

# Analysis of bubble-chamber data on neutrino-induced pion productions off the deuteron

Satoshi Nakamura

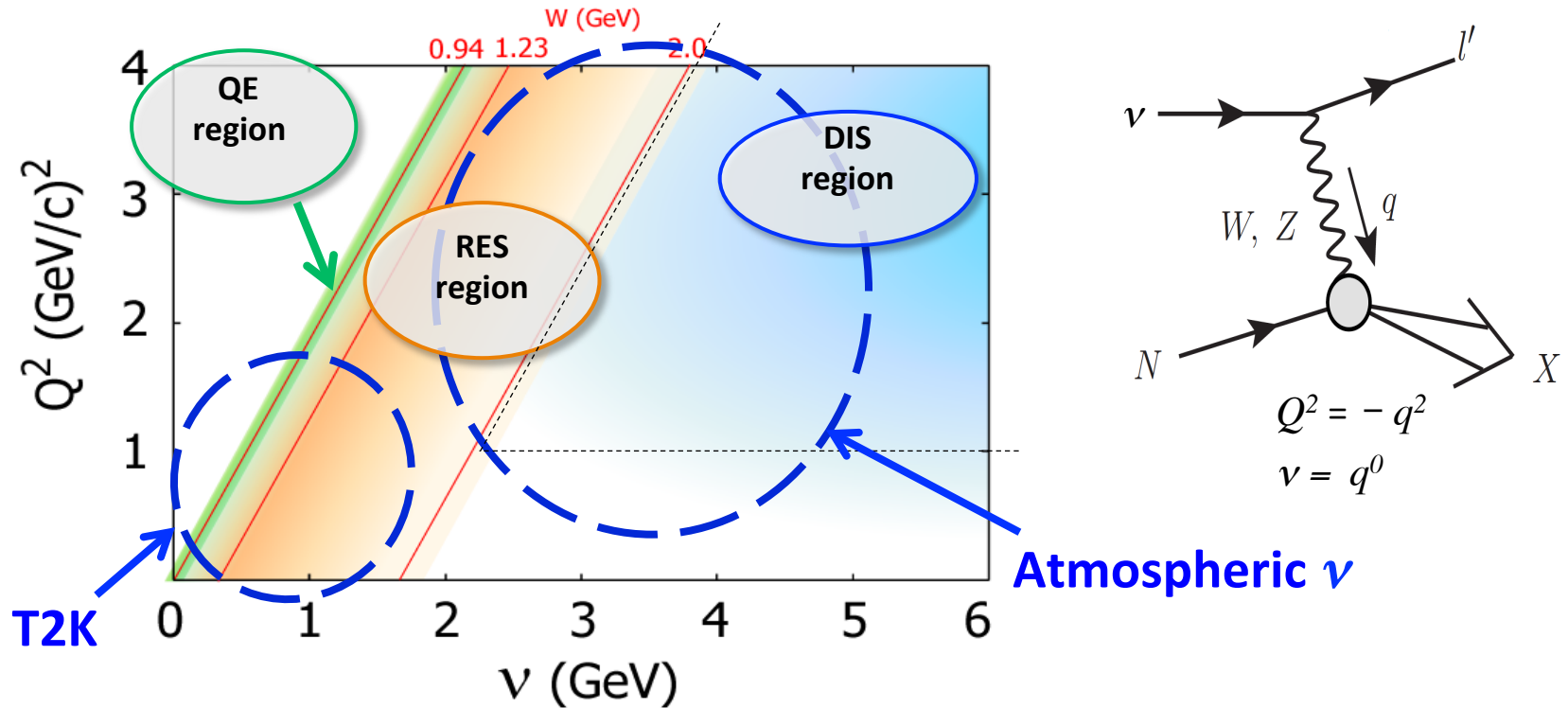
Universidade Cruzeiro do Sul, Brazil

Collaborators: H. Kamano (KEK), T. Sato (Osaka U.)

# Introduction

# Neutrino-nucleus scattering for $\nu$ -oscillation experiments

Wide kinematical region with different characteristic  $\rightarrow$  Different expertise need integrated



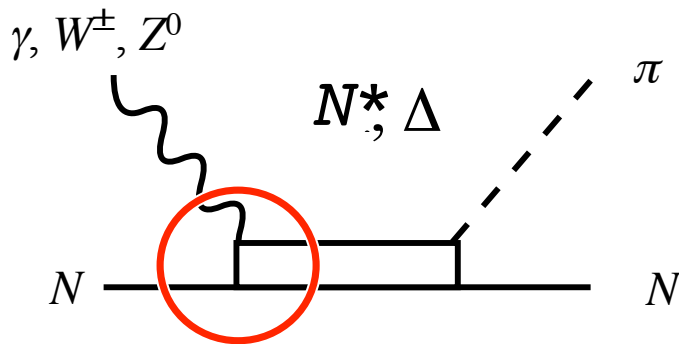
Collaboration at J-PARC Branch of KEK Theory Center

Current status reviewed in *Reports on Progress in Physics* **80** (2017) 056301

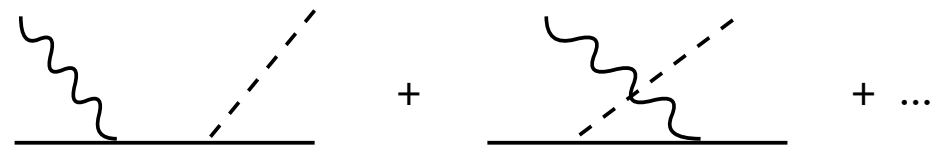
“Towards a Unified Model of Neutrino-Nucleus Reactions for Neutrino Oscillation Experiments”

# Theoretical description of elementary process in resonance region

## Resonance excitations



## Non-resonant mechanisms



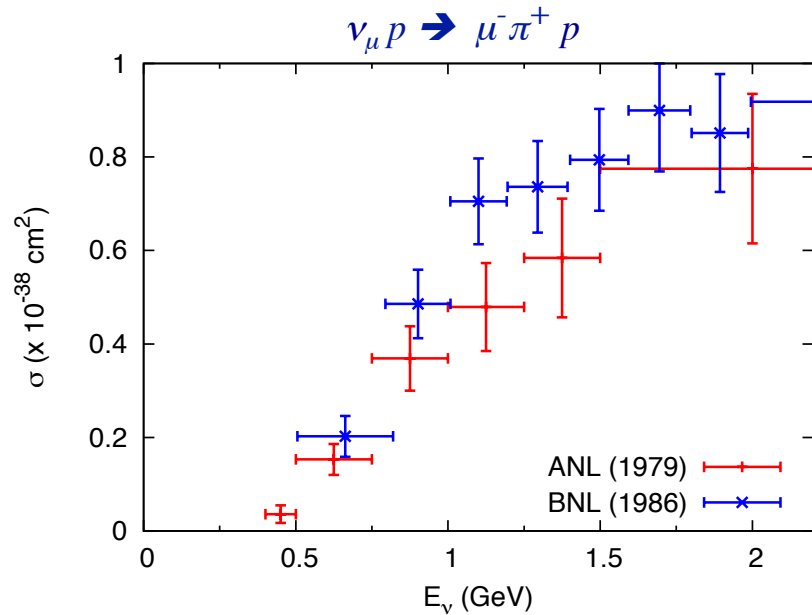
$\Delta(1232)$ -excitation is particularly important in  $\nu$ -induced  $1\pi$  productions

- Accurate determination of  $N$ - $\Delta(1232)$  transition strength is of vital importance
- Experimental inputs are needed to determine  $N$ - $\Delta(1232)$  transition strength

Vector current : photon- and electron-nucleon  $1\pi$  production data

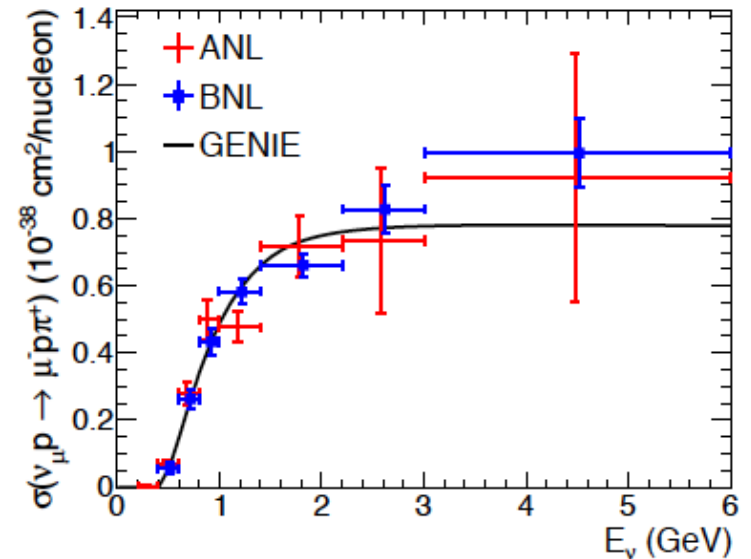
Axial current : neutrino-nucleon  $1\pi$  production data

# Neutrino interaction data in $\Delta(1232)$ region



- Data to fix  $g_{AN\Delta}$
- Discrepancy between BNL & ANL data
  - theoretical uncertainty in neutrino-nucleus cross sections

Wilkinson et al. PRD 90 (2014)



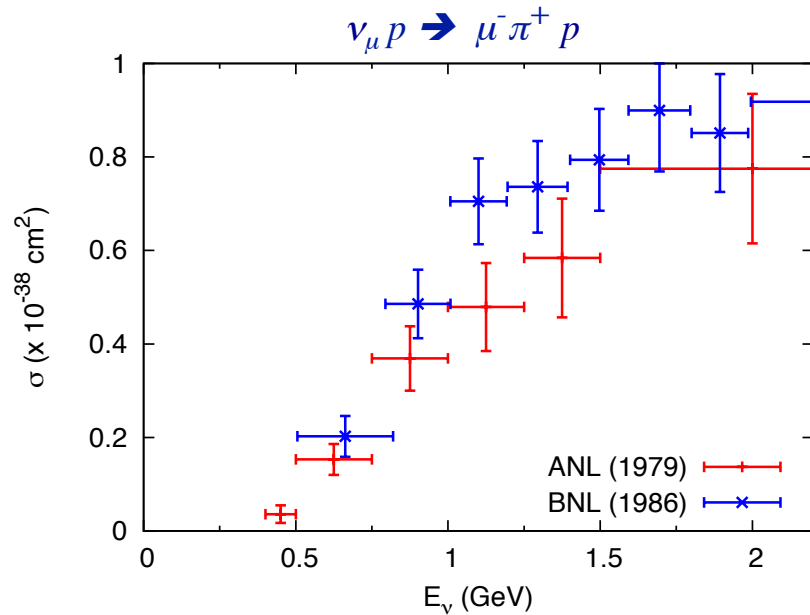
Reanalysis of original data

→ discrepancy resolved (probably)

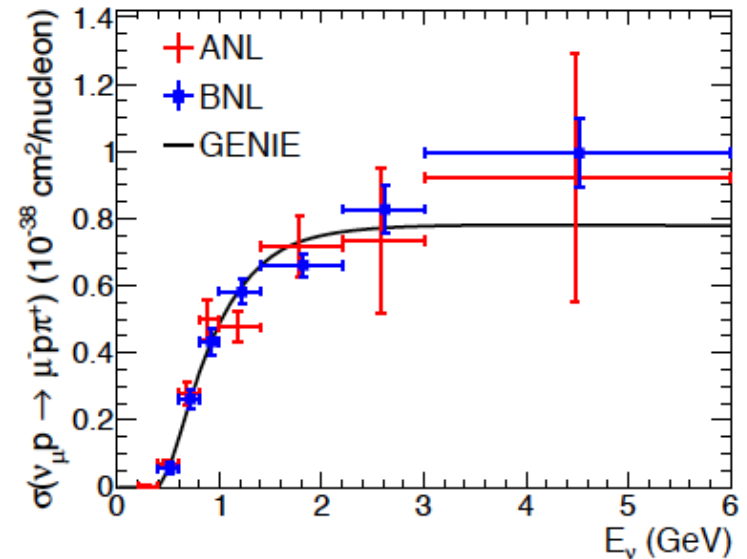
$$\frac{\sigma(\text{CC}1\pi; \text{data})}{\sigma(\text{CC}0\pi; \text{data})} \times \sigma(\text{CCQE}; \text{model})$$

Flux uncertainty is cancelled out

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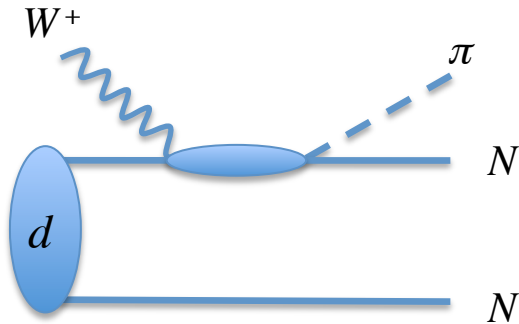
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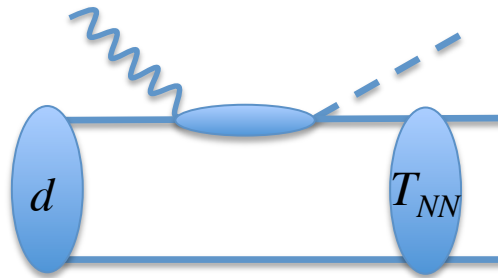
$\nu_\mu p \rightarrow \mu^- \pi^+ p$  data were extracted from  $\nu_\mu d \rightarrow \mu^- \pi^+ p n$  data    Nuclear effects matter ?

# Mechanisms (including nuclear effects) for $\nu_\mu d \rightarrow \mu^- \pi N N$

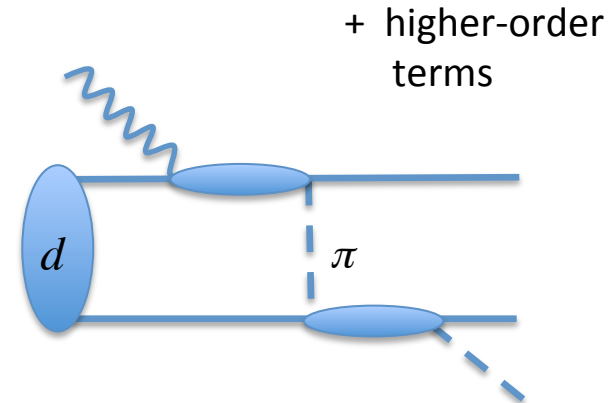
Impulse



$NN$  rescattering



$\pi N$  rescattering



## Nuclear effect managements

**Exp.** Quasi-free ( $\approx$  impulse) events were (supposedly) selected in ANL and BNL analyses

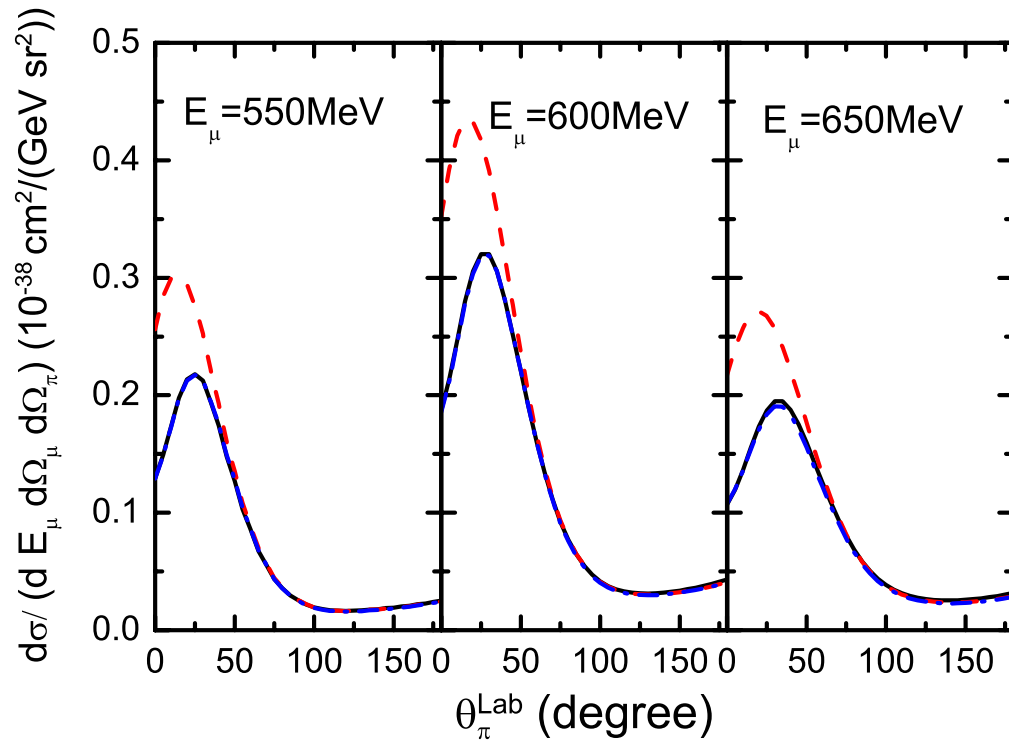
**Theory** Fermi motion considered in fixing  $g_{AN\Delta}$  Hernandez et al. (2010), Alam et al. (2016)

**Q:** Final state interactions (FSI) effects can be removed with simple kinematical cuts ?

FSI effects on  $\nu_\mu d \rightarrow \mu^- \pi N N$  have been explored with a dynamical model Wu et al. (2015)

# FSI effects on $\nu_\mu d \rightarrow \mu^- \pi^+ p n$

Wu, Lee, Sato, PRC91, 035203 (2015)



$$E_\nu = 1 \text{ GeV},$$

$$E_\mu = 550, 600, 650 \text{ MeV}, \theta_\mu = 25^\circ, \phi_{\pi^-} = 0^\circ$$

--- Impulse approx.

- · - NN FSI

— NN +  $\pi$ N FSI

Significant reduction due to NN FSI

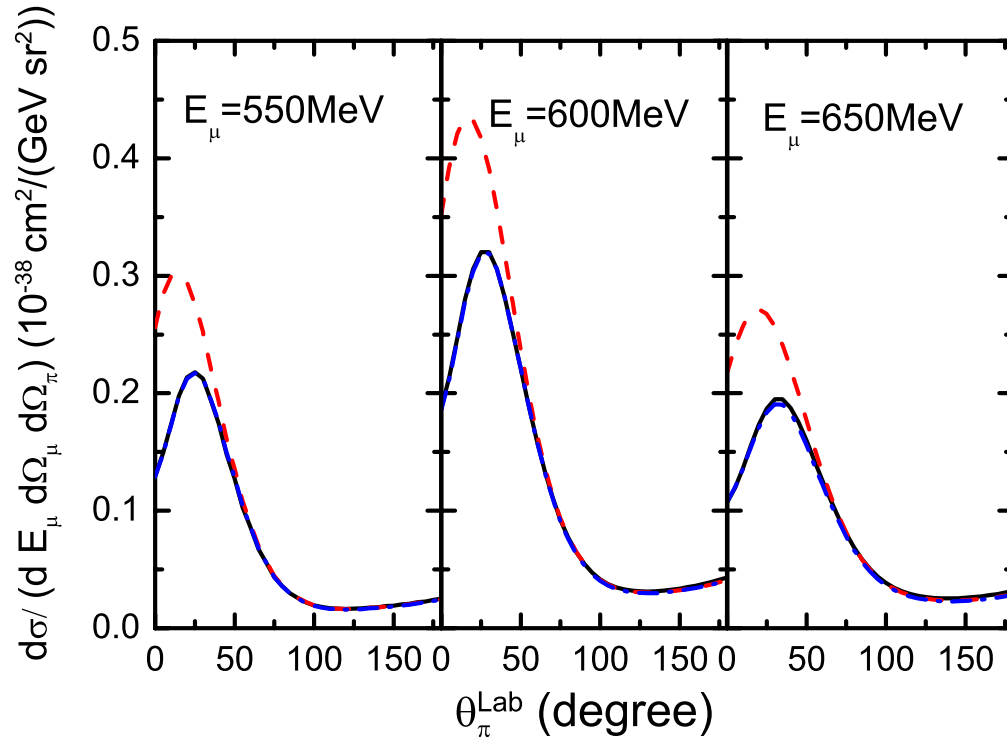


Orthogonality of  $pn$  and  $d$  wave functions



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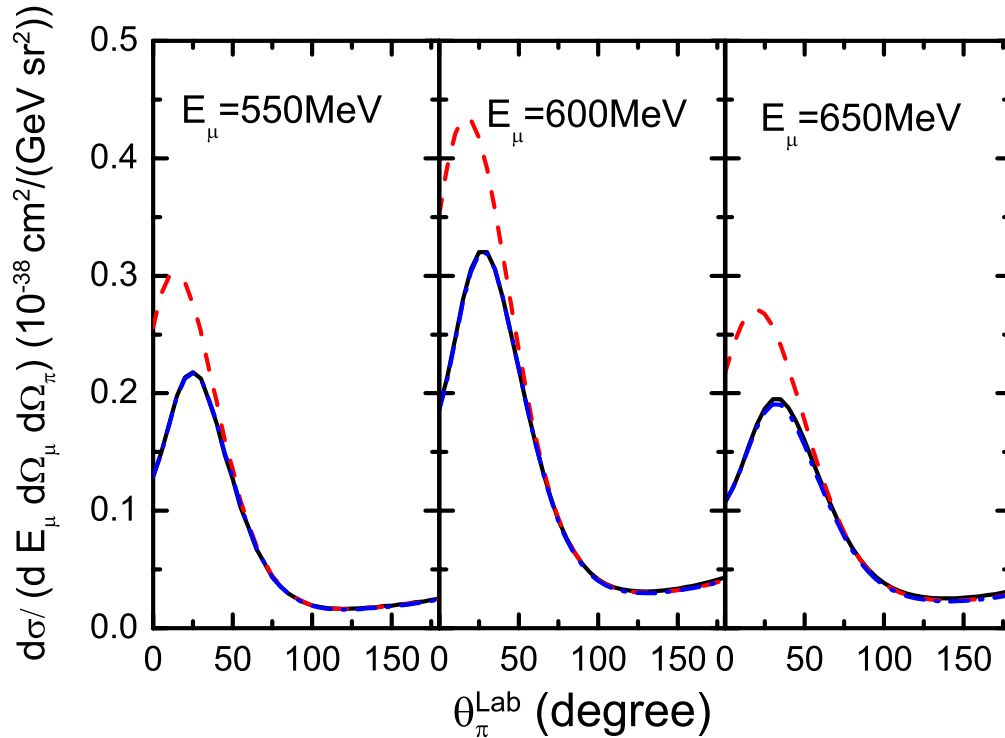
**Limitation of Wu et al.'s work** : A tiny fraction of the whole phase-space was covered

$$\sigma = \int dp_{N_1} dp_{N_2} dp_\mu dp_\pi \delta^{(4)}(P_i - P_f) |M|^2 \quad (7 \text{ dim. non-trivial numerical integral})$$

Numerically challenging problem

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Numerically challenging problem *will be managed in this work*

# This work

- Total ( $\sigma$ ) and single differential ( $d\sigma/dX$ ) cross sections for  $\nu_\mu d \rightarrow \mu^- \pi^+ p n$  with FSI are calculated with a dynamical model  
7-dim. numerical integral is managed with Monte-Carlo method
- FSI effects on  $\sigma$  and  $d\sigma/dX$  are examined
- How FSI could distort elementary  $\nu_\mu p \rightarrow \mu^- \pi^+ p$  and  $\nu_\mu n \rightarrow \mu^- \pi^+ n$  cross sections extracted from  $\nu_\mu d \rightarrow \mu^- \pi^+ p n$  cross sections

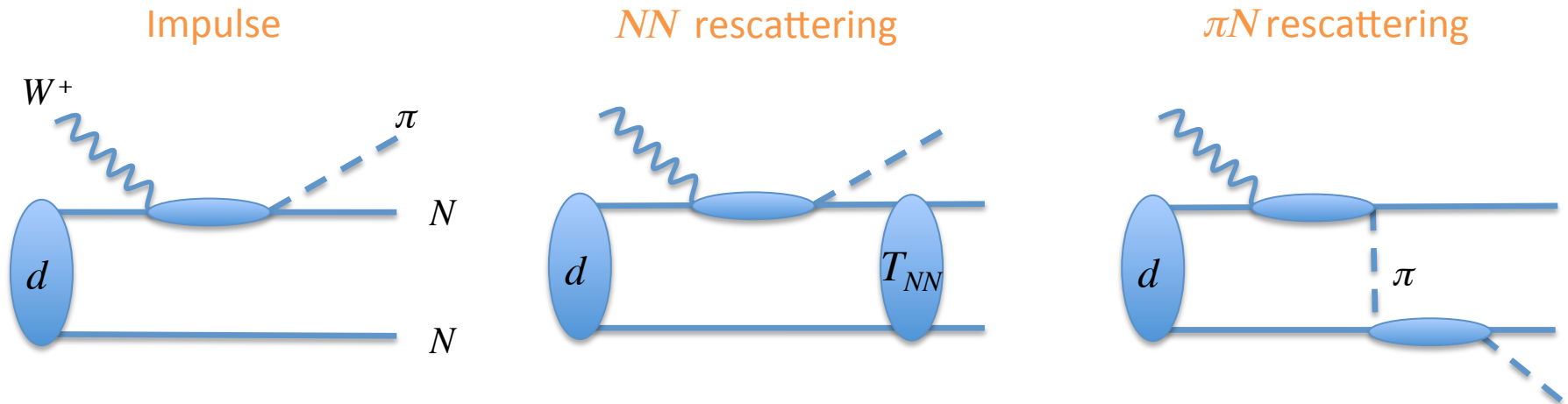
$\nu_{\mu} d \rightarrow \mu^{-} \pi^{+} p n$  reaction model

based on

dynamical coupled-channels model

# Model for $\nu_\mu d \rightarrow \mu^- \pi N N$

Multiple scattering theory truncated at the first-order rescattering



## Elementary amplitudes

$W^\pm N \rightarrow \pi N$ ,  $\pi N \rightarrow \pi N$  amplitude  $\leftarrow$  DCC model (SXN et al., PRD92 (2015))

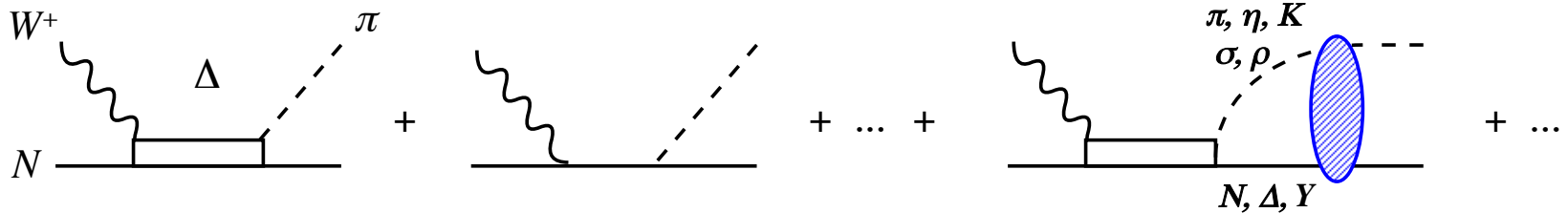
$T_{NN}$ , deuteron w.f.  $\leftarrow$  CD-Bonn potential (Machleidt et al., PRC 63 (2001))

3-dim. loop integral with off-shell amplitudes are numerically evaluated

The model has been validated in photo-induced pion productions

# Dynamical coupled-channels model

Kamano, SXN, Lee, Sato, PRC 88, 035209 (2013)  
 SXN, Kamano, Lee, Sato, PRD 92, 074024 (2015)



Developed through analyzing  $\gamma^{(*)}N, \pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$  data ( $\sim 25,000$  data pts.)

$\rightarrow$  Extended to  $\nu N \rightarrow l^- X$  ( $X = \pi N, \pi\pi N, \eta N, K\Lambda, K\Sigma$ )

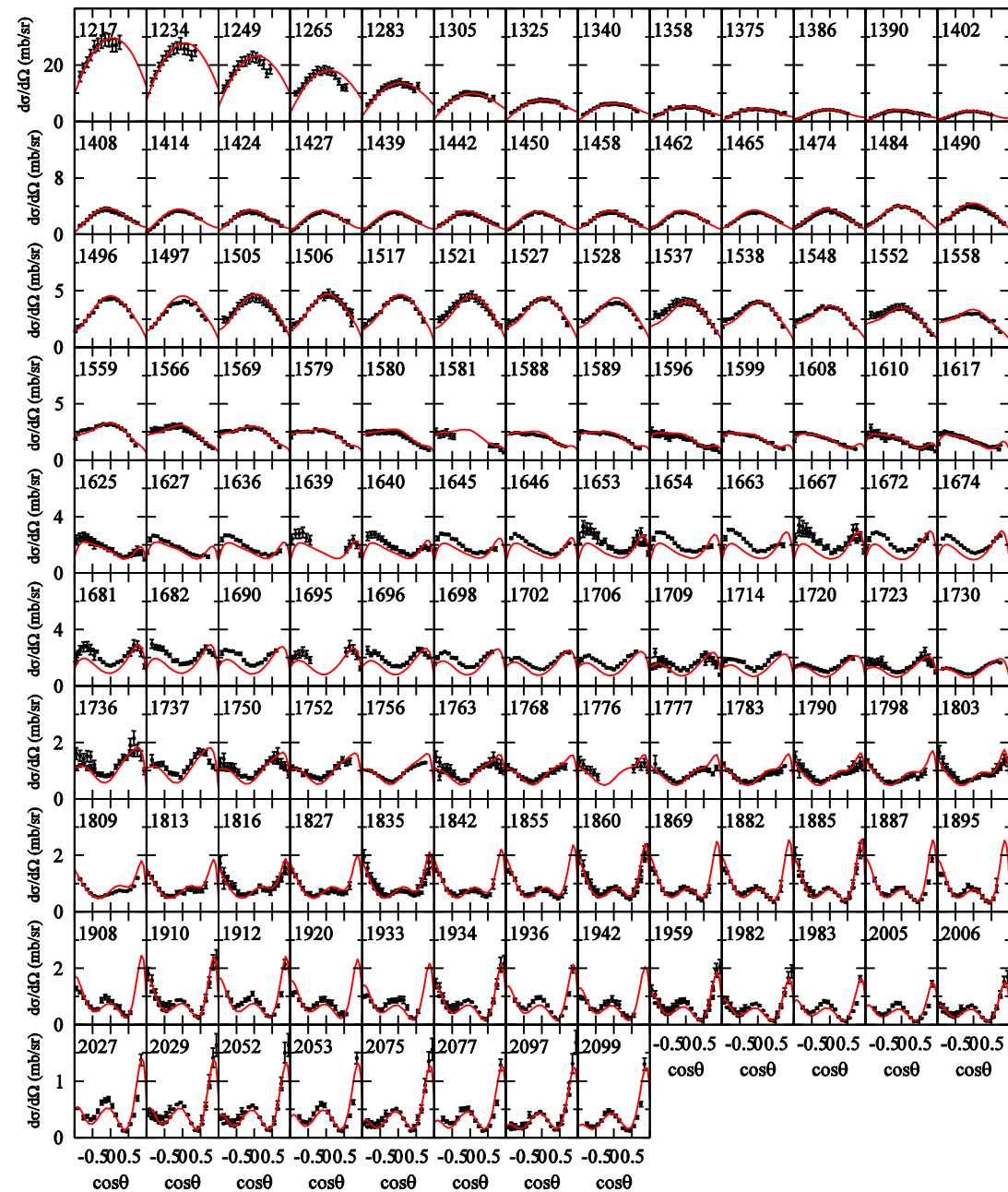
## Unique features

- Hadronic rescattering and channel-couplings are taken into account  
 $\leftarrow$  requirement from the unitarity
- Interference among resonant and non-resonant mechanisms are under control within the model
- One-pion AND two-pion productions for the whole resonance region are described

$\gamma p \rightarrow \pi^0 p$  $d\sigma/d\Omega$  for  $W < 2.1$  GeV

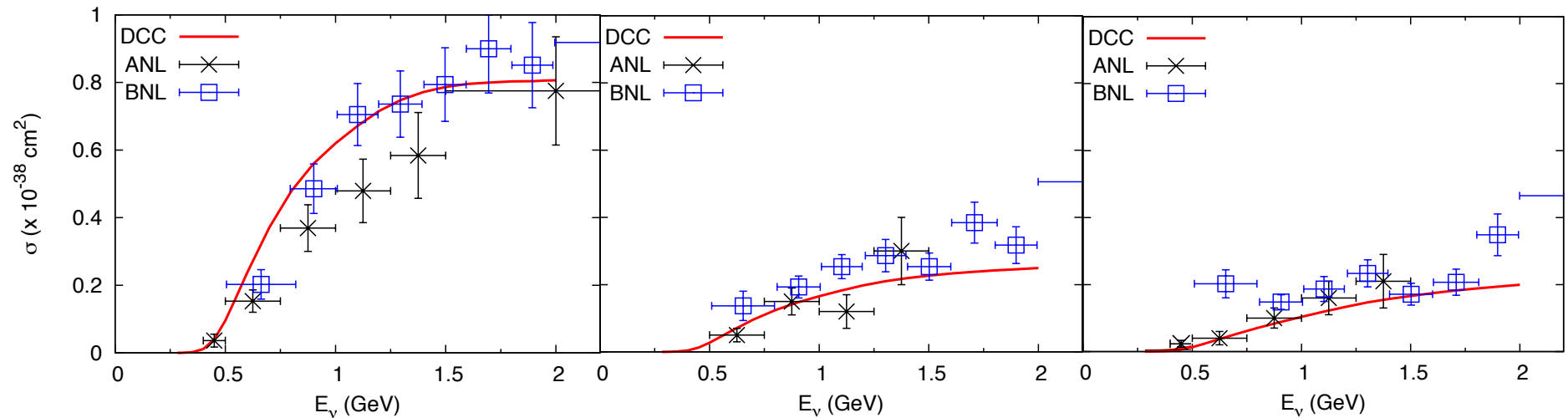
Comparison of DCC model with data

Kamano, Nakamura, Lee, Sato, PRC 88 (2013)

Vector current ( $Q^2=0$ ) for  $1\pi$ 

Production is well-tested by data

# Comparison of DCC model with single pion data



ANL Data : PRD **19**, 2521 (1979)

BNL Data : PRD **34**, 2554 (1986)

DCC model **prediction** is consistent with BNL data (before flux correction)

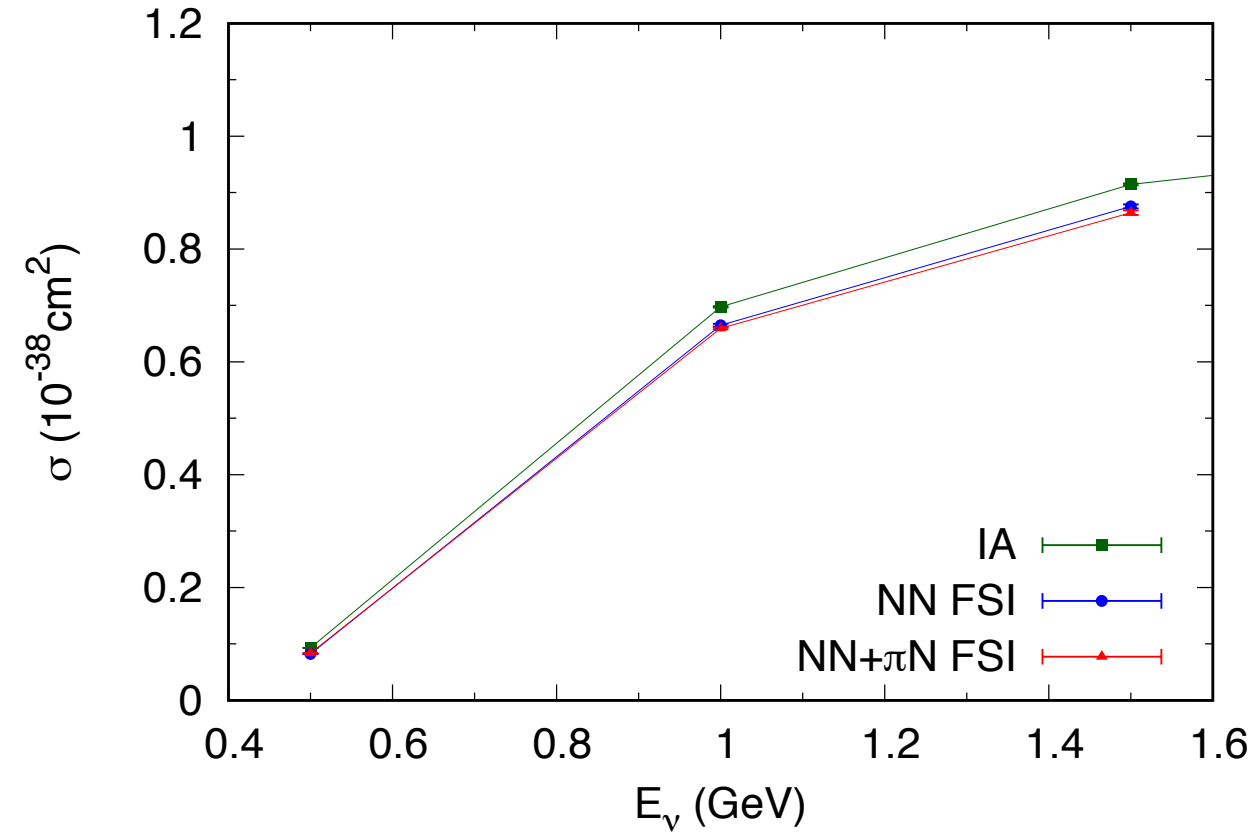
DCC model has flexibility to fit ANL data ( $ANN^*(Q^2)$ )

We will fit data after the issue of nuclear effects is clarified



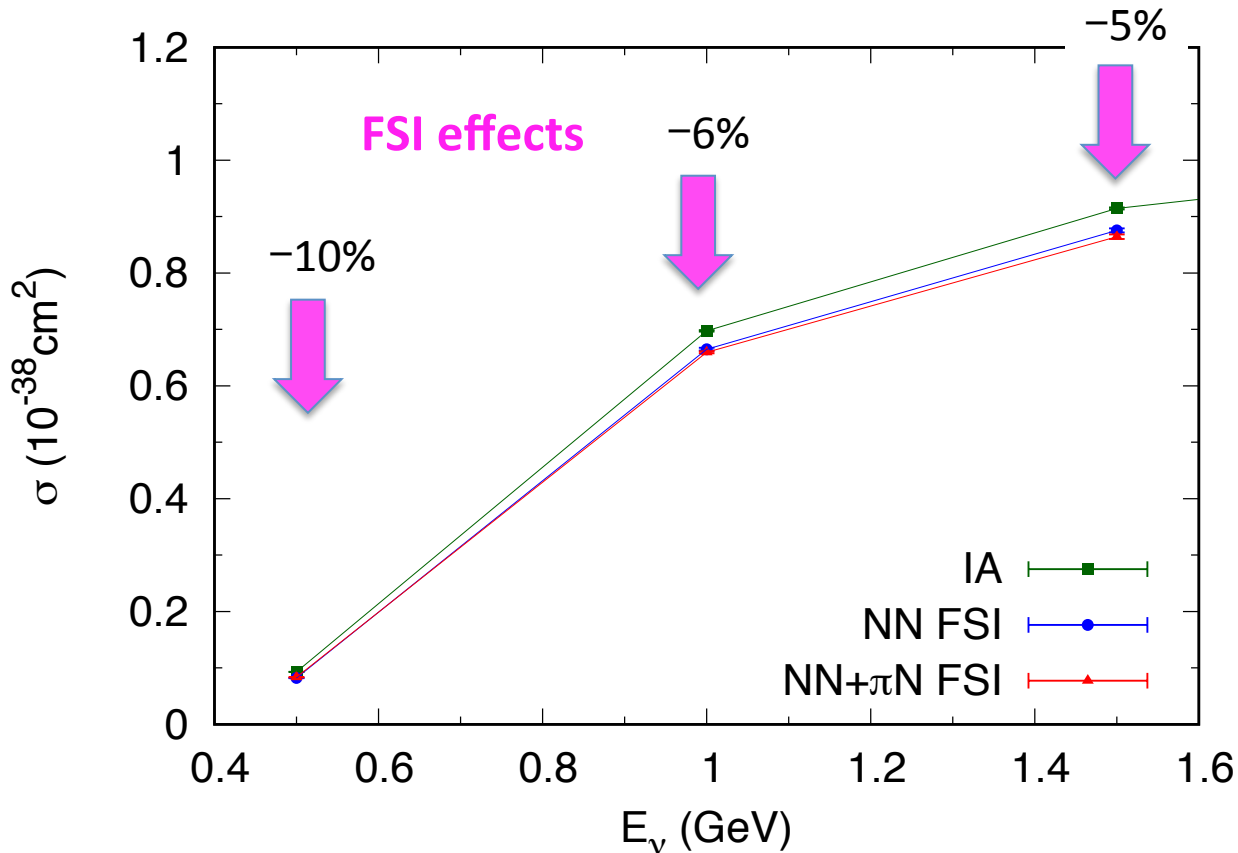
# Results

# $\nu_\mu d \rightarrow \mu^- \pi^+ p n$ total cross sections



**Caveat:** calculations have been done only at several  $E_\nu$  for a large computational cost; line is just for guiding your eyes

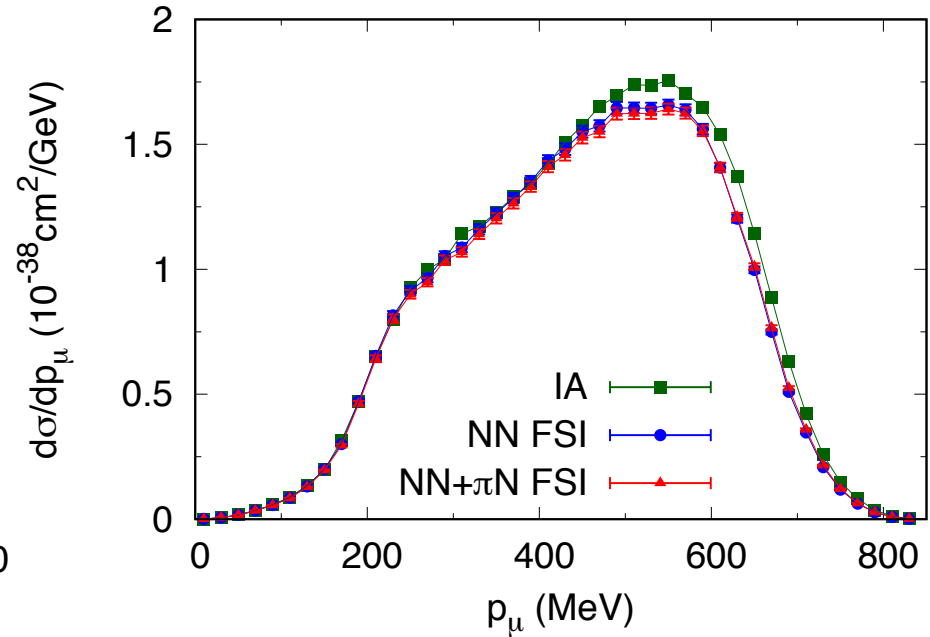
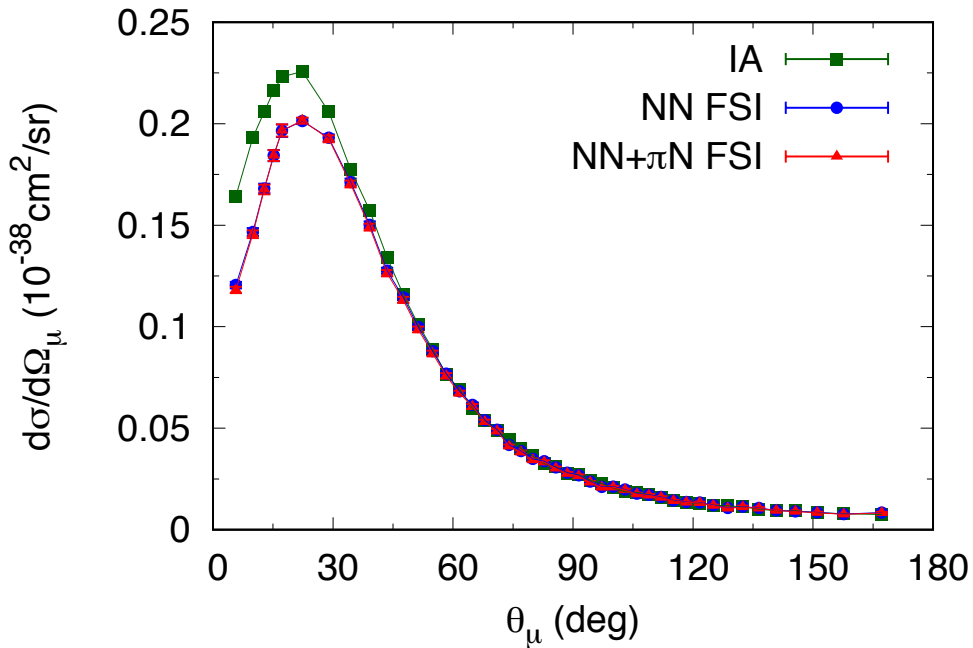
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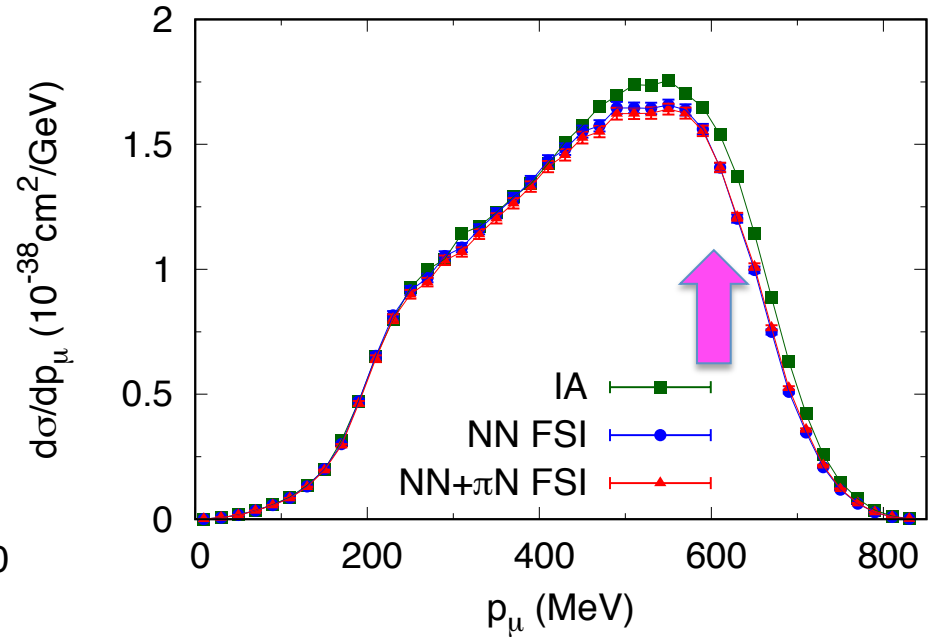
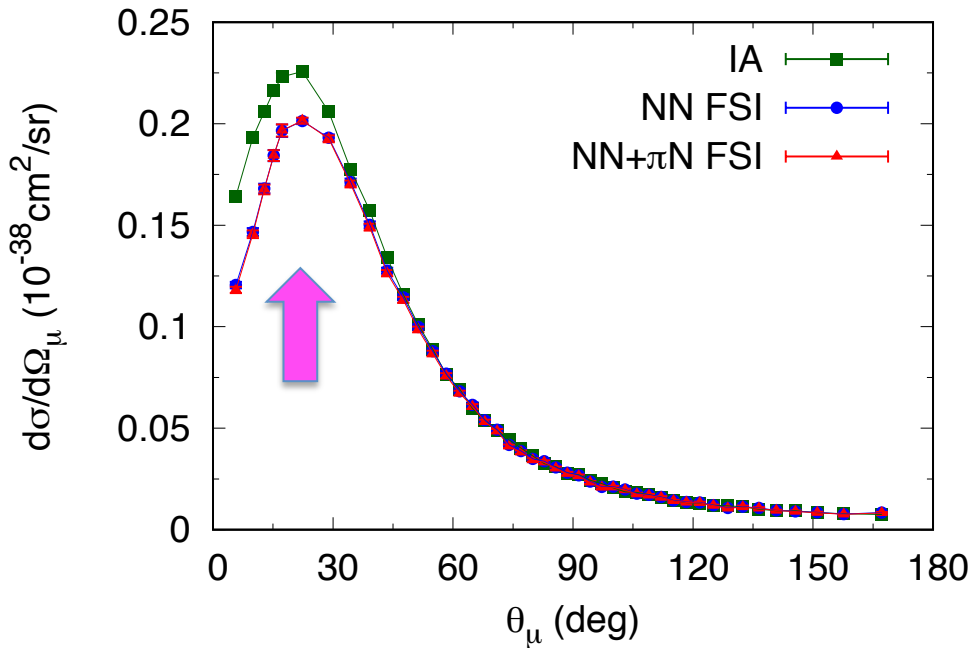
- (Mostly NN) FSI reduces  $\sigma$  by 10%, 6%, 5% at  $E_\nu = 0.5, 1, 1.5$  GeV
- $\pi$ N FSI hardly changes  $\sigma(\nu_\mu d \rightarrow \mu \pi^+ p n)$
- Much smaller FSI effects than what you might have expected from Wu et al.'s result ?

# $d\sigma/d\Omega_\mu$ and $d\sigma/dp_\mu$ for $\nu_\mu d \rightarrow \mu\pi^+ p n$

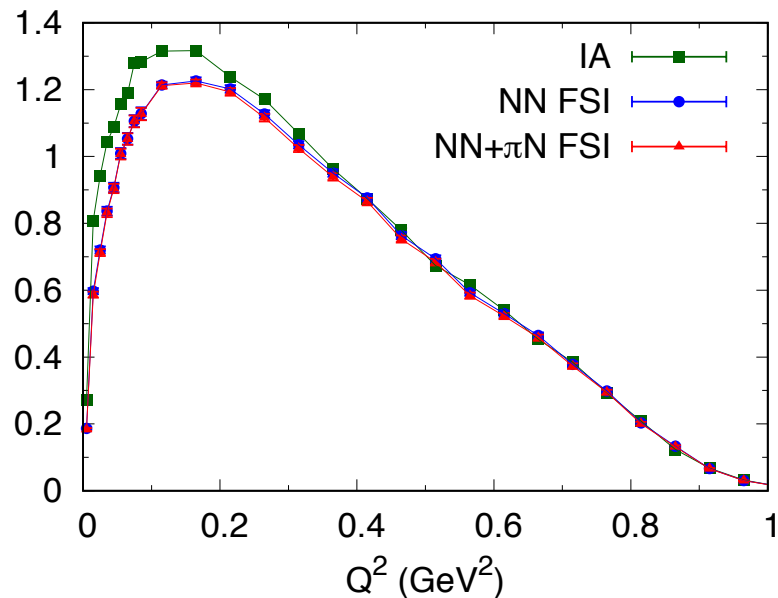
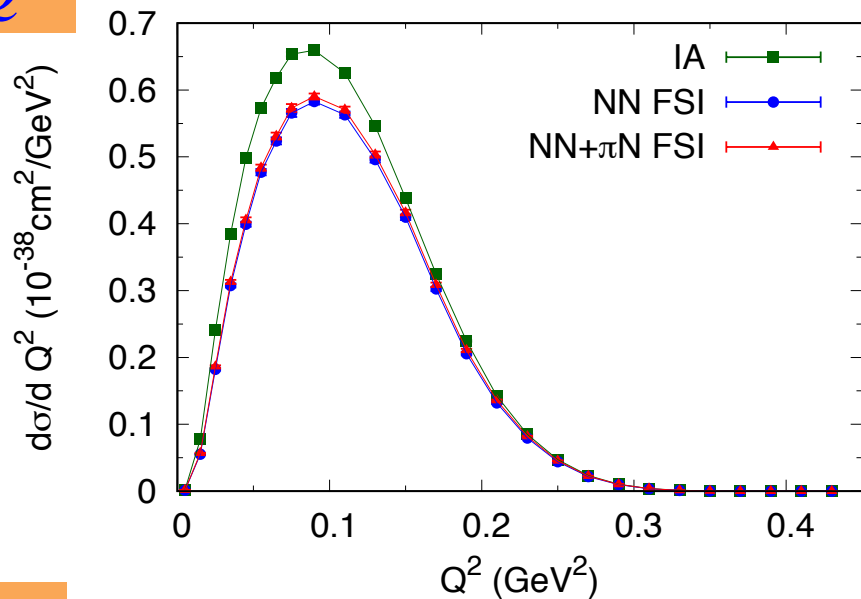
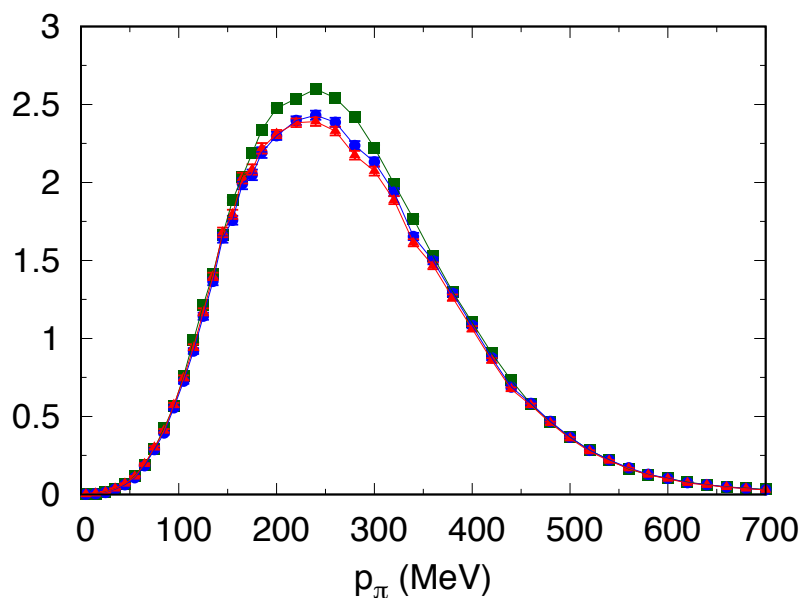
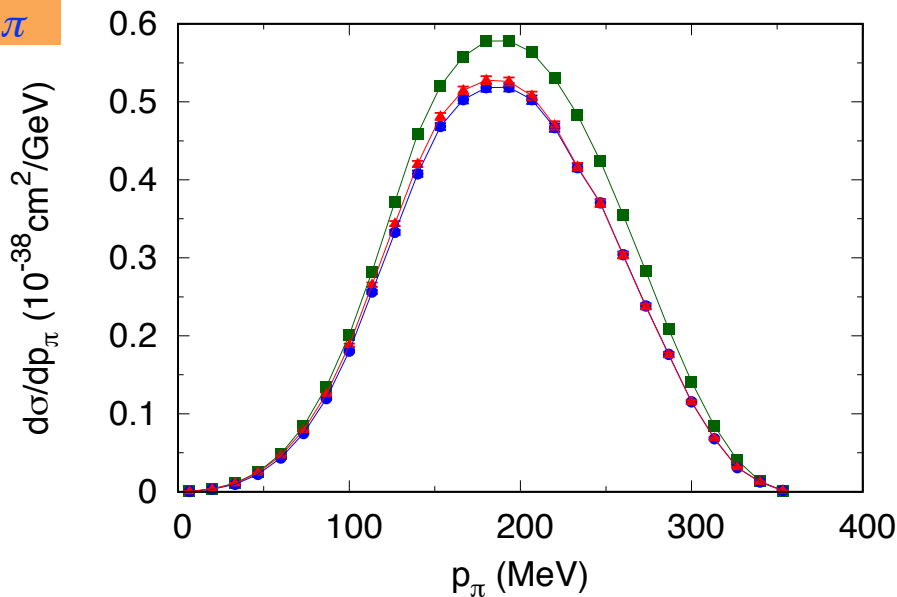


- Significant FSI effects are seen in narrow kinematical windows  
→ moderate reduction of total cross sections

# $d\sigma/d\Omega_\mu$ and $d\sigma/dp_\mu$ for $\nu_\mu d \rightarrow \mu\pi^+ p n$



- Significant FSI effects are seen in narrow kinematical windows  
→ moderate reduction of total cross sections
- Wu et al.'s calculation was at  $\theta_\mu = 25^\circ$  and  $E_\mu = 550, 600, 650$  MeV  
(region where quasi-free kinematics is important)

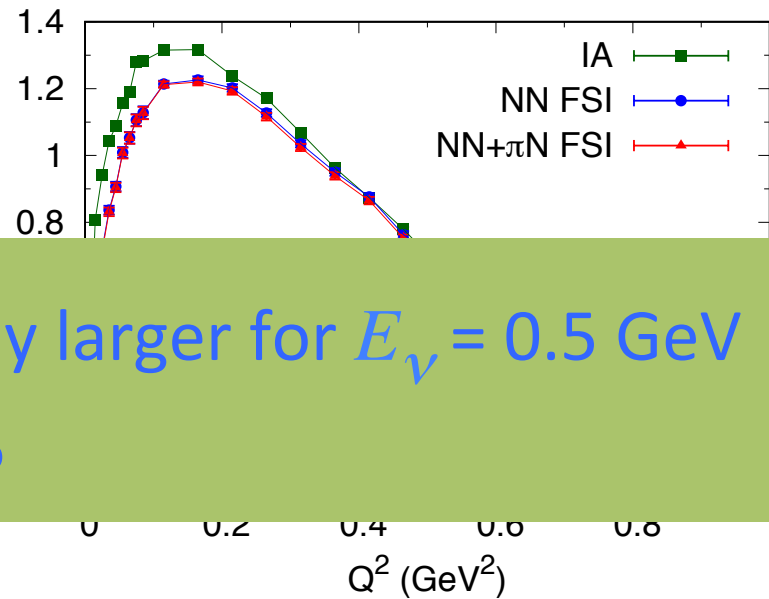
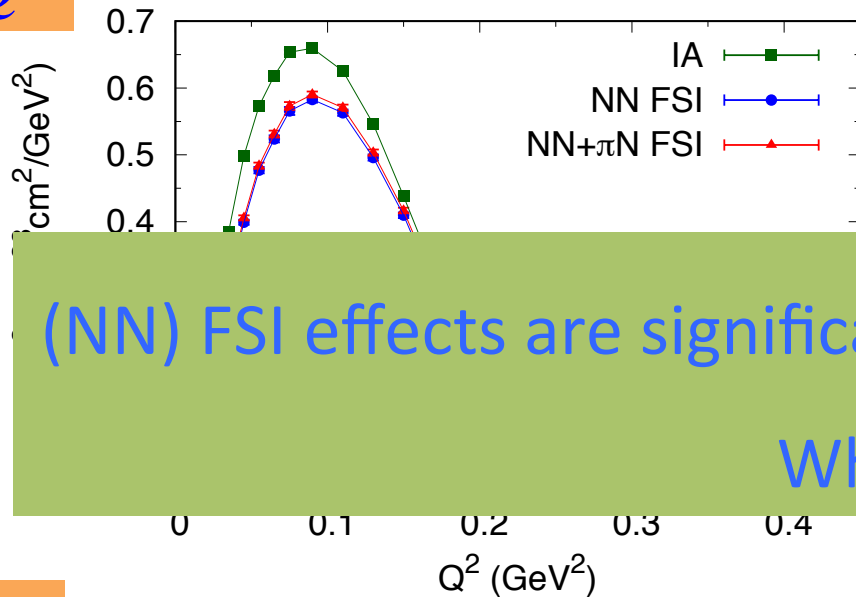
$E_\nu = 0.5 \text{ GeV}$ vs  $E_\nu = 1 \text{ GeV}$  $Q^2$  $p_\pi$ 

$E_\nu = 0.5 \text{ GeV}$

vs

$E_\nu = 1 \text{ GeV}$

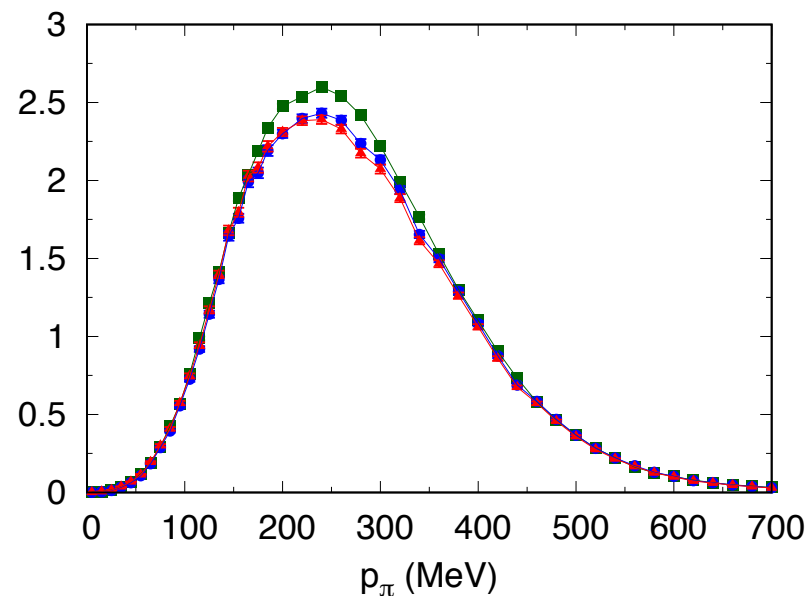
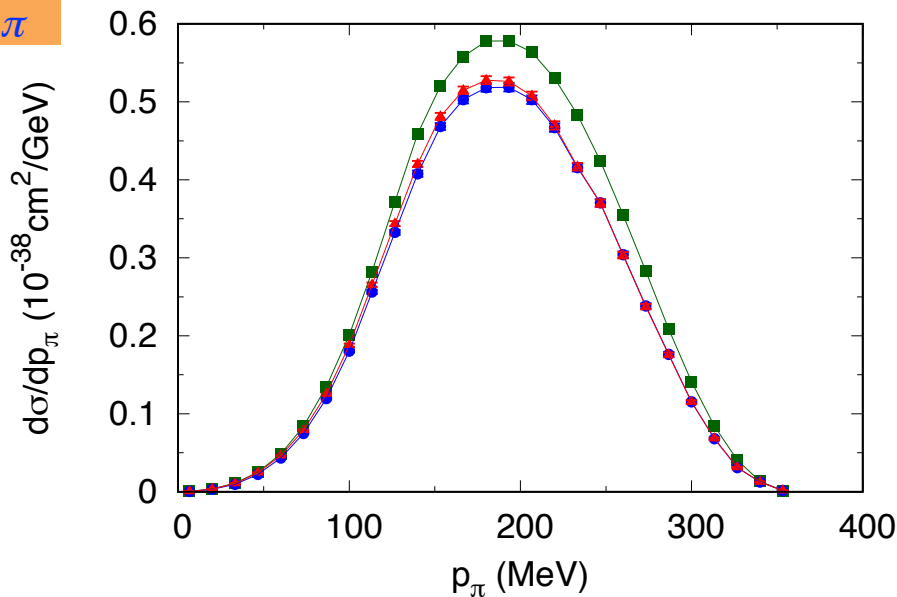
$Q^2$



(NN) FSI effects are significantly larger for  $E_\nu = 0.5 \text{ GeV}$

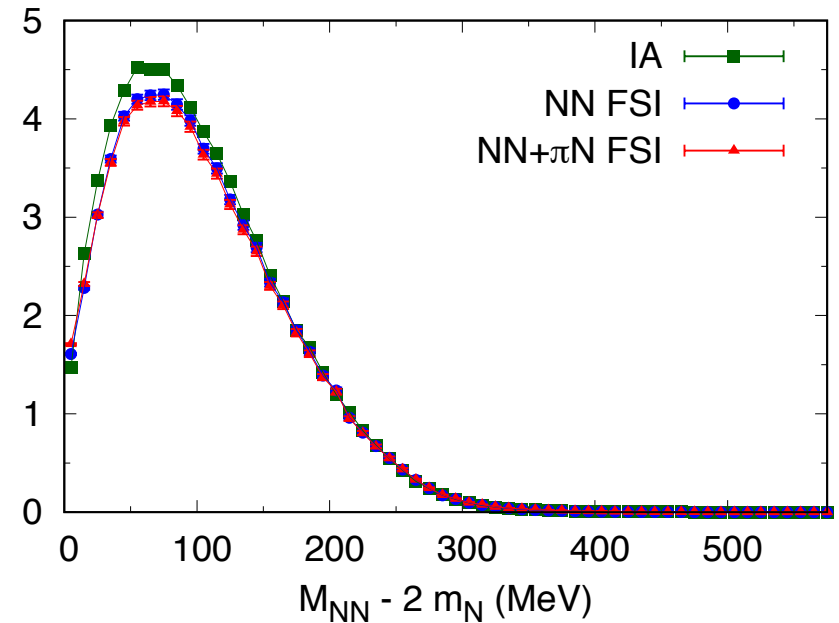
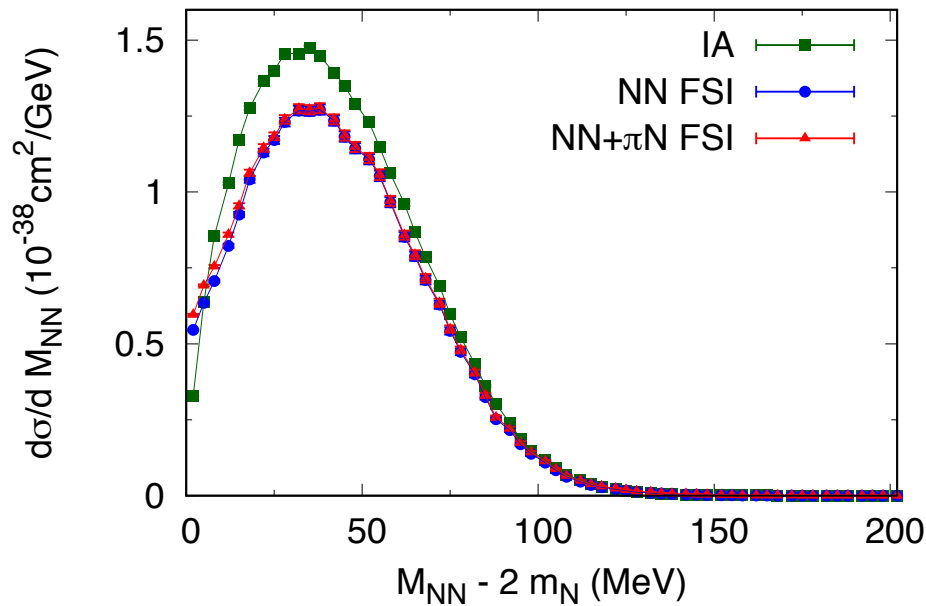
Why ?

$p_\pi$



$E_\nu = 0.5 \text{ GeV}$ 

vs

 $E_\nu = 1 \text{ GeV}$ 

- NN FSI effect is large at low NN energy region ( $\lesssim 50 \text{ MeV}$ ) where orthogonality between  $pn$  scattering states and deuteron is most effective
  - Low NN energy region occupies a relatively larger portion of phase-space for low  $E_\nu$
- Larger NN FSI effect for low  $E_\nu$



How could FSI distort elementary  $\nu$ - $p$  and  $\nu$ - $n$  cross sections extracted from  $\nu$ - $d$  cross sections ?

Q : How to extract  $\sigma(\nu_\mu p \rightarrow \mu \pi^+ p)$  and  $\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$  from  $\sigma(\nu_\mu d \rightarrow \mu \pi^+ pn)$  ?

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- A correct procedure

Develop a deuteron reaction model with FSI, and analyze  $\sigma(\nu_\mu d \rightarrow \mu \pi^+ pn)$

→ (correct)  $\nu_\mu p \rightarrow \mu \pi^+ p$  and  $\nu_\mu n \rightarrow \mu \pi^+ n$  elementary amplitudes

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- More common (and practical) procedure

Cutting out FSI-free  $\nu_\mu p \rightarrow \mu \pi^+ p$  and  $\nu_\mu n \rightarrow \mu \pi^+ n$  events (→ cross sections)

Easier ! Great ! But how ?

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(i) Follow ANL and BNL analyses → details lost in history 😞

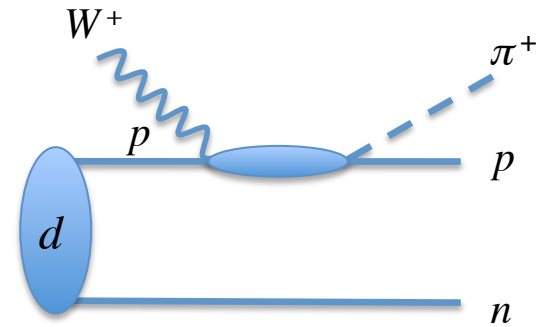
(ii) Our own *rough estimate* for here; more elaborate estimate under way

→ How cutting-out procedure would make a mistake in the presence of FSI

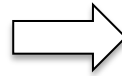
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How ?

Cutting out FSI-free  $\nu_\mu p \rightarrow \mu \pi^+ p$  cross section  
 $\approx$  Cutting out contribution from this mechanism  $\rightarrow$



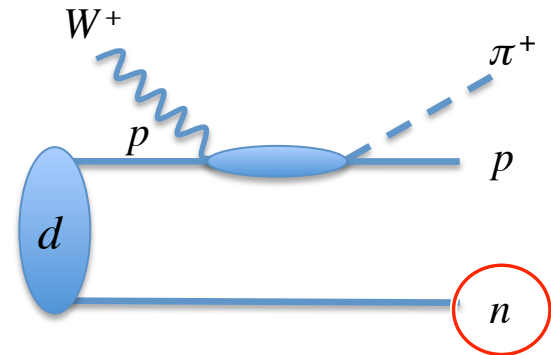
For doing this, we need to know how  
a cross section of this guy behaves ...



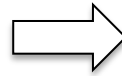
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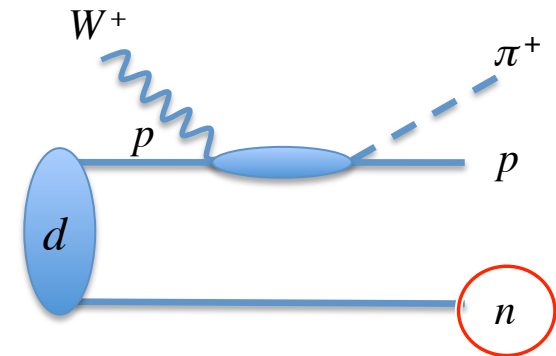


Momentum distribution of the **spectator**  
**neutron** should follow the Fermi motion ?

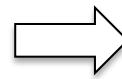
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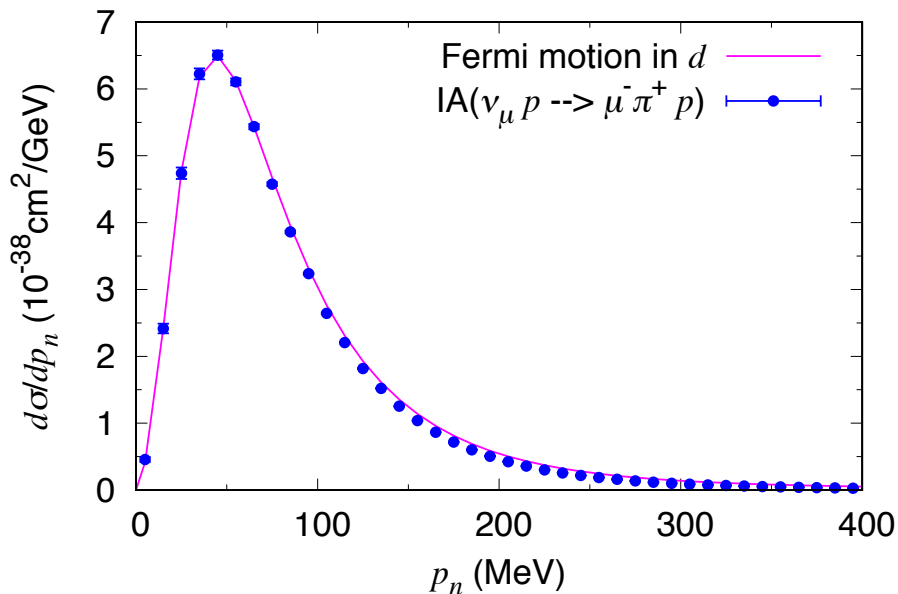
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Momentum distribution of the **spectator neutron** should follow the Fermi motion ?



That's right !

IA ( $\nu_\mu p \rightarrow \mu \pi^+ p$ ) contribution behaves:

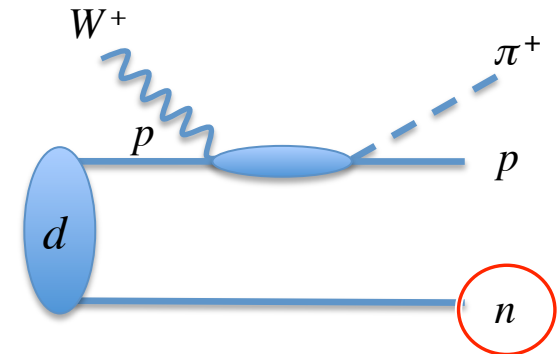
$$\frac{d\sigma}{dp_n} = \underbrace{N(p_n)}_{\approx 1} \sigma(\nu_\mu p \rightarrow \mu \pi^+ p) |\psi_d(p_n)|^2$$

But this is a phenomenological finding because  $\nu_\mu p \rightarrow \mu \pi^+ p$  amplitude depends on  $\vec{p}_n$ .

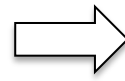
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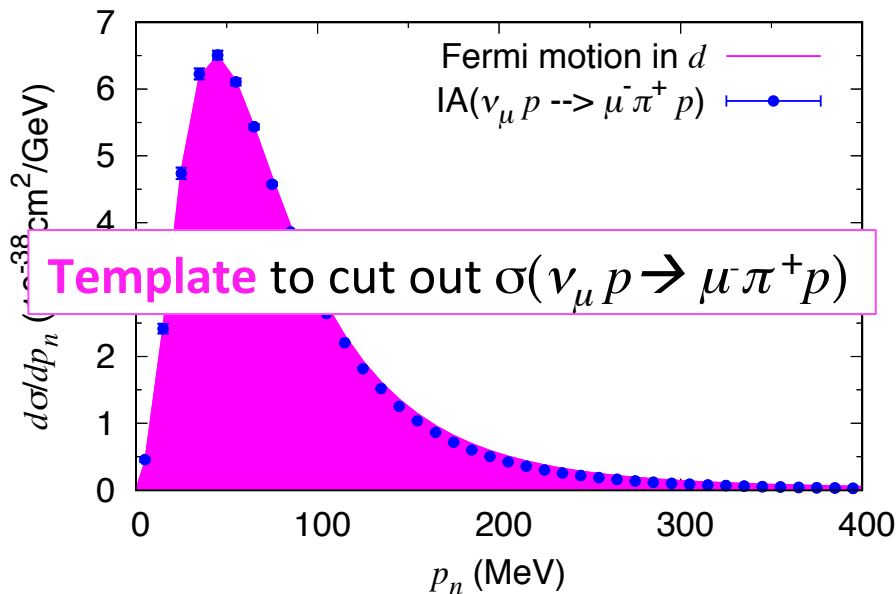
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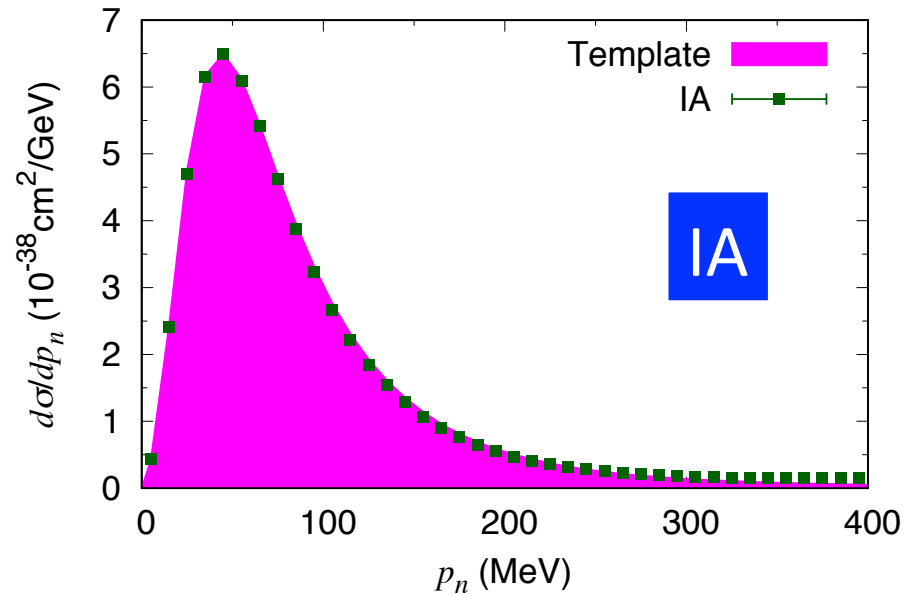
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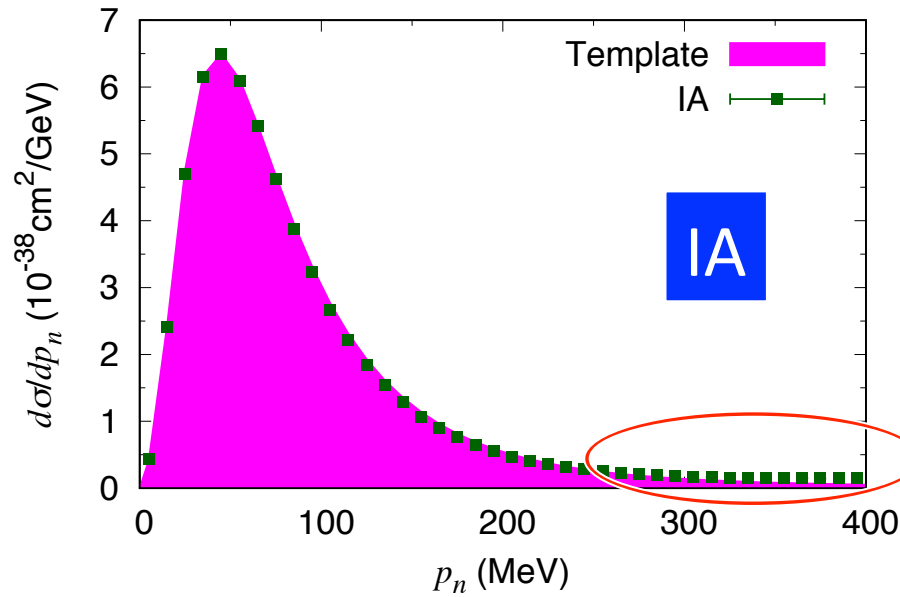


$\sigma(\nu_\mu p \rightarrow \mu \pi^+ p)$  from  $\sigma(\nu_\mu d \rightarrow \mu \pi^+ p n)$  at  $E_\nu = 1$  GeV



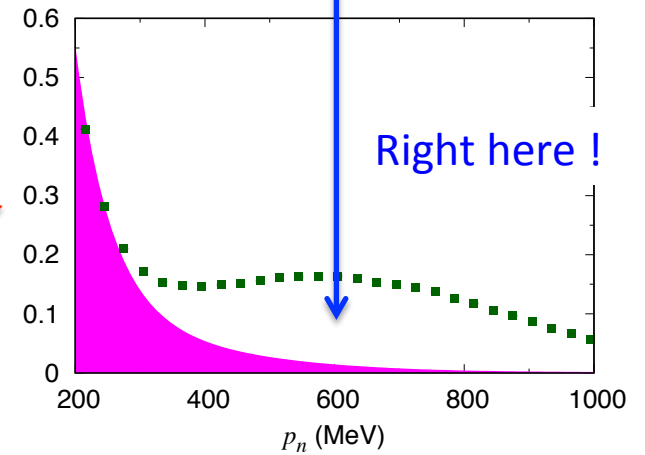
We cut out  $\sigma(\nu_\mu p \rightarrow \mu \pi^+ p)$  (< 1 % accuracy)  
... but where is  $\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$  ?

# $\sigma(\nu_\mu p \rightarrow \mu \pi^+ p)$ from $\sigma(\nu_\mu d \rightarrow \mu \pi^+ p n)$ at $E_\nu = 1$ GeV

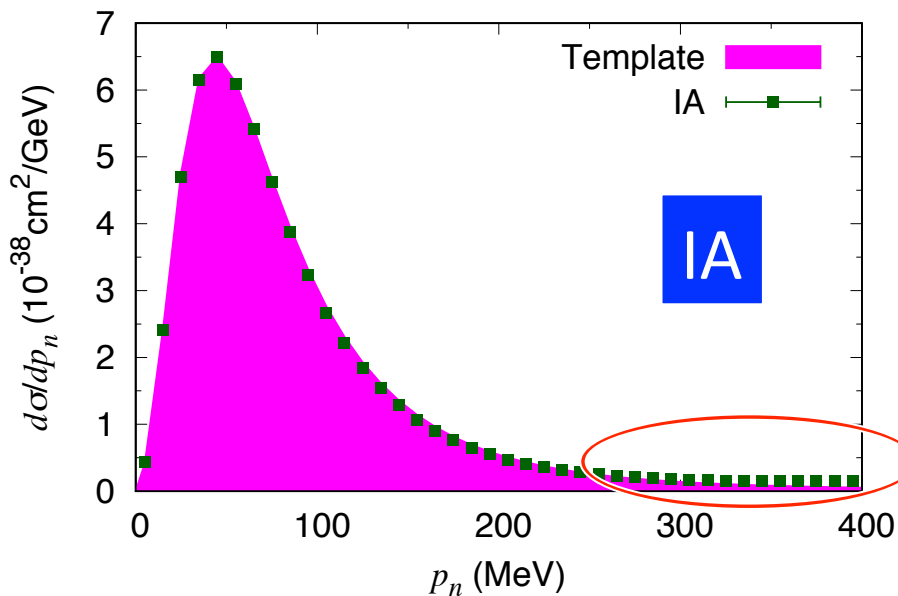


Zoom-in !

We cut out  $\sigma(\nu_\mu p \rightarrow \mu \pi^+ p)$  (< 1 % accuracy)  
 ... but where is  $\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$  ?

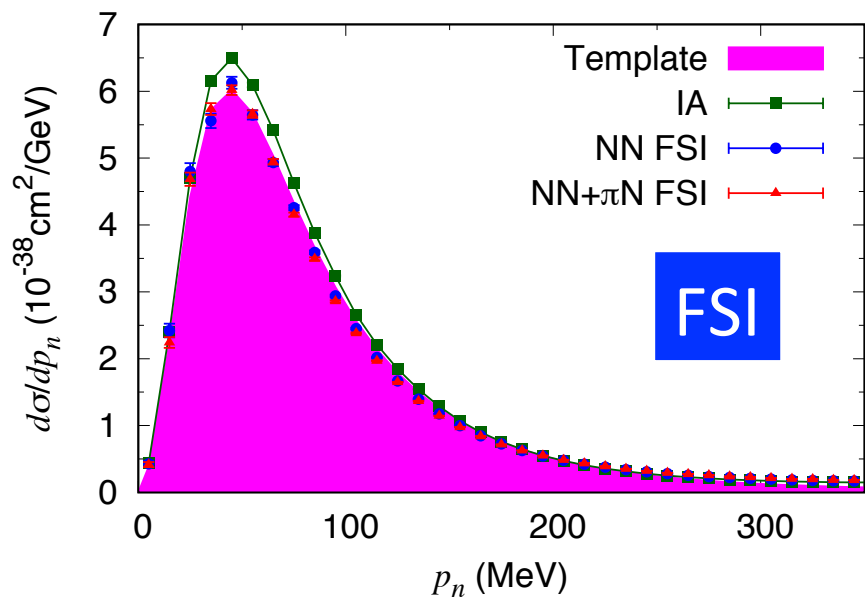
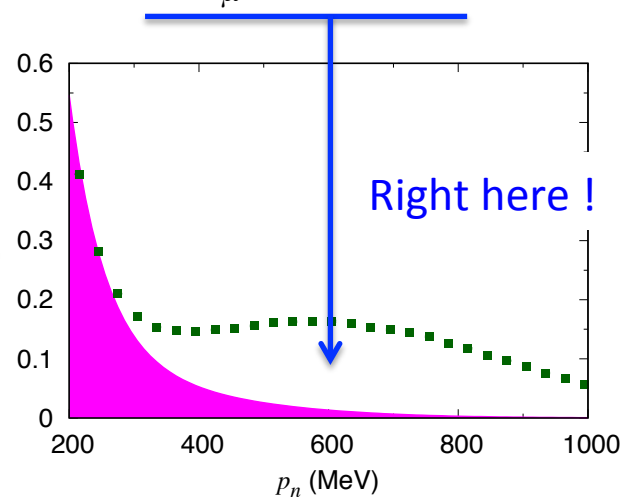


# $\sigma(\nu_\mu p \rightarrow \mu \pi^+ p)$ from $\sigma(\nu_\mu d \rightarrow \mu \pi^+ pn)$ at $E_\nu = 1$ GeV

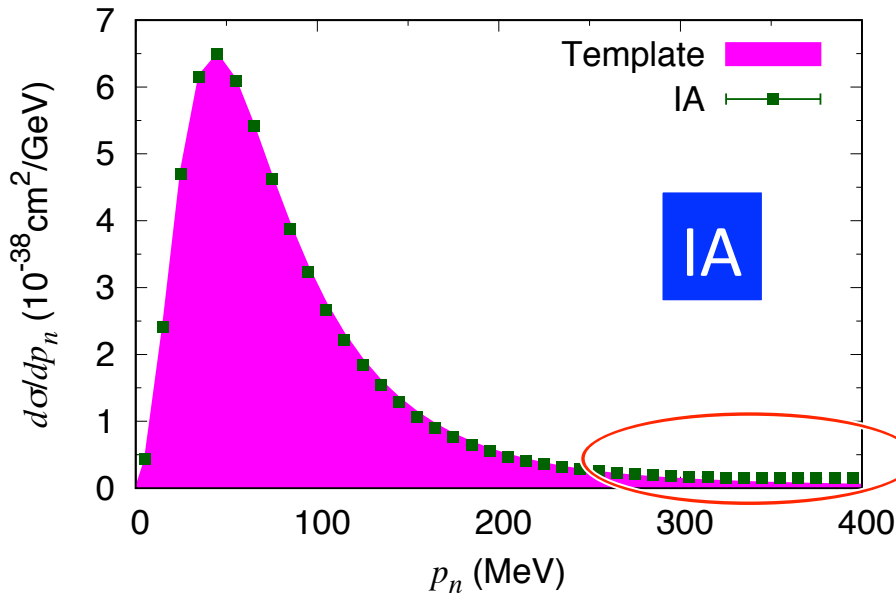


We cut out  $\sigma(\nu_\mu p \rightarrow \mu \pi^+ p)$  (< 1 % accuracy)  
 ... but where is  $\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$  ?

Zoom-in !

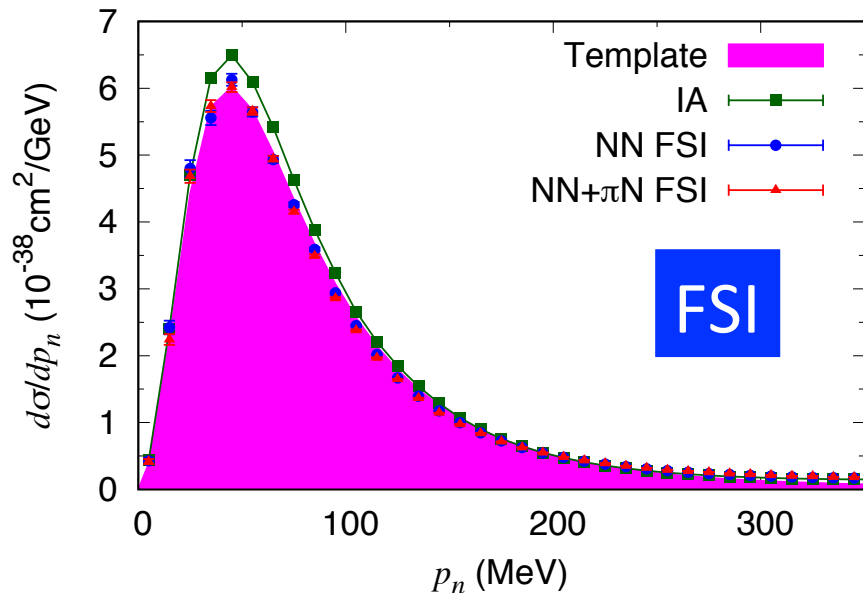
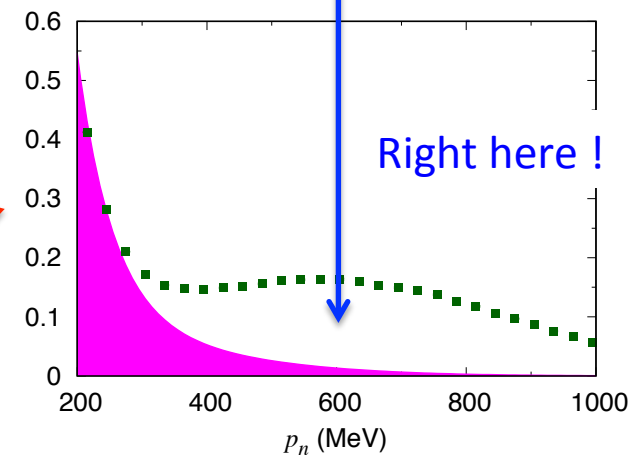


# $\sigma(\nu_\mu p \rightarrow \mu \pi^+ p)$ from $\sigma(\nu_\mu d \rightarrow \mu \pi^+ pn)$ at $E_\nu = 1$ GeV



We cut out  $\sigma(\nu_\mu p \rightarrow \mu \pi^+ p)$  (< 1 % accuracy)  
 ... but where is  $\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$  ?

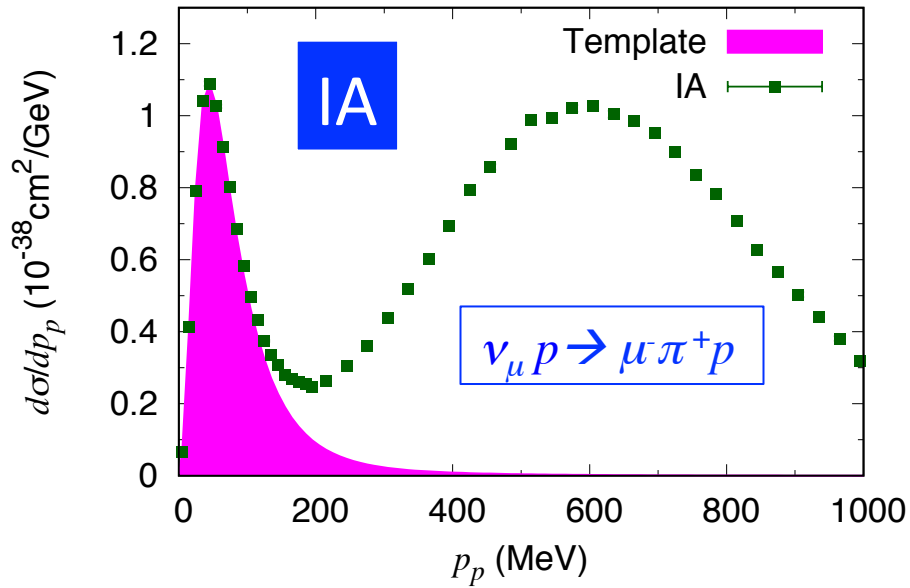
Zoom-in !



In the presence of FSI, the template method underestimates  $\sigma(\nu_\mu p \rightarrow \mu \pi^+ p)$  by **8 %**. Why ?

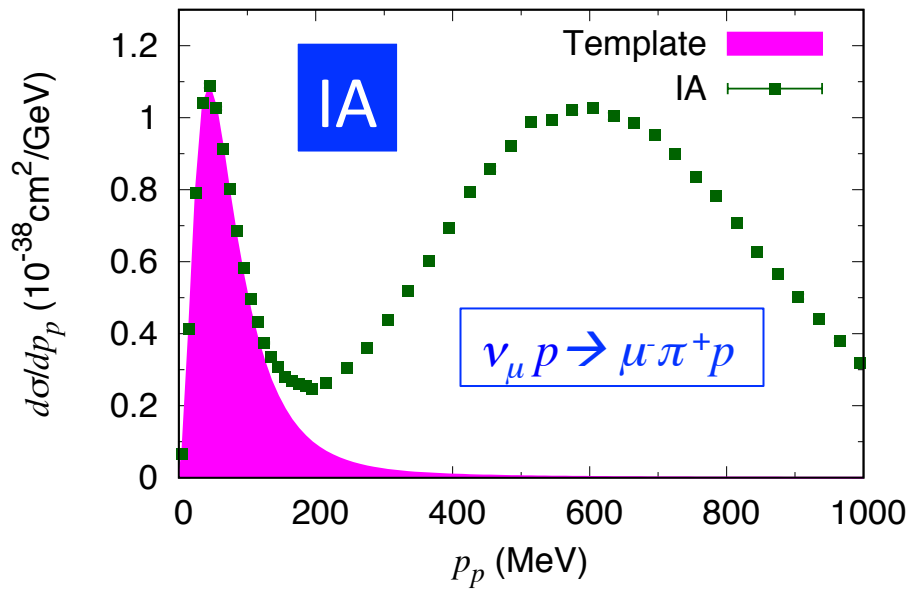
- FSI create events with large spectator momentum  $\rightarrow$  the template can remove it
- FSI reduce events with low spectator momentum  $\rightarrow$  the template can't compensate

$\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$  from  $\sigma(\nu_\mu d \rightarrow \mu \pi^+ pn)$  at  $E_\nu = 1$  GeV



**Method** : The template analysis done for spectator-**proton** momentum distribution

$\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$  from  $\sigma(\nu_\mu d \rightarrow \mu \pi^+ p n)$  at  $E_\nu = 1$  GeV

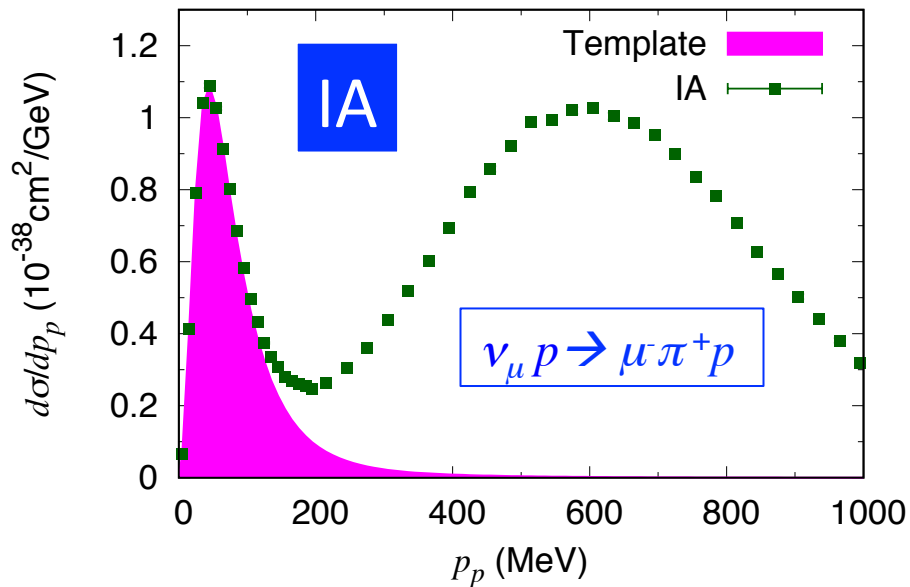


**Method** : The template analysis done for spectator-**proton** momentum distribution

$\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$  is cut out at 1% precision !

Unexpectedly successful because **mixture of**  
 $\nu_\mu p \rightarrow \mu \pi^+ p$  inside the template should have  
 been removed; interference is hard to handle

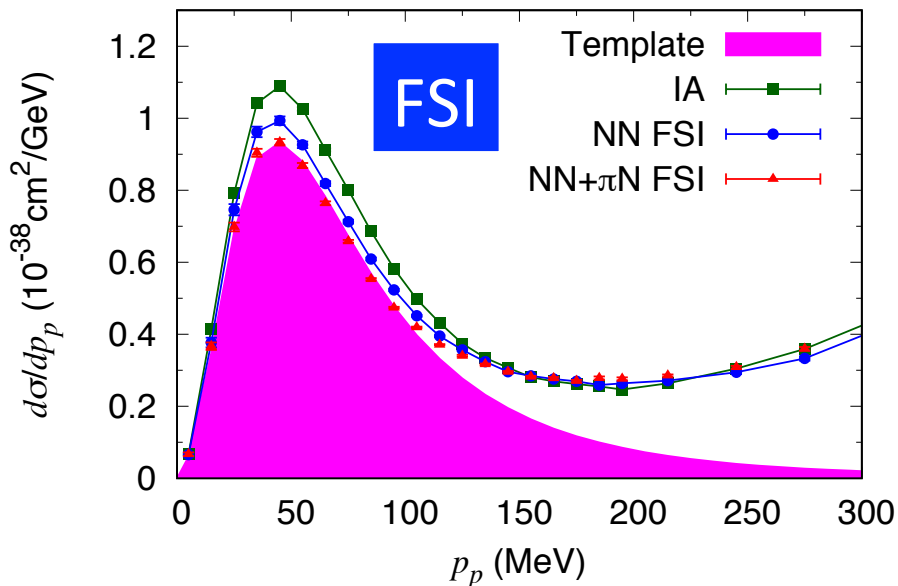
# $\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$ from $\sigma(\nu_\mu d \rightarrow \mu \pi^+ pn)$ at $E_\nu = 1$ GeV



**Method**: The template analysis done for spectator-**proton** momentum distribution

$\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$  is cut out at 1% precision !

Unexpectedly successful because **mixture of  $\nu_\mu p \rightarrow \mu \pi^+ p$**  inside the template should have been removed; interference is hard to handle



$\pi$ N and NN FSI effects are comparable size

$\pi$ N FSI is visible only where  $\nu$ - $p$  is suppressed

In the presence of FSI, the template method underestimates  $\sigma(\nu_\mu n \rightarrow \mu \pi^+ n)$  by **15%**

# Conclusion



# Conclusions

- Total ( $\sigma$ ) and single differential ( $d\sigma/dX$ ) cross sections for  $\nu_\mu d \rightarrow \mu^- \pi^+ p n$  with FSI are calculated for the first time
- FSI effects on  $\sigma$  and  $d\sigma/dX$  for  $\nu_\mu d \rightarrow \mu^- \pi^+ p n$  are examined; (NN) FSI reduces  $\sigma$  by 10%, 6%, and 5% at  $E_\nu = 0.5, 1, \text{ and } 1.5 \text{ GeV}$
- $\sigma(\nu_\mu p \rightarrow \mu^- \pi^+ p)$  and  $\sigma(\nu_\mu n \rightarrow \mu^- \pi^+ n)$  are extracted from  $\sigma(\nu_\mu d \rightarrow \mu^- \pi^+ p n)$  using Fermi motion-based template (temporary exercise)

If FSI is absent, the template method works well

If FSI is present, the template method significantly underestimates  $\sigma(\nu_\mu p \rightarrow \mu^- \pi^+ p)$  by 8%, and  $\sigma(\nu_\mu n \rightarrow \mu^- \pi^+ n)$  by 15%.

Very likely, other kinematical cut-based methods have the same problem because we cannot infer from data amount of reduction caused by FSI

Thank you very much  
for your attention

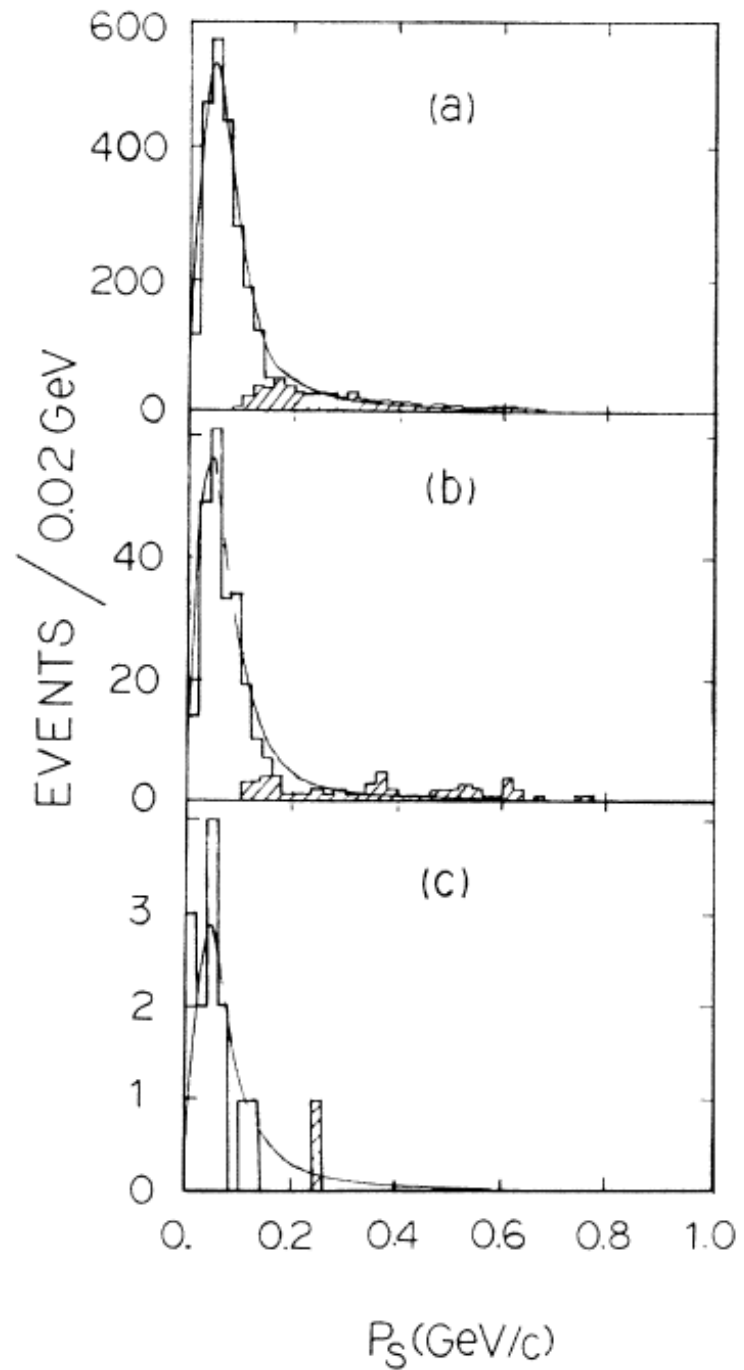
## Acknowledgments

- Financial support for this work  
KAKENHI JP25105010  
FAPESP 2016/15618-8
- Computing resource  
Blues: Laboratory Computing Resource Center  
at Argonne National Laboratory

BACKUP

# BNL analysis

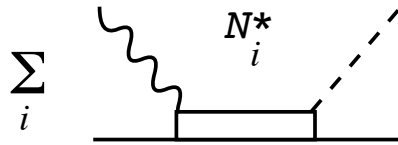
PRD 34, 2554 (1986)



# Previous models for $\nu$ -induced $1\pi$ production in resonance region

resonant only

Rein et al. (1981), (1987); Lalalulich et al. (2005), (2006)



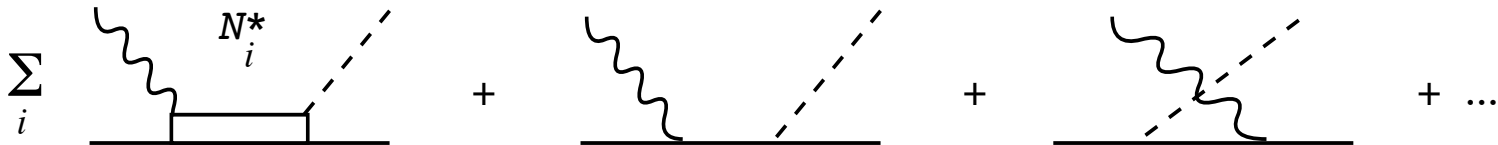
$VNN^*$  : helicity amplitudes listed in PDG

$ANN^*$  : quark model, PCAC relation to  $|\pi NN^*|$  (PDG)

relative phases among  $N^*$ 's are out of control

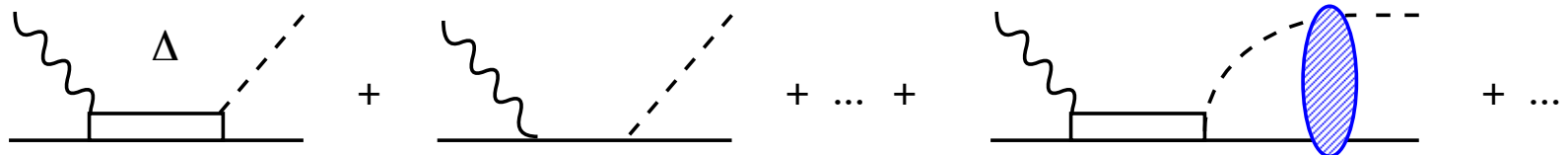
+ non-resonant (tree-level non-res)

Hernandez et al. (2007), (2010); Lalakulich et al. (2010)



+ rescattering ( $\pi N$  unitarity,  $\Delta(1232)$  region)

Sato, Lee (2003), (2005)



# DCC (Dynamical Coupled-Channel) model

Matsuyama et al., Phys. Rep. **439**, 193 (2007)

Kamano et al., PRC 88, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation for meson-baryon scattering

$$T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb}$$

$$\{a, b, c\} = \pi N, \eta N, \pi\pi N, \pi\Delta, \sigma N, \rho N, K\Lambda, K\Sigma$$

By solving the LS equation, coupled-channel unitarity is fully taken into account

# DCC (Dynamical Coupled-Channel) model

Matsuyama et al., Phys. Rep. **439**, 193 (2007)

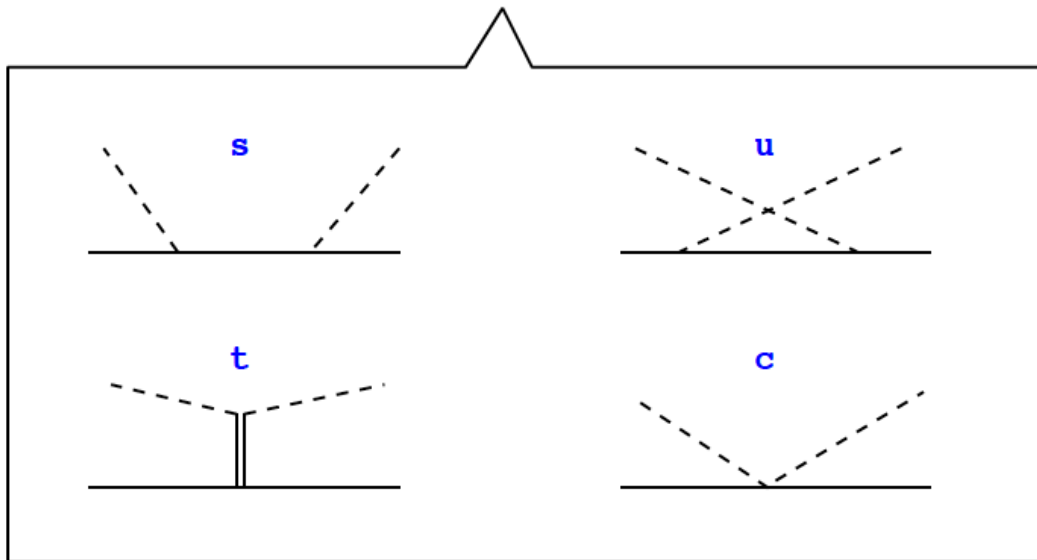
Kamano et al., PRC 88, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation for meson-baryon scattering

$$T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb}$$

$$\mathbf{V}_{ab} = \text{[diagram 1]} + \text{[diagram 2]}$$

The diagram shows the vertex  $\mathbf{V}_{ab}$  as the sum of two terms. The first term is a solid horizontal line with a black dot in the center, and two dashed lines meeting at the dot. The second term is a solid horizontal line with a rectangular box on top, and two dashed lines meeting at the top corners of the box. The text "bare N\*" is written above the box.



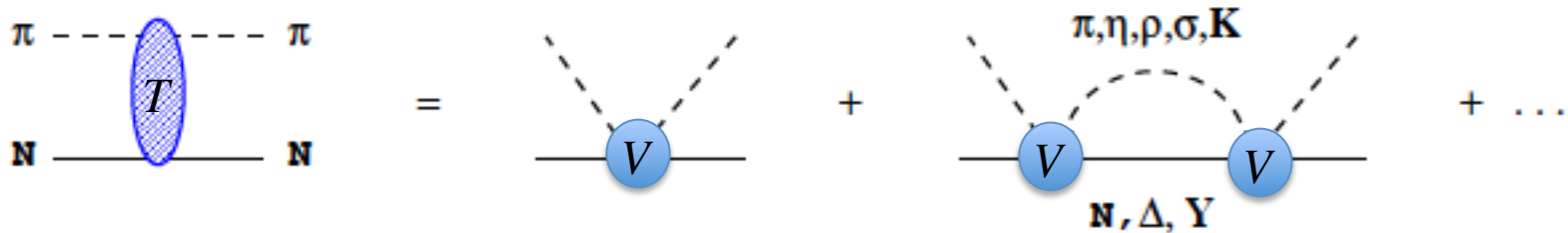
# DCC (Dynamical Coupled-Channel) model

Matsuyama et al., Phys. Rep. **439**, 193 (2007)

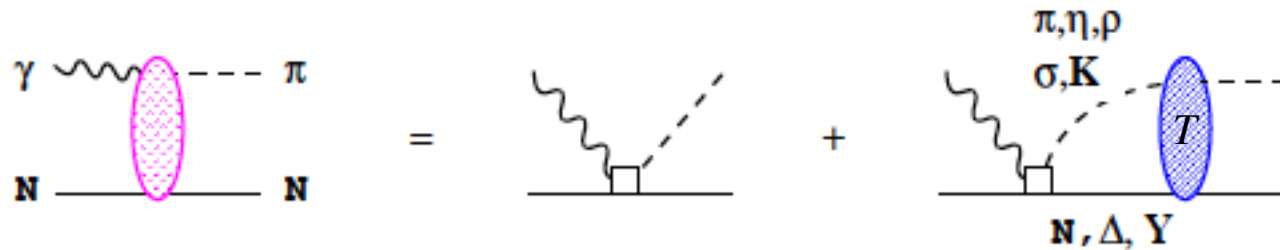
Kamano et al., PRC **88**, 035209 (2013)

Coupled-channel Lippmann-Schwinger equation for meson-baryon scattering

$$T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb}$$



In addition,  $\gamma N$ ,  $W^\pm N$ ,  $ZN$  channels are included perturbatively





# Relation between neutrino and electron (photon) interactions

Charged-current (CC) interaction (e.g.  $\nu_\mu + n \rightarrow \mu^- + p$ )

$$L^{cc} = \frac{G_F V_{ud}}{\sqrt{2}} [J_\lambda^{cc} \ell_{cc}^\lambda + h.c.] \quad J_\lambda^{cc} = V_\lambda - A_\lambda \quad \ell_{cc}^\lambda = \bar{\psi}_\mu \gamma^\lambda (1 - \gamma_5) \psi_\nu$$

Electromagnetic interaction (e.g.  $\gamma^{(*)} + p \rightarrow p$ )

$$L^{em} = e J_\lambda^{em} A_{em}^\lambda \quad J_\lambda^{em} = V_\lambda + V_\lambda^{IS}$$

$V$  and  $V^{IS}$  in  $J^{em}$  can be separately determined by analyzing photon ( $Q^2=0$ ) and electron reaction ( $Q^2 \neq 0$ ) data on both proton and neutron targets, because:

$$\langle p | V_\lambda | p \rangle = - \langle n | V_\lambda | n \rangle \quad \langle p | V_\lambda^{IS} | p \rangle = \langle n | V_\lambda^{IS} | n \rangle$$

Matrix element for the weak vector current is obtained from analyzing electromagnetic processes

$$\langle p | V_\lambda | n \rangle = \sqrt{2} \langle p | V_\lambda | p \rangle$$

# DCC model for axial current

Because neutrino reaction data are scarce, axial current cannot be determined phenomenologically

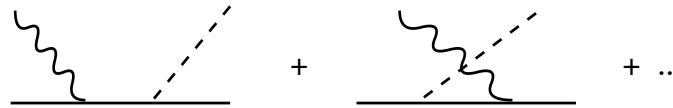
→ **Chiral symmetry** and **PCAC** (partially conserved axial current) are guiding principle

**PCAC relation**  $\langle X' | q \cdot A | X \rangle \sim i f_\pi \langle X' | T | \pi X \rangle$

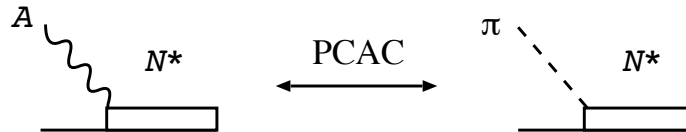
$Q^2=0$

non-resonant mechanisms

$$\partial_\mu \pi \rightarrow f_\pi A_\mu^{external}$$



resonant mechanisms



*Interference among resonances and background can be uniquely fixed within DCC model*

# DCC model for axial current

$Q^2 \neq 0$   $F_A(Q^2)$  : axial form factors

non-resonant mechanisms  $F_A(Q^2) = \left( \frac{1}{1 + Q^2 / M_A^2} \right)^2$   $M_A = 1.02 \text{ GeV}$

resonant mechanisms  $F_A(Q^2) = \left( \frac{1}{1 + Q^2 / M_A^2} \right)^2$

More neutrino data are necessary to fix axial form factors for  $ANN^*$

*Neutrino cross sections will be predicted with this axial current*

DCC analysis of  $\gamma N, \pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$

and electron scattering data

# DCC analysis of meson production data

Kamano, Nakamura, Lee, Sato, PRC 88 (2013)

Fully combined analysis of  $\gamma N, \pi N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$  data

$d\sigma / d\Omega$  and polarization observables ( $W \leq 2.1$  GeV)

~ 23,000 data points are fitted

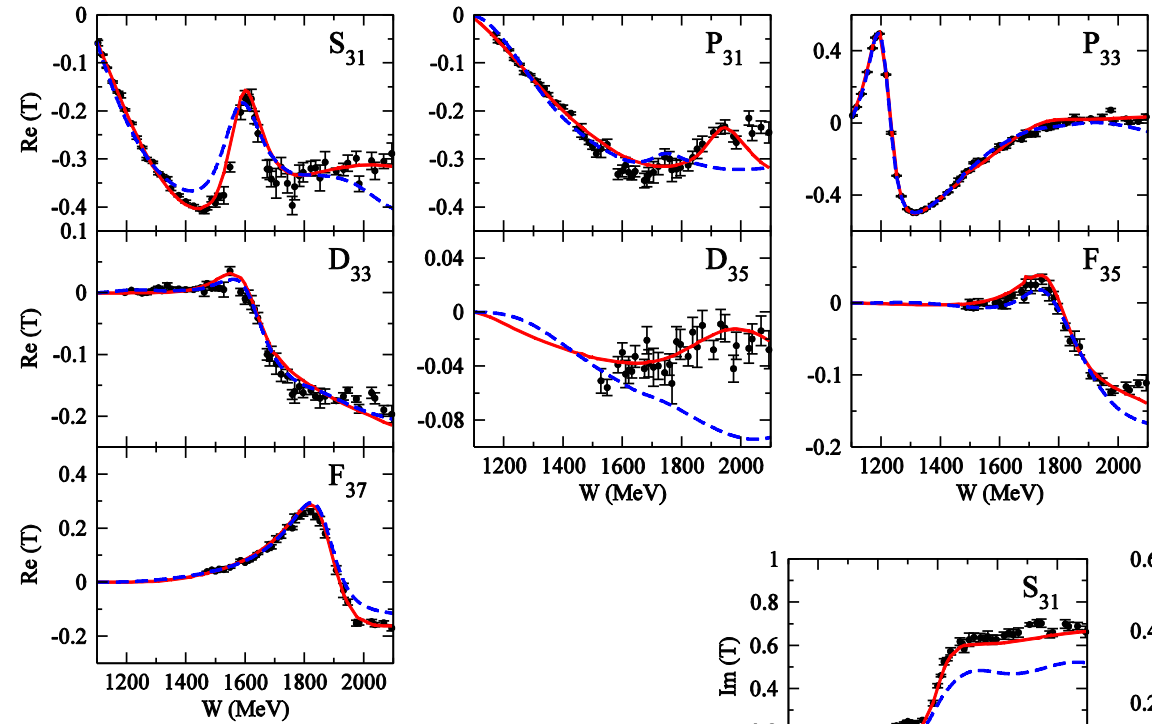
by adjusting parameters ( $N^*$  mass,  $N^* \rightarrow MB$  couplings, cutoffs)



Data for electron scattering on proton and neutron are analyzed by adjusting

$\gamma^* N \rightarrow N^*$  coupling strength at different  $Q^2$  values ( $Q^2 \leq 3$  (GeV/c)<sup>2</sup>)

# Partial wave amplitudes of $\pi N$ scattering



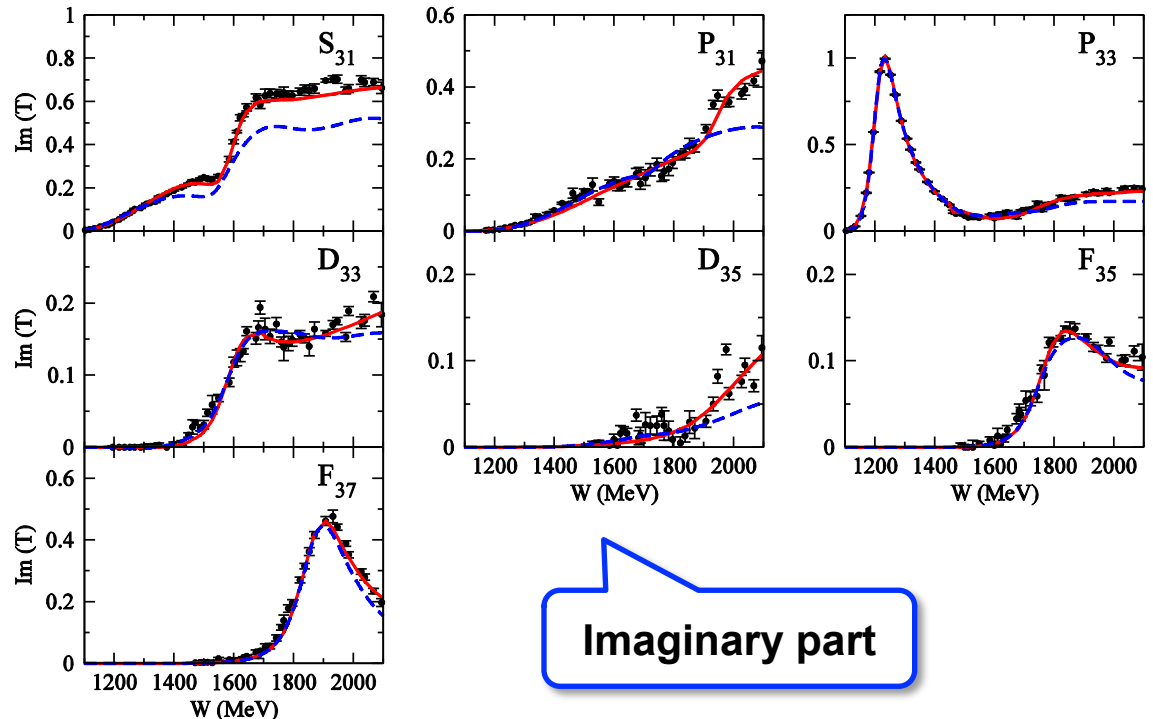
Real part

$$I = \frac{3}{2}$$

— Kamano, Nakamura, Lee, Sato,  
PRC 88 (2013)

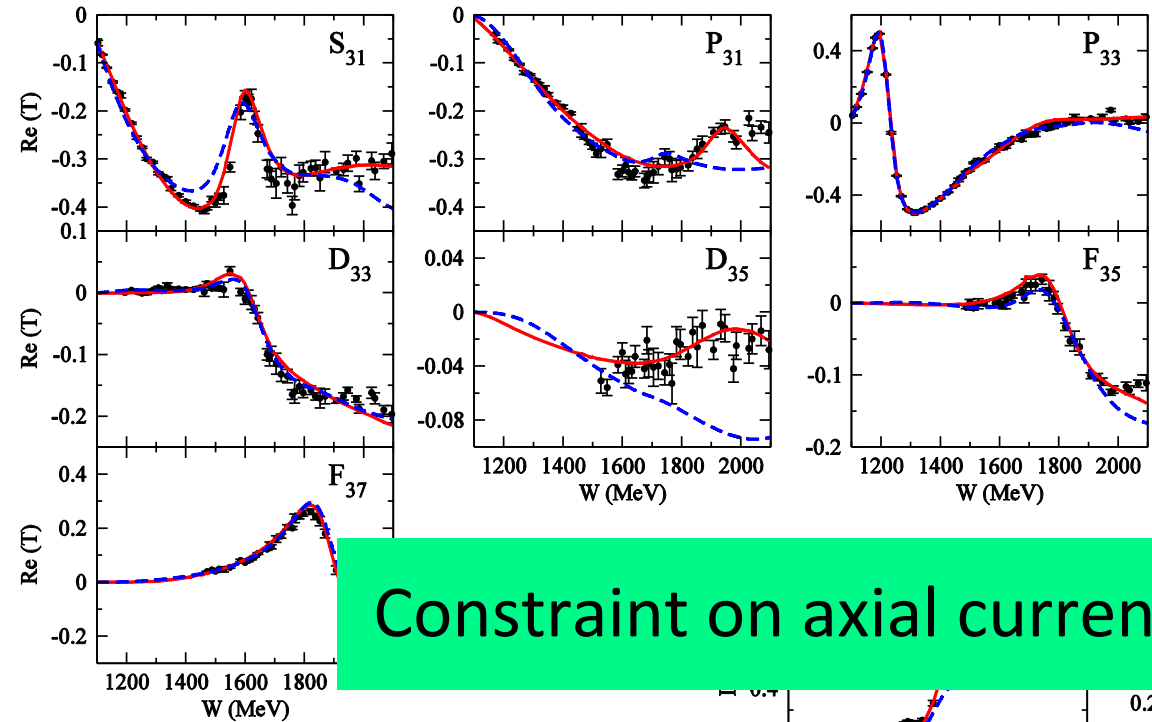
- - - Previous model  
(fitted to  $\pi N \rightarrow \pi N$  data only)  
[PRC 76 065201 (2007)]

Data: SAID  $\pi N$  amplitude



Imaginary part

# Partial wave amplitudes of $\pi N$ scattering



Real part

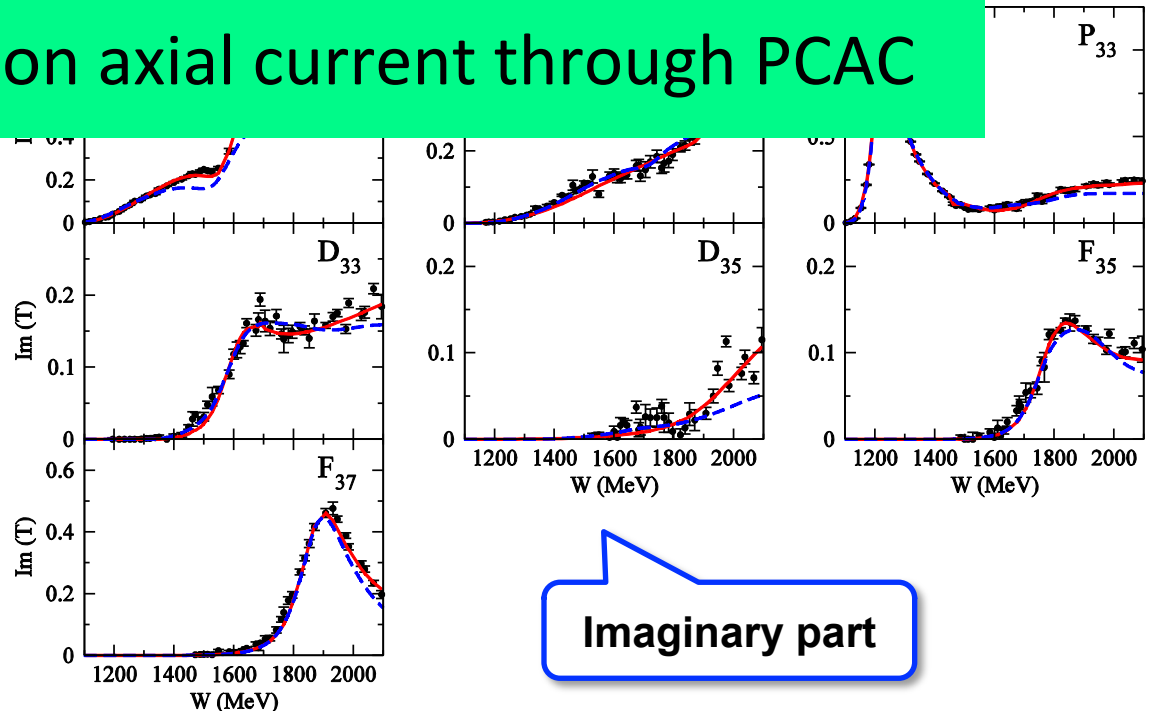
$$I = \frac{3}{2}$$

Constraint on axial current through PCAC

— Kamano, Nakamura, Lee, Sato,  
PRC 88 (2013)

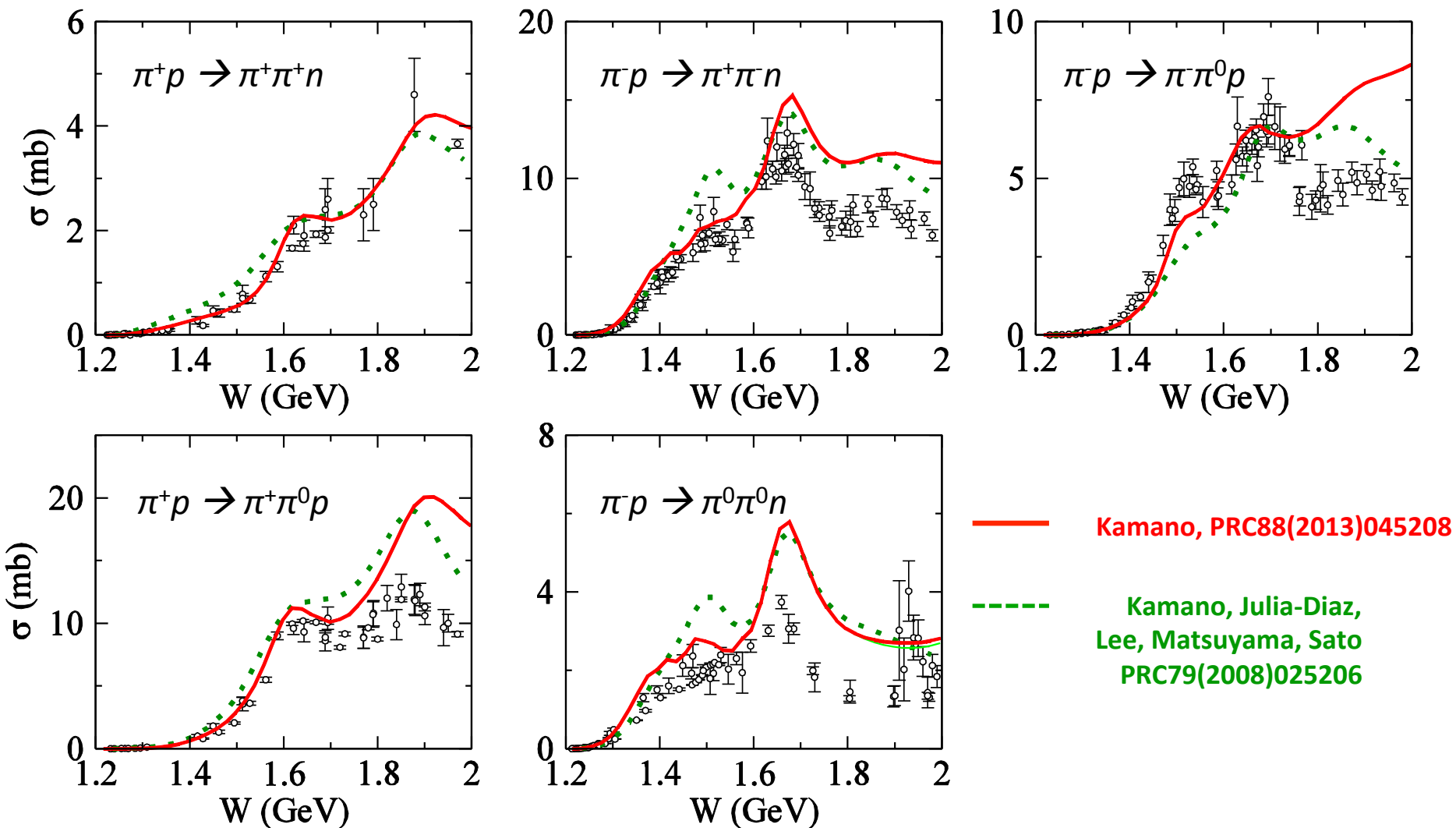
- - - Previous model  
(fitted to  $\pi N \rightarrow \pi N$  data only)  
[PRC76 065201 (2007)]

Data: SAID  $\pi N$  amplitude



Imaginary part

# Predicted $\pi N \rightarrow \pi\pi N$ total cross sections with our DCC model



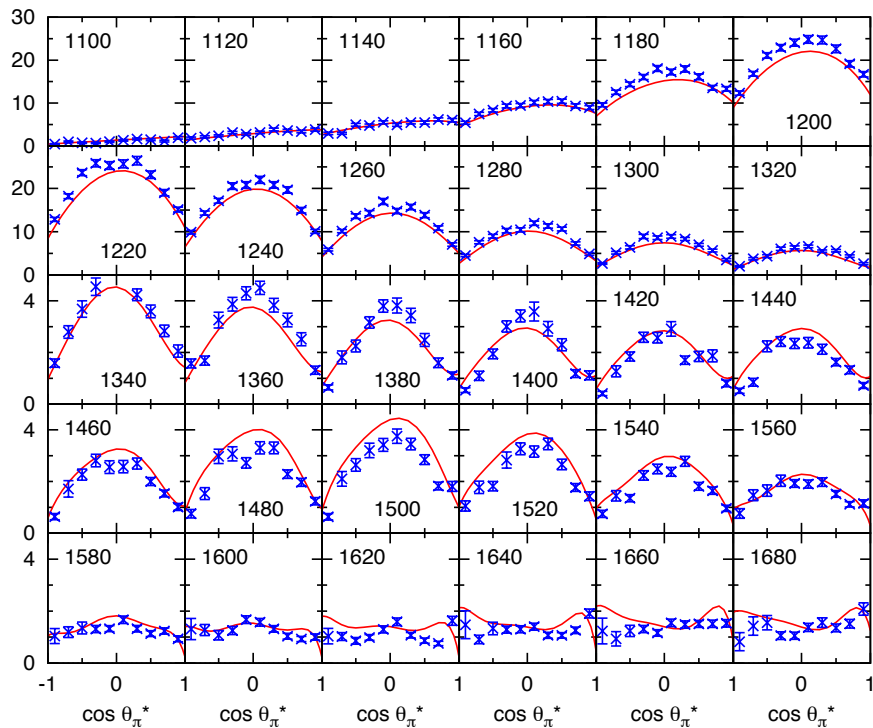


# Single $\pi$ production in electron-proton scattering

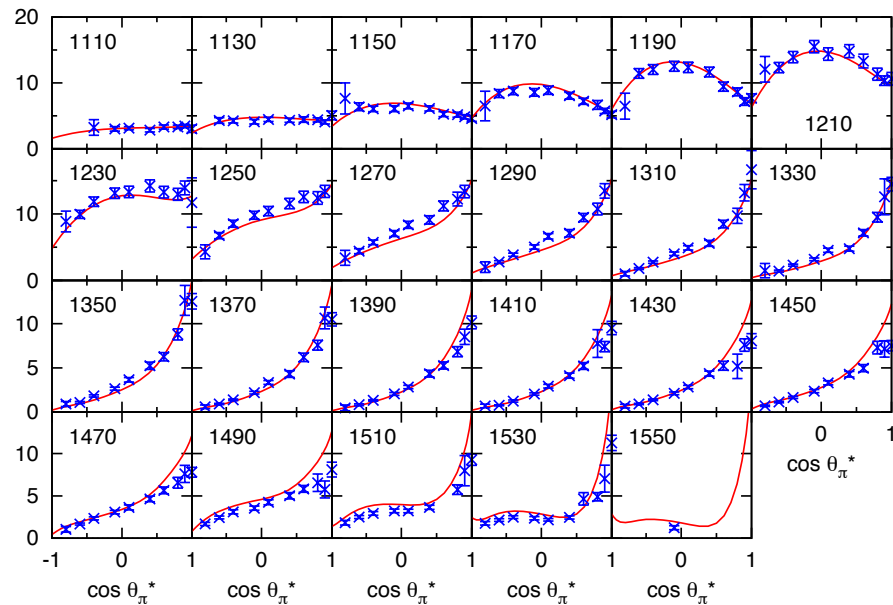
**Purpose** : Determine  $Q^2$ -dependence of vector coupling of  $p-N^*$  :  $V_{pN^*}(Q^2)$

$\sigma_T + \varepsilon \sigma_L$  for  $Q^2=0.40$  (GeV/c)<sup>2</sup> and  $W=1.1 - 1.68$  GeV

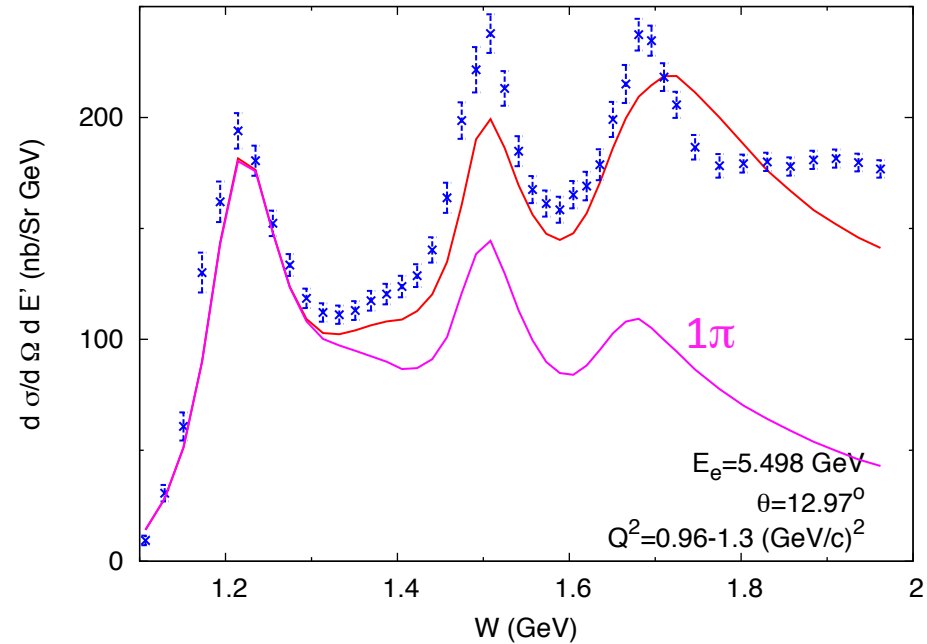
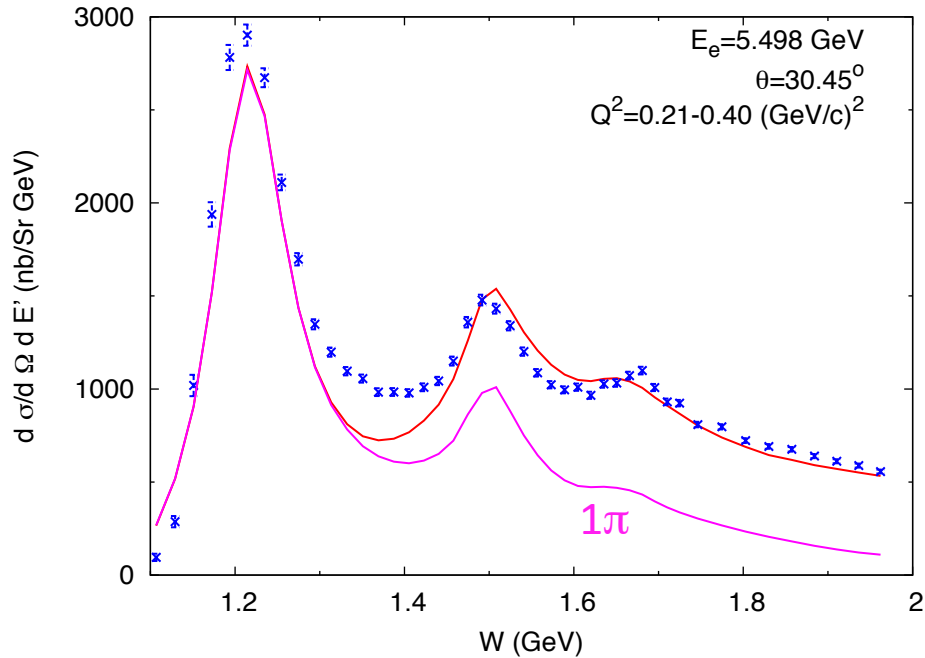
$p(e, e' \pi^0) p$



$p(e, e' \pi^+) n$



# Inclusive electron-proton scattering



Data: JLab E00-002 (preliminary)

- Reasonable fit to data for application to neutrino interactions
- Important  $2\pi$  contributions for high  $W$  region

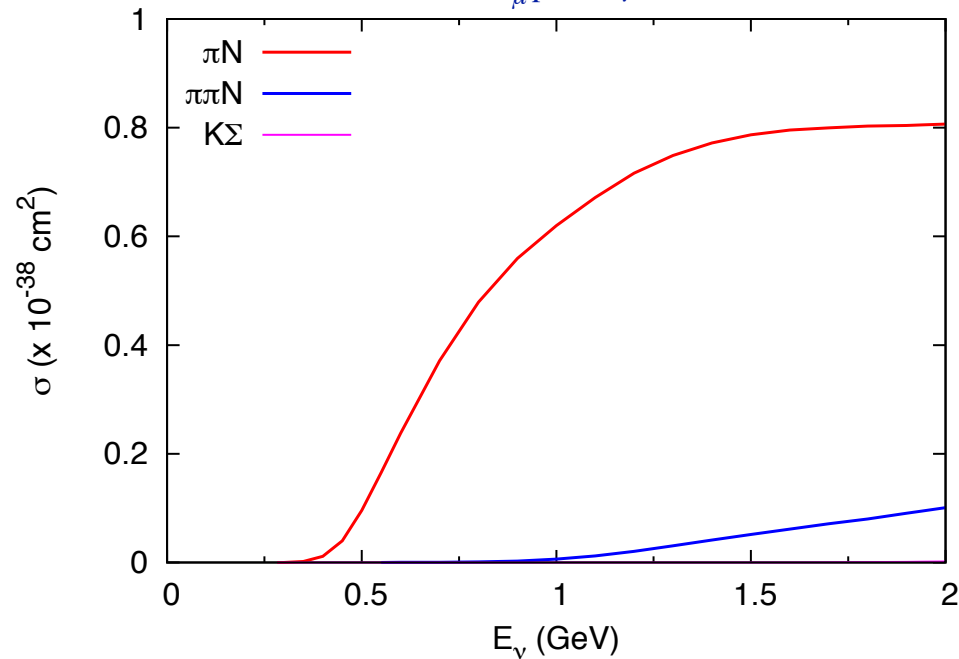
Similar analysis of **electron-neutron scattering** data has also been done

*DCC vector currents has been tested by data for whole kinematical region*

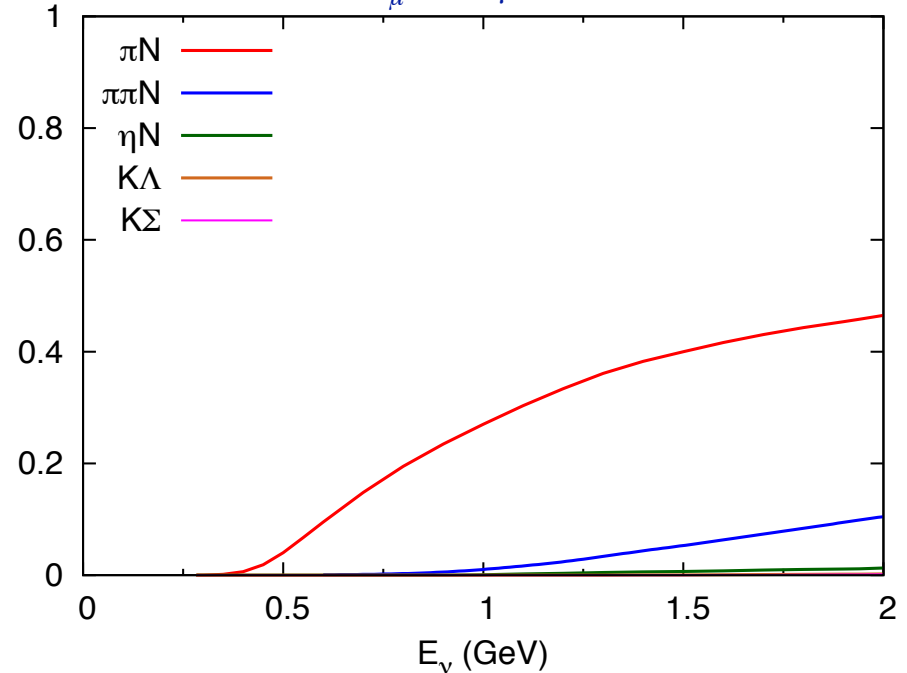
*relevant to neutrino interactions of  $E_\nu \leq 2$  GeV*

# Cross section for $\nu_\mu N \rightarrow \mu^- X$

$\nu_\mu p \rightarrow \mu^- X$

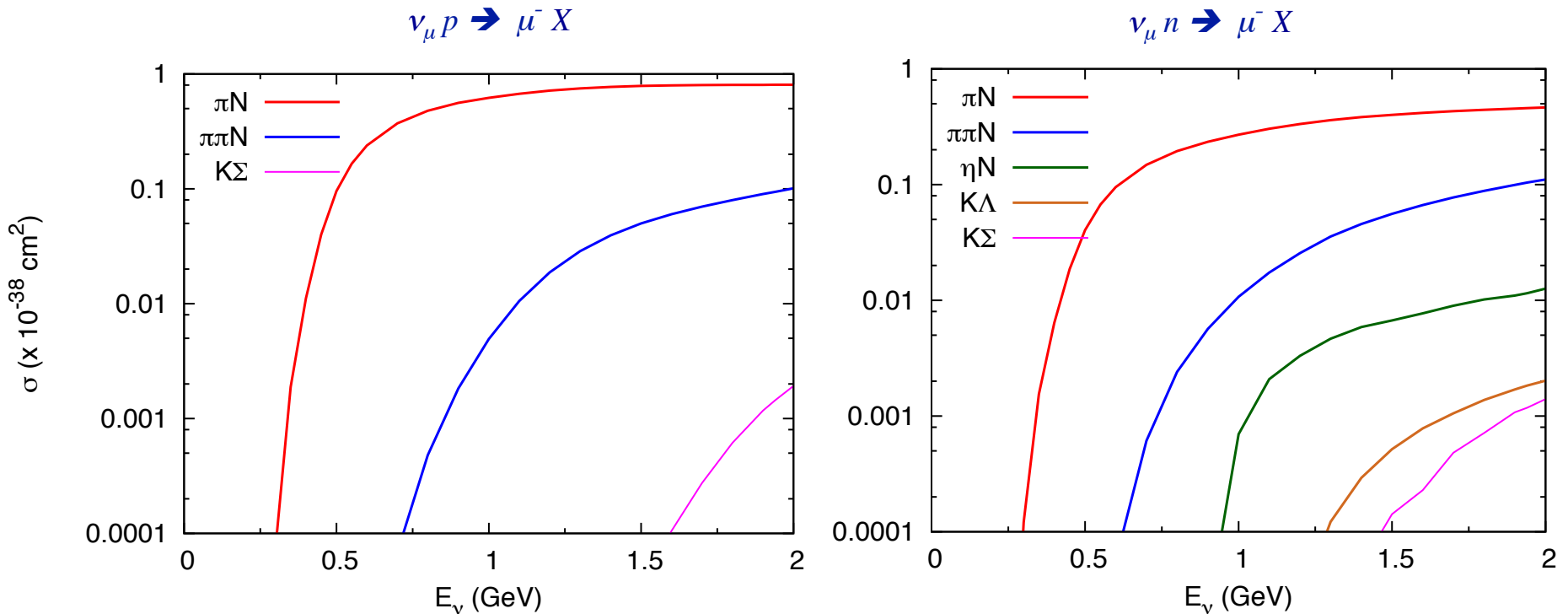


$\nu_\mu n \rightarrow \mu^- X$



- $\pi N$  &  $\pi\pi N$  are main channels in few-GeV region
- DCC model gives predictions for **all final states**
- $\eta N$ ,  $KY$  cross sections are  $10^{-1} - 10^{-2}$  smaller

# Cross section for $\nu_\mu N \rightarrow \mu^- X$



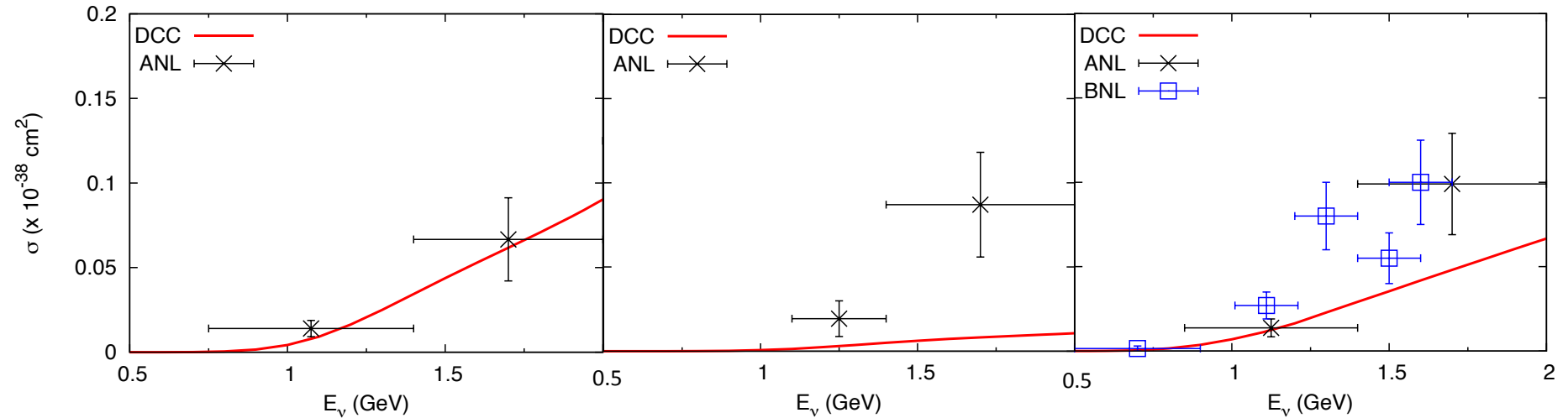
- $\pi N$  &  $\pi\pi N$  are main channels in few-GeV region
- DCC model gives predictions for **all final states**
- $\eta N$ ,  $KY$  cross sections are  $10^{-1} - 10^{-2}$  smaller

# Comparison with double pion data

$$\nu_{\mu} p \rightarrow \mu^{-} \pi^{+} \pi^{0} p$$

$$\nu_{\mu} p \rightarrow \mu^{-} \pi^{+} \pi^{+} n$$

$$\nu_{\mu} n \rightarrow \mu^{-} \pi^{+} \pi^{-} p$$



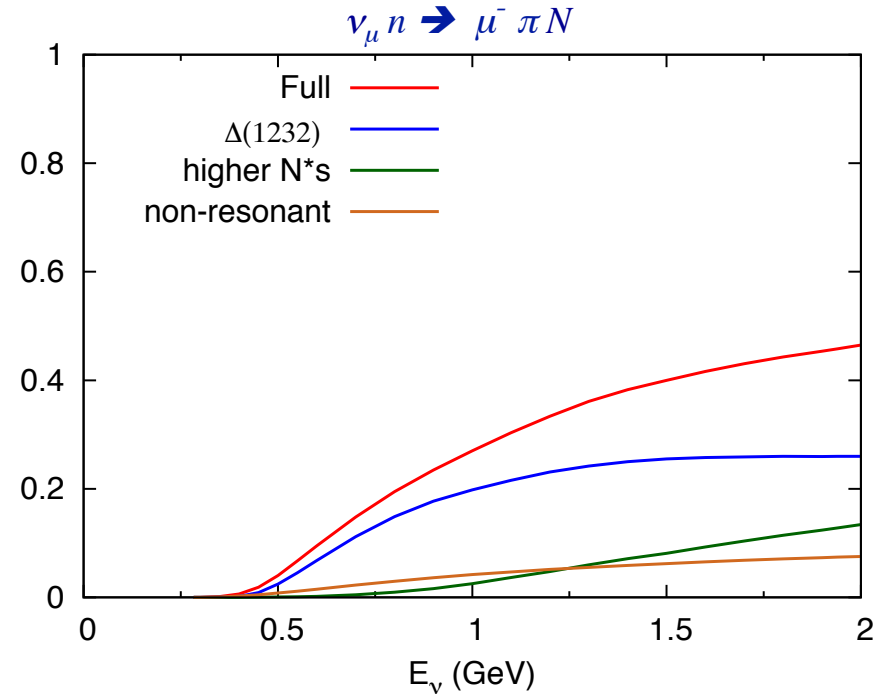
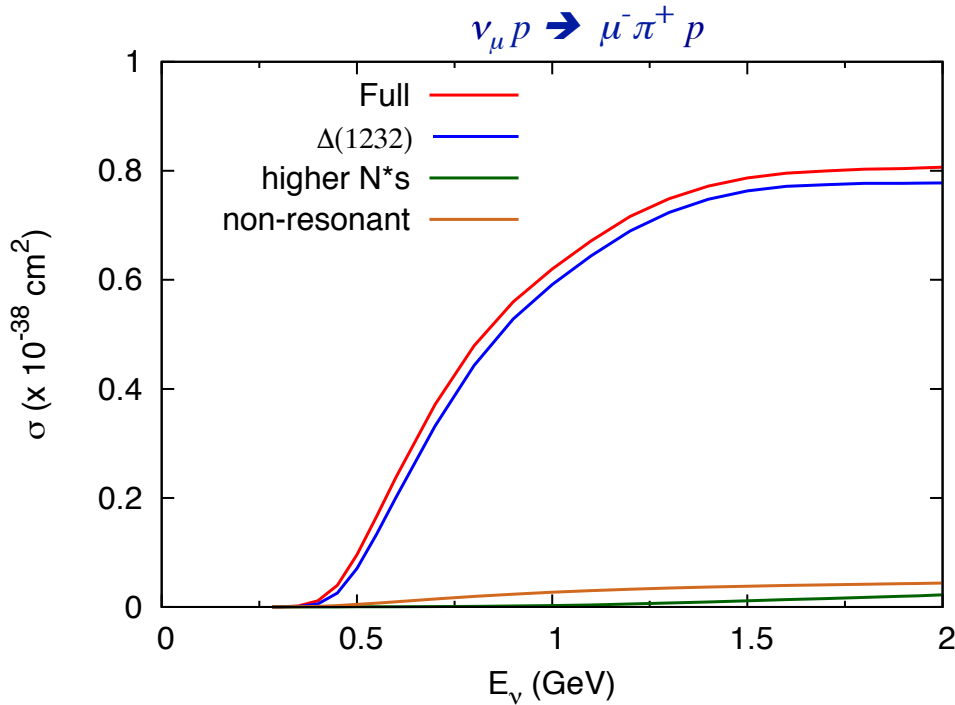
Fairly good DCC predication

ANL Data : PRD **28**, 2714 (1983)

BNL Data : PRD **34**, 2554 (1986)

First dynamical model for 2  $\pi$  production in resonance region

# Mechanisms for $\nu_\mu N \rightarrow \mu^- \pi N$



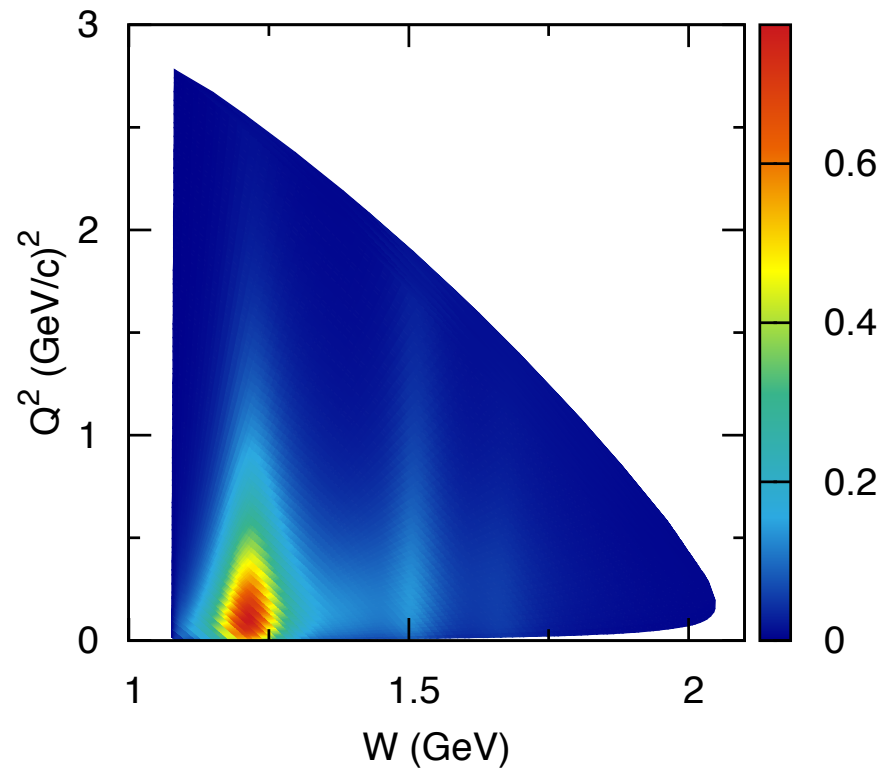
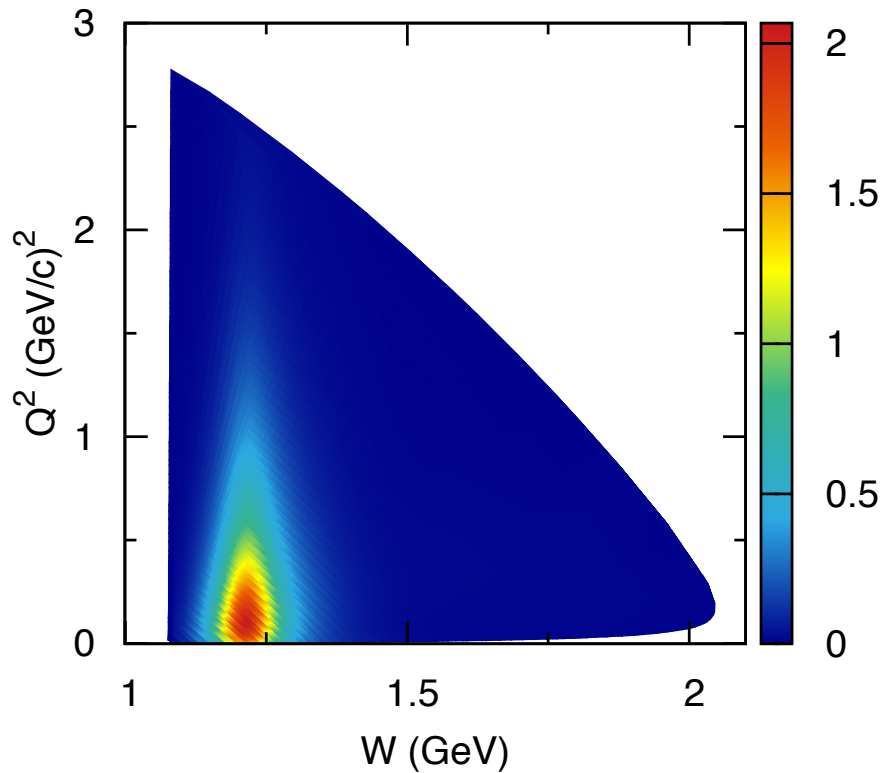
- $\Delta(1232)$  dominates for  $\nu_\mu p \rightarrow \mu^- \pi^+ p$  ( $I=3/2$ ) for  $E_\nu \leq 2$  GeV
- Non-resonant mechanisms contribute significantly
- Higher  $N^*$ s becomes important towards  $E_\nu \approx 2$  GeV for  $\nu_\mu n \rightarrow \mu^- \pi N$

$$d\sigma / dW dQ^2 \quad (\times 10^{-38} \text{ cm}^2 / \text{ GeV}^2)$$

$$E_\nu = 2 \text{ GeV}$$

$$\nu_\mu p \rightarrow \mu^- \pi^+ p$$

$$\nu_\mu n \rightarrow \mu^- \pi N$$

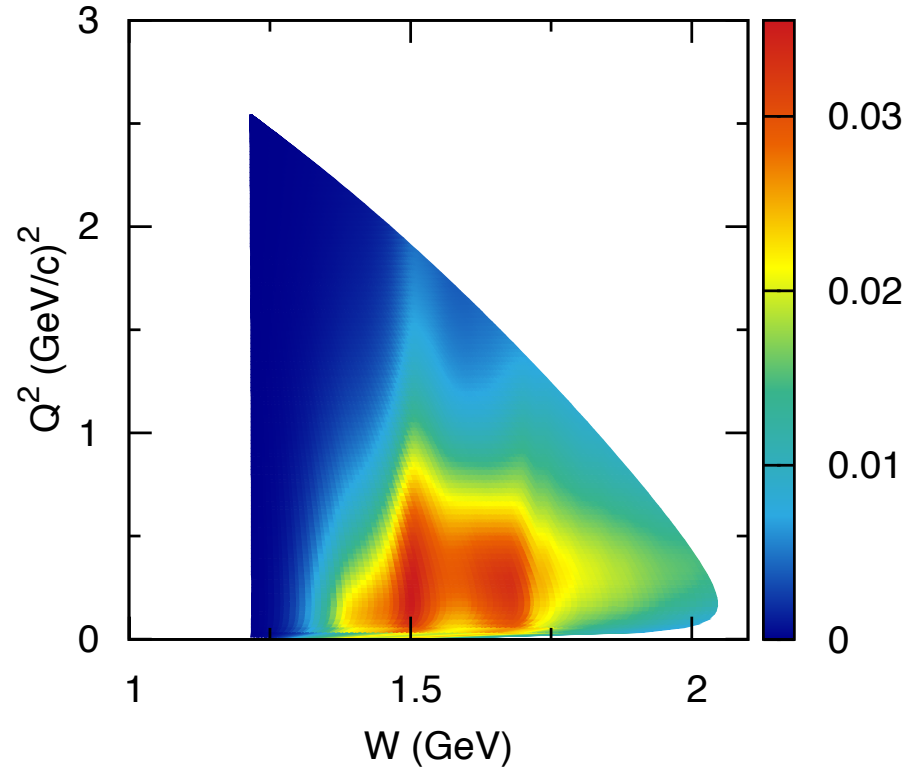
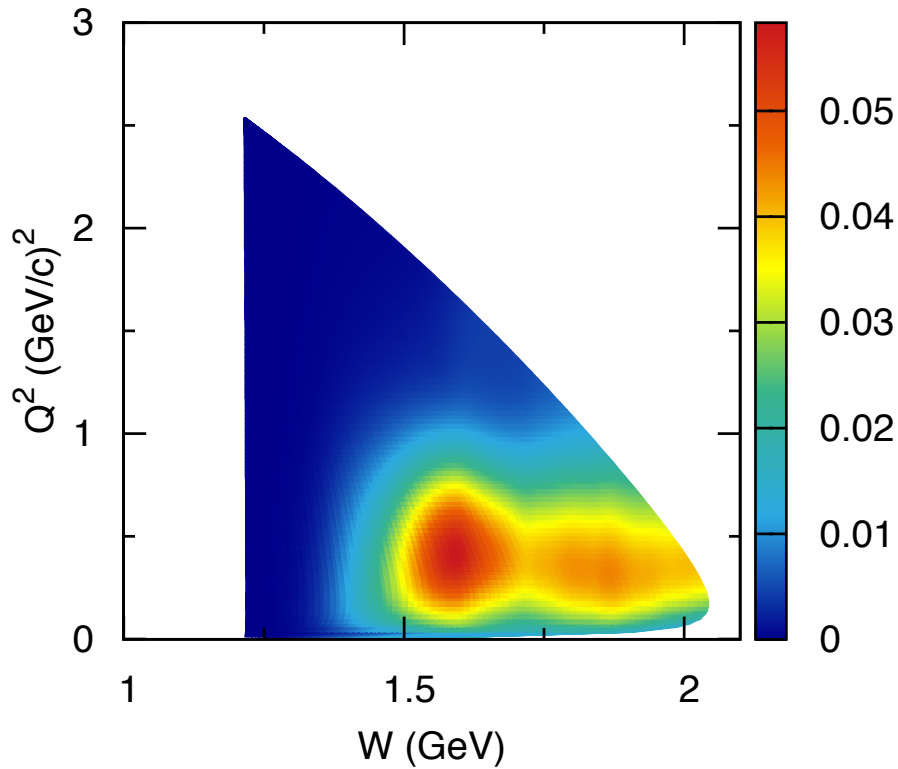


$$d\sigma / dW dQ^2 \quad (\times 10^{-38} \text{ cm}^2 / \text{GeV}^2)$$

$$E_\nu = 2 \text{ GeV}$$

$$\nu_\mu p \rightarrow \mu^- \pi^+ \pi^0 p$$

$$\nu_\mu n \rightarrow \mu^- \pi^+ \pi^- p$$

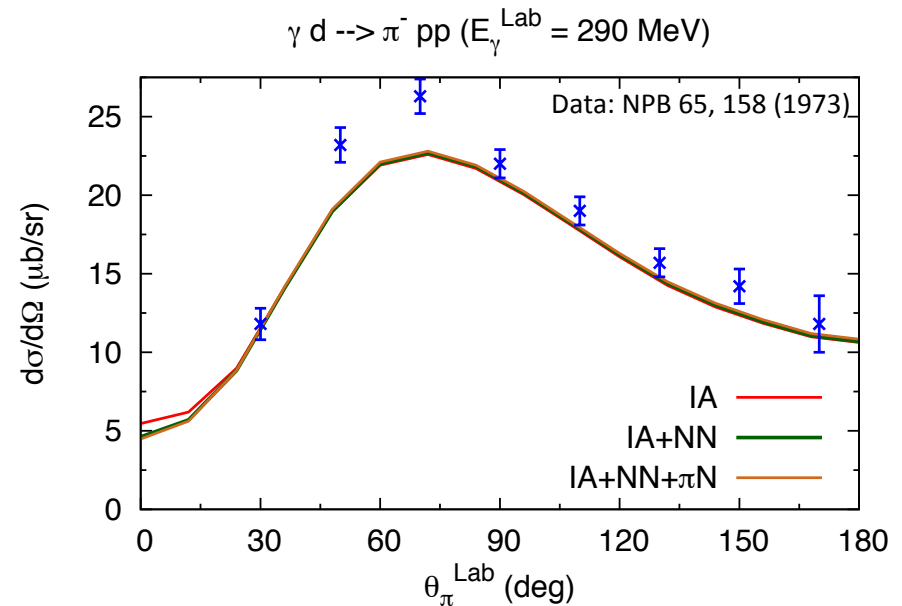
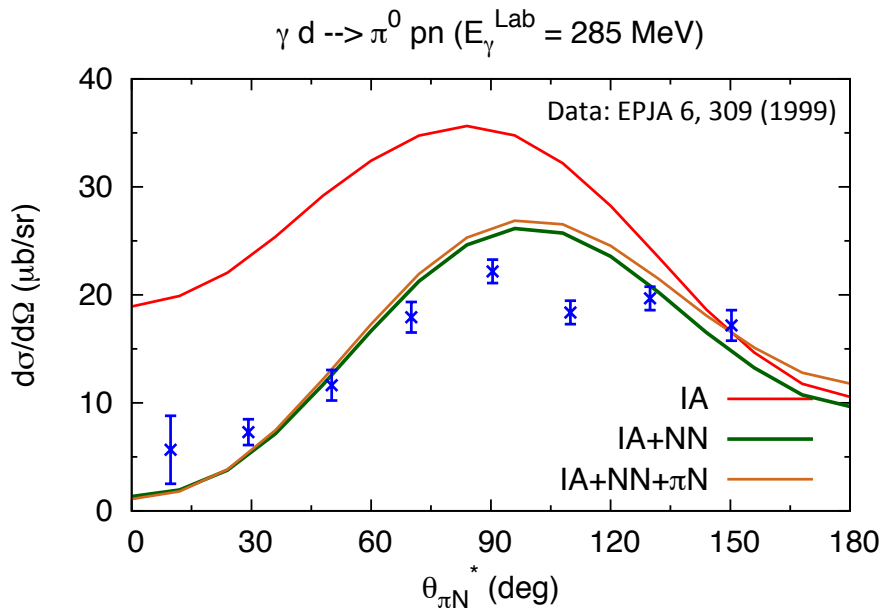




# $\gamma d \rightarrow \pi NN$

Purpose : test the soundness of the model

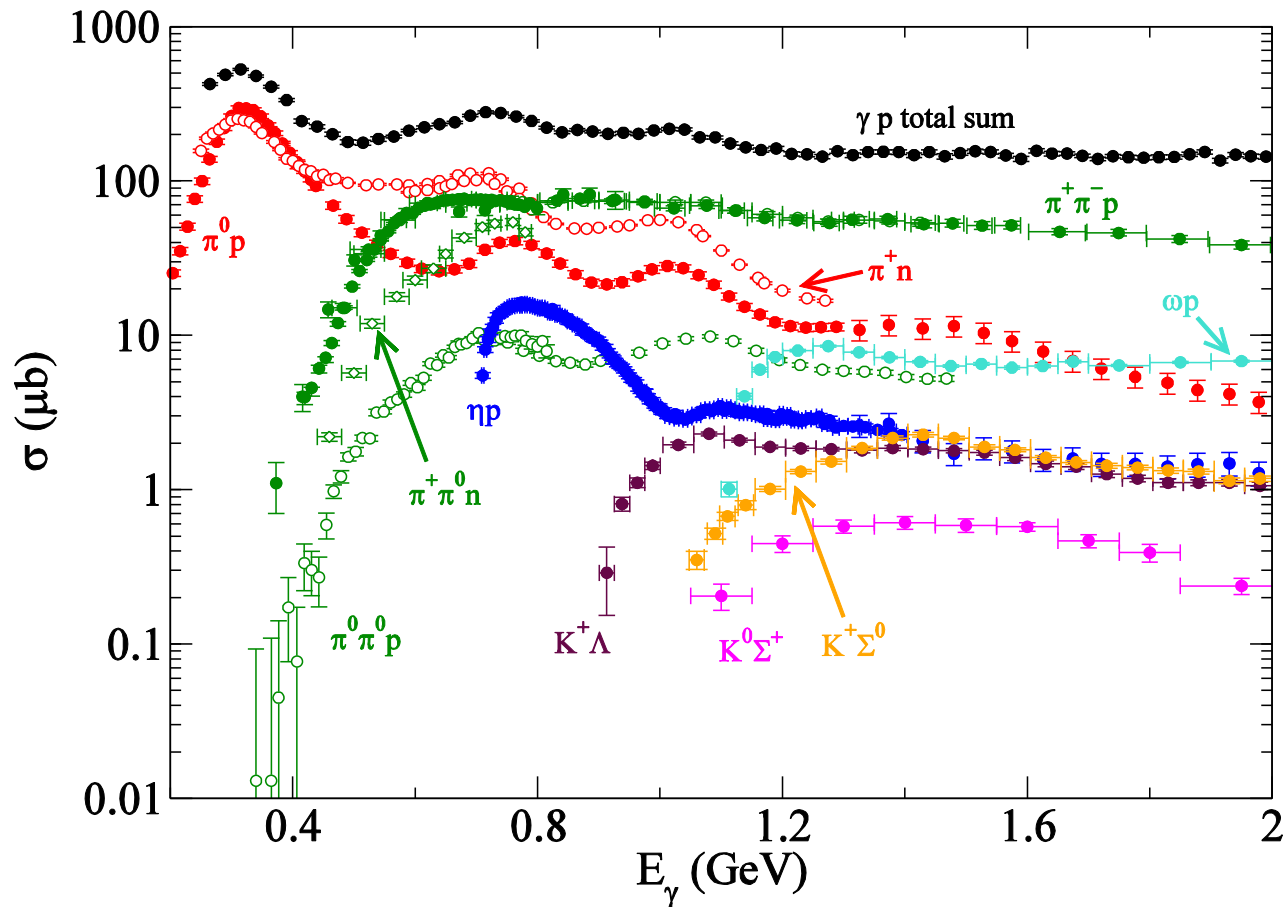
Wu, Sato, and Lee , PRC 91, 035203 (2015)



- Model prediction is reasonably consistent with data
- Large  $NN$  (small  $\pi N$ ) rescattering effect for  $\pi^0$  production  
orthogonality between deuteron and  $pn$  scattering wave functions
- Small rescattering effect for  $\pi^-$  production

# Resonance region (single nucleon)

$\gamma N \rightarrow X$



- Several resonances form characteristic peaks
- $2\pi$  production is comparable to  $1\pi$
- $\eta, K$  productions (multi-channel reaction)

# DCC (Dynamical Coupled-Channel) model

Matsuyama et al., Phys. Rep. **439**, 193 (2007)

Kamano et al., PRC 88, 035209 (2013)

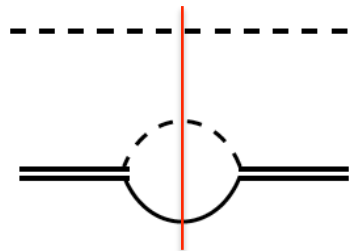
## Coupled-channel Lippmann-Schwinger equation

$$T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb}$$

$G_c =$



for stable channels



for unstable channels

# DCC (Dynamical Coupled-Channel) model

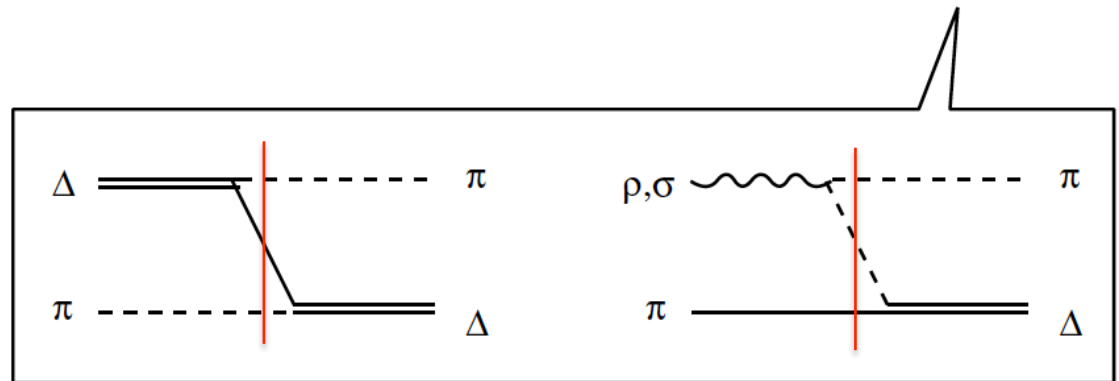
Matsuyama et al., Phys. Rep. **439**, 193 (2007)

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## Coupled-channel Lippmann-Schwinger equation

$$T_{ab} = V_{ab} + \sum_c V_{ac} G_c T_{cb}$$

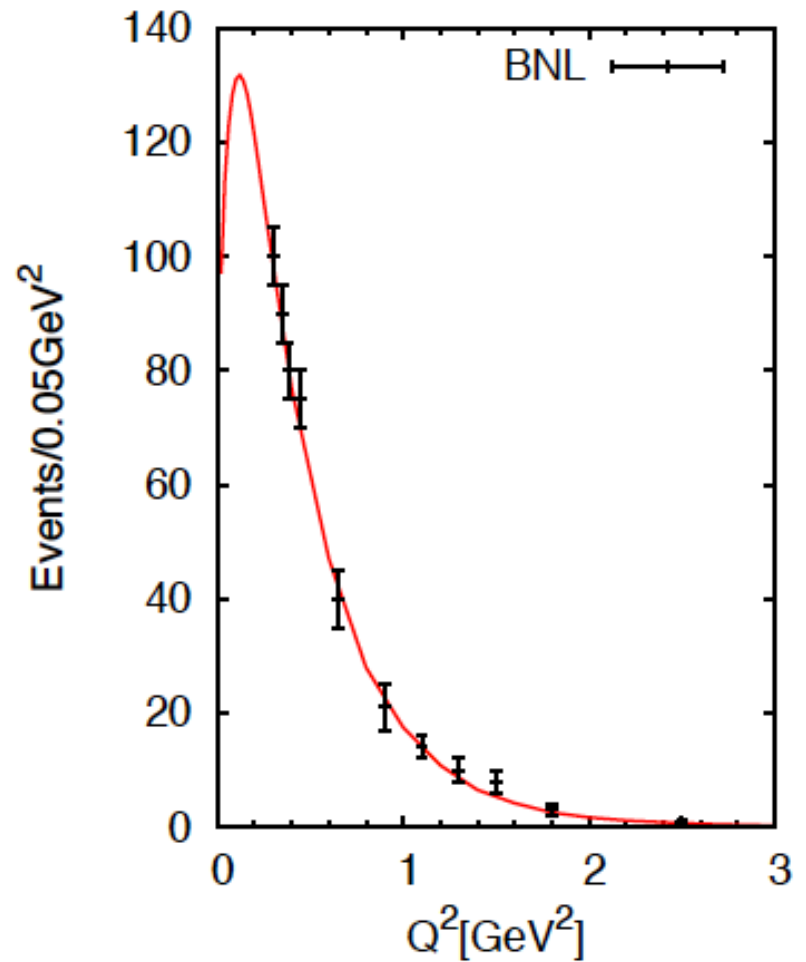
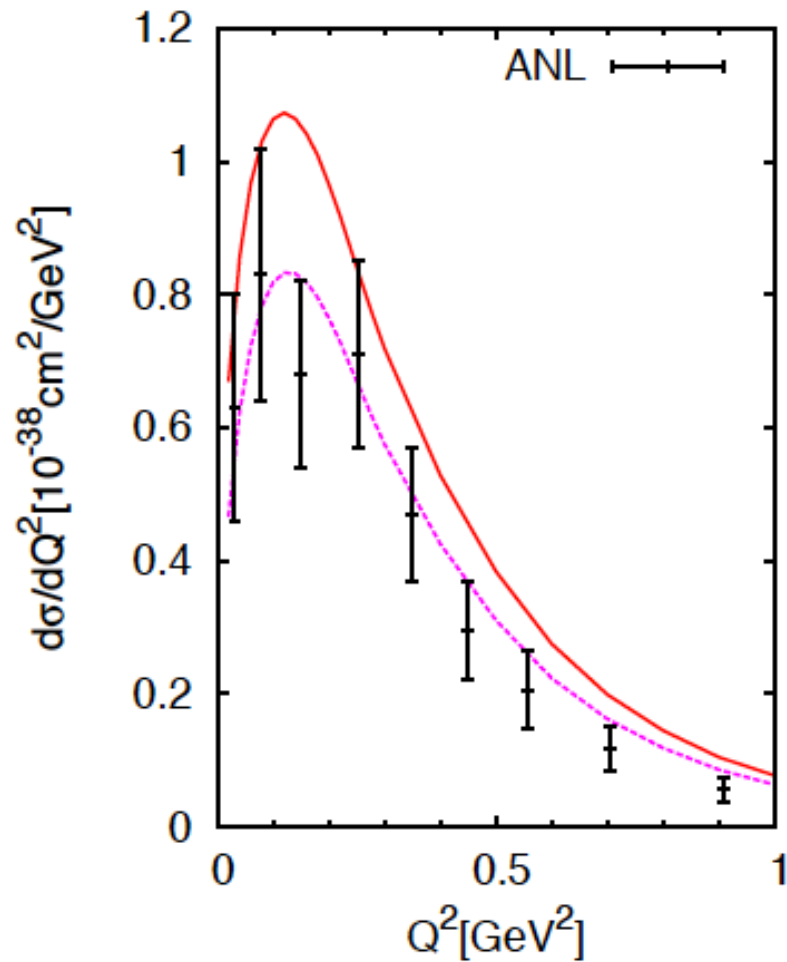
$$\mathbf{V}_{ab} = \text{[diagram 1]} + \text{[diagram 2]} + \mathbf{Z}$$



essential for three-body unitarity

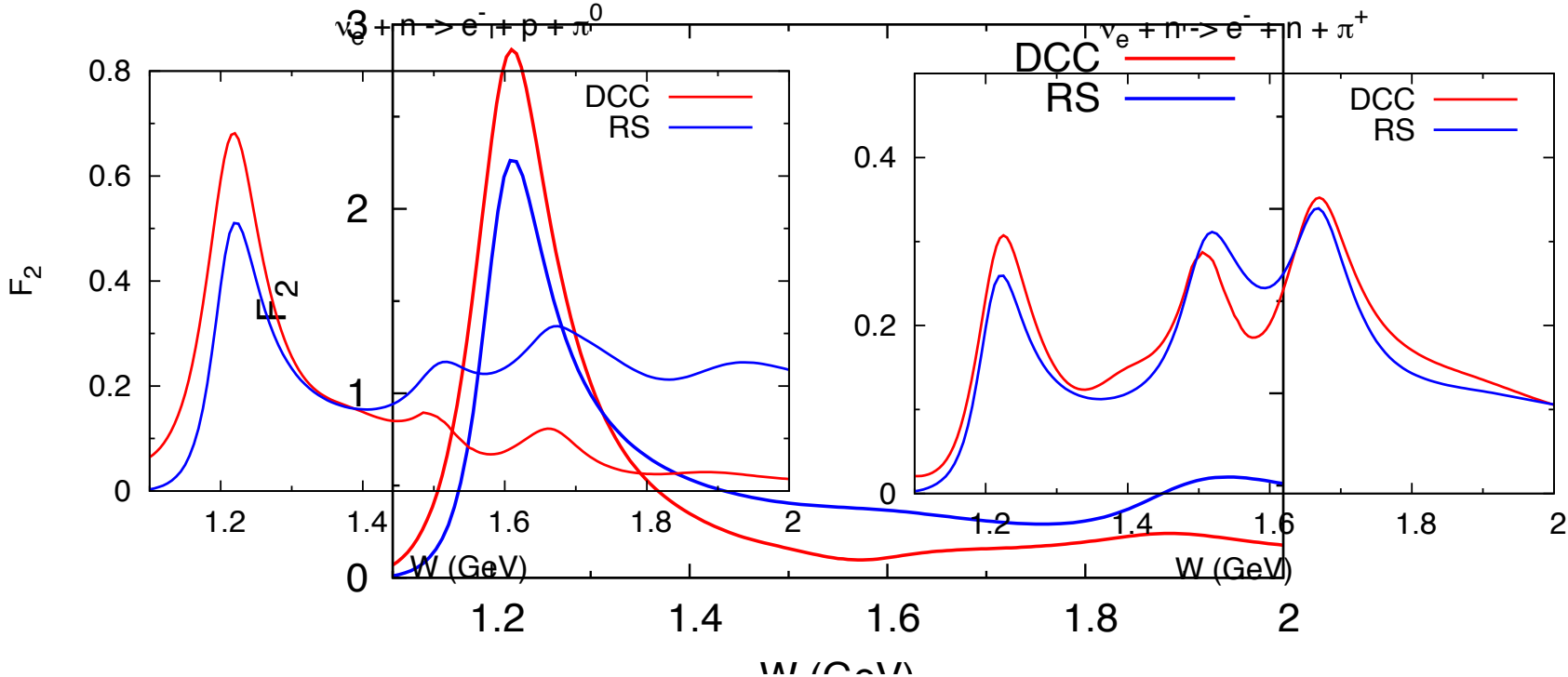
# $Q^2$ – dependence

$$\nu_\mu p \rightarrow \mu^- \pi^+ p$$



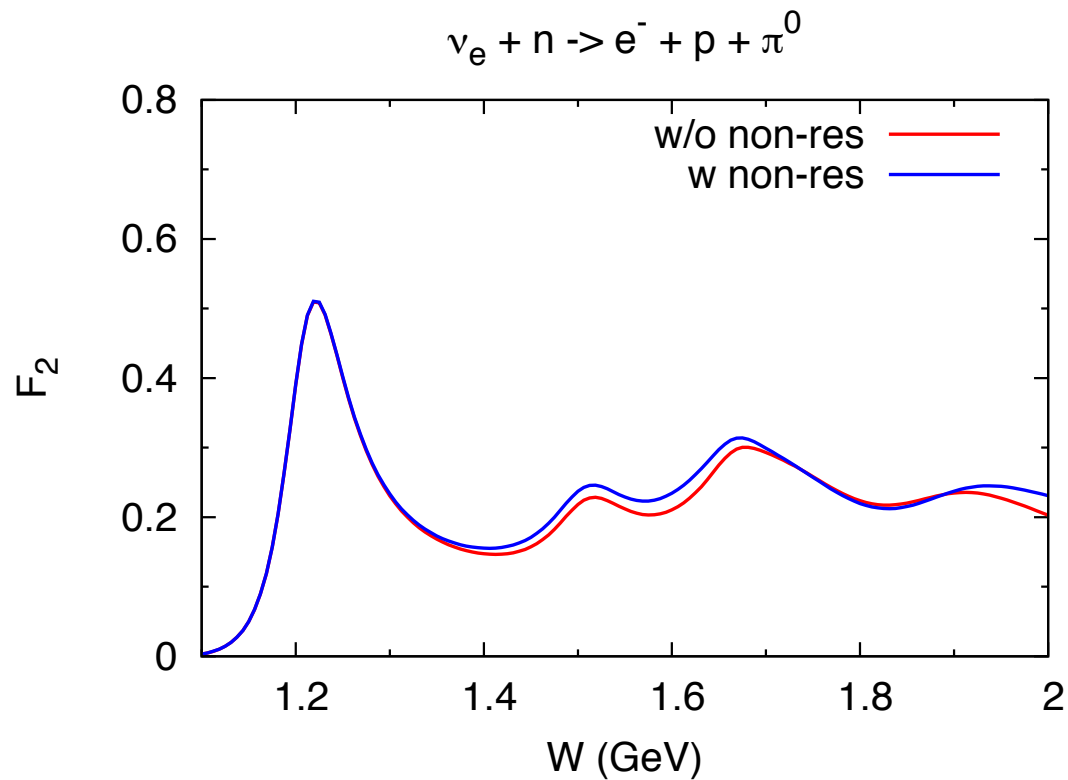
# Comparison with Rein-Sehgal model

$$\nu_e + p \rightarrow e^- + p + \pi^+$$



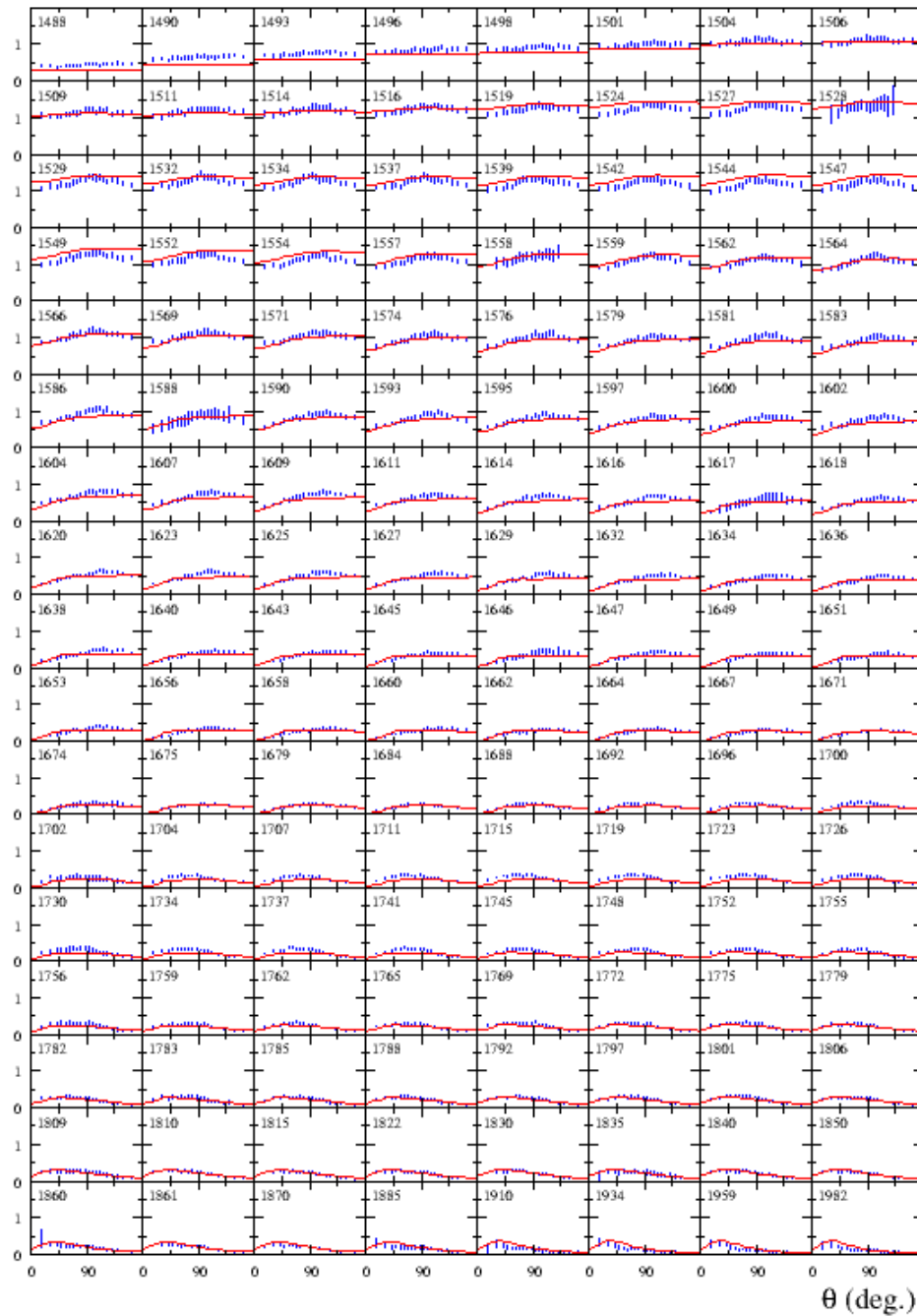
Comparison in whole kinematical region will be done  
after axial current model is developed

# $F_2$ from RS model



$d\sigma/d\Omega$  ( $\mu\text{b/sr}$ ) $\gamma p \rightarrow \eta p$ 

Kamano, Nakamura, Lee, Sato, arXiv:1305.4351

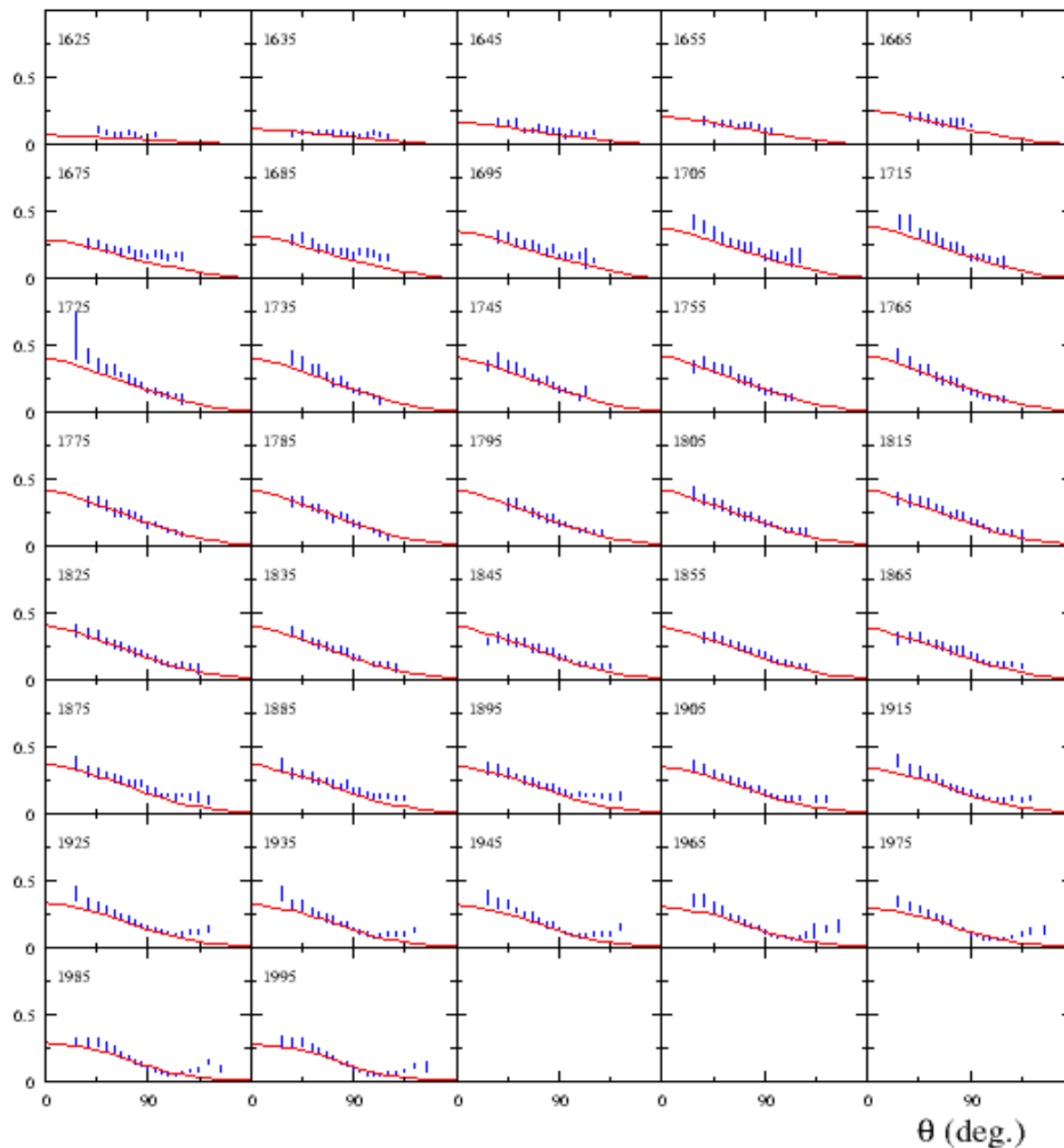


Vector current ( $Q^2=0$ ) for  $\eta$   
Production is well-tested by data



$d\sigma/d\Omega$  ( $\mu\text{b}/\text{sr}$ ) $\gamma p \rightarrow K^+ \Lambda$ 

Kamano, Nakamura, Lee, Sato, arXiv:1305.4351

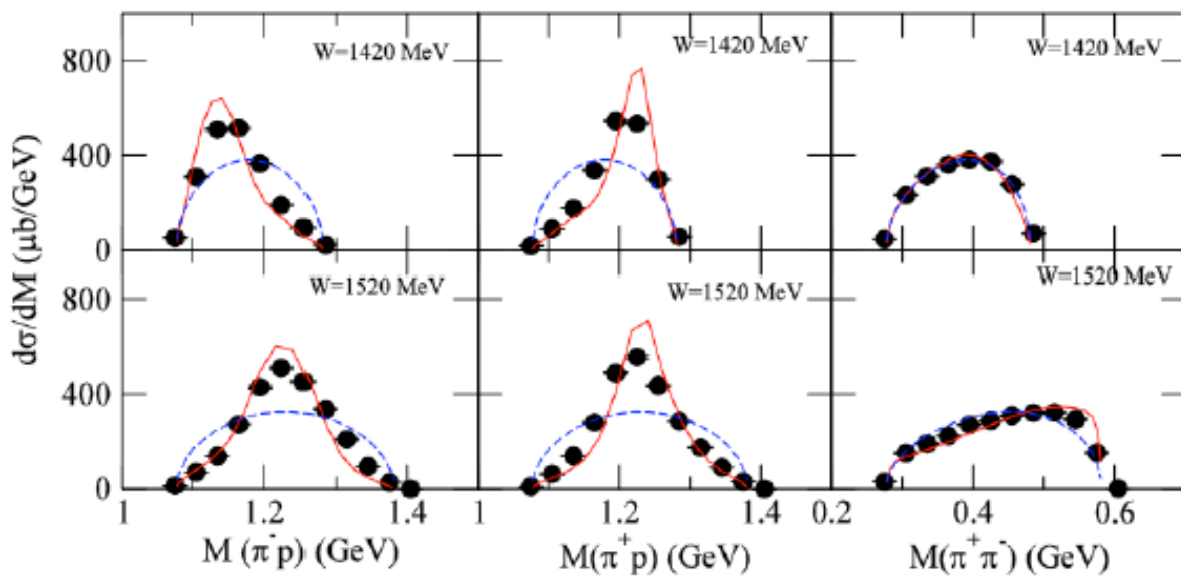
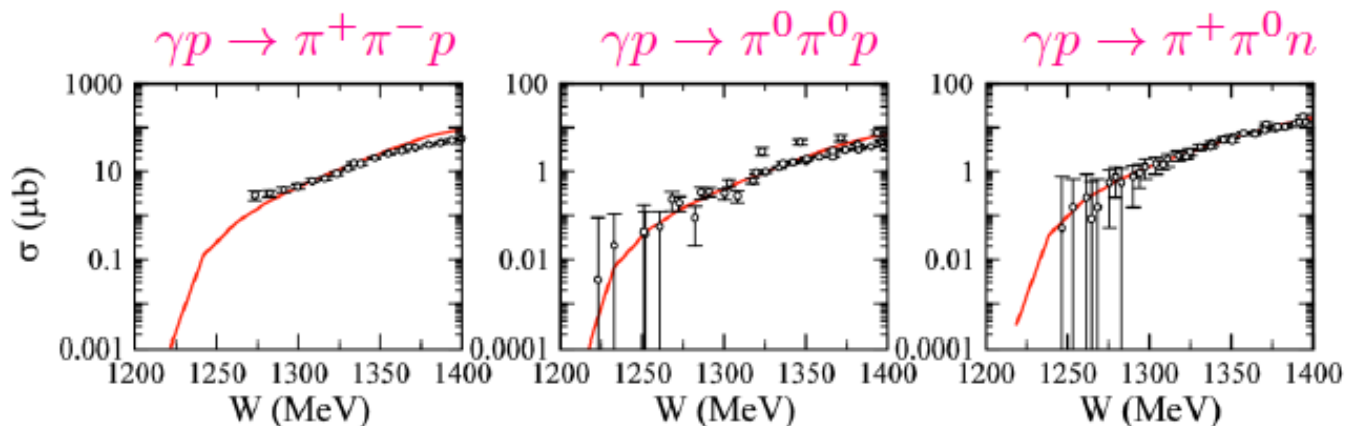


Vector current ( $Q^2=0$ ) for  $K$   
Production is well-tested by data

$$\gamma N \rightarrow \pi\pi N$$

(parameters had been fitted to  $\pi N, \gamma N \rightarrow \pi N$ )

Kamano, Julia-Diaz, Lee, Matsuyama, Sato, PRC80 065203 (2009)

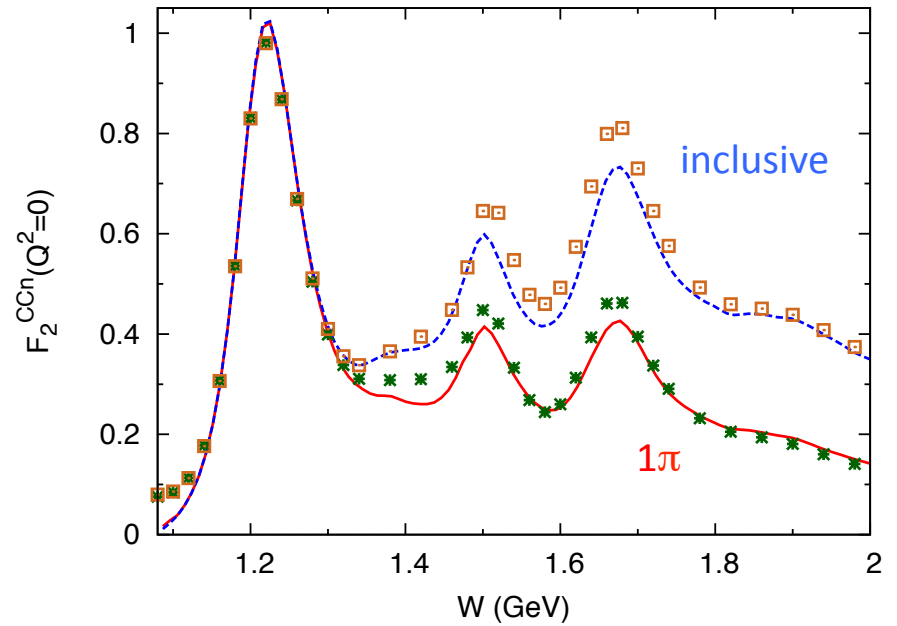
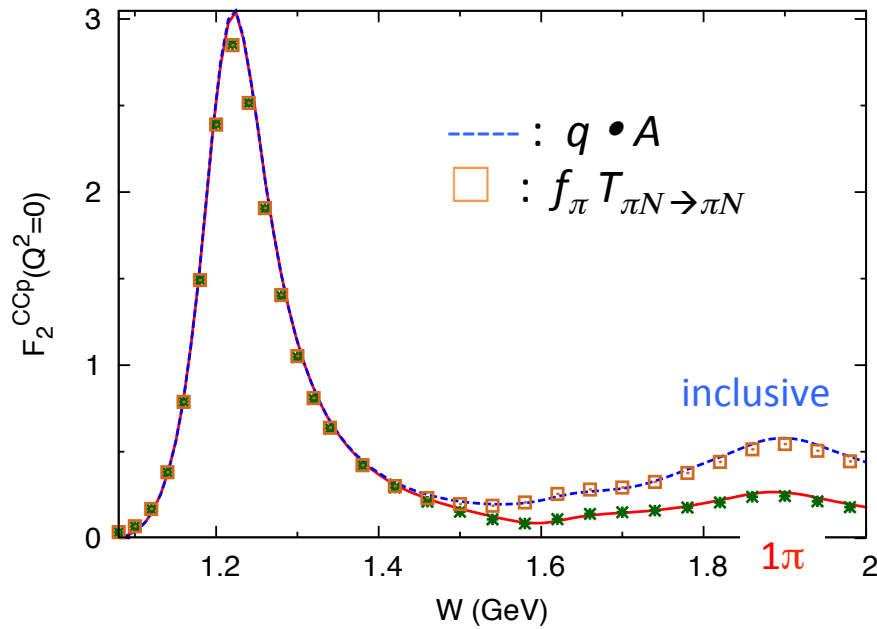


\* Good description near threshold

\* Good shape of invariant mass distribution

\* Total cross sections overestimate data for  $W \geq 1.5$  GeV

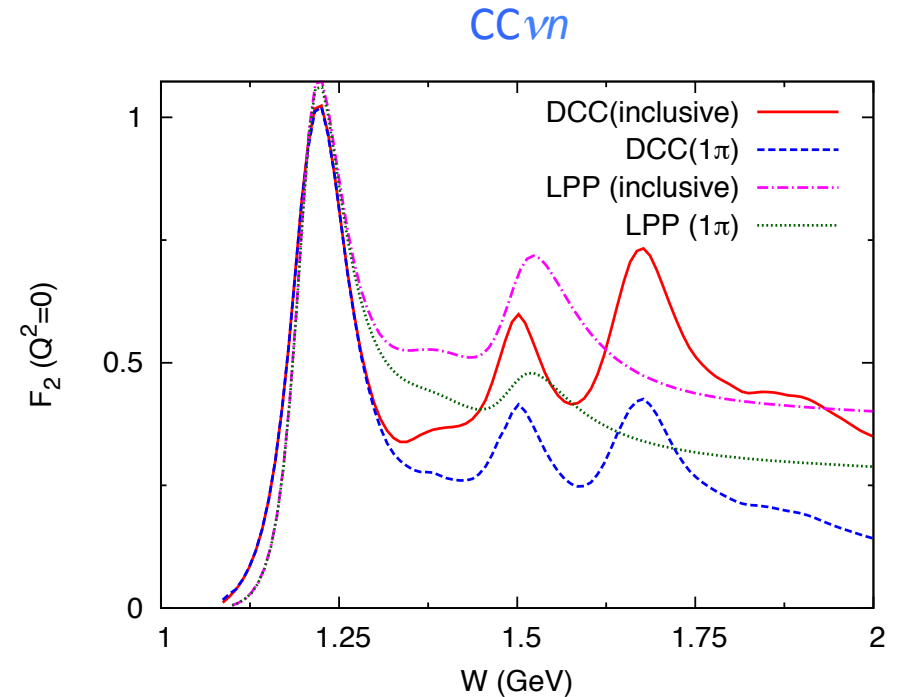
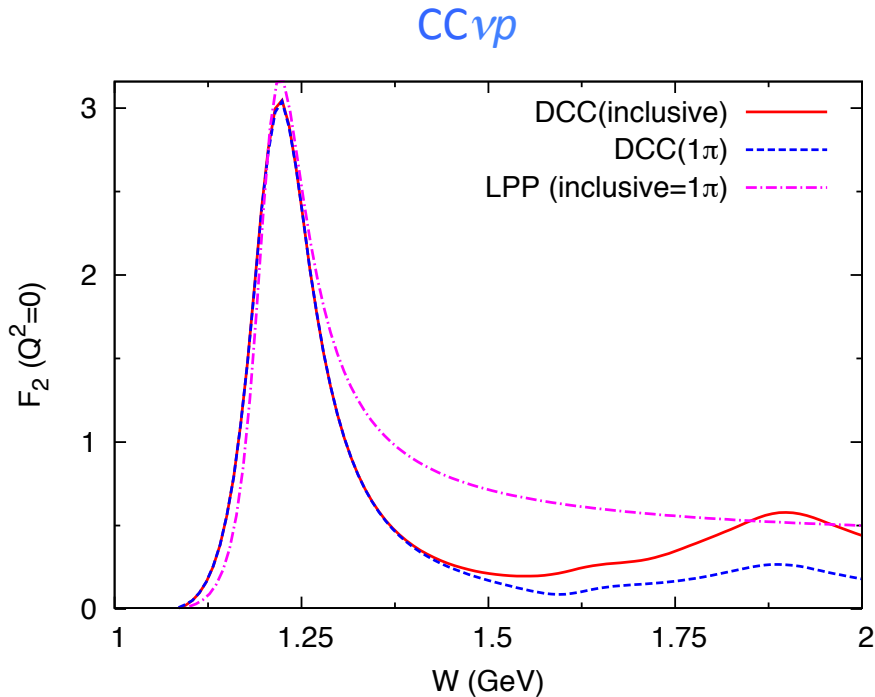
# $F_2(Q^2=0)$ from DCC model and PCAC



DCC model keeps good consistency with PCAC

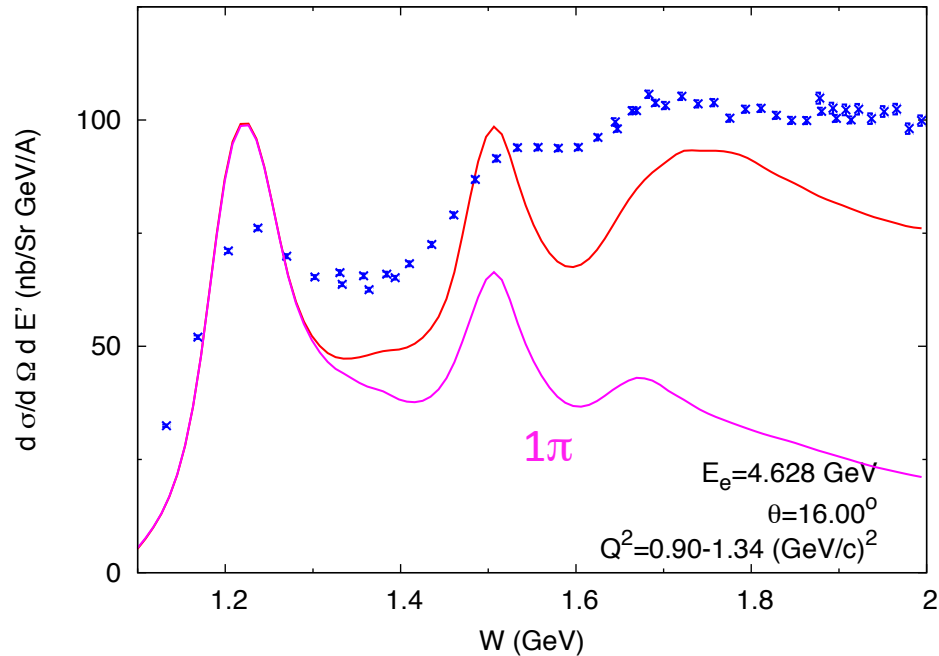
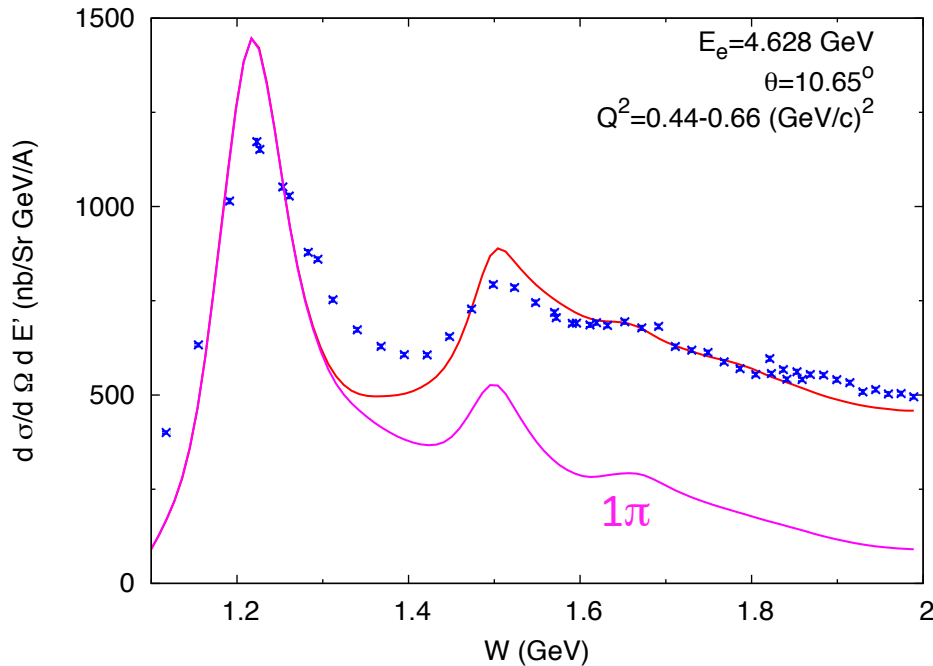
# Comparison with LPP model

LPP model : Lalakulich et al, PRD 74 (2006)



- Large difference beyond  $\Delta(1232)$  region
- Importance of consistency between axial-current and  $\pi N$  interaction

# Analysis result (inclusive $e^-d$ )



Data: NP Proc. Suppl. 159, 163 (2006)

- Our calculation :  $[\sigma(e^-p) + \sigma(e^-n)] / 2$
- Too sharp resonant peaks  $\rightarrow$  fermi motion smearing, other nuclear effects needed
- Reasonable starting point for application to neutrino interactions