Lattice QCD input for neutrino-nucleus interactions

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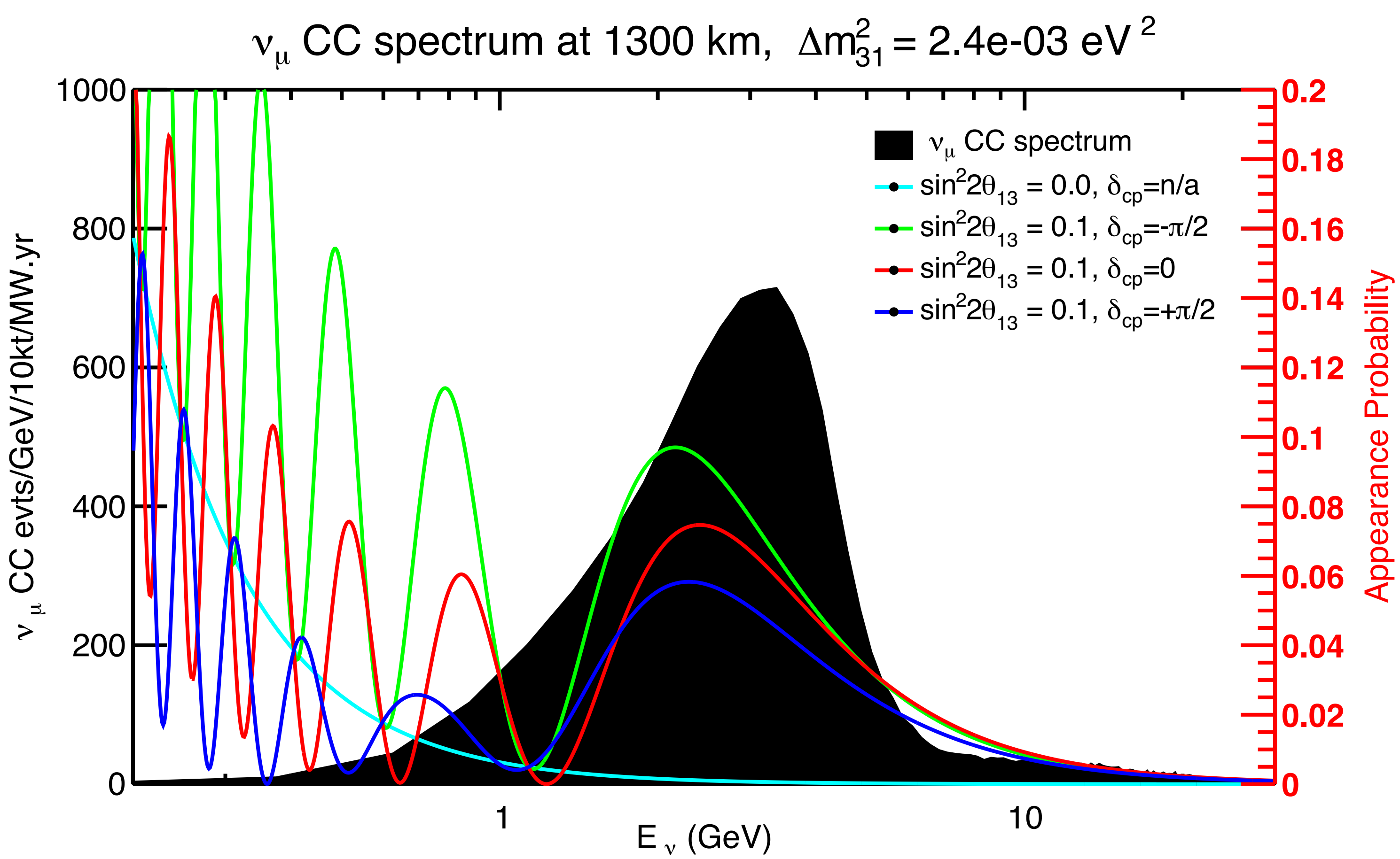
Long-baseline neutrino experiments

Neutrinos produced as secondary decay products of hadrons from primary reactions of protons with nuclei

→ neutrino energy must be reconstructed event-by-event from the final state of the reaction

DUNE
Need energy reconstruction to better than 100 MeV

Robust understanding of both nucleon and nuclear level amplitudes essential



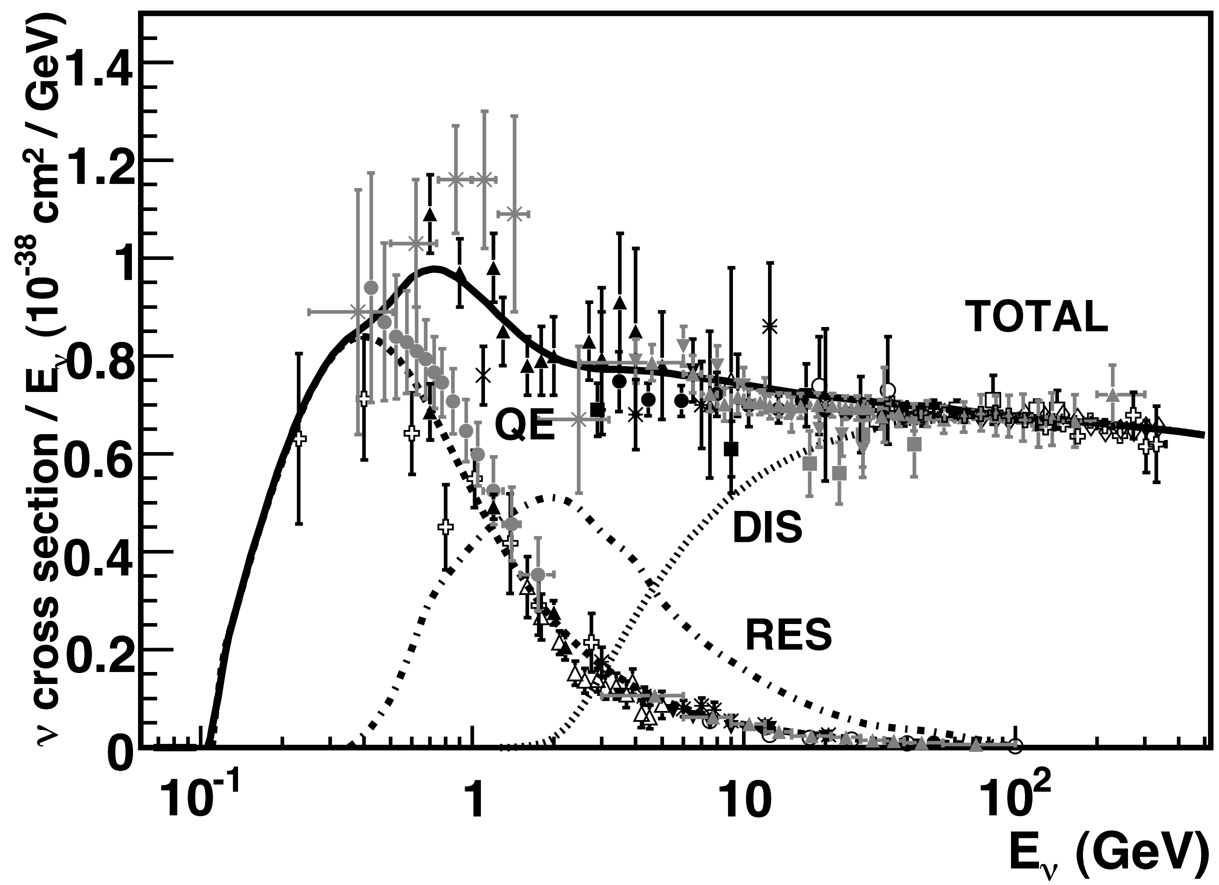
Adams C, et al. arXiv:1307.7335

Constraining ν-nucleus interactions

- For LBNEs neutrino energy distributions peak at 1-10 GeV
- Challenging region: several processes contribute
  - Quasielastic lepton scattering
  - Inelastic continuum / shallow-inelastic region
  - Resonances
- Lattice QCD can provide direct non-perturbative QCD predictions of nucleon and nuclear matrix elements

Neutrino charged-current cross-section

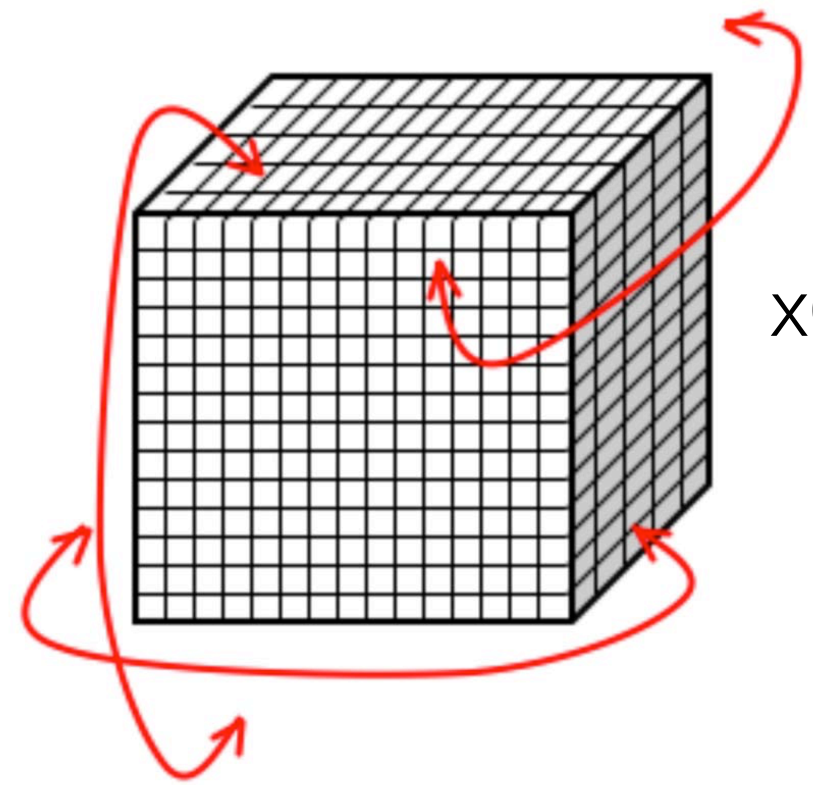
J.A. Formaggio, G.P. Zeller, Rev. Mod. Phys. 84 (2012) 1307



Lattice QCD

Calculate matrix elements directly from QCD

- Numerical first-principles approach
- Euclidean space-time $t \to i\tau$
  - Finite lattice spacing a
  - Volume $L^3 \times T \approx 32^3 \times 64$
  - Boundary conditions
- Finite but large number of d.o.f



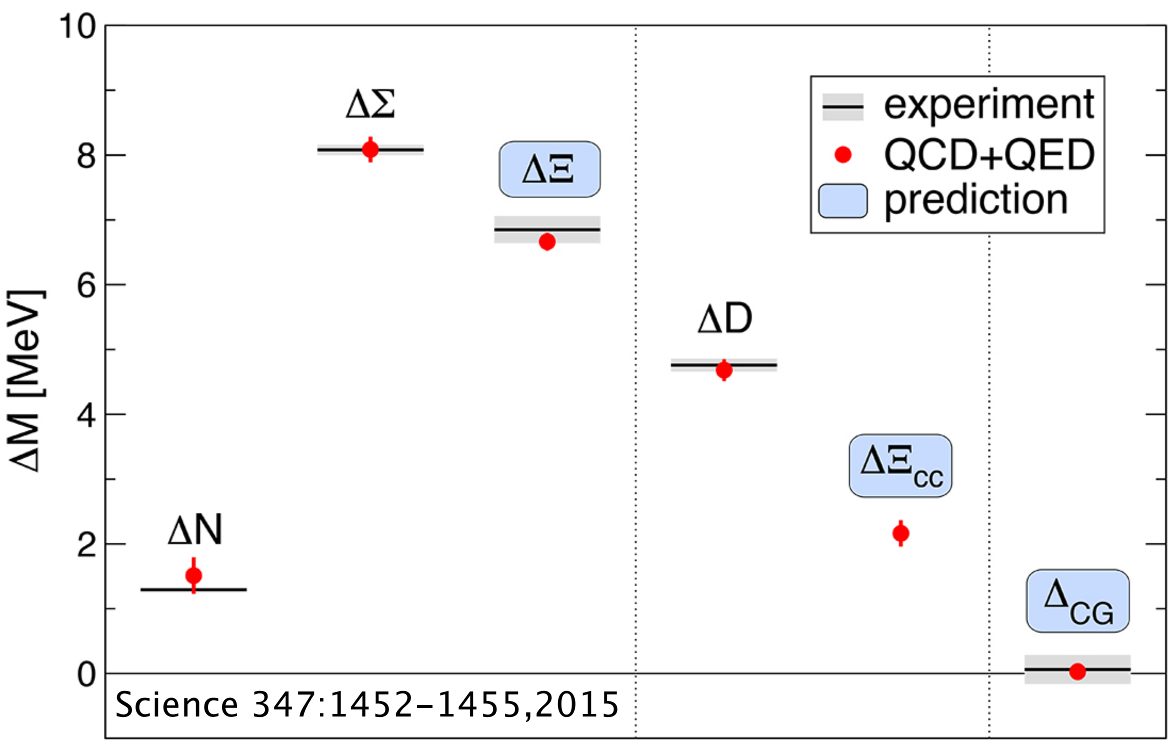
Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A\mathcal{D}\overline{\psi}\mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \quad \rightarrow \quad \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\mathrm{conf}}} \sum_{i}^{N_{\mathrm{conf}}} \mathcal{O}([U^i])$$

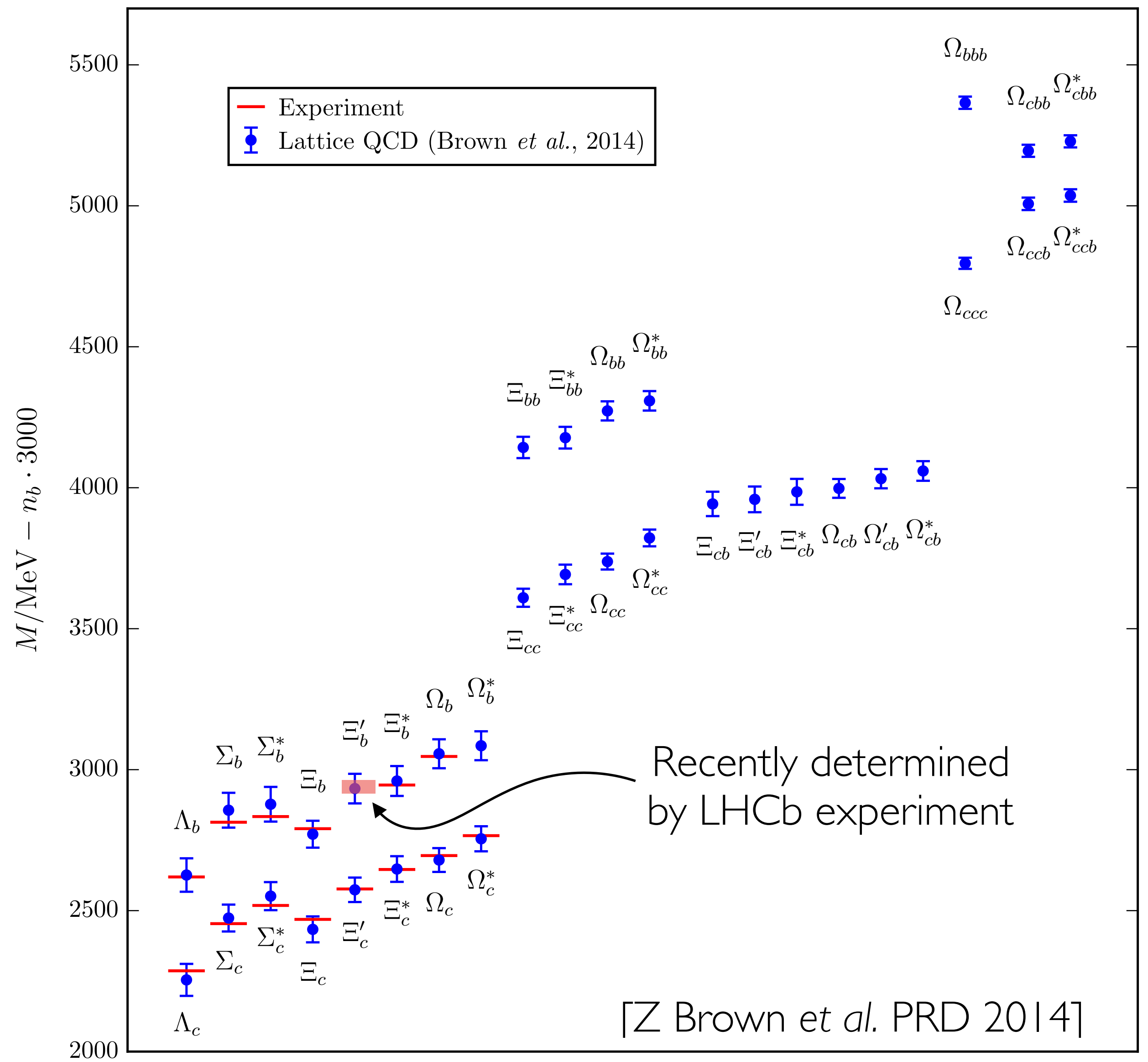
with field configurations U^i distributed according to $e^{-S[U]}$

Lattice QCD works

- Ground state hadron spectrum reproduced
- p-n mass splitting reproduced
- . . .



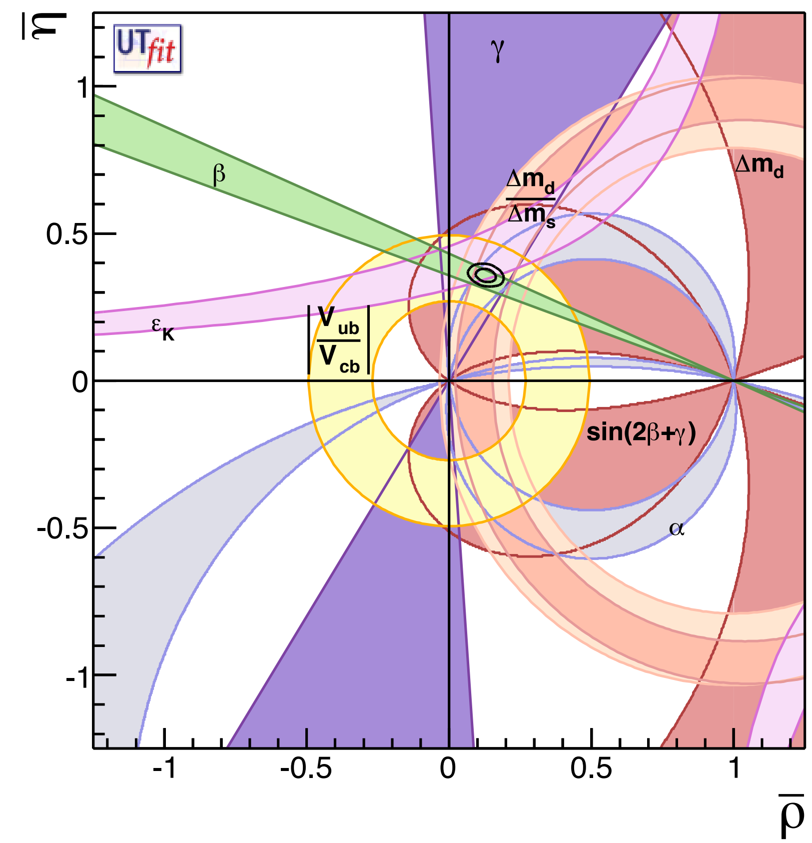
- Predictions for new states with controlled uncertainties

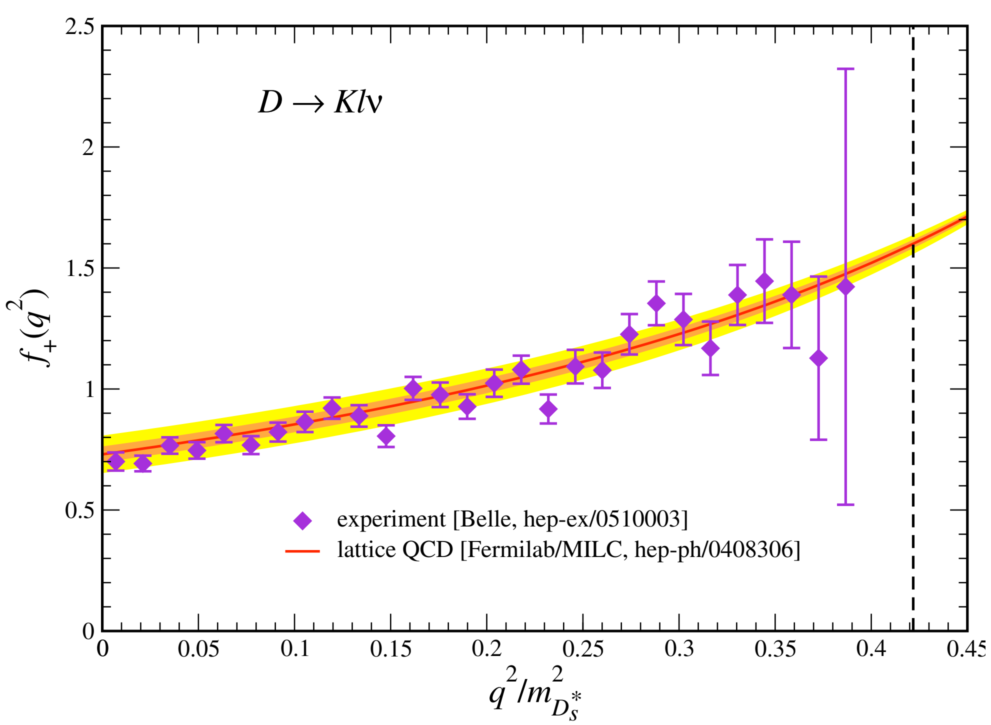


Lattice QCD for flavour physics

- For simple observables LQCD is precision science
  - Combine with experiment to determine SM parameters
  - Verify CKM paradigm
- SM predictions with reliable uncertainty quantification
- I.e., LQCD has had significant impact in flavour physics

Impact on neutrino program:
a current challenge to LQCD

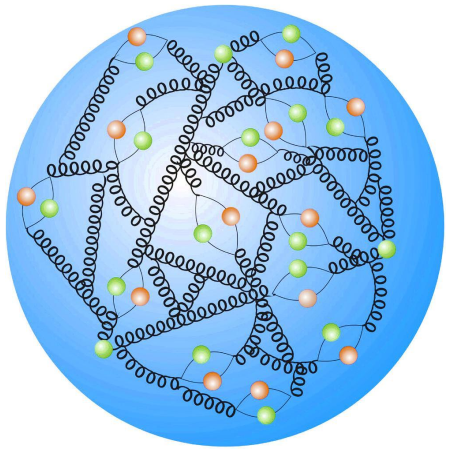




LQCD input for ν-nucleus interactions

1. **Directly access QCD single-nucleon form factors without nuclear corrections**

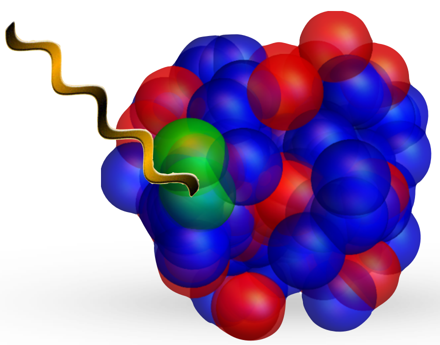
   Reliable calculations with fully-controlled uncertainties



2. **Calculate matrix elements in light nuclei from first principles**

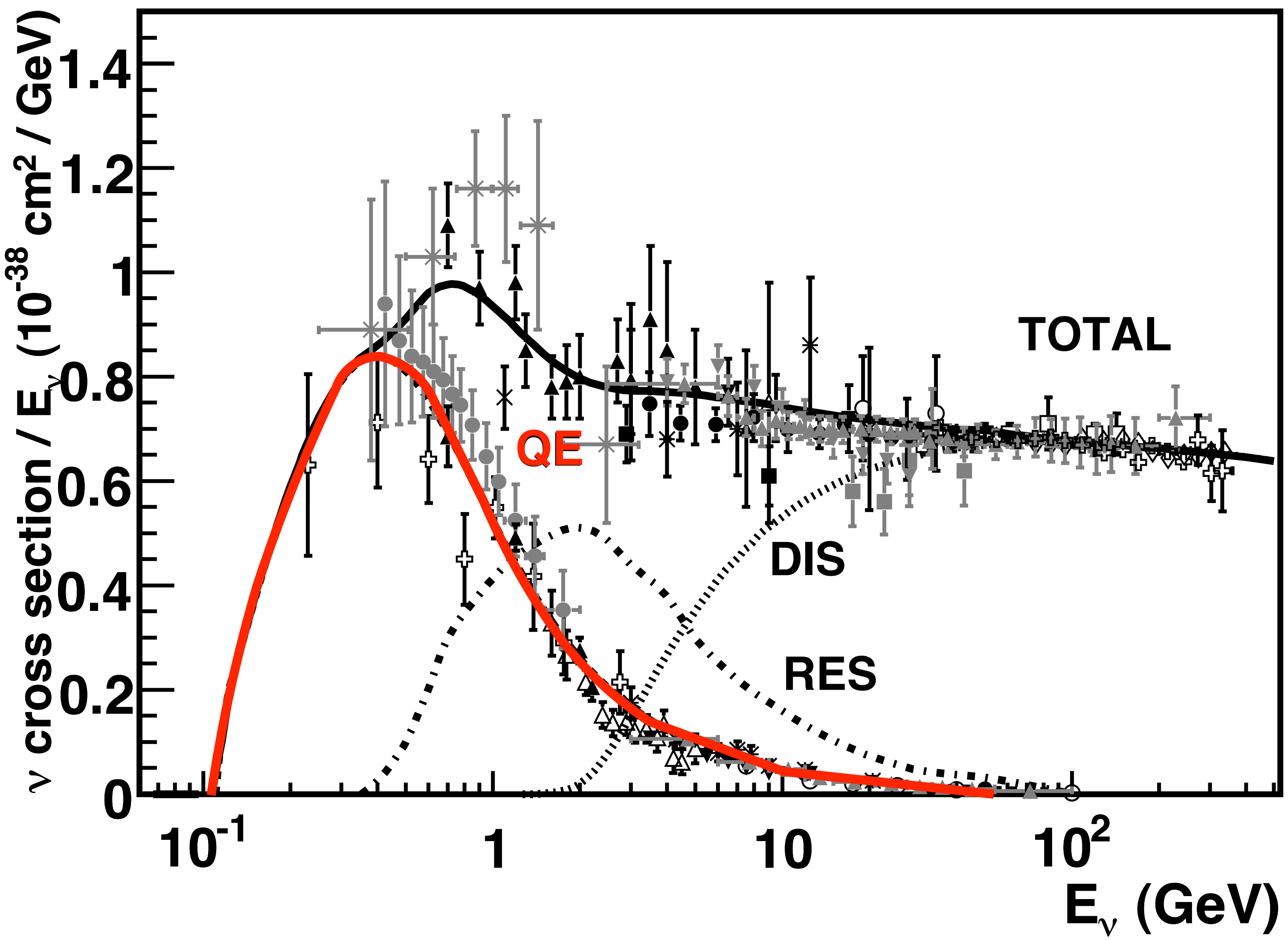
   ➡ EFT to reach heavy nuclear targets relevant to experiment

   First calculations of axial charge of light nuclei



Constraining ν-nucleus interactions

Neutrino charged-current
cross-section

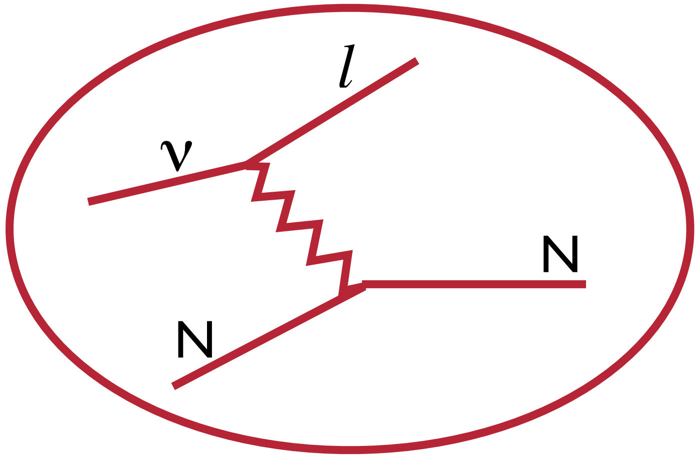


J.A. Formaggio, G.P. Zeller, Rev. Mod. Phys. 84 (2012) 1307

Quasi-elastic scattering

- Cross-section for quasi-elastic neutrino-nucleon scattering

$$\frac{d\sigma}{dQ^2} = \frac{G_f^2 M^2 \cos^2\theta_C}{8\pi E_\nu^2}\left[A \mp \frac{(s-u)}{M^2}B + \frac{(s-u)^2}{M^4}C\right]$$



$$A = \frac{(m^2+Q^2)}{M^2}\Big[(1+\tau)G_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2$$
$$- \frac{m^2}{4M^2}\left((F_1+F_2)^2 + (G_A+2G_P)^2 - \left(\frac{Q^2}{M^2}+4\right)G_P^2\right)\Big]$$

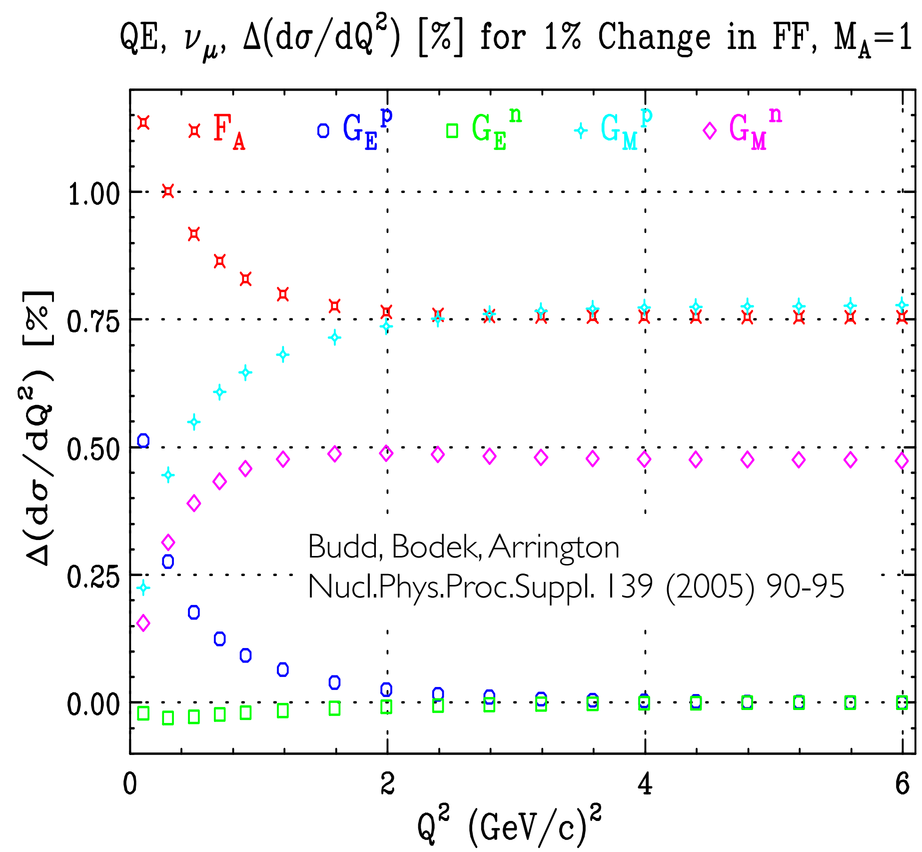
$$B = \frac{Q^2}{M^2}G_A(F_1+F_2)$$

$$C = \frac{1}{4}(G_A^2 + F_1^2 + \tau F_2^2)$$

G_A
- dominant contribution
- largest uncertainty

$F_{1,2}$ Well-determined from electron scattering expts

G_P can be related to G_A by pion pole dominance



Axial form factor

- Traditionally assumed to have dipole form

$$G_A(Q^2) = \frac{g_A}{\left(1 + Q^2 / M_A^2\right)^2}$$

- $g_A = 1.2671$ determined with high precision from nuclear beta decay

- axial mass M_A must be determined experimentally

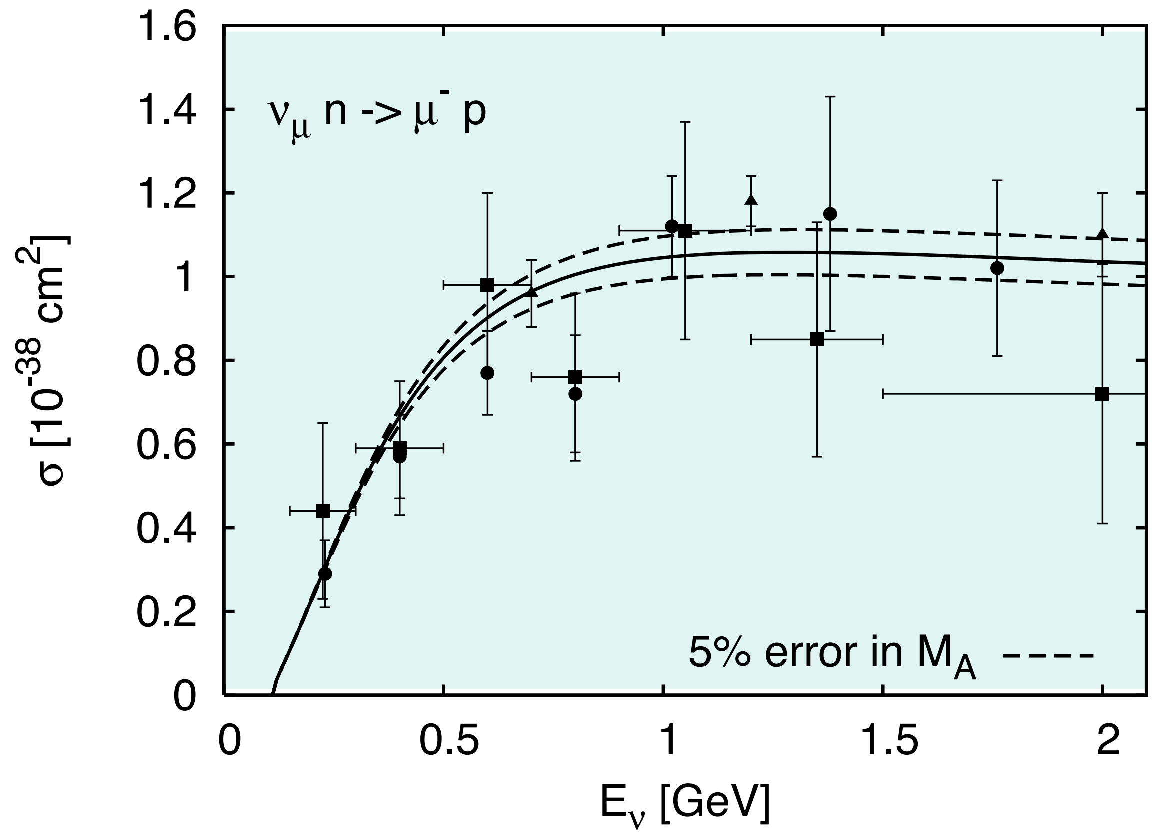
BUT

- Electromagnetic FFs show significant deviation from dipole parametrisation form

More general alternatives

- Model-indep z-expansion
  Bhattacharya et al, Phys.Rev. D84 (2011) 073006
- Direct LQCD results

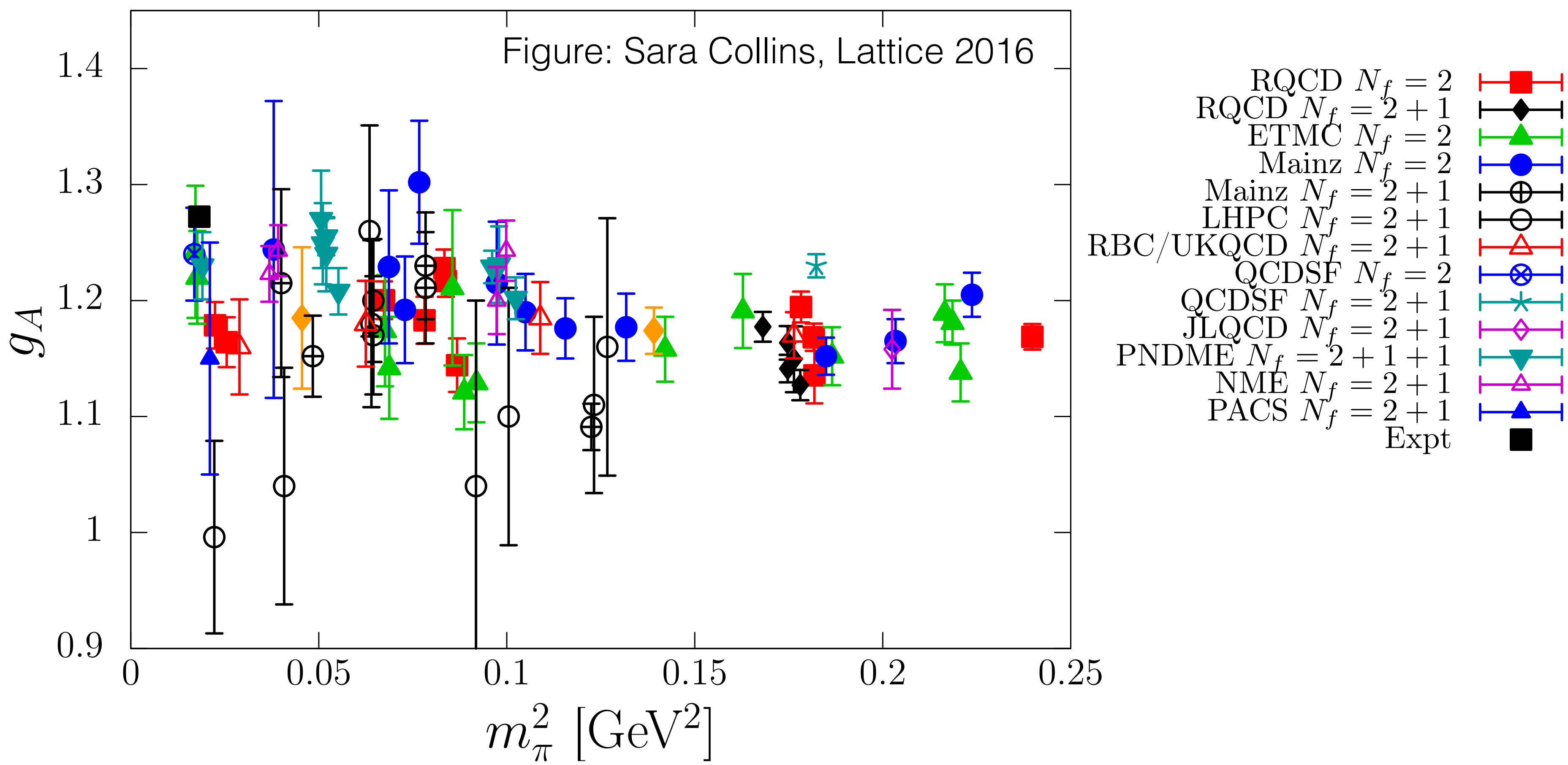
Total QE cross-section sensitive to the axial mass:



Mosel, Ann. Rev. Nucl. Part. Sci. 66, 171 (2016)

Nucleon Axial FFs from LQCD

- $g_A = G_A(Q^2 = 0)$ is a historically difficult calculation
- Recent calculations in agreement with experiment with fully-controlled uncertainties
- Q^2 -dependence well-determined in LQCD — competitive with experiment
- z-parameterisations remove assumption of dipole form



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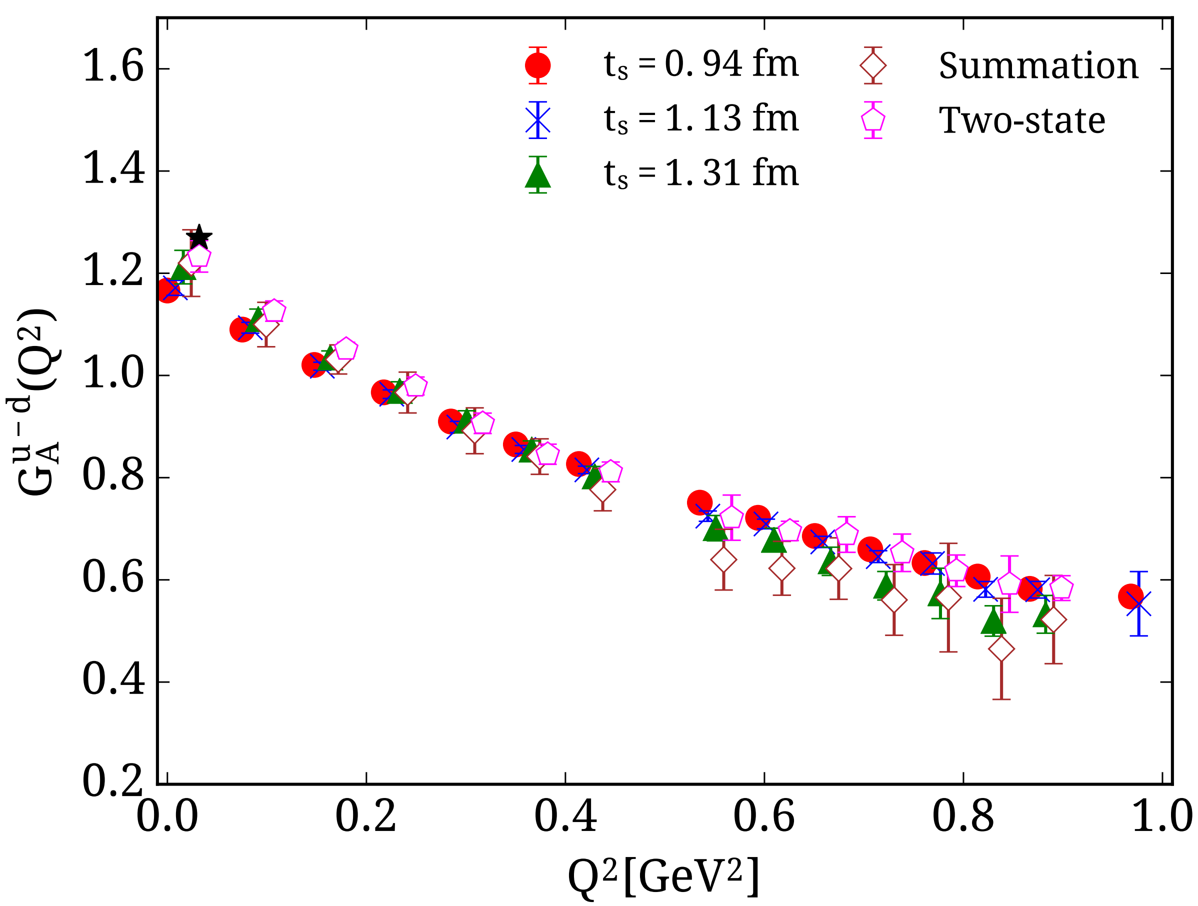
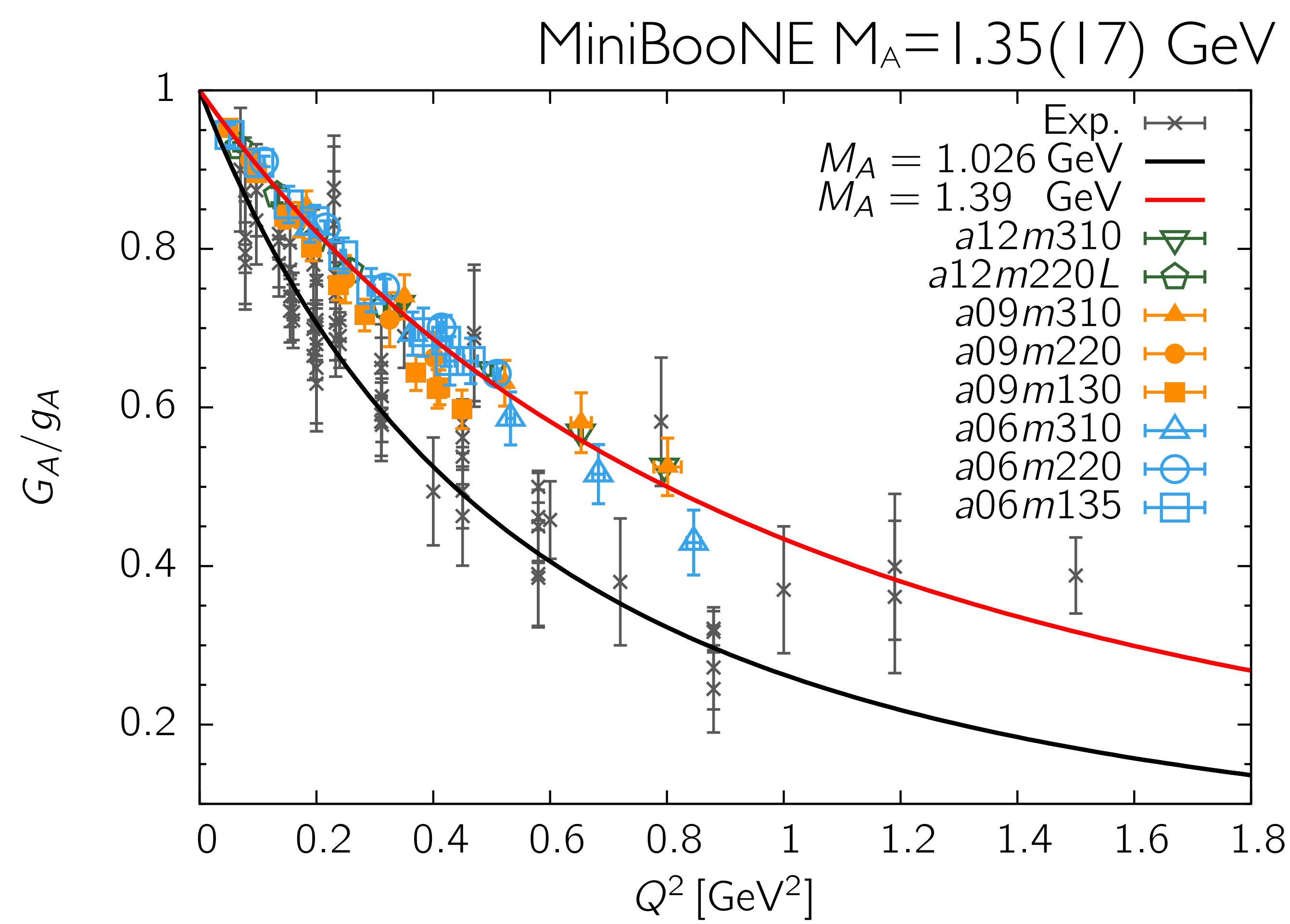


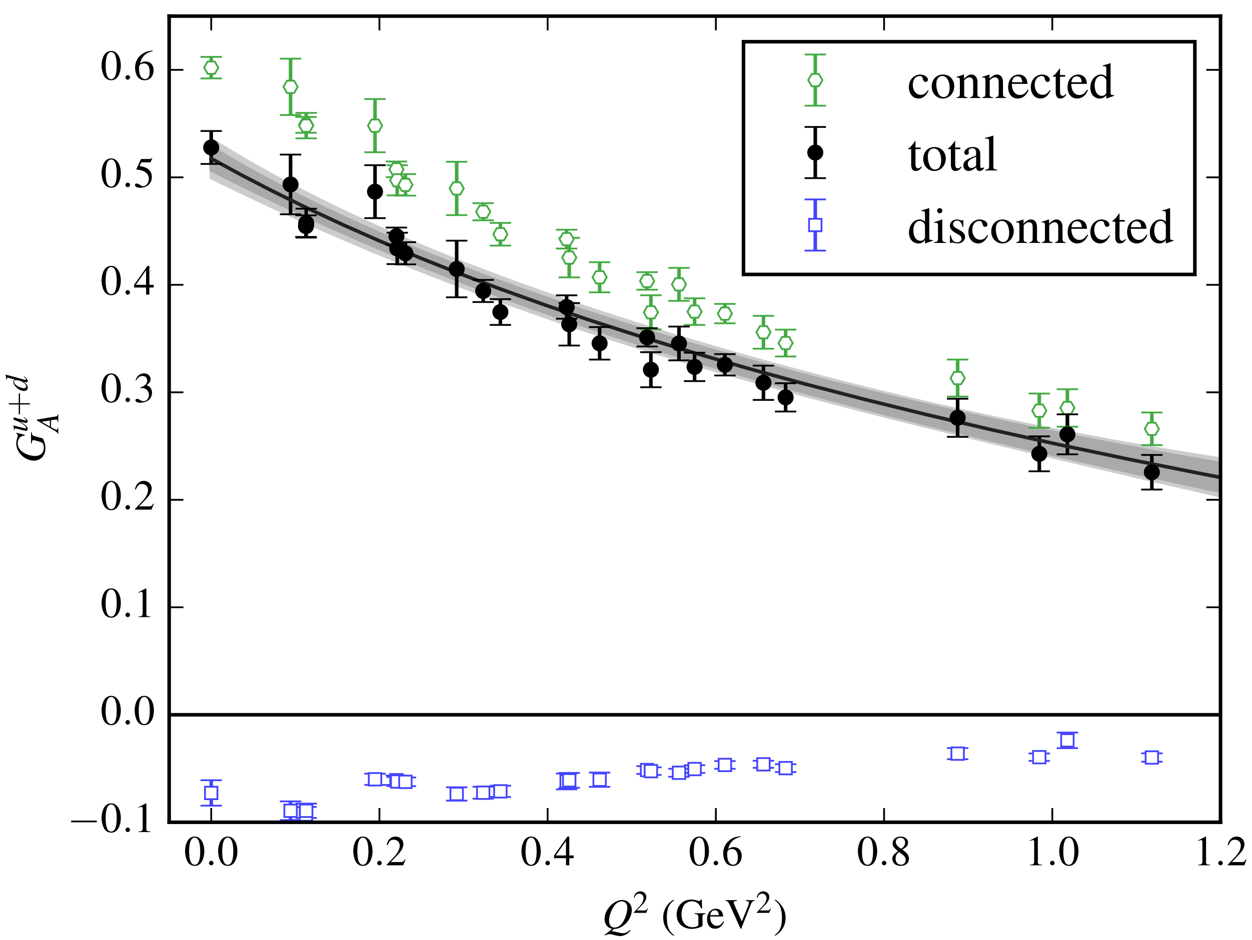
Alexandrou et al., arXiv:1705.03399

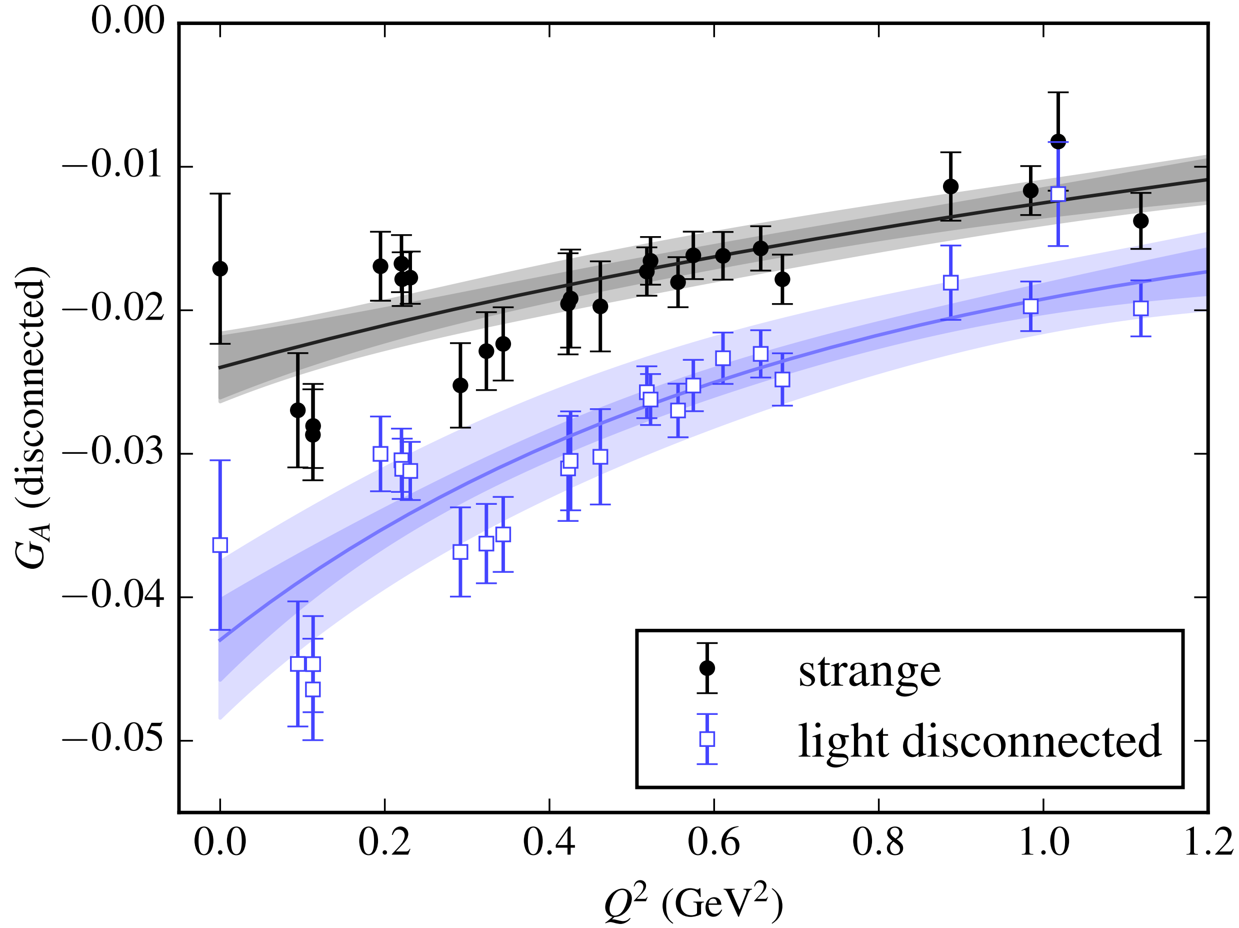


Gupta et al., arXiv:1705.06834

Nucleon Axial FFs from LQCD

- Strange quark contributions determined separately and can be isolated

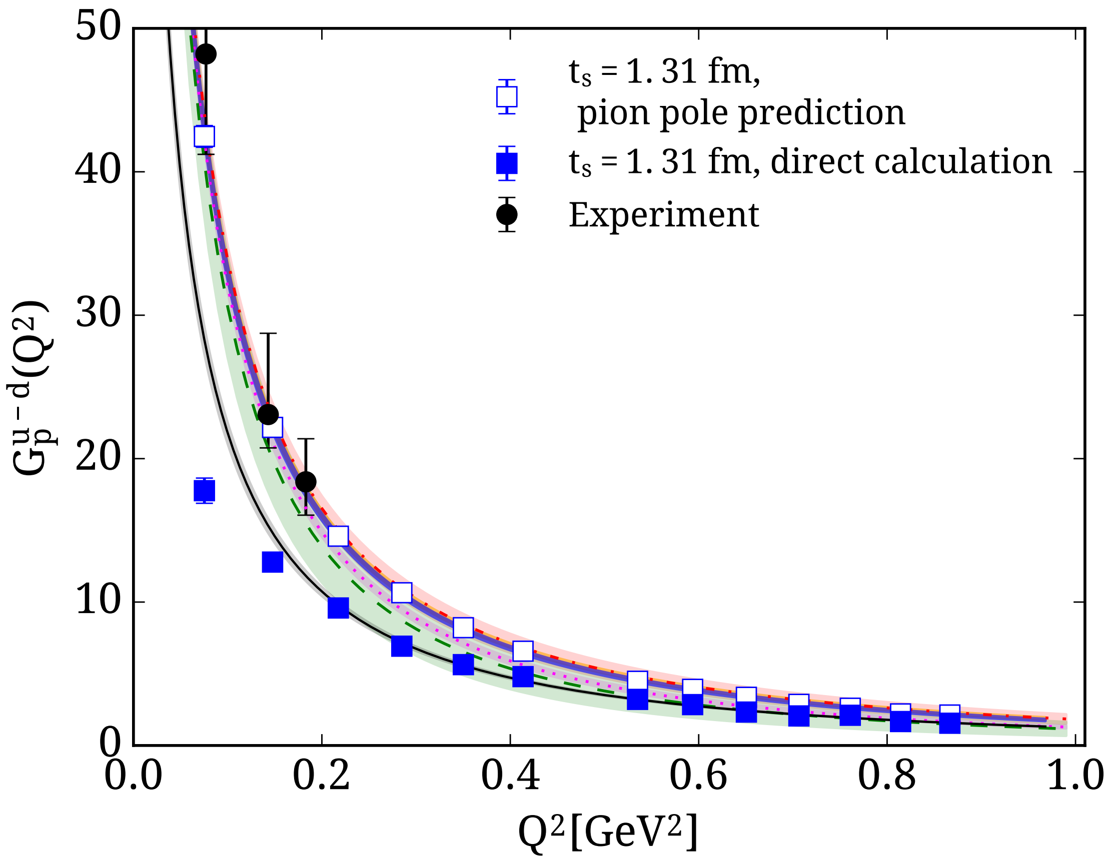




Green et al., Phys. Rev. D 95, 114502 (2017)

Nucleon pseudoscalar FF

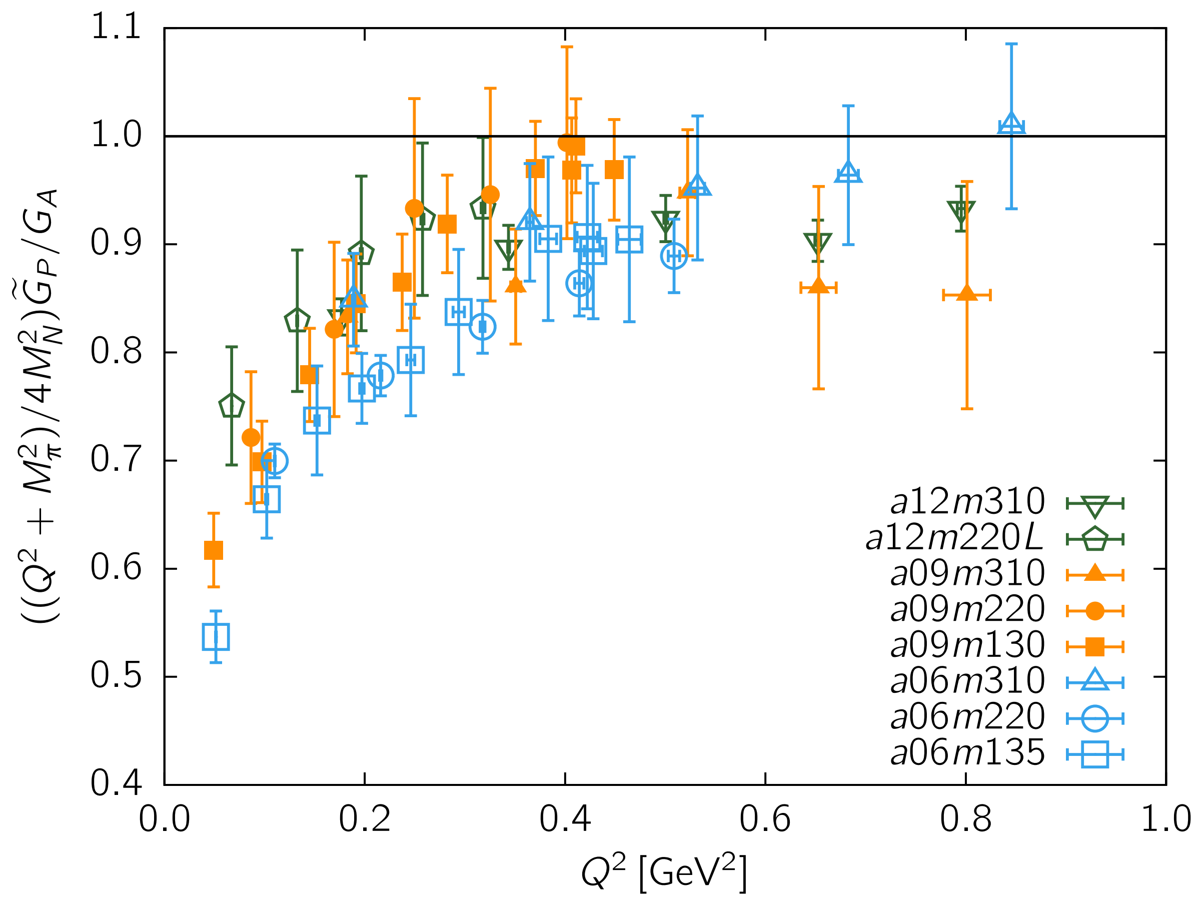
Calculations with controlled uncertainties in agreement with experiment



Alexandrou et al., arXiv:1705.03399

Deviations from pion-pole dominance ansatz at low-Q^2

$$\tilde{G}_P(Q^2) = G_A(Q^2)\left[\frac{4M_N^2}{Q^2 + M_\pi^2}\right]$$

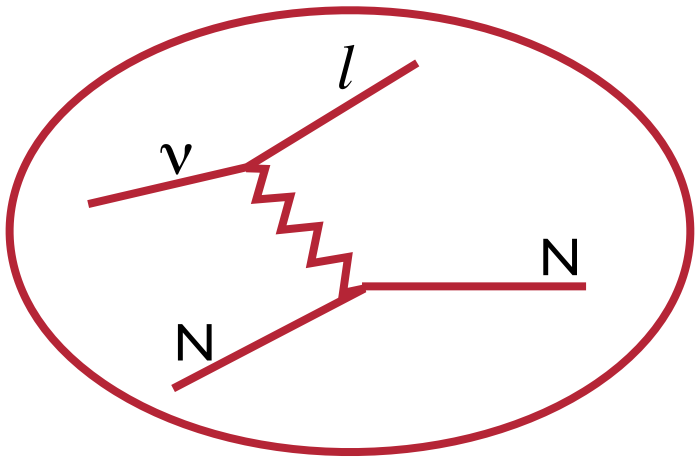


Gupta et al., arXiv:1705.06834

Quasi-elastic scattering

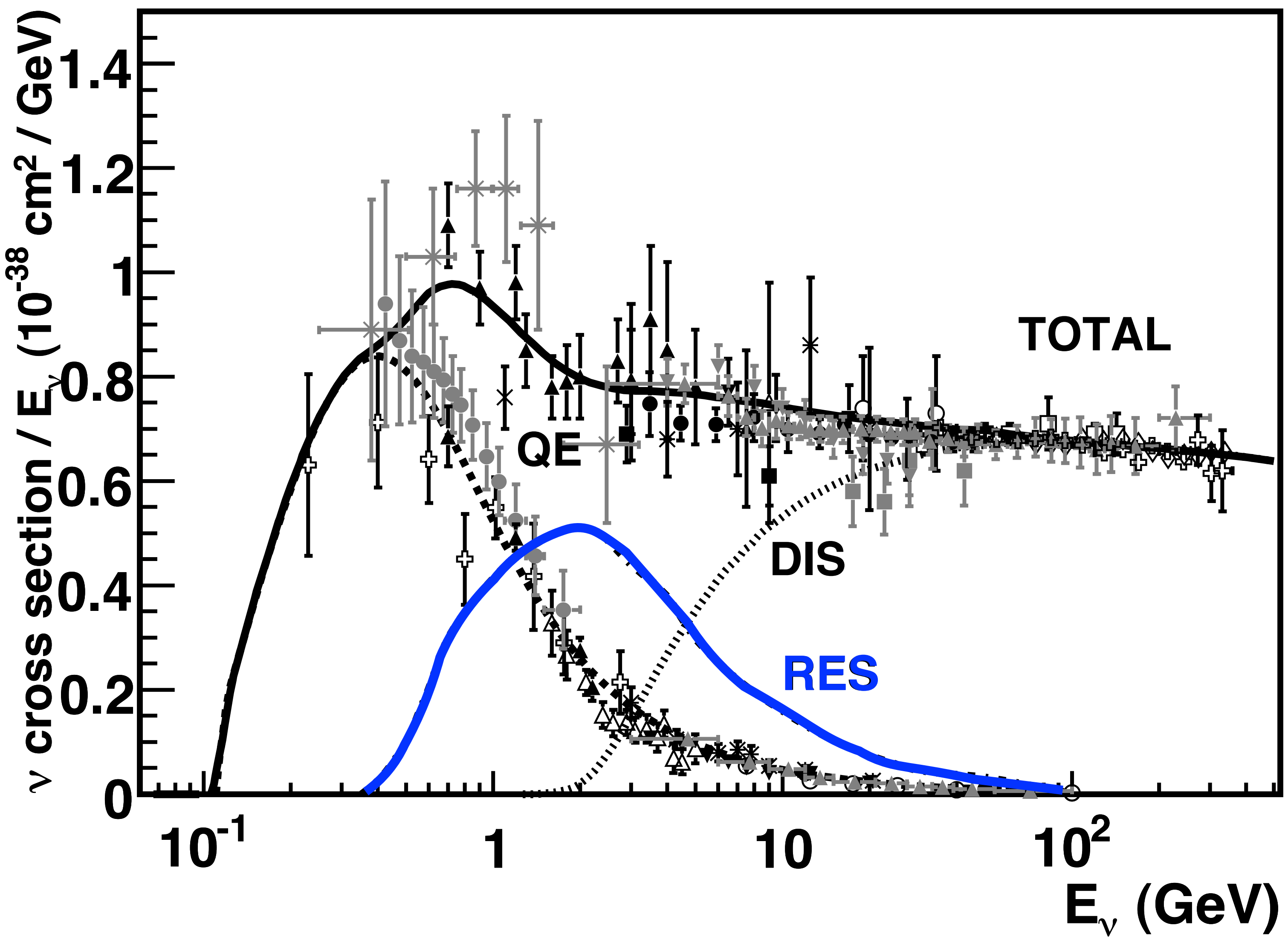
LQCD input for the quasi-elastic scattering region:

- Q^2 dependence of nucleon axial form factor
  - fully-controlled uncertainties
  - competitive with experiment
  - z parameterisation removes assumption of dipole form
- Nucleon pseudo scalar form factor
  - fully-controlled uncertainties
  - competitive with experiment
  - deviations from pion-pole ansatz observed



Constraining ν-nucleus interactions

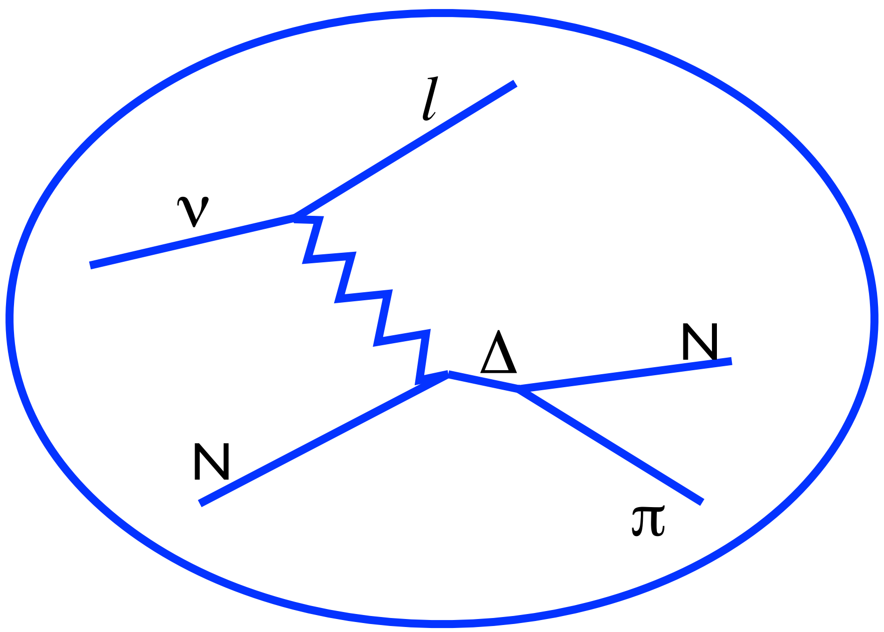
Neutrino charged-current cross-section



J.A. Formaggio, G.P. Zeller, Rev. Mod. Phys. 84 (2012) 1307

Resonance region

- Energies above ~200 MeV, inelastic excitations from pion production
- Dominant contribution from Δ resonance
- N*'s also important at high E_ν
- Very difficult to access experimentally
  Constrained only from PCAC
- QCD calculations possible
- Need to account for unstable nature of resonance: extract N→Nπ transition FFs

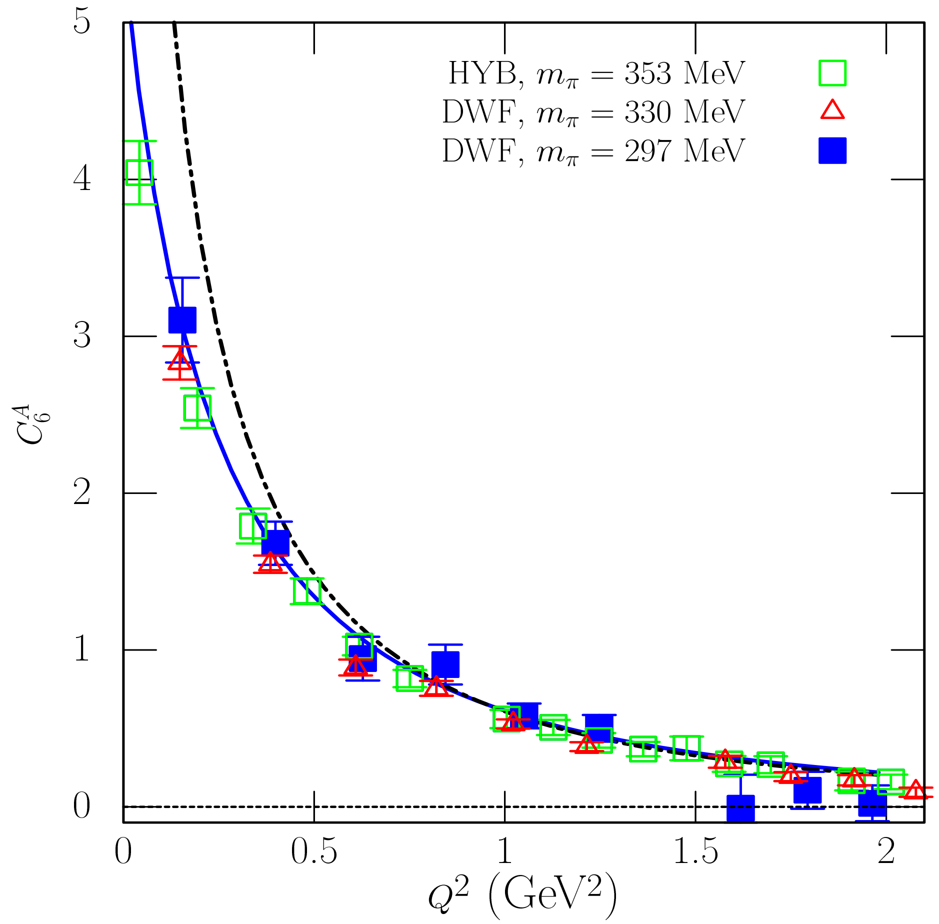


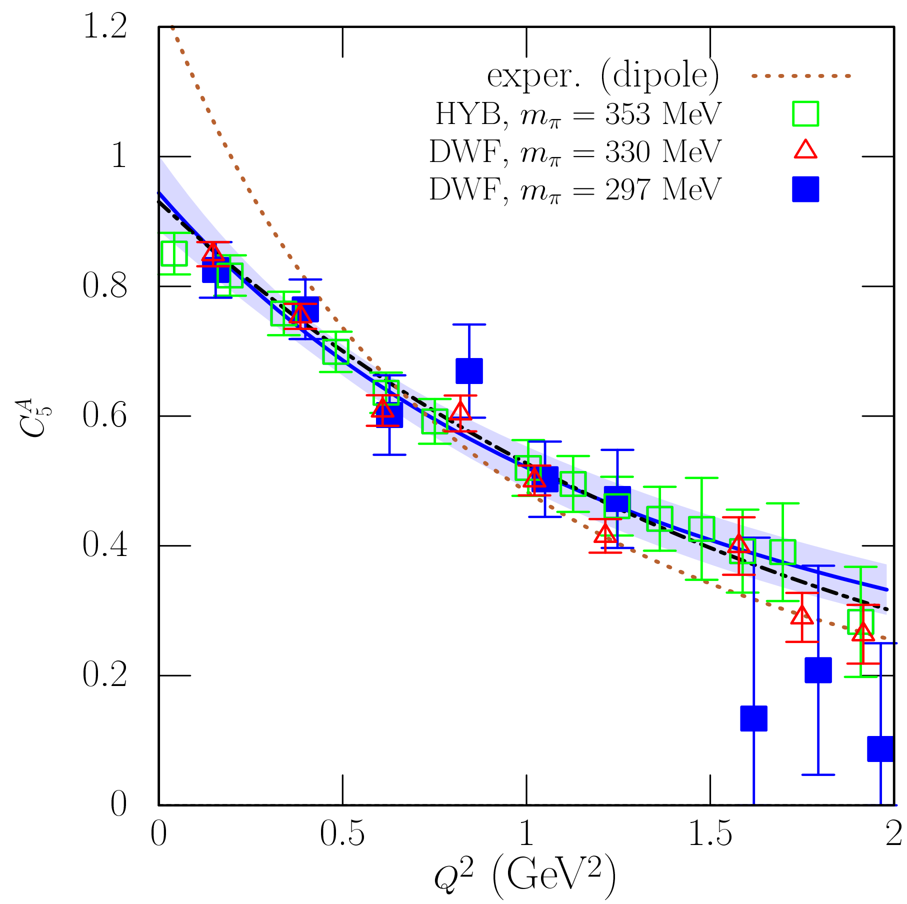
Resonance region

Lattice QCD calculation of axial N Δ transition form factor:

$$\langle \Delta(p',s')|A^3_\mu|N(p,s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta m_N}{E_\Delta(\mathbf{p}')E_N(\mathbf{p})}\right)^{1/2} \bar{u}^\lambda_{\Delta^+}(p',s') \left[\left(\frac{C^A_3(q^2)}{m_N}\gamma^\nu + \frac{C^A_4(q^2)}{m_N^2}p'^\nu\right)(g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu})\,q^\rho + C^A_5(q^2)g_{\lambda\mu} + \frac{C^A_6(q^2)}{m_N^2}q_\lambda q_\mu\right]u_P(p,s)$$

C Alexandrou et al., Phys.Rev. D83 (2011) 014501

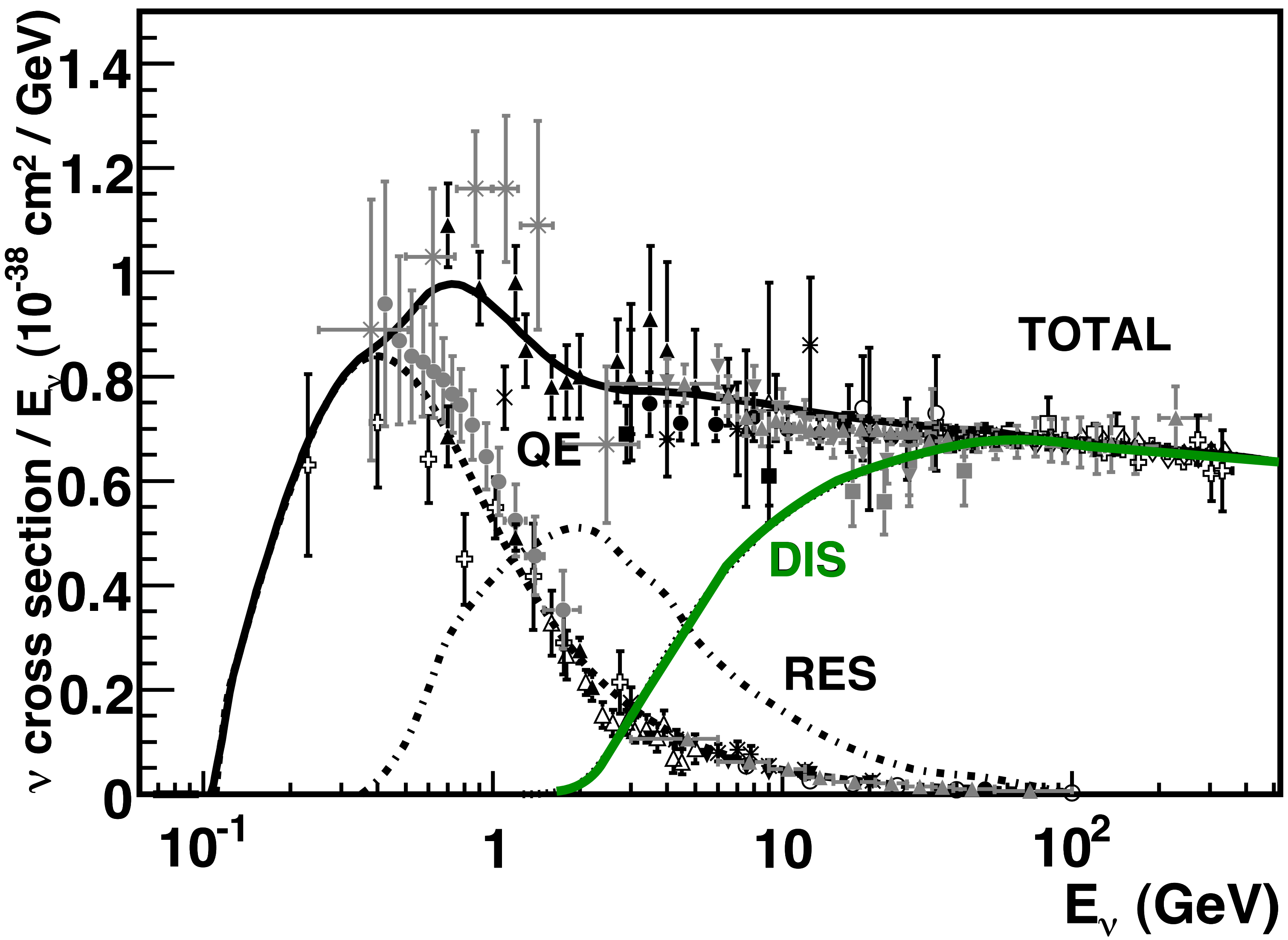




CAVEAT: Complexities at physical point with unstable resonances

Constraining ν-nucleus interactions

Neutrino charged-current
cross-section



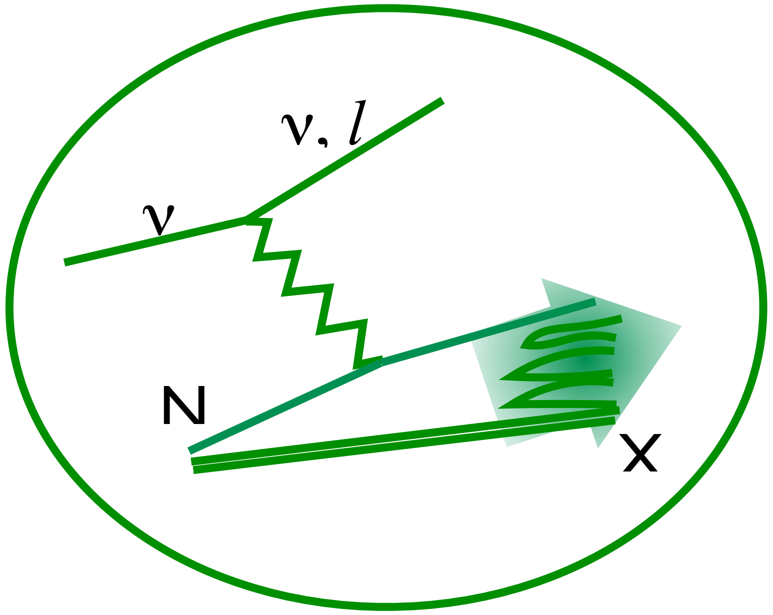
J.A. Formaggio, G.P. Zeller, Rev. Mod. Phys. 84 (2012) 1307

Shallow inelastic region

- In inelastic regime, quark PDFs of the nucleon control scattering cross-section
- In shallow inelastic region, both resonances and DIS are important
- Multi-meson channels may become important
- Nuclear effects are different in **ν**A vs. eA (MINER**ν**A)
- DIS structure functions accessible in lattice QCD
  - low moments of structure functions controlled

$$M_n = \int_{-1}^{1} x^n f(x) dx, \quad n \lessapprox 4$$

  - x-dependence difficult but promising

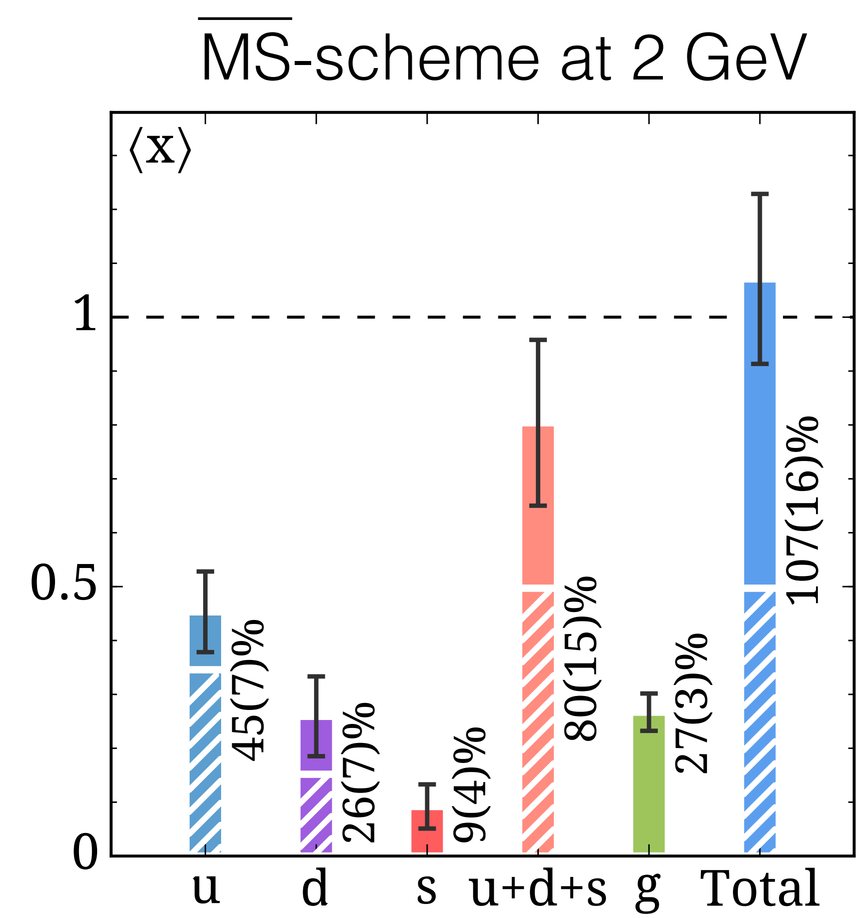


Nucleon PDFs

- Lattice QCD typically calculates low moments of PDFS
- Can separate and isolate contributions from
  - Strangeness
  - Charge symmetry violation
  - Gluons

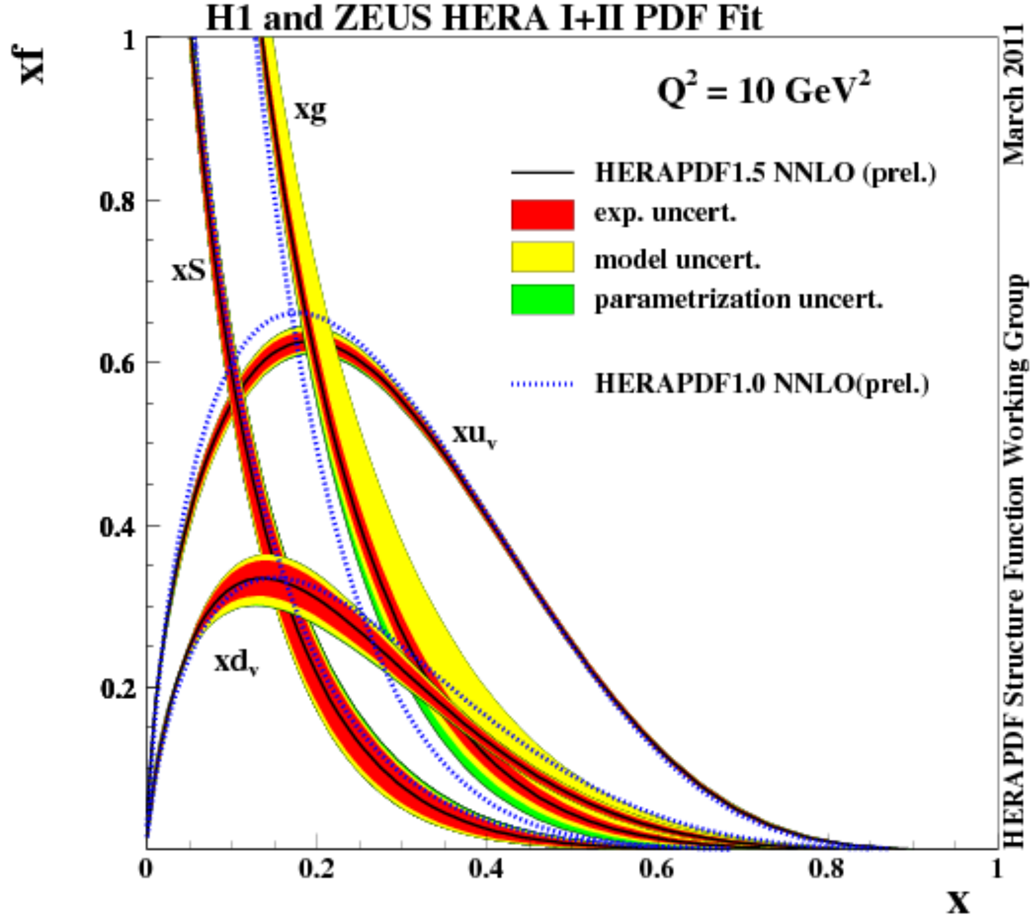
e.g., all terms of nucleon momentum decomposition calculated with controlled systematics

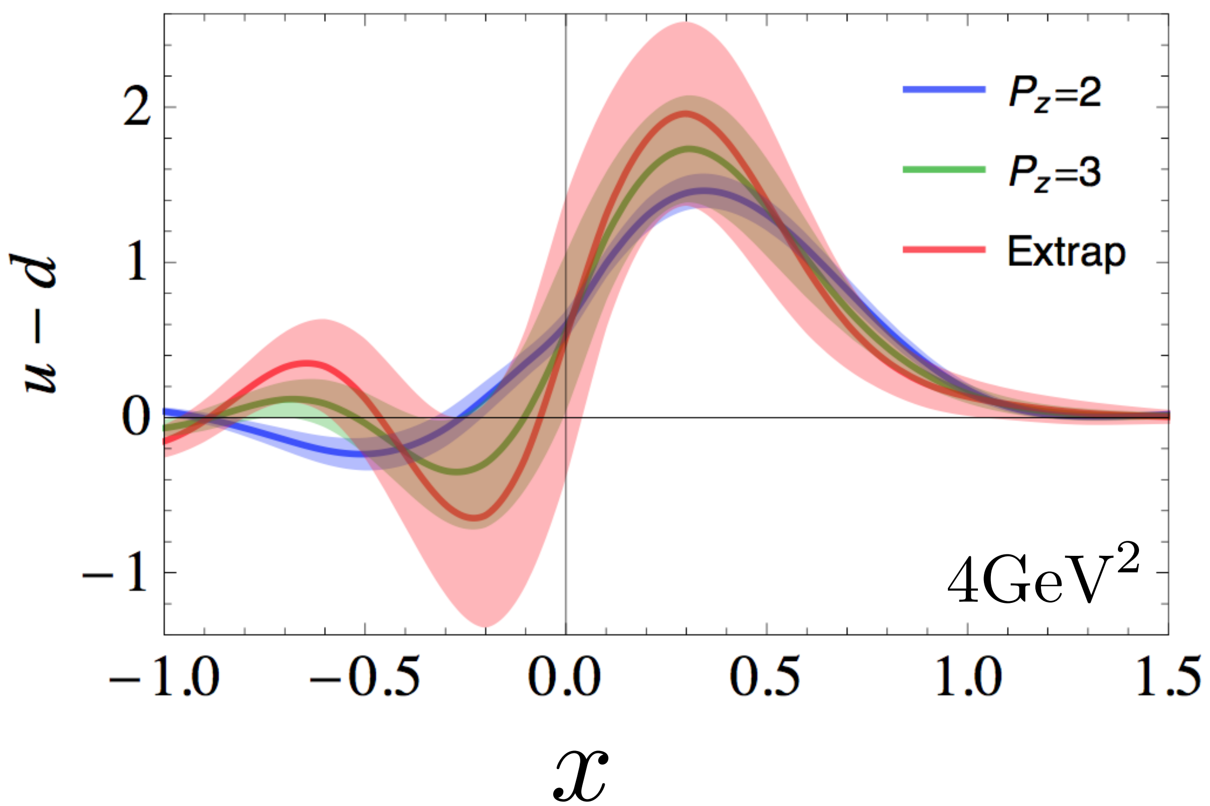
C. Alexandrou et al., arXiv:1706.02973



Nucleon PDFs

- First calculations of x-dependence of nucleon PDFs
- Rapid progress, but many systematics to be controlled
- Will not improve on experimental constraints in near future



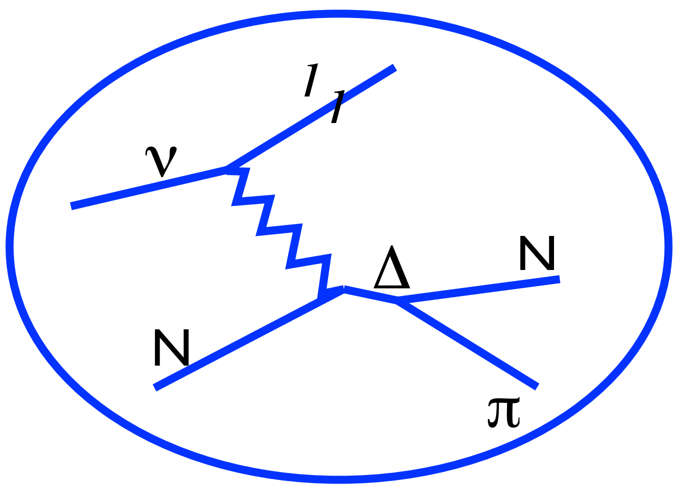


J-W Chen et al., arXiv:1706.01295

Resonance region

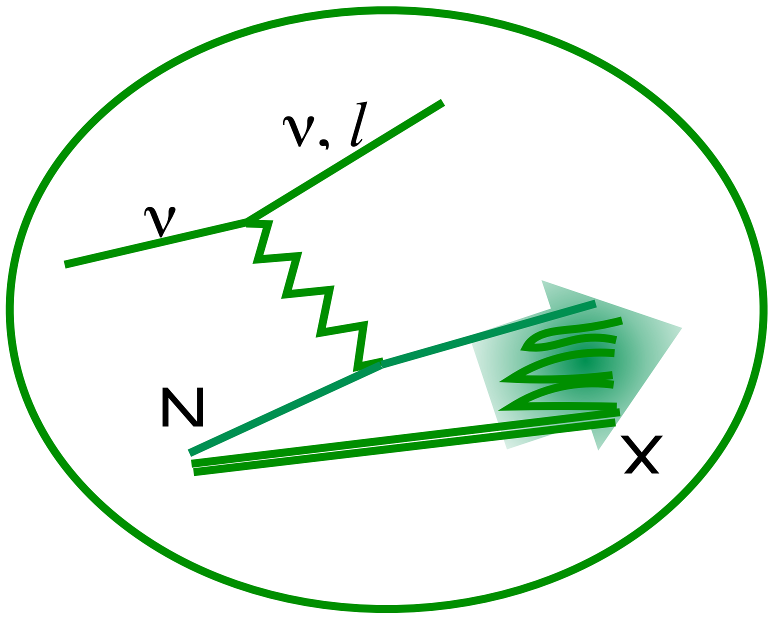
LQCD input for the resonance region:

- First calculations of axial transition form factors
  - resonances difficult for lattice QCD
  - currently: uncontrolled systematic uncertainties, unphysical values of quark masses
  - formalism in place to move to physical case



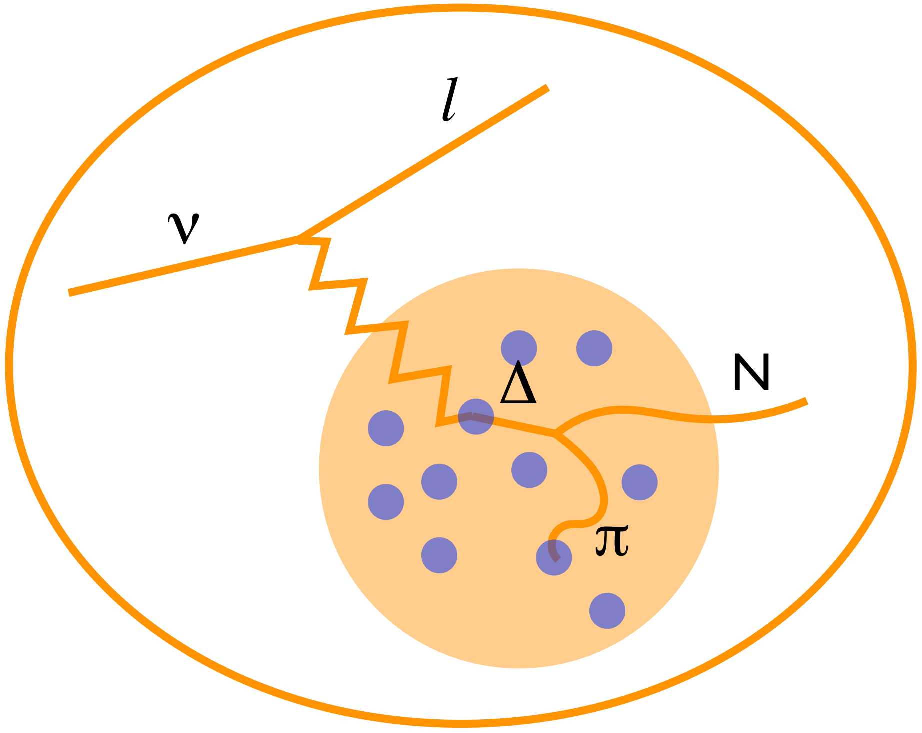
LQCD input for the inelastic scattering region:

- Much recent progress, but challenging region for direct input to neutrino program



Nuclear effects

- Targets are nuclei (C, Fe, Ar, Pb, H_2O) so how relevant are nucleon FFs, PDFs?
  - EMC effect
  - Quenching of g_A in GT transitions
- Experimental investigations: MINERνA



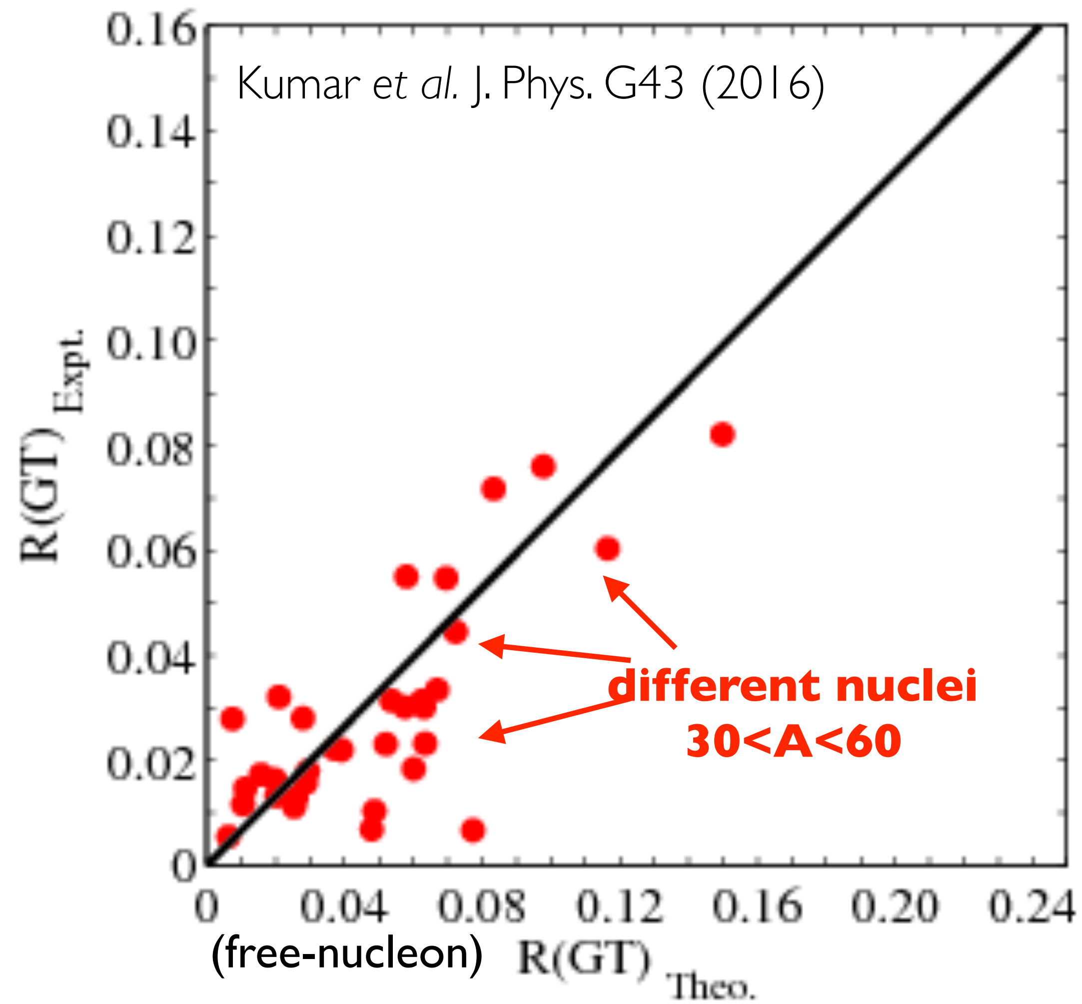
Calculate matrix elements in light nuclei from first principles

➡ EFT to reach heavy nuclear targets relevant to experiment

First calculations of axial charge of light nuclei

Nuclear effects

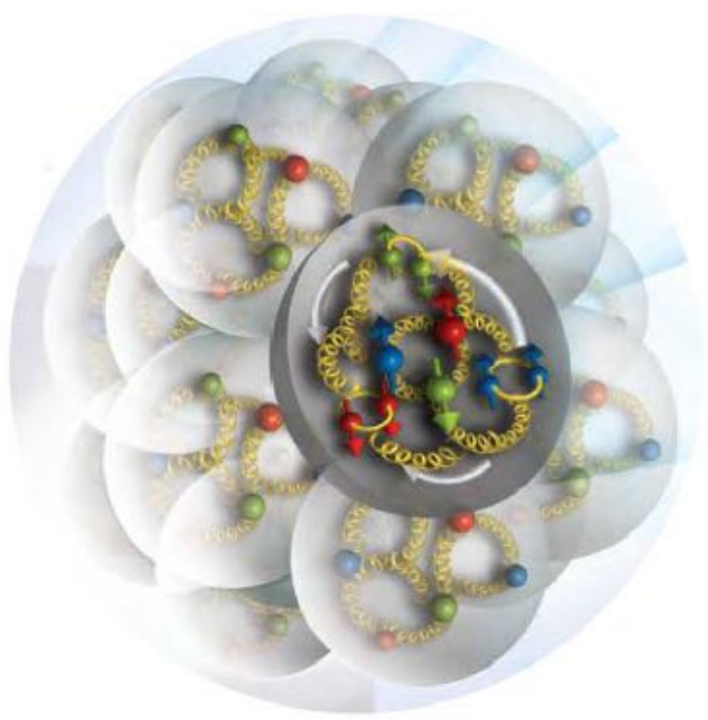
- Gamow-Teller transitions in nuclei are a stark example of problems
- Well-measured
- Best nuclear structure calculations are systematically off by 20–30%
    - Large range of nuclei (30<A<60) where spectrum is well described
    - QRPA, shell-model,...
    - Correct for it by "quenching" axial charge in nuclei ...



Nuclear physics from LQCD

Nuclei on the lattice

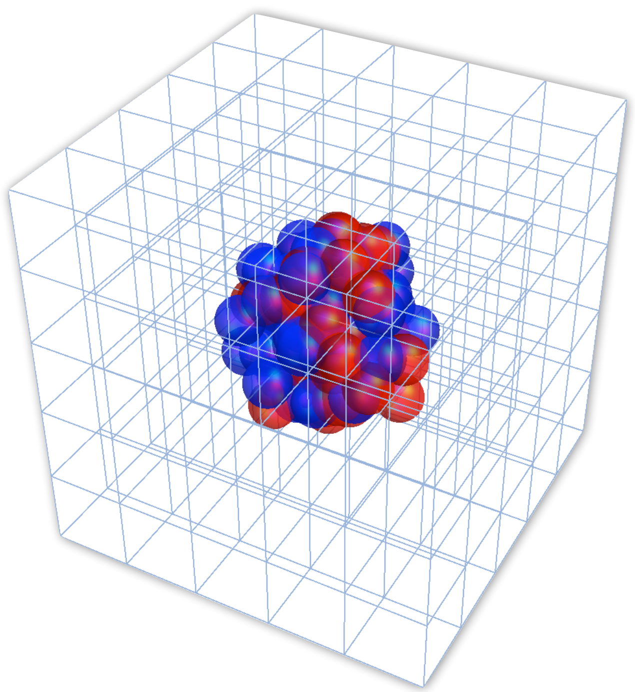
- Calculations of matrix elements of currents in light nuclei just beginning
- Deeply bound nuclei:
  use the same techniques as for single hadron matrix elements
- Near threshold states:
  need to be careful with volume effects

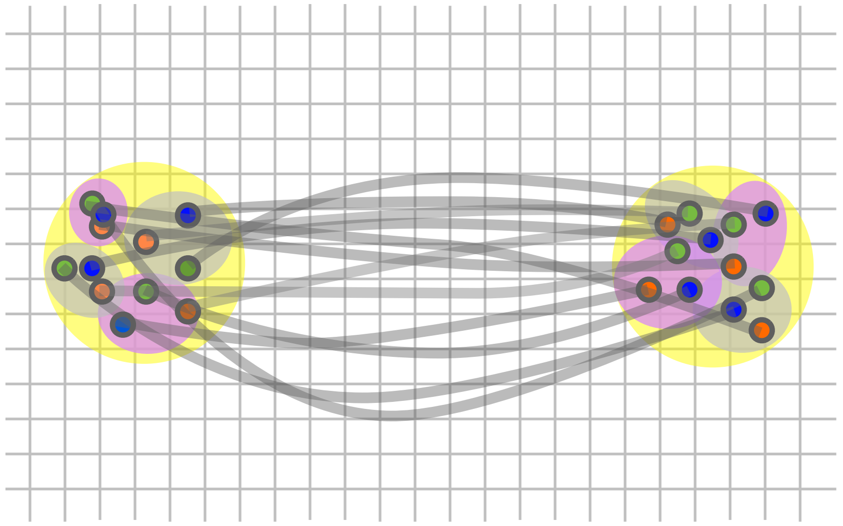


Nuclear physics from LQCD

Nuclei on the lattice

- Hard problem
  - Noise:
    Statistical uncertainty grows exponentially with number of nucleons
  - Complexity:
    Number of contractions grows factorially





[Detmold & Savage, Detmold & Orginos; Doi & Endres]

Unphysical nuclei

NPLQCD collaboration

- QCD with unphysical quark masses

  m_π~800 MeV, m_N~1,600 MeV

  m_π~450 MeV, m_N~1,200 MeV

- Spectrum of light nuclei (A<5)

  [PRD **87** (2013), 034506]

- Nuclear structure: magnetic moments, polarisabilities (A<5)

  [PRL **113**, 252001 (2014), PRD 92, 114502 (2015)]

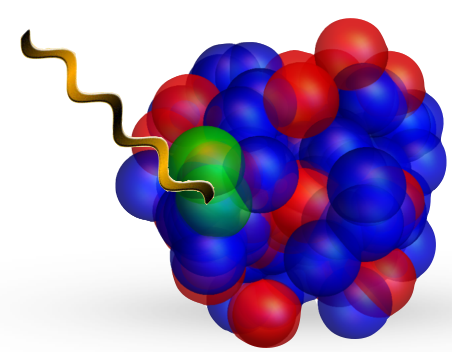
- First nuclear reaction: np→dγ

  [PRL **115**, 132001 (2015)]

- Proton-proton fusion and tritium β-decay

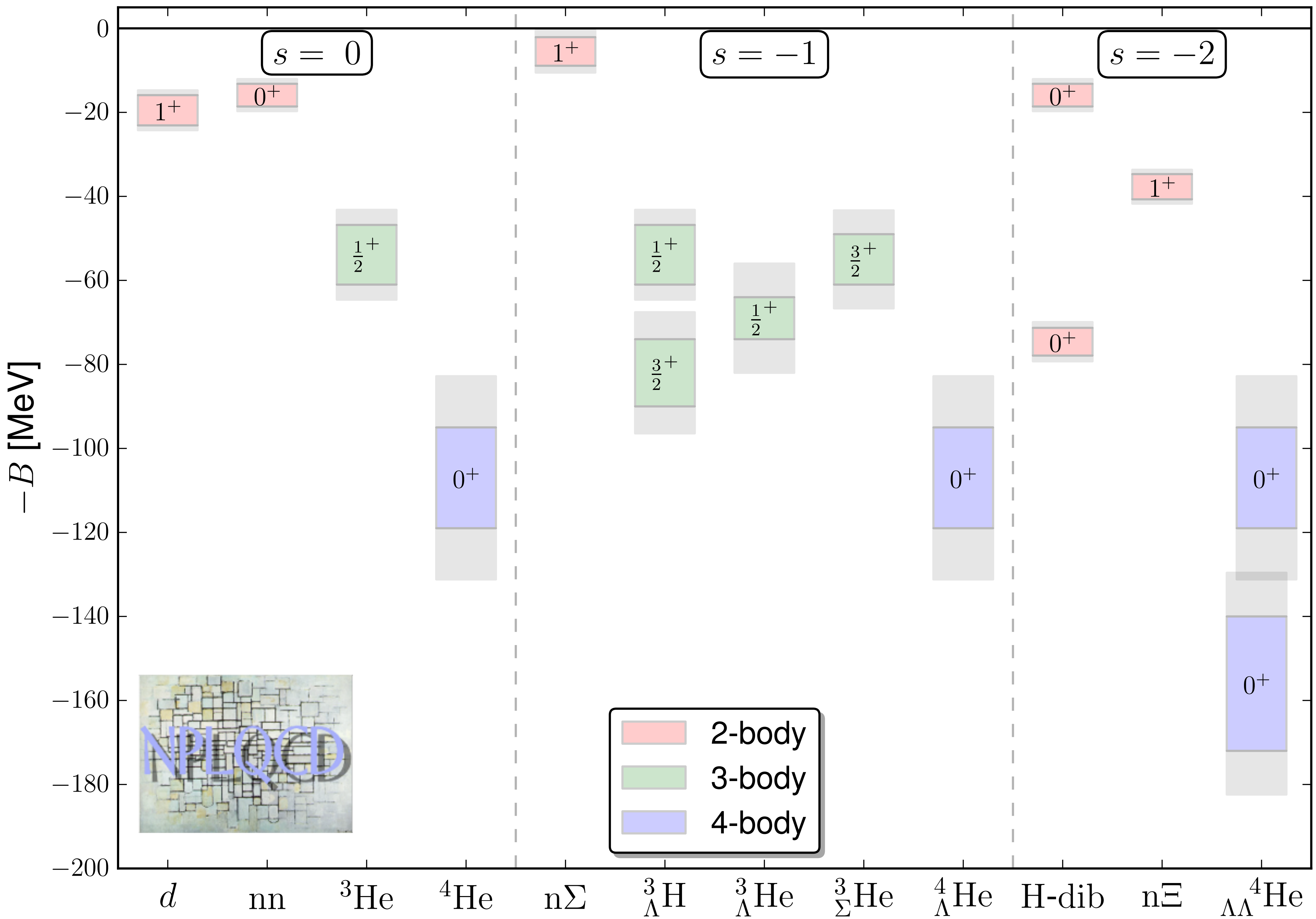
- Double β-decay

  m_π~800 MeV, m_N~1,600 MeV



Spectrum of light nuclei

@ m_π = 800 MeV



NPLQCD Phys.Rev. D87 (2013), 034506

Background field method

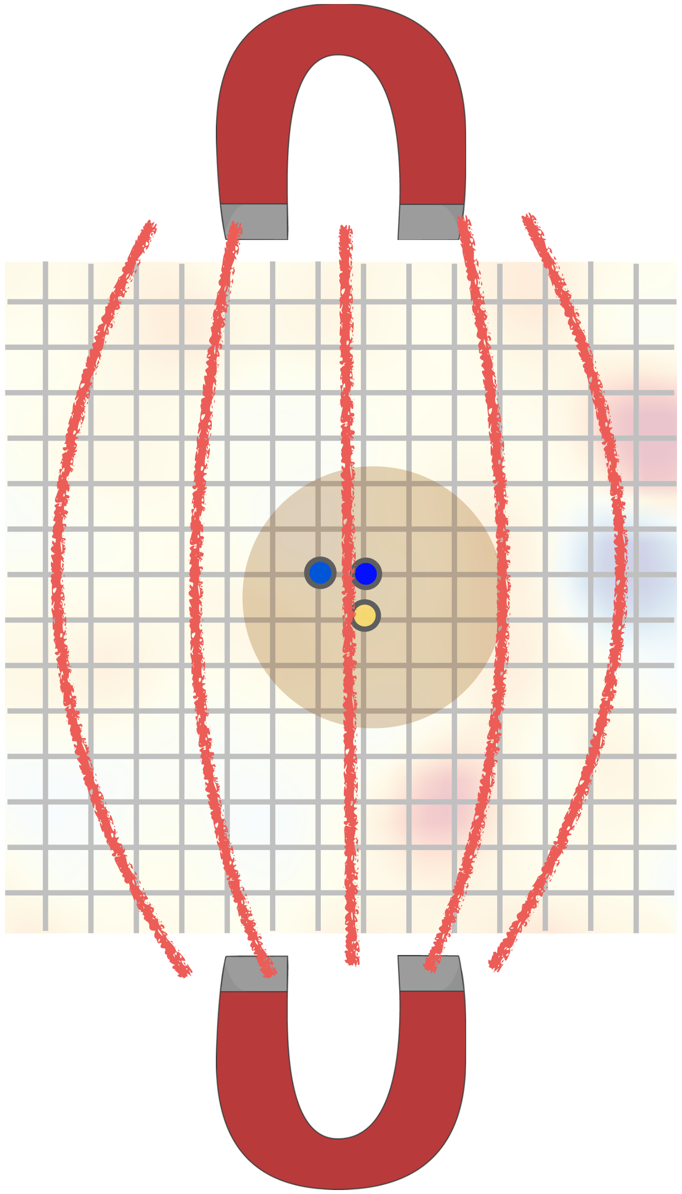
Hadron/nuclear energies are modified by presence of fixed/constant external fields

Example: fixed magnetic field

landau level — mag. mmt

$$E(\vec{B}) = \sqrt{M^2 + (2\boxed{n} + 1)|Qe\vec{B}|} - \boxed{\vec{\mu}} \cdot \vec{B}$$
$$-2\pi\boxed{\beta_{M0}}|\vec{B}|^2$$

mag. polarisability



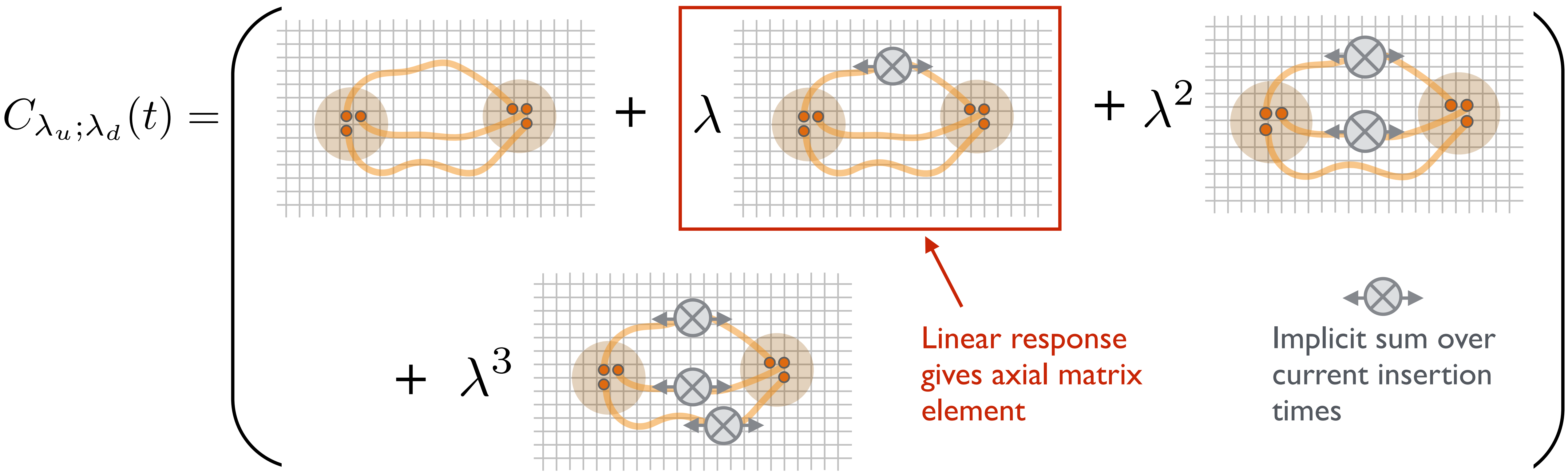
- Calculations with multiple fields
  ➡ extract coefficients of response
  e.g., magnetic moments, polarisabilities, …
- Not restricted to simple EM fields
  Axial MEs: uniform axial background field

[NPLQCD PRL **113**, 252001 (2014)]

Axial background field

Example: fixed magnetic field ➡ moments, polarisabilities

Axial MEs: fixed axial background field ➡ axial charges, other matrix elts.

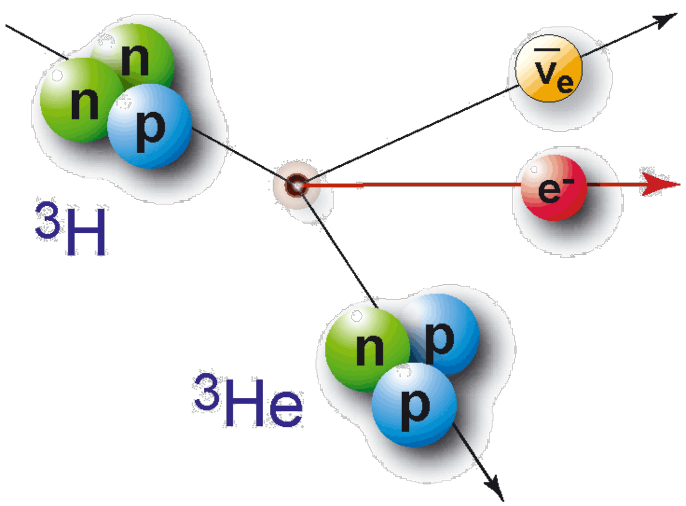


Second order piece: being used for calculations of double-beta decay

[NPLQCD Nucl. Phys. **A743**, 170 (2004)]

Tritium β-decay

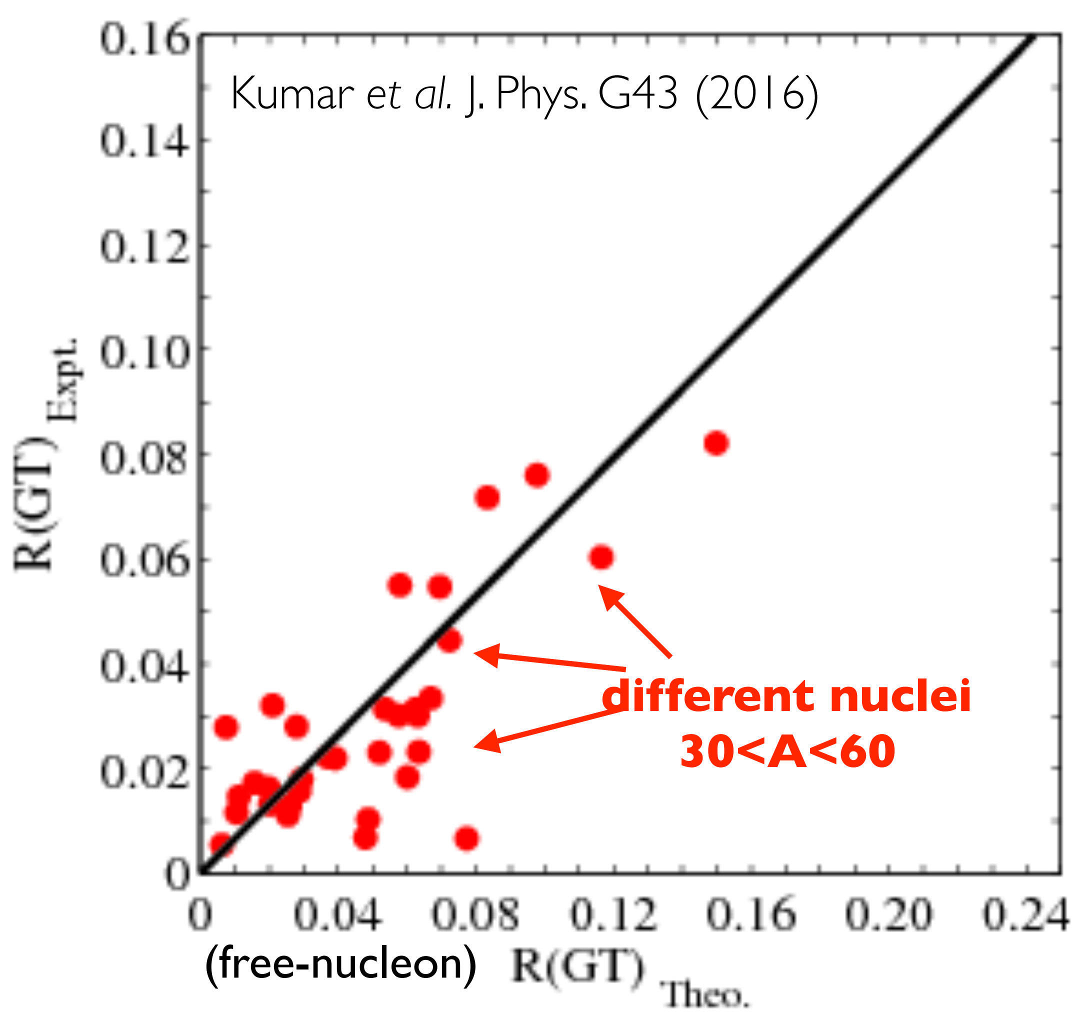
- Simplest semileptonic weak decay of a nuclear system



- Gamow-Teller (axial current) contribution to decays of nuclei not well-known from theory
- Understand multi-body contributions to $\langle \mathbf{GT} \rangle$ ➡ better predictions for decay rates of larger nuclei

We calculate

$$g_A \langle \mathbf{GT} \rangle = \langle ^3\mathrm{He} | \overline{\mathbf{q}} \gamma_{\mathbf{k}} \gamma_{\mathbf{5}} \tau^- \mathbf{q} | ^3\mathrm{H} \rangle$$



Tritium β-decay

half-life

vector ME

axial ME

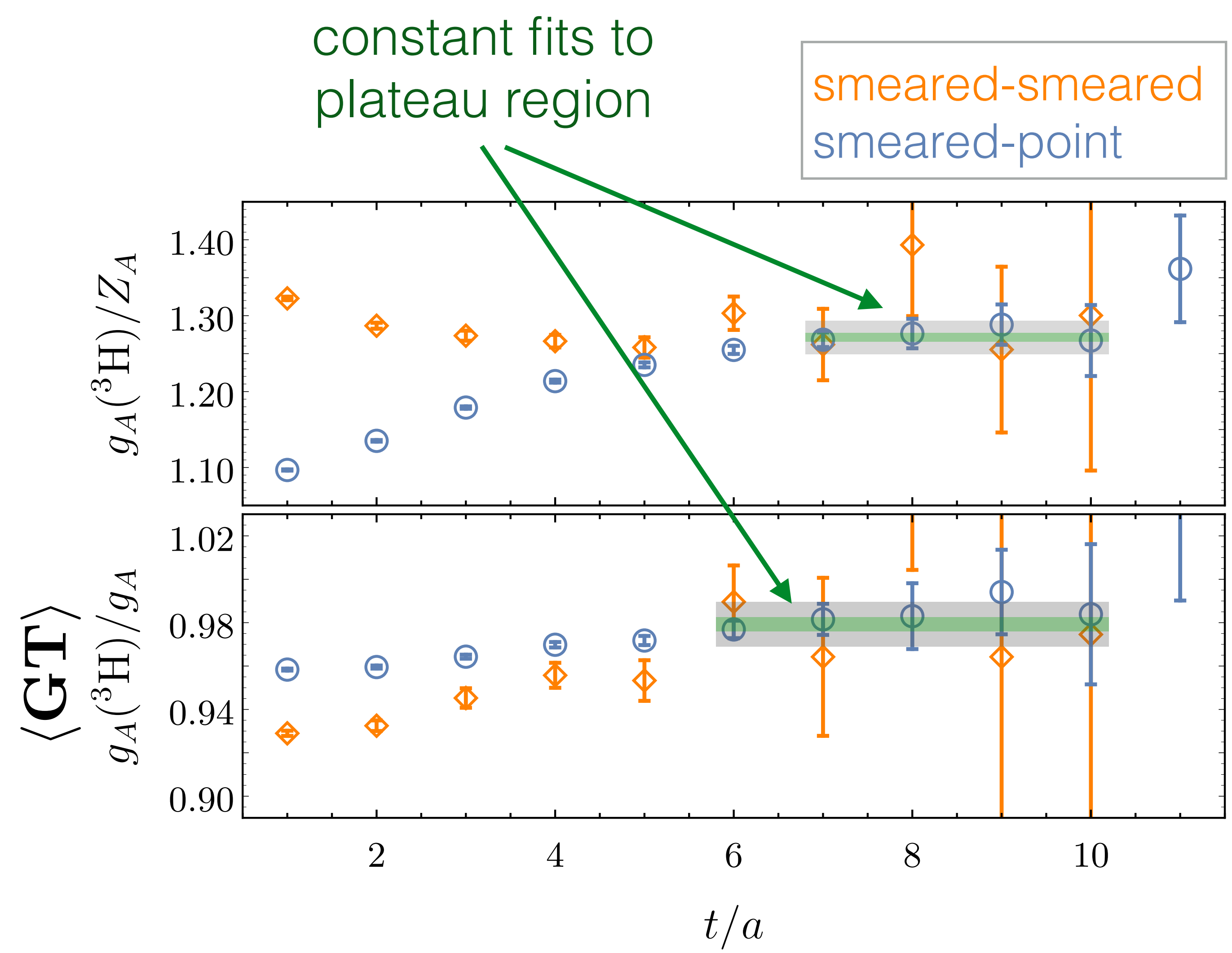
$$\frac{(1+\delta_R)f_V}{K/G_V^2}\, t_{1/2} = \frac{1}{\langle \mathbf{F}\rangle^2 + f_A/f_V\, g_A^2 \langle \mathbf{GT}\rangle^2}$$

known from theory or expt.

- Form ratios of compound correlators to cancel leading time-dependence:

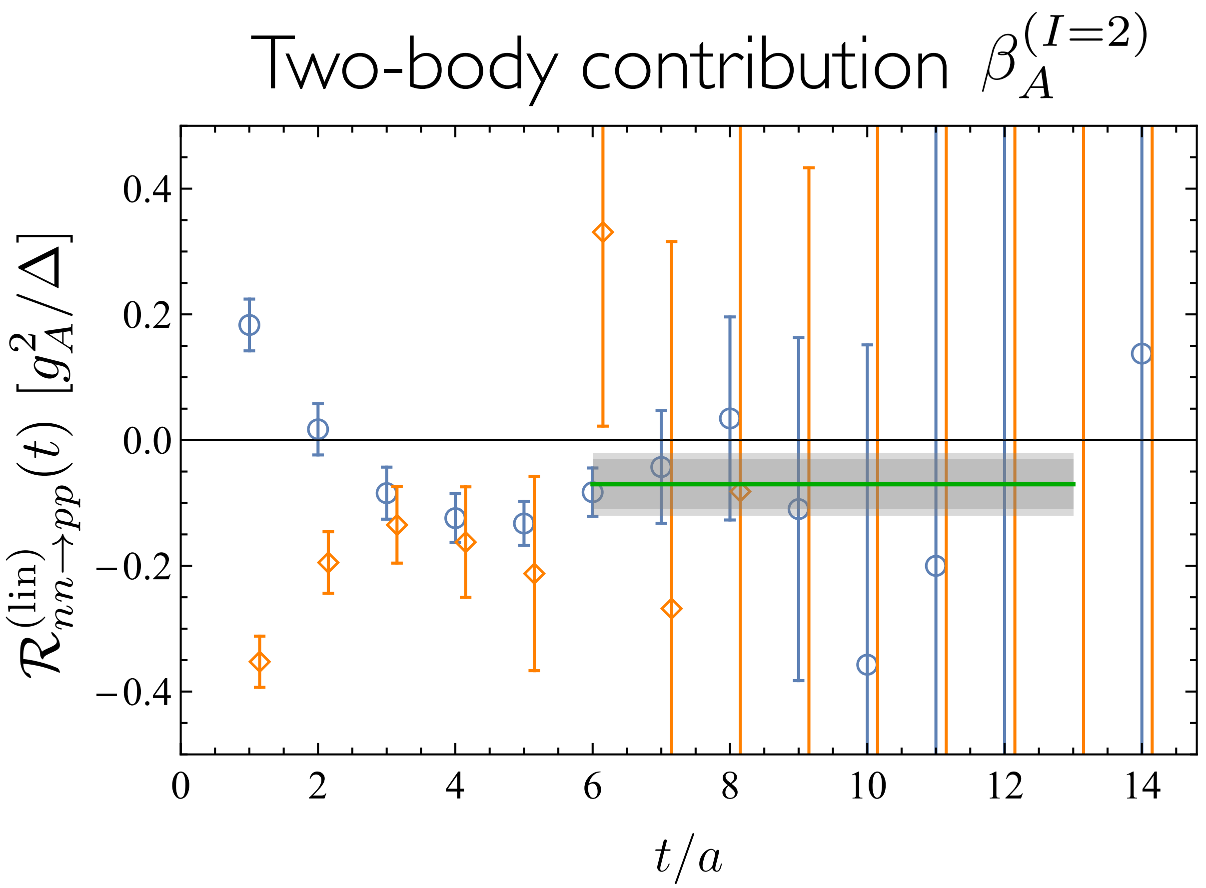
$$\frac{\overline{R}_{^3\mathrm{H}}(t)}{\overline{R}_p(t)} \xrightarrow{t\to\infty} \frac{g_A(^3\mathrm{H})}{g_A} = \langle \mathbf{GT}\rangle$$

- Ground state ME revealed through "effective ME plot"



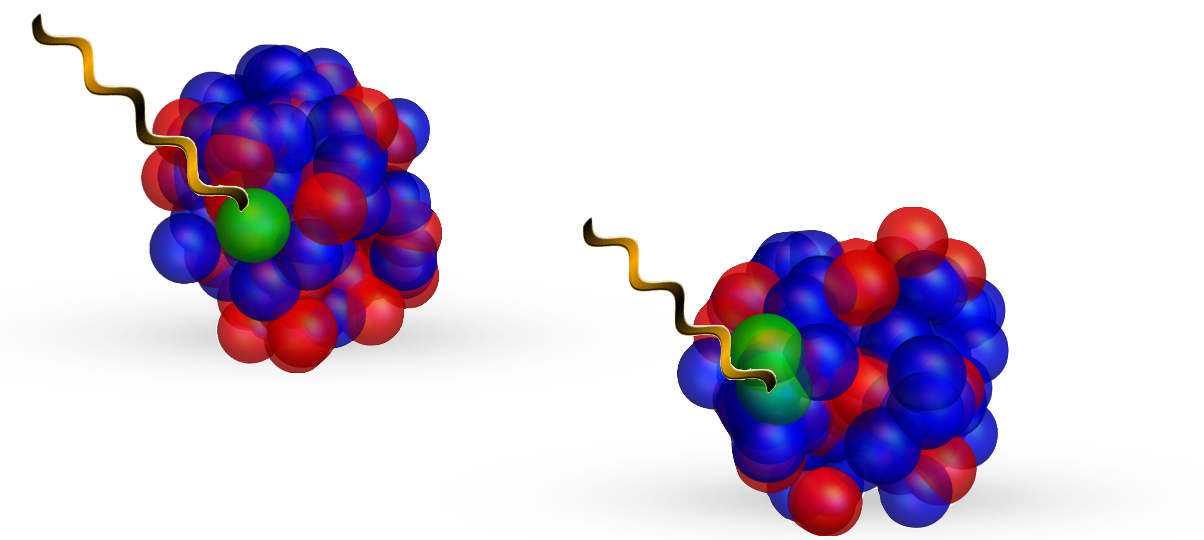
Tritium β-decay

- Take combinations to pull out isotensor axial polarisability (two body piece)
- Two-body contribution resolved from zero



Larger nuclei

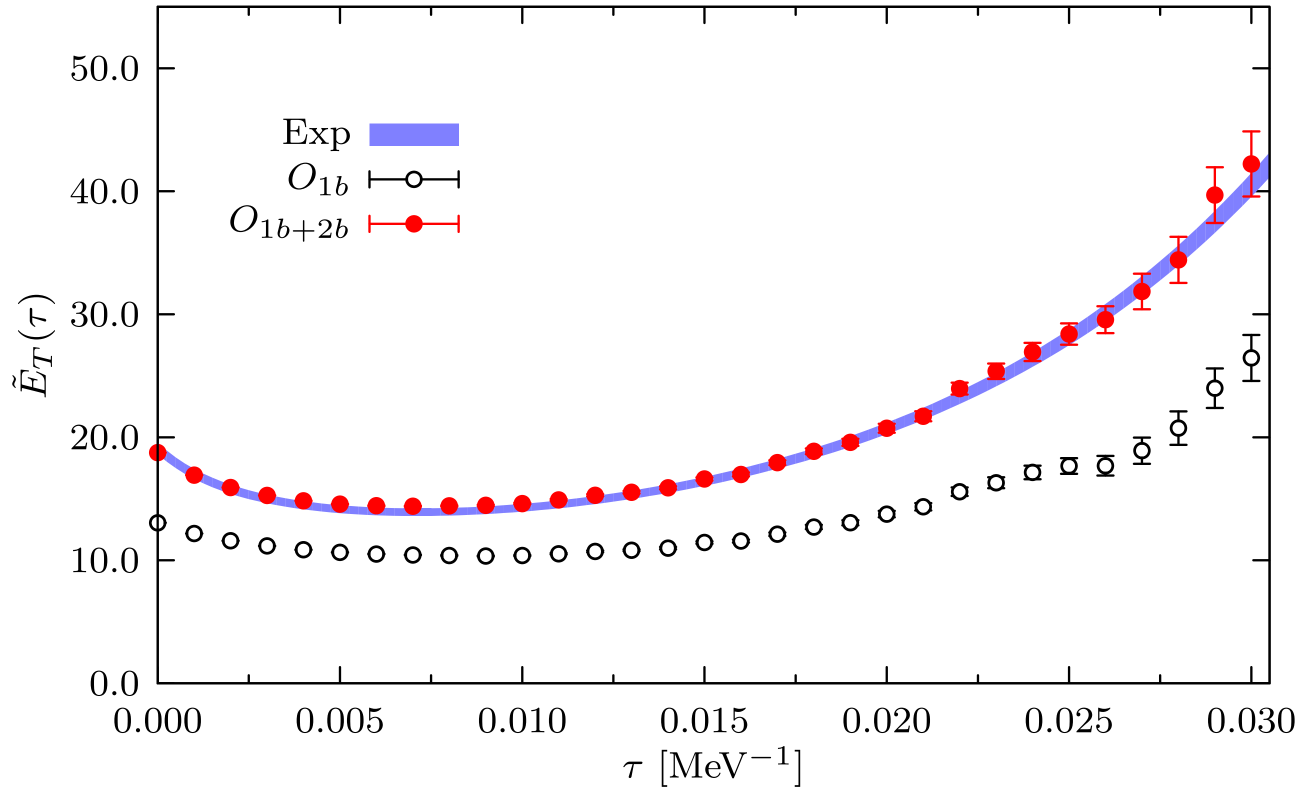
- What about larger (phenomenologically-relevant) nuclei?
- Nuclear effective field theory:
  - 1-body currents are dominant
  - 2-body currents are sub-leading *but non-negligible*



- Determine one body contributions from single nucleon
- Determine few-body contributions from A=2,3,4...
- Match EFT and many body methods to LQCD to make predictions for larger nuclei

Two-body effects

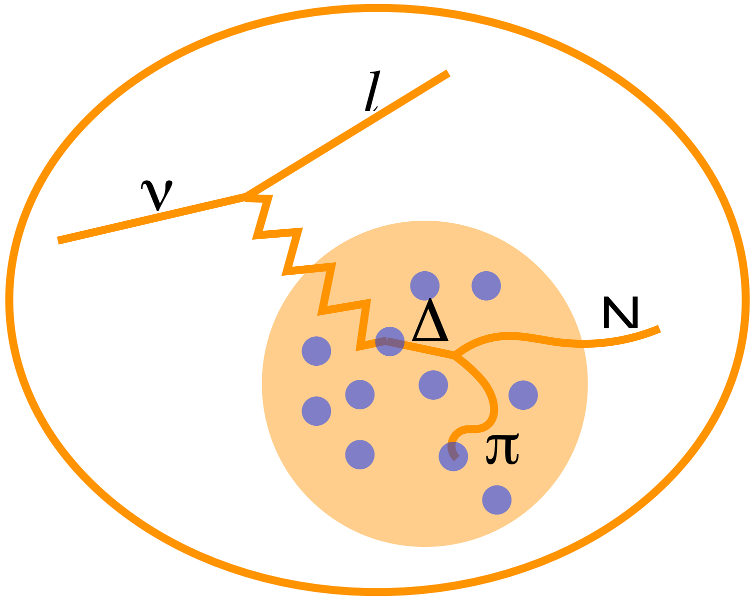
- EM transverse response function shows important two-body effects: ^{12}C at q = 570 MeV



Lovato et al., Phys. Rev. C 91, 062501 (2015)

- Expect to be similarly important for axial

Conclusion



- Lattice efforts have potential to impact ν energy determinations
- Precise determinations with controlled percent-level uncertainties within ~5 years
  - Axial and pseudoscalar FFs determined with momenta less than a few GeV
  - Large momentum FFs (> GeV) difficult. Novel ideas exist, need testing
- Early results with promising applications
  - Transition FFs
    Formalism exists but developments still necessary for higher states above Nππ inelastic threshold
  - Application of EFT using 2-, 3- body matrix elements to constrain nuclear effects