Determination of the axial mass parameter and the proton charge radius

Gil Paz

Department of Physics and Astronomy,

Wayne State University,

Detroit, Michigan, USA

Outline

- Introduction: the axial mass problem and the proton radius puzzle
- The z expansion
- Separating form factor and nuclear effects
- Conclusions and outlook

Introduction:

The axial mass problem
and
The proton radius puzzle

Quasiealstic $\nu - N$ scattering

- Quasiealstic $\nu - N$ scattering: $\nu_\ell + n \to \ell^- + p$
- At the quark level: $\nu_\ell + d \to \ell^- + u$
- Process "folded" twice

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$\Downarrow$ Form factor

Nucleon $\nu_\ell + n \to \ell^- + p$

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Nucleus: $\nu_\ell + \text{nucleus} \to \ell^- + \text{nucleus}'$

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- For precision need to control both *form factors* and *nuclear effects*

Quark $\rightarrow$ Nucleon

- The interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{ud}^* \, \bar{\ell}\gamma^\alpha(1-\gamma^5)\nu \, \bar{u}\gamma_\alpha(1-\gamma^5)d$$

Known current: $\bar{u}\gamma_\alpha(1-\gamma^5)d$

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- Parametrize $\langle p(p')|\bar{u}\gamma_\alpha(1-\gamma^5)d|n(p)\rangle$ by form factors:

F_1, F_2, F_p, F_A, functions of $q^2 = (p'-p)^2$

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- F_1, F_2 related via isospin to EM form factors: $F_{1,2} = F_{1,2}^p - F_{1,2}^n$
- F_P contribution suppressed by m_ℓ^2/m_N^2
- What do we know about $F_A(q^2)$?

Axial form factor

- Consider a small q^2 expansion of $F_A(q^2)$

- $F_A(0) = -1.269$ known from neutron decay
- Axial radius: $r_A \equiv \sqrt{\frac{6F_A'(0)}{F_A(0)}}$ or axial mass: $m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$

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- What can possibly go wrong?

The axial mass problem

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  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]
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  Nuclear effects, Form factors, or both...

- Discussion in literature focuses on nuclear effects:
  use dipole model for F_A and vary nuclear effects

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- What if we fix nuclear effects and vary F_A?

- How to vary F_A?

Proton charge radius

- Sachs electric and magnetic form factors

$$G_E^p(q^2) = F_1^p(q^2) + \frac{q^2}{4m_p^2} F_2^p(q^2) \qquad G_M^p(q^2) = F_1^p(q^2) + F_2^p(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- Proton charge radius : $r_E^p \equiv \sqrt{\frac{6G_E^{p\prime}(0)}{G_E^p(0)}}$

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- If you think the axial mass has a problem...

  Wait until you meet the proton radius puzzle!

The proton radius puzzle

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
  more recently $r_E^p = 0.84087(39)$ fm [Antognini et al. Science **339**, 417 (2013)]
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.87680(690)$ fm extracted mainly from electronic hydrogen
  more recently $r_E^p = 0.87510(610)$ fm [Mohr et al. RMP **88**, 035009 (2016)]
  spectroscopy alone: $r_E^p = 0.87590(770)$ fm [Mohr et al. RMP **88**, 035009 (2016)]

- **5σ discrepancy!** This is the proton radius puzzle

The z expansion

What did the PDG 2010 say?

K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010)

p CHARGE RADIUS

This is the rms charge radius, $\sqrt{\langle r^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.8768±0.0069	MOHR	08	RVUE	2006 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.897 ±0.018	BLUNDEN	05		SICK 03 + 2γ correction
0.8750±0.0068	MOHR	05	RVUE	2002 CODATA value
0.895 ±0.010 ±0.013	SICK	03		$e p \to e p$ reanalysis
0.830 ±0.040 ±0.040	[24] ESCHRICH	01		$e p \to e p$
0.883 ±0.014	MELNIKOV	00		1S Lamb Shift in H
0.880 ±0.015	ROSENFELDR.	00		$e p$ + Coul. corrections
0.847 ±0.008	MERGELL	96		$e p$ + disp. relations
0.877 ±0.024	WONG	94		reanalysis of Mainz $e p$ data
0.865 ±0.020	MCCORD	91		$e p \to e p$
0.862 ±0.012	SIMON	80		$e p \to e p$
0.880 ±0.030	BORKOWSKI	74		$e p \to e p$
0.810 ±0.020	AKIMOV	72		$e p \to e p$
0.800 ±0.025	FREREJACQ...	66		$e p \to e p$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$e p \to e p$

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

[24] ESCHRICH 01 actually gives $\langle r^2 \rangle = (0.69 \pm 0.06 \pm 0.06)$ fm^2.

Proton charge radius from scattering

- The PDG 2010 lists proton radii starting from 1963
- This is more then 50 years of radii extraction.
- You can find almost any value between 0.8-0.9 fm....
- Data sets have changed over the last 50 years
  but even using the same data sets different people get different values
- What is the problem?

Form Factors: What we don't know

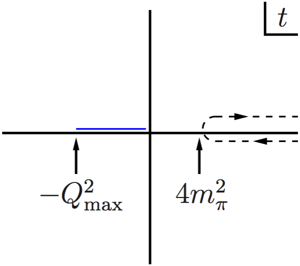
- The form factors are non-perturbative objects.
- **Nobody** knows the *exact* functional form of G_E^p and G_M^p
- They don't have to have a dipole/polynomial/spline or any other functional form
- Including such models can bias your extraction of r_E^p and r_M^p

Form Factors: What we do know

- Analytic properties of $G_E^p(t)$ and $G_M^p(t)$ are known
- They are analytic outside a cut $t \in [4m_\pi^2, \infty]$

  [Federbush, Goldberger, Treiman, Phys. Rev. **112**, 642 (1958)]

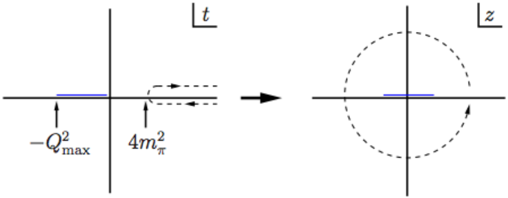
- $e - p$ scattering data is in $t < 0$ region



- z expansion: map domain of analyticity onto unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$



- Expand $G^p_{E,M}$ in a Taylor series in z: $G^p_{E,M}(q^2) = \sum_{k=0}^{\infty} a_k\, z(q^2)^k$

z expansion

- [Epstein, GP, Roy PRD **90**, 074027 (2014)]

z expansion

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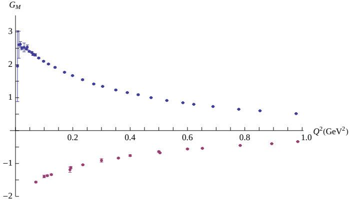
  $G_M(Q^2)$ for proton (blue, above axis) and neutron (red, below axis)



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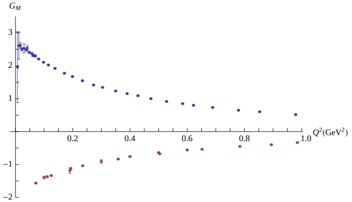
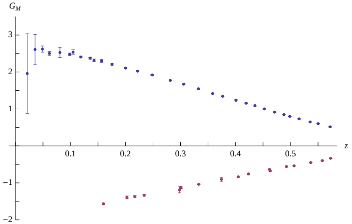


  $G_M(z)$ for proton (blue, above axis) and neutron (red, below axis)



- See also R.J. Hill talk at FPCP 2006 [hep-ph/0606023]

z expansion for bayons

- z expansion is **the** method for **meson** form factors, e.g. [Flavor Lattice Averaging Group, EPJ C **74**, 2890 (2014)]
- Starting in 2010 applied to **baryon** form factors [Hill, GP PRD **82** 113005 (2010)]
- Now applied successfully to extract r_E^p, r_M^p, r_M^n, m_A...

Example 1: r_E^p in PDG 2016

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

p CHARGE RADIUS

This is the rms electric charge radius, $\sqrt{\langle r_E^2 \rangle}$.

VALUE (fm)			*DOCUMENT ID*		*TECN*	*COMMENT*
0.8751	**±0.0061**		MOHR	16	RVUE	2014 CODATA value
0.84087	**±0.00026**	**±0.00029**	ANTOGNINI	13	LASR	μp-atom Lamb shift
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.895	±0.014	±0.014	[1] LEE	15	SPEC	Just 2010 Mainz data
0.916	±0.024		LEE	15	SPEC	World data, no Mainz
0.8775	±0.0051		MOHR	12	RVUE	2010 CODATA, $e p$ data
0.875	±0.008	±0.006	ZHAN	11	SPEC	Recoil polarimetry
0.879	±0.005	±0.006	BERNAUER	10	SPEC	$e p \to e p$ form factor
0.912	±0.009	±0.007	BORISYUK	10		reanalyzes old $e p$ data
0.871	±0.009	±0.003	HILL	10		z-expansion reanalysis
0.84184	±0.00036	±0.00056	POHL	10	LASR	See ANTOGNINI 13
0.8768	±0.0069		MOHR	08	RVUE	2006 CODATA value
0.844	$^{+0.008}_{-0.004}$		BELUSHKIN	07		Dispersion analysis
0.897	±0.018		BLUNDEN	05		SICK 03 + 2γ correction
0.8750	±0.0068		MOHR	05	RVUE	2002 CODATA value
0.895	±0.010	±0.013	SICK	03		$e p \to e p$ reanalysis

[Hill, GP PRD **82** 113005 (2010)]

[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

Example 2: r_M^p in PDG 2016

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

p MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*TECN*	*COMMENT*
0.776±0.034±0.017	[1] LEE	15	SPEC	Just 2010 Mainz data
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.914±0.035	LEE	15	SPEC	World data, no Mainz
0.87 ±0.02	EPSTEIN	14		Using $e\,p$, $e\,n$, $\pi\pi$ data
0.867±0.009±0.018	ZHAN	11	SPEC	Recoil polarimetry
0.777±0.013±0.010	BERNAUER	10	SPEC	$e\,p \to e\,p$ form factor
0.876±0.010±0.016	BORISYUK	10		Reanalyzes old $e\,p \to e\,p$ data
0.854±0.005	BELUSHKIN	07		Dispersion analysis

[1] Authors also provide values for a combination of all available data.

[Epstein, GP, Roy PRD **90**, 074027 (2014)]
[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

Example 3: r_M^n in PDG 2016

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

n MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)	*DOCUMENT ID*		*COMMENT*
$0.864^{+0.009}_{-0.008}$ OUR AVERAGE			
0.89 ± 0.03	EPSTEIN	14	Using $e p$, $e n$, $\pi\pi$ data
$0.862^{+0.009}_{-0.008}$	BELUSHKIN	07	Dispersion analysis

[Epstein, GP, Roy PRD **90**, 074027 (2014)]

Example 4: more z expansion for baryons

Partial list:

- Heavy-baryon form factors:
  - [Khodjamirian, Klein, Mannel Wang, JHEP **1109**, 106 (2011)]
  - [**Lattice**: Detmold, Lehner, Meinel PRD **92**, 034503 (2015)]
  - $|V_{ub}|/|V_{cb}|$ using baryonic decays
    [LHCb Collaboration, Nature Phys. **11**, 743 (2015)]
- Strange nucleon EM form factors:
  [**Lattice**: Green et al., Phys. Rev. D **92** 031501 (2015)]
- Nucleon EM form factors
  [**Lattice**: Alexandrou et al., arXiv:1706.00469]

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The bottom line

- Scattering:
  - World $e-p$ data [Lee, Arrington, Hill '15]
    $r_E^p = 0.918 \pm 0.024$ fm
  - Mainz $e-p$ data [Lee, Arrington, Hill '15]
    $r_E^p = 0.895 \pm 0.020$ fm
  - Proton, neutron and π data [Hill , GP '10]
    $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002$ fm
- Muonic hydrogen
  - [Pohl et al. Nature **466**, 213 (2010)]
    $r_E^p = 0.84184(67)$ fm
  - [Antognini et al. Science **339**, 417 (2013)]
    $r_E^p = 0.84087(39)$ fm
- Using z expansion, scattering disfavors muonic hydrogen
- Is there a problem with muonic hydrogen *theory*?

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- Using z expansion, scattering disfavors muonic hydrogen

- Is there a problem with muonic hydrogen *theory*? Potentially yes!
  [Hill, GP PRL **107** 160402 (2011)], [Hill, GP PRD **95**, 094017 (2017)]

Separating form factor and nuclear effects

z expansion

- Using the z expansion we can vary F_A
- Expand F_A in a Taylor series in z: $F_A = \sum_{k=0}^{\infty} a_k \, z(q^2)^k$
- $|z(q^2)| < 1$ so in practice we need only a few terms
- For the extraction to be independent of the # of parameters need to bound $|a_k|$: e.g. $|a_k| \leq 5$ and $|a_k| \leq 10$

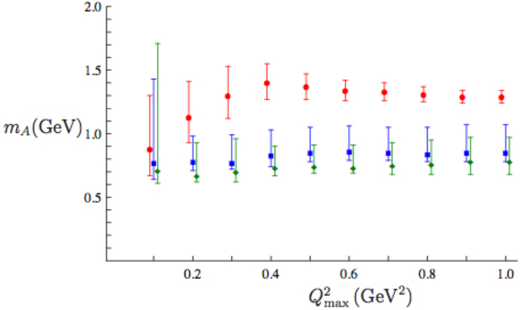
Example 1: m_A from MiniBooNE's neutrino data

- Example 1: extracting m_A using the z expansion
  [Bhattacharya, Hill, GP PRD **84** 073006 (2011)]
  from MiniBooNE's neutrino data
  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]

- Mostly follow MiniBooNE's analysis: use RFG as nuclear model

Neutrino: z expansion

- Our z expansion fit to MiniBooNE data (Assuming RFG):
  Red: dipole, Blue: z, $|a_k| \le 5$, Green: z, $|a_k| \le 10$



- Our fit using z expansion: $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV
  Our fit using dipole model: $m_A^{\rm dipole} = 1.29 \pm 0.05$ GeV
  MiniBooNE's fit: $m_A^{\rm dipole} = 1.35 \pm 0.17$ GeV

Example 2: m_A from MiniBooNE's antineutrino data

- Example 2: extracting m_A using the z expansion
  [Bhattacharya, GP, Tropiano, PRD **92**, 113011 (2015)]
  from MiniBooNE's antineutrino data
  [MiniBooNE Collaboration, PRD **88**, 032001 (2013)]

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- For scattering off mineral oil $C_n H_{2n+2}$ ($n \sim 30$):

- $\bar{\nu}$ can scatter off protons in C *and* protons in H

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- For scattering off mineral oil C_nH_{2n+2} ($n \sim 30$):
- $\bar{\nu}$ can scatter off protons in C *and* protons in H

- MiniBooNE used *different* $m_A^{\rm dipole}$ for C and H
  Problematic! the axial mass is a fundamental property of the nucleon
  Might hide nuclear effects in the axial mass

- We use the same F_A for C and H and extract *one* axial mass

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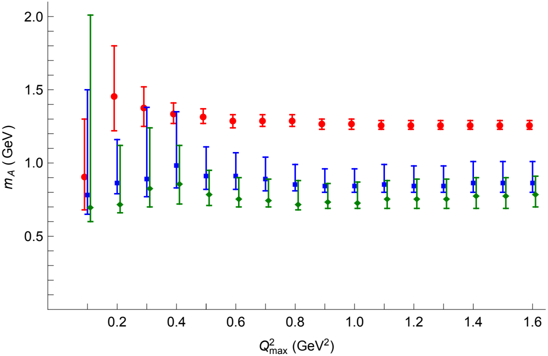
- Our z expansion fit to MiniBooNE data (Assuming RFG):
  Red: dipole, Blue: z, $|a_k| \leq 5$, Green: z, $|a_k| \leq 10$

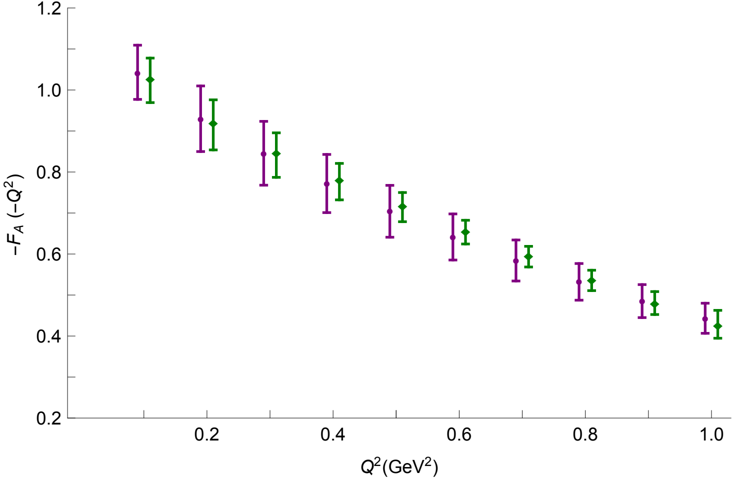


- Our fit using z expansion: $m_A = 0.84^{+0.12}_{-0.04} \pm 0.11$ GeV
  Our fit using dipole model: $m_A^{\rm dipole} = 1.27^{+0.03}_{-0.04}$ GeV

- Neutrino data z expansion: $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV
- Neutrino data dipole model: $m_A^{\rm dipole} = 1.29 \pm 0.05$ GeV

Comparing F_A from ν and $\bar{\nu}$ data

- We can also extract F_A directly from MiniBooNE data neutrino data (purple circles) and antineutrino data (green diamonds).



z expansion and possible nuclear effects

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- Crude qualitative check: multiply $d\sigma_c$ by a constant factor
  - factor > 1: error more symmetric, factor < 1: asymmetry grows
  - both cases little change in m_A^{dipole}

- Hint of a problem with nuclear model?
  Combine z expansion and correlated Fermi gas (CFG) nuclear model
  [Bhattacharya, Cohen, Hen, GP, Piasetzky, Tockstein, Wieske *(in progress)*]

Example 3: Deuterium target data

- Even more recently:
  z-expansion analysis of deuterium target data
  [Meyer, Betancourt, Gran, Hill PRD **93** 113015 (2016)]

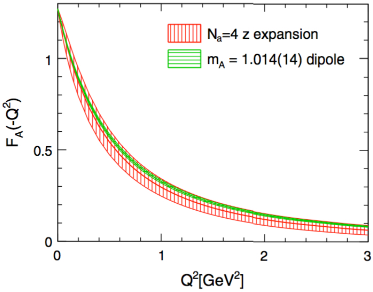
- They extract $r_A = \sqrt{12}/m_A$ and find $r_A^2 = 0.46(22)$ fm^2 or
  $r_A = 0.68^{+0.15}_{-0.19}$ fm

- Consistent with MiniBooNE neutrino and antineutrino data, e.g.
  $r_A = 0.81^{+0.05}_{-0.10} \pm 0.14$ fm
  [Bhattacharya, GP, Tropiano, PRD **92**, 113011 (2015)]

Example 3: Deuterium target data

- z-expansion analysis of deuterium target data
  [Meyer, Betancourt, Gran, Hill PRD **93** 113015 (2016)]

- They also extract the axial form factor



- $F_A = \sum_k a_k\, z(q^2)^k$

  $[a_1, a_2, a_3, a_4] = [2.30(0.13), -0.6(1.0), -3.8(2.5), 2.3(2.7)]$

Example 4: Lattice QCD

- Lattice QCD calculation of F_A using the z expansion is underway
  [Meyer, Hill, Kronfeld, Li, Simone, arXiv:1610.04593]

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- The future neutrino program requires *precision*
- Need better control of form factors, in particular F_A
- Axial mass problem: different m_A^{dipole} for different experiments
- Using z expansion: *consistent* values for axial mass:

- MiniBooNE ν data using the z expansion: $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV
  [Bhattacharya, Hill, GP PRD **84** 073006 (2011)]
- MiniBooNE $\bar{\nu}$ data using the z expansion: $m_A = 0.84^{+0.12}_{-0.04} \pm 0.11$ GeV
  [Bhattacharya, GP, Tropiano PRD **92** 113011 (2015)]
- Deuterium target data using the z expansion: $r_A^2 = 0.46(22)$ fm^2
  [Meyer, Betancourt, Gran, Hill PRD **93** 113015 (2016)]

Conclusions

- Next step: combine z expansion and CFG nuclear model [Bhattacharya, Cohen, Hen, GP, Piasetzky, Tockstein, Wieske *(in progress)*]
- The z expansion allows to control form factor effects *separately* from nuclear effects
- I invite the community to use the z expansion with a variety of nuclear models