

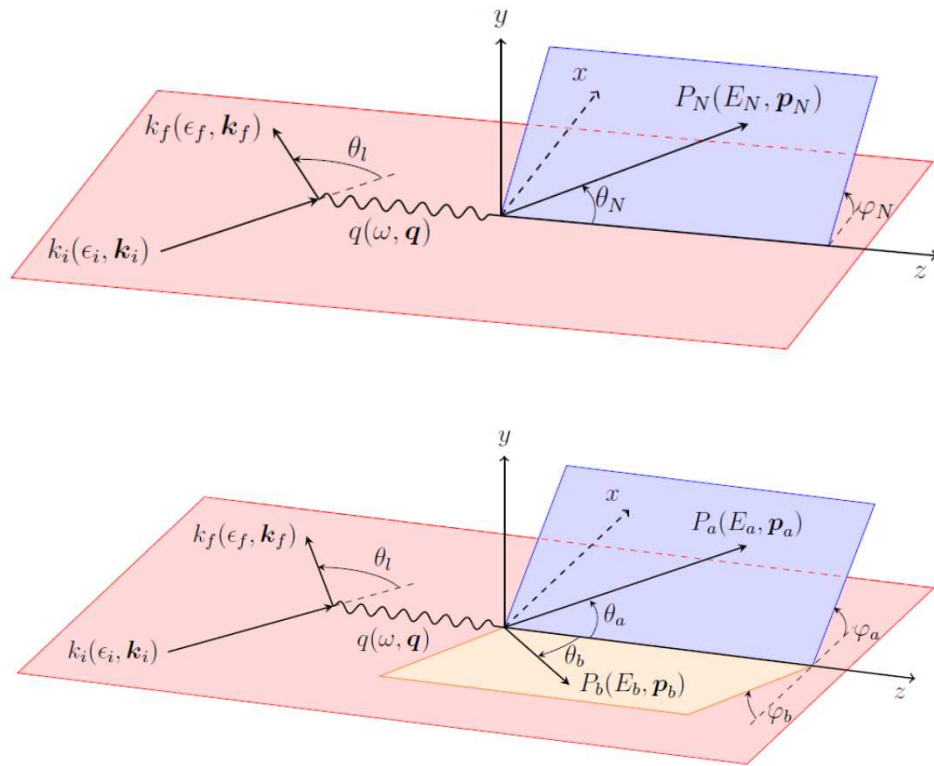
# CORRELATIONS IN QE(LIKE) NEUTRINO- NUCLEUS SCATTERING

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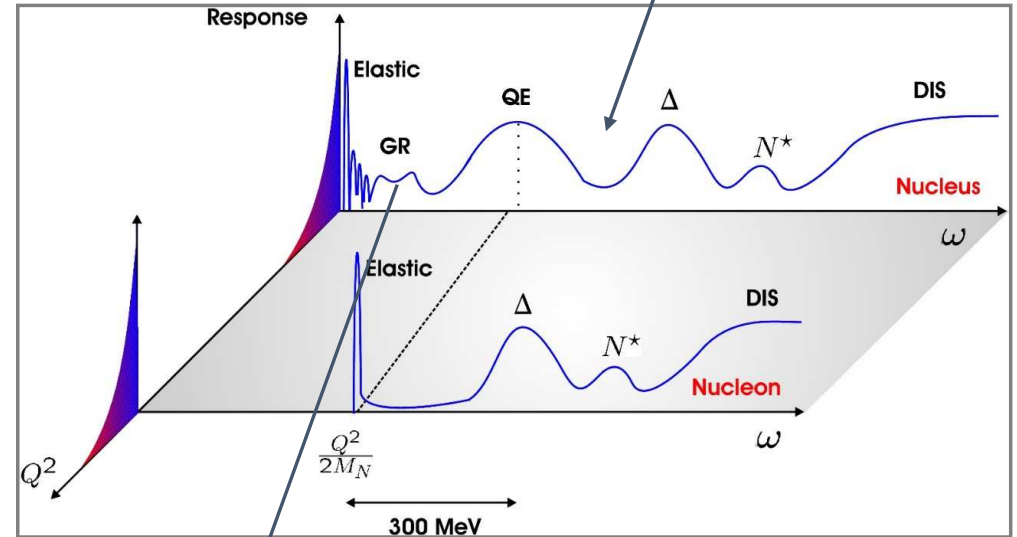
## Outline

- Detailed microscopic cross sections calculations for QE(-like) scattering
  - influence of long-range correlations
  - influence of short-range correlations in 1- and 2-nucleon knockout processes
  - Influence of seagull and pion-in-flight MEC contributions
- Scheme-dependent separation
  - Double counting

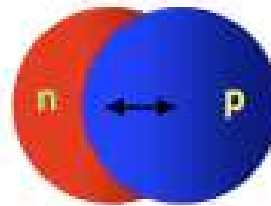
# Neutrino-hadron scattering



Dip region : multinucleon mechanisms

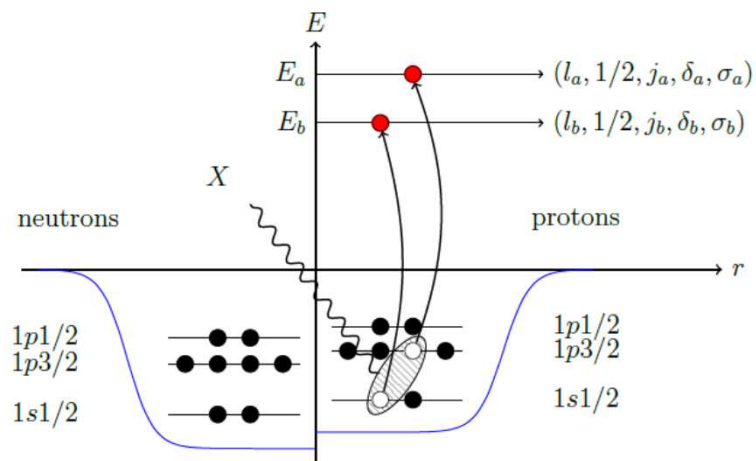
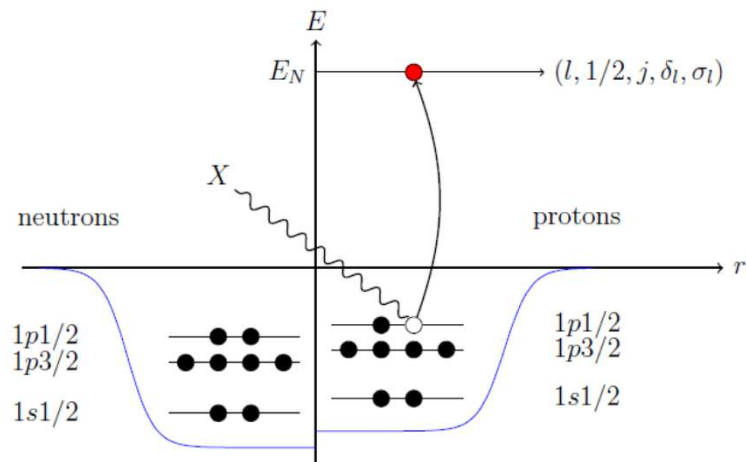


e.g.



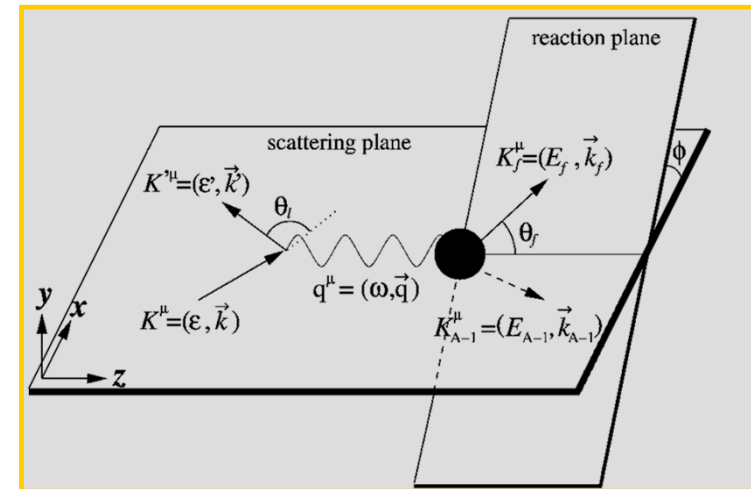
## Cross section calculations

- Starting point : mean-field nucleus with Hartree-Fock single-particle wave functions
- Skyrme SkE2 force used to build the potential
- Pauli blocking
- Binding



# Neutrino-nucleus interactions

$$\hat{H}_W = \frac{G}{\sqrt{2}} \int d\vec{x} \hat{j}_{\mu,lepton}(\vec{x}) \hat{j}^{\mu,hadron}(\vec{x})$$



Hadron current

$$J^\mu = F_1(Q^2)\gamma^\mu + i\frac{\kappa}{2M_N}F_2(Q^2)\sigma^{\mu\nu}q_\nu + G_A(Q^2)\gamma^\mu\gamma_5 + \frac{1}{2M_N}G_P(Q^2)q^\mu\gamma_5$$

Lepton tensor

$$l_{\alpha\beta} \equiv \sum_{s,s'} \overline{[\bar{u}_l \gamma_\alpha (1 - \gamma_5) u_l]}^\dagger [\bar{u}_\nu \gamma_\beta (1 - \gamma_5) u_\nu]$$

$$\vec{J}_V^\alpha(\vec{x}) = \vec{J}_{convection}^\alpha(\vec{x}) + \vec{J}_{magnetization}^\alpha(\vec{x})$$

$$\text{with } \vec{J}_c^\alpha(\vec{x}) = \frac{1}{2Mi} \sum_{i=1}^A G_E^{i,\alpha} \left[ \delta(\vec{x} - \vec{x}_i) \vec{\nabla}_i - \overleftarrow{\nabla}_i \delta(\vec{x} - \vec{x}_i) \right],$$

$$\vec{J}_m^\alpha(\vec{x}) = \frac{1}{2M} \sum_{i=1}^A G_M^{i,\alpha} \vec{\nabla} \times \vec{\sigma}_i \delta(\vec{x} - \vec{x}_i),$$

$$\vec{J}_A^\alpha(\vec{x}) = \sum_{i=1}^A G_A^{i,\alpha} \vec{\sigma}_i \delta(\vec{x} - \vec{x}_i),$$

$$J_V^{0,\alpha}(\vec{x}) = \rho_V^\alpha(\vec{x}) = \sum_{i=1}^A G_E^{i,\alpha} \delta(\vec{x} - \vec{x}_i),$$

$$J_A^{0,\alpha}(\vec{x}) = \rho_A^\alpha(\vec{x}) = \frac{1}{2Mi} \sum_{i=1}^A G_A^{i,\alpha} \vec{\sigma}_i \cdot \left[ \delta(\vec{x} - \vec{x}_i) \vec{\nabla}_i - \overleftarrow{\nabla}_i \delta(\vec{x} - \vec{x}_i) \right]$$

$$J_P^{0,\alpha}(\vec{x}) = \rho_P^\alpha(\vec{x}) = \frac{m_\mu}{2M} \sum_{i=1}^A G_P^{i,\alpha} \vec{\nabla} \cdot \vec{\sigma}_i \delta(\vec{x} - \vec{x}_i)$$

for NC reactions

$$G_E^{V,0} = \left( \frac{1}{2} - \sin^2 \theta_W \right) \tau_3 - \sin^2 \theta_W,$$

$$G_M^{V,0} = \left( \frac{1}{2} - \sin^2 \theta_W \right) (\mu_p - \mu_n) \tau_3 - \sin^2 \theta_W (\mu_p + \mu_n)$$

$$G^{A,0} = g_a \frac{\tau_3}{2} = -\frac{1.262}{2} \tau_3$$

for CC reactions

$$G_E^{V,\pm} = \tau_\pm$$

$$G_M^{V,\pm} = (\mu_p - \mu_n) \tau_\pm$$

$$G^{A,\pm} = g_a \tau_\pm = -1.262 \tau_\pm$$

$G = (1 + Q^2/M^2)^{-2}$   $Q^2$  dependence : dipole parametrization or BBBA07 :

## Inclusive QE 1-nucleon knockout cross sections

$$\frac{d^2\sigma}{d\Omega d\omega} = (2\pi)^4 k_f \varepsilon_f \sum_{s_f, s_i} \frac{1}{2J_i + 1} \sum_{M_f, M_i} |\langle f | \hat{H}_W | i \rangle|^2$$

$$\left( \frac{d^2\sigma_{i \rightarrow f}}{d\Omega d\omega} \right)_{\nu} = \frac{G^2 \varepsilon_f^2}{\pi} \frac{2 \cos^2 \left( \frac{\theta}{2} \right)}{2J_i + 1} \left[ \sum_{J=0}^{\infty} \sigma_{CL}^J + \sum_{J=1}^{\infty} \sigma_T^J \right]$$

$$\sigma_{CL}^J = \left| \left\langle J_f \left\| \hat{\mathcal{M}}_J(\kappa) + \frac{\omega}{|\vec{q}|} \hat{\mathcal{L}}_J(\kappa) \right\| J_i \right\rangle \right|^2$$

$$\sigma_T^J = \left( -\frac{q_\mu^2}{2|\vec{q}|^2} + \tan^2 \left( \frac{\theta}{2} \right) \right) \left[ \left| \left\langle J_f \left\| \hat{\mathcal{J}}_J^{mag}(\kappa) \right\| J_i \right\rangle \right|^2 + \left| \left\langle J_f \left\| \hat{\mathcal{J}}_J^{el}(\kappa) \right\| J_i \right\rangle \right|^2 \right]$$

$$\mp \tan \left( \frac{\theta}{2} \right) \sqrt{-\frac{q_\mu^2}{|\vec{q}|^2} + \tan^2 \left( \frac{\theta}{2} \right)} \left[ 2\Re \left( \left\langle J_f \left\| \hat{\mathcal{J}}_J^{mag}(\kappa) \right\| J_i \right\rangle \left\langle J_f \left\| \hat{\mathcal{J}}_J^{el}(\kappa) \right\| J_i \right\rangle^* \right) \right]$$

## 2-nucleon knockout cross sections

2-nucleon knockout :

$$\frac{d\sigma^X}{dE_f d\Omega_f dT_b d\Omega_b d\Omega_a} = \frac{p_a p_b E_a E_b}{(2\pi)^6} g_{rec}^{-1} \sigma^X \zeta$$

$$\times [v_{CC} W_{CC} + v_{CL} W_{CL} + v_{LL} W_{LL} + v_T W_T + v_{TT} W_{TT} + v_{TC} W_{TC}$$

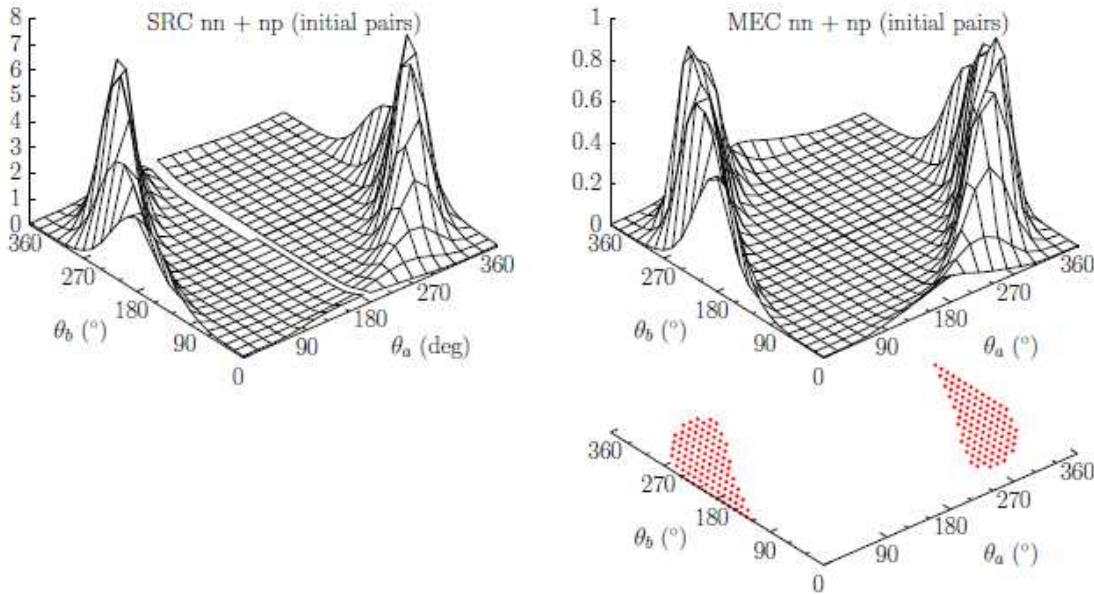
$$+ v_{TL} W_{TL} + h(v_{T'} W_{T'} + v_{TC'} W_{TC'} + v_{TL'} W_{TL'})],$$

with :

$$\mathcal{J}_\lambda = \langle \Phi_f^{(A-2)}(E^{exc}, J_R M_R); \mathbf{p}_a m_{s_a}; \mathbf{p}_b m_{s_b} | \hat{J}_\lambda(\mathbf{q}) | \Phi_{gs} \rangle$$

$$| \Phi^{2p2h} \rangle = | \Phi_f^{(A-2)}(E^{exc}, J_R M_R); \mathbf{p}_a m_{s_a}; \mathbf{p}_b m_{s_b} \rangle_{as}$$



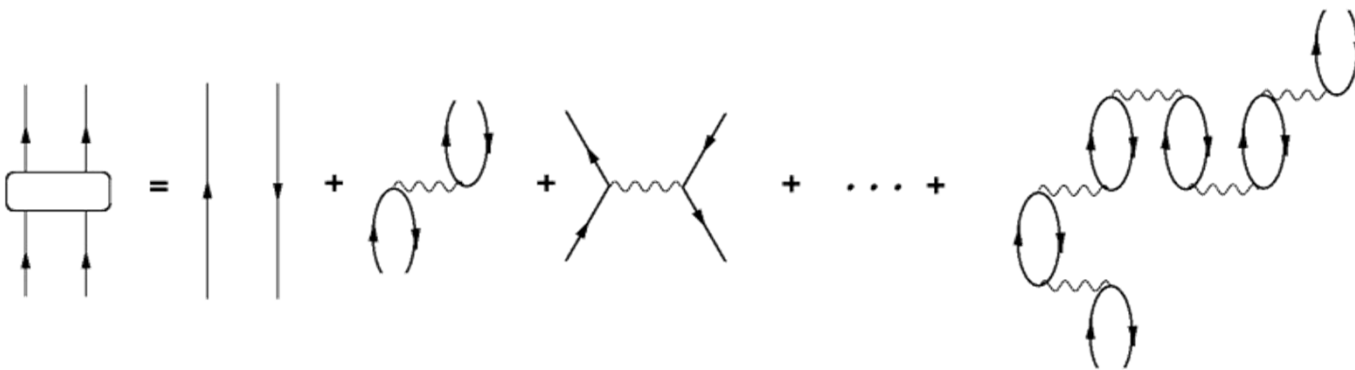


**Figure 4.5:** The  $^{12}\text{C}(\nu_\mu, \mu^- N_a N_b)$  cross section ( $N_a = p, N_b = p', n$ ) at  $\epsilon_{\nu_\mu} = 750$  MeV,  $\epsilon_\mu = 550$  MeV,  $\theta_\mu = 15^\circ$  and  $T_p = 50$  MeV for in-plane kinematics. Left with SRCs, right with MECs, the bottom plot shows the  $(\theta_a, \theta_b)$  regions with  $P_{12} < 300$  MeV/c.

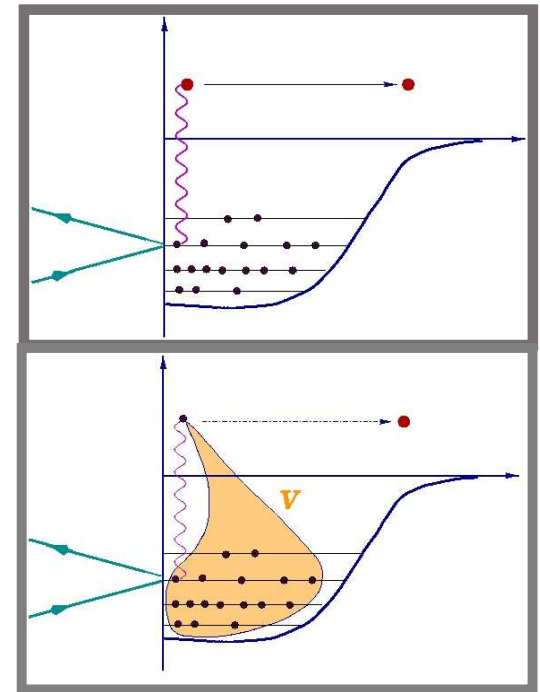
- Strength residing in restricted part of phase space
- $\mathbf{p}_b \approx \mathbf{p}_b^{ave}$
- Quasi-deuteron kinematics

# Long-range correlations : Continuum RPA

- Green's function approach
- Skyrme SkE2 residual interaction
- self-consistent calculations

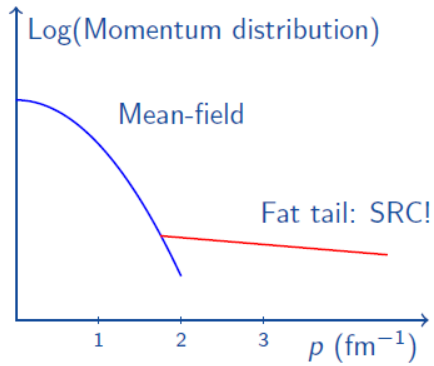


$$\Pi^{(RPA)}(x_1, x_2; \omega) = \Pi^{(0)}(x_1, x_2; \omega) + \frac{1}{\hbar} \int dx \int dx' \Pi^{(0)}(x_1, x; \omega) \tilde{V}(x, x') \Pi^{(RPA)}(x', x_2; \omega)$$



$$|\Psi_{RPA}\rangle = \sum_c \{ X_{(\Psi,C)} |ph^{-1}\rangle - Y_{(\Psi,C)} |hp^{-1}\rangle \} + \dots$$

## Short-range correlations

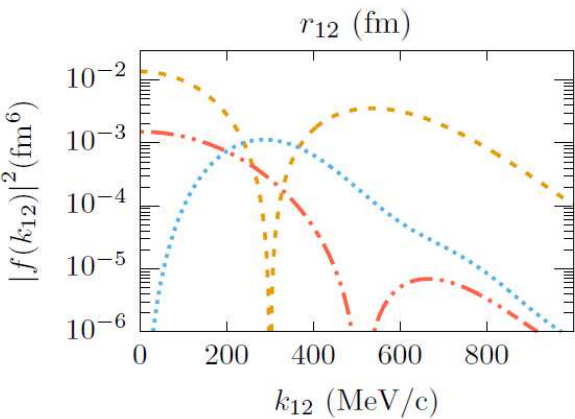
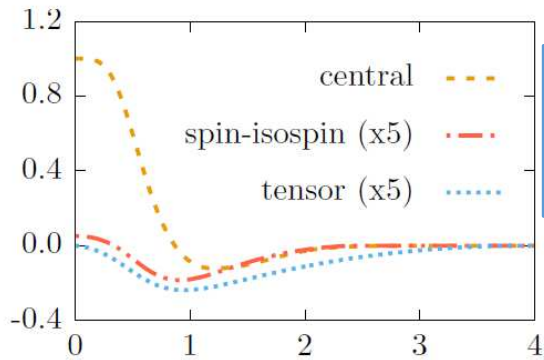


$$|\Psi\rangle = \frac{1}{\sqrt{\mathcal{N}}} \hat{\mathcal{G}} |\Phi\rangle \quad \text{with} \quad \hat{\mathcal{G}} \approx \hat{\mathcal{S}} \left( \prod_{i<j}^A [1 + \hat{l}(i, j)] \right)$$

$$\hat{l}(i, j) = -g_c(r_{ij}) + f_{\sigma\tau}(r_{ij}) (\vec{\sigma}_i \cdot \vec{\sigma}_j) (\vec{\tau}_i \cdot \vec{\tau}_j) + f_{t\tau}(r_{ij}) \hat{S}_{ij} (\vec{\tau}_i \cdot \vec{\tau}_j),$$

Shifting the complexity induced by correlations from the wave functions to the operators

$$\langle \Psi_f | \hat{J}_\mu^{\text{nucl}} | \Psi_i \rangle = \frac{1}{\sqrt{\mathcal{N}_i \mathcal{N}_f}} \langle \Phi_f | \hat{J}_\mu^{\text{eff}} | \Phi_i \rangle$$

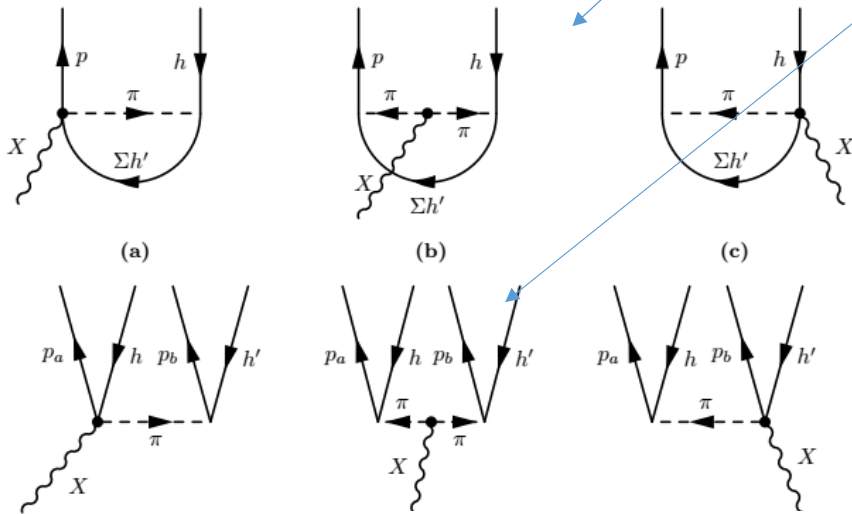


$$\hat{J}_\mu^{\text{eff}} \approx \sum_{i=1}^A \hat{J}_\mu^{[1]}(i) + \sum_{i<j}^A \hat{J}_\mu^{[1],\text{in}}(i, j) + \left[ \sum_{i<j}^A \hat{J}_\mu^{[1],\text{in}}(i, j) \right]^\dagger$$

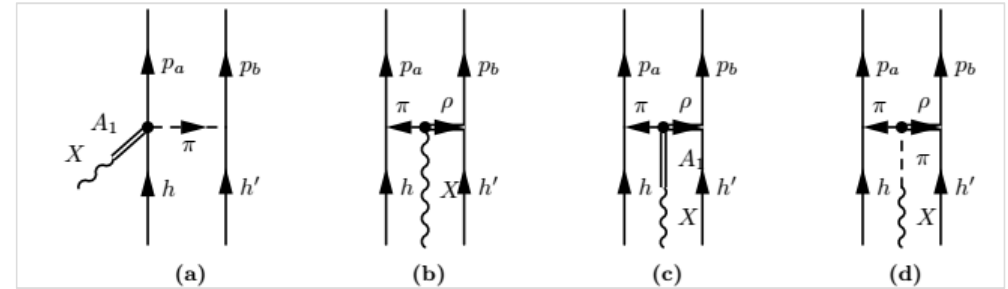
$$\hat{J}_\mu^{[1],\text{in}}(i, j) = \left[ \hat{J}_\mu^{[1]}(i) + \hat{J}_\mu^{[1]}(j) \right] \hat{l}(i, j)$$

Gearheart (1994) Pieper (1992)

# III. MEC in 1p1h and 2p2h

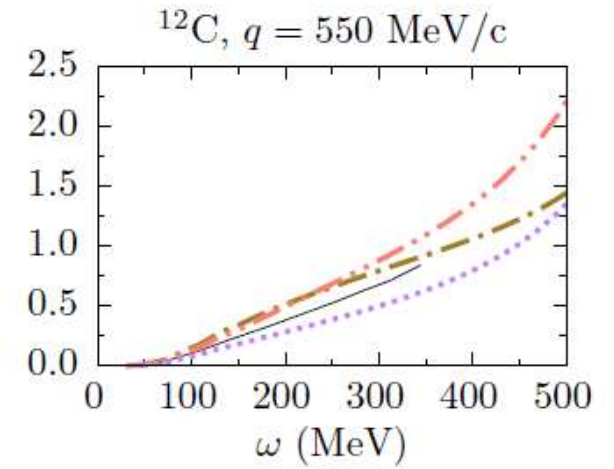
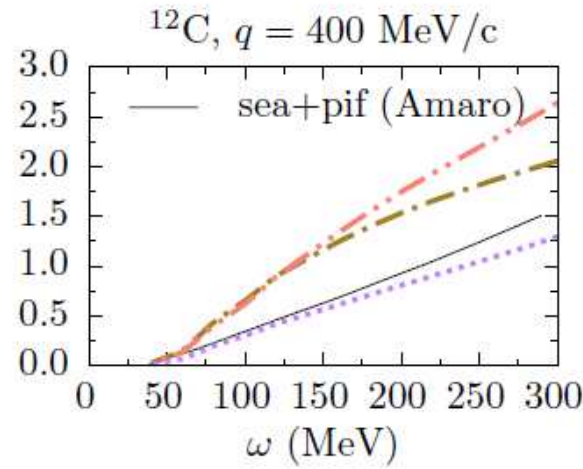
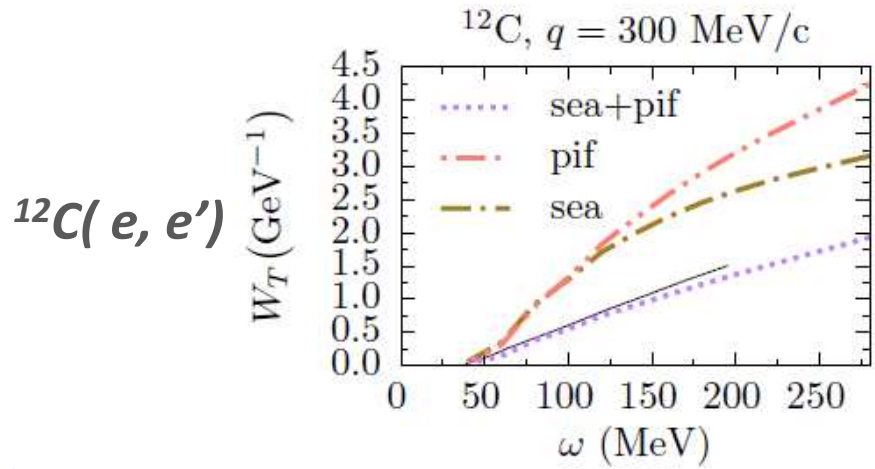


Axial contributions :



$$\hat{\rho}_A^{[2],\text{axi}}(\mathbf{q}) = \frac{i}{g_A} \left( \frac{f_{\pi NN}}{m_\pi} \right)^2 (\mathbf{I}_V) \left( F_\pi(q_2^2) \Gamma_\pi^2(q_2^2) \frac{\boldsymbol{\sigma}_2 \cdot \mathbf{q}_2}{q_2^2 + m_\pi^2} - F_\pi(q_1^2) \Gamma_\pi^2(q_1^2) \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q}_1}{q_1^2 + m_\pi^2} \right)$$

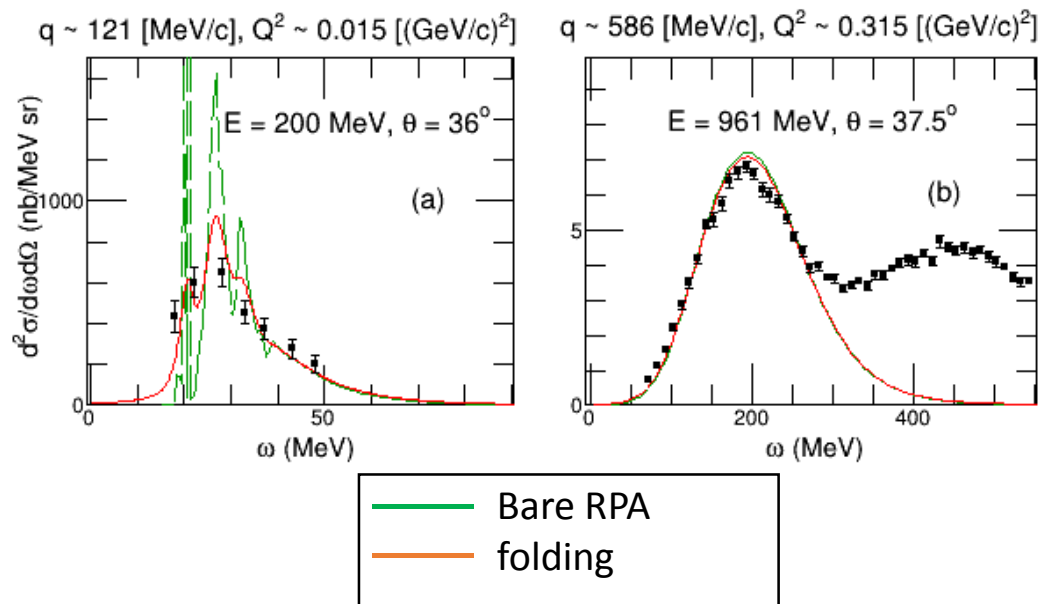
I. Towner, Nucl. Phys.A542, 631 (1992)



- Final state interactions :

- taken into account through the calculations of the wave function of the outgoing nucleon in the (real) nuclear potential generated using the Skyrme force

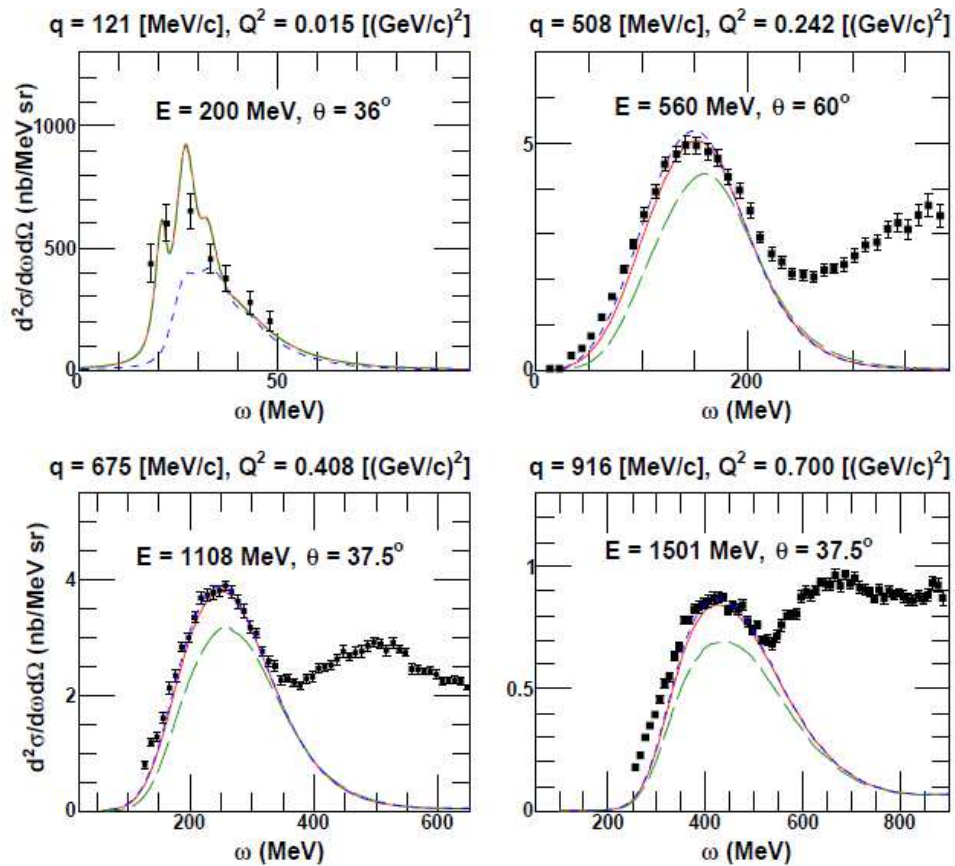
- influence of the spreading width of the particle states is implemented through a folding procedure



$$R'(q, \omega') = \int_{-\infty}^{\infty} d\omega R(q, \omega) L(\omega, \omega'),$$

$$L(\omega, \omega') = \frac{1}{2\pi} \left[ \frac{\Gamma}{(\omega - \omega')^2 + (\Gamma/2)^2} \right].$$

- Regularization of the residual interaction :

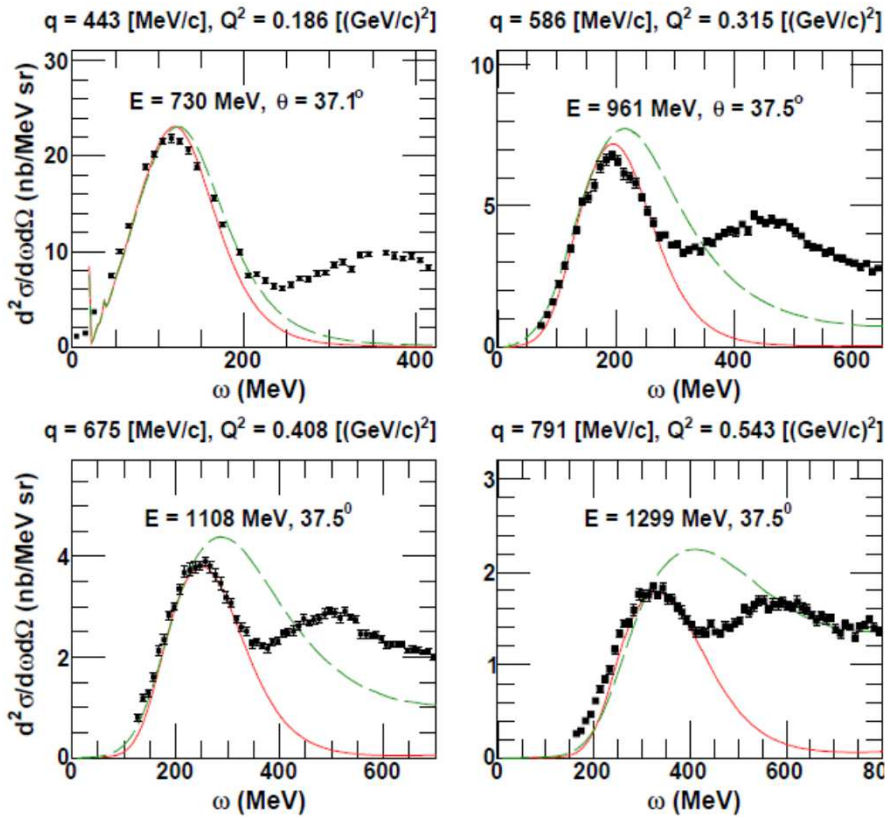
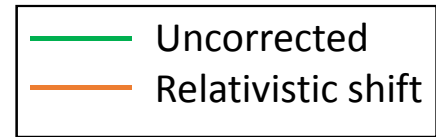


$$V(Q^2) \rightarrow V(Q^2 = 0) \frac{1}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2}$$

— Uncorrected  
 — dipole



•Relativistic corrections at higher energies (J. Jeschonnek and T. Donnelly, PRC57, 2438 (1998)):



Shift :

$$\lambda \rightarrow \lambda(\lambda + 1) \quad \lambda = \omega/2M_N$$

- The outgoing nucleon obtains the correct relativistic momentum
- $$p = \sqrt{T^2 + 2MT}$$
- Shifts the QE peak to the right relativistic position

Boost :

$$R_{CC}^V(q, \omega) \rightarrow \frac{q^2}{q^2 - \omega^2} R_{CC}^V(q, \omega),$$

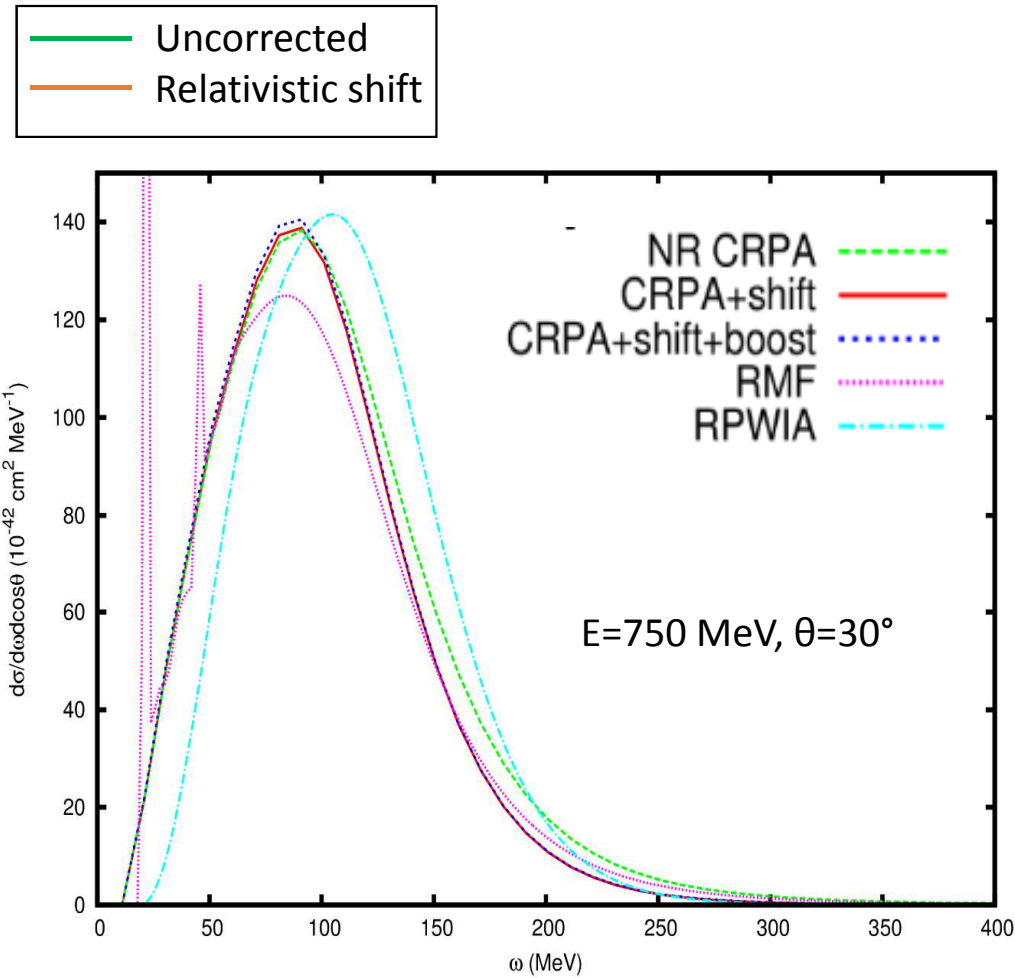
$$R_{LL}^A(q, \omega) \rightarrow \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R_{LL}^A(q, \omega),$$

$$R_{T}^V(q, \omega) \rightarrow \frac{q^2 - \omega^2}{q^2} R_{T}^V(q, \omega),$$

$$R_{T}^A(q, \omega) \rightarrow \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R_{T}^A(q, \omega),$$

$$R_{T'}^{VA}(q, \omega) \rightarrow \sqrt{\frac{q^2 - \omega^2}{q^2}} \sqrt{1 + \frac{q^2 - \omega^2}{4m^2}} R_{T'}^{VA}(q, \omega).$$

- Relativistic corrections at higher energies (J. Jeschonnek and T. Donnelly, PRC57, 2438 (1998)):



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Boost :

$$R_{CC}^V(q, \omega) \rightarrow \frac{q^2}{q^2 - \omega^2} R_{CC}^V(q, \omega),$$

$$R_{LL}^A(q, \omega) \rightarrow \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R_{LL}^A(q, \omega),$$

$$R_T^V(q, \omega) \rightarrow \frac{q^2 - \omega^2}{q^2} R_T^V(q, \omega),$$

$$R_T^A(q, \omega) \rightarrow \left(1 + \frac{q^2 - \omega^2}{4m^2}\right) R_T^A(q, \omega),$$

$$R_{T'}^{VA}(q, \omega) \rightarrow \sqrt{\frac{q^2 - \omega^2}{q^2}} \sqrt{1 + \frac{q^2 - \omega^2}{4m^2}} R_{T'}^{VA}(q, \omega).$$



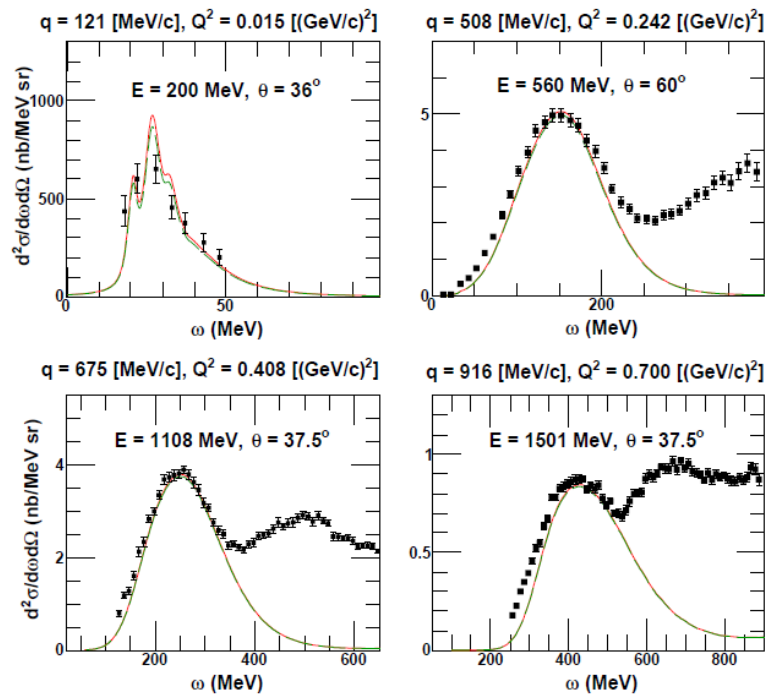
•Coulomb correction for the outgoing lepton in charged-current interactions :

✓ Low energies : Fermi function

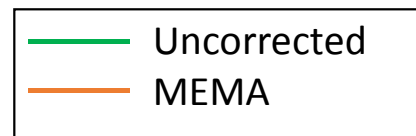
$$F(Z', E) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}} \quad \eta \sim \mp Z' \alpha$$

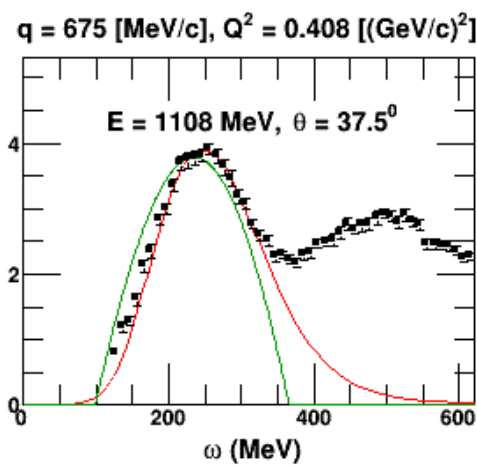
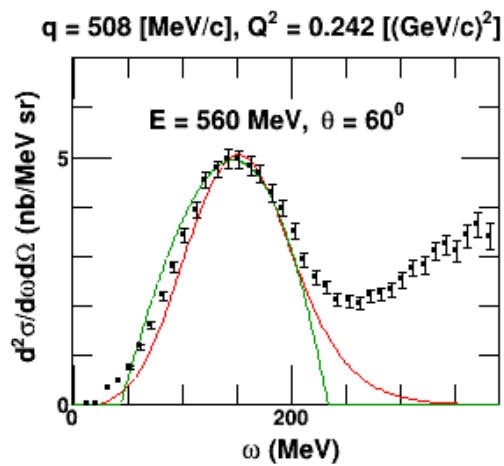
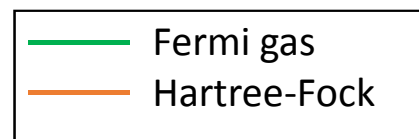
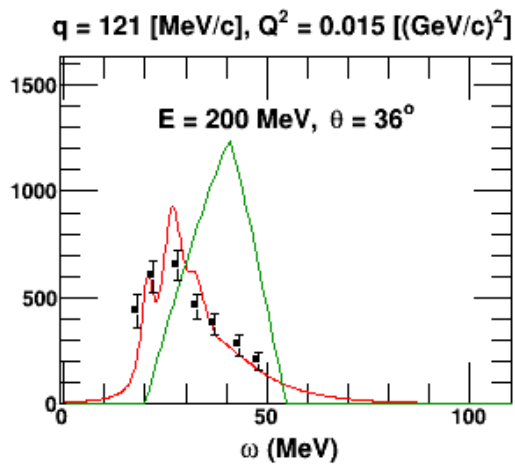
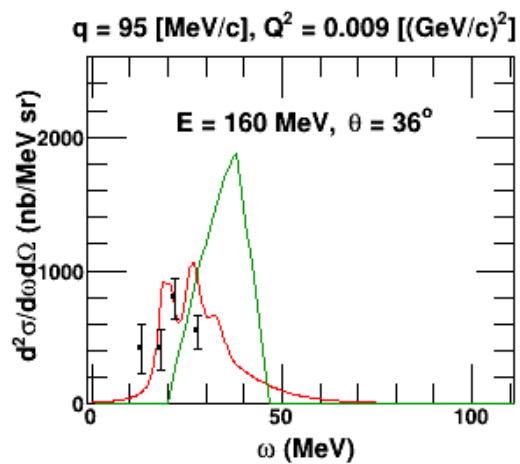
✓ High energies : modified effective momentum approximation (J. Engel, PRC57,2004 (1998))

$$q_{eff} = q + 1.5 \left( \frac{Z' \alpha \hbar c}{R} \right), \quad \Psi_l^{eff} = \zeta(Z', E, q) \Psi_l,$$



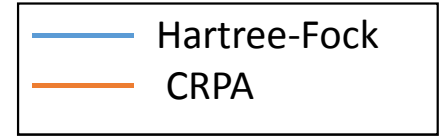
$$\zeta(Z', E, q) = \sqrt{\frac{q_{eff} E_{eff}}{qE}}$$



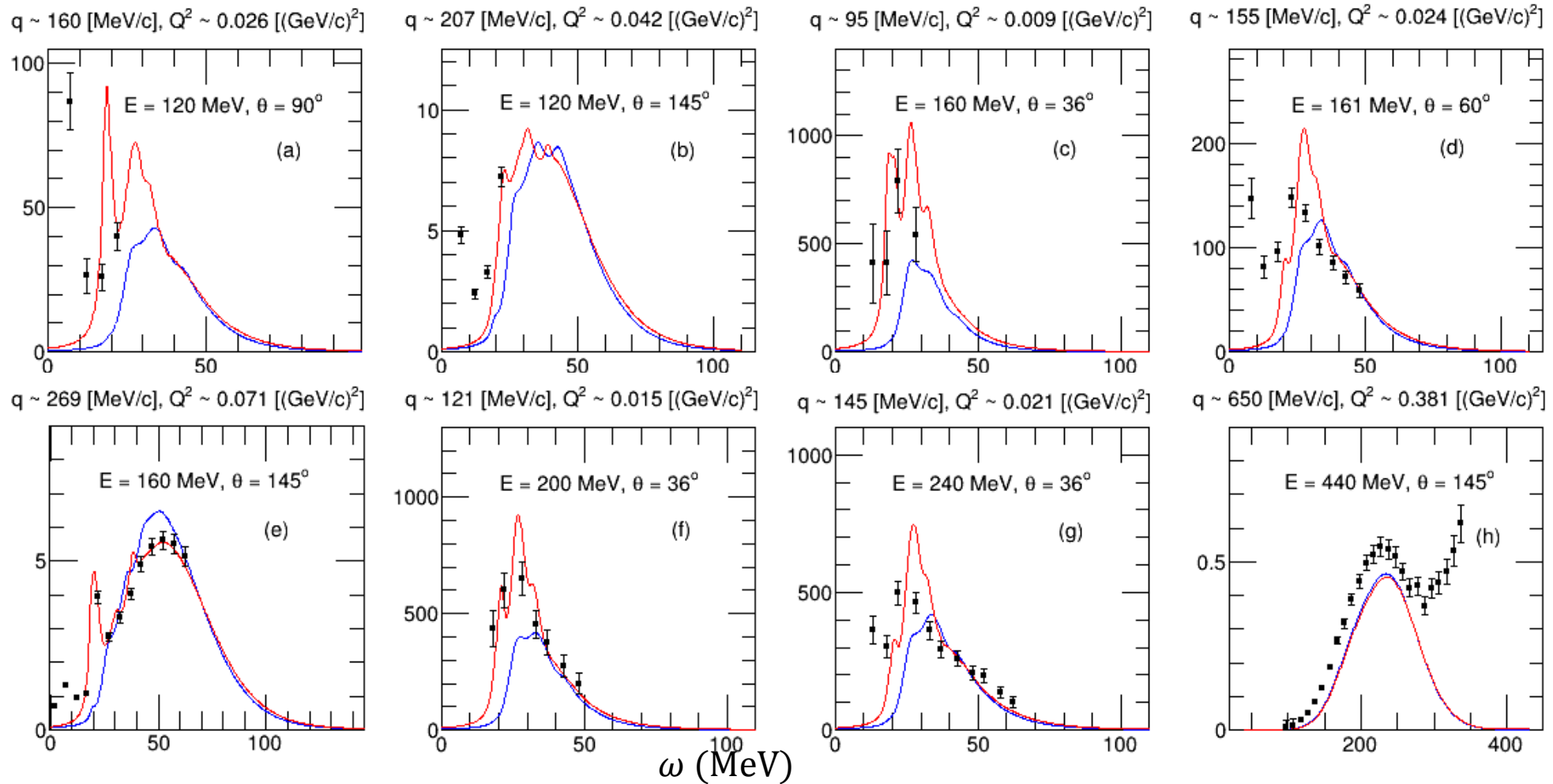


# CRPA : Comparison with electron scattering data

$^{12}\text{C}(e, e')$

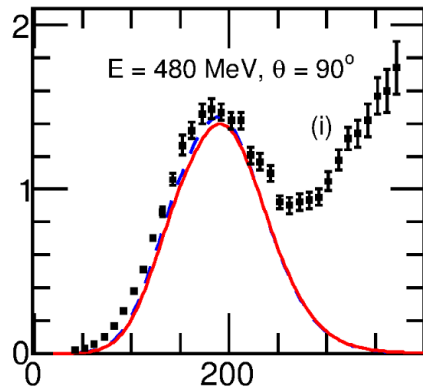


$d^2\sigma/d\omega d\Omega(\text{nb/MeV sr})$

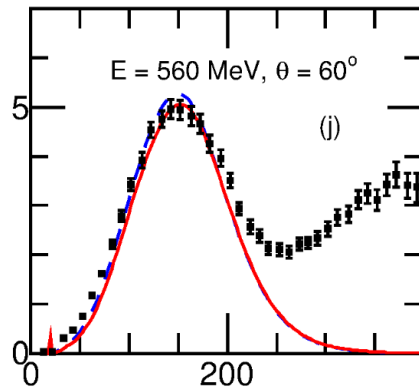


$d^2\sigma/d\omega d\Omega$  (nb/MeV sr)

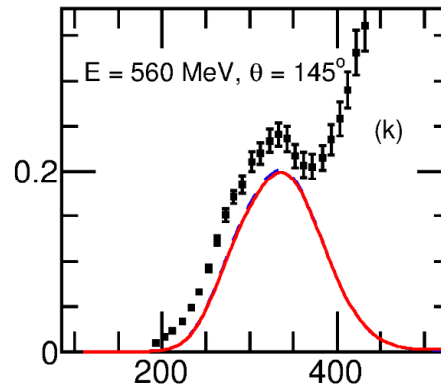
$q \sim 576$  [MeV/c],  $Q^2 \sim 0.305$  [(GeV/c) $^2$ ]



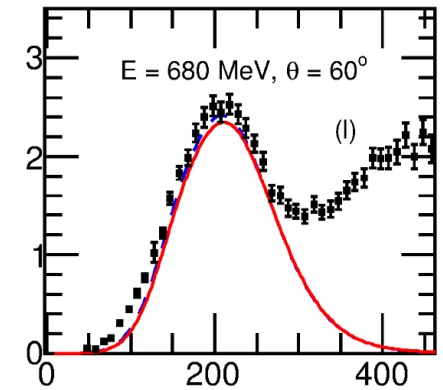
$q \sim 508$  [MeV/c],  $Q^2 \sim 0.242$  [(GeV/c) $^2$ ]



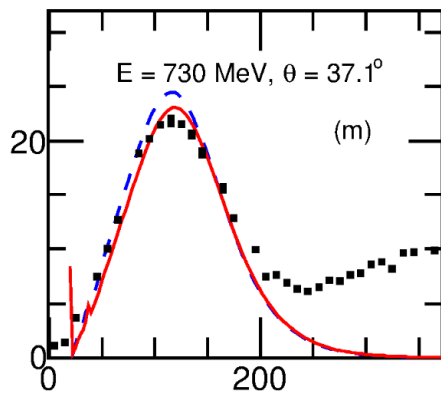
$q \sim 795$  [MeV/c],  $Q^2 \sim 0.548$  [(GeV/c) $^2$ ]



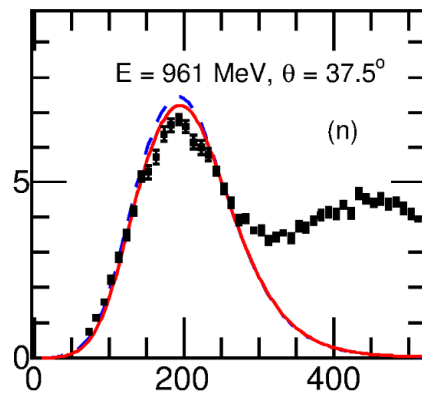
$q \sim 610$  [MeV/c],  $Q^2 \sim 0.340$  [(GeV/c) $^2$ ]



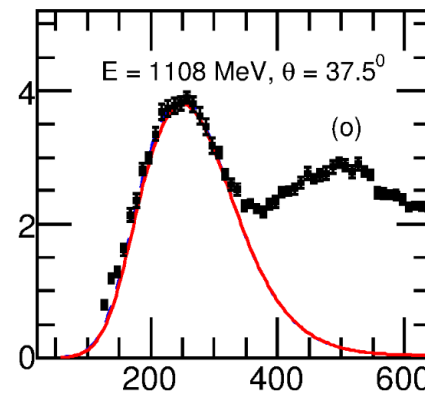
$q \sim 443$  [MeV/c],  $Q^2 \sim 0.186$  [(GeV/c) $^2$ ]



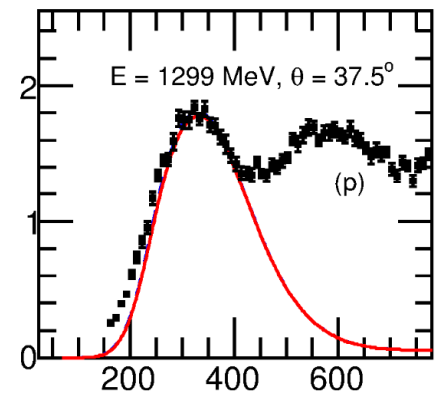
$q \sim 586$  [MeV/c],  $Q^2 \sim 0.315$  [(GeV/c) $^2$ ]



$q \sim 675$  [MeV/c],  $Q^2 \sim 0.408$  [(GeV/c) $^2$ ]



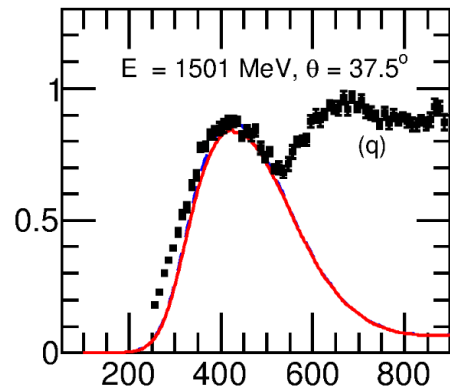
$q \sim 791$  [MeV/c],  $Q^2 \sim 0.543$  [(GeV/c) $^2$ ]



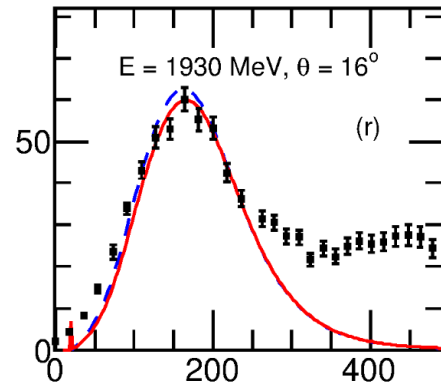
$\omega$  (MeV)

$d^2\sigma/d\omega d\Omega$  (nb/MeV sr)

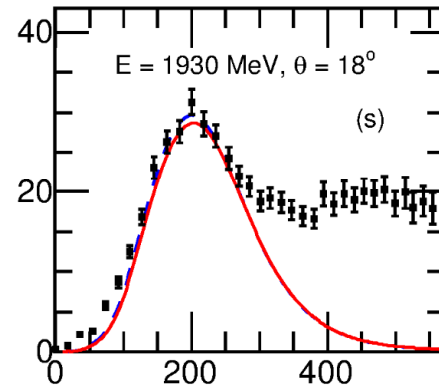
$q \sim 916$  [MeV/c],  $Q^2 \sim 0.700$  [(GeV/c) $^2$ ]



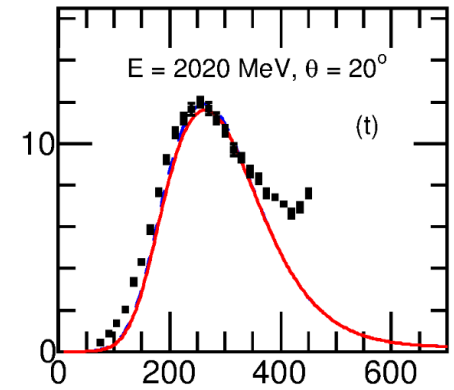
$q \sim 536$  [MeV/c],  $Q^2 \sim 0.267$  [(GeV/c) $^2$ ]



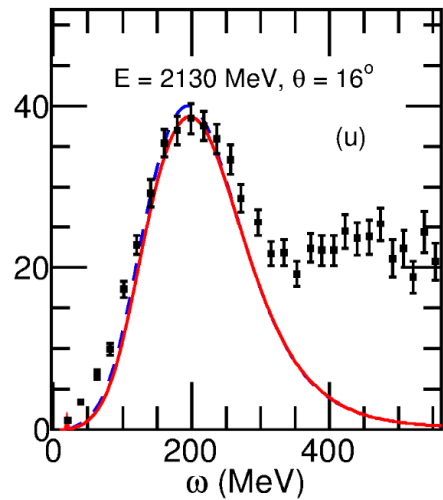
$q \sim 601$  [MeV/c],  $Q^2 \sim 0.331$  [(GeV/c) $^2$ ]



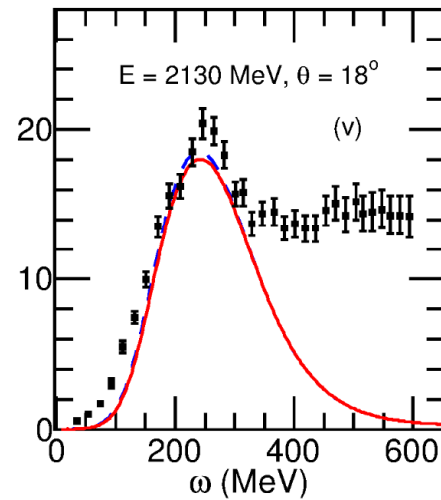
$q \sim 700$  [MeV/c],  $Q^2 \sim 0.436$  [(GeV/c) $^2$ ]



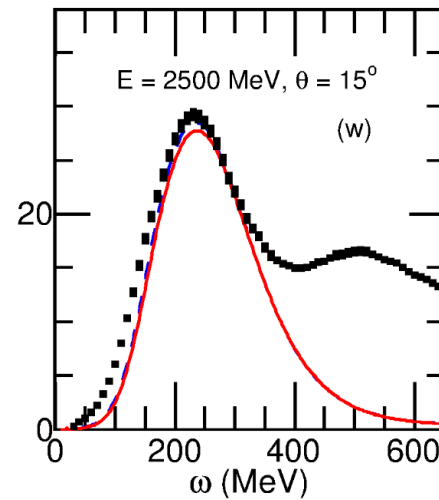
$q \sim 594$  [MeV/c],  $Q^2 \sim 0.323$  [(GeV/c) $^2$ ]



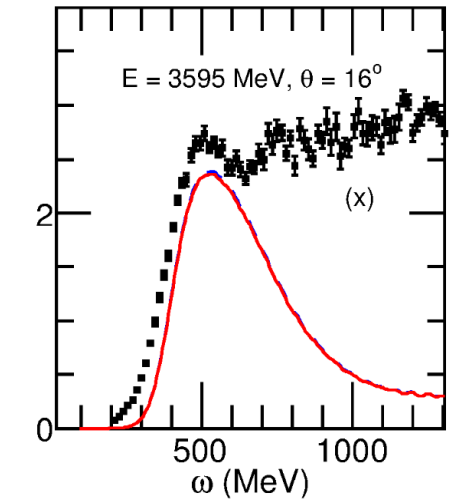
$q \sim 667$  [MeV/c],  $Q^2 \sim 0.399$  [(GeV/c) $^2$ ]



$q \sim 658$  [MeV/c],  $Q^2 \sim 0.391$  [(GeV/c) $^2$ ]

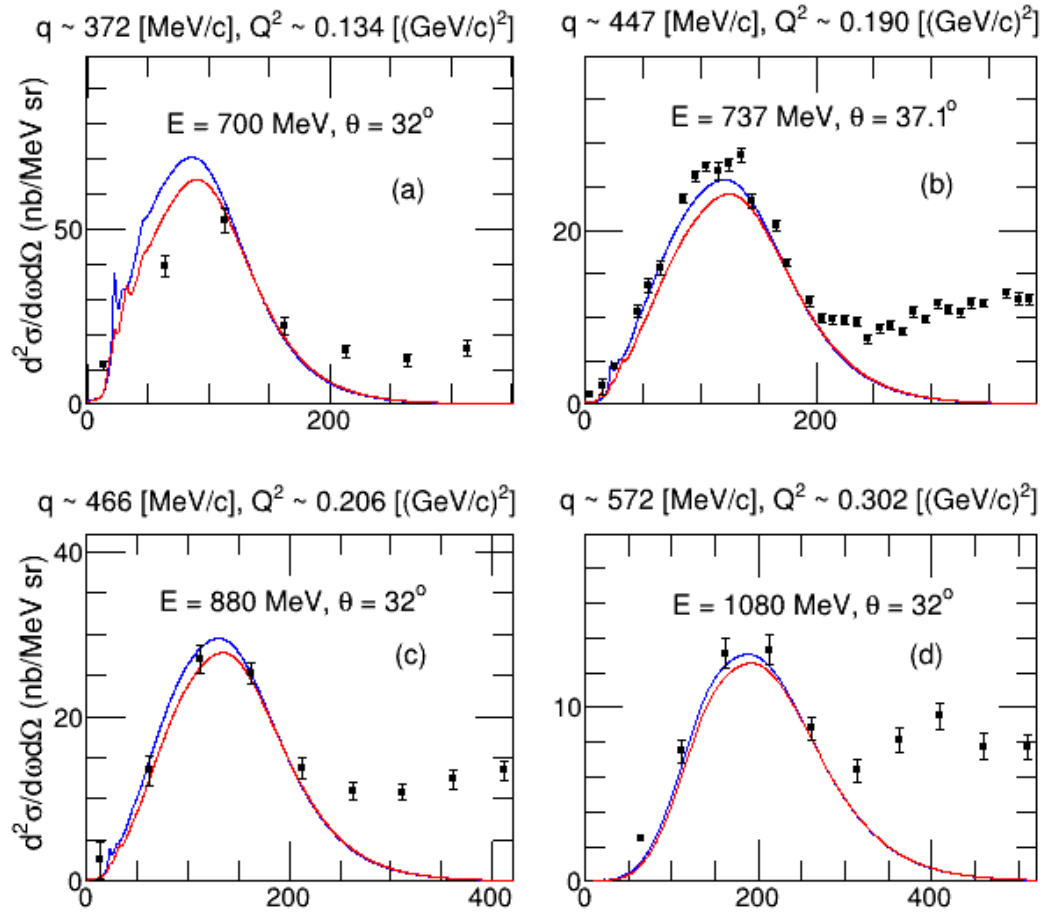


$q \sim 1043$  [MeV/c],  $Q^2 \sim 0.872$  [(GeV/c) $^2$ ]



$\omega$  (MeV)

# $^{16}\text{O}(e, e')$

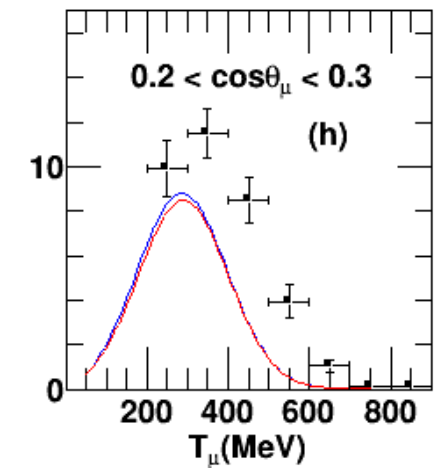
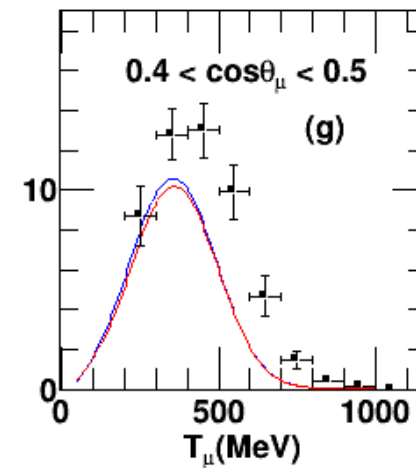
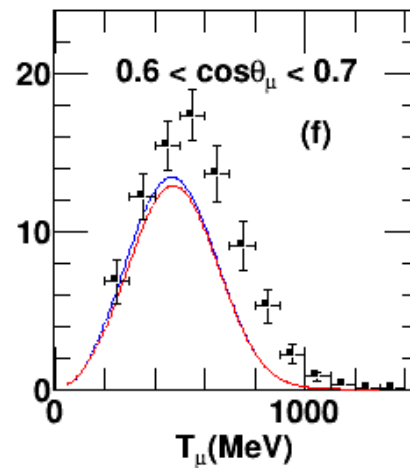
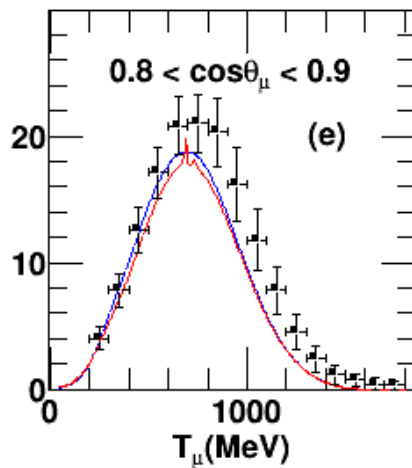
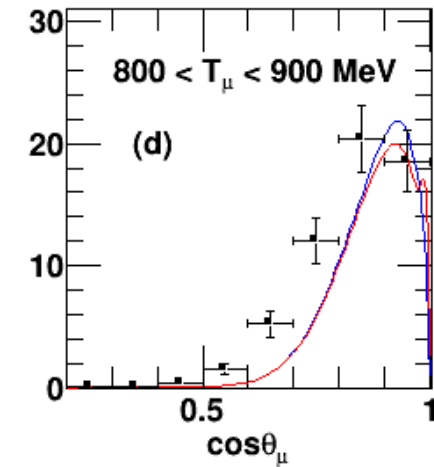
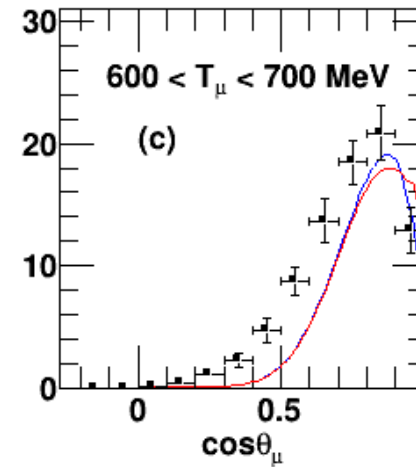
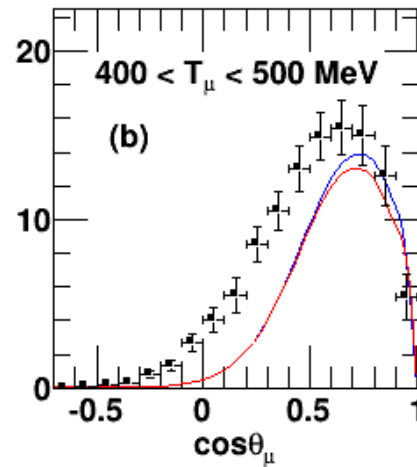
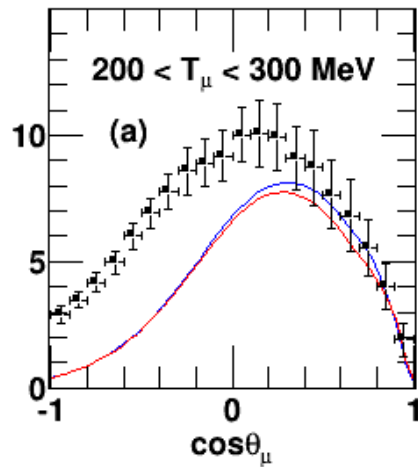


- Good overall agreement with e-scattering data

P. Barreau et al., Nucl. Phys. A402, 515 (1983), J. S. O'Connell et al., Phys. Rev. C35, 1063 (1987), R. M. Sealock et al., Phys. Rev. Lett.62, 1350 (1989), D. S. Bagdasaryan et al., YERPHI-1077-40-88 (1988), D. B. Day et al., Phys. Rev. C 48, 1849 (1993), D. Zeller, DESY-F23-73-2 (1973).

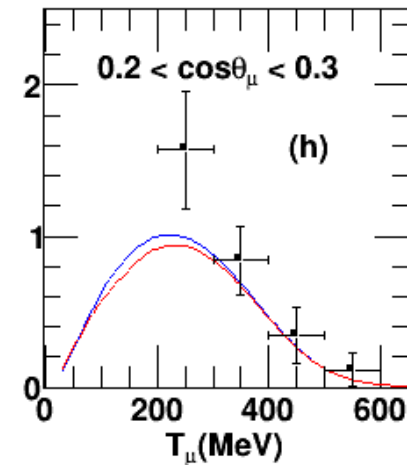
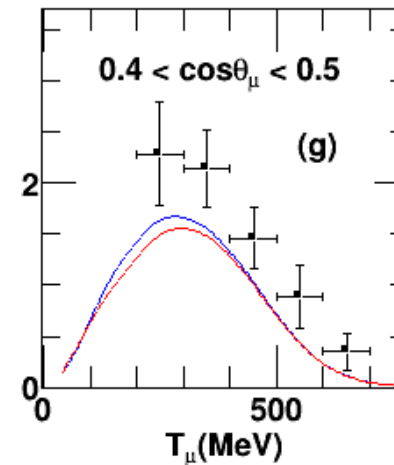
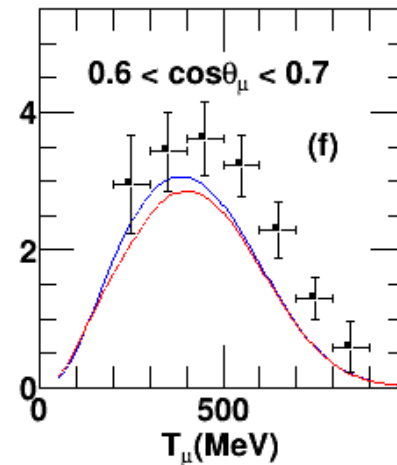
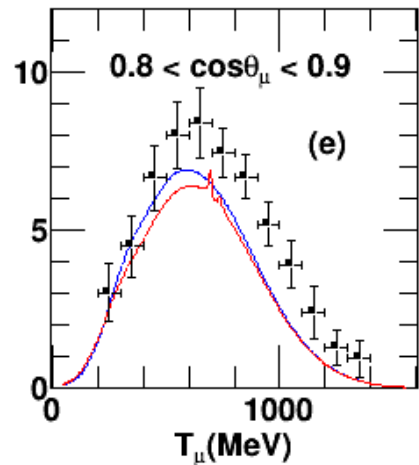
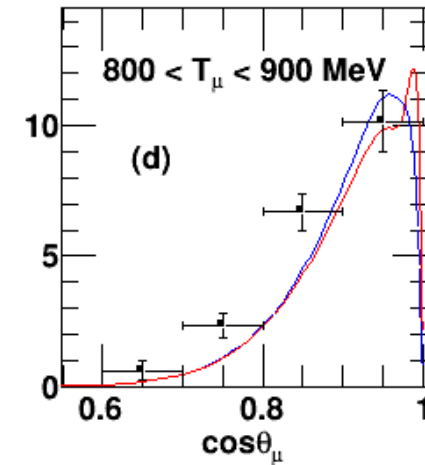
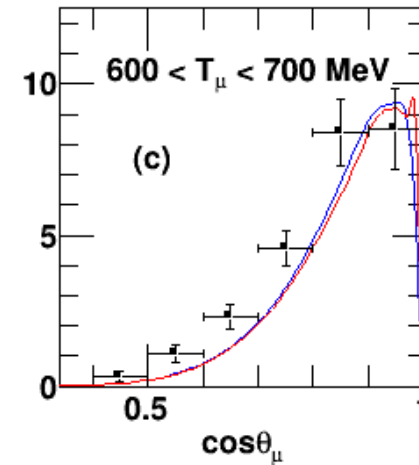
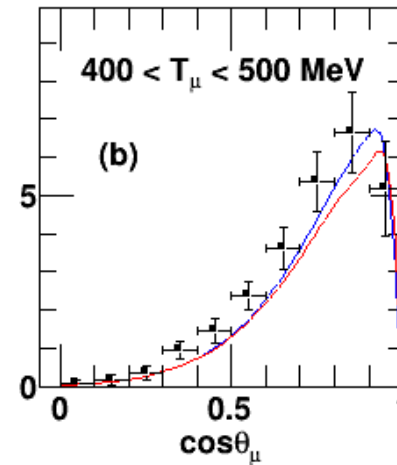
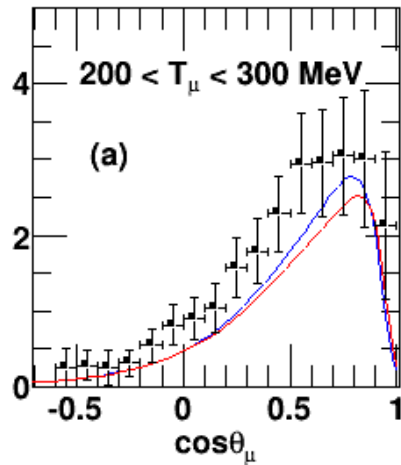
## MiniBooNe $\nu_\mu$

- Satisfactory general agreement
- Good agreement for forward scattering
- Missing strength for low  $T_\mu$ , backward scattering



## MiniBooNe $\bar{\nu}_\mu$

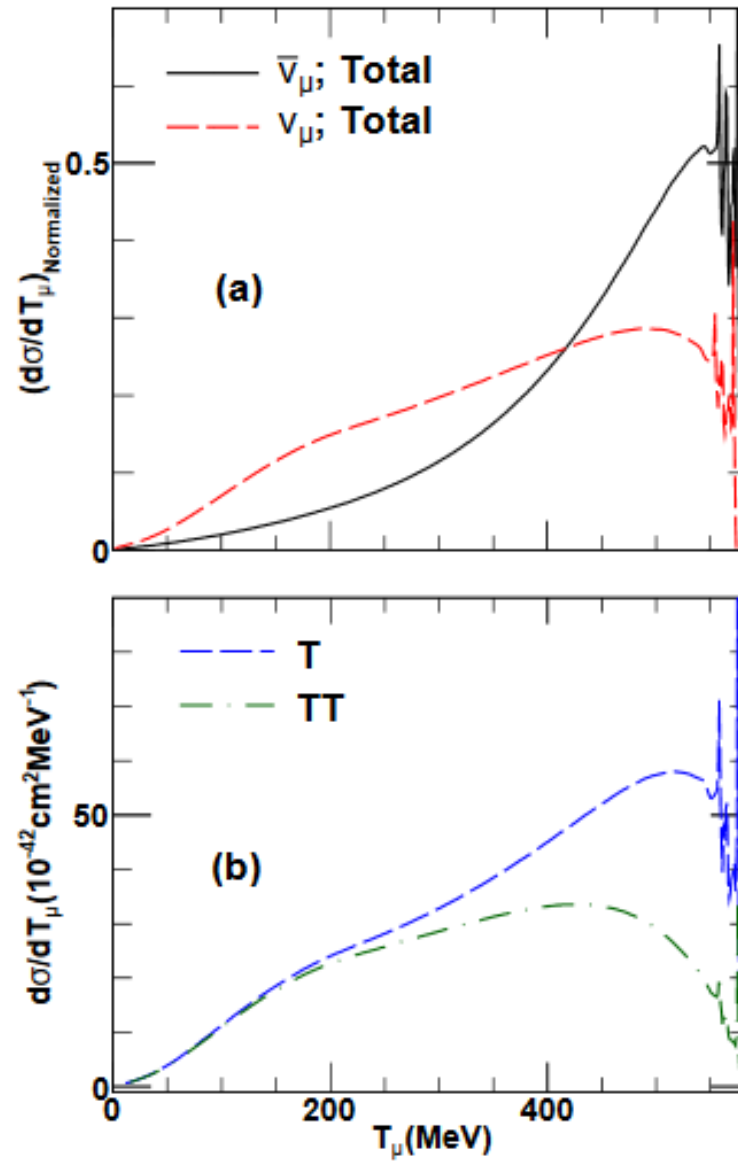
- Good general agreement
- Good agreement for forward scattering
- Missing strength for high  $T_\mu$ , backward scattering
- Better agreement with data than neutrino cross sections



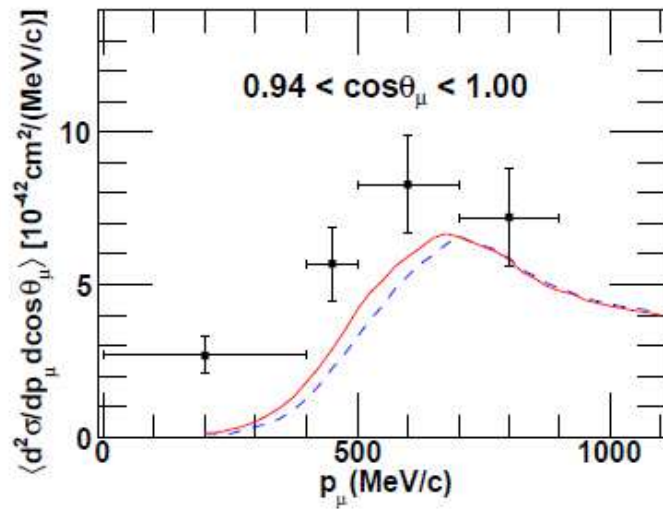
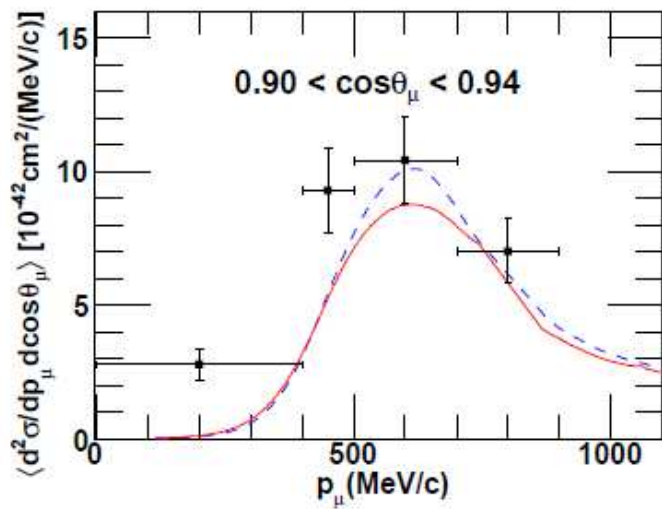
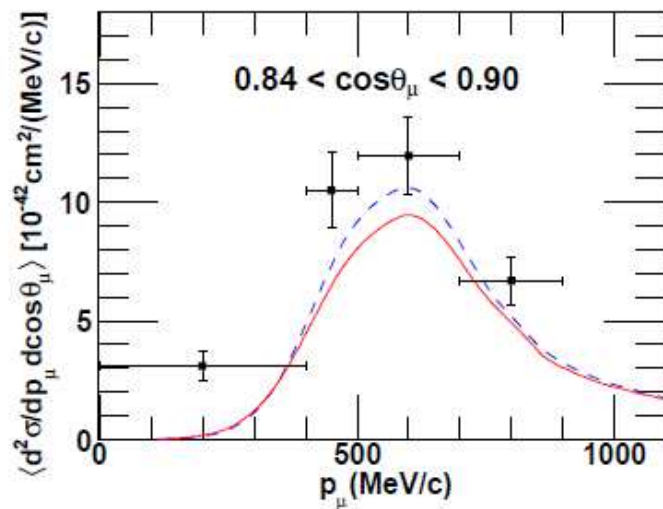
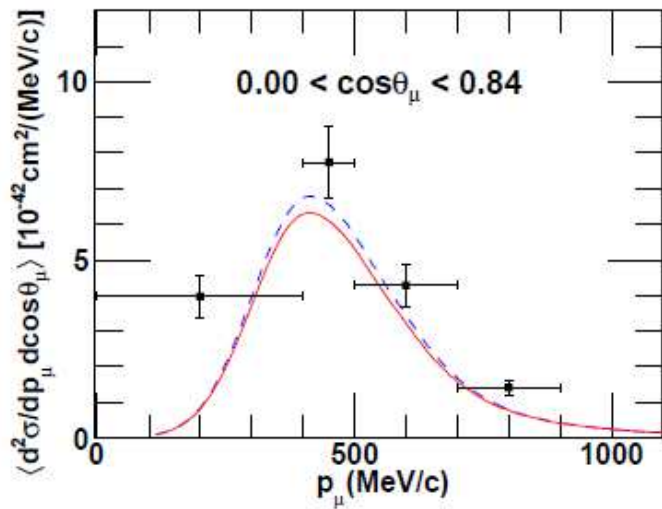


neutrino vs anti  
neutrinos

E=700 MeV

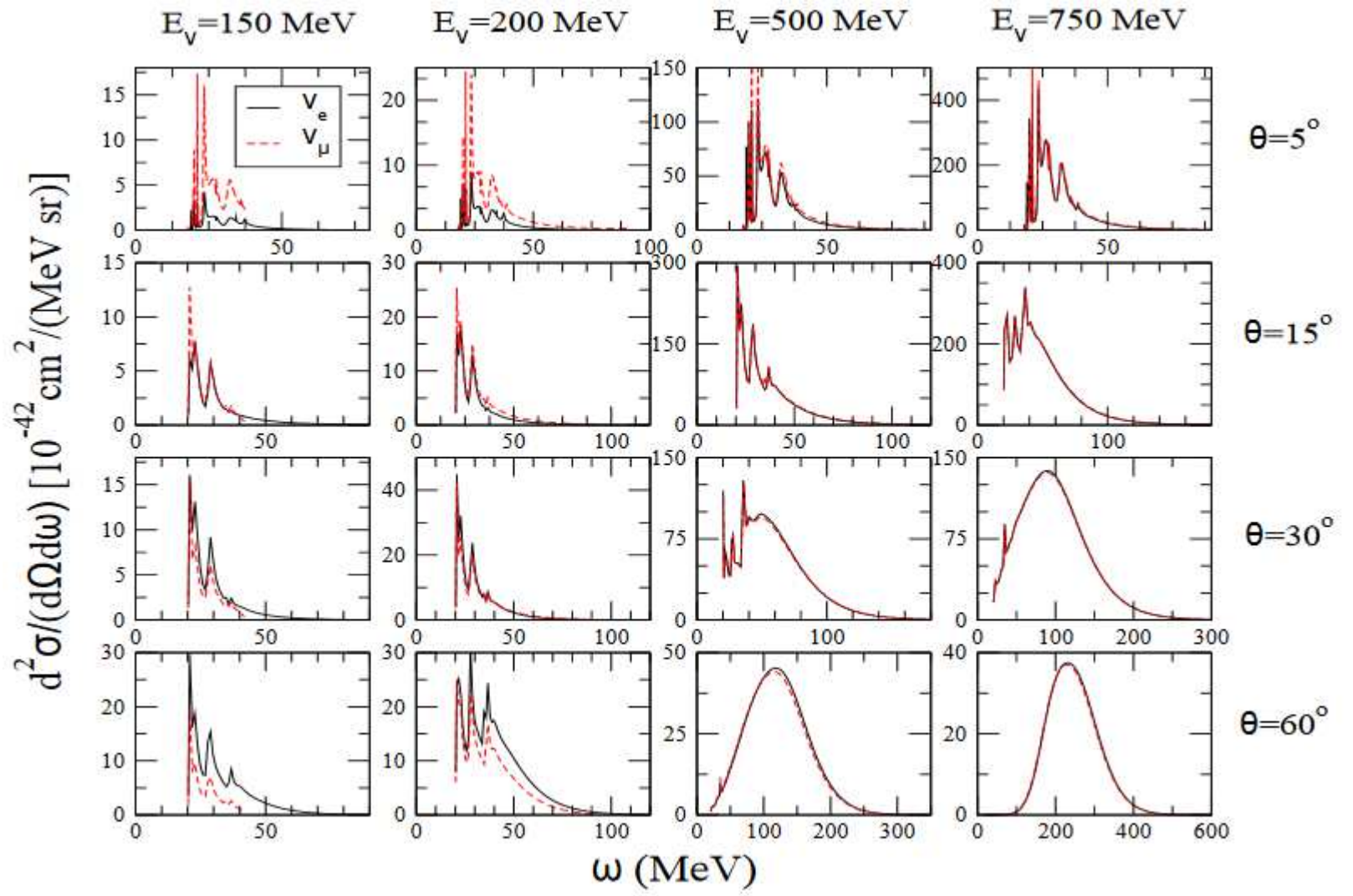


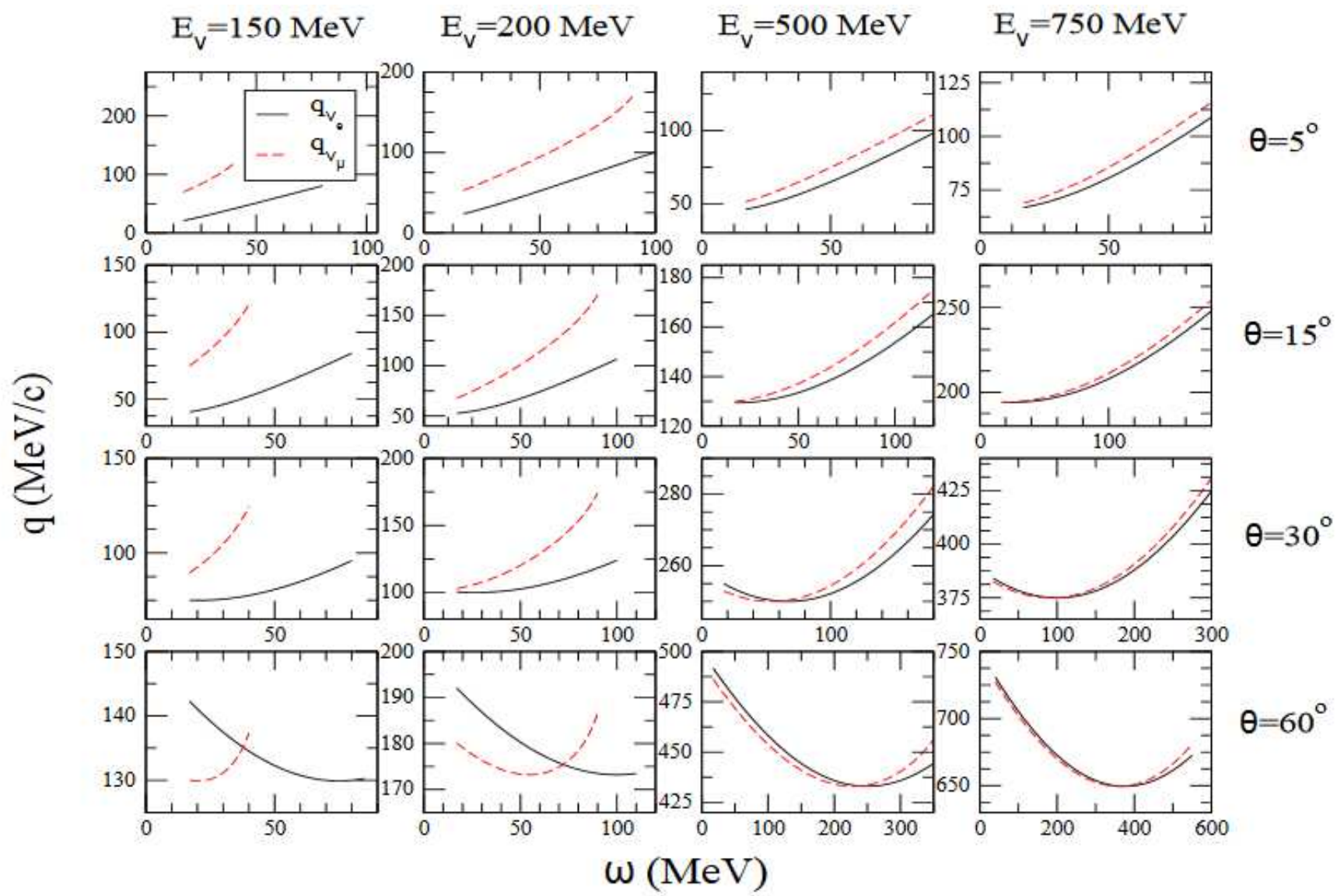
V. Pandey et al, Phys. Rev. C 92, 024606 (2015)



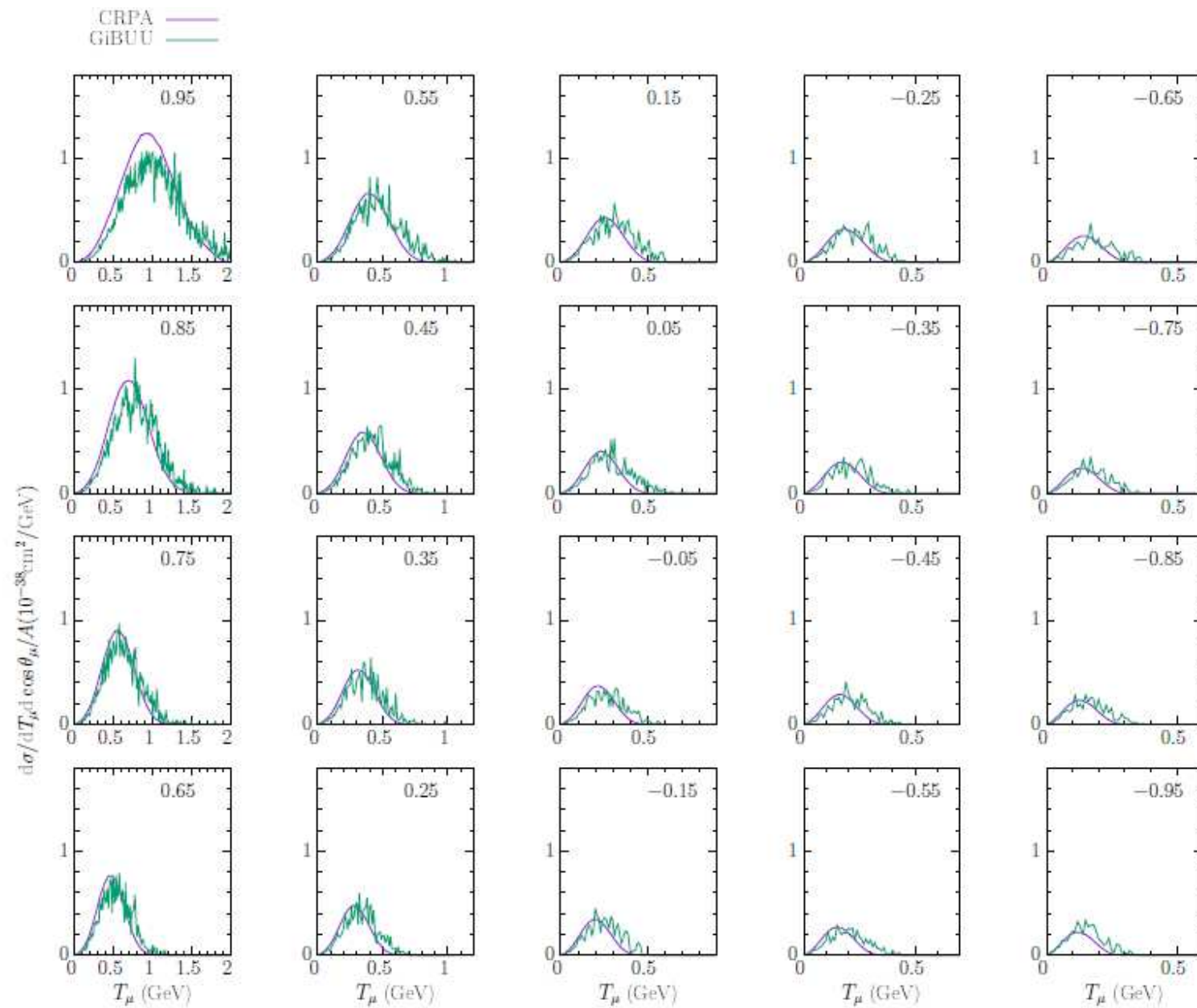
- T2K  $\nu_\mu$
- General agreement quite good
  - Missing strength for low  $p_\mu$

electron vs muon  
neutrinos



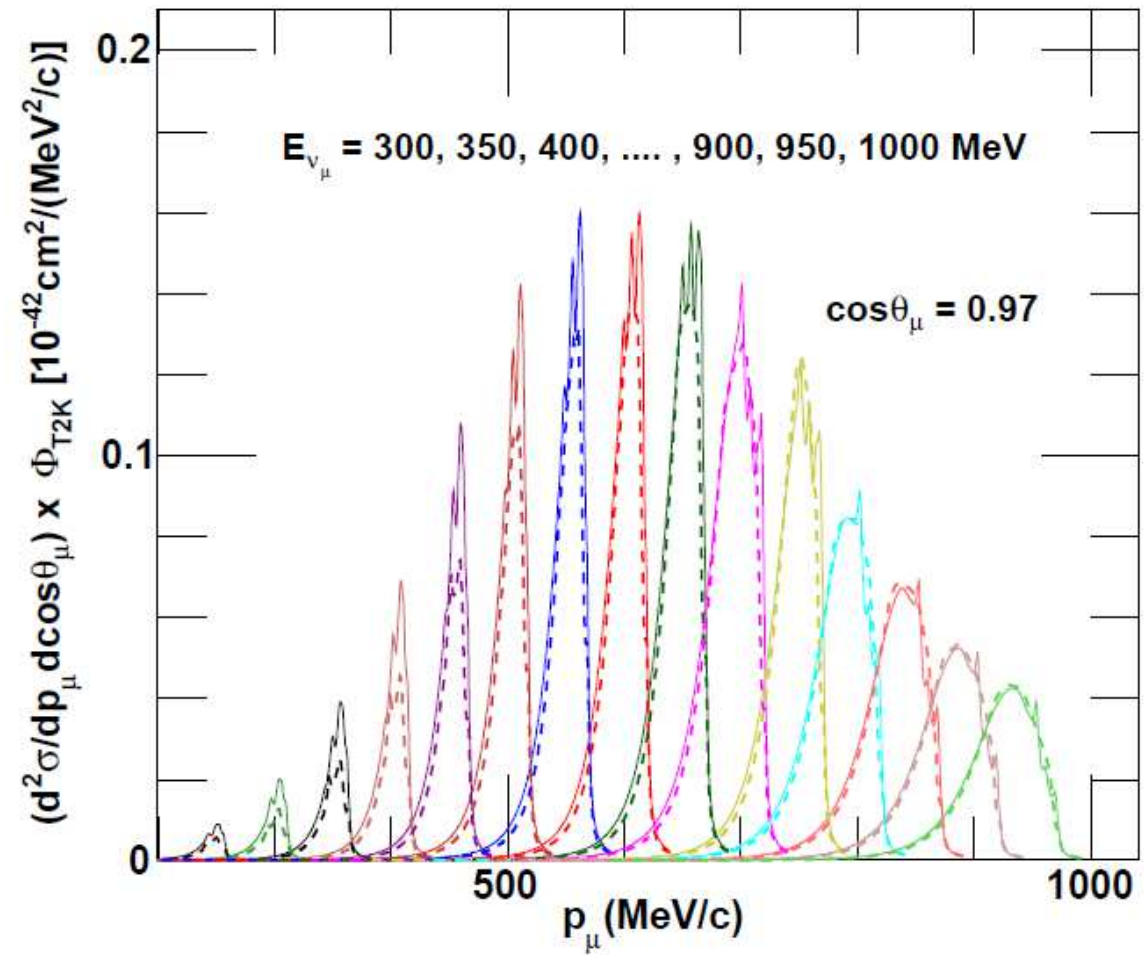


$^{40}\text{Ar}$



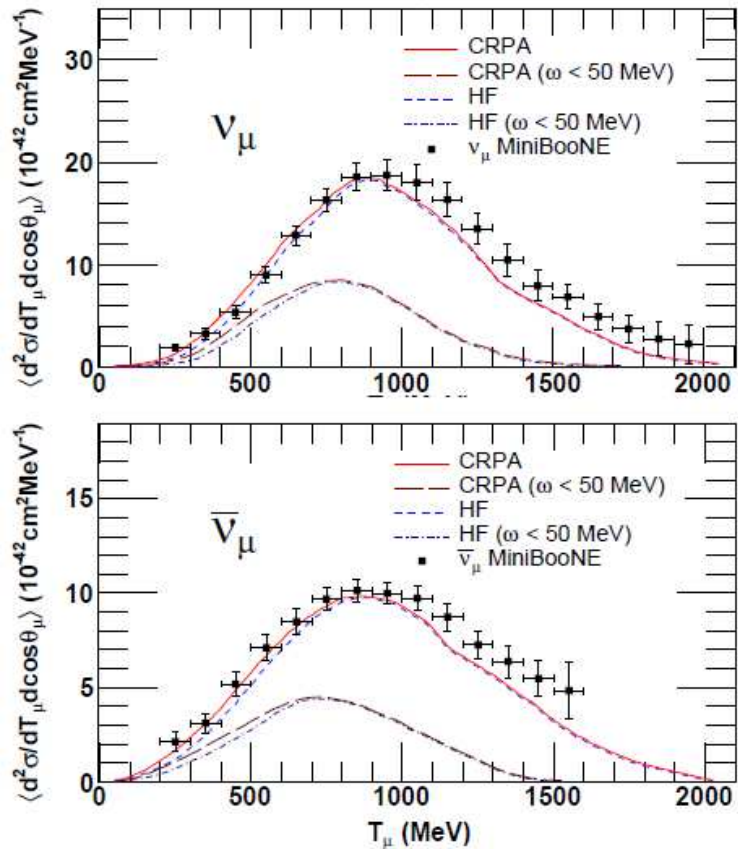


## Forward scattering



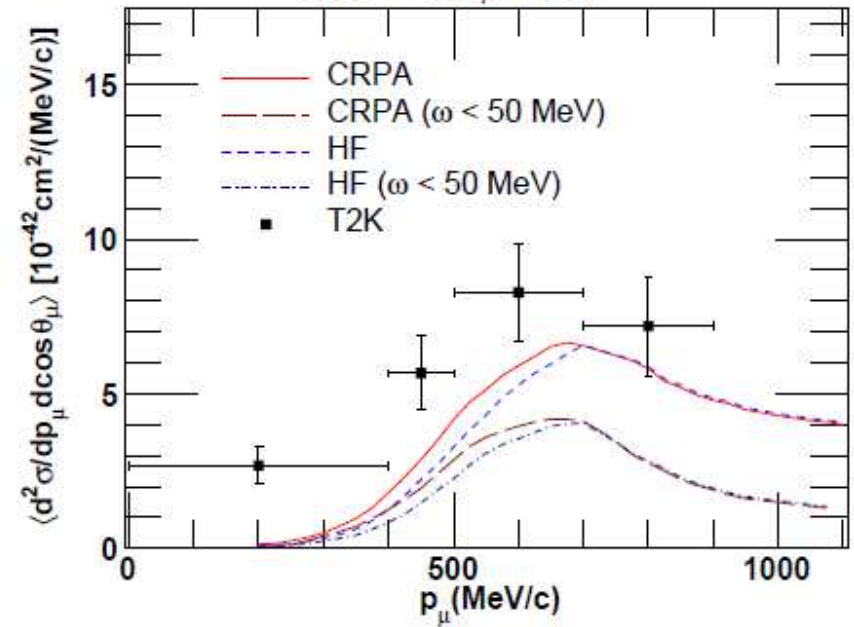
## MiniBooNe

$0.9 < \cos\theta_\mu < 1.0$

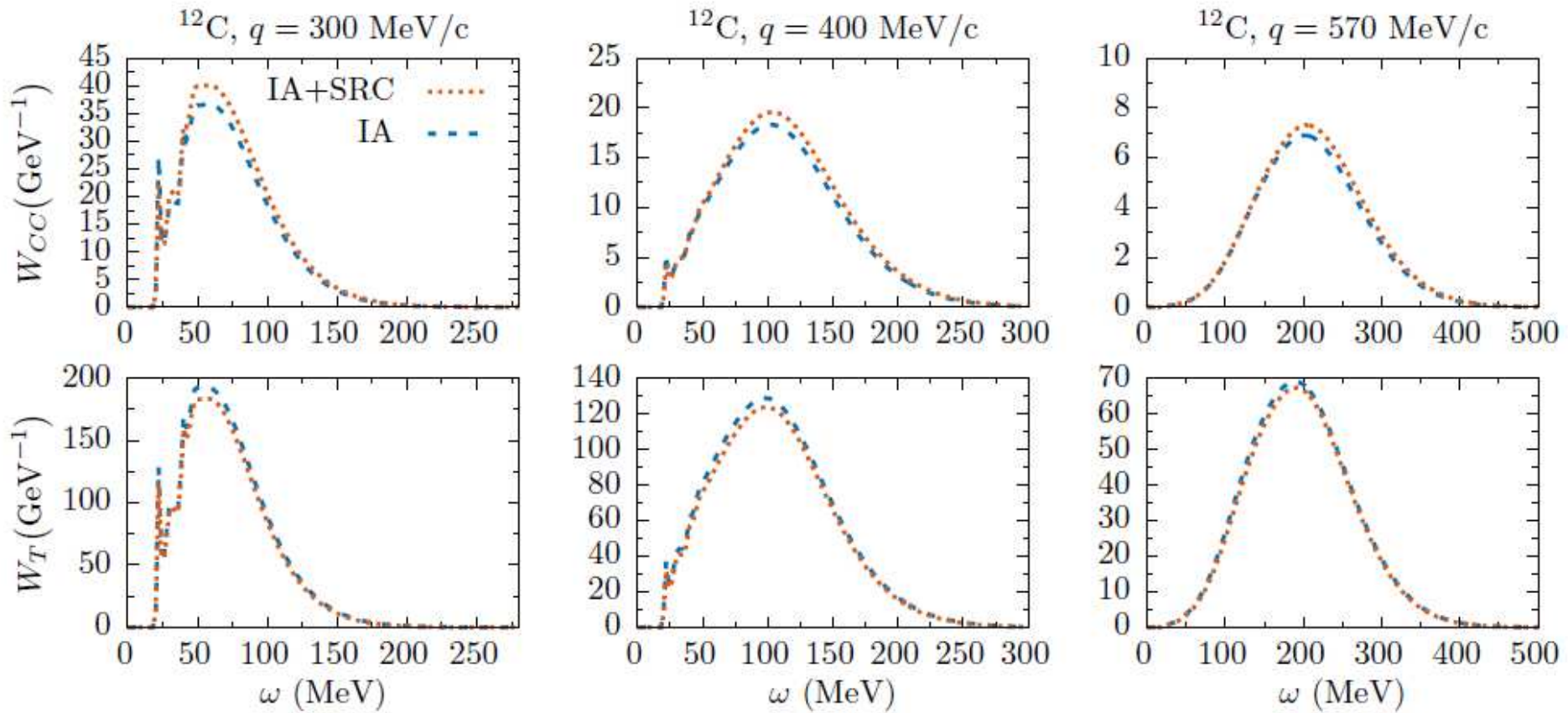


## T2K

$0.94 < \cos\theta_\mu < 1.00$



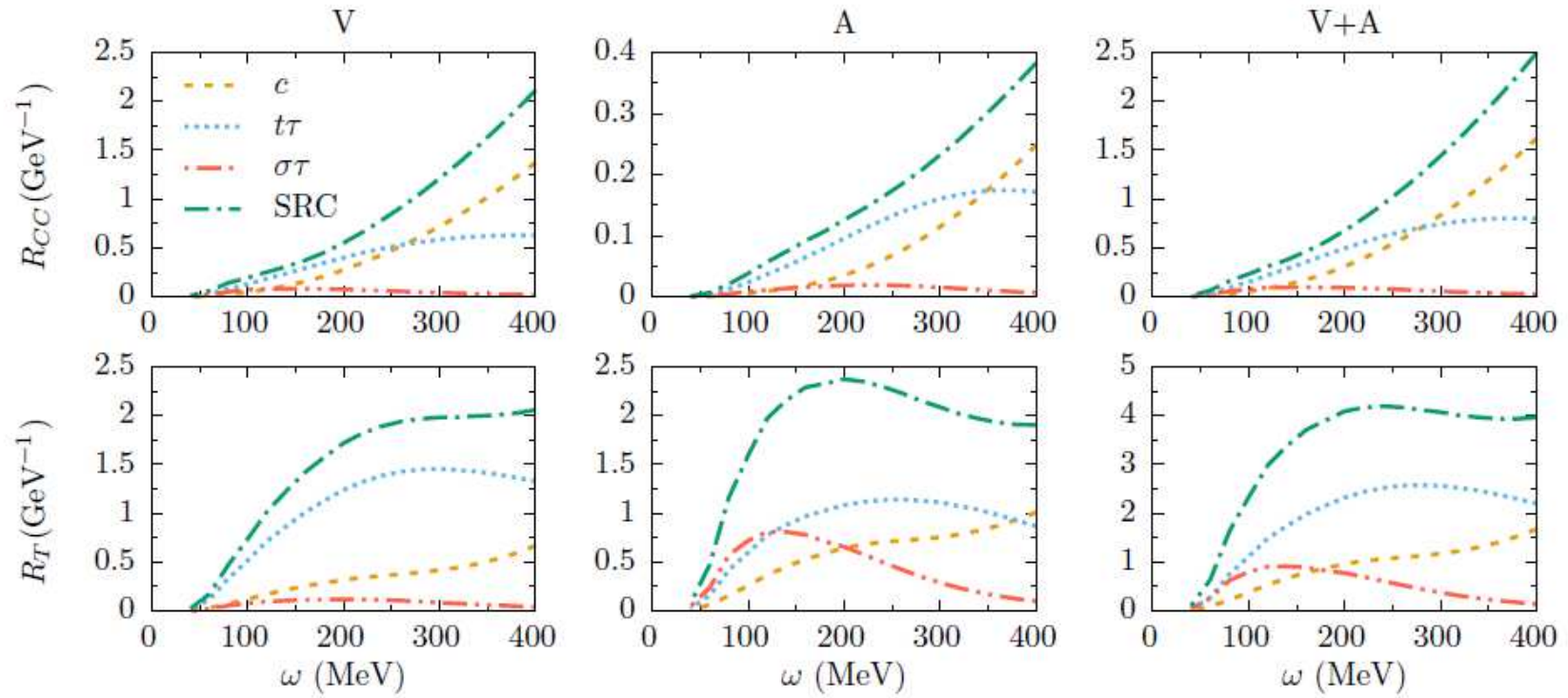
# SRC neutrinos 1p1h



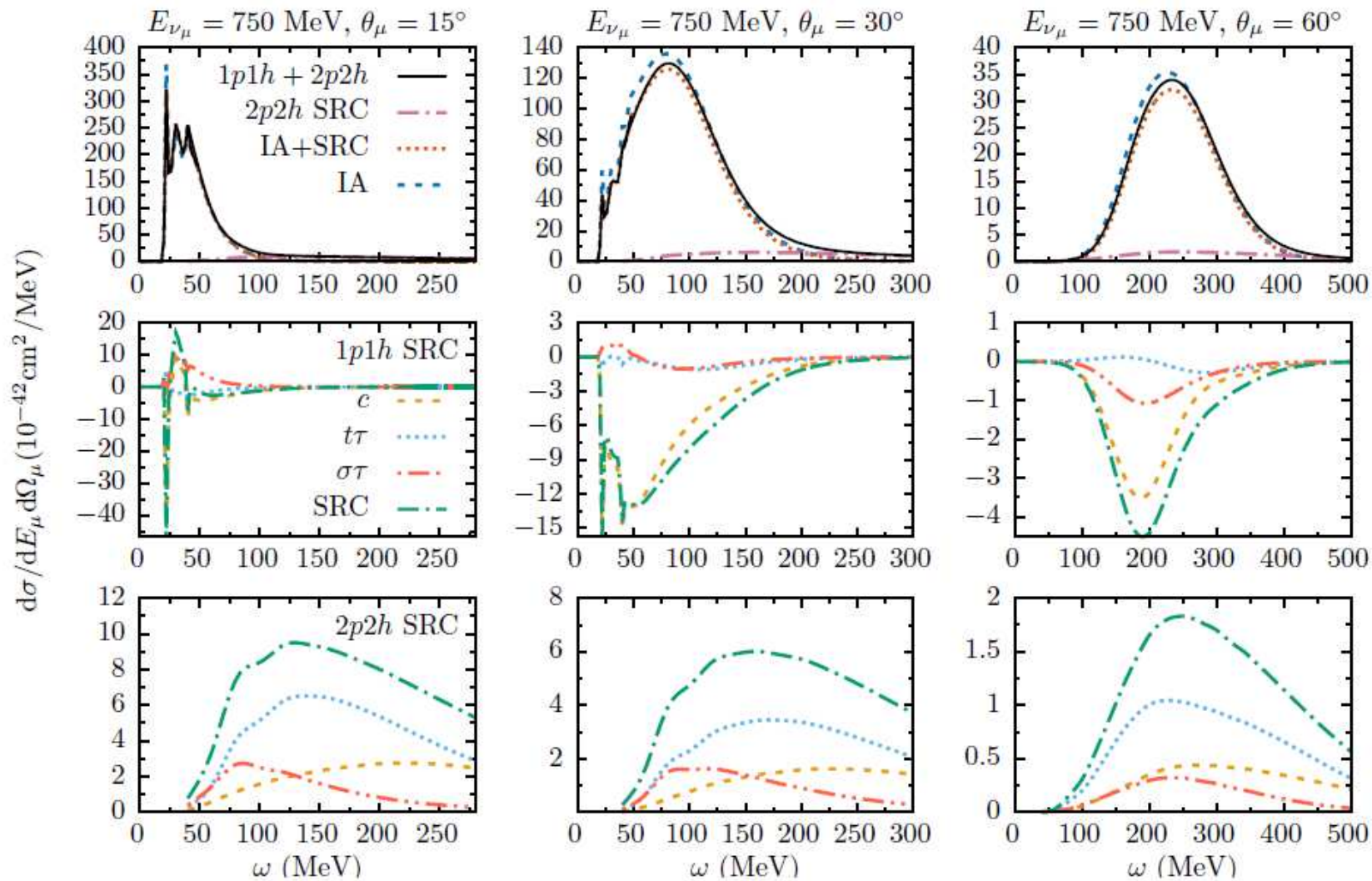


# SRC neutrinos 2p2h

$q = 400 \text{ MeV}/c$ .

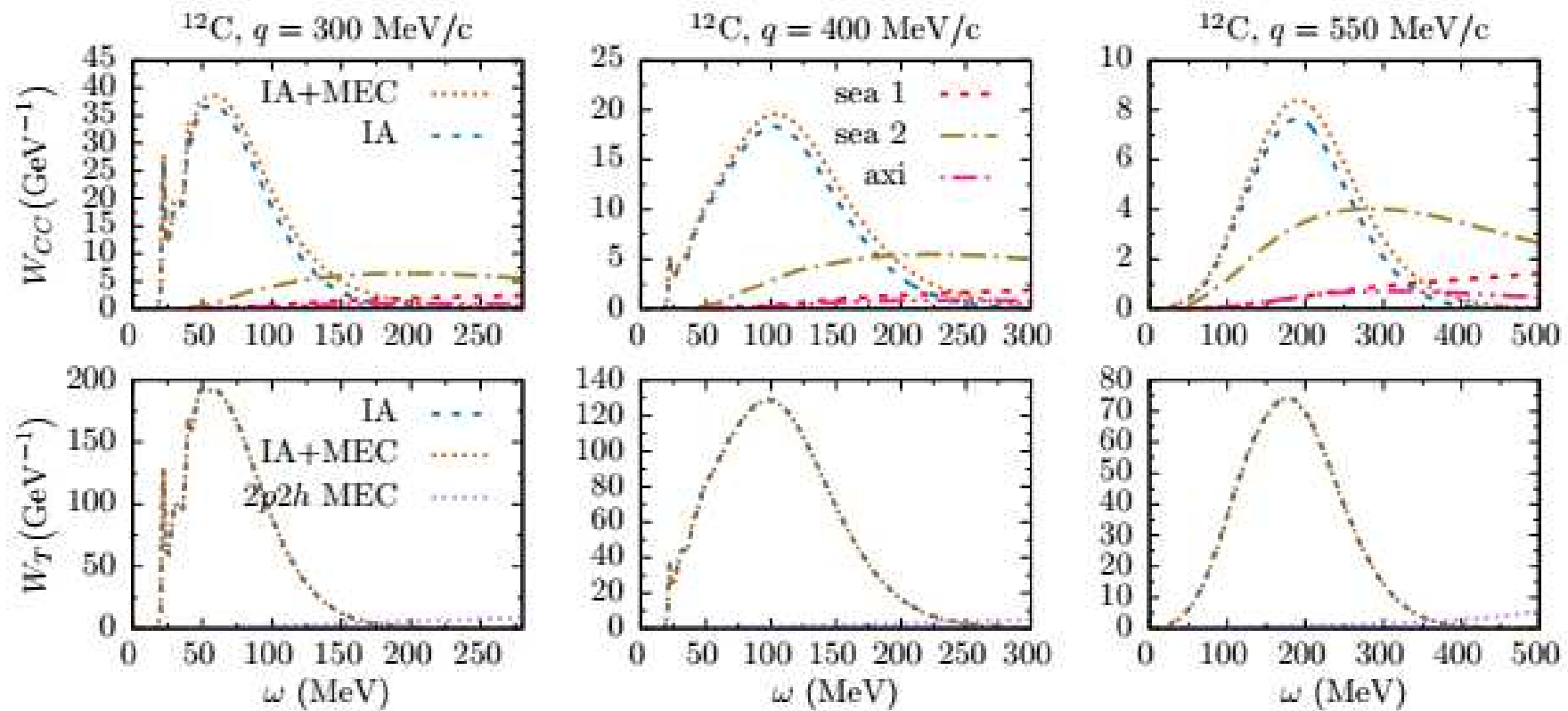


# SRC neutrinos 1p1h+2p2h

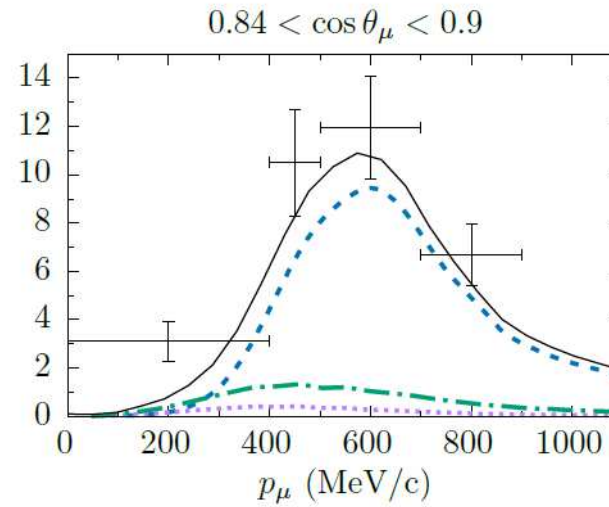
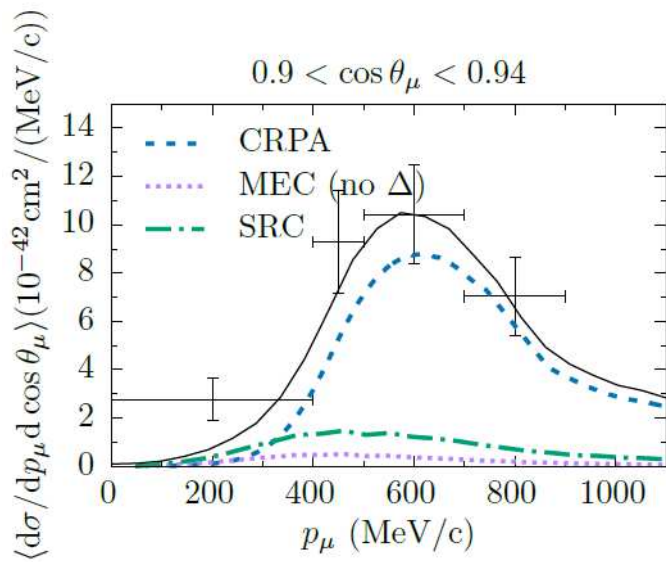


T. Van Cuyck et al  
 Phys. Rev. C 94,  
 024611 (2016)

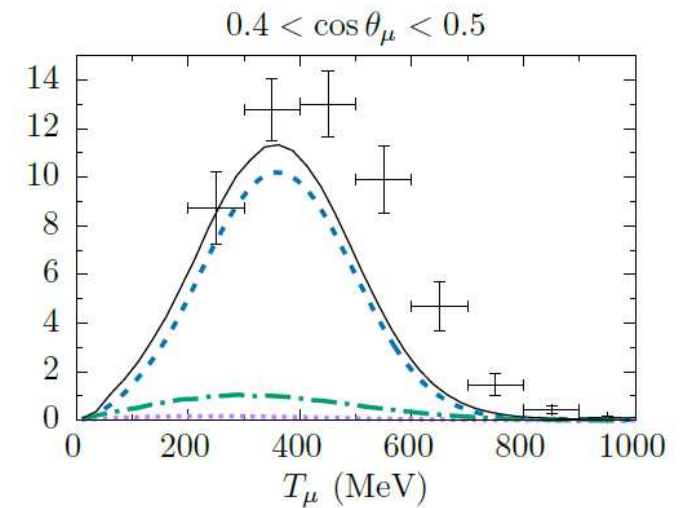
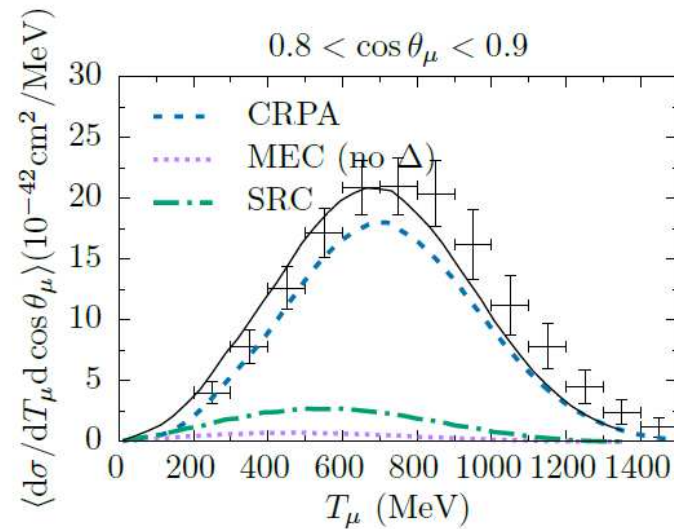
# MEC neutrinos 2p2h



T2K



MiniBooNe



## Summary

- Long- and short-range correlations in QE-like cross sections
- CRPA calculations provide extra strength for forward scattering arising from low-energy excitations
- This might affect CCQE neutrino cross sections as measured by MiniBooNe, T2K, ...
- SRC and MEC affect 1- and 2-nucleon knockout processes