About "Last year's words"

– Mean-field singlet, triplet, and mixed-spin pairing in deformed heavy nuclei

Spin-Triplet Pairing in Heavy Nuclei Is Stable against Deformation

Georgios Palkanoglou[1,2], Michael Stuck[1], and Alexandros Gezerlis[1]

Show more

Phys. Rev. Lett. **134**, 032501 – **Published 23 January, 2025**

– Comparison with LEFT shows exciting exotic pairing and clustering in neutrons

See Dean's talk

– TRIUMF experiment to find triplet pairing gaps and the proton-drip, in Lanthanides

S1936	Beam Development of Light Lanthanides for Nuclear Structure Investigations Approaching N=Z	A.A. Kwiatkowski, E. Leistenschneider, G. Palkanoglou

TRIUMF

Putting the subleading three-nucleon contact force to (good) use

Georgios Palkanoglou
TRIUMF

Collaborator: Petr Navratil

PAINT25

February 28, 2025

Discovery, accelerated

TRIUMF

Outline

- Introduction
  - “Why any/more many-nucleon forces?”
  - Subleading three-nucleon forces at N3LO and N4LO
- The subleading (N4LO) 3-nucleon contacts and the isospin projections
  - Partial wave decomposition
  - Implementation in the No-Core Shell Model
  - Effects on ^{3}H
- *Summary & Outlook*

Discovery, accelerated

Outline

- Introduction
  - "Why any/more many-nucleon forces?"
  - Subleading three-nucleon forces at N3LO and N4LO
- The subleading (N4LO) 3-nucleon contacts and the isospin projections
  - Partial wave decomposition
  - Implementation in the No-Core Shell Model
  - Effects on ^{3}H
- *Summary & Outlook*

Some history

All realistic *NN* forces underbind the triton [7], and small differences among them can be traced to nonlocalities.

[*J.L. Friar et al, PRC (1999)*]

Overbound nuclear matter (wrong saturation density)

[*A. Akmal et al, Phys. Rev. C* ***58*** *(1998)*]

- *Fujita-Miyazawa*: One of the earliest (1957)
- *UIX*: starting from Fujita-Miyazawa
- *Tucson-Melbourne* / *Brazil*

⋮

- Chiral 3NFs: N2LO (1994), N3LO ($\sim$ 2010), N4LO (2011).

TRIUMF

Why better 3NF?

Outstanding problems

- $\mathrm{A_y}$ ***puzzle***: long-standing underestimation of the polarization asymmetry in N-d scattering by $30\ \%$.

  Known since 1990s: ascribed to (lack of) spin-orbit 3NFs.

- ***Space-star anomaly***: similar discrepancy in the break-up channel of N-d.

2NFs + modifications offer no resolution

3NFs up to N3LO have offer no resolution

[*J. Golak et al, EPJA (2014)*]

[*H. Witala et al, PRC (2021)*]

Prospects

- **Scattering exp.**: d-p, p-^{3}He, etc. scattering can provide high quality data.

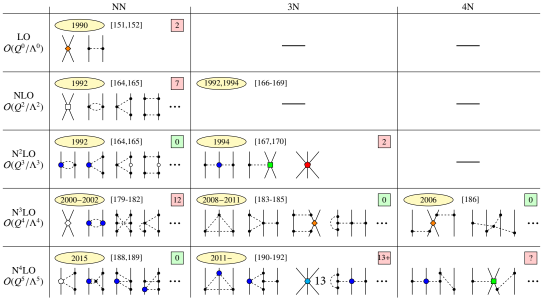
  [*K. Sekiguchi, FBS (2024)*]

  Possible fit of the 14 LECs for 3NF like the 24 LECs for 2NF (at N4LO).

  This will require controlled regulator effects, etc.

Discovery, accelerated

N4LO three-nucleon contacts



Adapted from [K. Hebeler, Phys. Rept. (2021)]

Three-nucleon force LECs

N2LO $\#2 \; : \quad c_D, c_E$

N3LO $\#0$

N4LO $\#13 \; : \quad E_i$

[*Girlanda et al, PRC (2011)*]

Outline

- Introduction
  - “Why any/more many-nucleon forces?”
  - Subleading three-nucleon forces at N3LO and N4LO
- The subleading (N4LO) 3-nucleon contacts and the isospin projections
  - Partial wave decomposition
  - Implementation in the No-Core Shell Model
  - Effects on ^{3}H
- *Summary & Outlook*

Discovery, accelerated

TRIUMF

N4LO three-nucleon contacts

Thirteen new contacts at N4LO, originally in momentum space ($\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}'_i$):

$$O_1 = \sum_{i\neq j\neq k} \left(-k_i^2\right) \ , \quad O_2 = \sum_{i\neq j\neq k} \left(-k_i^2\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \ , \quad O_3 = \sum_{i\neq j\neq k} \left(-k_i^2\right) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \ ,$$

$$O_4 = \sum_{i\neq j\neq k} \left(-k_i^2\right) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$O_5 = \sum_{i\neq j\neq k} \left(-3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j + k_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right) \ , \quad O_6 = \sum_{i\neq j\neq k} \left(-3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j + k_i^2 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$O_7 = \sum_{i\neq j\neq k} \left(-\frac{i}{4}\mathbf{k}_i\right) \times (\mathbf{p}_i - \mathbf{p}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \ , \quad O_8 = \sum_{i\neq j\neq k} \left(-\frac{i}{4}\mathbf{k}_i\right) \times (\mathbf{p}_i - \mathbf{p}_j) \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$O_9 = \sum_{i\neq j\neq k} (-\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j) \ , \quad O_{10} = \sum_{i\neq j\neq k} (-\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$O_{11} = \sum_{i\neq j\neq k} (-\mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i) \ , \quad O_{12} = \sum_{i\neq j\neq k} (-\mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

$$O_{13} = \sum_{i\neq j\neq k} (-\mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k \ .$$

[*Girlanda et al, PRC* ***84*** *(2011)*]

Discovery, accelerated

TRIUMF

N4LO three-nucleon contacts: Regulators

These will have to be regulated

$$V(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_3') \to F(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) V(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; \mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_3') F(\mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_3')$$

with choices

$$\text{non-local regulator}: \quad F(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \exp\left[-\frac{1}{4}\left(\pi_1^2 + \pi_2^2\right)^2 / \Lambda^4\right]$$

$$\text{local regulator}: \quad F(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \prod_i \exp\left(-k_i^4 / \Lambda^4\right)$$

Discovery, accelerated

N4LO three-nucleon contacts

Instructive to look at them in coordinate space with a local regulator depending on momentum exchange

$$
\begin{aligned}
V_{\text{cont}}^{(\text{N4LO})} = \sum_{i\neq j\neq k} \Bigg\{ & \left(E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j\right) \left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + \left(E_5 + E_6 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\right) S_{ij} \left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik}) \\
& + \left(E_7 + E_8 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k\right) (\mathbf{L}\cdot\mathbf{S})_{ij} \frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
& + \left(E_9 + E_{10} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k\right) \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \\
& + \left(E_{11} + E_{12} \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k + E_{13} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\right) \boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{ij} \boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik}) \Bigg\}
\end{aligned}
$$

where

$$
Z_0(r) = \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{r}} F(\mathbf{k}^2; \Lambda)
$$

N4LO three-nucleon contacts

We won't (over)fit $\mathbf{13}$ new contacts.

We'll take a subset from an isospin basis (separating by system relevance).

$$\sum_{i=1}^{13} E_i O_i = \sum_{T=1/2}^{3/2} \sum_i h_i^{(T)} D_i^{(T)} \quad (1)$$

$$D_i^{(T)} = \sum_j c_{ij}^{(T)} O_j \quad (2)$$

where $h_i^{(T)}$ are new LECs.

Two isospin channels: $D_i^{(1/2)}, D_i^{(3/2)}$, defined from the solutions

$$P_T D_i^{(T)} P_T = D_i^{(T)} \quad (3)$$

where P_T are isospin projection operators.

TRIUMF

N4LO three-nucleon contacts

$$O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8, O_9, O_{10}, O_{11}, O_{12}, O_{13} \longrightarrow$$

$$D_1^{(1/2)} = -O_1 + O_2$$
$$D_2^{(1/2)} = -O_7 + O_8$$
$$D_i^{(1/2)} = O_i \,, \quad i = 3, 4, 5, 6$$
$$D_7^{(1/2)} = \frac{1}{2} O_1 + O_7 + O_9$$
$$D_8^{(1/2)} = \frac{1}{2} O_1 - O_7 + O_{10}$$
$$D_9^{(1/2)} = \frac{1}{2} O_1 - O_7 + O_{11}$$
$$D_{10}^{(1/2)} = \frac{1}{2} O_1 - O_7 + O_{12}$$
$$D_{11}^{(1/2)} = \frac{1}{2} O_1 - O_7 + O_{13}$$
$$D_1^{(3/2)} = O_2 - \frac{1}{3}(O_4 + O_6) - (O_3 + O_5) - \frac{3}{2} O_9 - \frac{1}{2} O_{10} + O_{13}$$
$$D_2^{(3/2)} = -2O_2 + 2(O_3 + O_5) + \frac{2}{3}(O_4 + O_6) + 3O_9 + O_{10} + 3O_{11} + O_{12}$$

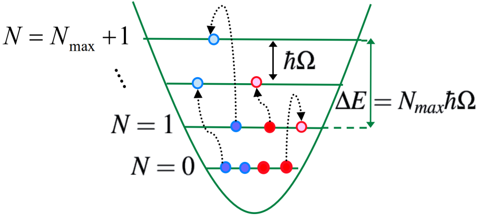
Originally derived in [*Alessia Nasoni, MSc Thesis*]

Discovery, accelerated

No Core Shell Model

Solution via a systematic expansion on A-body harmonic oscillator states.

Characterized by the HO frequency ($\hbar\Omega$, energy spacing of excitations) and number of excitations ($N_{\max}$).



[*Navratil et al, Phys.Rev.C* ***61*** *(2000)*]

[*Barrett et al. PPNP* ***69*** *(2013)*]

$$\left|\Psi_A^{J^\pi T}\right\rangle = \sum_{N=0}^{N_{\max}} \sum_n c_{Nn}^{J^\pi T} \left|\Phi_{Nn}^{J^\pi T}\right\rangle$$

TRIUMF

The basis for decomposition

Choosing the standard Jj-coupled three-body partial wave basis: [*W. Glöckle (1983)*]

$$|N\alpha JT\rangle = |N[(ls)j(\mathcal{L}\mathcal{S})\mathcal{J}]JT(t\tau)\rangle$$

$$|N\alpha JT\rangle = |N[(l_{ij}s_{ij})j_{ij}(\mathcal{L}_k\mathcal{S}_k)\mathcal{J}_k]JT(t_{ij}\tau_k)\rangle \ , \quad \text{i.e., only antisymm. in } (ij)$$

Each three-body partial wave is specified by $J, T, \mathcal{P} = (-1)^{l+\mathcal{L}}$ and the quantum numbers $\alpha = (l, s, j\mathcal{L}, \mathcal{J}, t)$.

Discovery, accelerated

N4LO 3NF: Partial Waves

$$O_1 = \sum_{i\neq j\neq k} \left(-k_i^2\right) \ , \quad O_2 = \sum_{i\neq j\neq k} \left(-k_i^2\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \ ,$$

$$V_{1,2} = \sum_{i\neq j\neq k} \left(E_1 + E_2 \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j\right) \left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}} \right] Z_0(r_{ik})$$

Similar to c_E. Partial wave decomposition

$$E_1 W_1 + E_2 W_2 = \delta_{ss'} \left[E_1 \delta_{tt'} + E_2 \hat{t}\hat{t}' (-1)^{t+t'+T+\frac{1}{2}} \begin{Bmatrix} t & t' & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} t & t' & 1 \\ \frac{1}{2} & \frac{1}{2} & T \end{Bmatrix} \right] \times$$
$$\times \hat{j}\hat{j}'\hat{\mathcal{J}}\hat{\mathcal{J}}'\hat{l}'\hat{\mathcal{L}}'(-1)^{J-\frac{1}{2}\mathcal{J}'-\mathcal{J}+l+\mathcal{L}+s} \times$$
$$\times \sum_X (-1)^X \hat{X}^2 \begin{Bmatrix} l & s & j \\ j' & X & l' \end{Bmatrix} \begin{Bmatrix} X & j & j' \\ J & \mathcal{J}' & \mathcal{J} \end{Bmatrix} \begin{Bmatrix} \mathcal{L}' & 1/2 & \mathcal{J}' \\ \mathcal{J} & X & \mathcal{L} \end{Bmatrix} C^{l'Xl}_{000} C^{\mathcal{L}'X\mathcal{L}}_{000} \times$$
$$\times \int d\xi_{12}\xi_1^2\xi_2^2 R_{nl}\left(\xi_1, b\right) R_{\mathcal{N}\mathcal{L}}\left(\xi_1, b\right) R_{n'l'}\left(\xi_1', b\right) R_{\mathcal{N}'\mathcal{L}'}\left(\xi_1, b\right)$$
$$\times Z_0^{(2)}\left(\sqrt{2}\xi_1; \Lambda\right) Z_{0,X}\left(\sqrt{\frac{1}{2}}\xi_1, \sqrt{\frac{3}{2}}\xi_2; \Lambda\right) \ .$$

N4LO 3NF: Partial Waves

$$O_3 + O_5 = \tilde{O}_5 = \sum_{i \neq j \neq k} (-3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j) \ , \quad O_4 + O_6 = \tilde{O}_6 = \sum_{i \neq j \neq k} (-3\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_i \cdot \boldsymbol{\sigma}_j) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \ .$$

Similar to c_D. Partial wave decomposition

$$E_5 \tilde{W}_5 + E_6 \tilde{W}_6 = -36 \delta_{ss'} \left[E_5 \delta_{tt'} + E_6 3 \hat{t}\hat{t}' (-1)^{t+t'+T+\frac{1}{2}} \begin{Bmatrix} t & t' & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \begin{Bmatrix} t & t' & 1 \\ \frac{1}{2} & \frac{1}{2} & T \end{Bmatrix} \right] \hat{j}\hat{j}' \hat{\mathcal{J}} \hat{\mathcal{J}}' \hat{s}\hat{s}'$$
$$\times (-1)^{J-\mathcal{J}+s+j'} \begin{Bmatrix} s & s' & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \hat{l}' \hat{\mathcal{L}}' \sum_{K=0,2} \hat{K} (-1)^{K/2} \sum_{VX} (-1)^V \hat{V} \hat{X}^2 C^{11K}_{000} C^{Vl'l}_{000} C^{XKV}_{000} C^{X\mathcal{L}'\mathcal{L}}_{000}$$
$$\times \sum_Z \hat{Z}^2 \begin{Bmatrix} l & s & j \\ l' & s' & j' \\ V & 1 & Z \end{Bmatrix} \begin{Bmatrix} \mathcal{L} & \frac{1}{2} & \mathcal{J} \\ \mathcal{L}' & \frac{1}{2} & \mathcal{J}' \\ X & 1 & Z \end{Bmatrix} \begin{Bmatrix} j & j' & Z \\ \mathcal{J}' & \mathcal{J} & J \end{Bmatrix} \begin{Bmatrix} V & 1 & Z \\ 1 & X & K \end{Bmatrix}$$
$$\times \int d\xi_{12} \xi_1^2 \xi_2^2 R_{nl}(\xi_1, b) R_{\mathcal{N}\mathcal{L}}(\xi_2, b) R_{n'l'}(\xi_1, b) R_{\mathcal{N}'\mathcal{L}'}(\xi_2, b) f_K\left(\sqrt{2}\xi_1; \Lambda\right) Z_{0,X}\left(\sqrt{\frac{1}{2}}\xi_1, \sqrt{\frac{3}{2}}\xi_2; \Lambda\right)$$

N4LO 3NF: Partial Waves

$$O_9 = \sum_{i\neq j\neq k} \left(-\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j\right) \ , \quad O_{10} = \sum_{i\neq j\neq k} \left(-\mathbf{k}_i \cdot \boldsymbol{\sigma}_i \mathbf{k}_j \cdot \boldsymbol{\sigma}_j\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$$

Similar to c_1. Partial wave decomposition

$$E_9 W_9 + E_{10} W_{10} = -6\left[E_5 \delta_{tt'} + E_6 \left\langle \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \right\rangle\right] (-1)^{s+j'+J-\mathcal{J}} \hat{l}' \hat{\mathcal{L}}' \hat{s} \hat{s}' \hat{j} \hat{j}' \hat{\mathcal{J}} \hat{\mathcal{J}}' \begin{Bmatrix} 1 & s & s' \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix}$$

$$\times \sum_{XVRY} \sum_{K_3=0}^{1} (-1)^Y \hat{X}^2 \hat{V} \hat{R} \hat{Y} \widehat{1-K_3} \left[\binom{3}{2K_3}\right]^{\frac{1}{2}} C^{XK_3Y}_{000} C^{X1-K_3R}_{000} C^{Y1V}_{000} C^{l'Vl}_{000} C^{\mathcal{L}'R\mathcal{L}}_{000} \times$$

$$\times \begin{Bmatrix} Y & X & K_3 \\ 1-K_3 & 1 & R \end{Bmatrix} \begin{Bmatrix} Y & j' & j \\ J & \mathcal{J} & \mathcal{J}' \end{Bmatrix} \begin{Bmatrix} j & l & s \\ j' & l' & s' \\ Y & V & 1 \end{Bmatrix} \begin{Bmatrix} \mathcal{J} & \mathcal{L} & \frac{1}{2} \\ \mathcal{J}' & \mathcal{L}' & \frac{1}{2} \\ Y & R & 1 \end{Bmatrix}$$

$$\times \int d\xi_{12} \xi_1^2 \xi_2^2 R_{nl}(\xi_1) R_{\mathcal{N}\mathcal{L}}(\xi_2) R_{n'l'}(\xi_1) R_{\mathcal{N}'\mathcal{L}'}(\xi_2) f_1\left(\sqrt{2}\xi_1\right) \left(\sqrt{\frac{1}{2}}\xi_1\right)^{K_3} \left(\sqrt{\frac{3}{2}}\xi_2\right)^{1-K_3}$$

$$\times f_{1,X}\left(\sqrt{\frac{1}{2}}\xi_1, \sqrt{\frac{3}{2}}\xi_2; \Lambda\right)$$

N4LO 3NF: Partial Waves

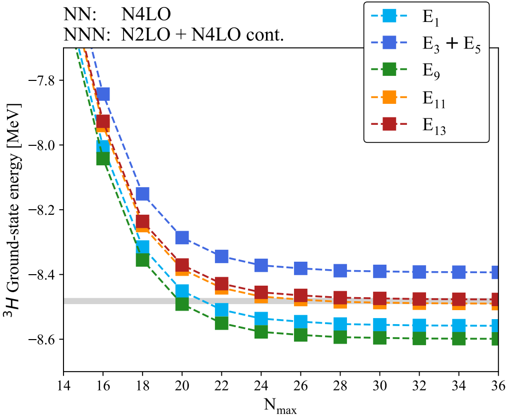
$$O_{11} = \sum_{i\neq j\neq k} \left(-\mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i\right) \,, \quad O_{12} = \sum_{i\neq j\neq k} \left(-\mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \,, \quad O_{13} = \sum_{i\neq j\neq k} \left(-\mathbf{k}_i \cdot \boldsymbol{\sigma}_j \mathbf{k}_j \cdot \boldsymbol{\sigma}_i\right) \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_k$$

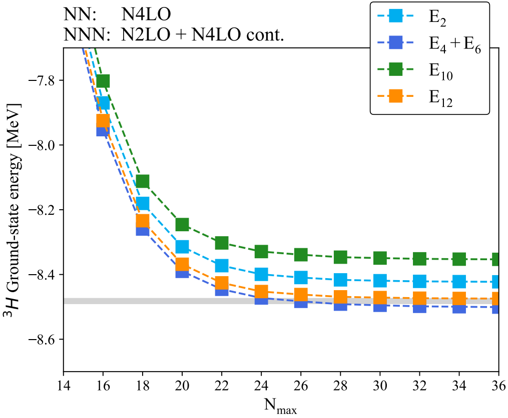
Partial wave decomposition

$$\begin{aligned}
E_{11}W_{11} + E_{12}W_{12} + E_{13}W_{13} &= -6\left[E_{11}\delta_{tt'} + E_{12}\langle \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3\rangle + E_{13}\langle \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3\rangle\right] \\
&\times \begin{Bmatrix} 1 & s & s' \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix} \hat{l}'\hat{\mathcal{L}}'\hat{s}\hat{s}'\hat{j}\hat{j}'\hat{\mathcal{J}}\hat{\mathcal{J}}'(-1)^{s+j'+J-\mathcal{J}} \sum_{VR} \hat{V}\hat{R} C^{l' V l}_{000} C^{\mathcal{L}' R \mathcal{L}}_{000} \sum_{Y} \hat{Y} C^{Y 1 V}_{000} \times \\
&\times \sum_{G} (-1)^{G} \hat{G}^2 \begin{Bmatrix} 1 & V & G \\ 1 & R & Y \end{Bmatrix} \begin{Bmatrix} G & j' & j \\ J & \mathcal{J} & \mathcal{J}' \end{Bmatrix} \begin{Bmatrix} j & l & s \\ j' & l' & s' \\ G & V & 1 \end{Bmatrix} \begin{Bmatrix} \mathcal{J} & \mathcal{L} & \frac{1}{2} \\ \mathcal{J}' & \mathcal{L}' & \frac{1}{2} \\ G & R & 1 \end{Bmatrix} \times \\
&\times \sum_{K_3=0}^{1} \left[\binom{3}{2K_3}\right]^{\frac{1}{2}} \widehat{1-K_3} \sum_{X} \hat{X}^2 C^{X K_3 Y}_{000} C^{X\, 1-K_3\, R}_{000} \begin{Bmatrix} Y & X & K_3 \\ 1-K_3 & 1 & R \end{Bmatrix} \times \\
&\times \int d\xi_{12} \xi_1^2 \xi_2^2 R_{nl}\left(\xi_1\right) R_{\mathcal{N}\mathcal{L}}\left(\xi_2\right) R_{n'l'}\left(\xi_1\right) R_{\mathcal{N}'\mathcal{L}'}\left(\xi_2\right) f_1\left(\sqrt{2}\xi_1\right) \left(\sqrt{\frac{1}{2}}\xi_1\right)^{K_3} \left(\sqrt{\frac{3}{2}}\xi_2\right)^{1-K_3} \\
&\times f_{1,X}\left(\sqrt{\frac{1}{2}}\xi_1, \sqrt{\frac{3}{2}}\xi_2; \Lambda\right)
\end{aligned}$$

TRIUMF

N4LO three-nucleon contacts at ^{3}H

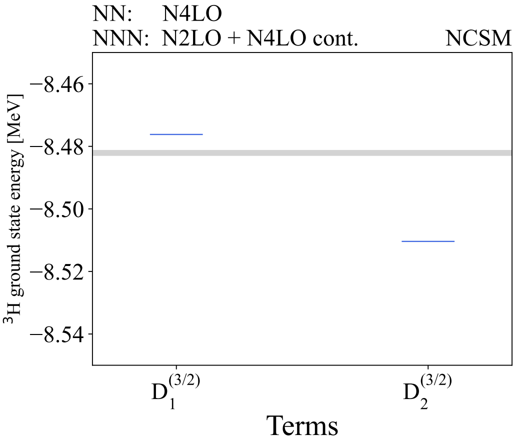




[G. P. and P. Navratil, in preparation (2025)].

Discovery, accelerated

Triton ground state



$T = 3/2$ terms shouldn't contribute at a $T = 1/2$ ground state

[G. P. and P. Navratil, in preparation (2025)]

TRIUMF

Regulator effects

Typically **one** of the **three** possible terms is taken (rest related by permutations)

$$\sum_{i\neq j\neq k} W_{ijk} = W_1 + W_2 + W_3$$

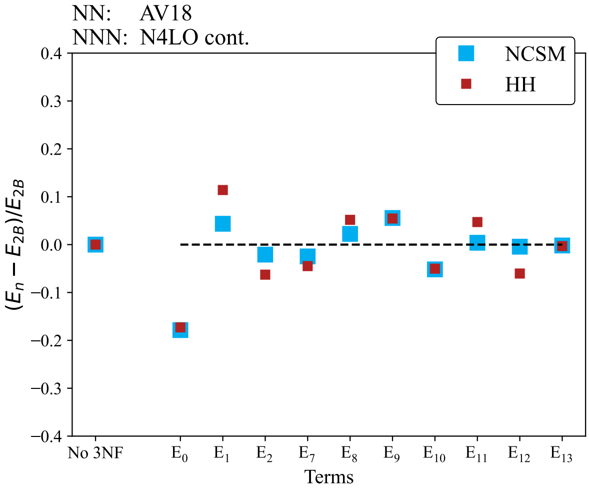
For a properly antisymmetrized many-body wavefunction, they are equal so

$$\langle N\alpha JT|\sum_{i\neq j\neq k} W_{ijk}\,|N'\alpha' JT\rangle = 3\langle N\alpha JT|\,W_2\,|N'\alpha' JT\rangle$$

But the local regulator breaks this.

Discovery, accelerated

Comparison with HH

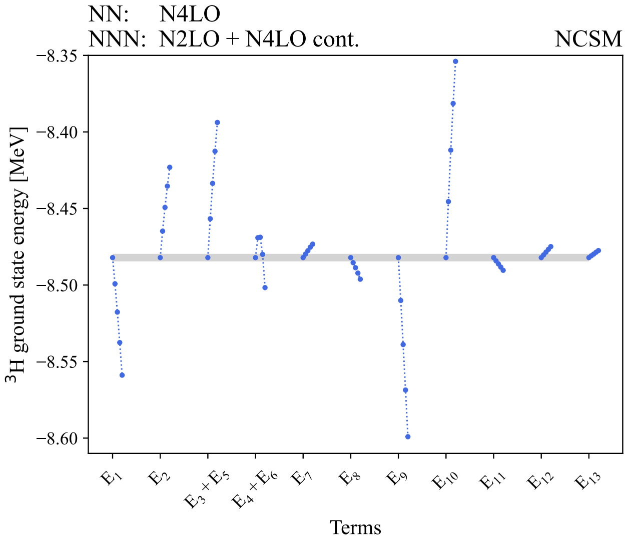


With only a 2NF: varying degree of agreement due to regulator effects.

■ *from Luca Girlanda*

[G. P. and P. Navratil, in preparation (2025)]

Varying the LECS



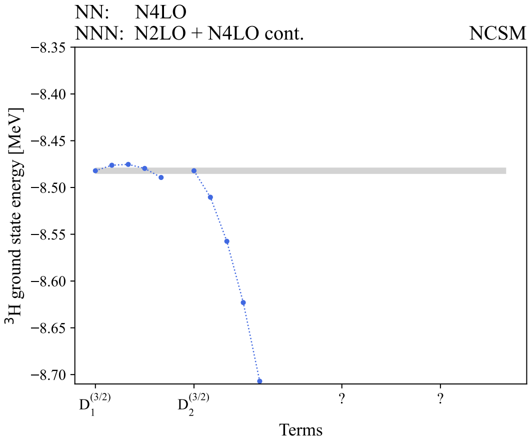
Varying the dimensionless LECs:

$$c_{E_i} = \frac{E_i}{F_\pi^4 \Lambda_\chi^3} = 0.25,\ 0.5,\ 0.75,\ 1.0$$

Discovery, accelerated

TRIUMF

N4LO $T = 3/2$ 3NF contacts



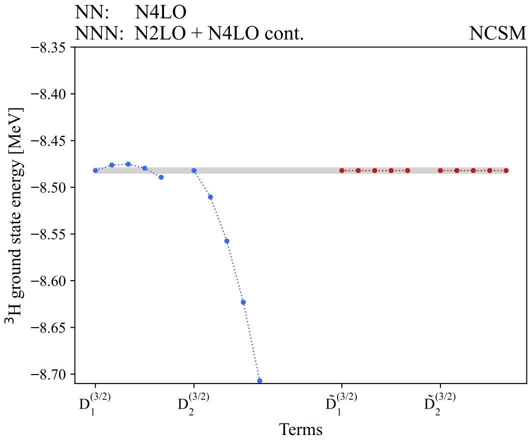
But they **shouldn't**: we have $T = 1/2$ components.

What to do?

1. Complete all $3 \times 13 = 39$ combinations
2. Take a shortcut, see results, and re-evaluate

Discovery, accelerated

N4LO $T = 3/2$ 3NF contacts



But they **shouldn't**: we have $T = 1/2$ components which we can kill:

$$\tilde{D}_i^{(3/2)} = P_{3/2} D_i^{(3/2)} P_{3/2}$$

$$P_{3/2} = \frac{1}{2} + \frac{1}{6}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 + \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3)$$

TRIUMF

Summary & Next steps

Summary

- We have a consistent implementation of the subleading 3NF contacts at the N4LO.
- Given available data, these terms can now be fitted.
- A way of "purifying" $D_1^{(3/2)}$, $D_2^{(3/2)}$ operators from $T = 1/2$ contributions. These can imminently be fitted to light nuclei.

Next steps

0. Tune the $D_i^{(3/2)}$ combinations to light nuclei (i.e., *our job*)

1. Explore their importance for neutron rich-er nuclei

2. Neutron drops

3. Explore other subleading 3NFs (see Wouter's talk and posters)

Discovery, accelerated

Thank you.

Discovery, accelerated