

Global analysis from DIS to the small- Q^2 region

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in the Few-GeV Region (NuInt 2017)**

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<https://nuint2017.physics.utoronto.ca/>

June 26, 2017

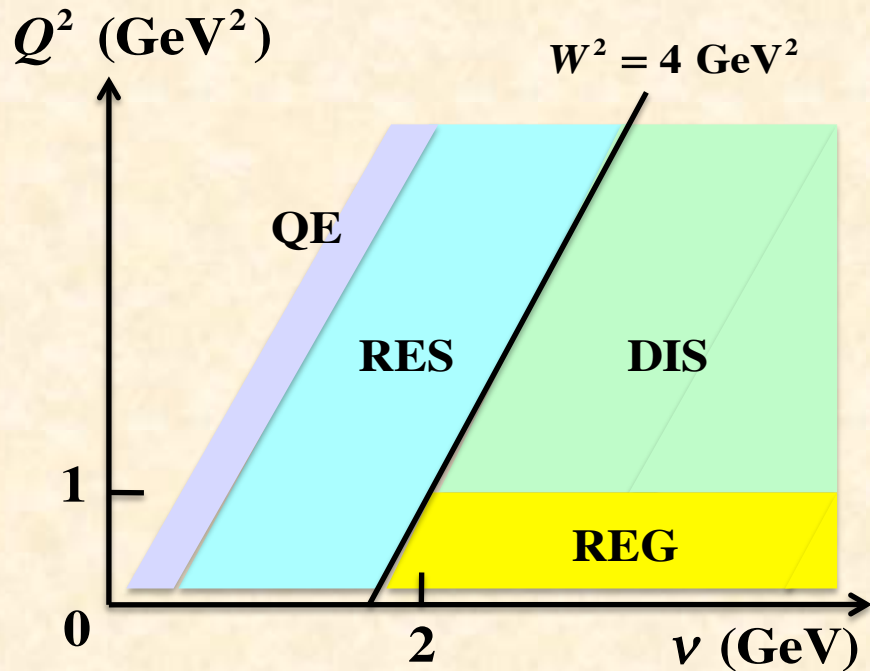
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Motivation

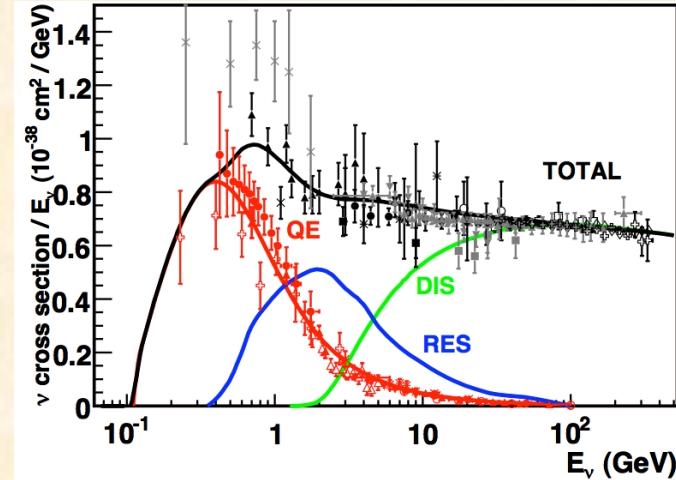
Motivation

Kinematical regions of neutrino-nucleus scattering



Depending on the neutrino beam energy, different physics mechanisms contribute to the cross section.

- QE (Quasi elastic)
- RES (Resonance)
- DIS (Deep inelastic scattering)
- REG (Regge)



J.L. Hewett *et al.*, arXiv:1205.2671,
 Proceedings of the 2011 workshop
 on Fundamental Physics at the Intensity Frontier

ν flux		16%
ν flux and cross section	w/o ND measurement	21.8%
	w/ ND measurement	2.7%
ν cross section due to difference of nuclear target btw. near and far		5.0%
Final or Secondary Hadronic Interaction		3.0%
Super-K detector		4.0%
total	w/o ND measurement	23.5%
	w/ ND measurement	7.7%

ν interactions

A.K.Ichikawa@KEK workshop 2015

ν -interaction collaboration at J-PARC

Toward Unified Description of Lepton-Nucleus Reactions from MeV to GeV Region

Top Page | Research Projects | Participants | Collaboration Meeting | Publications | Links | To Japanese Page

What's New

- 03/31/2016 Publications updated.
- 04/20/2014 Publications updated.
- 12/27/2013 Collaboration Meeting updated.
- 12/27/2013 Publications updated.
- 12/18/2013 Links updated.
- 10/01/2013 Site opens!

Recent breakthrough measurements of the neutrino mixing angle revealed that θ_{13} is non-zero, that opened a possibility of CP violation in the lepton sector. The major interests of the neutrino physics is now the determination of the leptonic CP phase and the neutrino mass hierarchy. To extract such neutrino properties successfully from the data, a precise knowledge of the neutrino-nucleus reactions (Fig. 1) is becoming a crucial issue. The kinematic regions relevant to the neutrino parameter searches extend over the quasi-elastic, resonance, and deep inelastic scatterings (Fig. 2) regions. The objective of the project is to construct a unified neutrino reaction model which describes the wide energy region by forming a new collaboration of experimentalists and theorists in different fields.

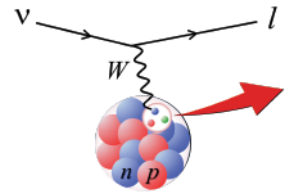


Fig. 1. Neutrino-nucleus reaction

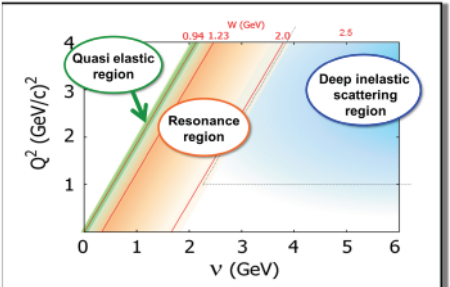
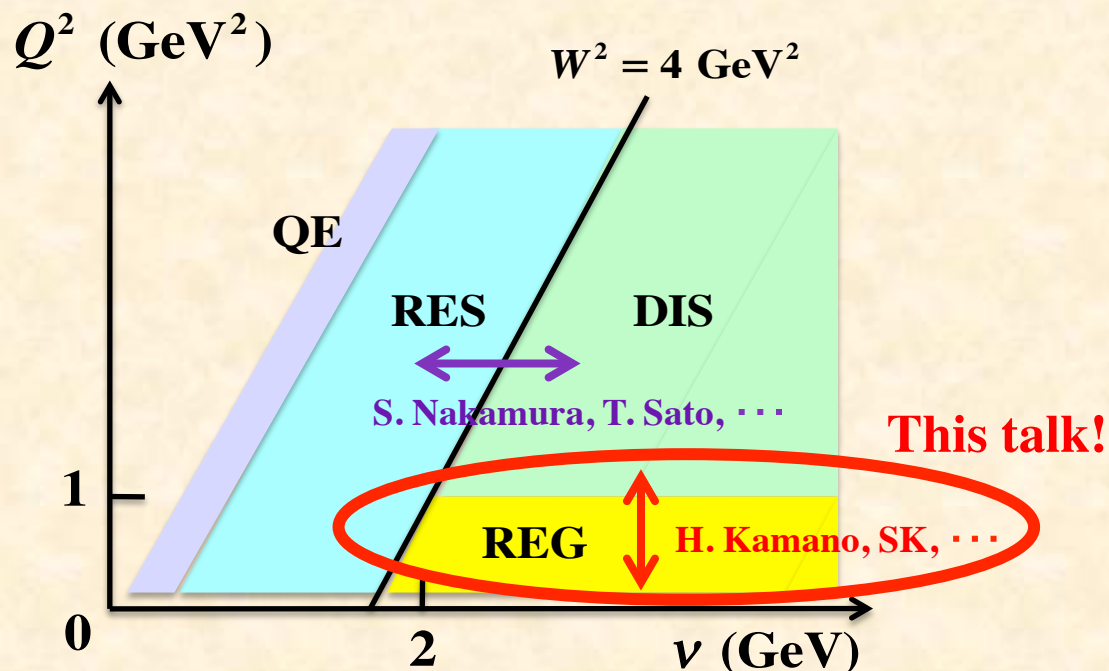


Fig. 2. Kinematical region relevant to neutrino oscillation experiment

Activities at the J-PARC branch, KEK theory center
<http://j-parc-th.kek.jp/html/English/e-index.html>

Y. Hayato, M. Hirai, W. Horiuchi, H. Kamano, S. Kumano, T. Murata, S. Nakamura, K. Saito, M. Sakuda, T. Sato
http://nuint.kek.jp/index_e.html



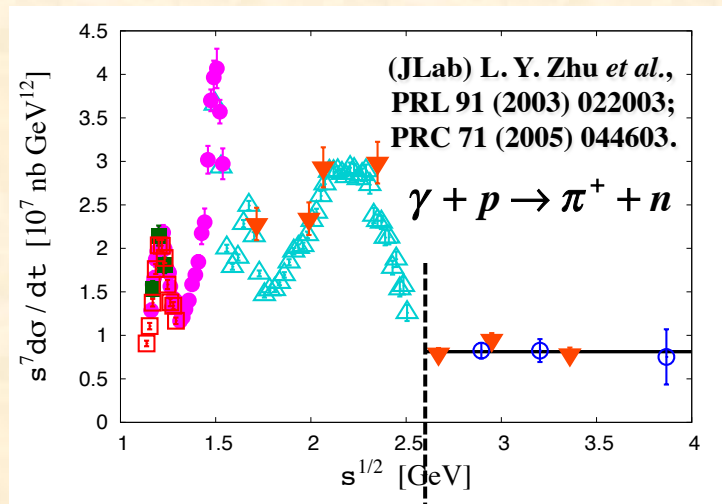
For the details, see

Towards a unified model of neutrino-nucleus reactions for neutrino oscillation experiments, S. X. Nakamura, H. Kamano, Y. Hayato, M. Hirai, W. Horiuchi, S. Kumano, T. Murata, K. Saito, M. Sakuda, T. Sato, and Y. Suzuki, Rep. Prog. Phys. 80 (2017) 056301.

General motivation

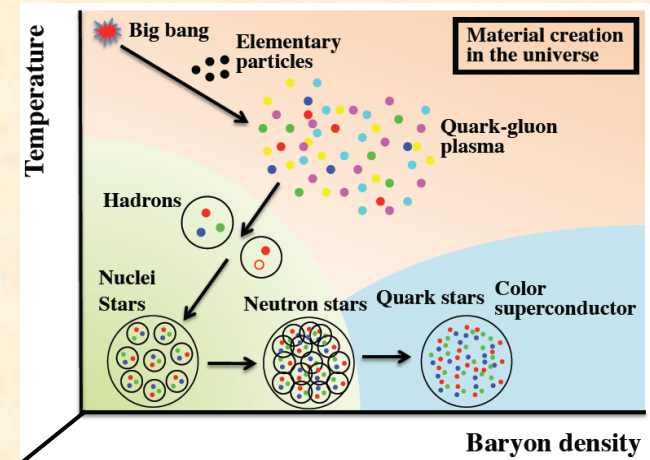
Ultimate purpose of Theoretical nuclear physics
 = Describe hadronic many-body systems
 in the whole phase diagram
 from low to high energies.

Transition from hadron to quark-gluon d.o.f.:
 H. Kawamura, S. Kumano, T. Sekihara,
 Phys. Rev. D 88 (2013) 034010.

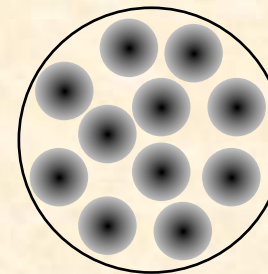


Low energies:
 Hadron degrees
 of freedom
 (Resonances)

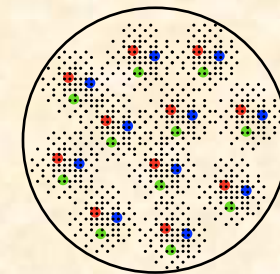
High energies:
 Quark-gluon
 degrees of freedom
 (Perturbative QCD:
 Constituent-counting rule)



Q^2



Low energies:
 Hadron degrees
 of freedom



High energies:
 Quark-gluon
 degrees of freedom

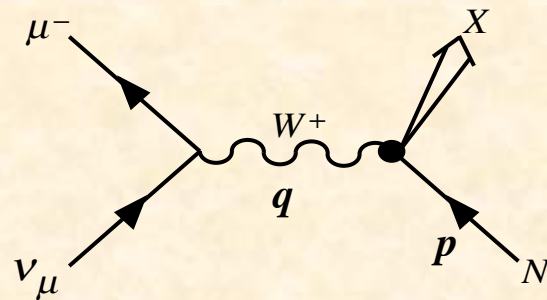
Nuclei should be described by quark and gluon
 degrees of freedom at high energies.

Structure functions of nucleon and nuclei

Deep inelastic scattering (DIS)

A nucleon is broken up by a high-energy neutrino.

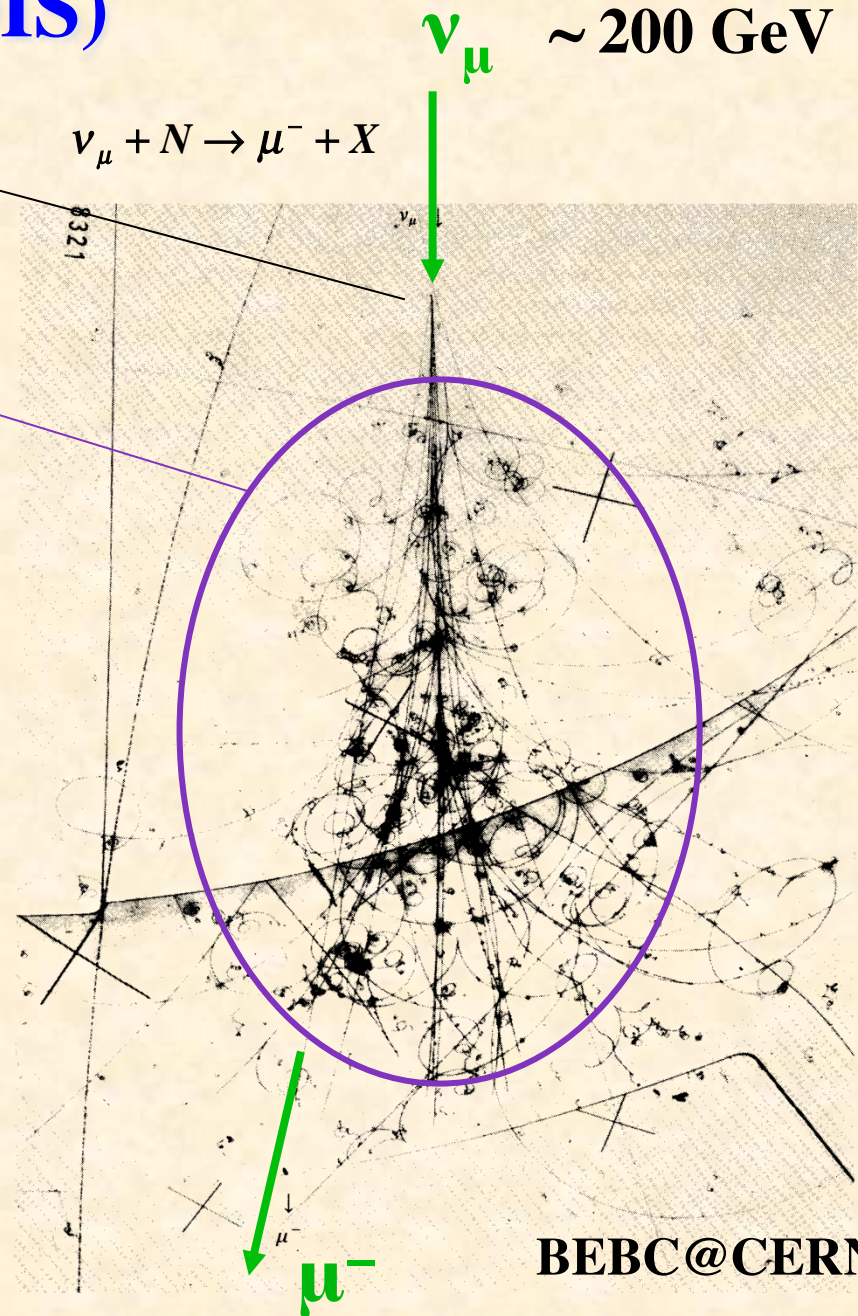
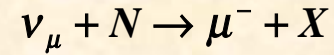
Hadrons are produced; however, these are not usually measured. (inclusive reaction)



Momentum transfer: $q^2 = (k - k')^2 = -Q^2$

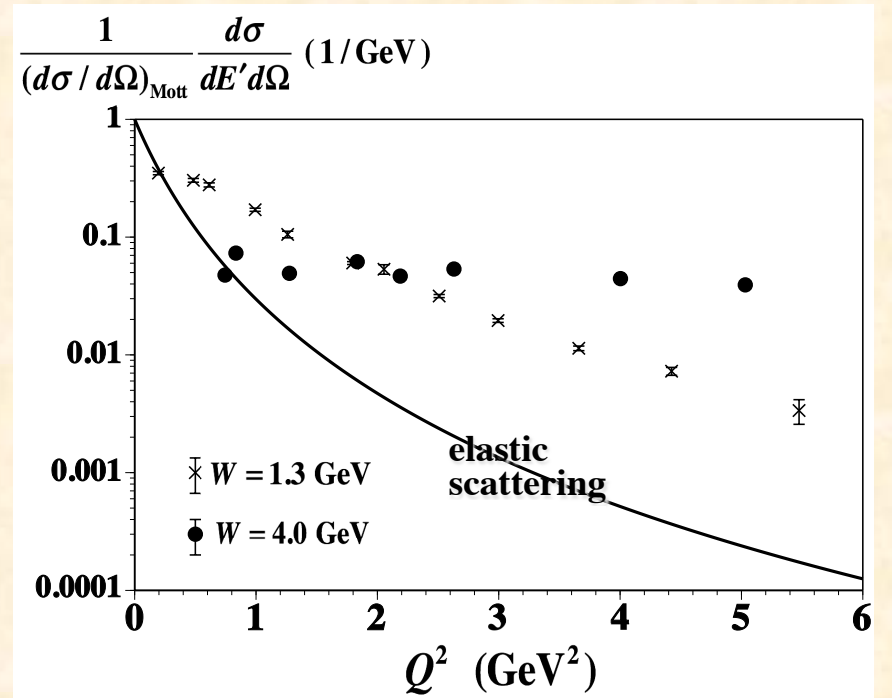
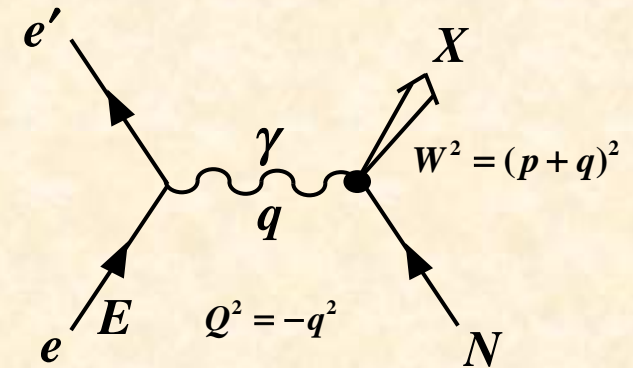
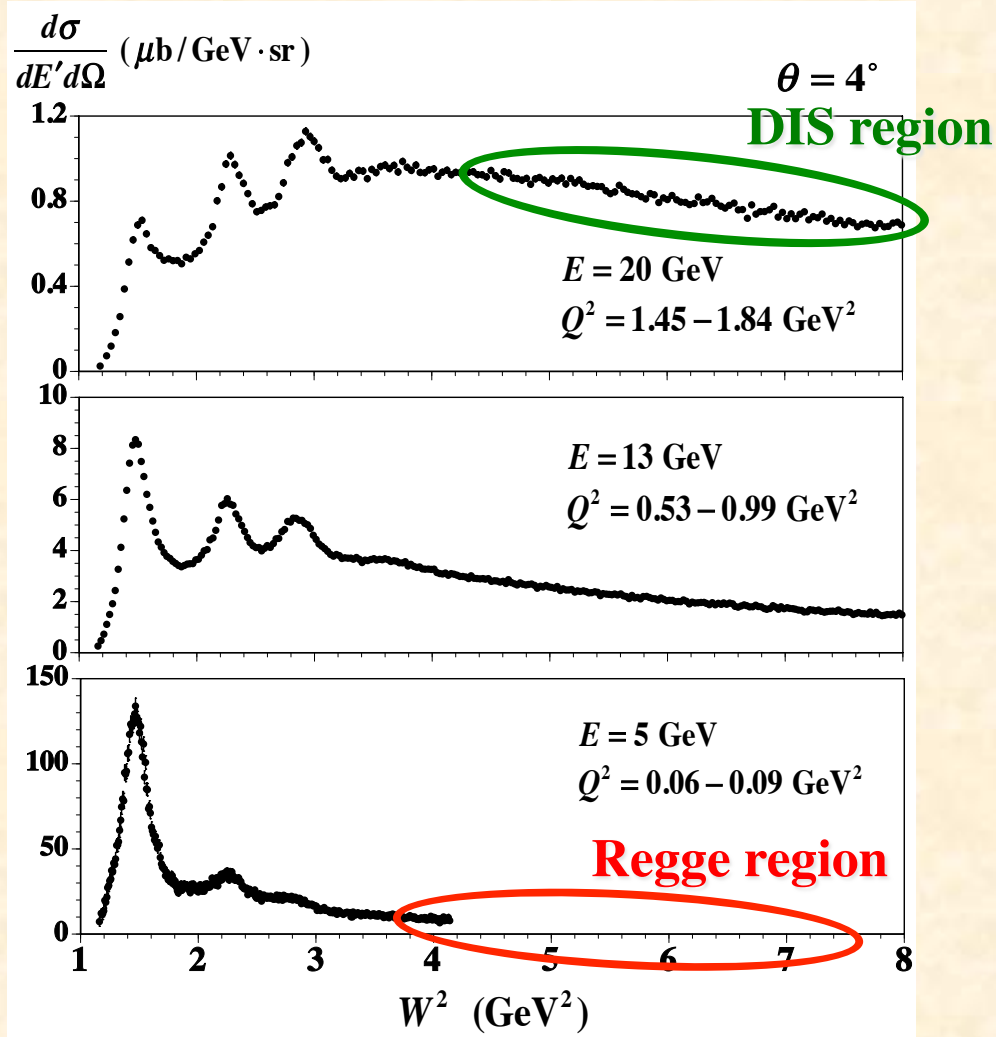
Bjorken scaling variable: $x = \frac{Q^2}{2p \cdot q}$

Invariant mass: $W^2 = p_X^2 = (p + q)^2$



BEBC@CERN

Lepton scattering

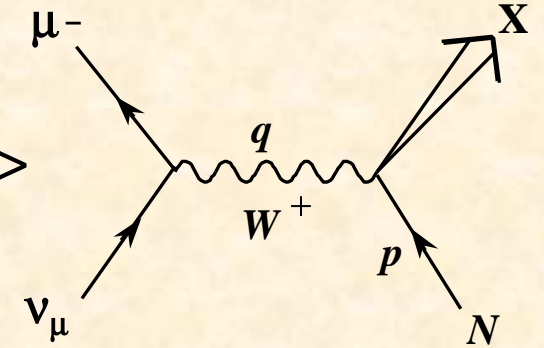


Neutrino deep inelastic scattering (CC: Charged Current)

$$d\sigma = \frac{1}{4k \cdot p} \frac{1}{2} \sum_{spins} \sum_X (2\pi)^4 \delta^4(k + p - k' - p_X) |M|^2 \frac{d^3k'}{(2\pi)^3 2E'}$$

$$M = \frac{1}{1 + Q^2/M_W^2} \frac{G_F}{\sqrt{2}} \bar{u}(k', \lambda') \gamma^\mu (1 - \gamma_5) u(k, \lambda) \langle X | J_\mu^{CC} | p, \lambda_p \rangle$$

$$\frac{d\sigma}{dE' d\Omega} = \frac{G_F^2}{(1 + Q^2/M_W^2)^2} \frac{k'}{32\pi^2 E} L^{\mu\nu} W_{\mu\nu}$$



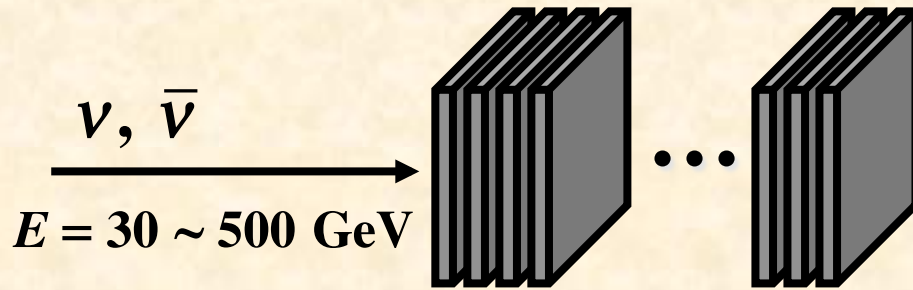
$$L^{\mu\nu} = 8 \left[k^\mu k'^\nu + k'^\mu k^\nu - k \cdot k' g^{\mu\nu} + i \underline{\varepsilon^{\mu\nu\rho\sigma}} k_\rho k'_\sigma \right], \quad \varepsilon_{0123} = +1$$

$$W_{\mu\nu} = -W_1 \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + W_2 \frac{1}{M^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) + \frac{i}{2M^2} \underline{W_3 \varepsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma}$$

$$MW_1 = F_1, \quad \nu W_2 = F_2, \quad \nu W_3 = F_3, \quad x = \frac{Q^2}{2p \cdot q}, \quad y = \frac{p \cdot q}{p \cdot k}$$

$$\frac{d\sigma_{\nu, \bar{\nu}}^{CC}}{dx dy} = \frac{G_F^2 (s - M^2)}{2\pi (1 + Q^2/M_W^2)^2} \left[x y^2 F_1^{CC} + \left(1 - y - \frac{M x y}{2E} \right) F_2^{CC} \pm x y \left(1 - \frac{y}{2} \right) \underline{F_3^{CC}} \right]$$

Neutrino DIS experiments

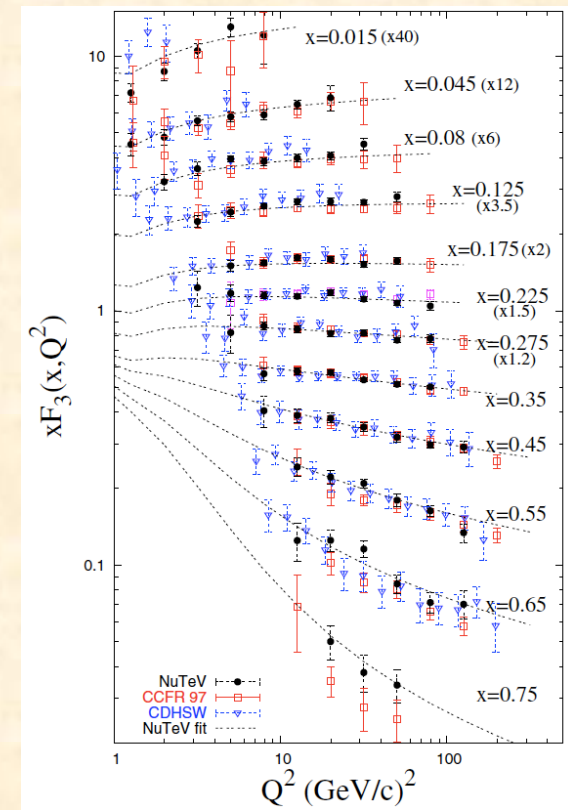
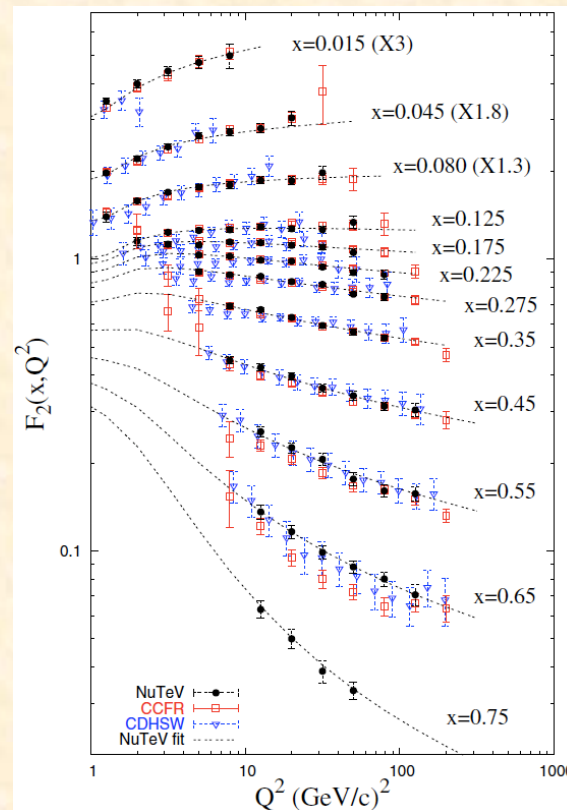


Huge Fe target (690 ton)

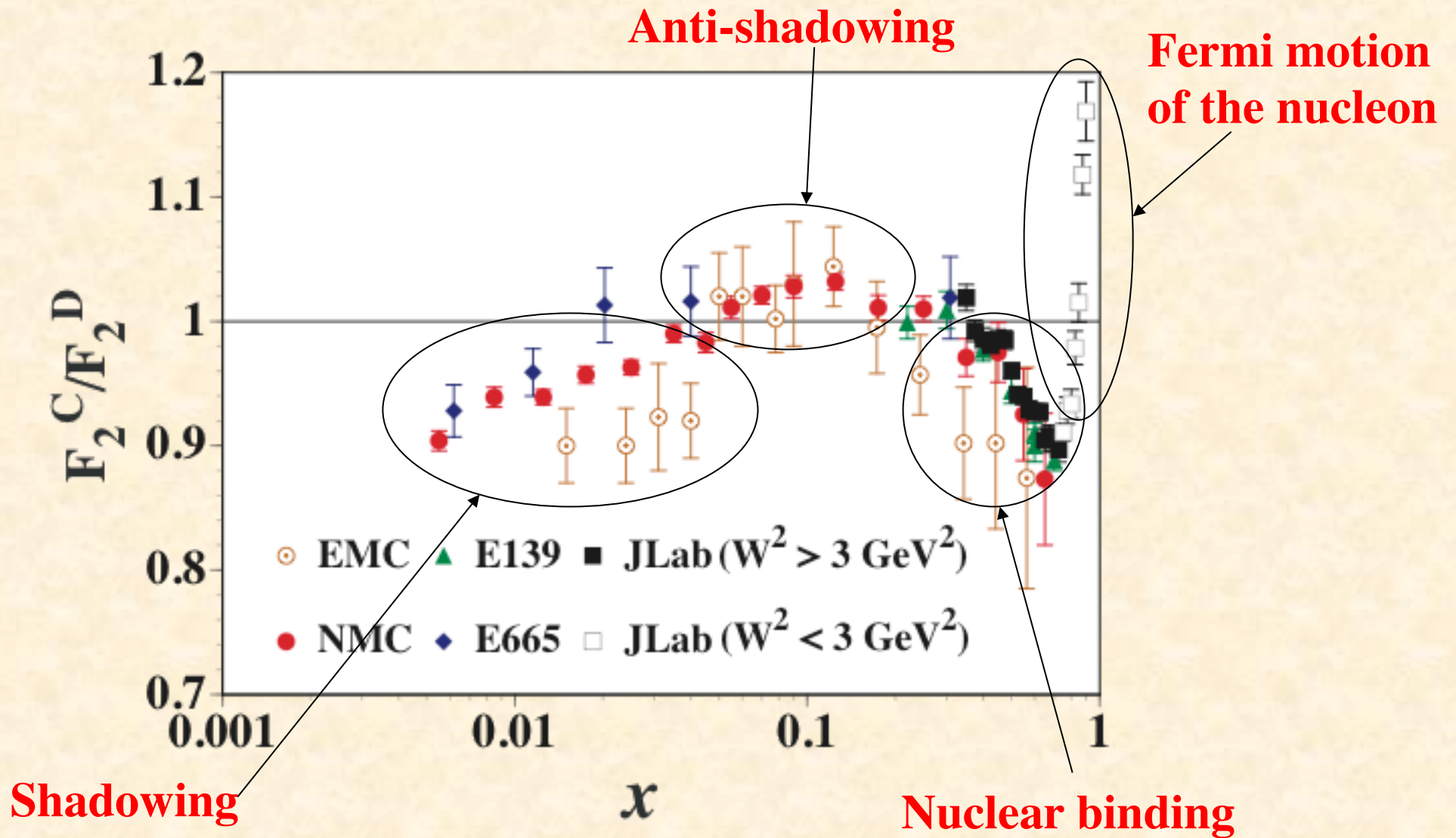
Experiment	Target	ν energy (GeV)
CCFR	Fe	30-360
CDHSW	Fe	20-212
CHORUS	Pb	10-200
NuTeV	Fe	30-500

MINERvA (He, C, Fe, Pb), ...

M. Tzanov *et al.* (NuTeV),
 PRD74 (2006) 012008.



Nuclear modifications of structure function F_2

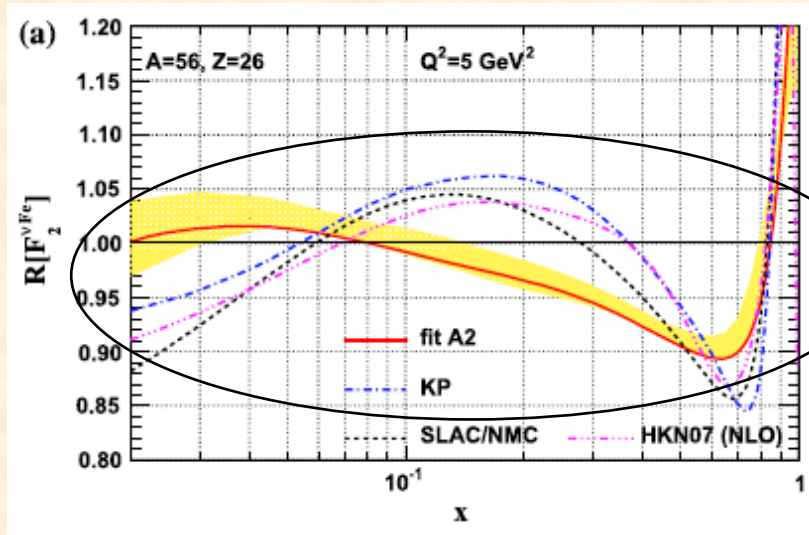


D. F. Geesaman, K. Saito, A. W. Thomas,
Ann. Rev. Nucl. Part. Sci. 45 (1995) 337

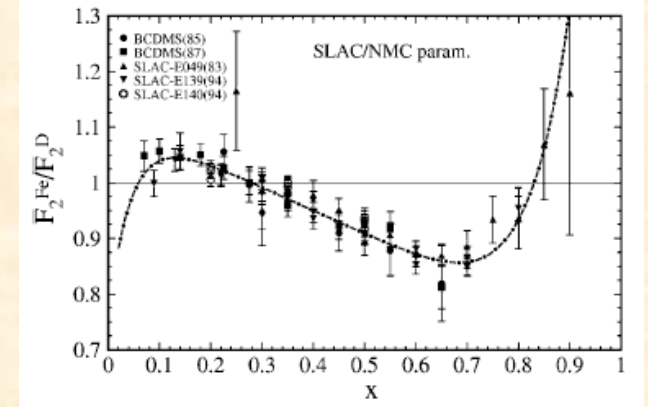
Analysis of CTEQ-2008 (Schienbein *et al.*)

I. Schienbein *et al.*,
PRD 77 (2008) 054013

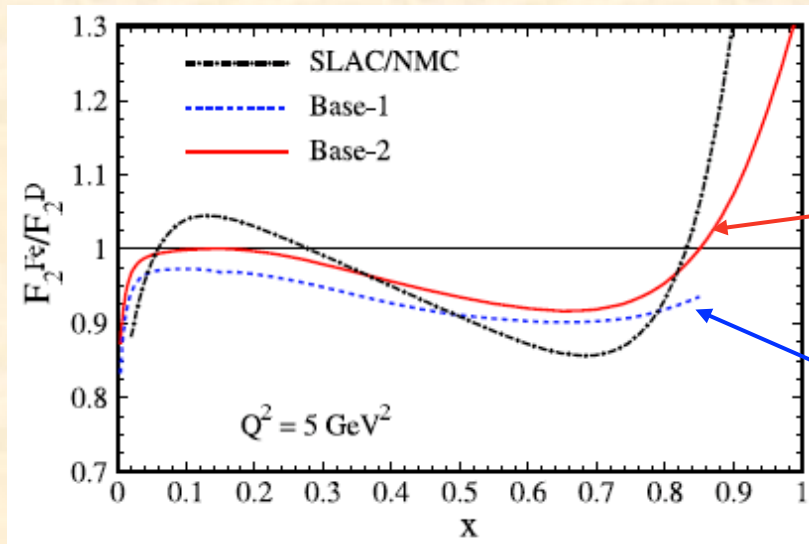
Charged-lepton scattering



Differences from typical NPDFs.



Neutrino scattering



Base-1 • remove CCFR data

• incorporate deuteron corrections

Base-2 corresponds to CTEQ6.1M with $s \neq \bar{s}$

• include CCFR data

Charged-lepton correction factors are applied.

• $s \neq \bar{s}$

Base-2: Using current nucleonic PDFs, they (and MRST) obtained very different corrections from charged-lepton data.

Base-1: However, it depends on the analysis method for determining “nucleonic” PDFs.

Recent progress on neutrino DIS \Leftrightarrow Charged DIS

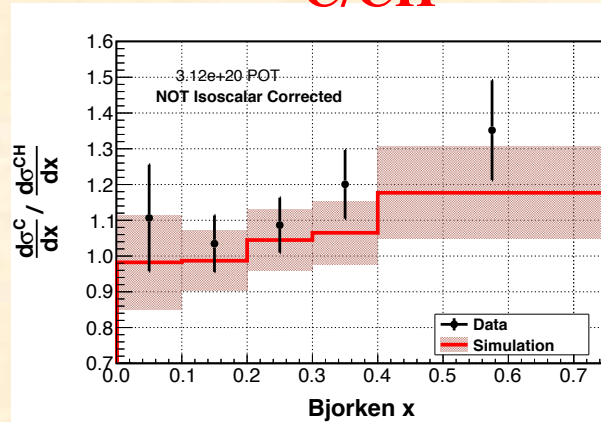
Measurements by Minerva

B. G. Tice *et al.*, PRL 112 (2014) 231801;

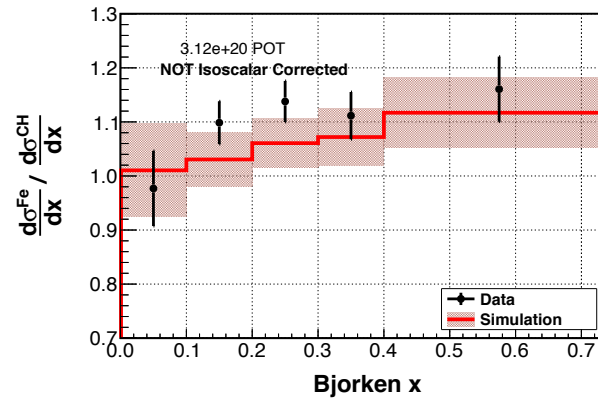
J. Mousseau *et al.*, PRD 93 (2016) 071101(R).

Different shadowing from charged-lepton case?!

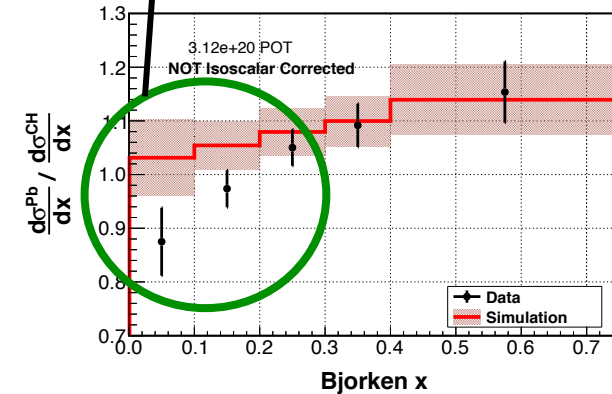
C/CH



Fe/CH

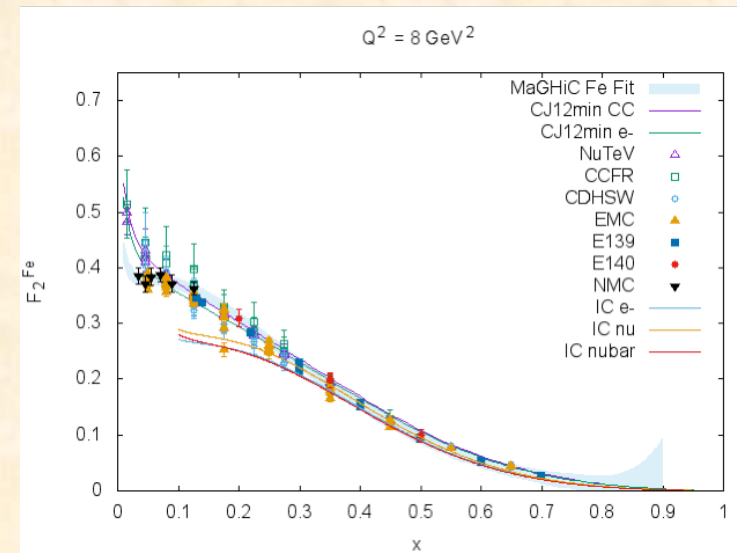


Pb/CH



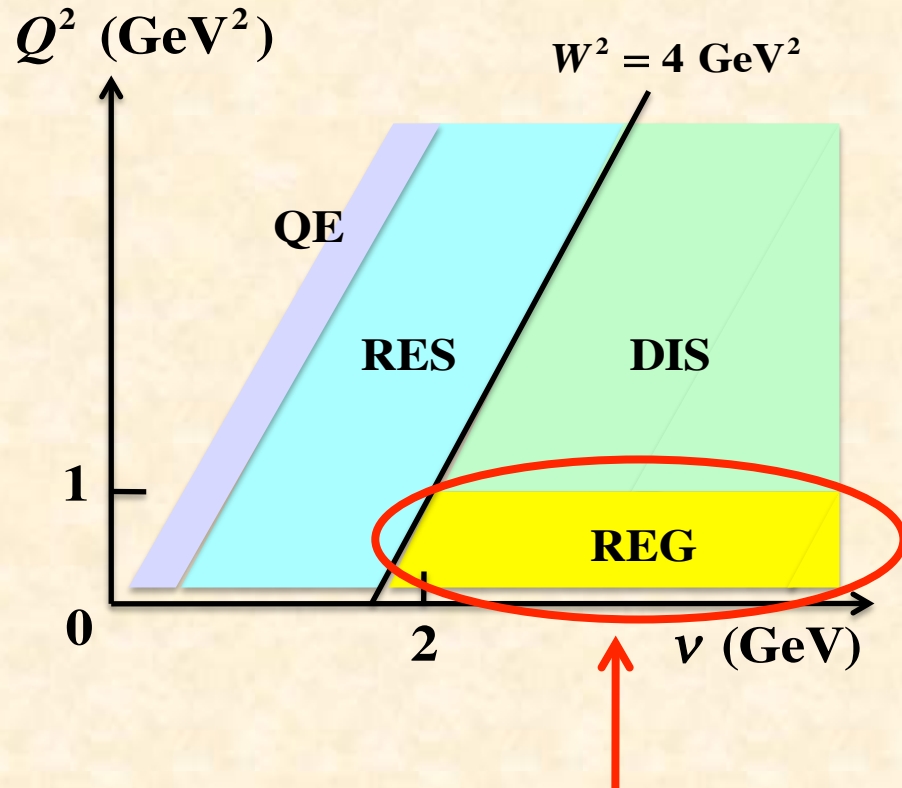
N. Kalantarians, E. Christy, and C. Keppel,
arXiv:1706.02002

According to this analysis, both structure functions
are same except for the small- x region ($x < 0.05$).



Small Q^2 region

$Q^2 \rightarrow 0$ region: Theoretical background



References:

A. Donnachie and P. V. Landshoff, Z. Phys. C 61 (1994) 139
 B. Z. Kopeliovich, Nucl. Phys. B 139 (2005) 219;
 S. A. Kulagin and R. Petti, Phys. Rev. D 76 (2007) 094023.

$$F_{T,L} = \frac{\gamma}{\pi} Q^2 \sigma_{T,L}, \quad \gamma = \frac{|\vec{q}|}{q_0} = \sqrt{1 + \frac{Q^2}{\nu^2}}$$

$\sigma_{T,L}$ = Total ν cross section

$$\sim \sum_f (2\pi)^4 \delta(p + q - p_f) \left| \langle f | \epsilon_{T,L} \cdot J(0) | p \rangle \right|^2$$

$F_{T,L}$ = transverse, longitudinal cross section

Vector current conservation: $q_\mu W^{\mu\nu} = 0$

$$\Rightarrow F_L^V \sim Q^2 F_T^V \text{ as } Q^2 \rightarrow 0$$

PCAC (Partially Conserved Axial-vector Current):

$$\partial_\mu A^\mu(x) = f_\pi m_\pi^2 \pi(x), \quad A^\mu = \text{Axial-vector current,}$$

f_π = Pion-decay constant, π = Pion field

$$\Rightarrow F_L^A \sim \frac{f_\pi^2}{\pi} \sigma_\pi \text{ as } Q^2 \rightarrow 0,$$

Pion-scattering cross section: σ_π

$Q^2 \rightarrow 0$ region: Practical descriptions in ν reactions

$$F_{1,2,3}^{\nu A}(x, Q^2 \rightarrow 0)$$

(1) **FLUKA**, G. Battistoni *et al.*,

Acta Phys. Pol. B 40 (2009) 2431

$$F_{2,3}(x, Q^2) = \frac{2Q^2}{Q_0^2 + Q^2} F_{2,3}(x, Q_0^2)$$

(2) **A. Bodek and U.-K. Yang**, arXiv:1011.6592

charged-lepton:

$$F_2^{e/\mu}(x, Q^2 < 0.8 \text{ GeV}^2) = K_{valence}^{vector}(Q^2) F_{2,LO}^{valence}(\xi_w, Q^2 = 0.8 \text{ GeV}^2) \\ + K_{sea}^{vector}(Q^2) F_{2,LO}^{sea}(\xi_w, Q^2 = 0.8 \text{ GeV}^2)$$

$$K_{valence}^{vector}(Q^2) = \frac{Q^2}{Q^2 + C_s}, \quad K_{sea}^{vector}(Q^2) = [1 - G_D^2(Q^2)] \frac{Q^2 + C_{v2}}{Q^2 + C_{v1}}$$

$$G_D(Q^2) = \frac{1}{(1 + Q^2 / 0.71)^2}, \quad \xi_w = \frac{2x(Q^2 + M_f^2 + B)}{Q^2 [1 + \sqrt{1 + 4M^2 x^2 / Q^2}] + 2Ax}$$

neutrino:

Separate $F_i^\nu(x, Q^2)$ into vector and axial-vector parts.

$F_i^\nu(x, Q^2)_{\text{vector}} \rightarrow Q^2 \rightarrow 0$ ($Q^2 \rightarrow 0$) as the charged-lepton case.

$F_i^\nu(x, Q^2)_{\text{axial-vector}} \neq 0$ ($Q^2 \rightarrow 0$) due to PCAC.

Actual expressions are slightly complicated (see the original paper).

Summary on duality:

W. Melnitchouk, R. Ent, C. E. Keppel,
Phys. Rept. 406 (2005) 127.

Introduction to the Regge theory

Scattering amplitude for $a + b \rightarrow c + d$ expanded by t -channel partial wave amplitudes $a_l(t)$

$$A_{ab \rightarrow cd}(s, t) = \sum_{l=0}^{\infty} (2l+1) a_l(t) P_l(1 + 2s/t), \quad \cos \theta_t = 1 + 2s/t, \quad P_l: \text{Legendre polynomials}$$

If it is dominated by a single resonance with mass m_J and spin J [note $P_l(z) \sim z^l$]

$$A_{ab \rightarrow cd}(s, t) \sim \frac{1}{t - m_J^2} \left(\frac{2s}{t} \right)^J$$

Let us assume that the angular momentum l is mathematically a complex variable.

Then, it is expressed by a complex integral s

$$A_{ab \rightarrow cd}(s, t) = \frac{1}{2i} \oint_C dl (2l+1) \frac{a(l, t)}{\sin(\pi l)} P(l, 1 + 2s/t).$$

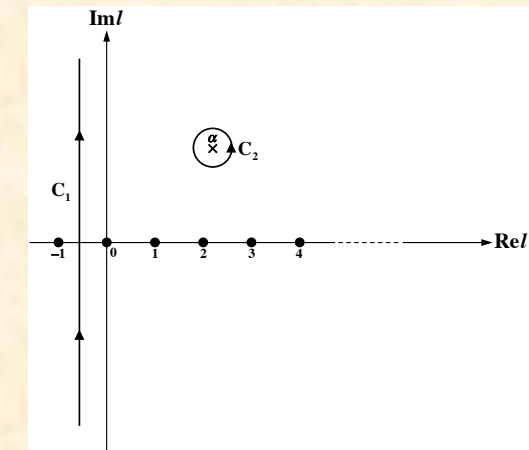
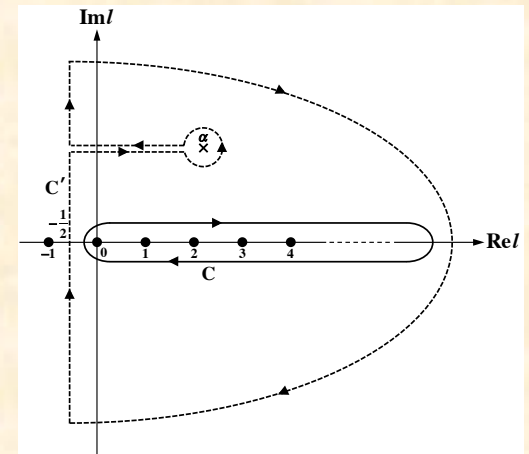
If there were one pole at $l = \alpha$ with $a(l, t) = \frac{\tilde{\beta}(t)}{1 - \alpha(t)}$, we have

$$A_{ab \rightarrow cd}(s, t) = \{2\alpha(t) + 1\} \frac{\pi \tilde{\beta}(t)}{\sin\{\pi\alpha(t)\}} P(\alpha(t), 1 + 2s/t).$$

In the high-energy limit $s \gg |t|$, $P(l, z) \sim z^l$ and the amplitude becomes

$$A_{ab \rightarrow cd}(s, t) \sim \beta(t) \left(\frac{s}{s_0} \right)^{\alpha(t)} \Leftrightarrow A_{ab \rightarrow cd}(s, t) \sim \frac{1}{t - m_J^2} \left(\frac{2s}{t} \right)^J.$$

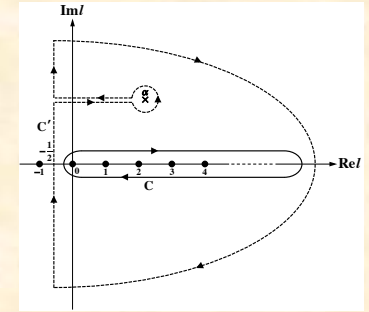
The scattering amplitude is expressed by the pole, which is called the Regge pole.



In the high-energy limit $s \gg |t|$, $P(l, z) \sim z^l$ and the amplitude becomes

$$A_{ab \rightarrow cd}(s, t) \sim \beta(t) \left(\frac{s}{s_0} \right)^{\alpha(t)} \Leftrightarrow A_{ab \rightarrow cd}(s, t) \sim \frac{1}{t - m_J^2} \left(\frac{2s}{t} \right)^J.$$

The scattering amplitude is expressed by the pole, which is called the Regge pole, with the mass $t = m_J^2$ and $J = \alpha(t = m_J^2)$.



Experimental data indicate

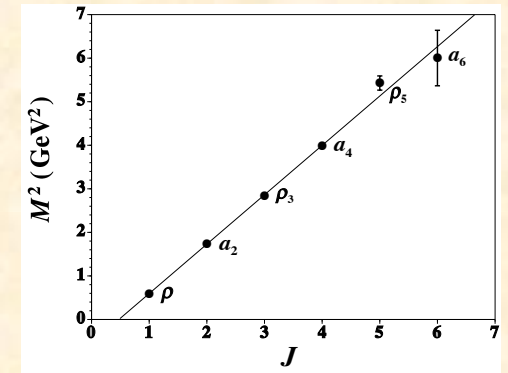
$$\alpha(t) = \alpha(0) + \alpha' t.$$

The total cross section is expressed by the forward elastic scattering (optical theorem),

$$\sigma_{tot} \propto s^{\alpha(0)-1}.$$

For the ρ - a_2 (ω - f_2) trajectory with $I = 1$ ($I = 0$), $C = \text{odd}$, and natural parity [$J = (-1)^J$], $\alpha(0) = 0.55$.

Other trajectories have smaller $\alpha(0)$.



Experimental measurements of pp and $p\bar{p}$ did not agree with

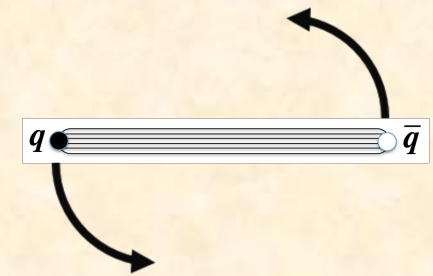
$$\sigma_{tot} \propto s^{0.55-1}.$$

\Rightarrow We need to introduce the Pomeron ($I = 0$, $C = \text{even}$: vacuum quantum number) with $\alpha(0) - 1 \approx 0$:

$$\alpha_{tot} = C_P s^{\alpha_P(0)-1} + C_R s^{\alpha_R(0)-1}.$$

$$\Rightarrow F_2 \sim C_P x^{1-\alpha_P(0)} + C_R x^{1-\alpha_R(0)}$$

$$\Rightarrow \text{Typical functional form: } F_2 = c_P x^{a_P} (1-x)^{b_P} + c_R x^{a_R} (1-x)^{b_R}$$



$$\begin{aligned} H &= p + \sigma r^n = J / r + \sigma r^n, & J &= pr \\ \frac{dH}{dr} &= -\frac{J}{r^2} + n\sigma r^{n-1} = 0 & \Rightarrow J &= n\sigma r^{n+1} \\ M &= n\sigma r^n + \sigma r^n = (n+1)\sigma r^n = (n+1)\sigma \left(\frac{J}{n\sigma} \right)^{n/(n+1)} \\ M^2(n=1) &= 4\sigma J, & \text{consistent with linear confining potential} \end{aligned}$$

Our analysis method

- There are accurate structure-function (or PDF) code in the DIS region for both nucleon and nuclei: $F_2(x, Q^2)$ at $Q^2 \geq Q_0^2 = 1 \sim 2 \text{ GeV}^2$, $W^2 \geq W_0^2 \sim 4 \text{ GeV}^2$.
- We use a DIS code and extrapolate it to the Regge region. So far, our analysis is on charged-lepton F_2 .

$$F_2(x, Q^2) = w(x, Q^2; x_0, Q_0^2) F_2(x_0, Q_0^2), \quad w(x, Q^2; x_0, Q_0^2) = \frac{F_2^{\text{REG}}(x, Q^2)}{F_2^{\text{REG}}(x_0, Q_0^2)}$$

$F_2^{\text{REG}}(x, Q^2) = \text{structure function valid in the Regge region.}$

- We parametrize $F_2^{\text{REG}}(x, Q^2)$ based on the Regge + Pomeron picture.

References for the nucleon:

A. Donnachie and P. V. Landshoff, ZP C61 (1994) 139; PLB 518 (2001) 63;

H. Abramowicz, E. Levin, A. Levy, U. Maor (ALLM), PLB 269 (1991) 465; hep-ph/9712415;

I. Abt, A. M. Cooper-Sarkar, B. Foster, V. Myronenko, K. Wichmann, M. Wing, PRD 94, 034032 (2016).

$$F_2^{\text{REG}}(x, Q^2) = \frac{Q^2}{m_0^2 + Q^2} [F_2^P(x, Q^2) + F_2^R(x, Q^2)], \quad F_2^V(x, Q^2) = c_V x_V^{a_V(t)} (1-x)^{b_V(t)}, \quad V = P, R$$

$$t = \ln \left[\frac{\ln\{(Q^2 + \mu_0^2) / \Lambda^2\}}{\ln(\mu_0^2 / \Lambda^2)} \right], \quad \frac{1}{x_V} = 1 + \frac{W^2 - M_N^2}{Q^2 + m_V^2}$$

$$\text{ALLM: } d = d_1 + d_2 \frac{1}{1+t^{d_3}} \quad (d = a_P, c_P), \quad e = e_1 + e_2 t^{e_3} \quad (d = b_P, a_R, b_R, c_R)$$

Introduce m_0^2 so as to have $F_2^{\text{REG}}(x, Q^2) \sim Q^2$ at $Q^2 \rightarrow 0$.

Note $x = \frac{Q^2}{2M_{N,V}} \rightarrow 0$ as $Q^2 \rightarrow 0$, whereas there is x_{\min} in practical structure-function (PDF) codes.

$F_2^{\text{DIS}}(x \rightarrow 0, Q^2 \rightarrow 0)$ cannot be calculated.

\Rightarrow We may modify $x \rightarrow x_V$.

$$F_2(x, Q^2) = w(x, Q^2; x_0, Q_0^2) F_2(x_0, Q_0^2), \quad w(x, Q^2; x_0, Q_0^2) = \frac{F_2^{\text{REG}}(x, Q^2)}{F_2^{\text{REG}}(x_0, Q_0^2)}$$

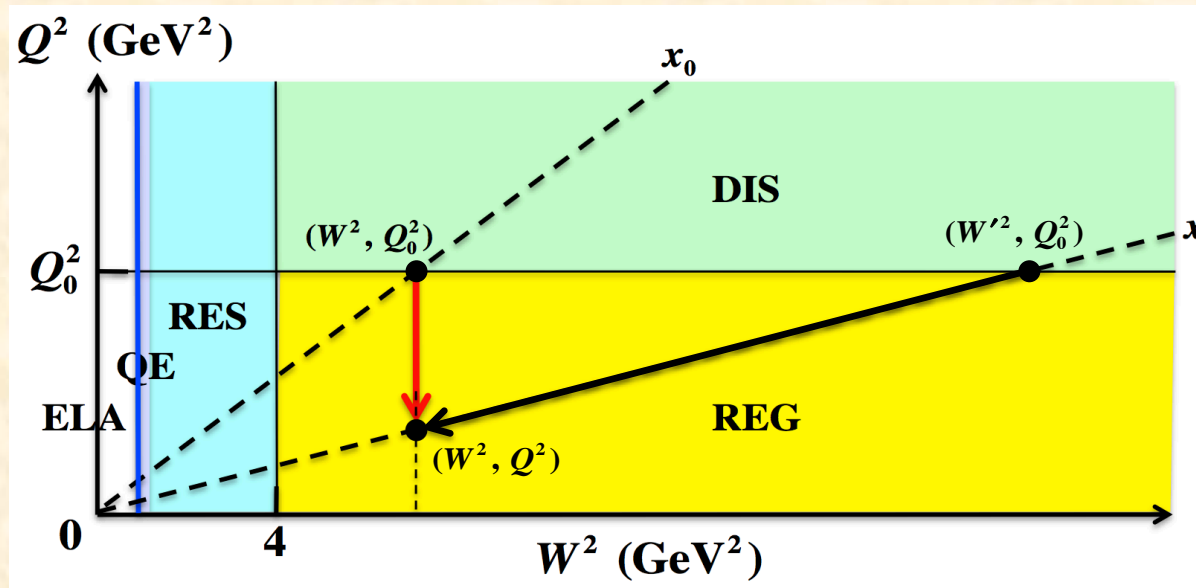
$$F_2^{\text{REG}}(x, Q^2) = \frac{Q^2}{m_0^2 + Q^2} [F_2^P(x, Q^2) + F_2^R(x, Q^2)], \quad F_2^V(x, Q^2) = c_V x_V^{a_V(t)} (1-x)^{b_V(t)}, \quad V = P, R$$

If we use $F_2(x, Q^2) = w(Q^2; Q_0^2) F_2(x, Q_0^2)$, the extrapolation path is long and there is a limit x_{\min} .

The extrapolation function w depends on x according to experimental measurements.

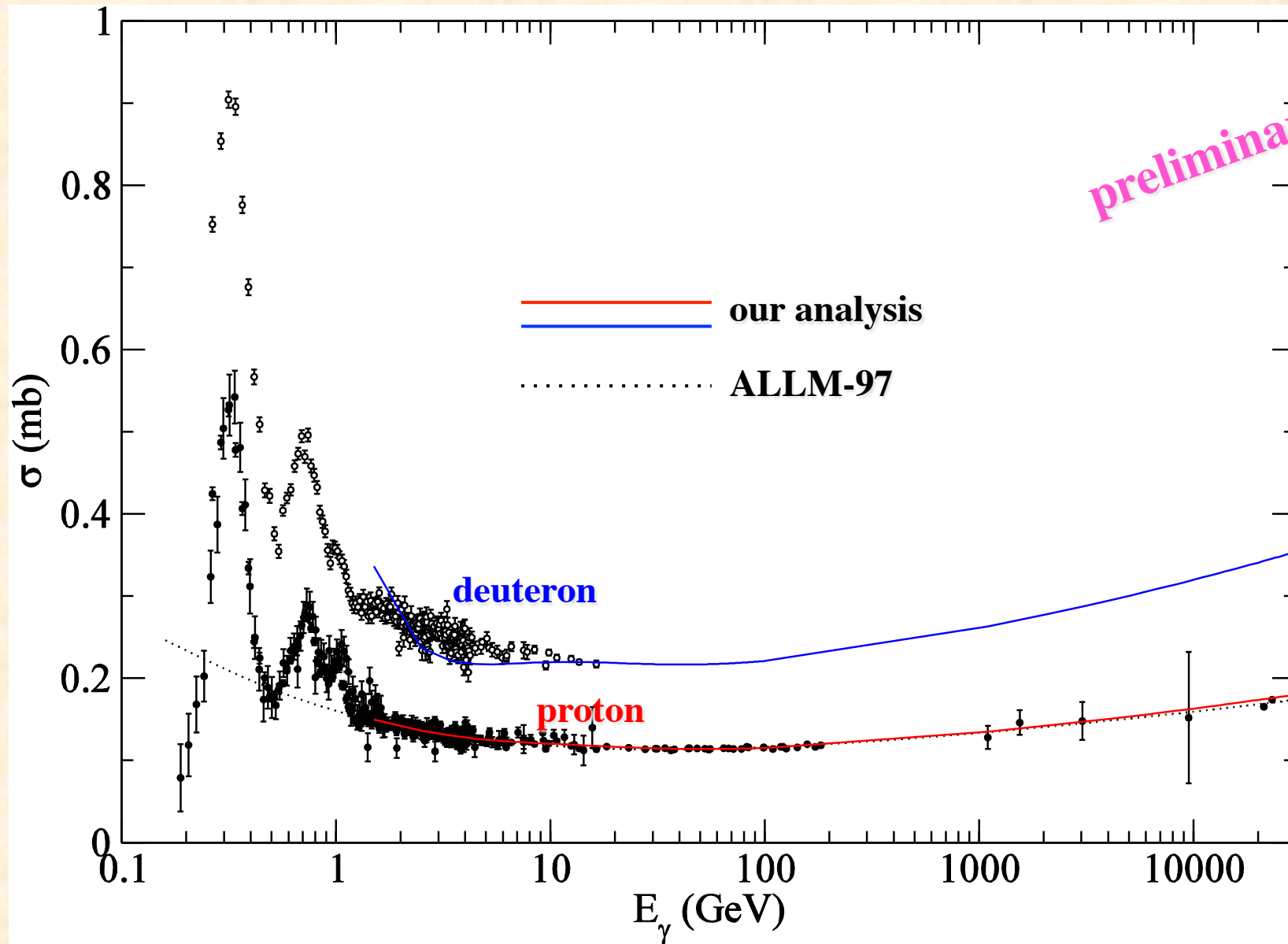
$$W^2 = (p+q)^2 = M_N^2 - Q^2 + 2p \cdot q = M_N^2 - Q^2 + \frac{Q^2}{x}$$

$$\Rightarrow \frac{1}{x} = 1 + \frac{W^2 - M_N^2}{Q^2} \quad \Rightarrow \frac{1}{x_0} \equiv 1 + \frac{W^2 - M_N^2}{Q_0^2}$$



Experiment / Publication	Year
SLAC	1992
Fermilab-E665	1996
NMC	1997
H1-ZEUS	2010
HERMES	2011
JLab-C	2015
PDG2016- γp	2016

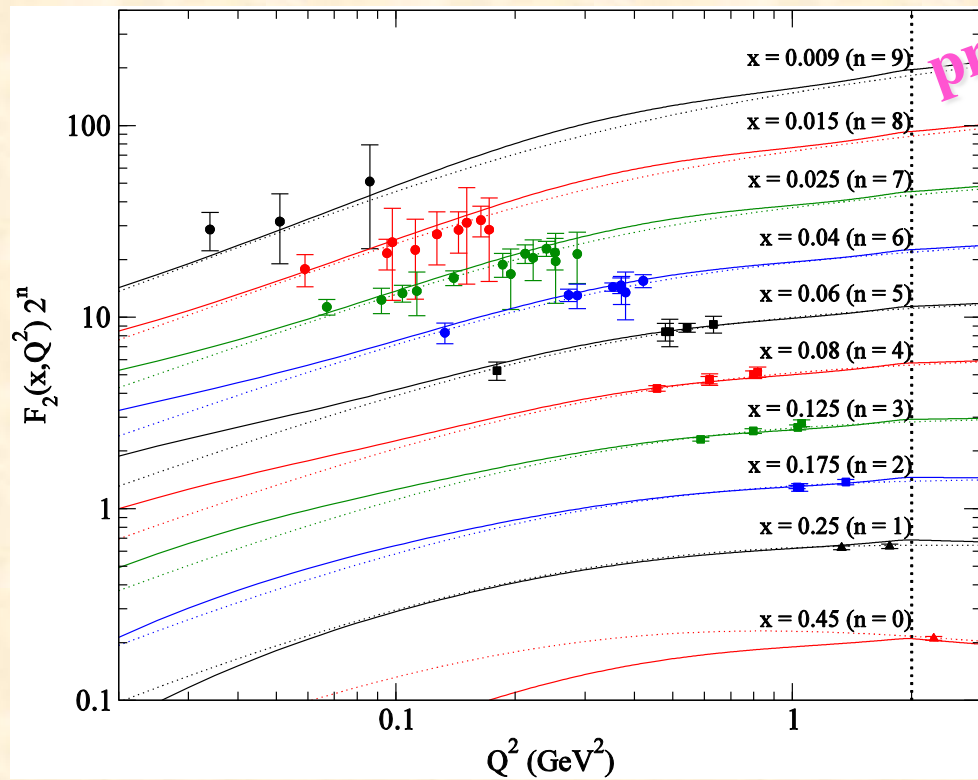
Photoproduction ($\gamma + p, d \rightarrow X$) cross sections



Comparison with JLab/SLAC data

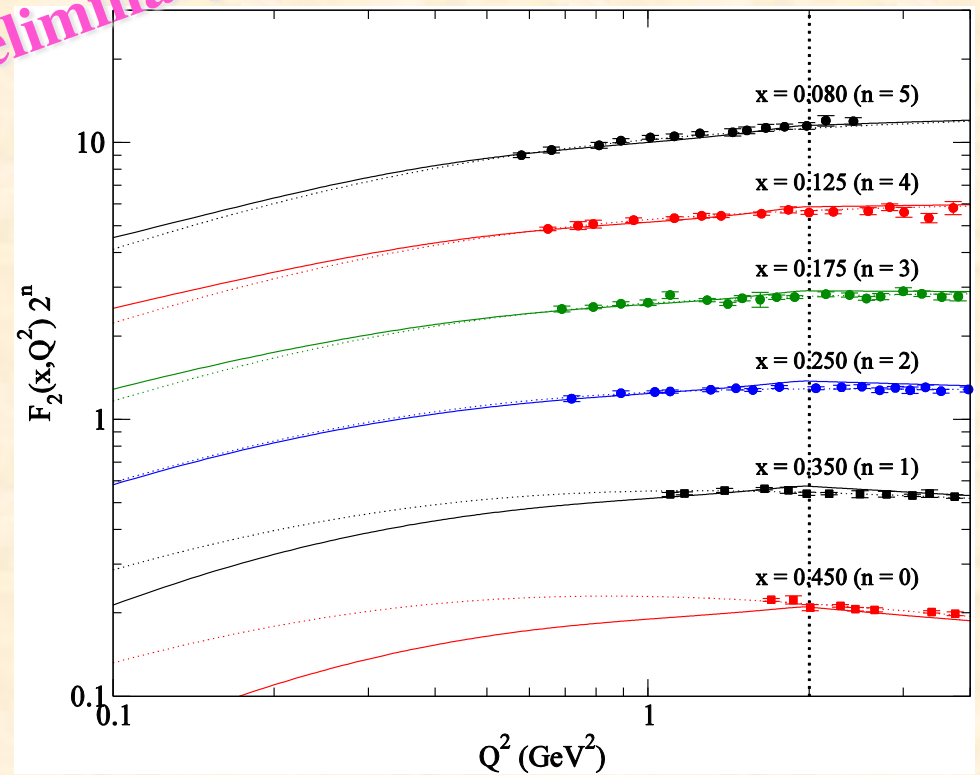
—— our analysis
..... ALLM-97

JLab



preliminary

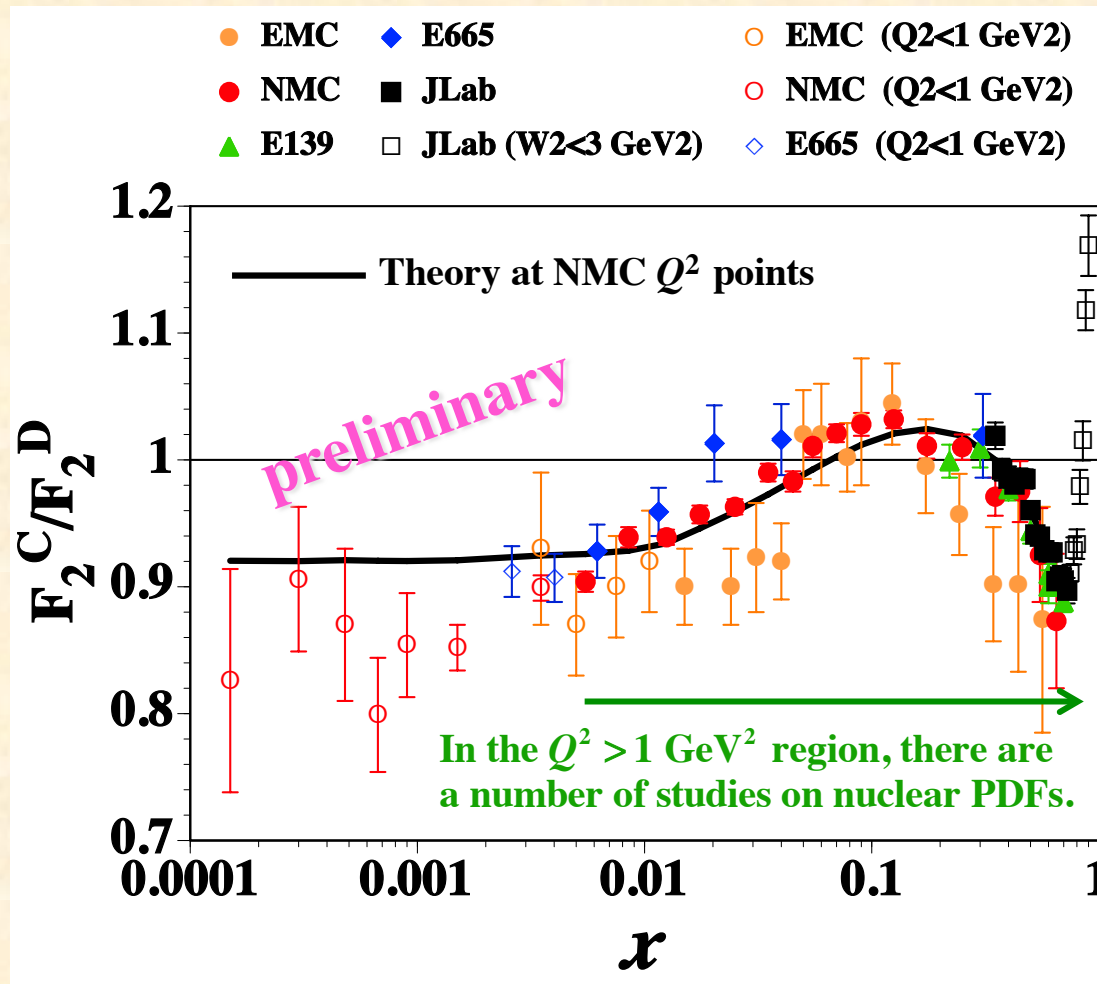
SLAC



Comparison with nuclear data on F_2^A / F_2^D

$$F_2^A(x, Q^2) = w(x, Q^2; x_0, Q_0^2) F_2^A(x_0, Q_0^2)$$

At this stage, we use $w(x, Q^2; x_0, Q_0^2)$ obtained for the nucleon, so that nuclear modifications are constrained only in $F_2^A(x_0, Q_0^2)$.



Nuclear $F_2^A(x_0, Q_0^2)$ from
M. Hirai, S. Kumano, T.-H. Nagai,
Phys. Rev. C 76 (2007) 065207.

Summary

- **Lepton-nucleus cross sections should be understood in the wide kinematical regions for neutrino experiments.**
Especially, the order of **5%** accuracy is needed for future oscillation measurements.
 - **There are significant studies in the quasi-elastic, resonance, and DIS regions *separately*.**
→ It is desirable to have a unified code for calculating the cross sections.
 - **The Regge region ($W^2 \geq 4 \text{ GeV}^2$, $Q^2 < 1 \text{ GeV}^2$) is not well investigated.**
We tried to extrapolate the charged-lepton DIS structure functions to the Regge region by a global analysis of existing experimental data with $W^2 \geq 4 \text{ GeV}^2$ and $Q^2 < 1 \text{ GeV}^2$.
→ **Need further efforts on elaboration.**
→ **Nuclear modifications.**
→ **Neutrino-nucleus structure functions.**
- Our studies will be submitted for publication in the near future.**

The End

The End