^{12}$C electromagnetic response

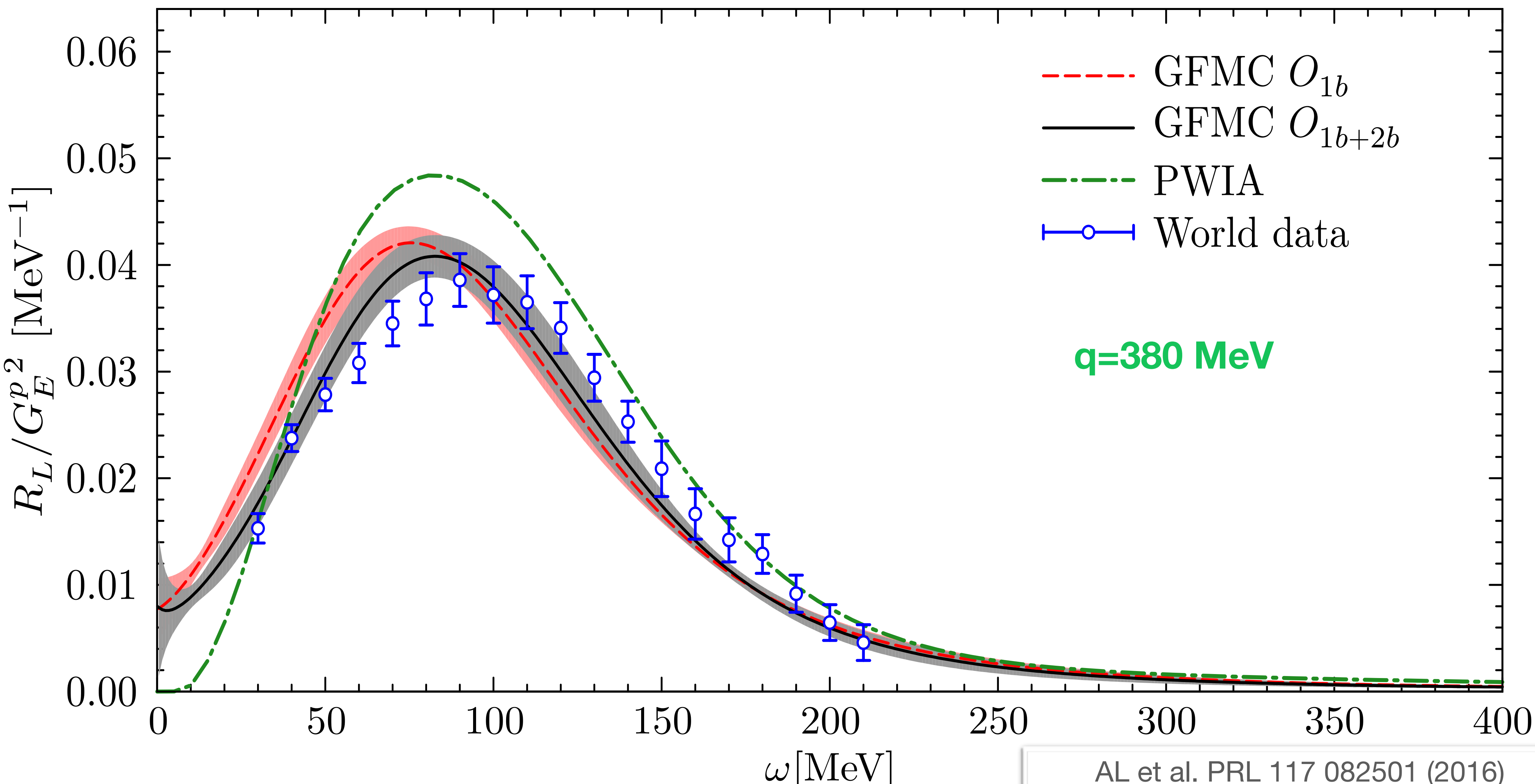
- We inverted the electromagnetic Euclidean response of $^{12}$C
- Very good agreement with the experimental data. Small contribution from two-body currents.




AL et al. PRL 117 082501 (2016)

# $^{12}$C electromagnetic response

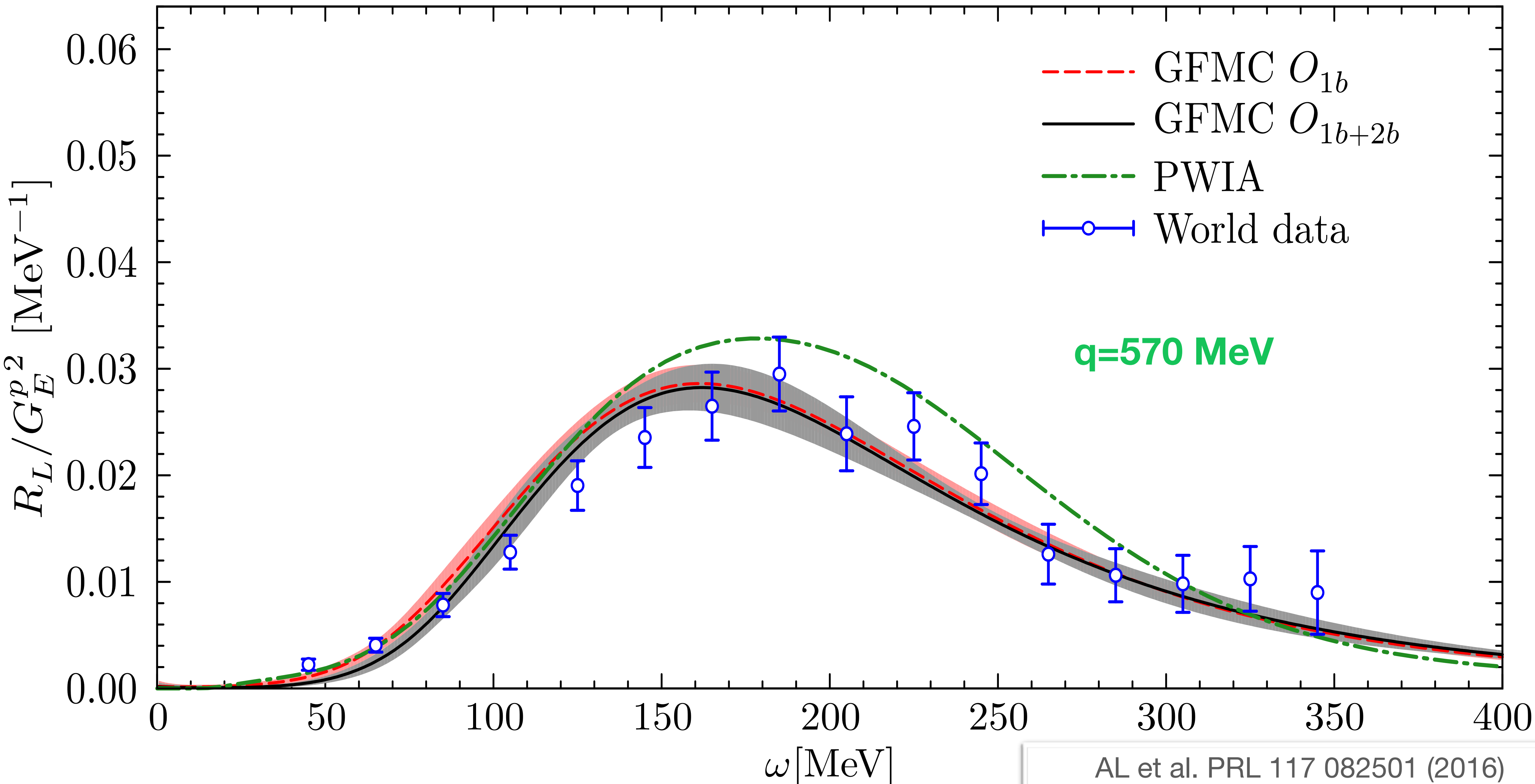
- We inverted the electromagnetic Euclidean response of $^{12}$C
- Very good agreement with the experimental data. Small contribution from two-body currents.




AL et al. PRL 117 082501 (2016)

# $^{12}$C electromagnetic response

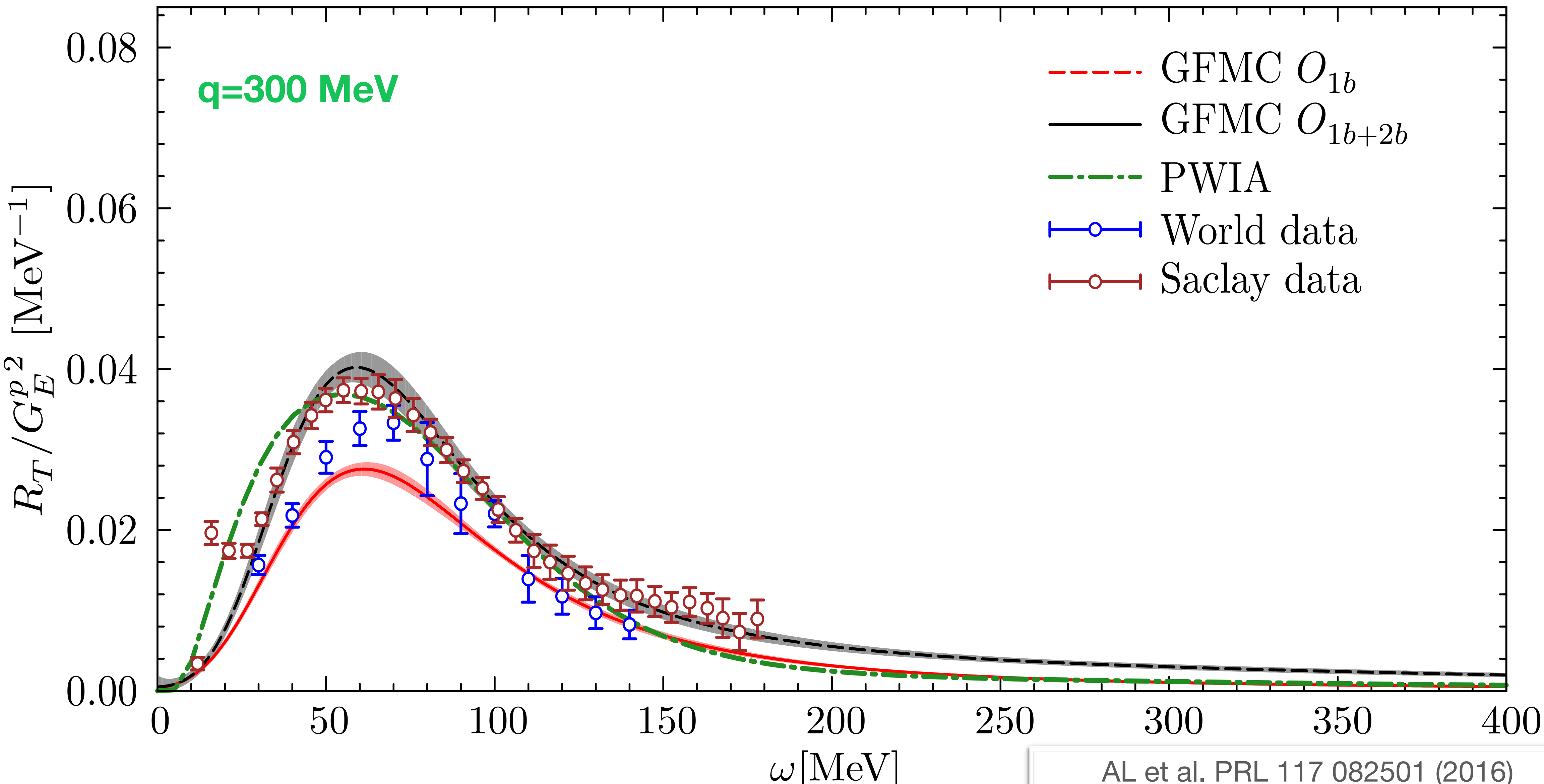
- We inverted the electromagnetic Euclidean response of $^{12}$C
- Very good agreement with the experimental data. Small contribution from two-body currents.




AL et al. PRL 117 082501 (2016)

# $^{12}$C electromagnetic response

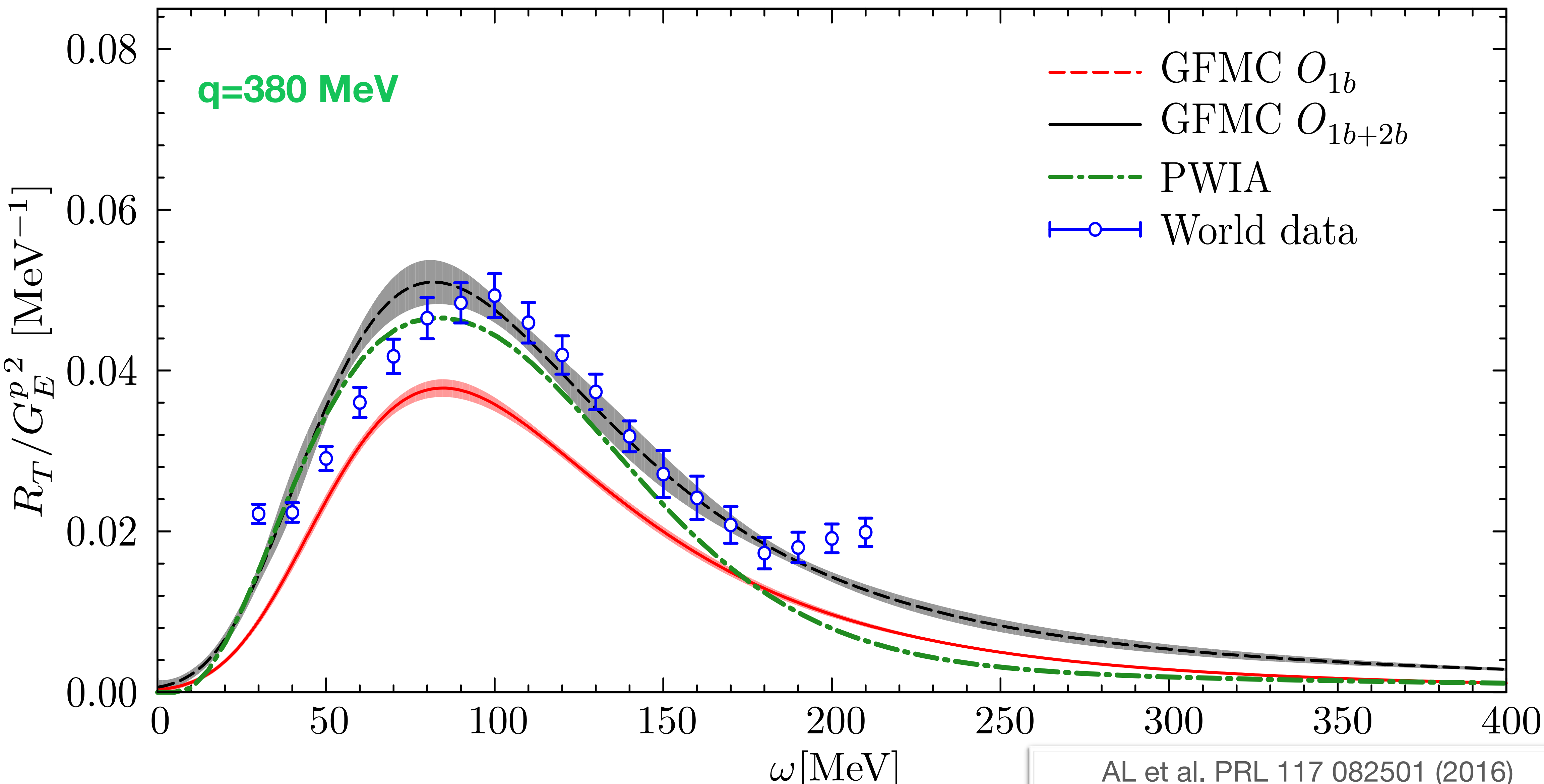
- We inverted the electromagnetic Euclidean response of $^{12}$C
- Very good agreement with the experimental data once two-body currents are accounted for




AL et al. PRL 117 082501 (2016)

# $^{12}$C electromagnetic response

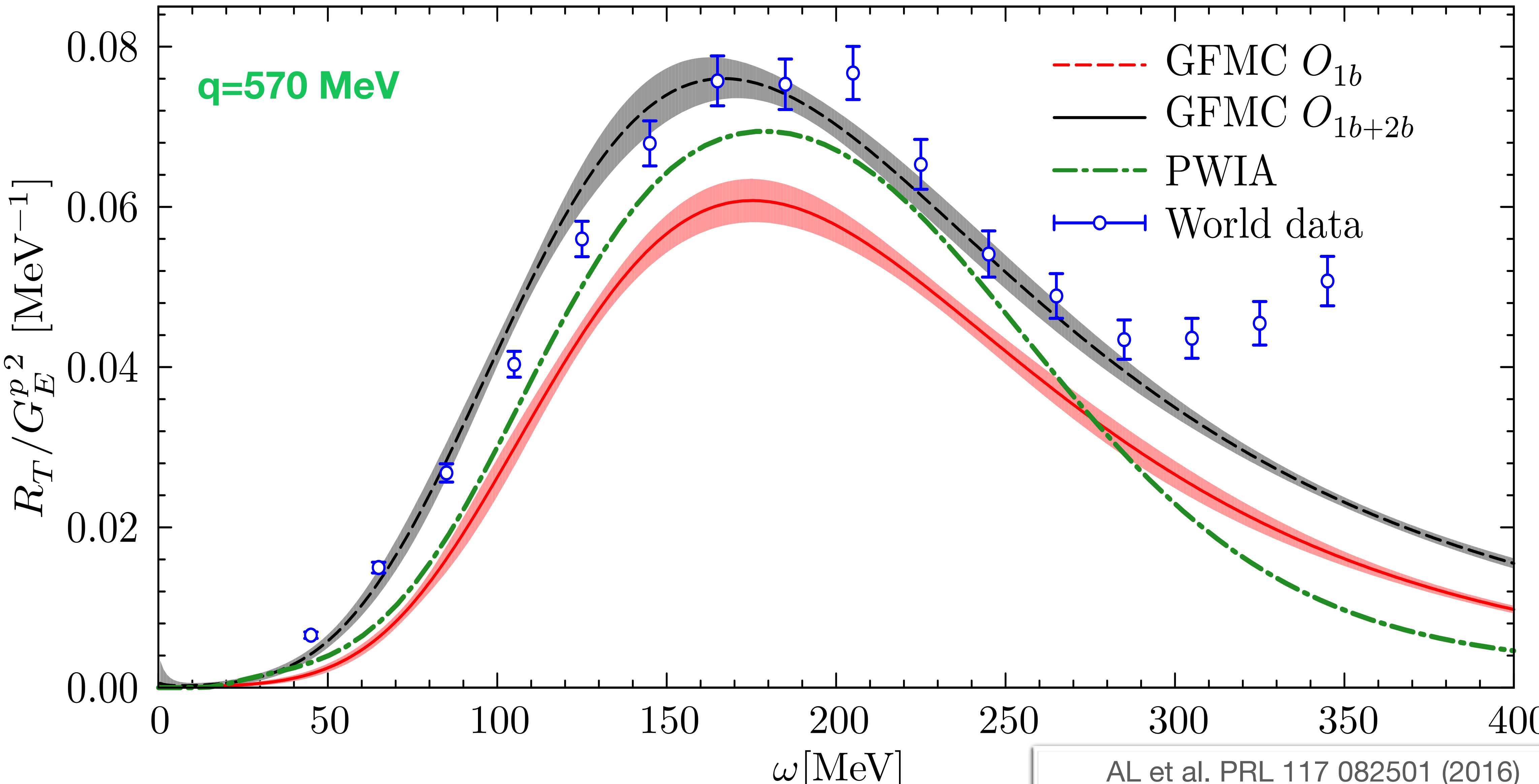
- We inverted the electromagnetic Euclidean response of $^{12}$C
- Very good agreement with the experimental data once two-body currents are accounted for




AL et al. PRL 117 082501 (2016)

# $^{12}$C electromagnetic response

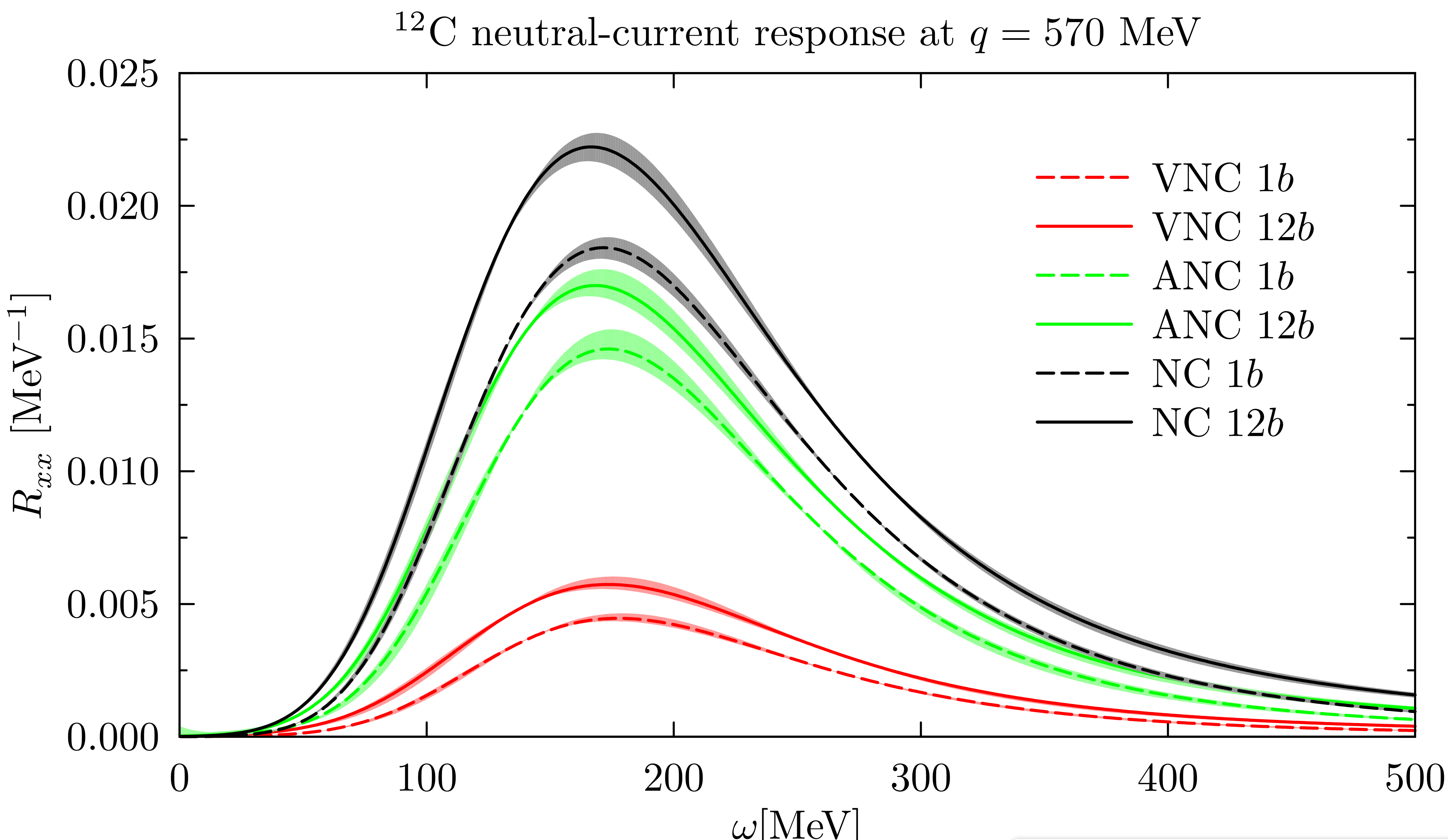
- We inverted the electromagnetic Euclidean response of $^{12}$C
- Very good agreement with the experimental data once two-body currents are accounted for




AL et al. PRL 117 082501 (2016)

# $^{12}C$ neutral-current response

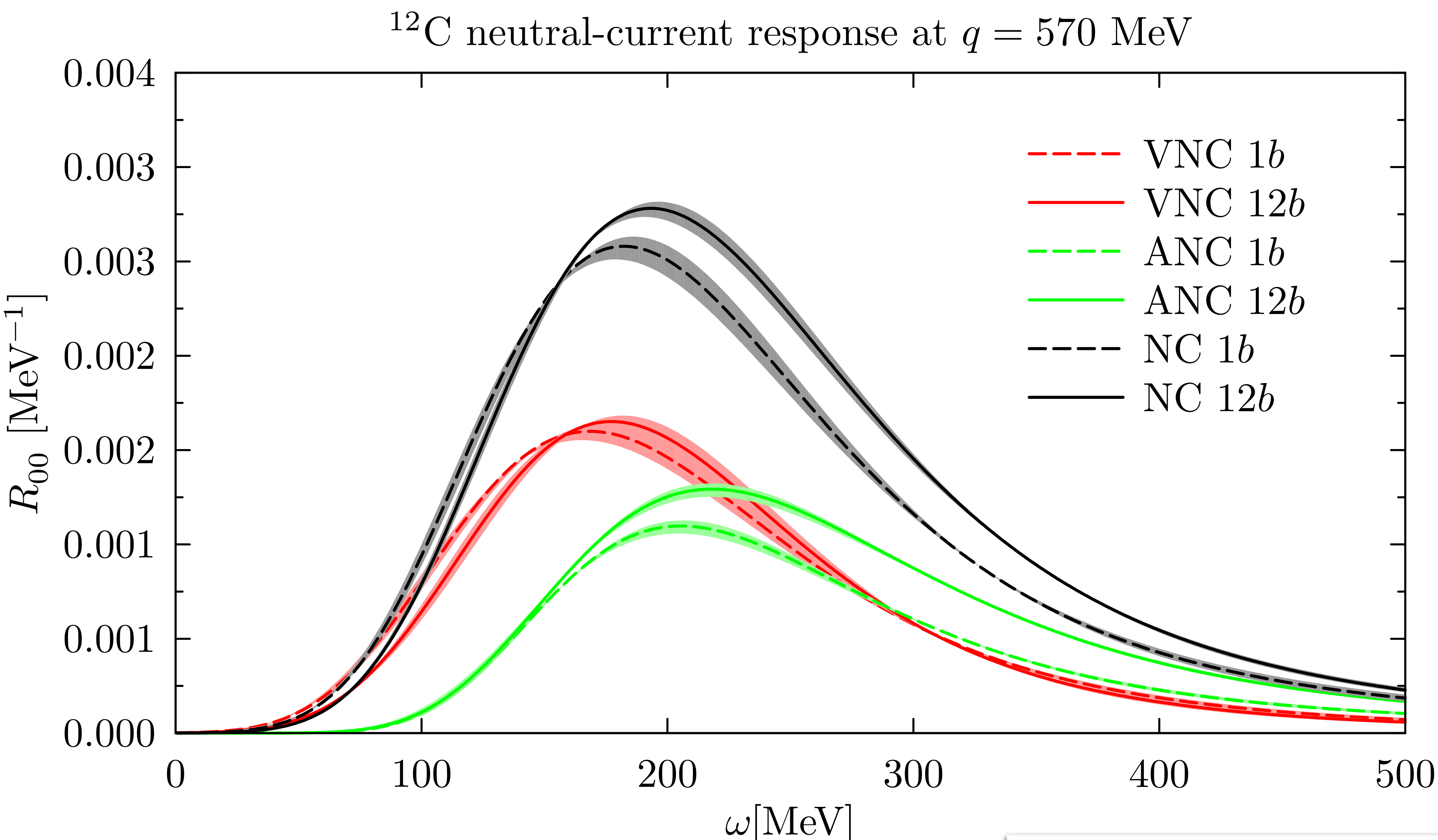
- We were recently able to invert the neutral-current Euclidean responses of $^{12}C$




AL et al. in preparation

# $^{12}$C neutral-current response

- We were recently able to invert the neutral-current Euclidean responses of $^{12}$C




AL et al. in preparation

# Scaling properties of the GFMC responses

- Within the global Fermi gas model, an analytic expression for the response functions can be derived

$$R_{\alpha\beta}(\omega,\mathbf{q}) \simeq \int d\mathbf{k}\,\theta(k_F - |\mathbf{k}|)\theta(|\mathbf{k}+\mathbf{q}| - k_F)\langle \mathbf{k}|j_\alpha^\dagger|\mathbf{k}+\mathbf{q}\rangle\langle \mathbf{k}+\mathbf{q}|j_\beta|\mathbf{k}\rangle\delta(\omega - E_{\mathbf{p}+\mathbf{q}} + E_{\mathbf{p}})$$

- The longitudinal and transverse response functions have the same functional form: “nuclear dynamics” is singled-out from single-nucleon transitions

$$R_{L,T} = \boxed{\frac{3\xi_F}{2m\eta_F^3}(1-\psi^2)\theta(1-\psi^2)} \times \boxed{G_{L,T}}$$

- The scaling functions are defined as

$$f_{L,T}(\psi) = k_F \times \frac{R_{L,T}}{G_{L,T}} \quad \rightarrow \quad f_{L,T}^{GFG}(\psi) = \frac{3\xi_F}{2\eta_F^2}(1-\psi^2)\theta(1-\psi^2)$$

- Scaling properties of nuclear response functions highlight interesting information about nuclear structure and dynamics.

# Scaling properties of the GFMC responses

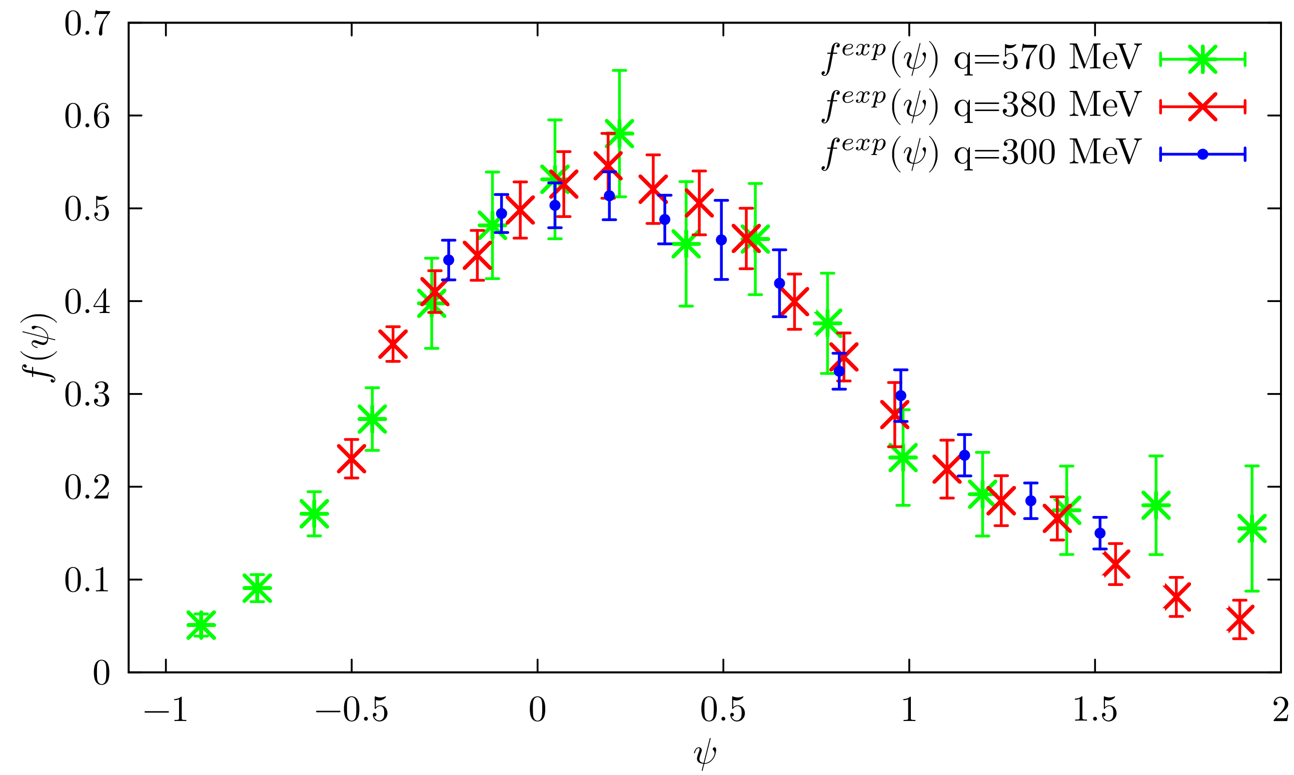
- The experimental longitudinal response functions of $^4$He and $^{12}$C exhibit scaling of the first kind

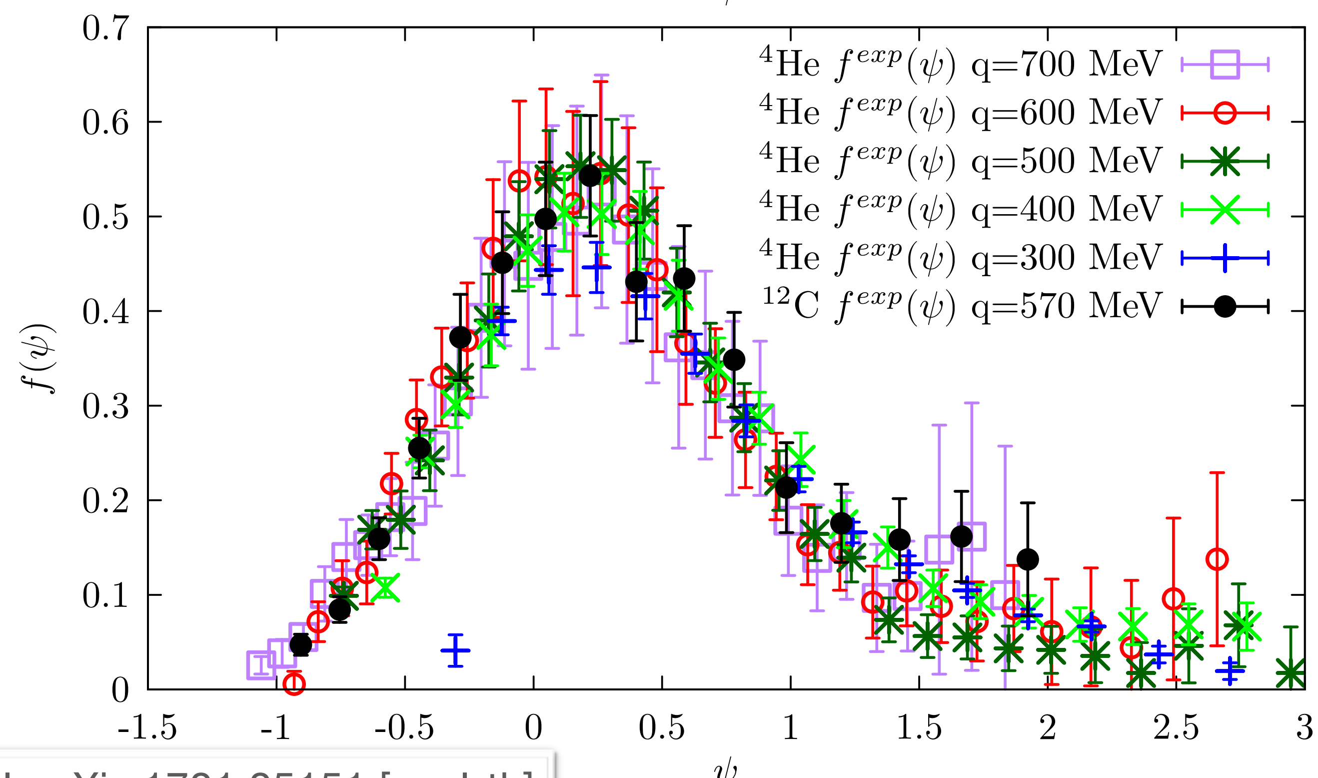
$$f_{L,T}(\psi) = k_F \times \frac{R_{L,T}}{G_{L,T}}$$

- By properly choosing $k_F$, scaling of the second kind between $^4$He and $^{12}$C also occurs

- In both cases the scaling function is very different from the one of the global Fermi gas model

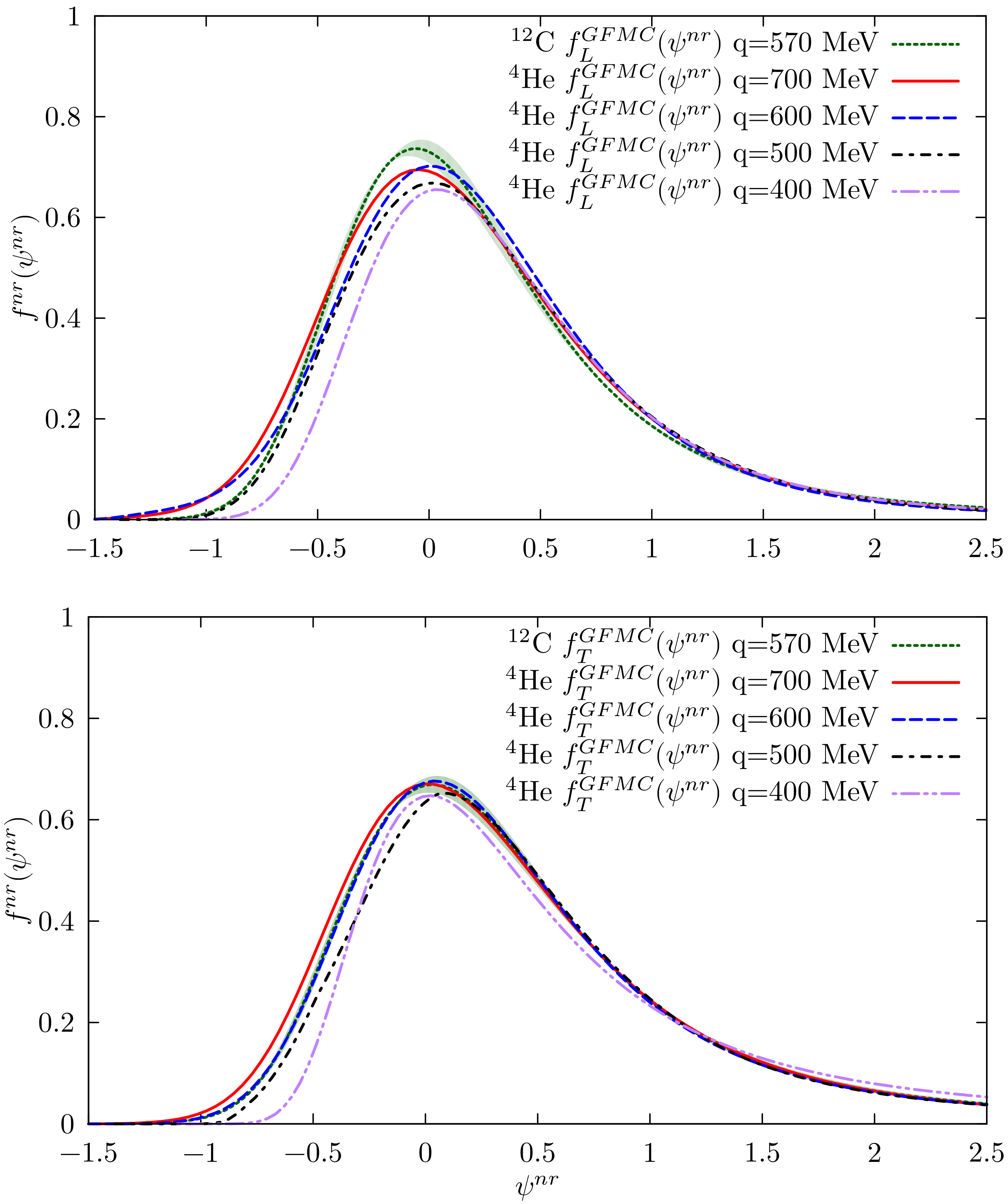
- The scaling functions exhibit a clearly asymmetric shape, with a tail extending in the region $\psi > 0$




N. Rocco et al. arXiv:1701.05151 [nucl-th]

# Scaling properties of the GFMC responses




- We studied the GFMC non-relativistic scaling functions of $^{4}$He and $^{12}$C

$$f_{L,T}(\psi^{nr}) = k_F \times \frac{R_{L,T}}{G^{nr}_{L,T}}$$

- The results are consistent with scaling of zeroth, first and second kinds

- Despite the non relativistic nature of the calculation, all the scaling functions are strongly asymmetric, with a tail extending to the large ψ region.

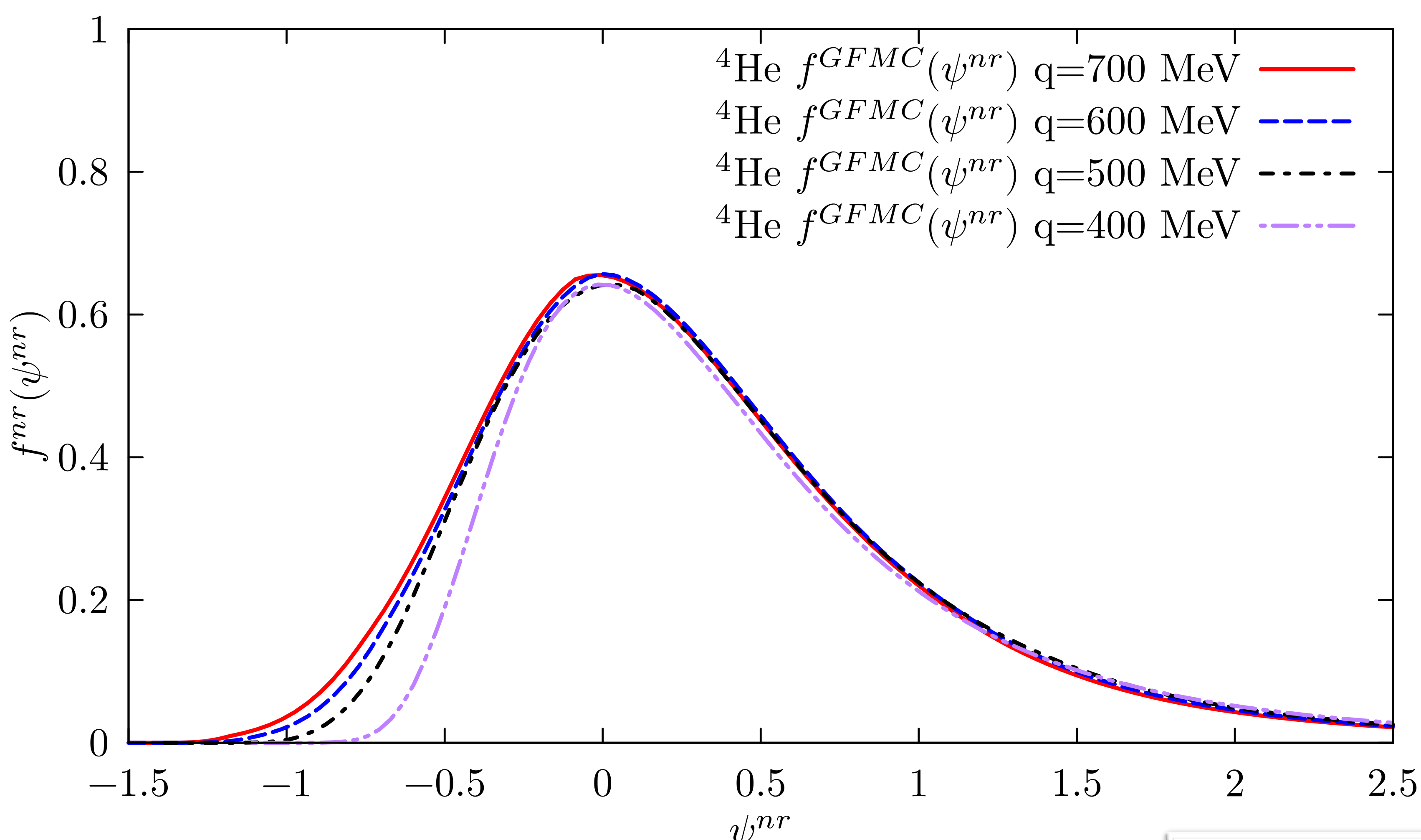
N. Rocco et al. arXiv:1701.05151 [nucl-th]

# Scaling properties of the GFMC responses

- We identified the longitudinal scaling function with the proton-density response

$$R_{p(n)} \equiv \sum_f \langle 0|\varrho^\dagger_{p(n)}(\mathbf{q})|f\rangle\langle f|\varrho_{p(n)}(\mathbf{q})|0\rangle\,\delta(E_0+\omega-E_f)$$

$$\varrho_{p(n)} \equiv \sum_i e^{i\mathbf{q}\cdot\mathbf{r}_i}\frac{(1\pm\tau_{i,z})}{2} \longleftrightarrow \varrho \simeq \sum_i e^{i\mathbf{q}\cdot\mathbf{r}_i}\left[G_E^p\frac{(1+\tau_{i,z})}{2}+G_E^n\frac{(1+\tau_{i,z})}{2}\right]$$



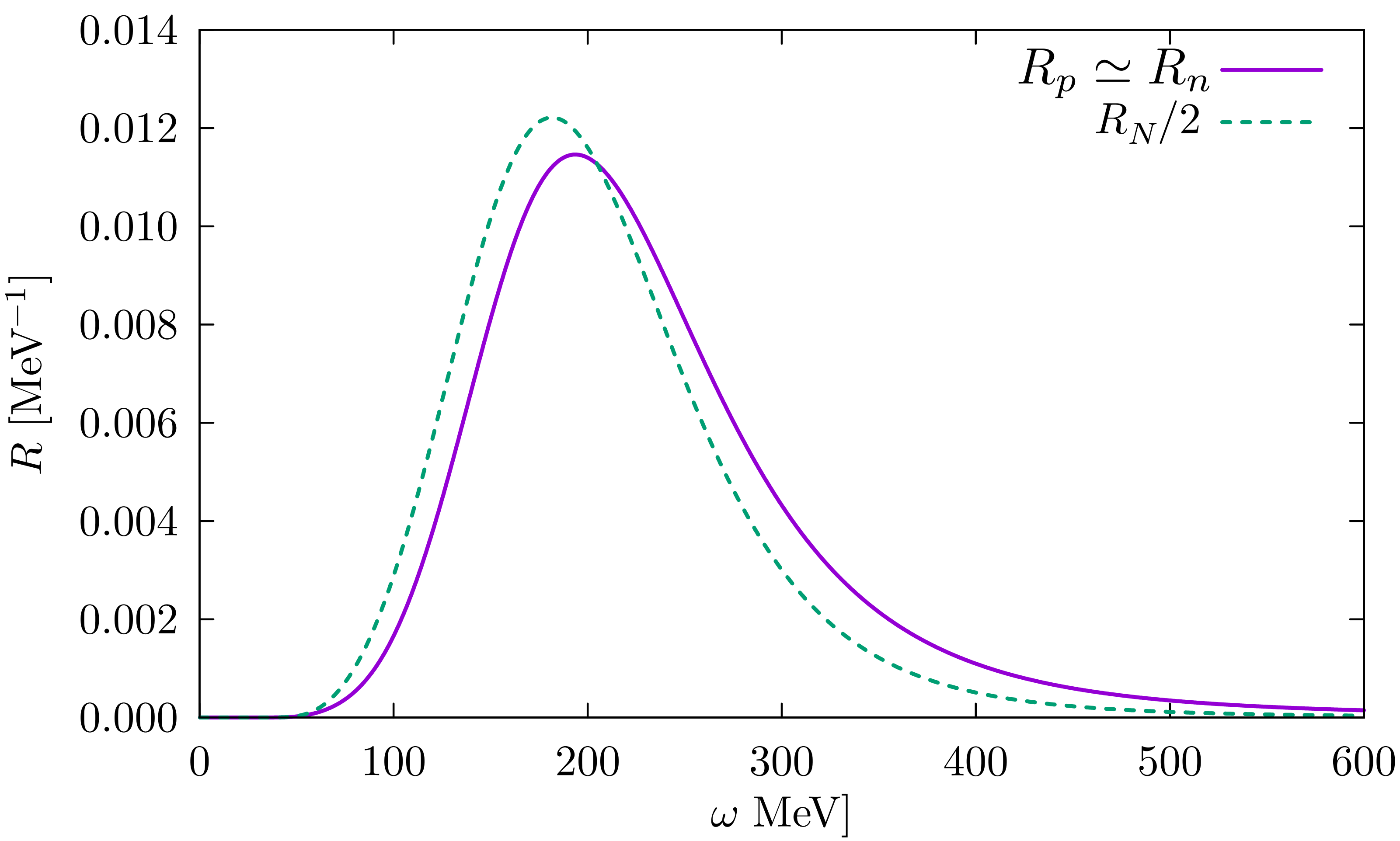

N. Rocco et al. arXiv:1701.05151 [nucl-th]

# Nuclear dynamics surprises

- Beyond impulse approximation effects are important. Particularly enlightening is the comparison between the nucleon and proton responses

$$R_{N,p} \equiv \sum_f \langle 0|\rho^\dagger_{N,p}(\mathbf{q})|f\rangle\langle f|\rho_{N,p}(\mathbf{q})|0\rangle\delta(E_0+\omega-E_f)$$

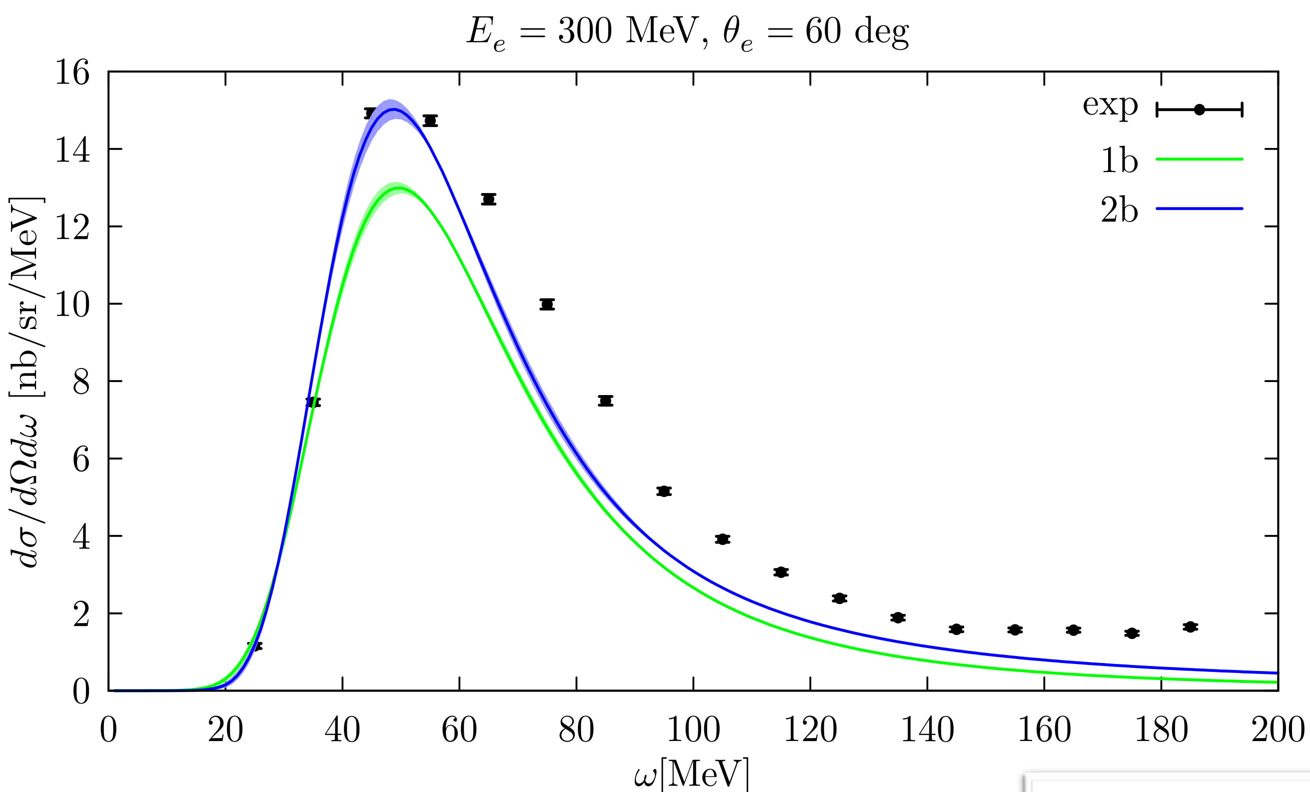
$$\rho_p(\mathbf{q}) = \sum_i e^{i\mathbf{q}\mathbf{r}_i}\frac{1+\tau_{i,z}}{2} \qquad \rho_N(\mathbf{q}) = \sum_i e^{i\mathbf{q}\mathbf{r}_i} = \rho_n + \rho_p$$

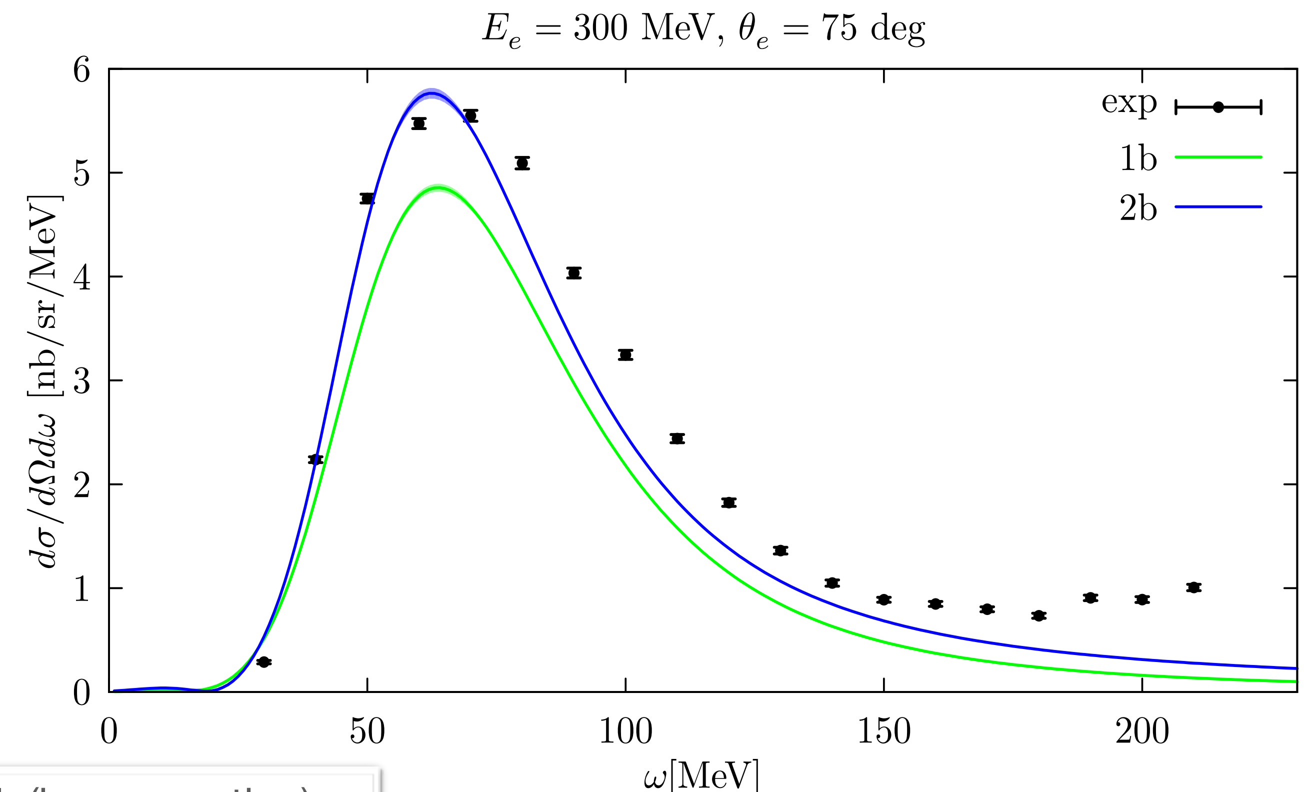



- In the impulse approximation the nucleon and the proton responses coincide

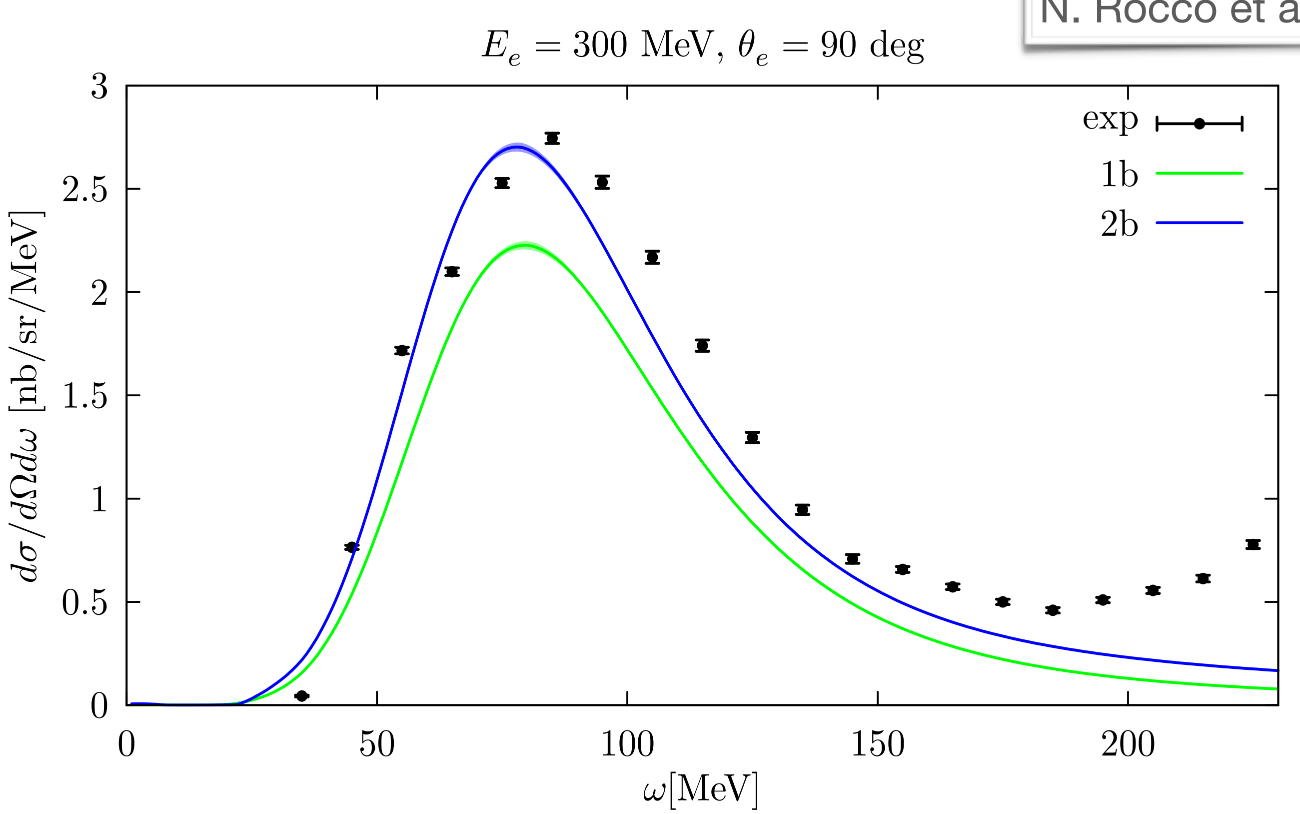
- GFMC results demonstrate the importance of the charge-exchange character of the nucleon-nucleon force

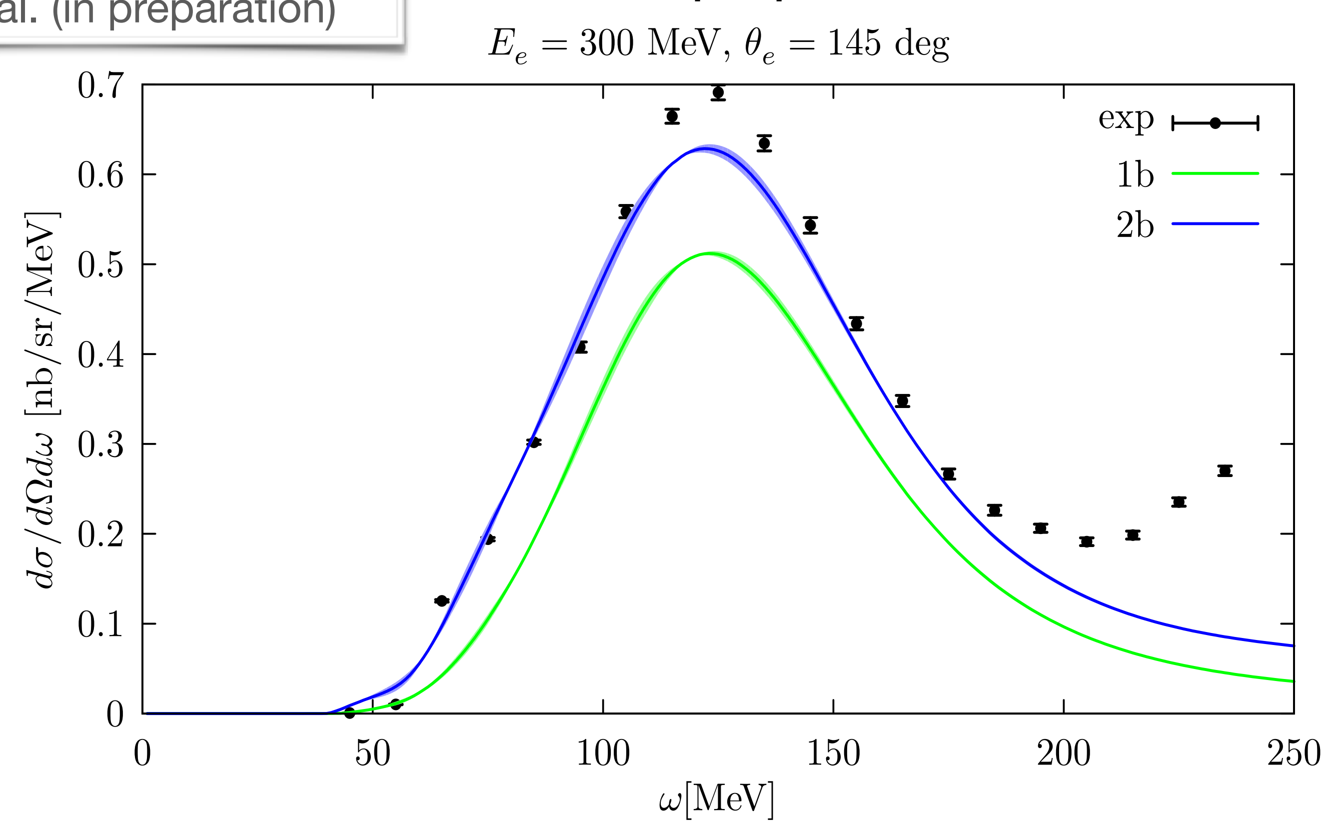
# GFMC $^4$He cross sections



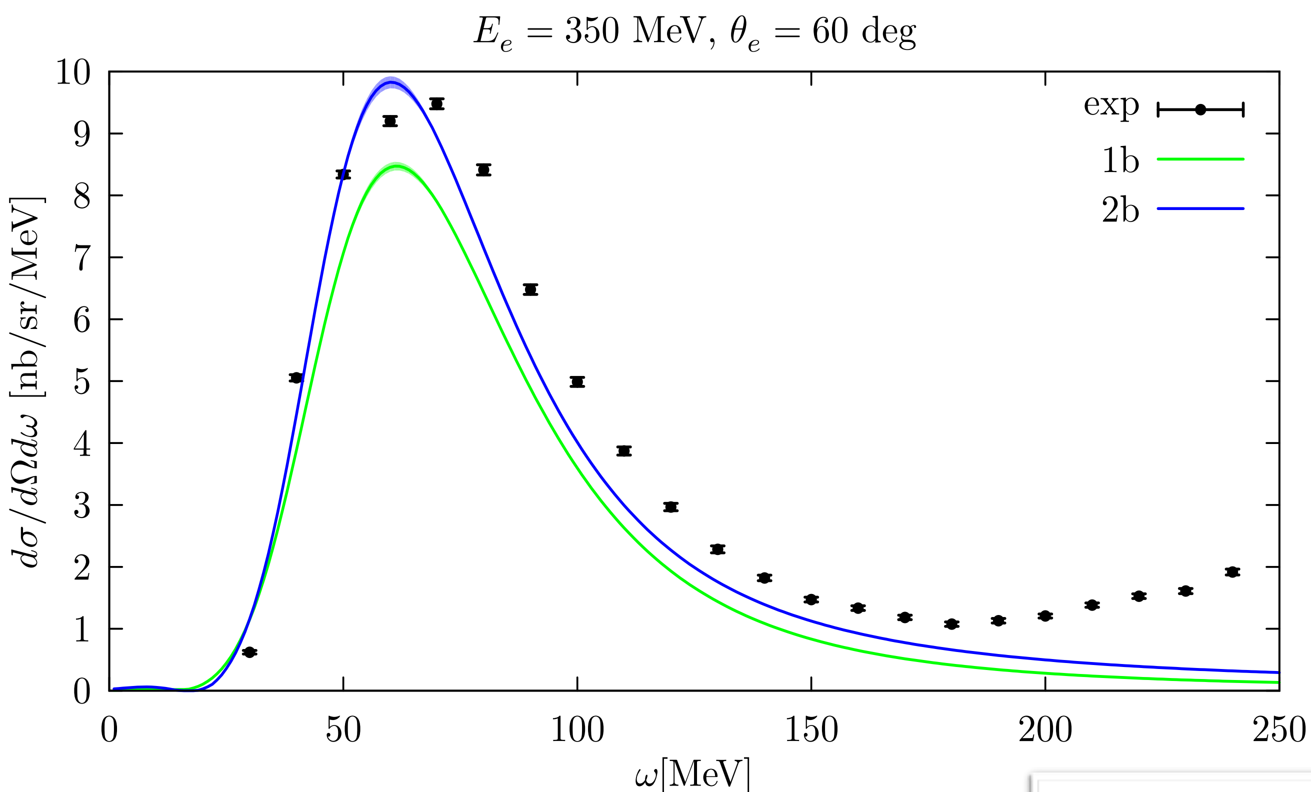



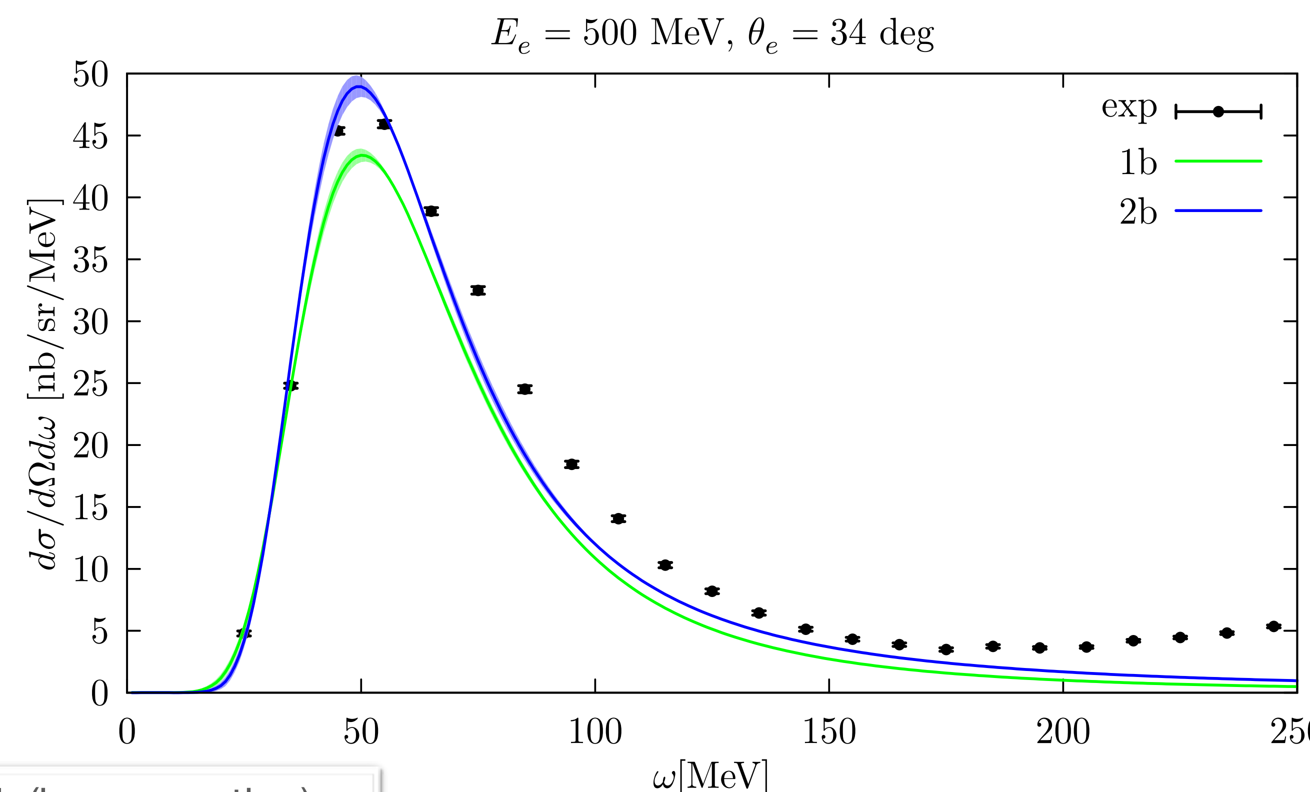

N. Rocco et al. (in preparation)



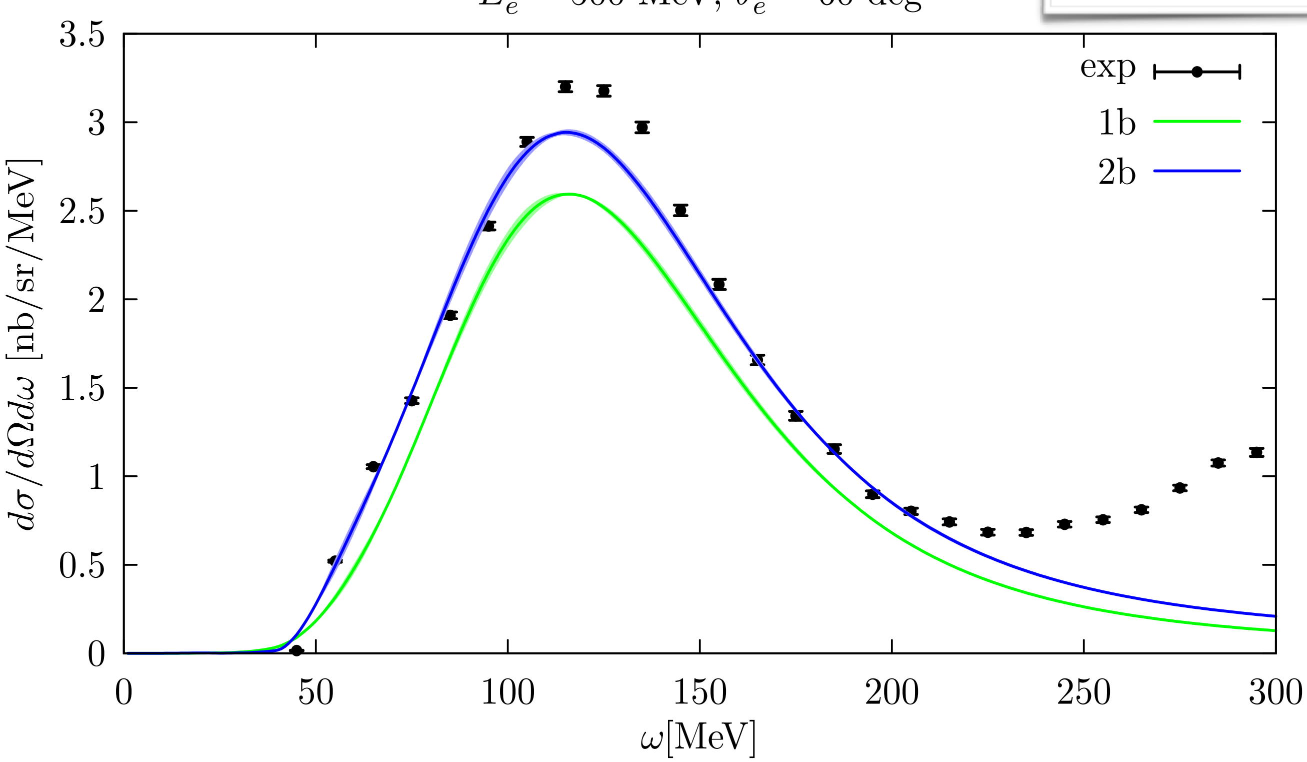



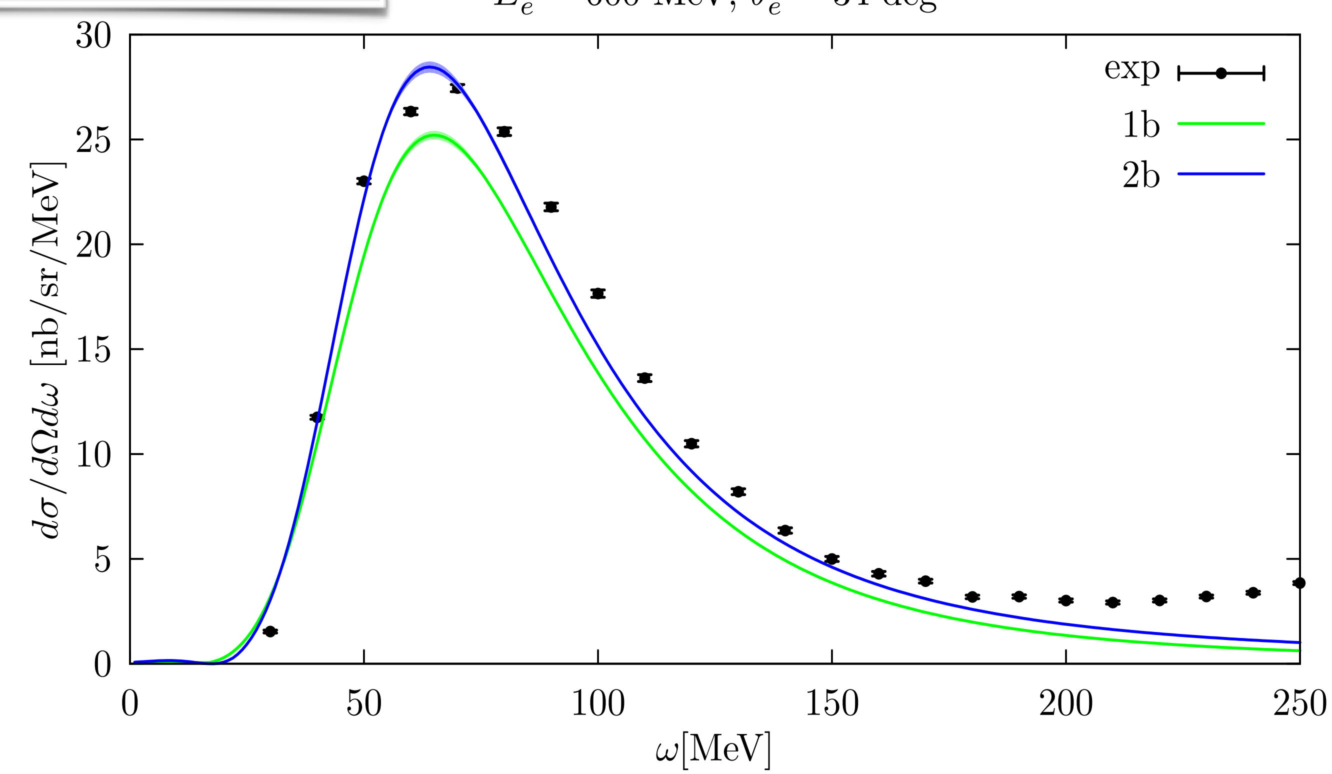
# GFMC $^4$He cross sections



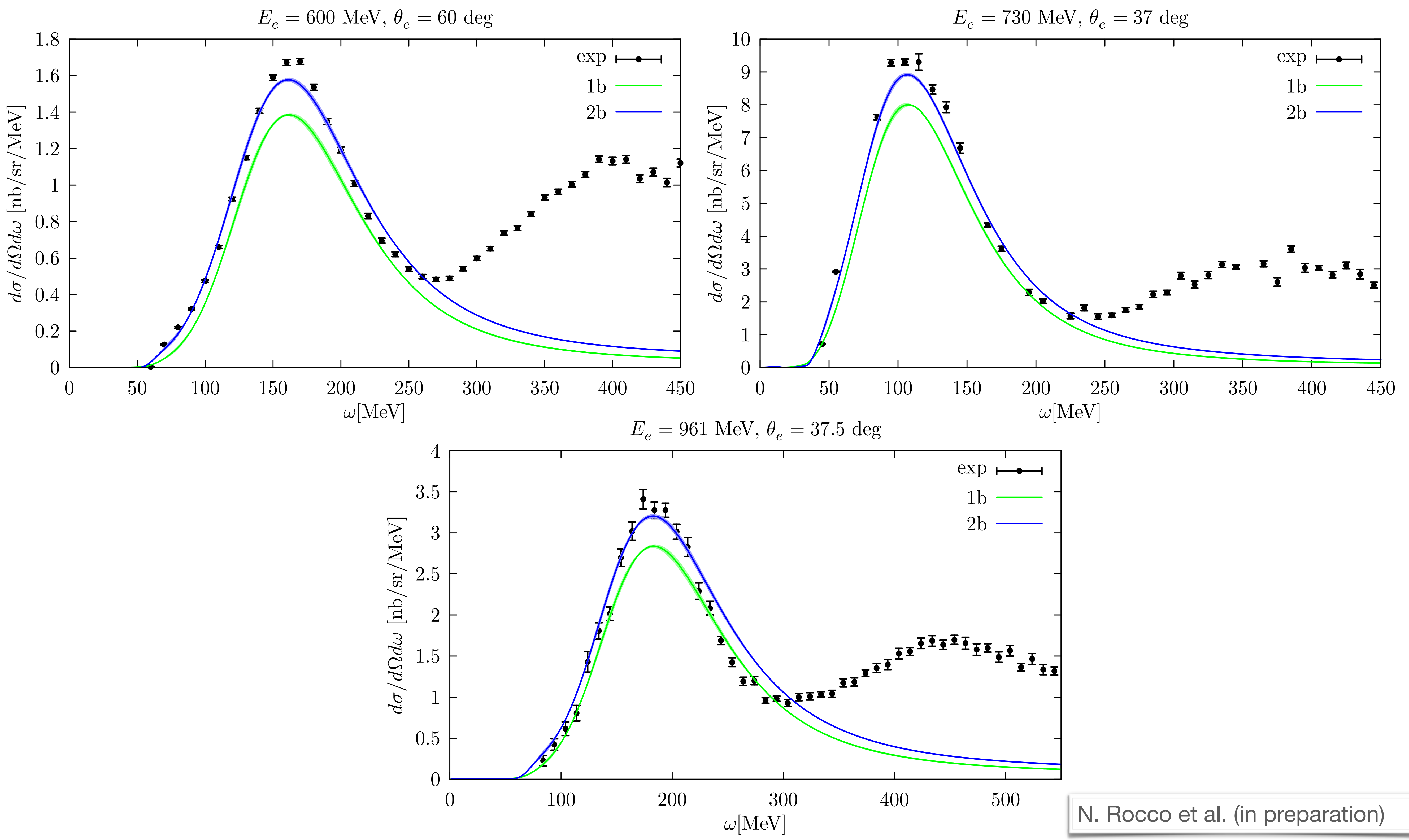




N. Rocco et al. (in preparation)



# GFMC $^4$He cross sections




N. Rocco et al. (in preparation)

# Conclusions

- The two-body currents enhancement is effective in the entire energy transfer domain.

- $^4$He and $^{12}$C results for the electromagnetic response obtained using Maximum Entropy technique are in very good agreement with experimental data.

- Two-body current contributions enhance the longitudinal and transverse axial responses

- Quantum Monte Carlo is suitable to compute cross-sections, not only responses

**Disclaimer**

- The continuity equation only constraints the longitudinal components of the current
- The transverse component and the axial terms are phenomenological (the coupling constant is fitted on the tritium beta-decay)
- Two- and three- body forces not fully consistent

The theoretical error arising from modeling the nuclear dynamics cannot be properly assessed

# Conclusions

•The two-body currents enhancement is effective in the entire energy transfer domain.

• $^4$He and $^{12}$C results for the electromagnetic response obtained using Maximum Entropy technique are in very good agreement with experimental data.

• Two-body current contributions enhance the longitudinal and transverse axial responses

• Quantum Monte Carlo is suitable to compute cross-sections, not only responses

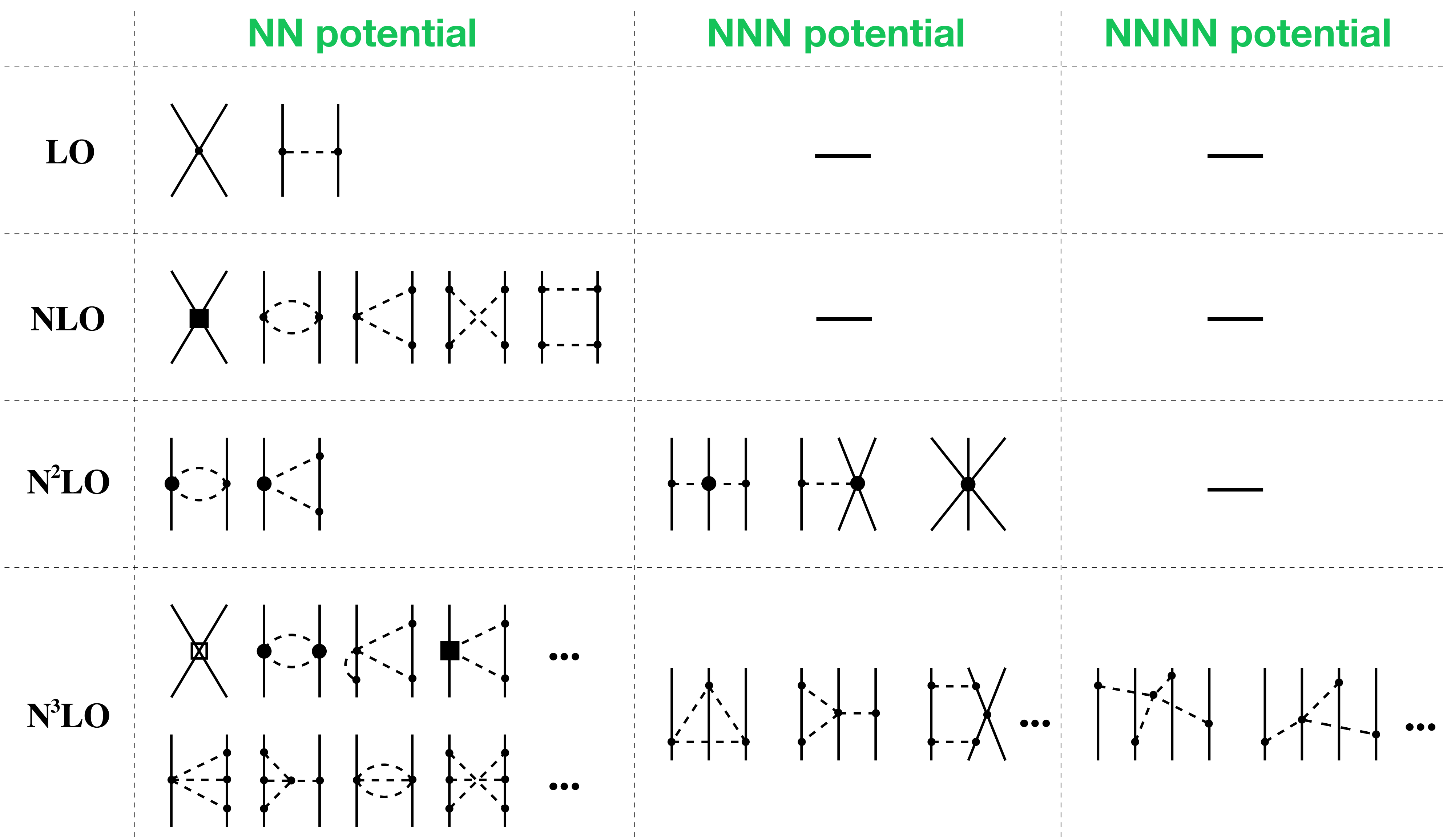
**Disclaimer**

- The continuity equation only constraints the longitudinal components of the current
- The transverse component and the axial terms are phenomenological (the coupling constant is fitted on the tritium beta-decay)
- Two- and three- body forces not fully consistent

The theoretical error arising from modeling the nuclear dynamics cannot be properly assessed

# Chiral EFT

In chiral-EFT, the symmetries of quantum chromodynamics, in particular its approximate chiral symmetry, are employed to systematically constrain classes of Lagrangians describing the interactions of baryons with pions as well as the interactions of these hadrons with electroweak fields



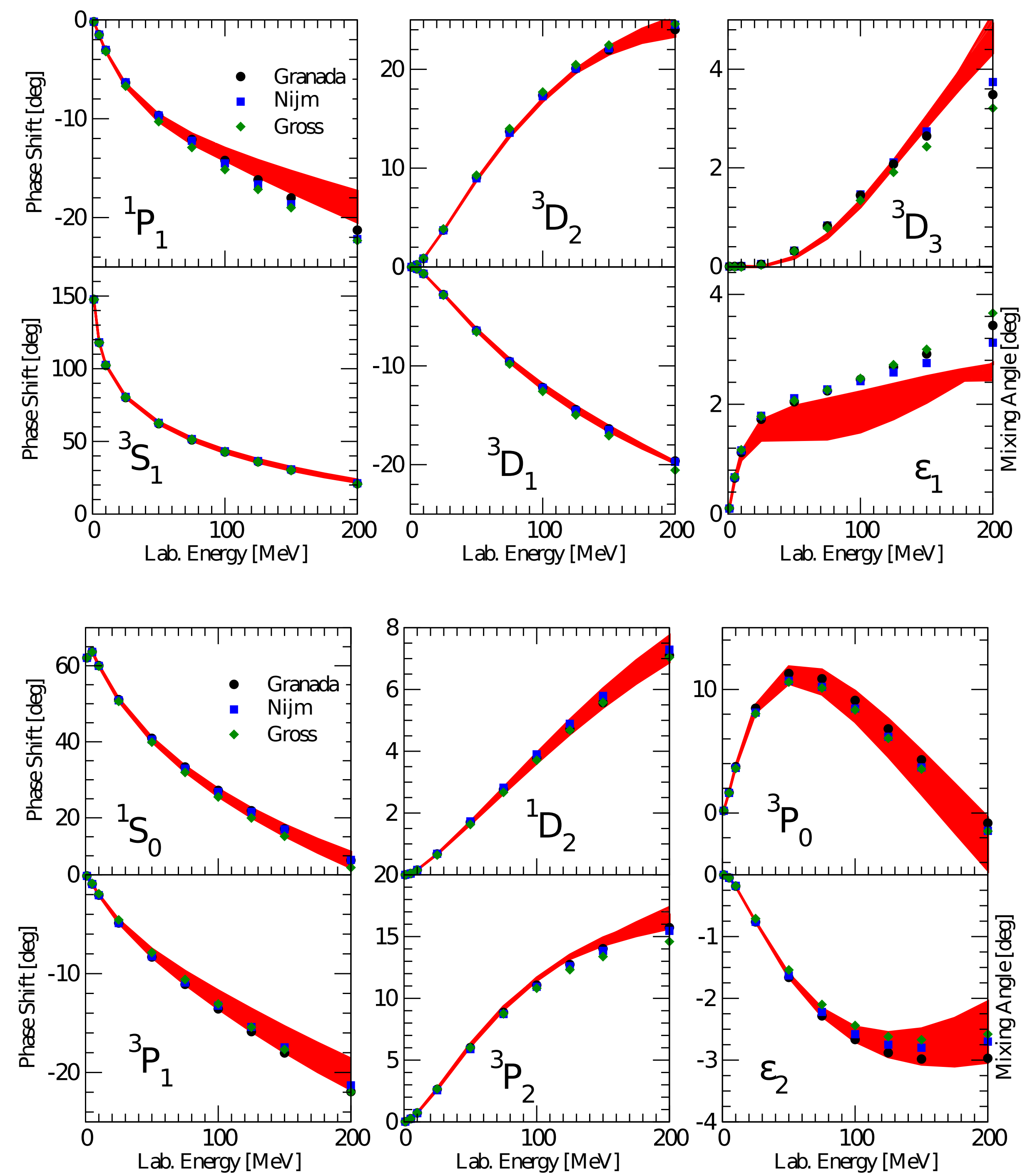
# Δ-full local chiral potential

M. Piarulli, et al. PRC 94 (2016) 054007

We have complemented the historical “Argonne” approach by considering a local chiral Δ -full potential giving an excellent fit to the NN scattering data that can be readily used in QMC.

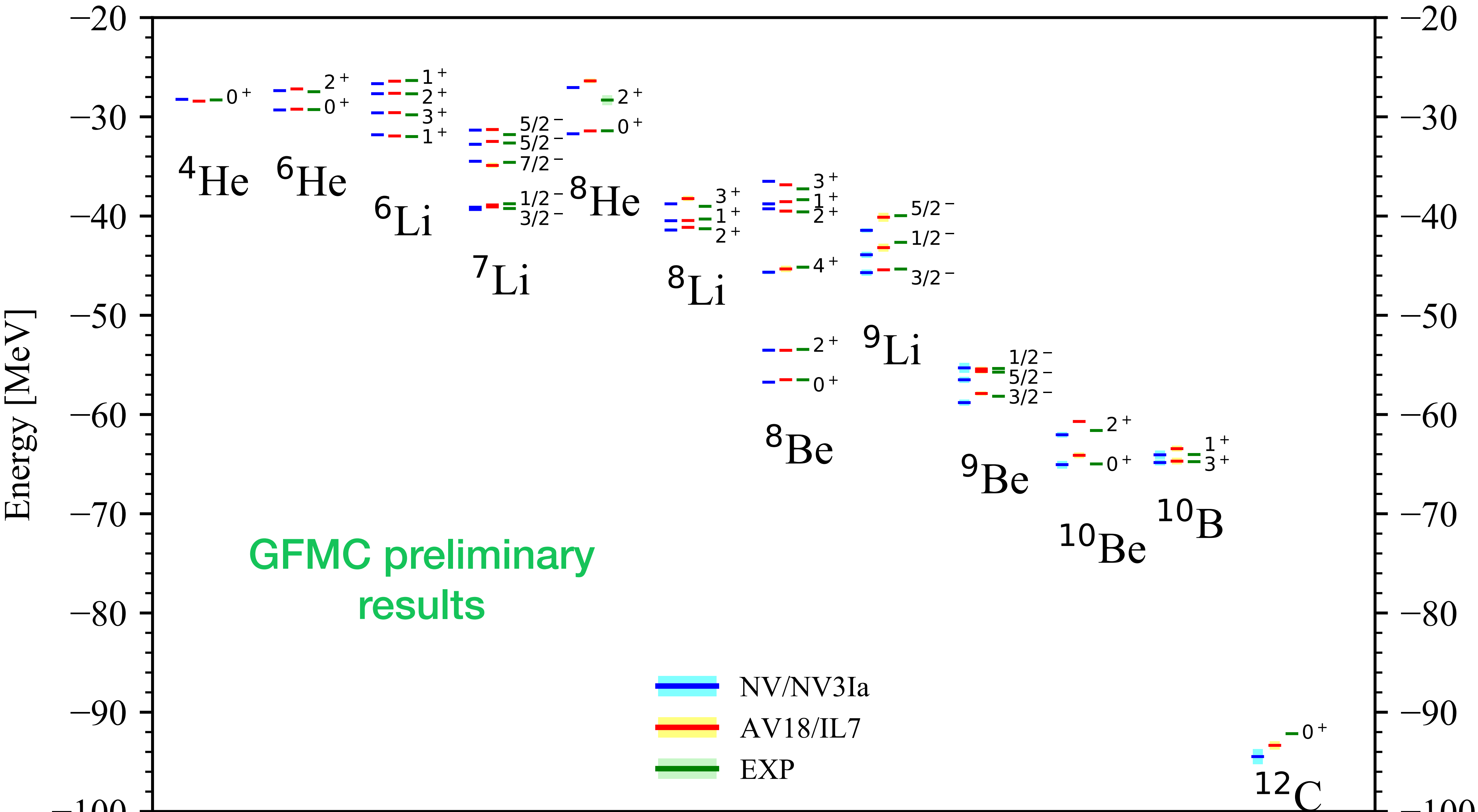
- Closer connection with QCD
- Consistent MEC being constructed
- Reliable theoretical uncertainty estimation

| model | order | $E_{\rm Lab}$ (MeV) | $N_{pp+np}$ | $\chi^2$/datum |
|---|---|---|---|---|
| $b$ | LO | 0–125 | 2558 | 59.88 |
| $b$ | NLO | 0–125 | 2648 | 2.18 |
| $b$ | N2LO | 0–125 | 2641 | 2.32 |
| $b$ | N3LO | 0–125 | 2665 | 1.07 |
| $a$ | N3LO | 0–125 | 2668 | 1.05 |
| $c$ | N3LO | 0–125 | 2666 | 1.11 |
| $\widetilde{a}$ | N3LO | 0–200 | 3698 | 1.37 |
| $\widetilde{b}$ | N3LO | 0–200 | 3695 | 1.37 |
| $\widetilde{c}$ | N3LO | 0–200 | 3693 | 1.40 |



# Δ-full local chiral potential

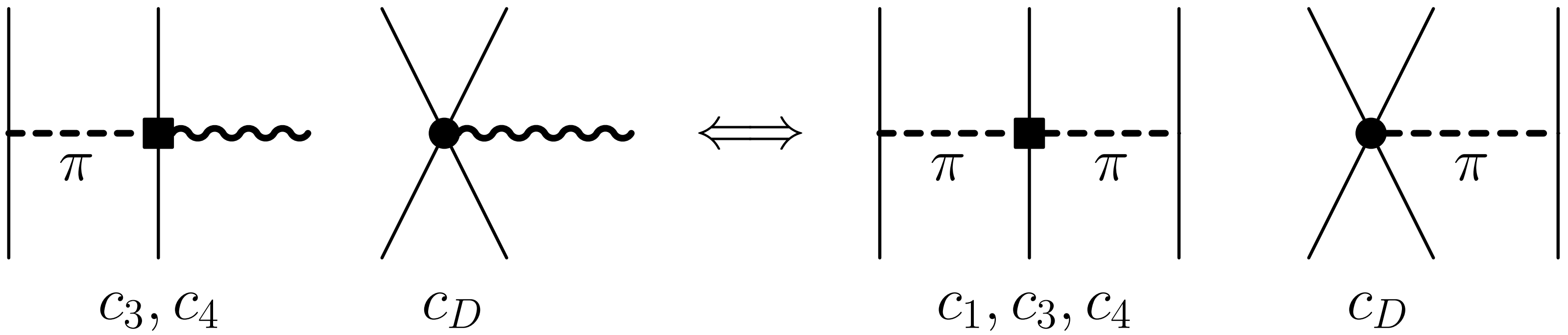
The experimental A≤12 ground- and excited state energies are very well reproduced by the local NN+NNN chiral interaction




M. Piarulli et al., in preparation

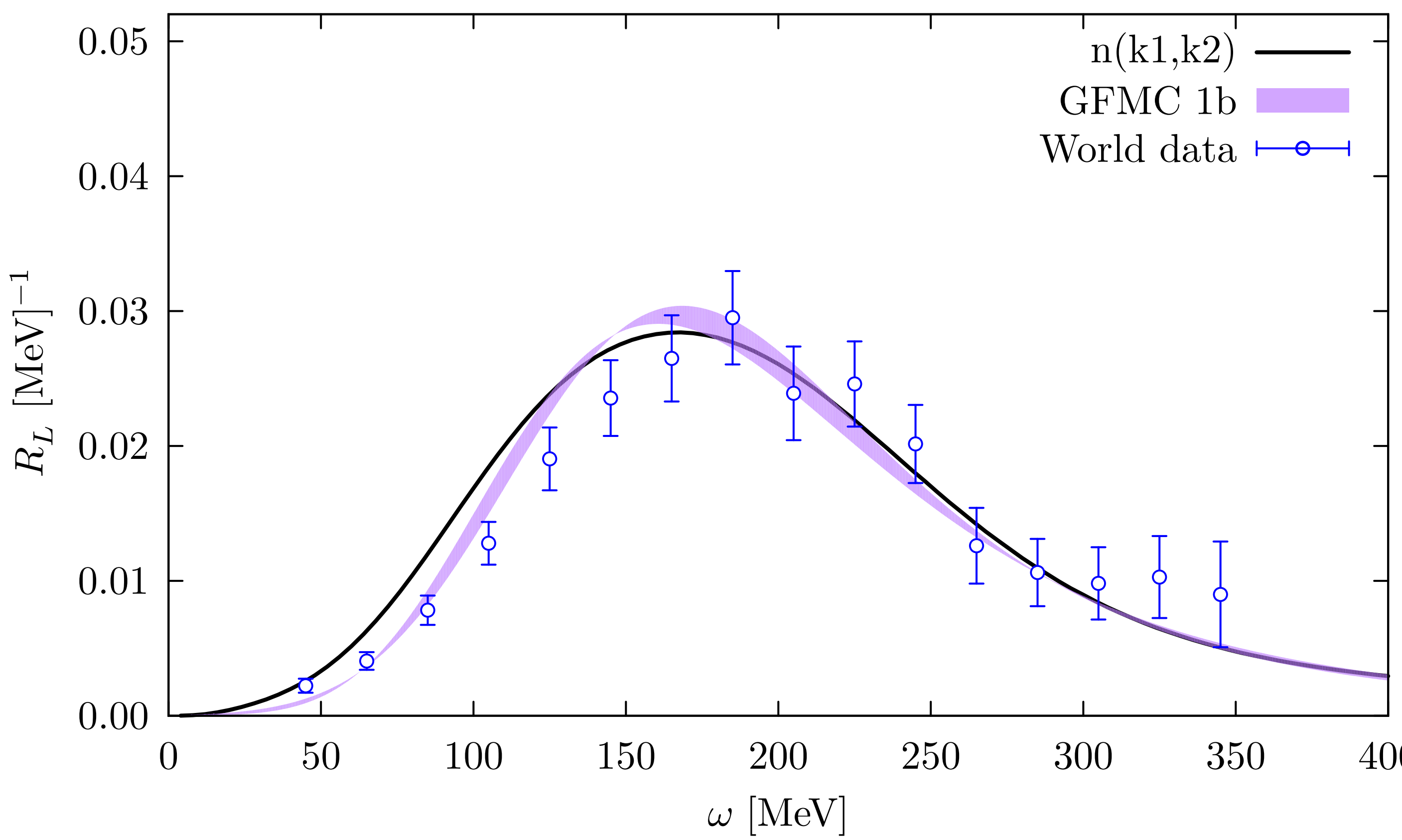
# To-do list

- Calculation of the charged-current response functions of $^{12}C$
- Implement consistent Δ-full potential and currents. Note that some of the three-body force parameters also enter the two-body axial current




H. W. Hammer et al., RMP 85, 197 (2013)

- Devise computationally affordable yet accurate strategies to tackle larger and isospin-asymmetric nuclei



Thank you