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Jet broadening in a viscous nuclear medium

University of Regina

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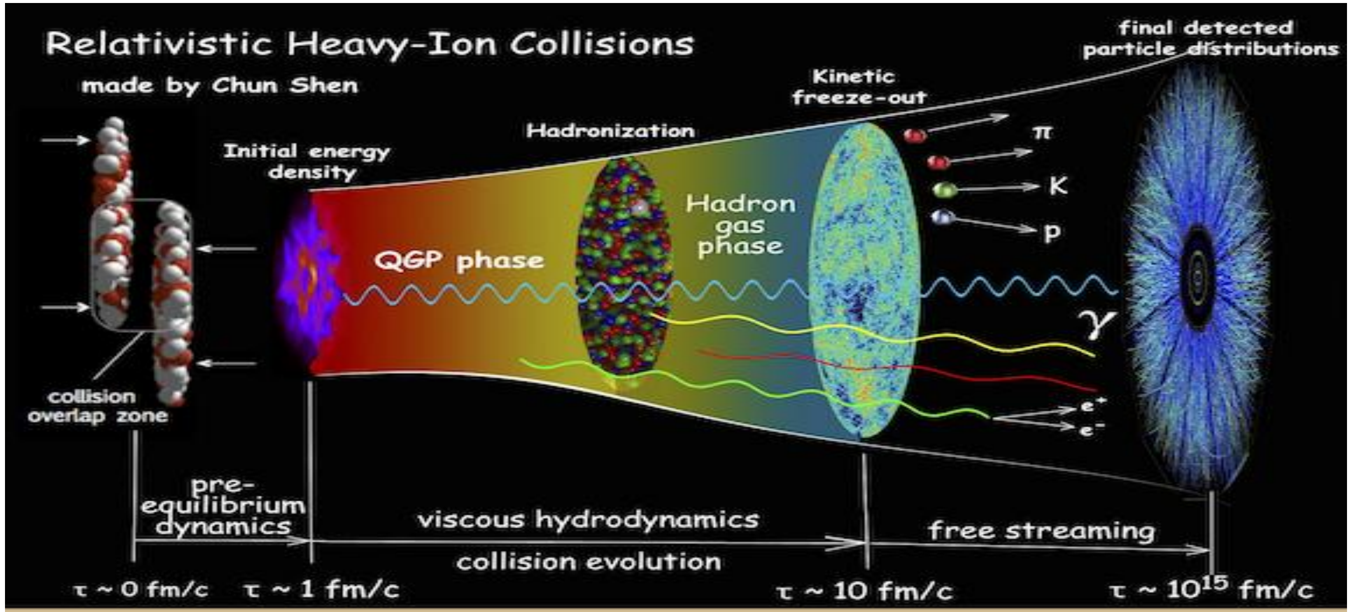
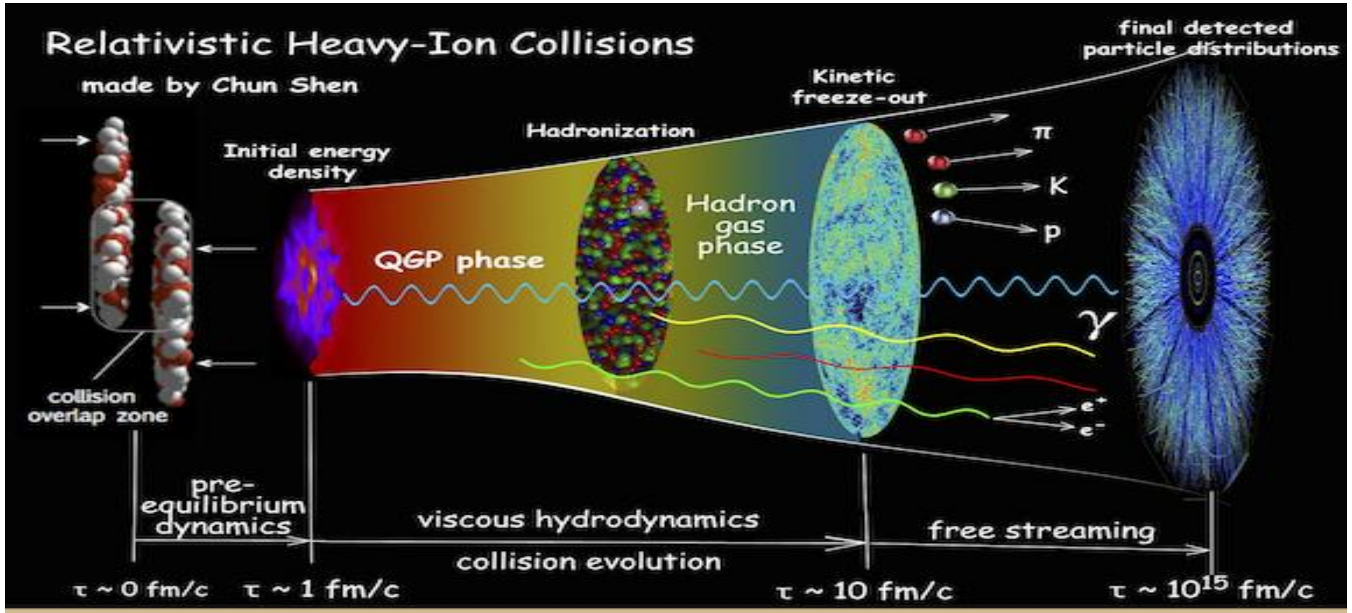
Quark-Gluon Plasma



An illustration of relativistic heavy-ion collisions.

- The goal of heavy-ion collisions is to melt the nucleus to form a system where quarks and gluons are quasi-free particles, i.e. to create a QGP.
- During the pre-equilibrium stage, the quarks and gluons produced will have very different energy scales.
- High energy partons give jets, and only the lower energies will form the QGP.

Quark-Gluon Plasma



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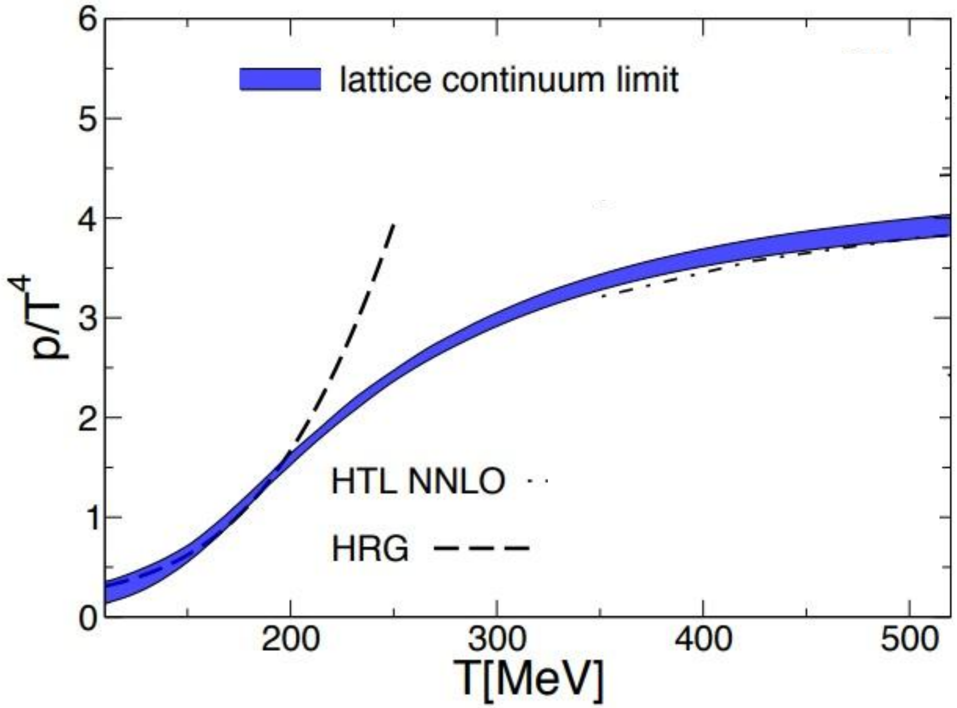

Borsányi, Szabolcs, et al. *Physics Letters B* 730 (2014): 99-104.

- Focusing on lower energy partons that participate in forming the QGP, a good description of their evolution is through hydrodynamics.
- To solve the fluid equations of motion, an Equation of State (EoS) is required, that couples pressure to temperature.
- The EoS is given by Lattice QCD.

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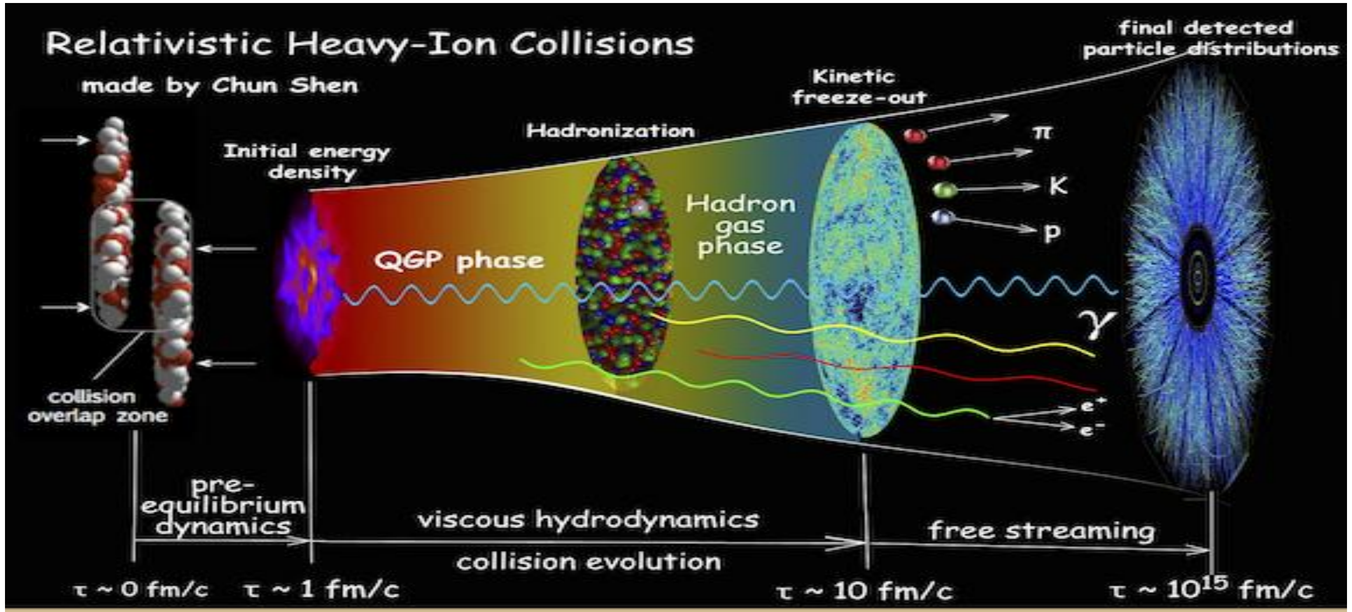
Quark-Gluon Plasma



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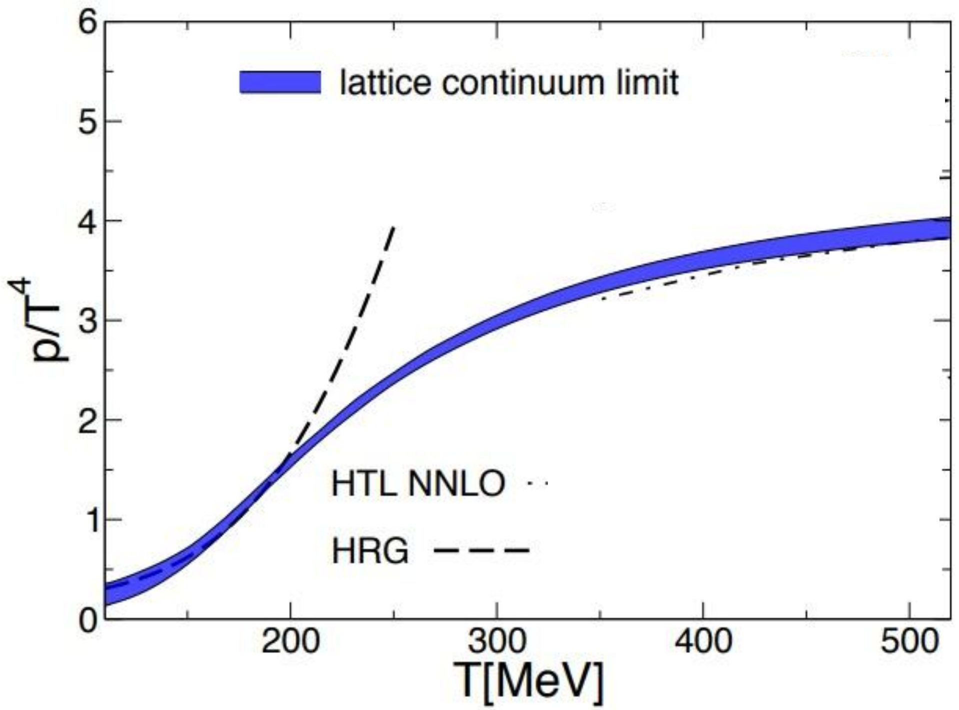

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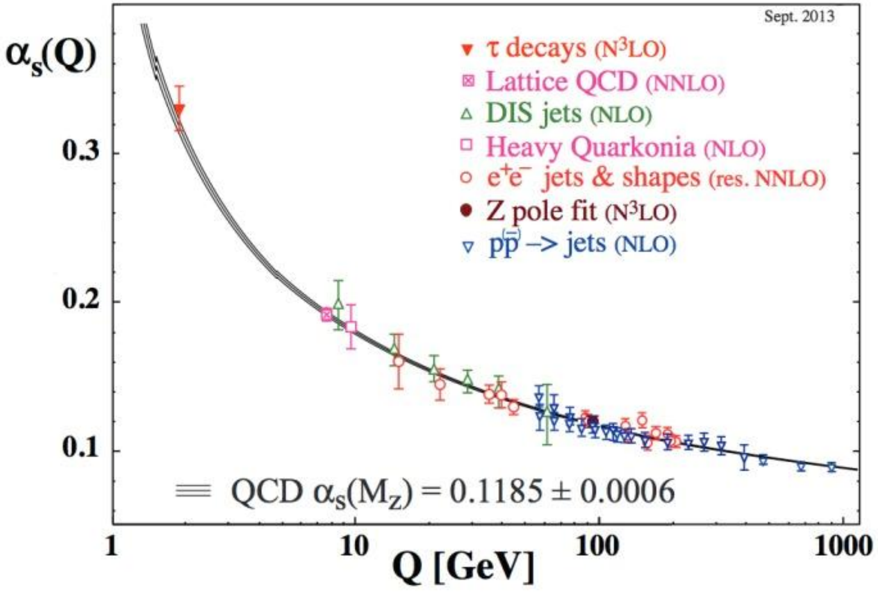
- At T<200 MeV (~ 2 Trillion K), the QCD equation of state can be described using hadronic degrees of freedom (i.e. EoS is perturbative in the effective theory of hadrons).
- At T>350, MeV behavior is similar to pQCD.
- Thus, we have a good understanding of the QCD EoS: at high temperatures the EoS is described by quarks and gluons, but as we go to lower temperatures a continuous transition to hadrons is seen.

QCD Coupling and Asymptotic Freedom

- Jet partons have much higher energy scales than the temperature of the QGP.
- At those energy scales, there is a significant running of the strong coupling.
- In fact, for $\mathrm{Q} > \sim 10$ GeV, α_s is small enough for perturbation theory to work. This is because of the asymptotic freedom of QCD.

$$\alpha_s(Q^2) \sim \frac{1}{\ln Q^2/\Lambda_{QCD}^2}$$



Particle Data Group, K. Olive et al., Chin.Phys. C38, 090001 (2014).

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- In fact, for $Q > \sim 10$ GeV, α_s is small enough for perturbation theory to work. This is because of the asymptotic freedom of QCD.
- Partons at these high energies, will see a very dilute QGP, given that α_s is so small.
- This allows the use of perturbation theory when describing how jet partons interact with QGP partons.
- Let's explore how jet partons evolve in the vacuum vs the QGP medium.

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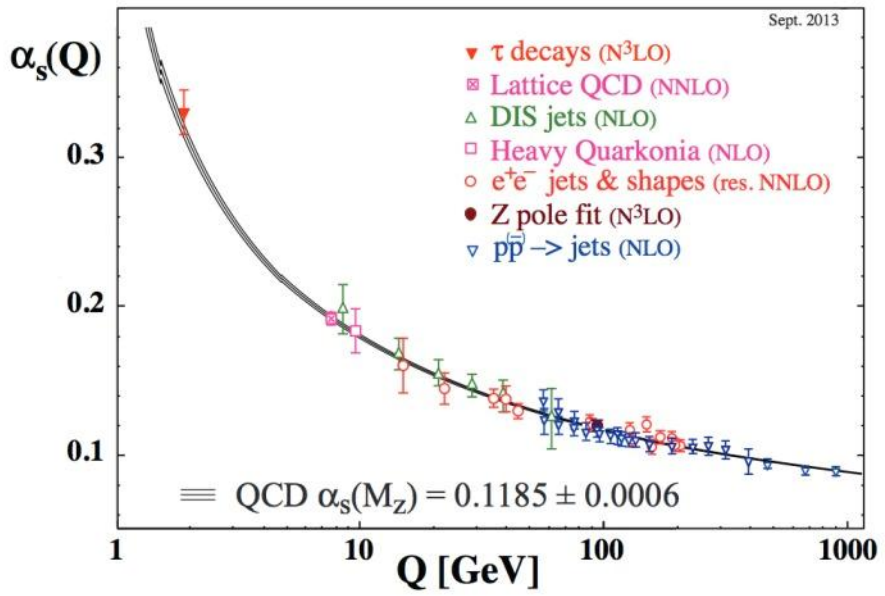

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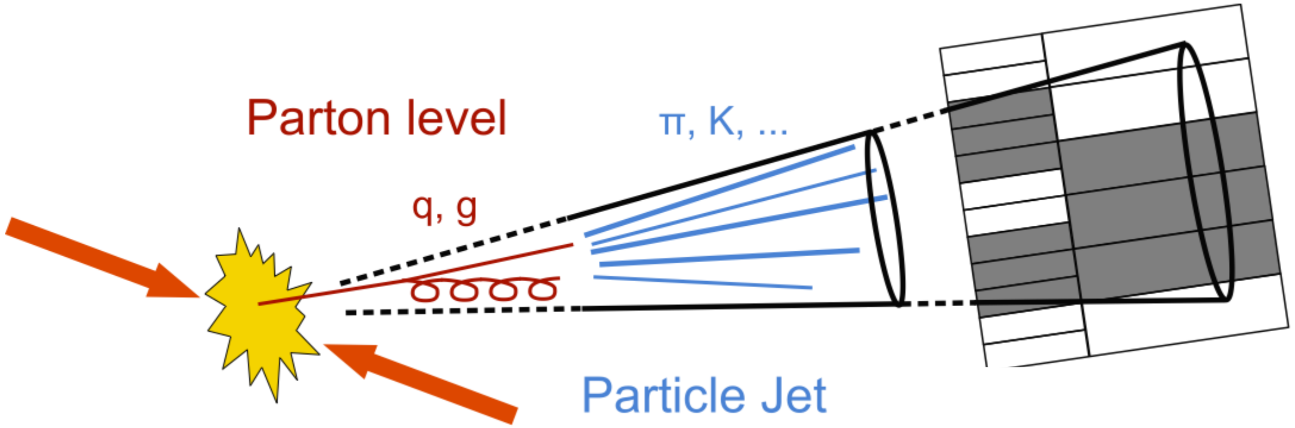
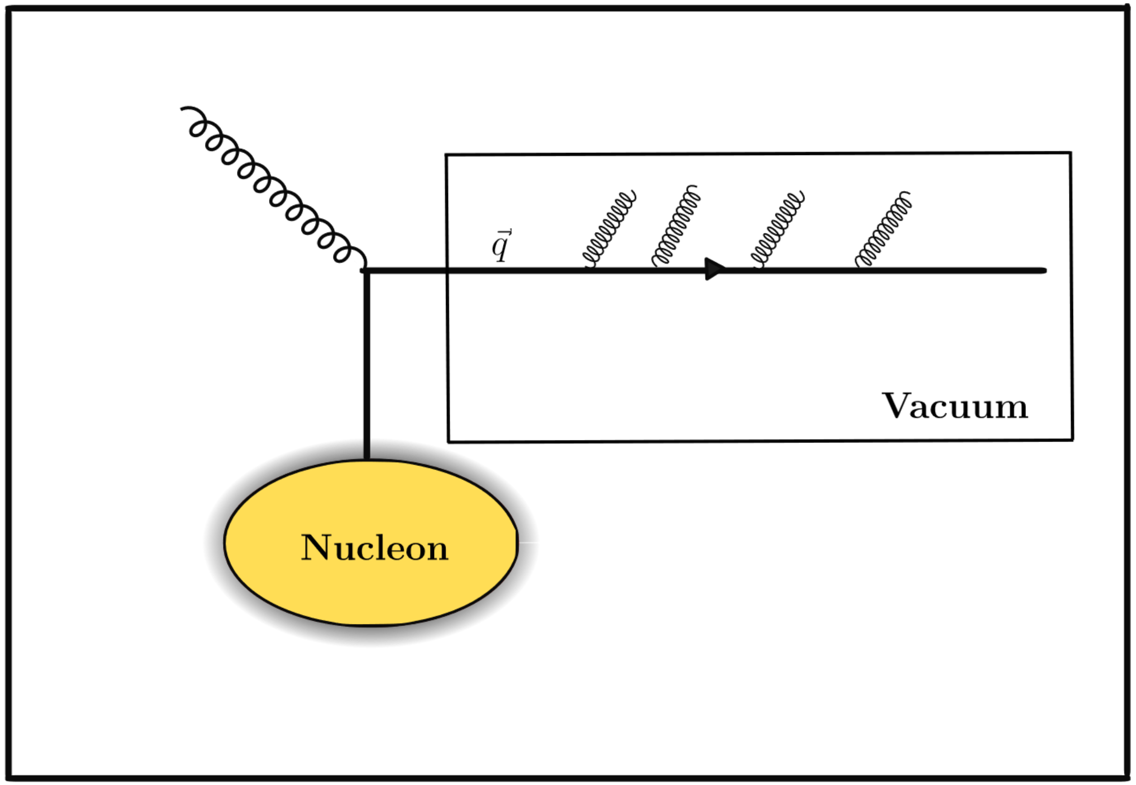
Jets in Vacuum

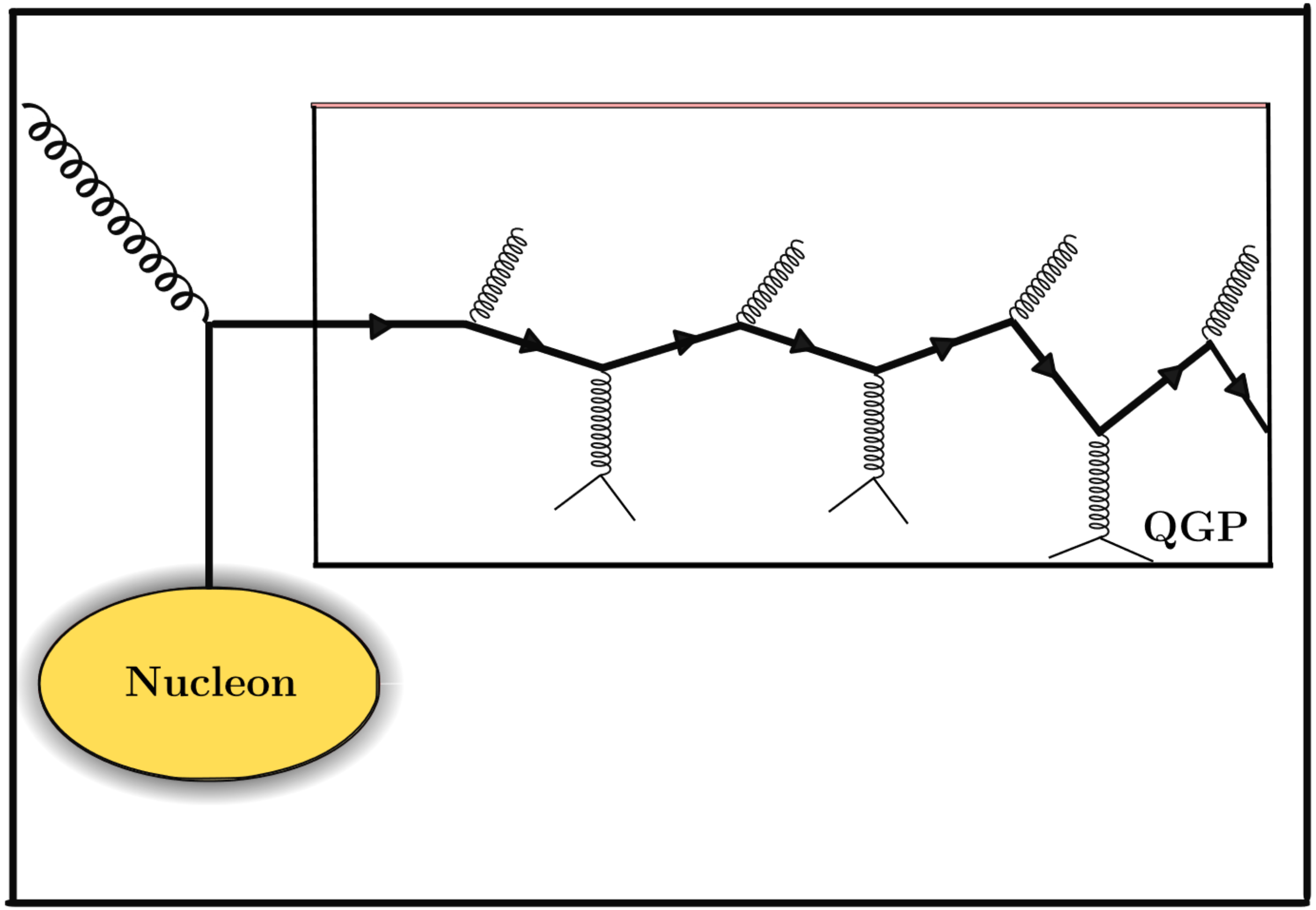


https://cms.cern/news/jets-cms-and-determination-their-energy-scale

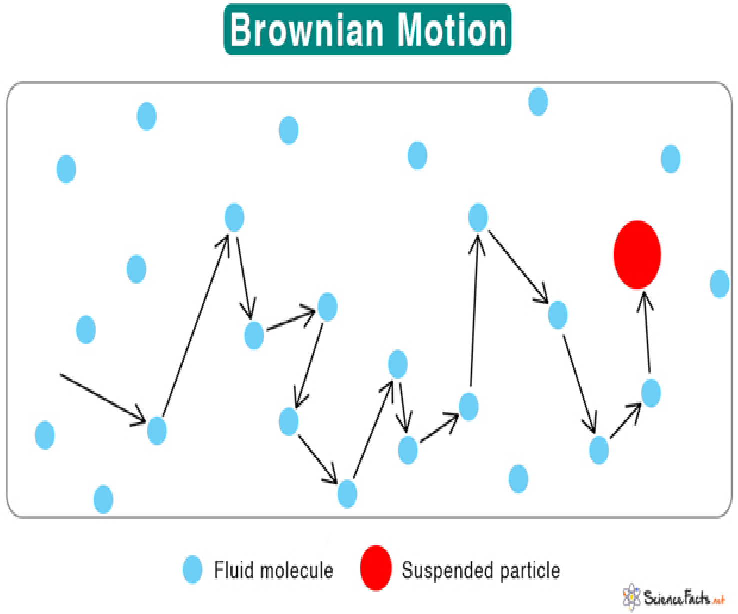
- **The evolution of a highly virtual quark in the vacuum is dominated by Bremsstrahlung radiation of gluons.**
- Extensively studied in electron-positron collisions in vacuum. Thus jets are vacuum-calibrated probes.
- Heavy quarks are further unique jet probes as we can tag on them.



Jets in Medium



- Jet passing through QGP deviates from its path due to collisions with particles in the medium.
- These collisions would ultimately lead to Brownian motion of particles in the jet… however we have hadronization
- Collision can be described using the transport coefficient $\hat{q}$: the average squared transverse momentum broadening per unit length, in the medium.



Boltzmann Equation of Parton Transport

- The multiple scattering of jet partons in the QGP is described by the Boltzmann equation.
- Governs the time evolution of distribution functions of particles in a medium and the jet.
- Applicable for weakly interacting and dilute systems.
- In thermal equilibrium, f is known $f_0 = [\exp\beta(p \cdot u) \pm 1]^{-1}$

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla f = C[f]$$

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$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m}\cdot\nabla f = C[f]$$

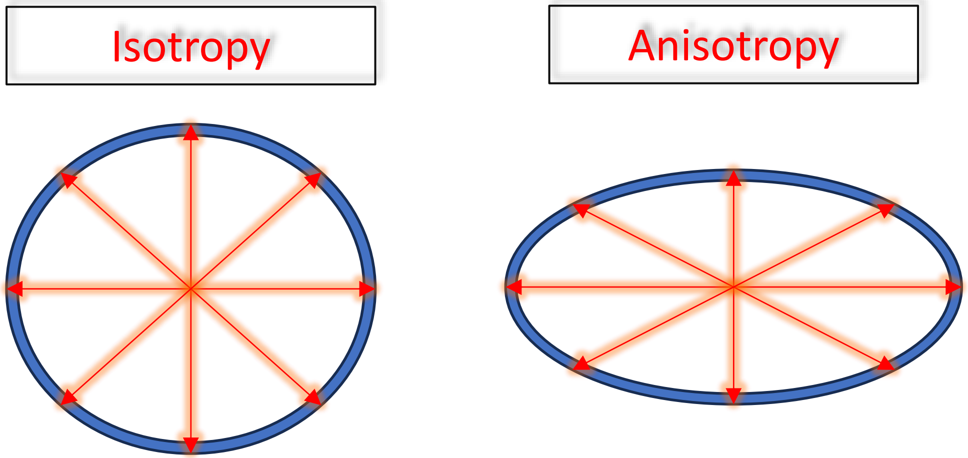
$$\frac{d^4N}{d^4x} = dR$$

$$= \int \frac{d^3p_1}{(2\pi)^3 2E_1} f(p_1) \frac{d^3p_2}{(2\pi)^3 2E_2} f(p_2) \frac{d^3p_3}{(2\pi)^3 2E_3}[1 \pm f(p_3)] \frac{d^3p_4}{(2\pi)^3 2E_4}[1 \pm f(p_4)]|M|^2 (2\pi)^4 \delta^{(4)}(\mathrm{p}_1 + \mathrm{p}_2 - \mathrm{p}_3 - \mathrm{p}_4)$$

Thermal Equilibrium and Isotropy

Thermal Equilibrium → Isotropic System

Thermal Non-Equilibrium → Anisotropic System

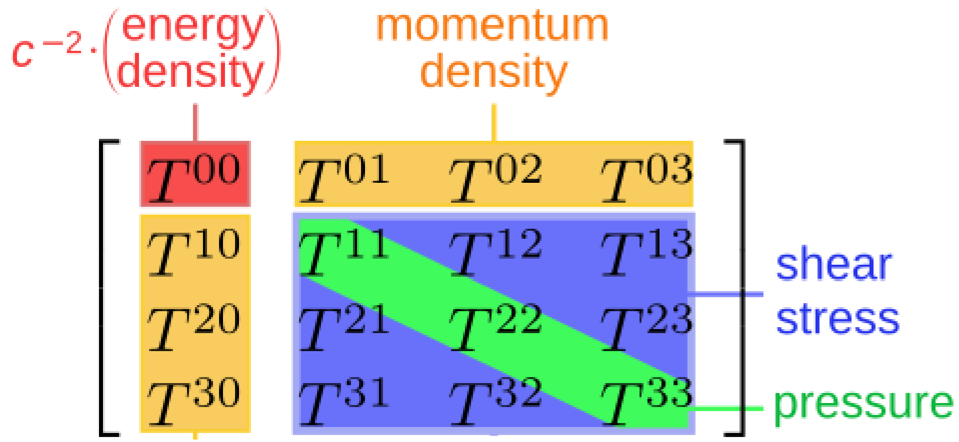


- System in thermal equilibrium is isotropic (spherically symmetric) in momentum space.
- The Boltzmann equation allows for the system to go away from thermal equilibrium.
- Going off-equilibrium manifests as the momentum distribution becoming anisotropic.
- An anisotropic momentum distribution of particles at the microscopic scale generates viscous effects in the (macroscopic) fluid.
- The goal of my work is to constrain the viscosity of the QGP by studying how jets are quenched in the QGP.

Relativistic Hydrodynamics and Viscosity

- The link between the microscopic distribution function f and the macroscopic stress-energy tensor is given by:

$$T^{\mu\nu}_{ideal} + \pi^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 2E} p^\mu p^\nu f_0(p \cdot u) + \int \frac{d^3p}{(2\pi)^3 2E} p^\mu p^\nu \delta f(p \cdot u)$$



arXiv:0909.0754 **[nucl-th]**

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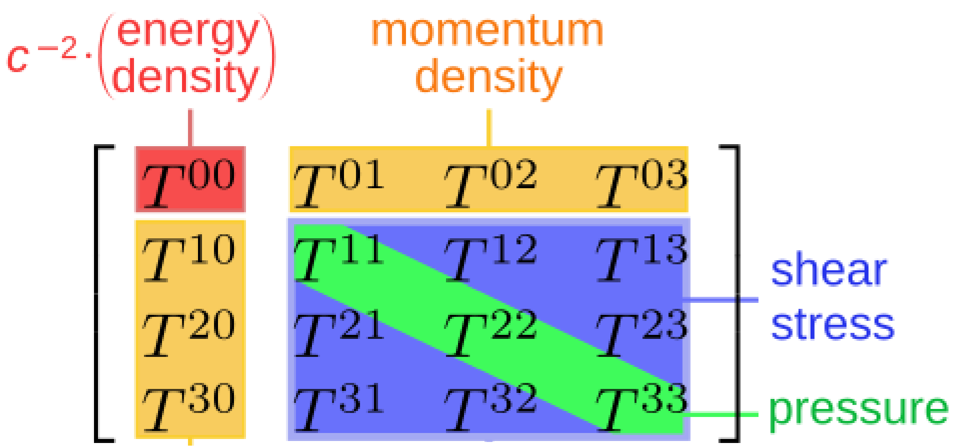
- The equilibrium (ideal) part of stress-energy tensor is given by f_0
  - For an ideal fluid, only diagonal terms remain in the stress-energy tensor.
- The viscous part of the stress-energy tensor is encoded in δf.
  - For non-ideal fluids, off-diagonal terms become important.



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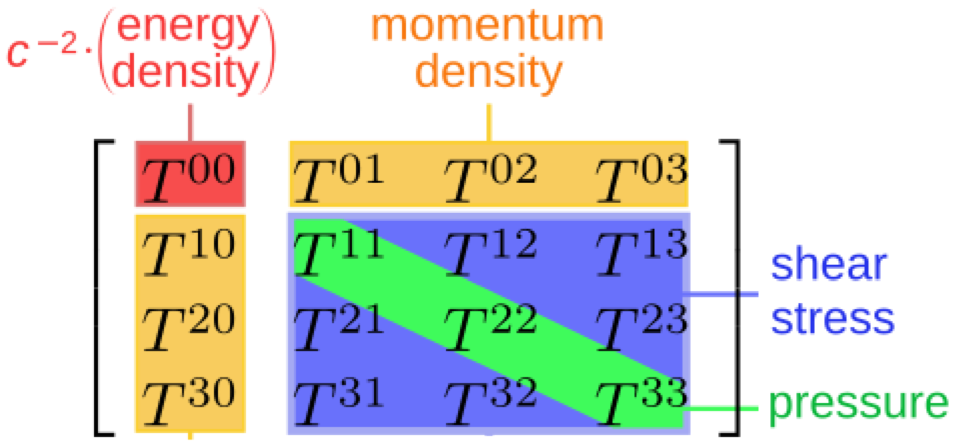
- The viscous part of the stress-energy tensor is encoded in δf.
  - For non-ideal fluids, off-diagonal terms become important.
- These off-diagonal contributions are also present in the scattering rate, through the viscous correction to the distribution function:

$$\frac{d^4N}{d^4x} = dR(f_1, f_2, f_3, f_4) \rightarrow dR(f_1, f_2 + \delta f_2, f_3, f_4 + \delta f_4)$$

where,

$$\delta f(p) = \frac{C}{2} f(p)\big(1 \pm f(p)\big) \frac{p^\alpha p^\beta}{T^2} \frac{\pi_{\alpha\beta}}{\epsilon + P}$$



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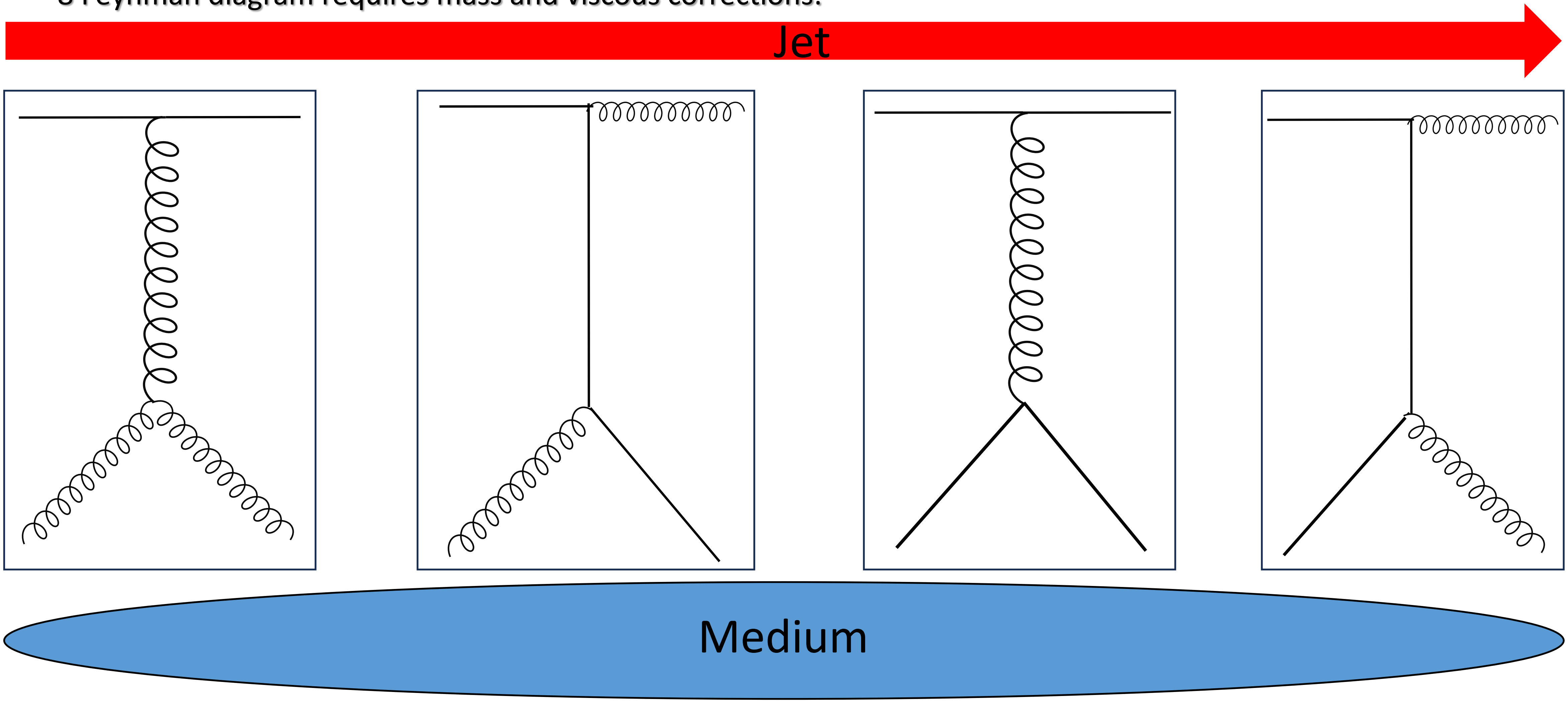
Scattering Channels for Jet-Medium Interactions

- 8 Feynman diagram requires mass and viscous corrections:



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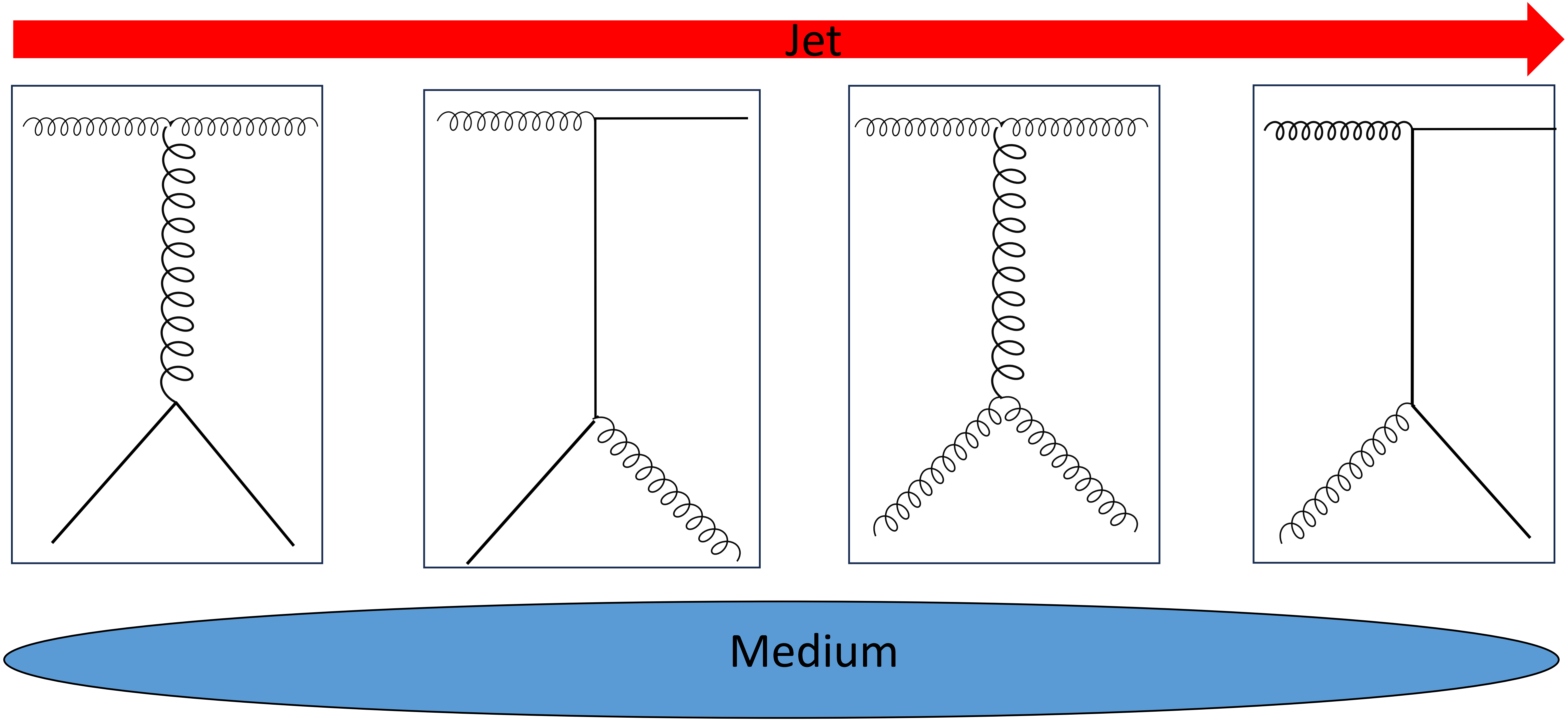
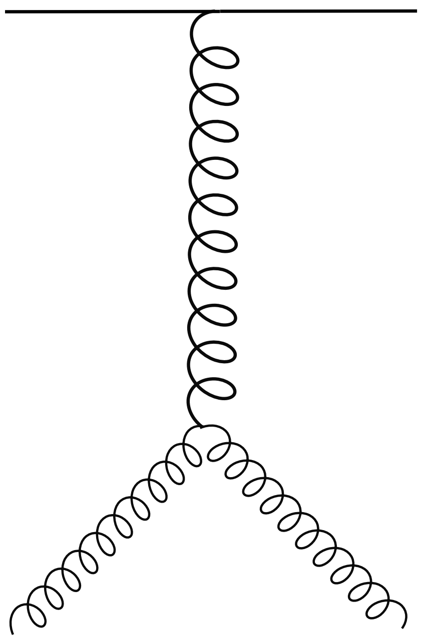
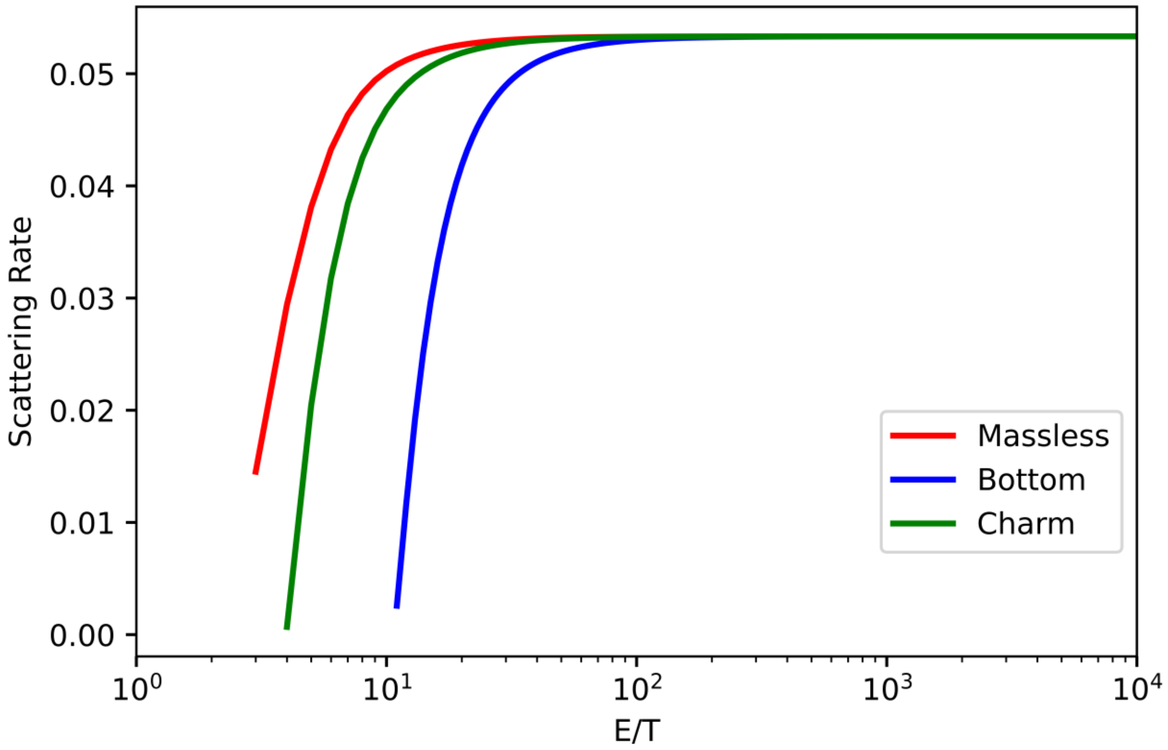


Results

Quark-Gluon scattering in t channel

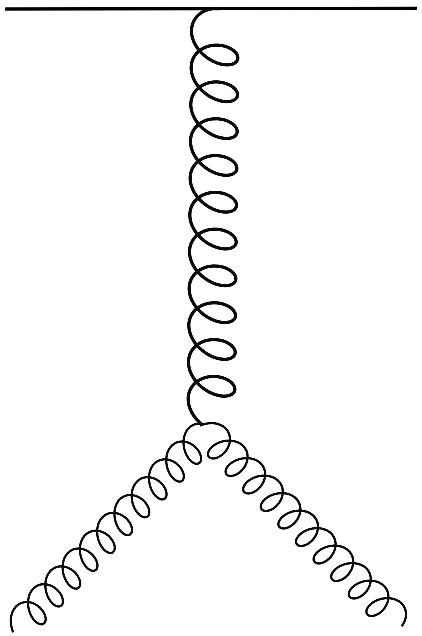


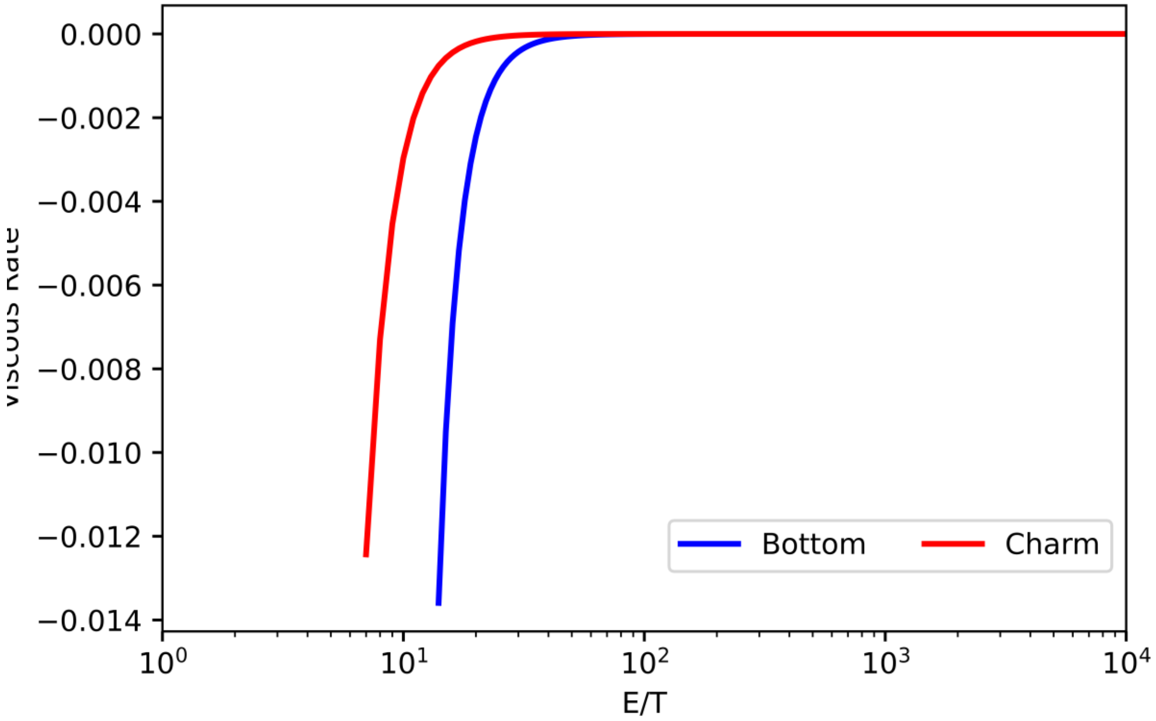


- Rates of gluon scattering and light quark scattering are at hand.
- Heavy flavour mass significantly affects the inviscid rate.

Results

Quark-Gluon scattering in t channel





- No viscous correction for light flavour quark and gluon scatterings in Forward Scattering Approx.
- The viscous correction with heavy flavour masses is small.

Summary and Outlook

- Computed rates for gluons, light and heavy quarks.
- Heavy quark mass scale affects the ideal rate.
- Viscous effects are only observed for heavy flavour.
- Next steps:
  - Implement these viscous rates inside of Monte Carlo event generators for jets.
  - Heavy flavour jets will be used to constrain shear viscosity of QGP.

Thank You

Questions?

Future Prospects

- Phenomenological study based on the modified rate and transverse momentum broadening.

$$dR = \int \frac{d^3p_1}{(2\pi)^3 2E_1} f(p_1) \frac{d^3p_2}{(2\pi)^3 2E_2} f(p_2) \frac{d^3p_3}{(2\pi)^3 2E_3} [1 \pm f(p_3)] \frac{d^3p_4}{(2\pi)^3 2E_4} [1 \pm f(p_4)] |M|^2 (2\pi)^4 \delta^{(4)} (\mathrm{p}_1 + \mathrm{p}_2 - \mathrm{p}_3 - \mathrm{p}_4)$$

- Rate expressed in terms of *s* and *t* :

$$R \approx \frac{1}{\sqrt{\lambda}} \int \frac{d^3p_2}{2E_2} f(p_2) \int dt \frac{1}{4} |M(s,t)|^2 \{1 \pm f[p_4(s,t)]\}$$

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$$\frac{d^3R}{d^3p_1} = \frac{1}{(2\pi)^8 16E_1} \int \frac{d^3p_2}{E_2} f(p_2) \int \frac{d^3p_3}{E_3} \int \frac{d^3p_4}{E_4} (1 \pm f(p_4)) |\mathcal{M}|^2 \delta^{(4)}(p_1 + p_2 - p_3 - p_4).$$

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E_1 is fixed. $s = (p_1 + p_2)^2$

To fix *s*, sample $E_2, cos\theta_2, \phi_2$:

$$\int \sqrt{(E_2^2 - m_2^2)} dE_2 f(p_2) \int dcos\theta_2 \int d\phi_2$$

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To fix *s*, sample $E_2, cos\theta_2, \phi_2$:

$\int \sqrt{(E_2^2 - m_2^2)} dE_2 f(p_2) \int dcos\theta_2 \int d\phi_2$

t $(= (p_1 - p_3)^2)$ sampling:

$t = [t_-, t_+]$

Future Prospects

- Viscous corrected rate sampling is more challenging.

$$\delta f(p) = \frac{C}{2} f(p)\left(1 \pm f(p)\right) \frac{p^\alpha p^\beta}{T^2} \frac{\pi_{\alpha\beta}}{\epsilon + \mathcal{P}}.$$

- Angular dependency in the rate from δf needs to be considered while sampling.
- Extend the theory from Forward Scattering Approximation to a generalized scattering theory.
- Photon production in QGP with mass and viscous correction is another prospect for the developed theoretical technique.

Boltzmann Equation of Parton Transport

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- The scattering rate of a single jet parton with momentum p_1

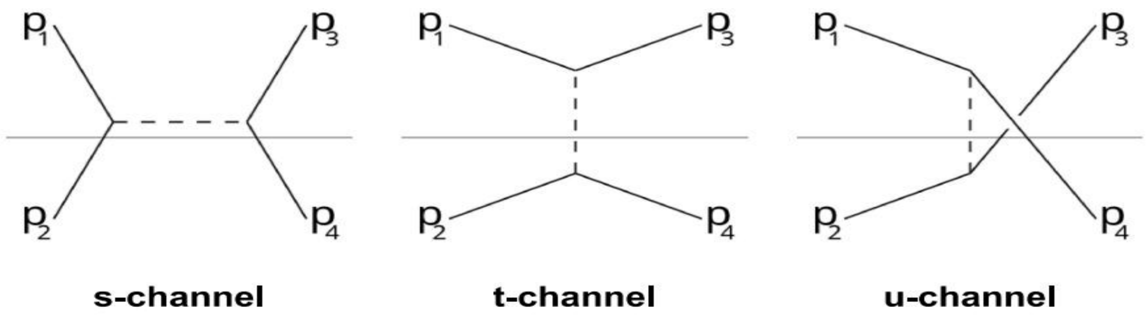
$$\frac{d^7N}{d^4x d^3p_1} = \frac{d^3R}{d^3p_1} = \frac{1}{(2\pi)^8 16E_1} \int \frac{d^3p_2}{(2\pi)^3 2E_2} f(p_2) \int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} [1 \pm f(p_4)] |M|^2 \delta^4 (\mathrm{p}_1 + \mathrm{p}_2 - \mathrm{p}_3 - \mathrm{p}_4)$$

Forward Scattering Approximation

- According to Forward Scattering Approximation:

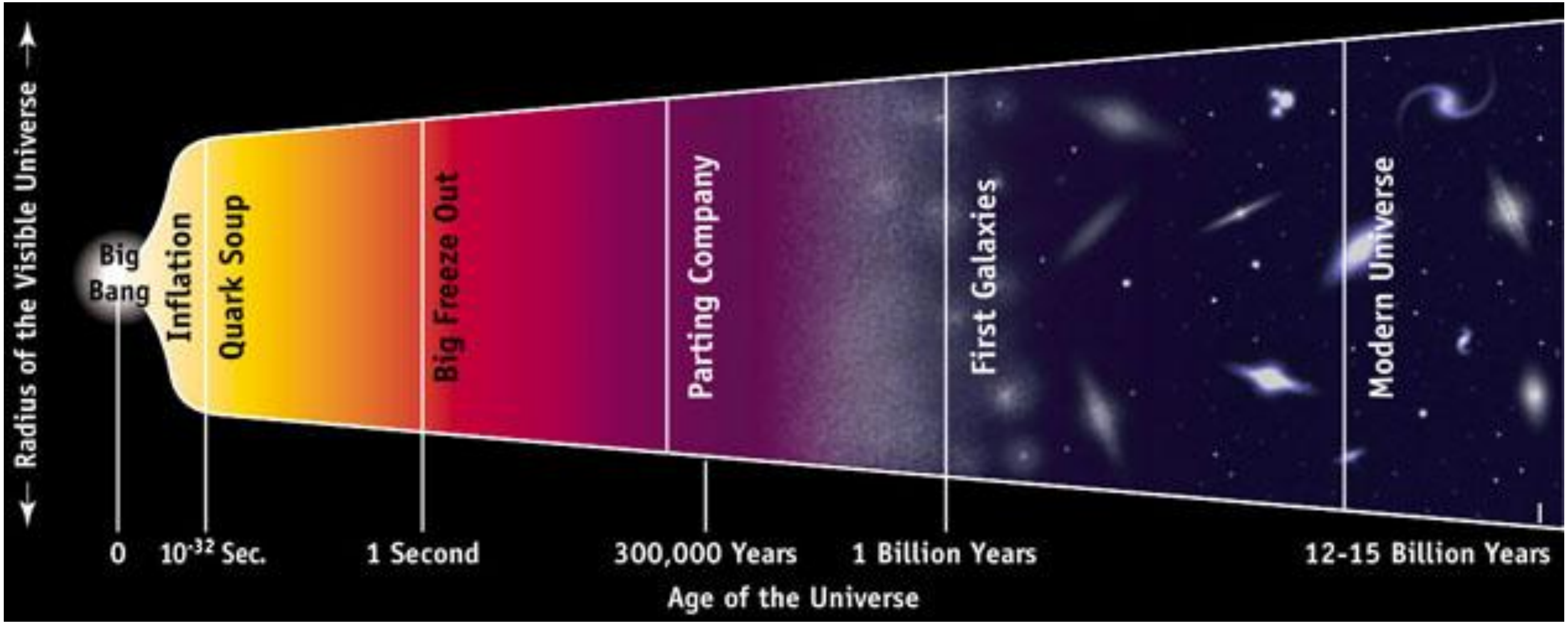
$$p_1 \gg p_2 \gg q \quad \Rightarrow \quad p_1 \approx p_3$$

- Consequently, we will neglect terms such as $\frac{q^0}{E_1}, \frac{q^0}{E_2}$, etc.
- Light quarks (u,d,s) masses are neglected. Charm and Bottom quark, masses retained.
- Mandelstam *s channel* diagram contributions are neglected in matrix element.



- $s = (p_1 + p_2)^2 = (p_3 + p_4)^2$
- $t = (p_1 - p_3)^2 = (p_4 - p_2)^2$
- $u = (p_1 - p_4)^2 = (p_3 - p_2)^2$

The Big-Bang and Early Universe



https://cmb.physics.wisc.edu/pub/tutorial/bigbang.html

- Explains the evolution of the universe from an extremely hot and dense state.
- Isotropic and homogenous.
- Quark Gluon Plasma – State of matter filled in the universe at the first microsecond.