Neutrino Theory

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The Plan

1. Overview and subtleties of ν oscillation
2. What is the ν-A cross section problem?
3. Probing BSM + astrophysics in ν experiments

Overview of the standard neutrino oscillation paradigm

Most Established Aspect of Neutrinos

The standard neutrino oscillation paradigm

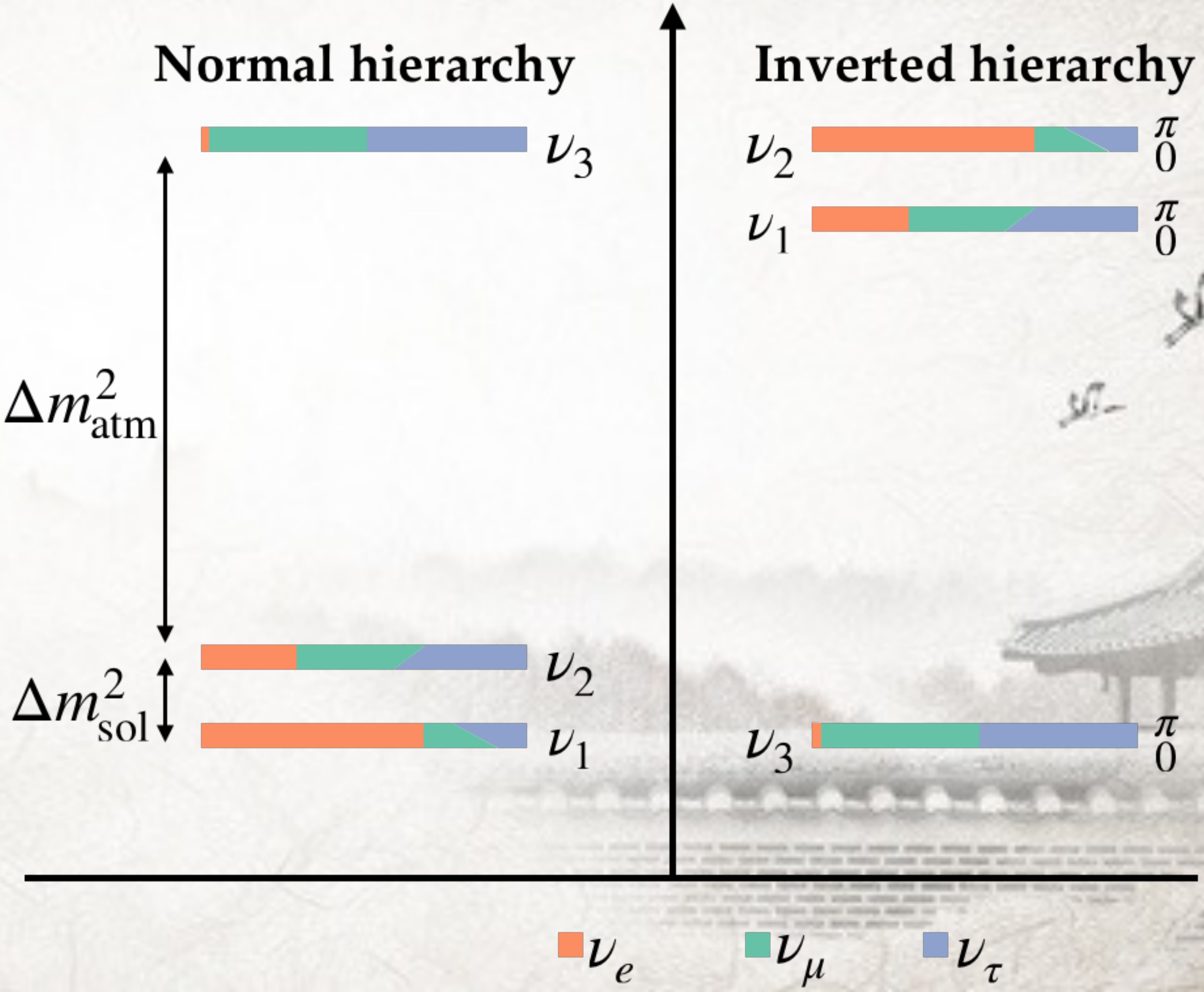
PMNS mixing matrix:

$\theta_{12}, \theta_{13}, \theta_{23}, \delta_{\mathrm{CP}}$

Mass-square differences:

$\Delta m^2_{21}, \Delta m^2_{32},$

mass hierarchy



3×3 Unitary Transformation

Standard Model: 3 neutrinos

$$|\nu_\alpha\rangle = \sum_{k=1}^{3} U_{\alpha k}^* |\nu_k\rangle$$

Flavor　　　　Mass

Why is there a star on the U element?

Unitary Pontecorvo–Maki–Nakagawa–Sakata matrix

Why do we use four real numbers to parameterize a 3*3 unitary matrix?

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\boxed{\theta_{23}}$　　$\boxed{\theta_{13}, \delta_{\text{CP}}}$　　$\boxed{\theta_{12}}$

Standard Derivation of ν Oscillation in Vacuum

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Appearance channel: $\alpha \neq \beta$
Disappearance channel: $\alpha = \beta$

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For two-level system: $(\alpha = e, \mu, k = 1, 2)$

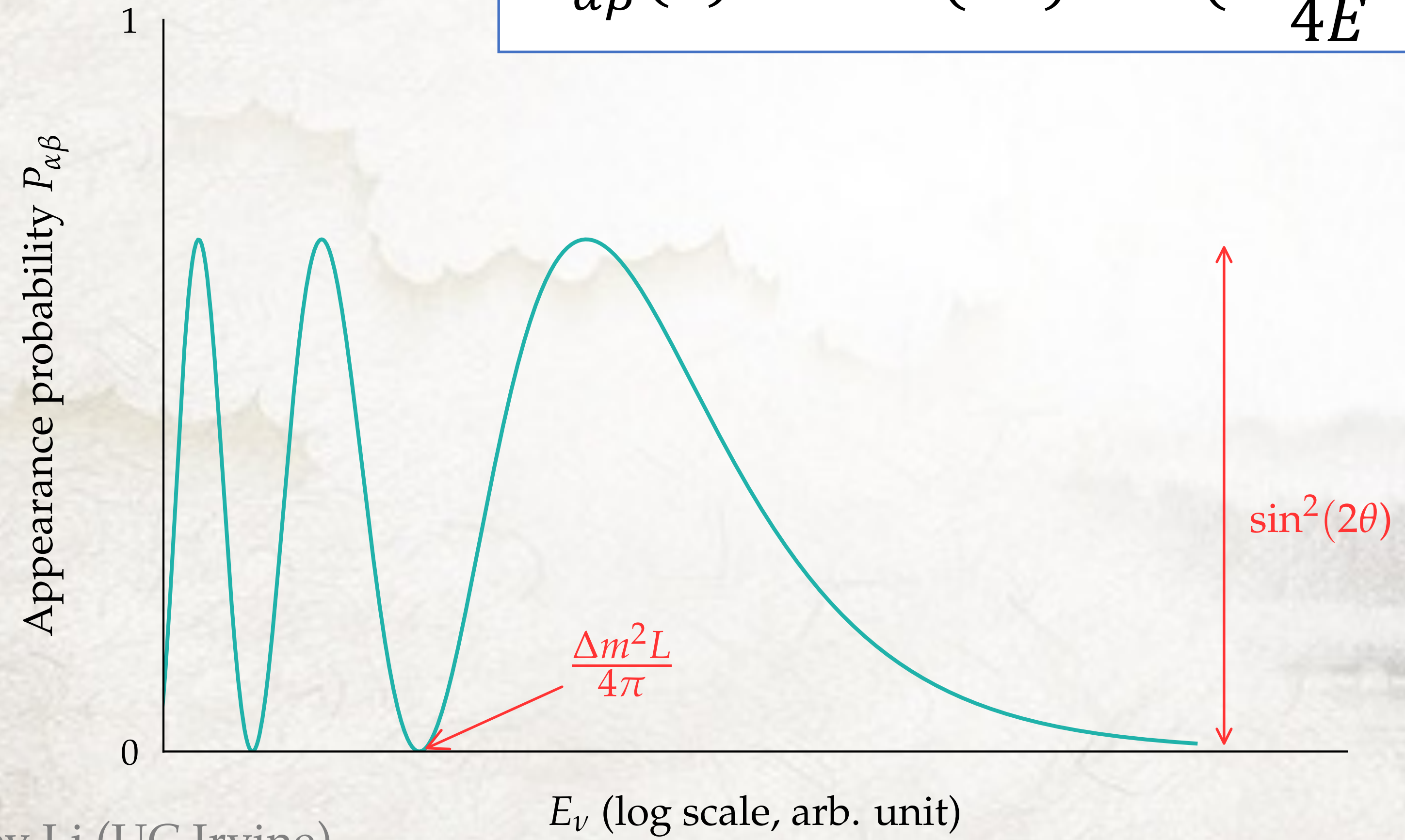
$$\boxed{P_{\alpha\beta}(L) = \sin^2(2\theta)\sin^2(\frac{\Delta m^2_{kj}\,L}{4E})}$$

What's wrong with my derivation?

Basic Features of Oscillation Measurements

For two-level system: $(\alpha = e, \mu, k = 1, 2)$

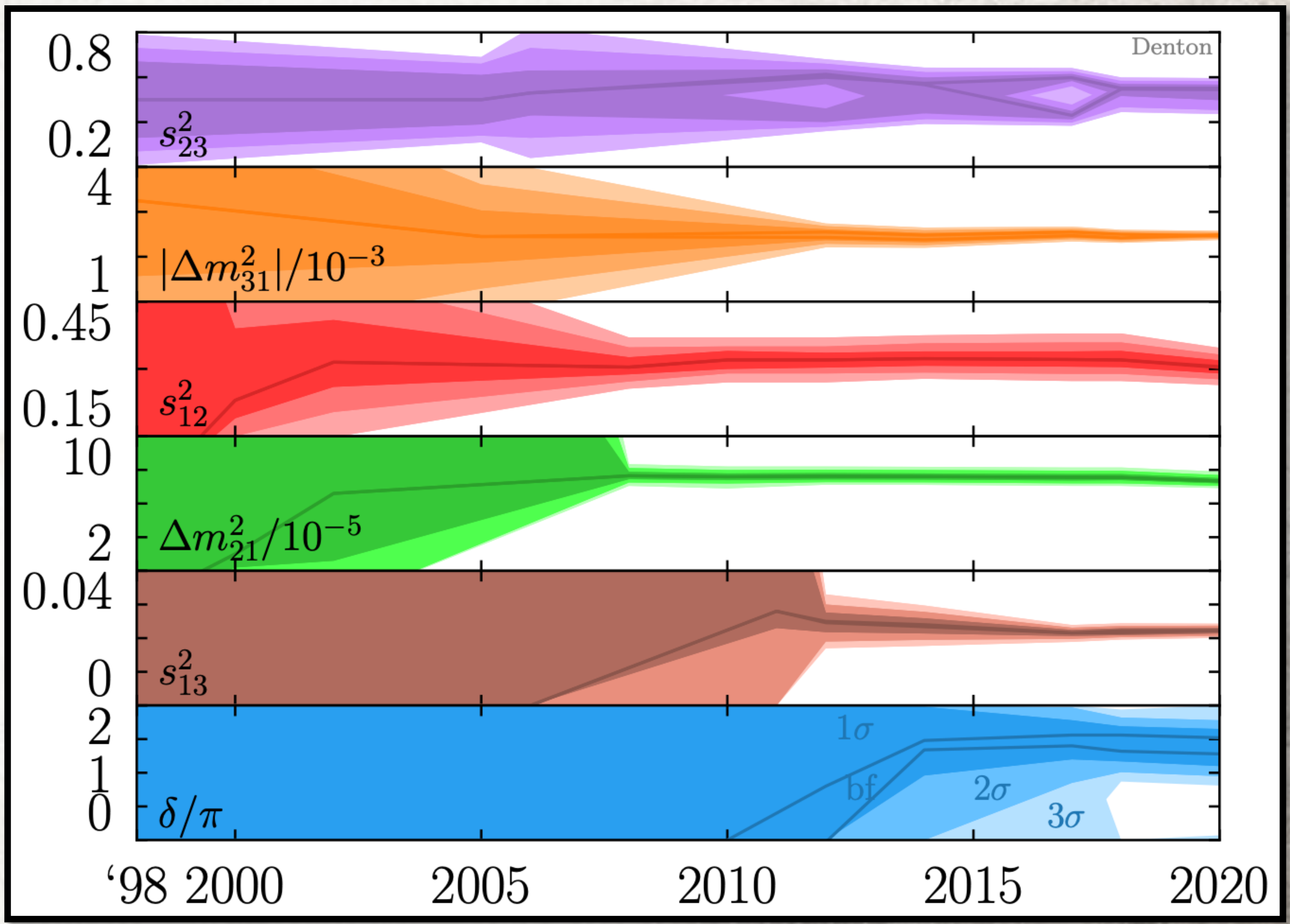
$$P_{\alpha\beta}(L) = \sin^2(2\theta)\sin^2\left(\frac{\Delta m_{kj}^2 L}{4E}\right)$$



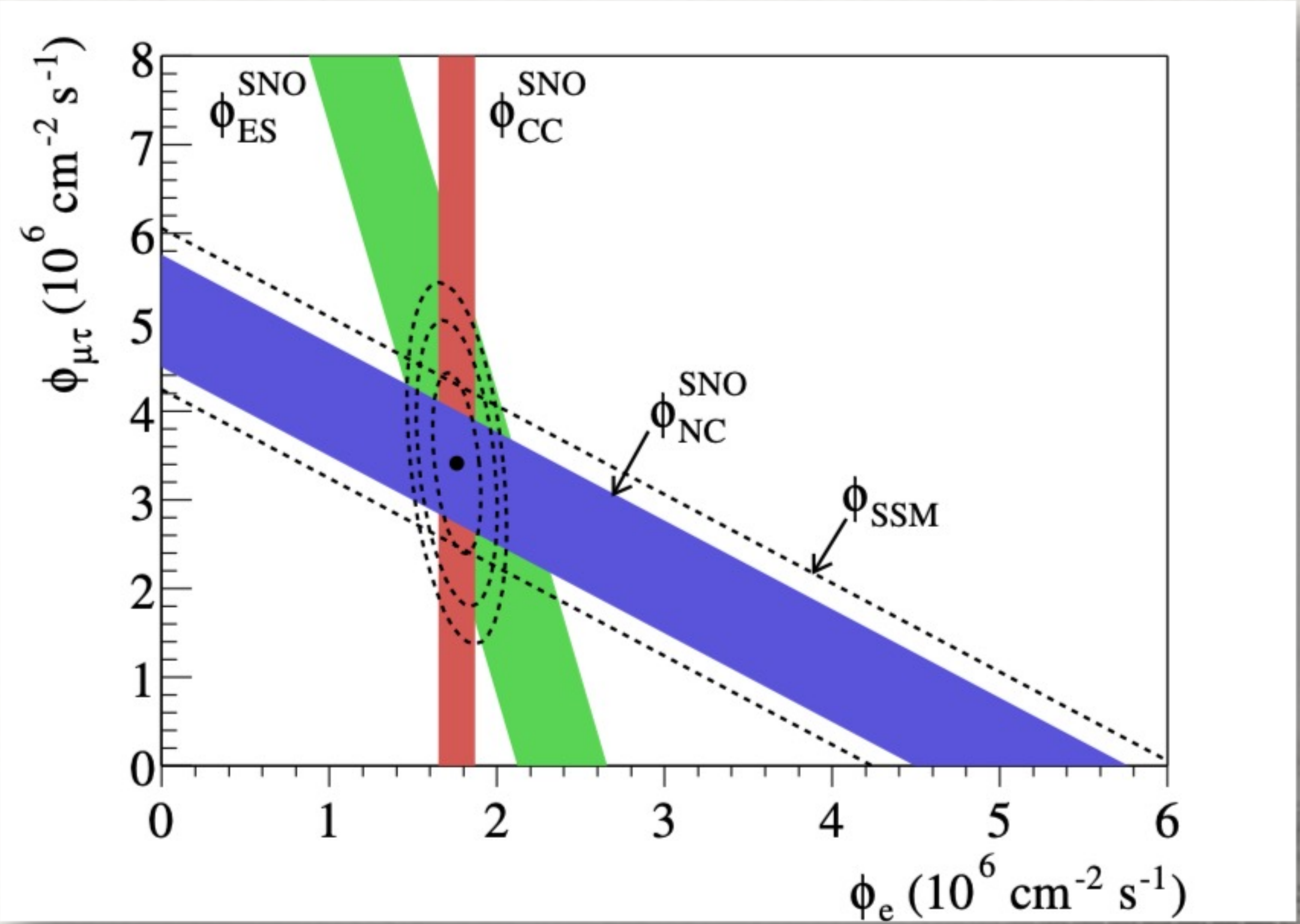
- Amplitude: angle
- Frequency: Δm^2
- No sign on Δm^2

Summary of Measurements

Goal: measure $U_{\alpha k}$
or
θ_{12}, θ_{13}, θ_{23}, δ_{CP},
Δm^2_{21}, Δm^2_{32}

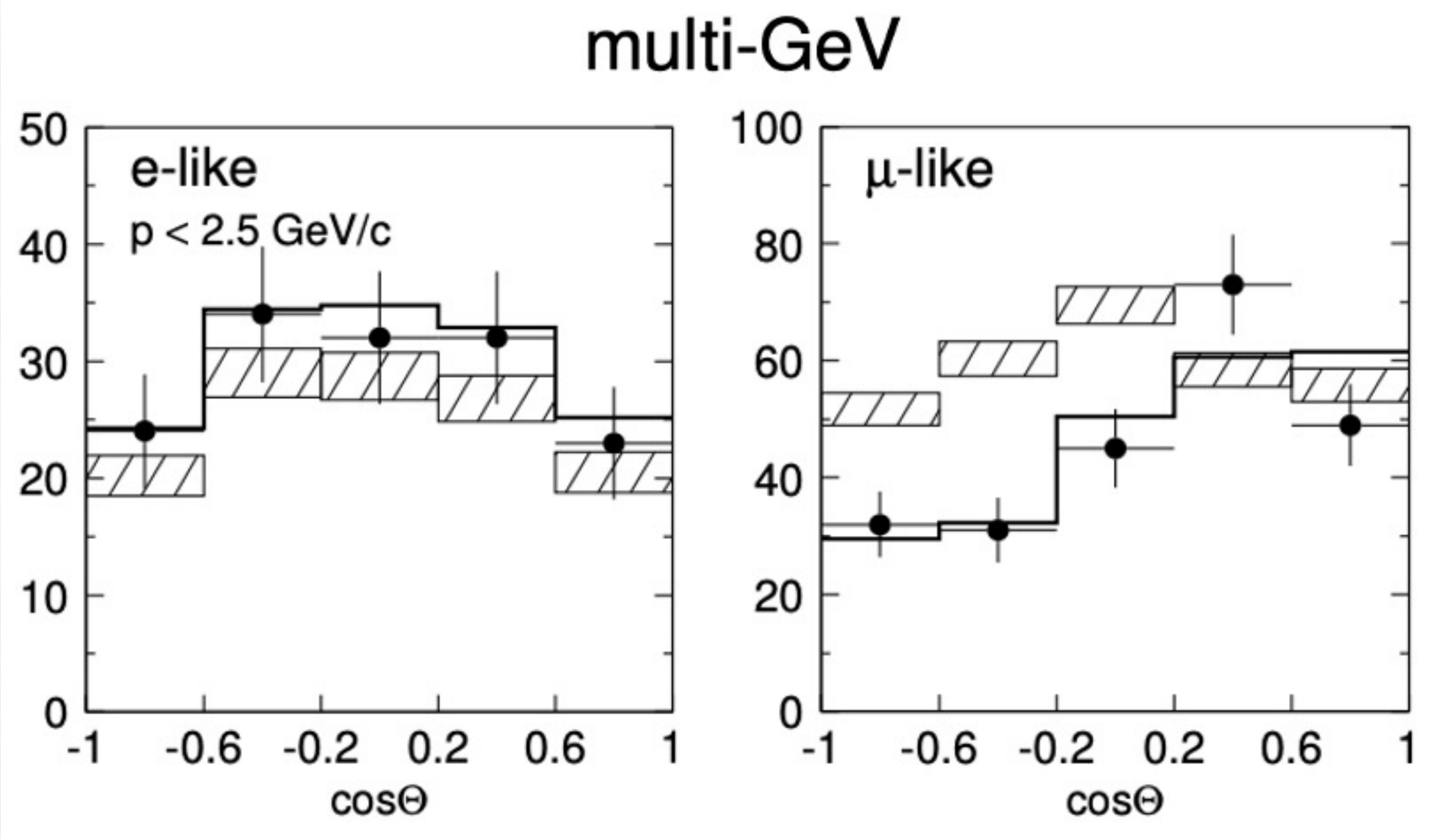


SNO Solar Neutrino Measurement



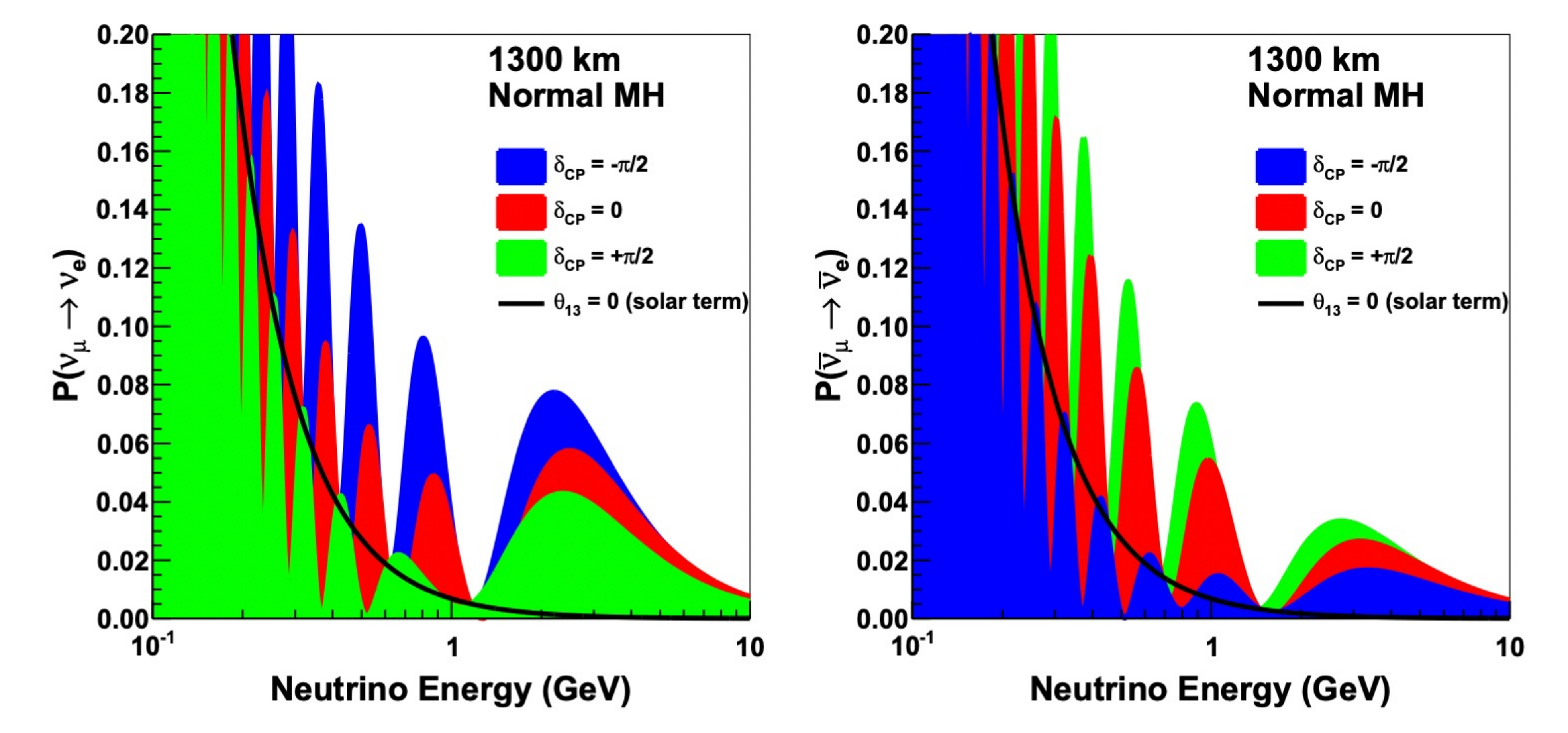
SNO 2016

Super-Kamiokande Atmospheric Neutrino Measurement



Super-Kamiokande 1988

DUNE Spectra



DUNE 2015

Bi-Probability Plot