Single Pion Production
in
Neutrino-Nucleon Interaction

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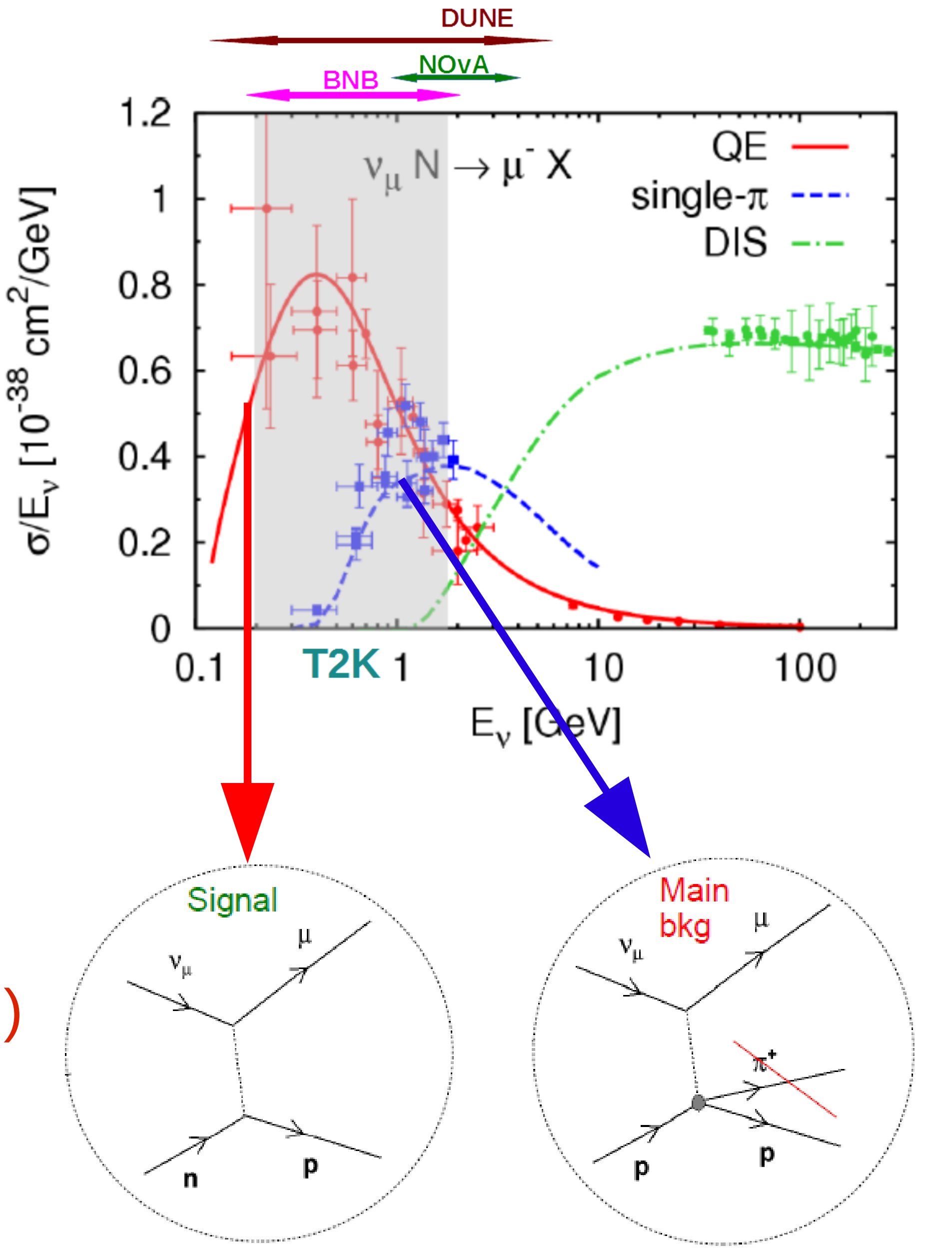
Why Single Pion Production Model?

- Single pion production has significant contribution at intermediate energy.
- Good model is very crucial for precision neutrino oscillation measurements.
- Main goal is to develop a single pion production model suitable for Neutrino generators (NEUT)

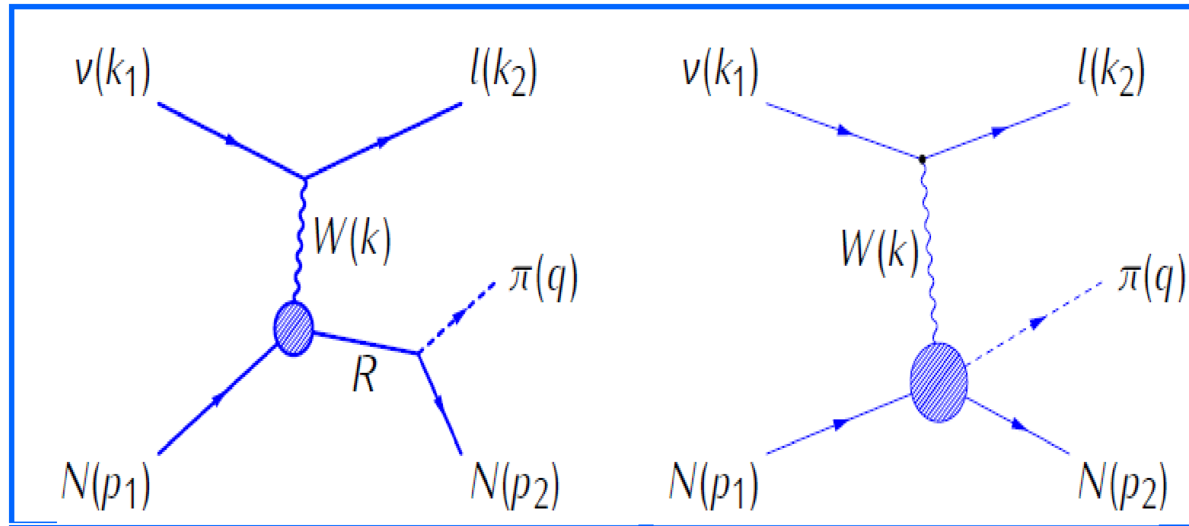


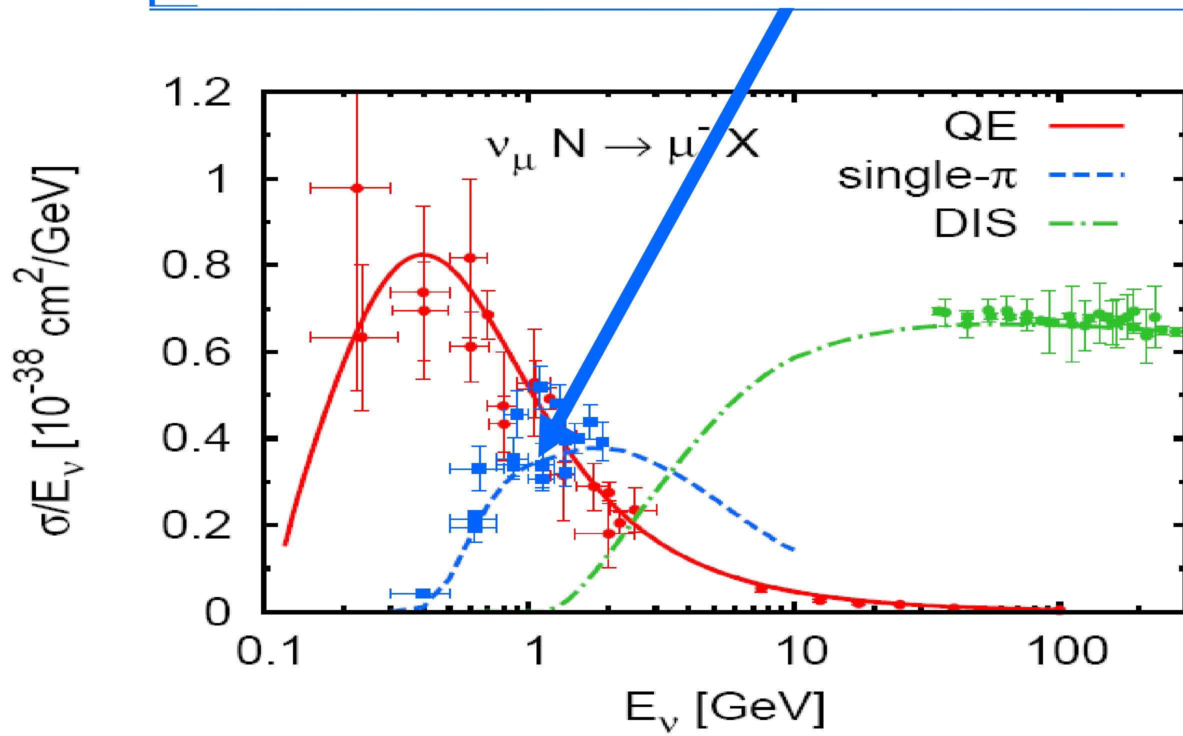
> Fast enough
> Precise enough (with no assumption)
> Full kinematics
> Valid up to W=2 GeV (for NEUT)

Charged Current Single-Pion Production

$$\nu p \rightarrow \mu^- p \pi^+$$
$$\nu n \rightarrow \mu^- p \pi^0$$
$$\nu n \rightarrow \mu^- n \pi^+$$

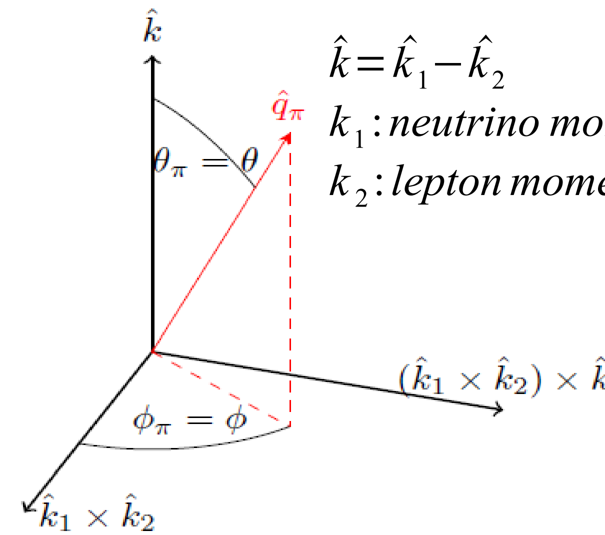
$$\bar{\nu} n \rightarrow \mu^+ n \pi^-$$
$$\bar{\nu} p \rightarrow \mu^+ n \pi^0$$
$$\bar{\nu} p \rightarrow \mu^+ p \pi^-$$





Rein Model

- Define a suitable framework; Adler frame
- Calculate both resonant and non-resonant interactions
- Add them coherently to include the interference effects



$\hat{k} = \hat{k}_1 - \hat{k}_2$

k_1: *neutrino momentum*

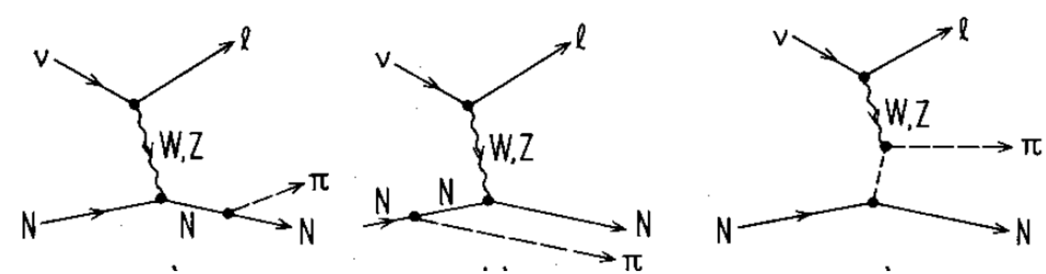
k_2: *lepton momentun*

➔Resonant Interaction

- **Rein-Sehgal** model is based on **helicity amplitudes** derived in a relativistic quark model
- The helicity amplitudes depend on the spin projection of the initial and final particle.

➔Nonresonant Interaction

Born graphs based on linear sigma model.



➔The lepton is assumed to be massless

D. Rein, Z.Phys. C – Particles and Fields 35,43-64 (1987)

Rein-Sehgal model

Rein-Sehgal[1] (RS) is default model in the **NEUT**

- Easy to be implemented in generators
- Valid up to W<2 GeV region
- The RS model in NEUT includes the lepton mass and Graczyk-Sobczyk[2] form-factor

- Not a full kinematic model $d\sigma/dW\,dQ^2$
  The helicity amplitudes are **not** a function of pion angles
- Pion angles is described by density matrix. NEUT **only** implemented the Δ resonance.

The RS model is improved by including the pion angles in helicity amplitudes

Output of the modified RS model

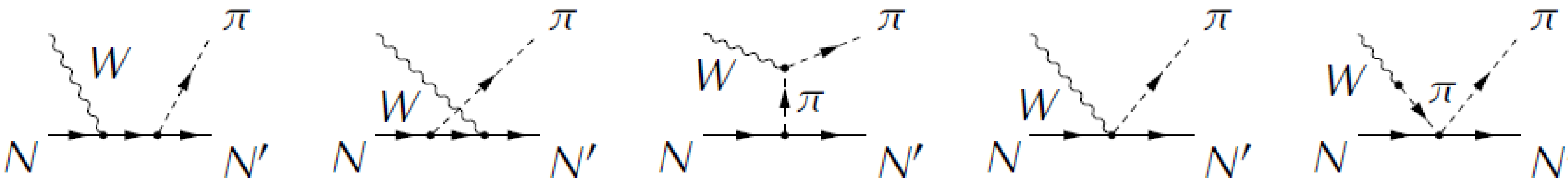
$$d\sigma/dW\,dQ^2\,d\Omega_\pi$$

Resonance	M_R	Γ_0	χ_E
$P_{33}(1232)$	1232	117	1
$P_{11}(1440)$	1430	350	0.65
$D_{13}(1520)$	1515	115	0.60
$S_{11}(1535)$	1535	150	0.45
$P_{33}(1600)$	1600	320	0.18
$S_{31}(1620)$	1630	140	0.25
$S_{11}(1650)$	1655	140	0.70
$D_{15}(1675)$	1675	150	0.40
$F_{15}(1680)$	1685	130	0.67
$D_{13}(1700)$	1700	150	0.12
$D_{33}(1700)$	1700	300	0.15
$P_{11}(1710)$	1710	100	0.12
$P_{13}(1720)$	1720	250	0.11
$F_{35}(1905)$	1880	330	0.12
$P_{31}(1910)$	1890	280	0.22
$P_{33}(1920)$	1920	260	0.12
$F_{37}(1950)$	1930	285	0.40

1. D. Rein and L. M. Sehgal, Annals Phys. 133 (1981) 79.
2. K. M. Graczyk and J. T. Sobczyk, Phys. Rev. D 77 (2008) 053001.

non-resonant background is defined by a set of diagrams determined by HNV model based on non-linear sigma model.

E. Hernandez, J. Nieves and M. Valverde, Phys. Rev. D **76** (2007) 033005

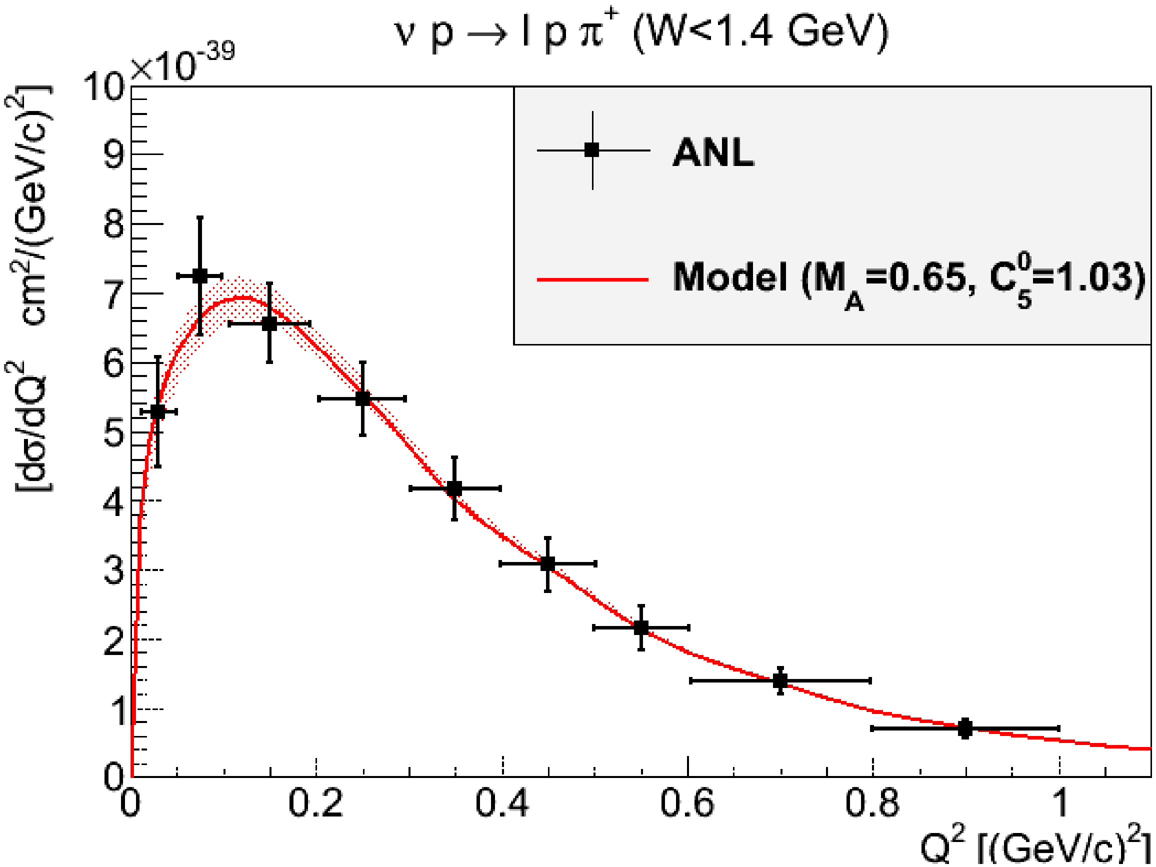


- Helicity amplitudes of above diagrams are calculated in the Adler frame.

- the model is based on chiral symmetry and it is not reliable at high energy and high W.
- A practical solution (suggested by J. Nieves) is to multiply a form-factor by the virtual pion propagator.
- To preserve the conservation of vector current we have to multiply the same form-factor to other amplitudes
- The nonresonant background has no contribution at W>1.6 GeV

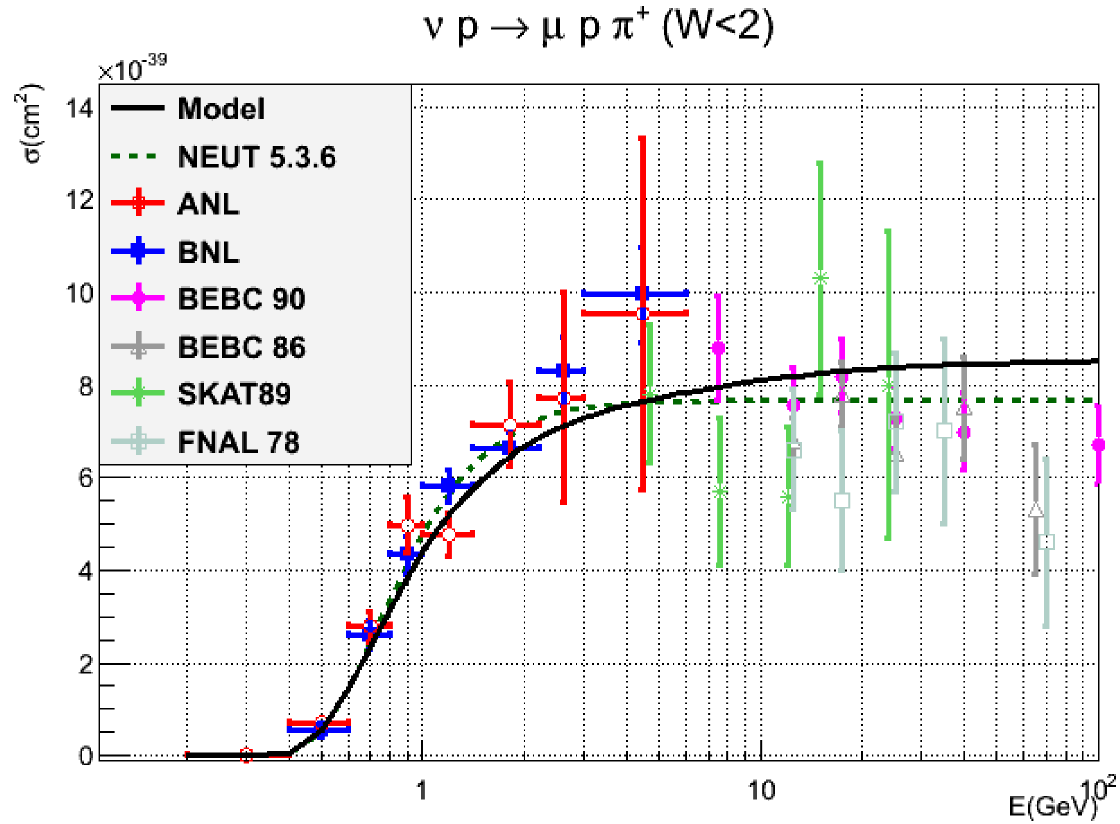
Verifying the model is difficult with limited data sets!

- Existing bubble chamber data on "free" nucleon are old and with large error and it is very unlikely to be improved :(
- We extract the dipole axial form-factor from the bubble chamber data.

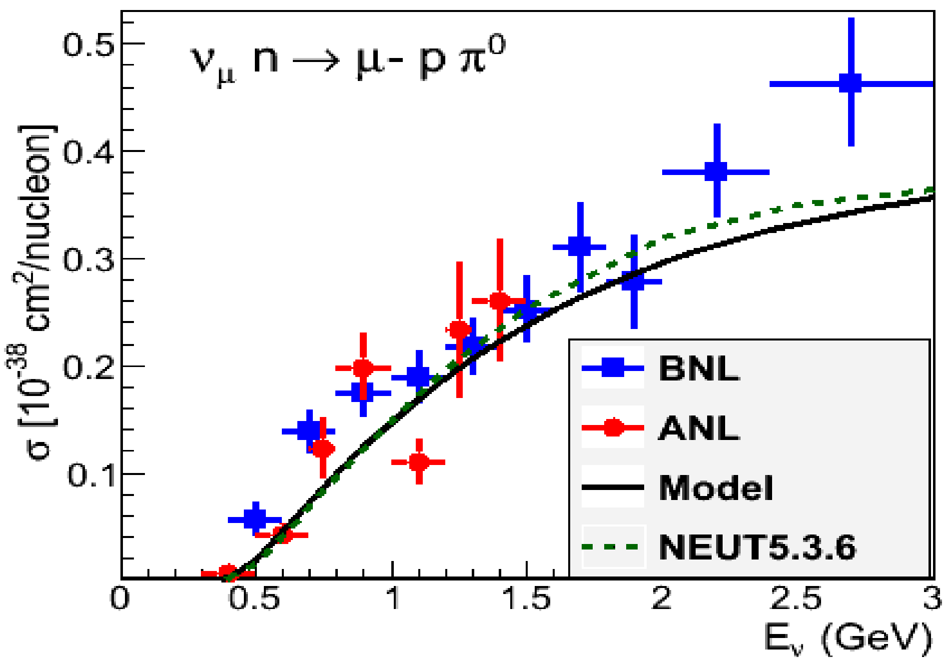


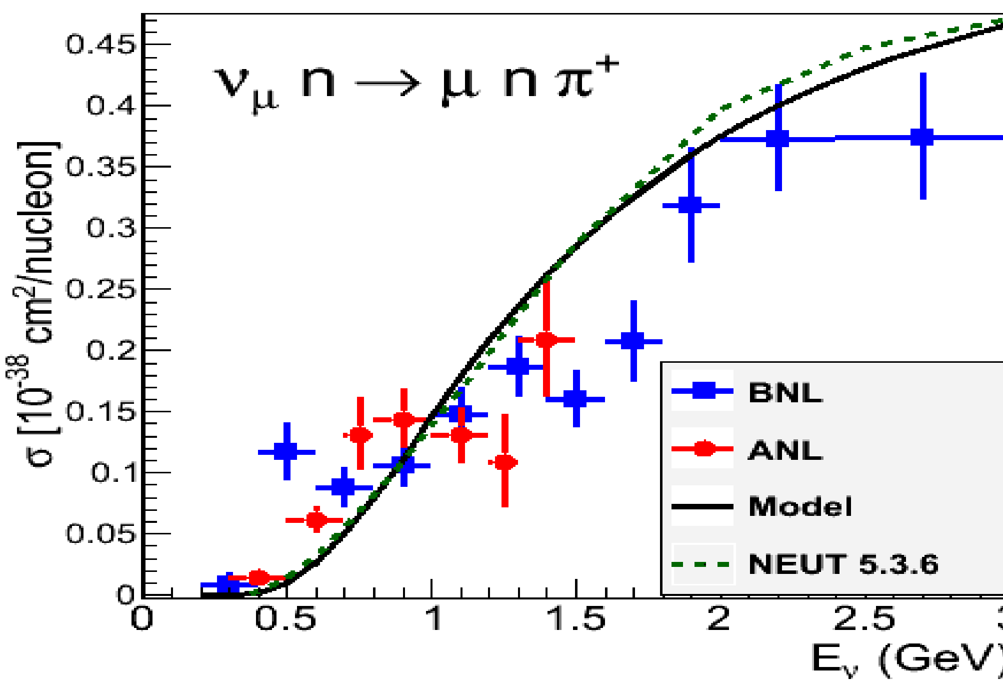
G. M. Radecky et al., Phys. Rev. D 25 (1982) 1161 [Phys. Rev. D 26 (1982) 3297].

Total Cross-section
for
charged current
neutrino interaction

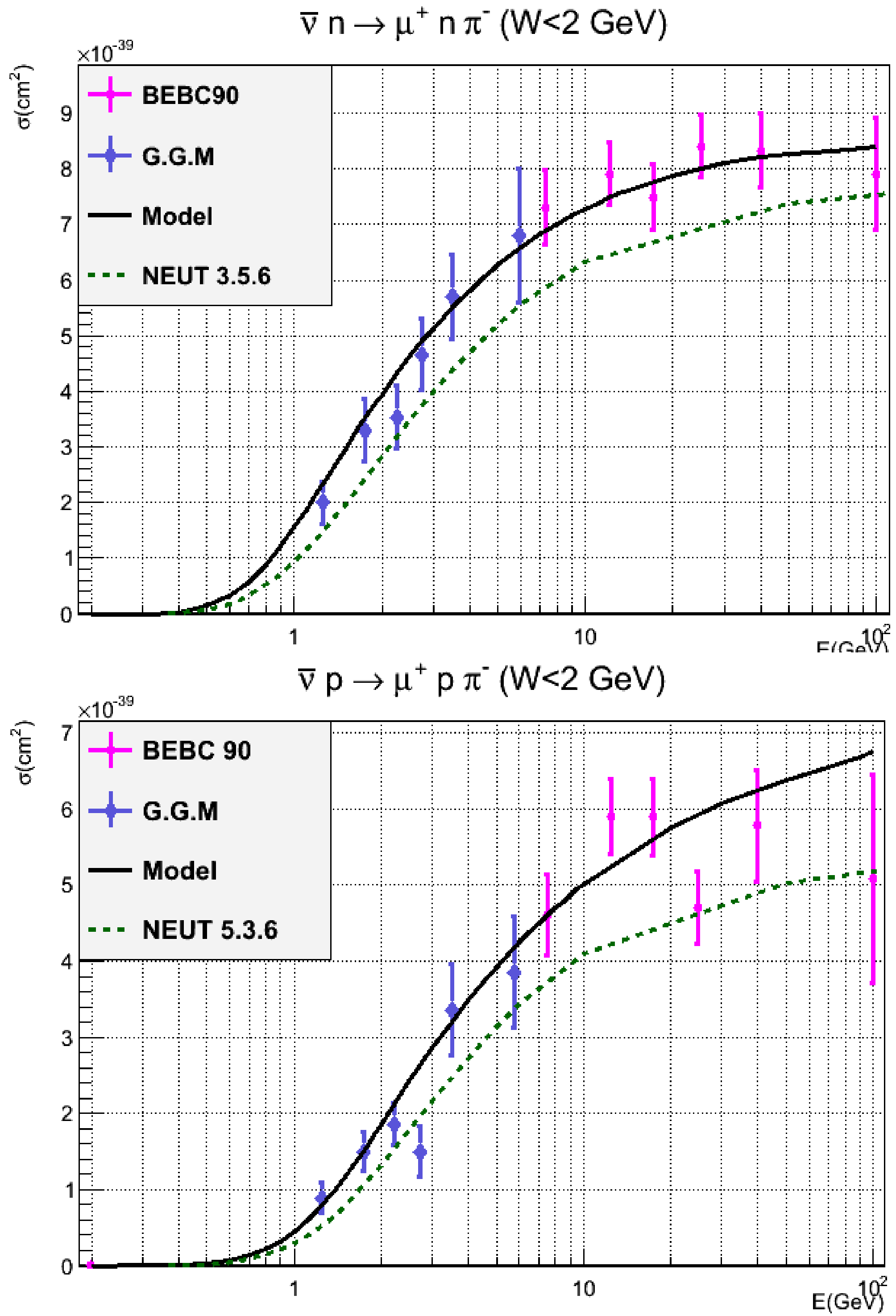


ANL & BNL data is reanalyzed by
C. Wilkinson et al., Phys. Rev. D 90 (2014) 11, 112017





Total Cross-section
for
charged current
anti-neutrino interactions

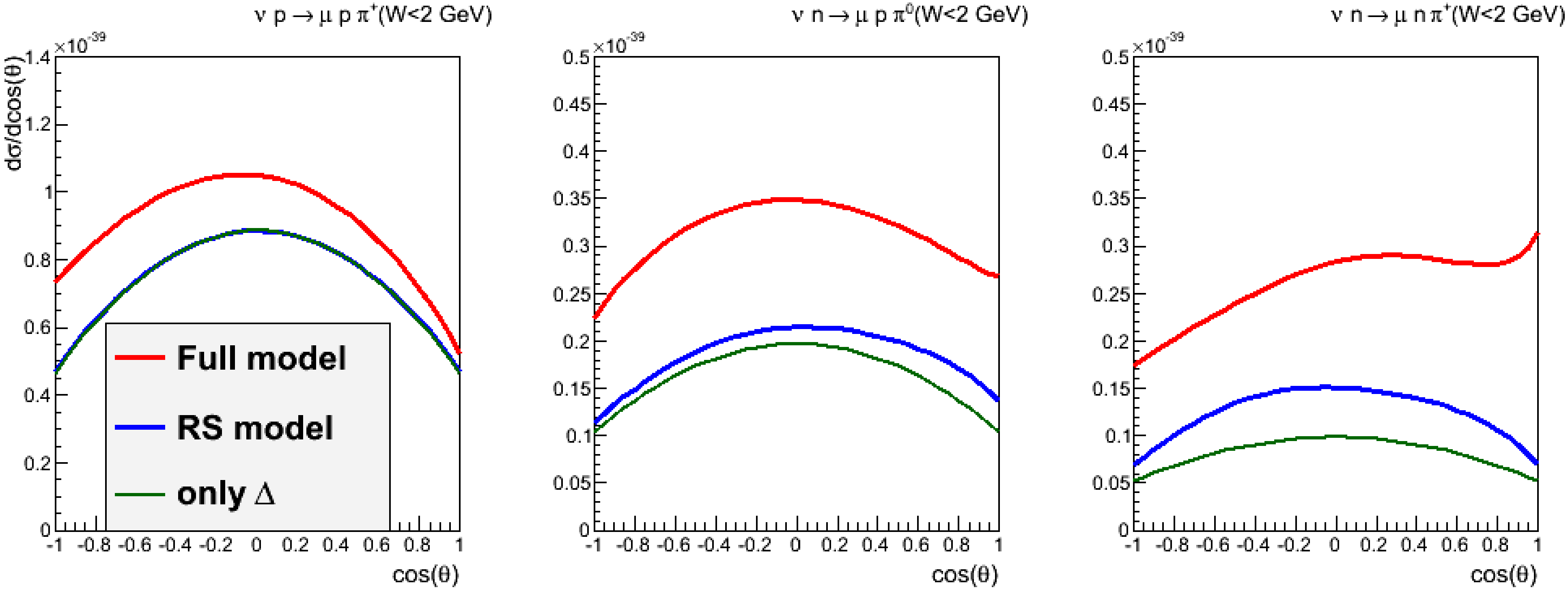


Data from

D. Allasia et al., Nucl. Phys. B 343 (1990) 285.
T. Bolognese, J. P. Engel, J. L. Guyonnet and J. L. Riester, Phys. Lett. 81B (1979) 393.

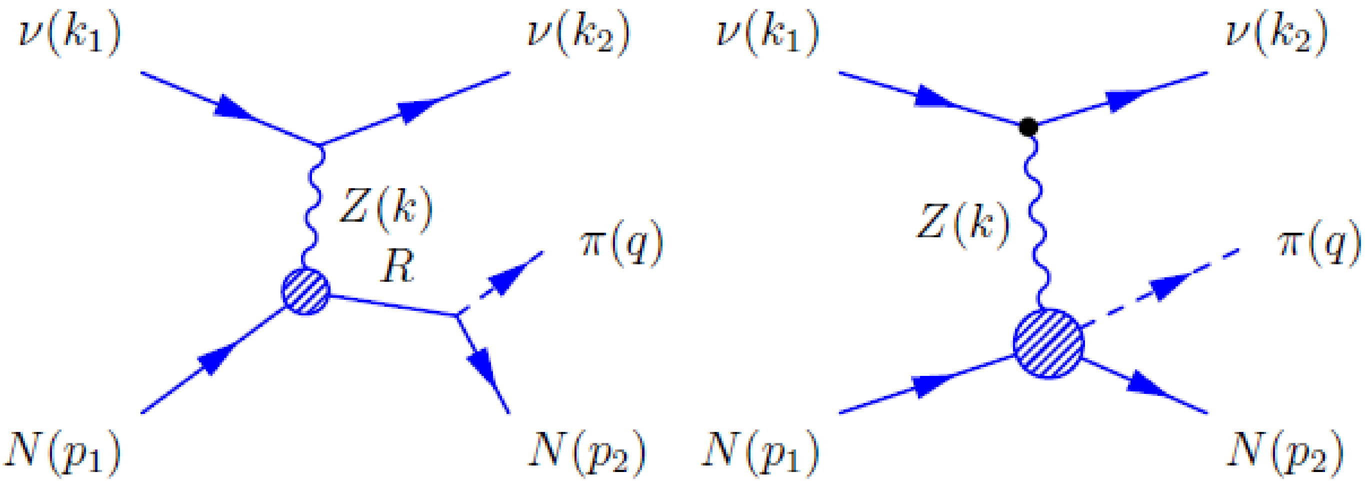
Angular Distribution In the Adler frame

For T2K energy



- "Only Δ" has a symmetric distribution.
- Any deviation from the symmetric distribution comes from the interference effects.

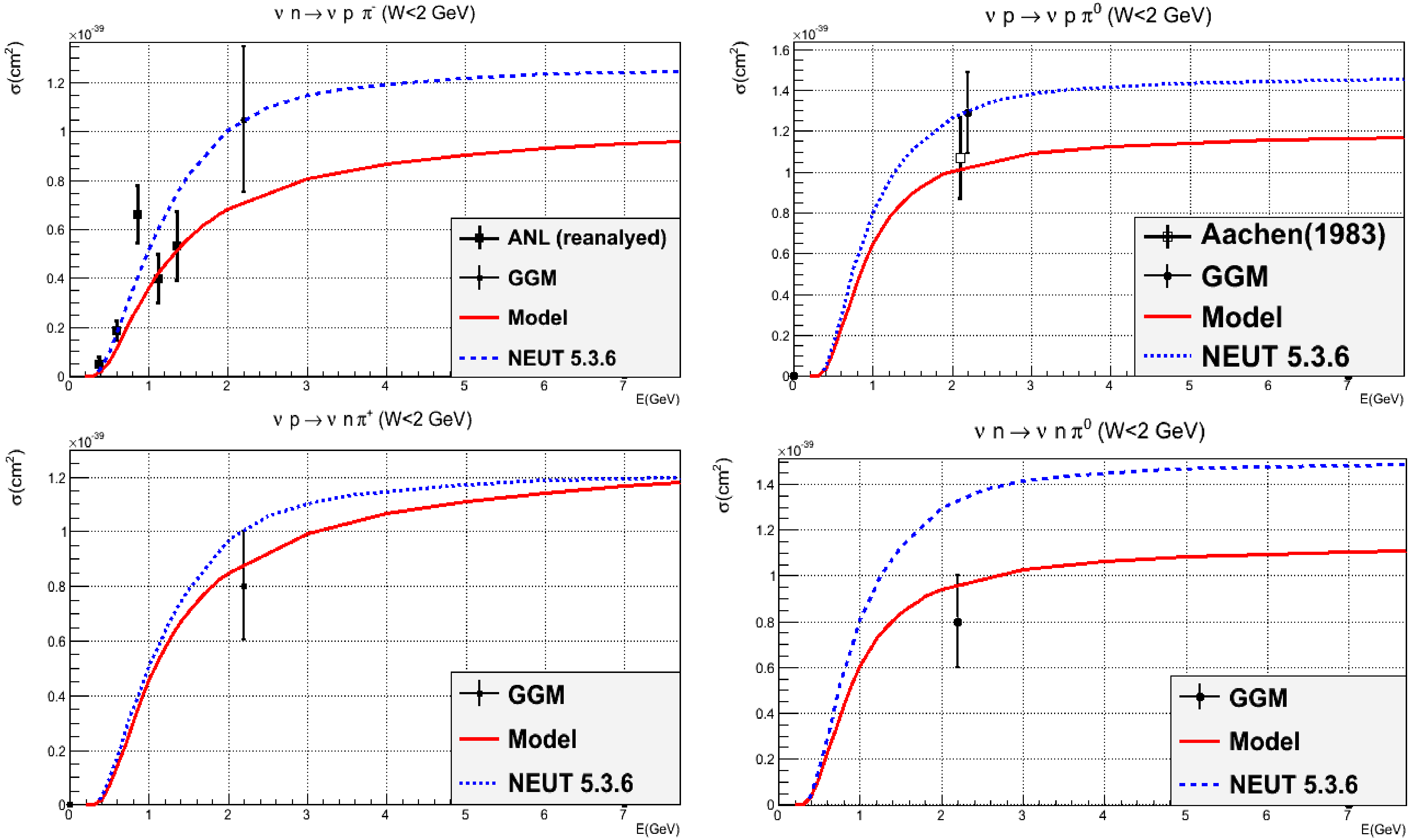
Neutral Current Single-Pion Production



$\nu p \rightarrow \nu p \pi^0$	$\bar{\nu} p \rightarrow \bar{\nu} p \pi^0$
$\nu p \rightarrow \nu n \pi^+$	$\bar{\nu} p \rightarrow \bar{\nu} n \pi^+$
$\nu n \rightarrow \nu n \pi^0$	$\bar{\nu} n \rightarrow \bar{\nu} n \pi^0$
$\nu n \rightarrow \nu p \pi^-$	$\bar{\nu} n \rightarrow \bar{\nu} p \pi^-$

Total Cross-section

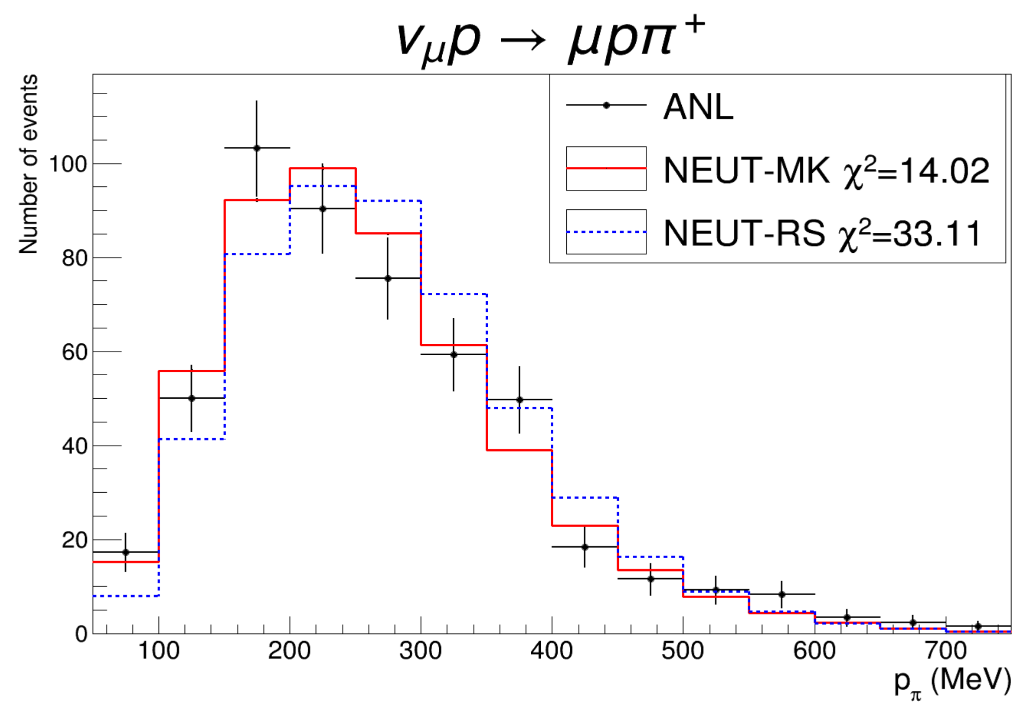
for neutral current neutrino interactions



Data from J. A. Formaggio and G. P. Zeller, Rev. Mod. Phys. 84 (2012) 1307
ANL data is reanalyzed based on C. Wilkinson et al., Phys. Rev. D 90 (2014) 11, 112017

The NEUT Implementation MK-Model

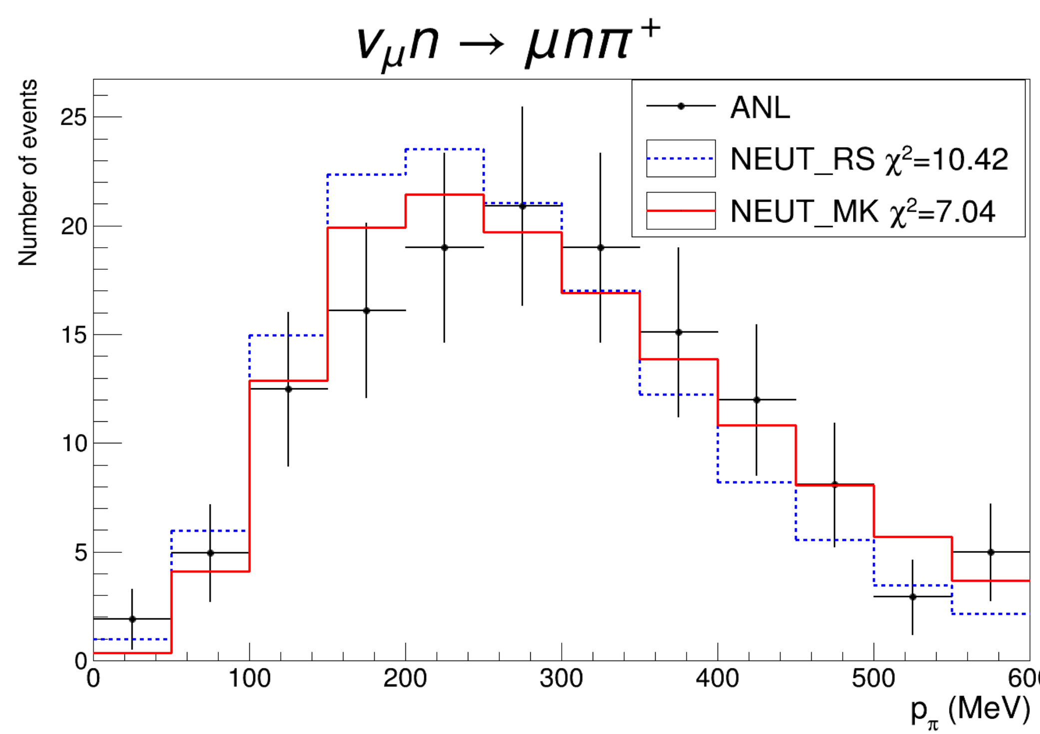
Clarence Wret and I implemented the model in NEUT



Pion momentum in the lab frame

Data comparison can be done

- In any frame
- With any target
- With any selection



Data comparison with **NUISANCE**

P. Stowell et al., JINST 12 (2017) no.01, P01016

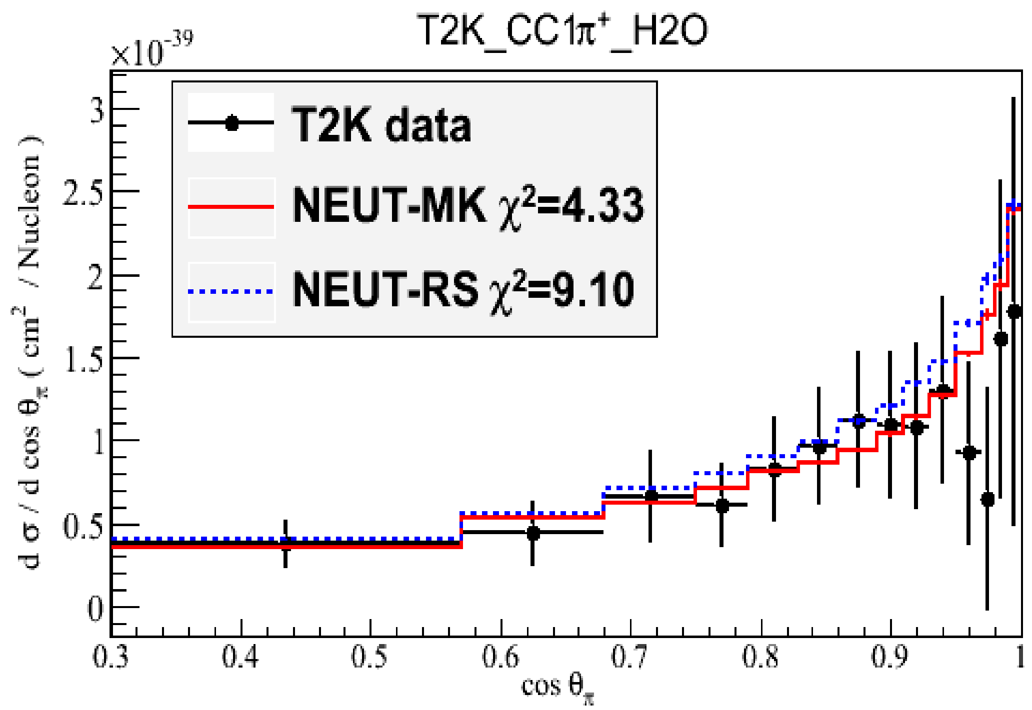
T2K data on Water

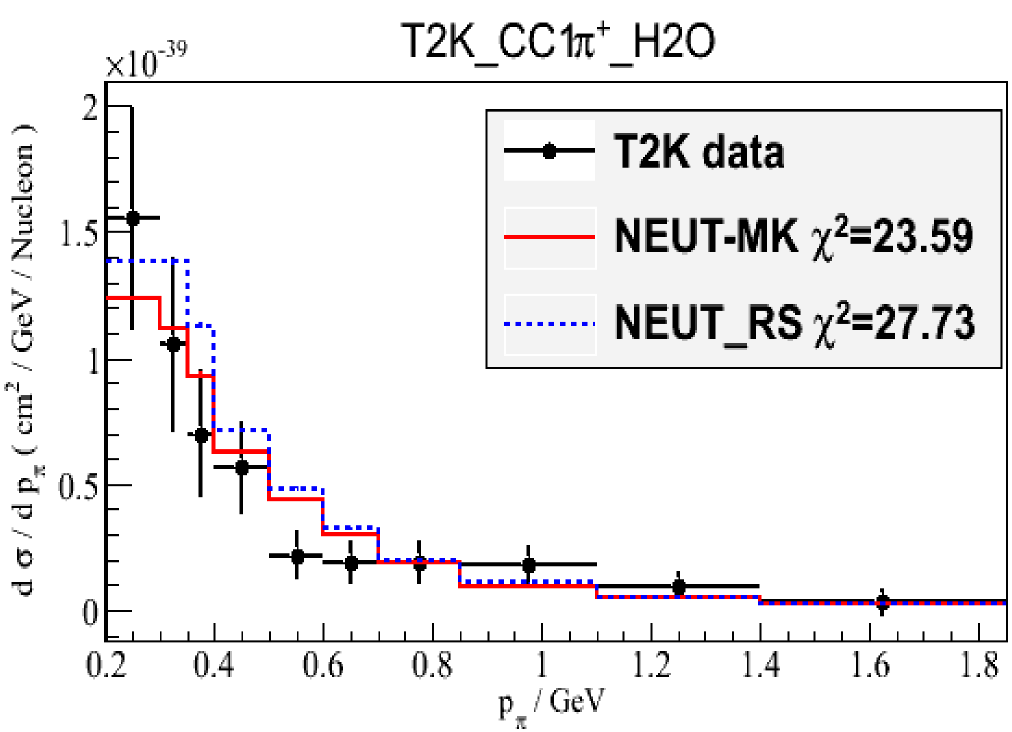
Pion kinematics

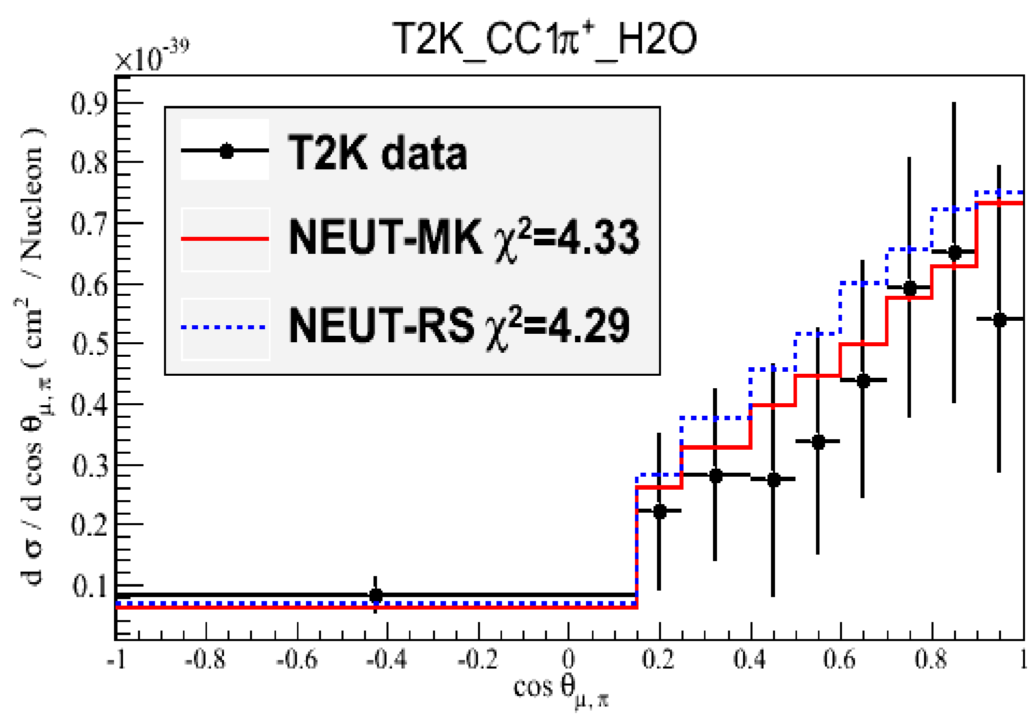
$$\nu_\mu + H_2O \rightarrow \mu^- \pi^+ + X$$

$p_\mu > 200\ MeV/c$
$p_\pi > 200\ MeV/c$
$\cos\theta_\mu > 0.3$
$\cos\theta_\pi > 0.3$

K. Abe et al. [T2K Collaboration], Phys. Rev. D 95 (2017) no.1, 012010.







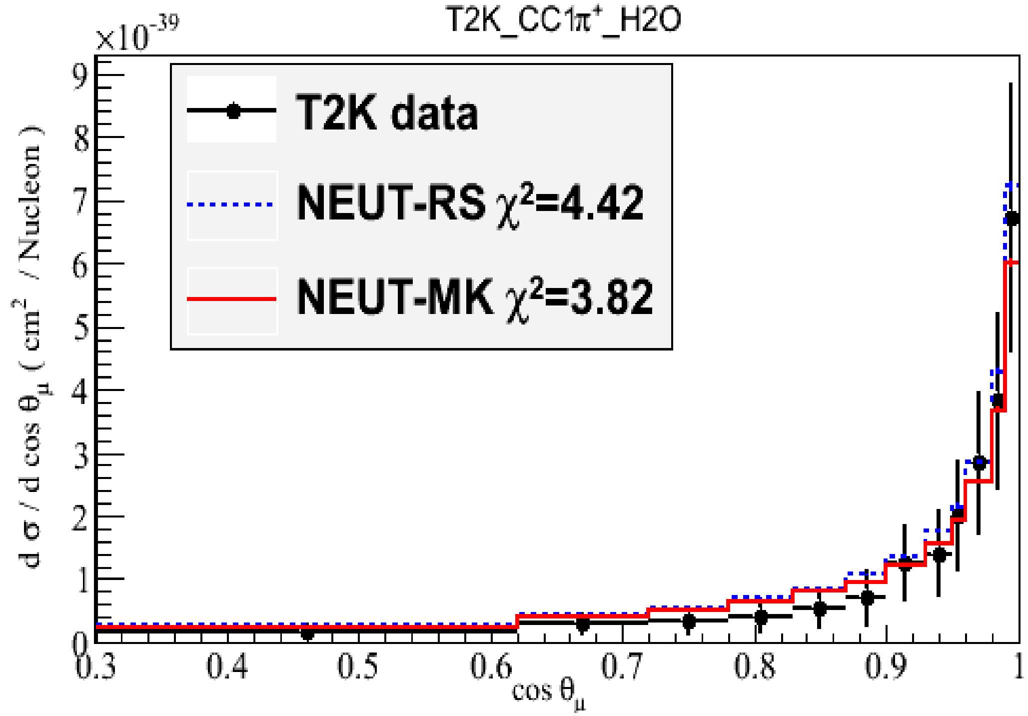
T2K data on Water

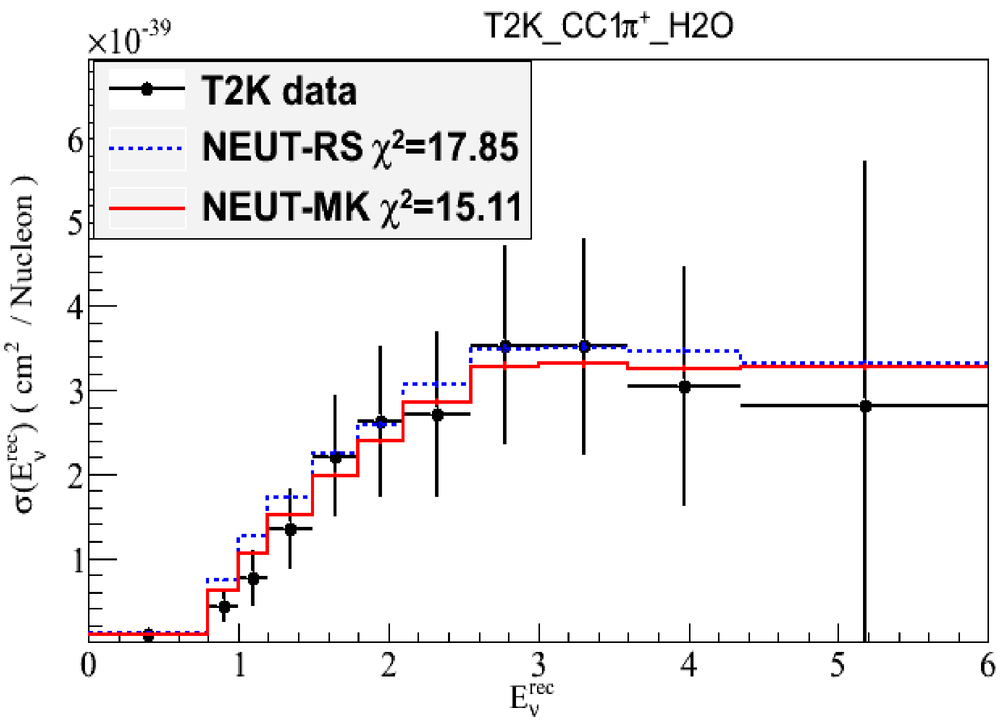
Muon kinematics

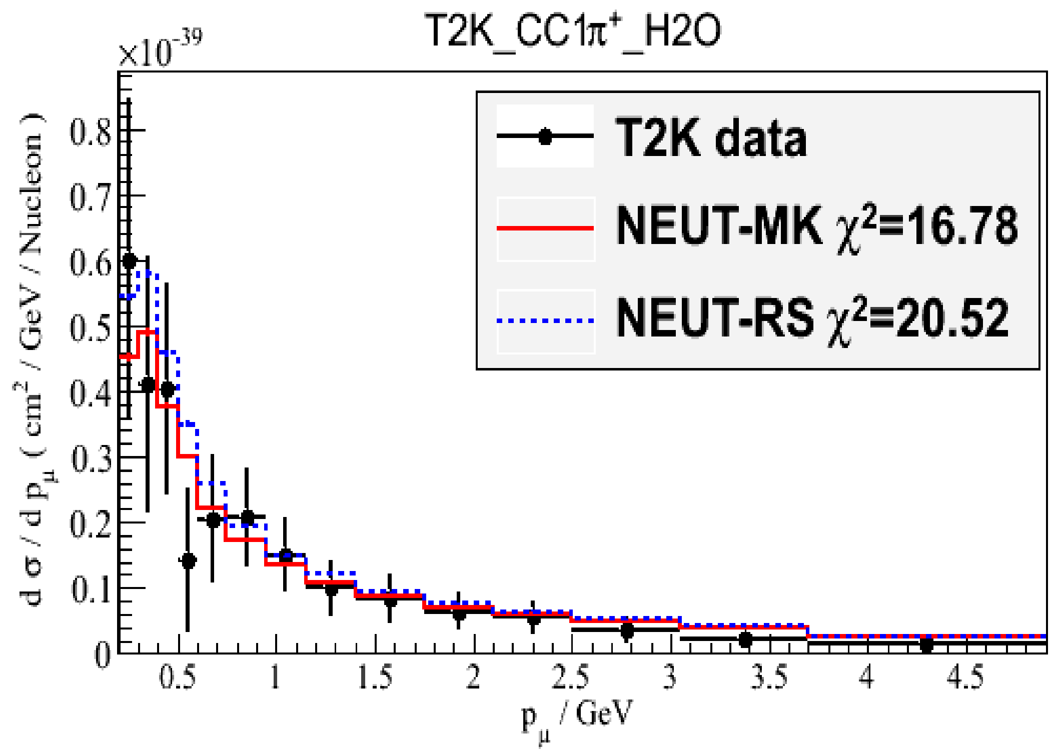
$$\nu_\mu + H_2O \rightarrow \mu^- \pi^+ + X$$

$$p_\mu > 200\ MeV/c$$
$$p_\pi > 200\ MeV/c$$
$$\cos\theta_\mu > 0.3$$
$$\cos\theta_\pi > 0.3$$

K. Abe et al. [T2K Collaboration], Phys. Rev. D 95 (2017) no.1, 012010.







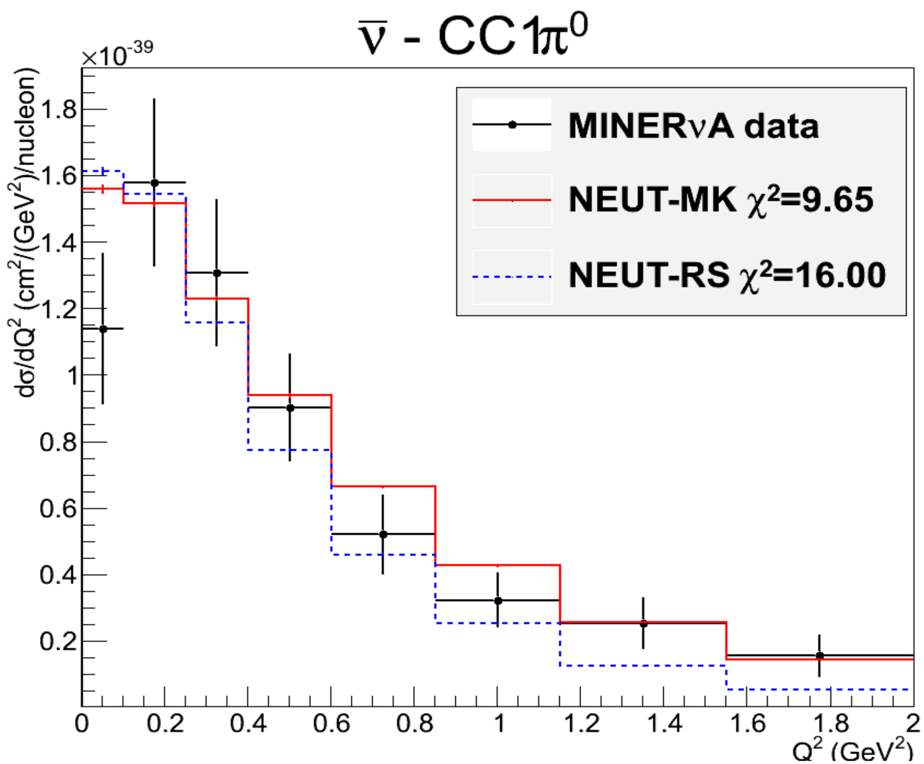
MINERvA data 2016

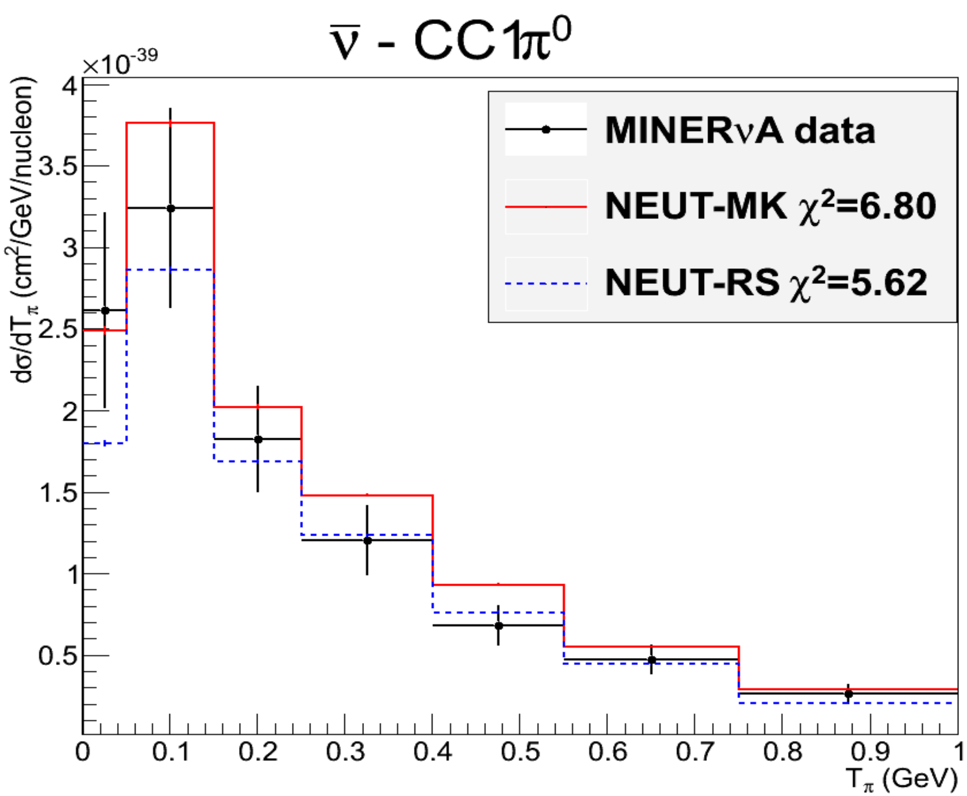
CH target

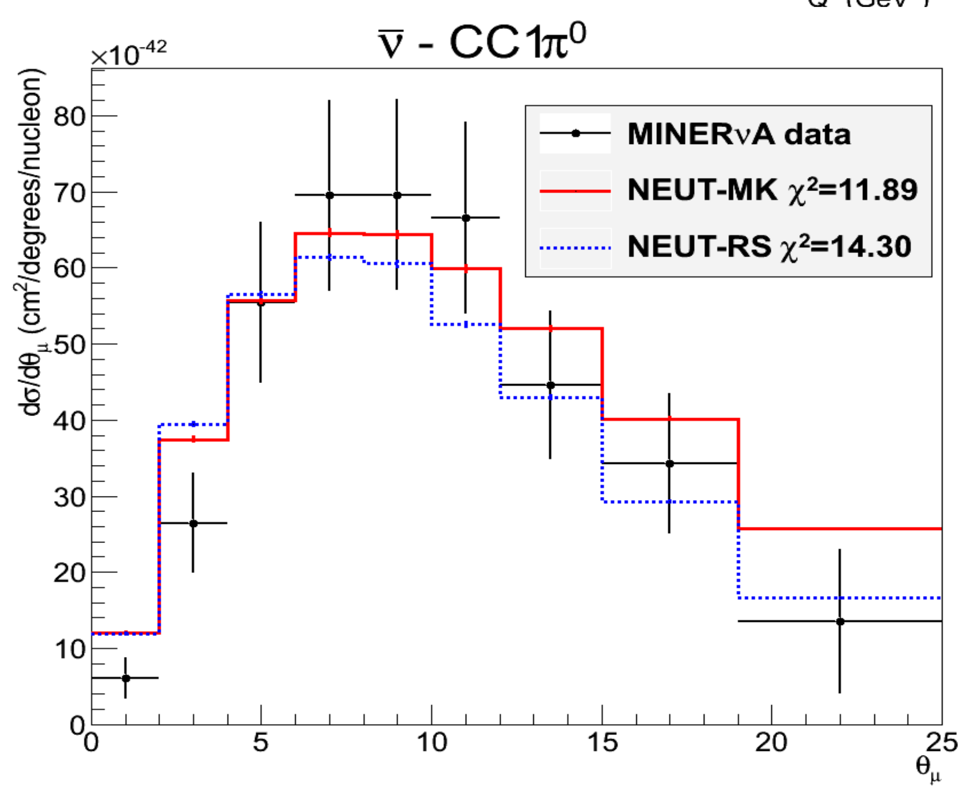
W<1.8 GeV

$$\bar{\nu}_\mu + \mathrm{CH} \rightarrow \mu^+ + \pi^0 + X'$$

C. L. McGivern et al. [MINERvA Collaboration], Phys.Rev. D 94 (2016) no.5, 052005







Summary

- The model consists of resonant and nonresonant interactions, **including the interference effects**.
- The **lepton mass** is included
- The model shows good agreements with bubble chamber data.
- The model is implemented in NEUT and it is satisfactory:

✔ It is as fast as the Rein-Sehgal model.
✔ It is valid up to W=2 GeV.
✔ It is a full kinematics model $d\sigma / dW\, dQ^2\, d\Omega_\pi$

- It will soon be tested in T2K analyses, hopefully improved data/MC agreement and reducing systematic uncertainties propagated to the oscillation measurements.

Thank you!

Thanks **J .Zmuda** and **R. GONZALEZ JIMENEZ**
for validating my results.

Backup

General framework: how to calculate the helicity amplitudes?

$$\mathscr{M}_{CC}(\nu N \to l_\lambda N'\pi) = \frac{G_F}{\sqrt{2}} \cos\theta_C \langle N'\pi | \varepsilon_\lambda^\rho J_\rho | N \rangle$$

$$= \frac{G_F}{\sqrt{2}} \cos\theta_C \langle N'\pi | C_{L_\lambda} e_L^\rho J_\rho + C_{R_\lambda} e_R^\rho J_\rho + C_\lambda e_\lambda^\rho J_\rho | N \rangle$$

Hadron current $J_\alpha = J_\alpha^V - J_\alpha^A$

Lepton current

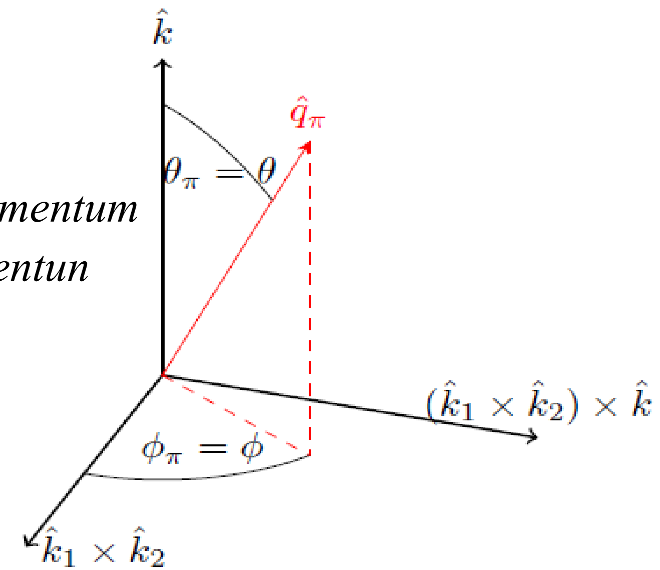
$$\epsilon^\alpha = \bar{u}_l(k_2)\gamma^\alpha(1-\gamma_5)u_\nu(k_1)$$

can be interpreted as the intermediate gauge boson's polarization vector.

$\hat{k} = \hat{k}_1 - \hat{k}_2$

k_1: *neutrino momentum*

k_2: *lepton momentun*



$$\epsilon_\lambda^\rho = C_{L_\lambda} e_L^\rho + C_{R_\lambda} e_R^\rho + C_\lambda e_\lambda^\rho$$

Four different polarizations $e_{\lambda_k}^\rho$

$$e_L^\rho = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & -i & 0 \end{pmatrix}$$

$$e_R^\rho = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -1 & -i & 0 \end{pmatrix}$$

$$e_\lambda^\rho = \frac{1}{\sqrt{|(\varepsilon_\lambda^0)^2 - (\varepsilon_\lambda^3)^2|}}\begin{pmatrix} \varepsilon_\lambda^0 & 0 & 0 & \varepsilon_\lambda^3 \end{pmatrix}$$

Hadronic Current

- Dirac equation allows us to have 16 independent Lorentz covariance
- Conservation of vector current reduce the number of $O(V_i)$ to six.

$$J_V^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{6} V_i \; \bar{u}_N(p_2) O^{\lambda_k}(V_i) u_N(p_1)$$

Dirac Spinors

$$J_A^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{8} A_i \; \bar{u}_N(p_2) O^{\lambda_k}(A_i) u_N(p_1)$$

O(V) and O(A) are 4*4 matrices in terms of Dirac matrices and particle's 4-momenta.

we can rewrite the right hand side of above eq. in terms of 2*2 matrices

$$J_V^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{6} \mathscr{F}_i \; \chi_2^* \; \Sigma_i^{\lambda_k} \; \chi_1$$

Pauli spinors

$$J_A^\rho e_\rho^{\lambda_k} = \sum_{i=1}^{8} \mathscr{G}_i \; \chi_2^* \; \Lambda_i^{\lambda_k} \; \chi_1$$

Helicity Amplitudes

$$\tilde{F}^{\lambda_k}_{\lambda_2,\lambda_1} = \langle \, N\pi | \; e^\rho_{\lambda_k} J^V_\rho \; | N \, \rangle$$

$$\tilde{G}^{\lambda_k}_{\lambda_2,\lambda_1} = \langle \, N\pi | \; e^\rho_{\lambda_k} J^A_\rho \; | N \, \rangle$$

Can be defined by knowing the helicity of incident and outgoing nucleons and gauge boson's polarization. 16 helicity amplitudes for each vector and axial.

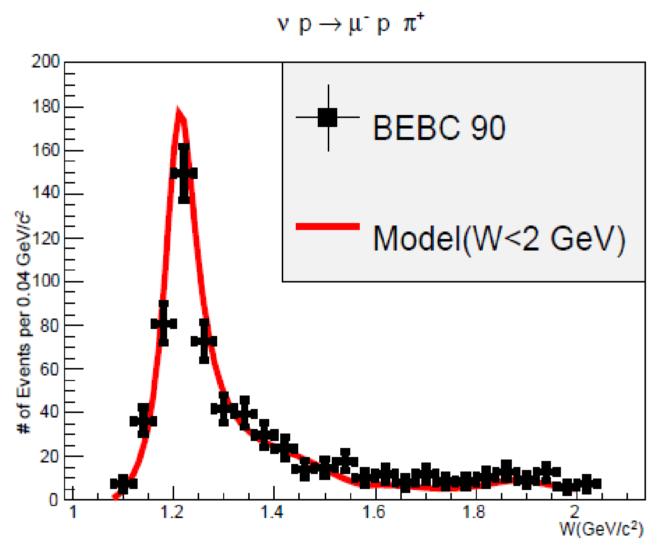
Cross-Section

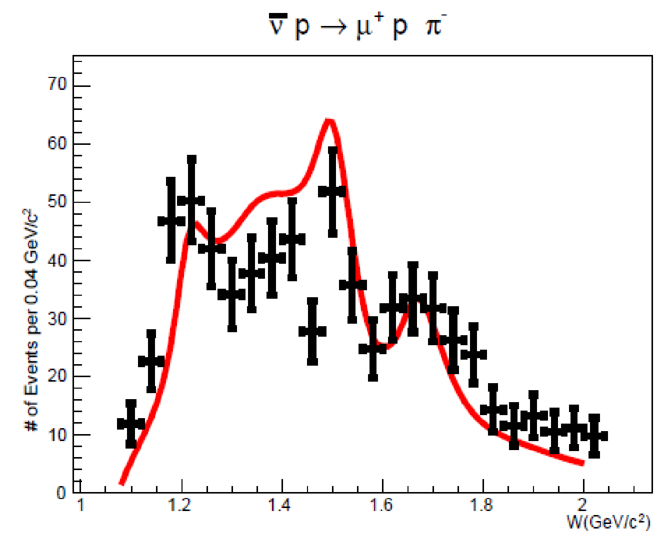
One needs to calculate the helicity amplitudes for resonant and nonresonant interactions.

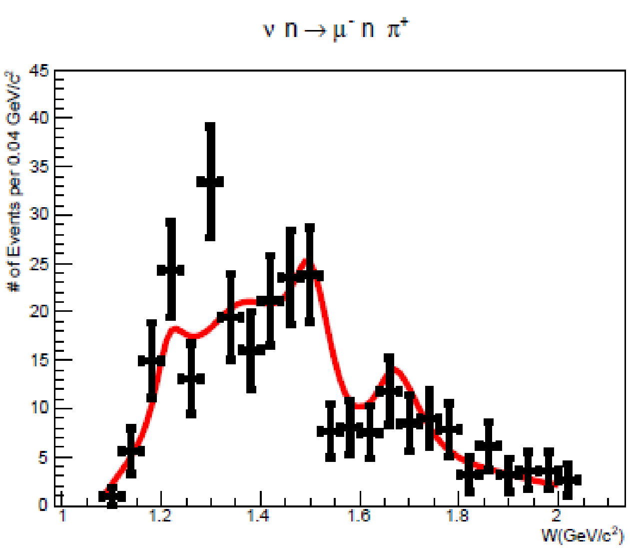
$$\frac{d\sigma(\nu N \to l N\pi)}{dk^2 dW d\Omega_\pi} = \frac{G_F^2}{2}\frac{1}{(2\pi)^4}\frac{|\mathbf{q}|}{4}\frac{-k^2}{(k^L)^2}\sum_{\lambda_2,\lambda_1}\left\{ \left|C_{L_-}(\tilde{F}^{eL}_{\lambda_2\lambda_1} - \tilde{G}^{eL}_{\lambda_2\lambda_1}) + C_{R_-}(\tilde{F}^{eR}_{\lambda_2\lambda_1} - \tilde{G}^{eR}_{\lambda_2\lambda_1}) + C_-(\tilde{F}^{e-}_{\lambda_2\lambda_1} - \tilde{G}^{e-}_{\lambda_2\lambda_1})\right|^2 + \left|C_{L_+}(\tilde{F}^{eL}_{\lambda_2\lambda_1} - \tilde{G}^{eL}_{\lambda_2\lambda_1}) + C_{R_+}(\tilde{F}^{eR}_{\lambda_2\lambda_1} - \tilde{G}^{eR}_{\lambda_2\lambda_1}) + C_+(\tilde{F}^{e+}_{\lambda_2\lambda_1} - \tilde{G}^{e+}_{\lambda_2\lambda_1})\right|^2 \right\}$$

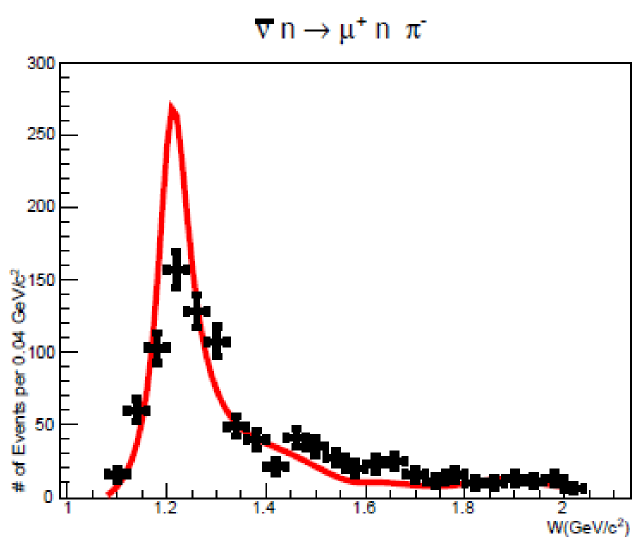
- Helicity amplitudes are a functions of $W, Q^2 = -k^2$ and pion angles in the Adler frame (θ, φ)
- The cross-section is used in several papers, but the lepton mass was ignored.

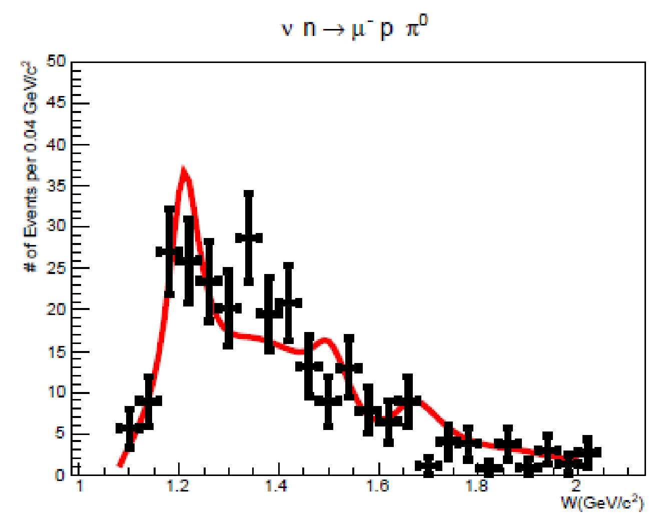
W -distribution
BEBC data
Hydrogen target



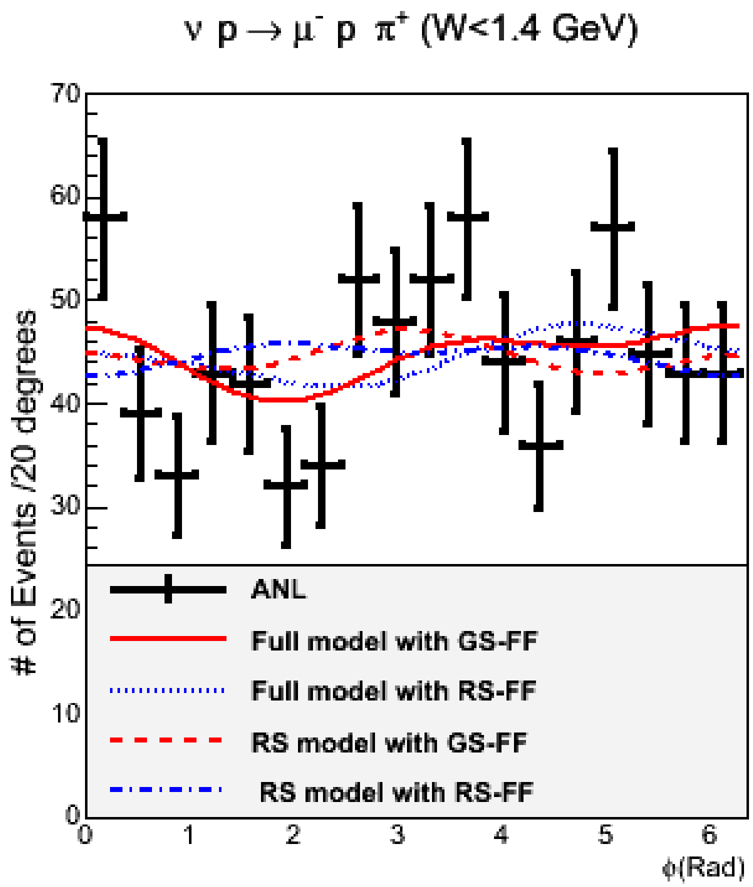


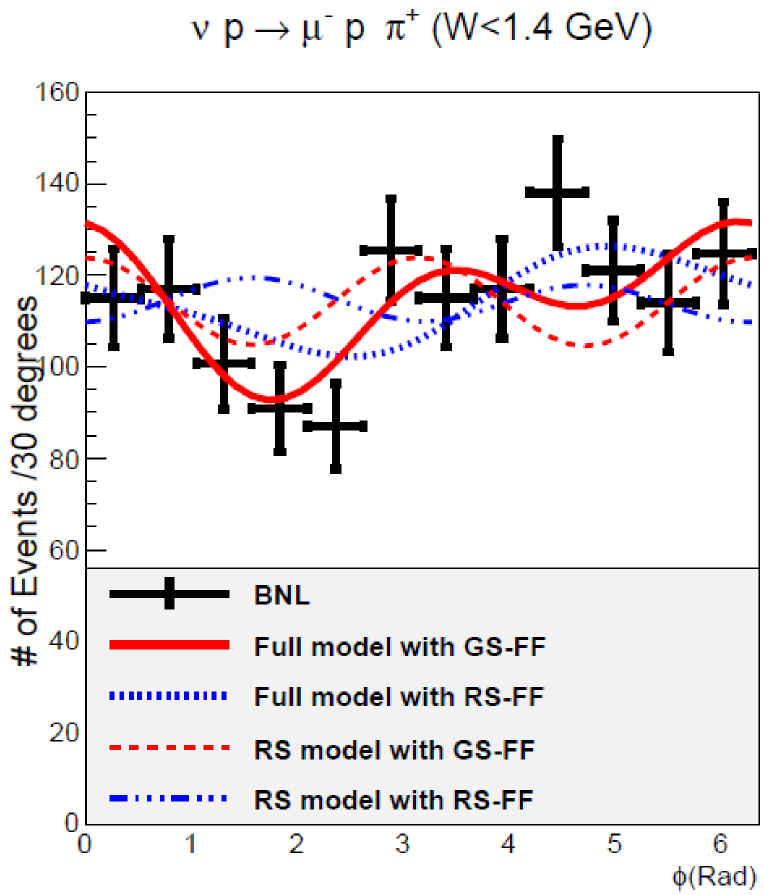






Pion azimuthal angle

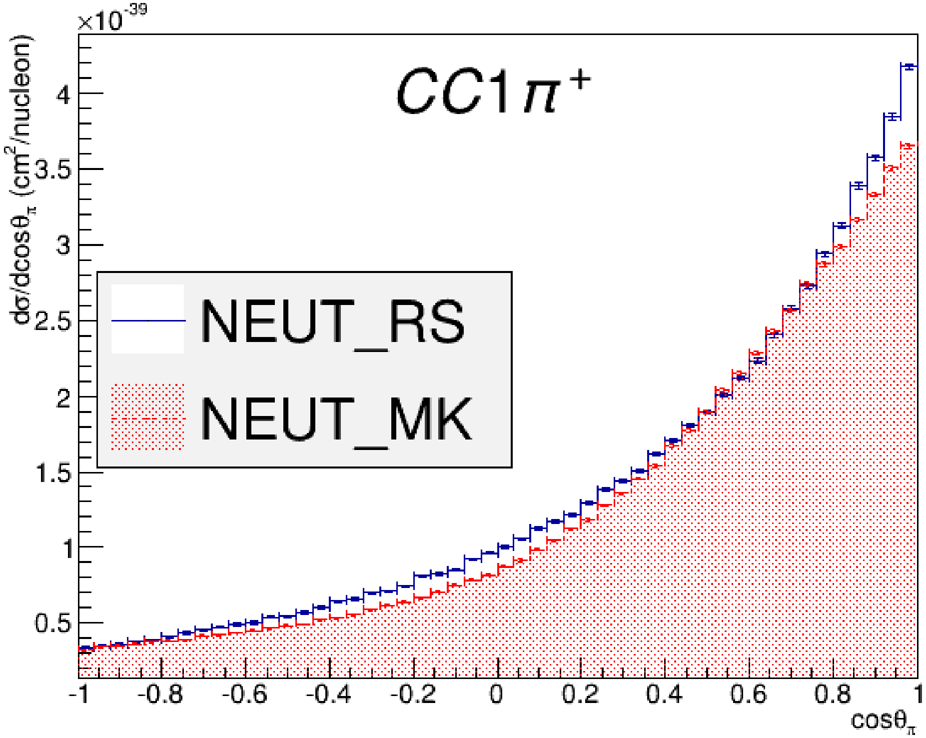


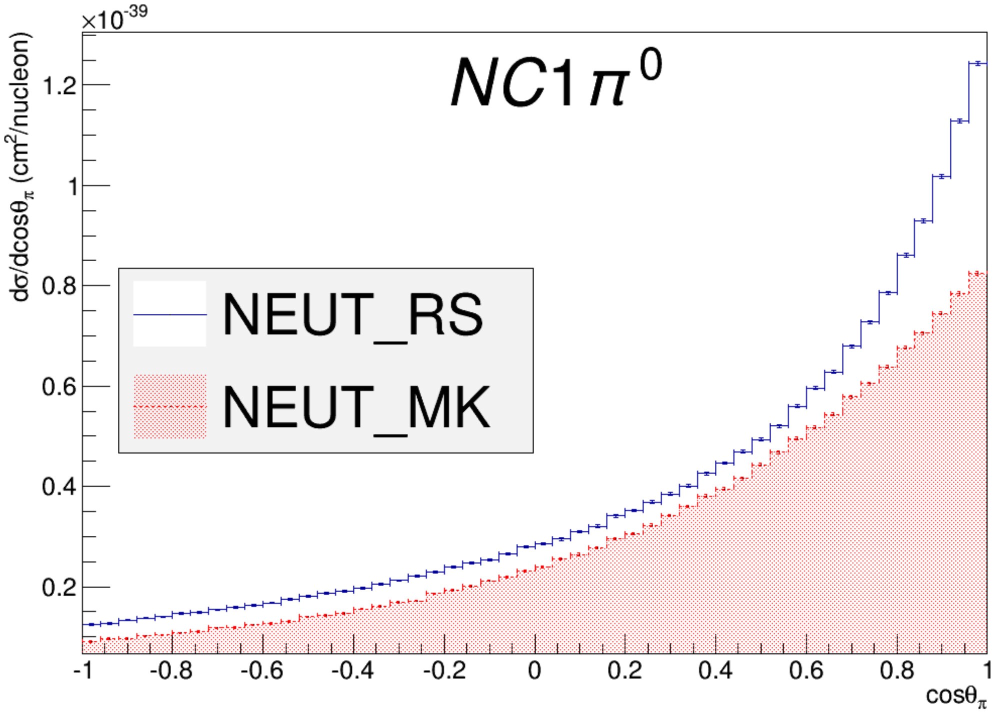


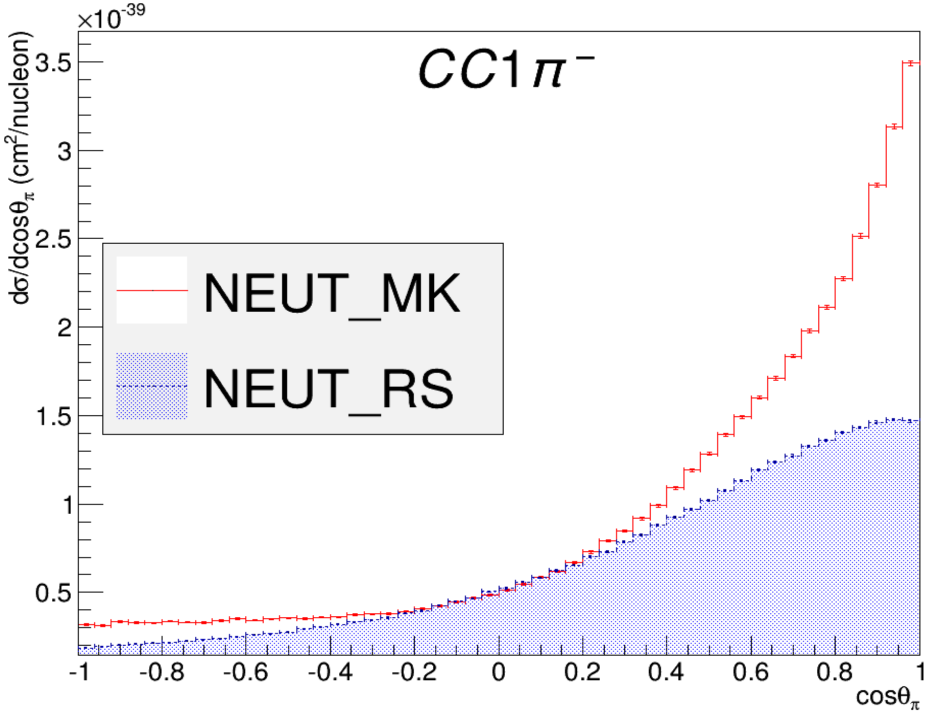
Pion polar angle in the lab frame

Nucleon target

Different samples







MiniBooNE data on CH2