Lecture 1. after slides.

History discovery of ν oscillation $\begin{cases} \text{solar } \nu. \\ \text{atmospheric } \nu. \end{cases}$

Solar : nuclear fusion produce ν_e.

e.g. $p+p \to d+e^+ +\nu_e$. "pp neutrino"
$^8B \to {}^8Be^* + e^+ + \nu_e$ "8B neutrino".

a bunch of such reactions, all produce ν_e
with energy between 0.1 – 20 MeV.

John Bahcall & Ray Davis. proposed to measure. 60 s.

Homestake experiment.
detection channel: $\nu_e + {}^{37}Cl \to e + {}^{37}Ar$.
count # of ^{37}Ar

result: $N_{\text{experiment measured}}$
$\simeq \frac{1}{3} N_{\text{theory prediction}}$.

⇒ "solar neutrino problem"

60 s – 2000. convince the community "the problem" is real.

definitive proof of the solution. SNO.

sol: N_{theory} is correct.

$N_{exp} \simeq \frac{1}{3} N_{th}$ because $\sim 1/3$ remained as ~~we~~ ν_e, and $2/3$ of ν_e oscillated to $\nu_\mu + \nu_\tau$

Why was SNO convincing?

3 channels:
$$\begin{cases} \nu + d \to \nu + p + n & (\text{NC, measure } \phi_{e+\mu+\tau}) \\ \nu_e + d \to e + p + p & (\text{CC, measure } \phi_e) \\ \nu + e \to \nu + e & (\text{mix, " some combination of } \phi_e, \phi_\mu + \phi_\tau) \end{cases}$$

$\Rightarrow$ the ~~was~~ well recognized SNO result plot.

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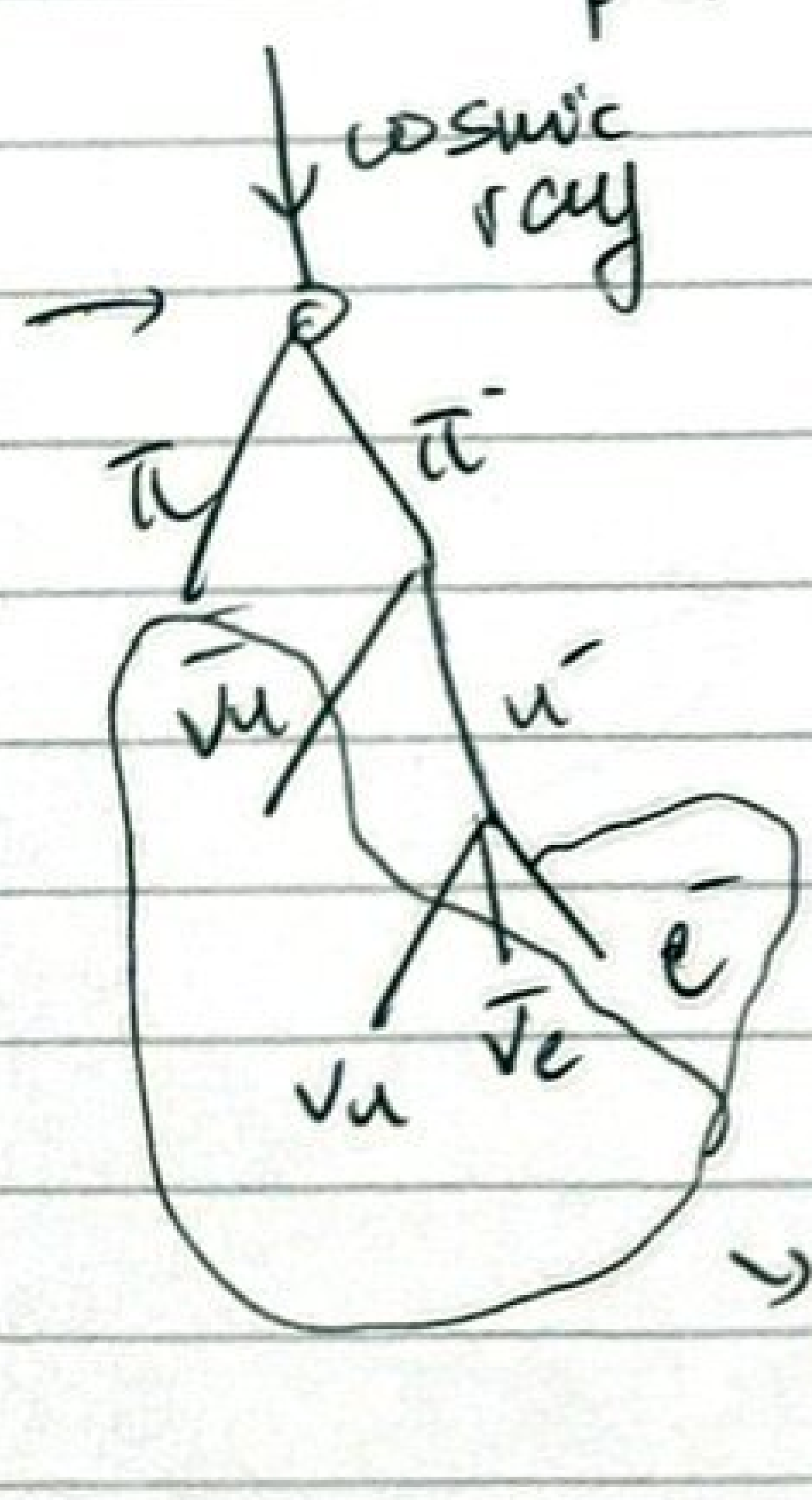
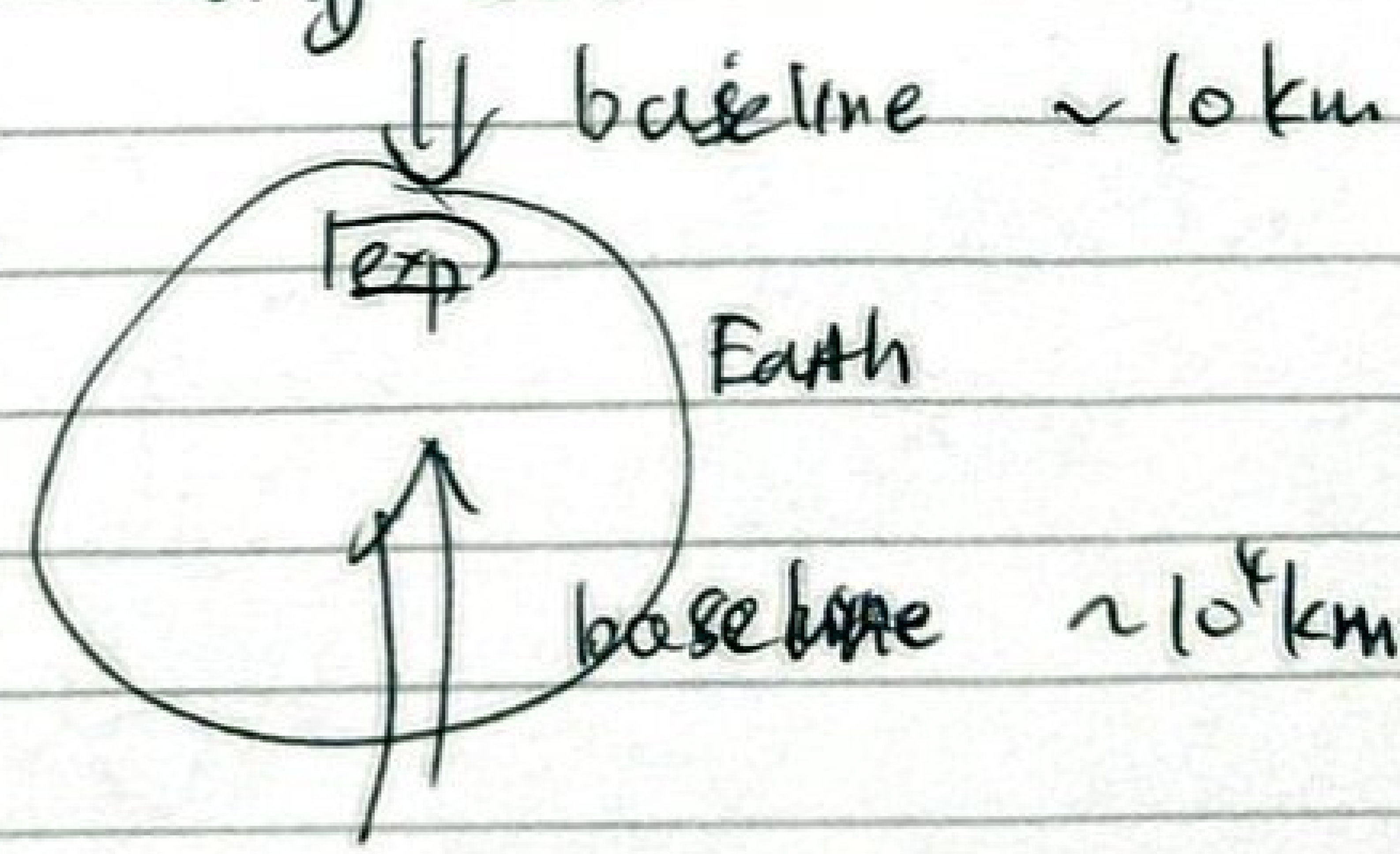
atmospheric ν oscillation.



~~without osci~~

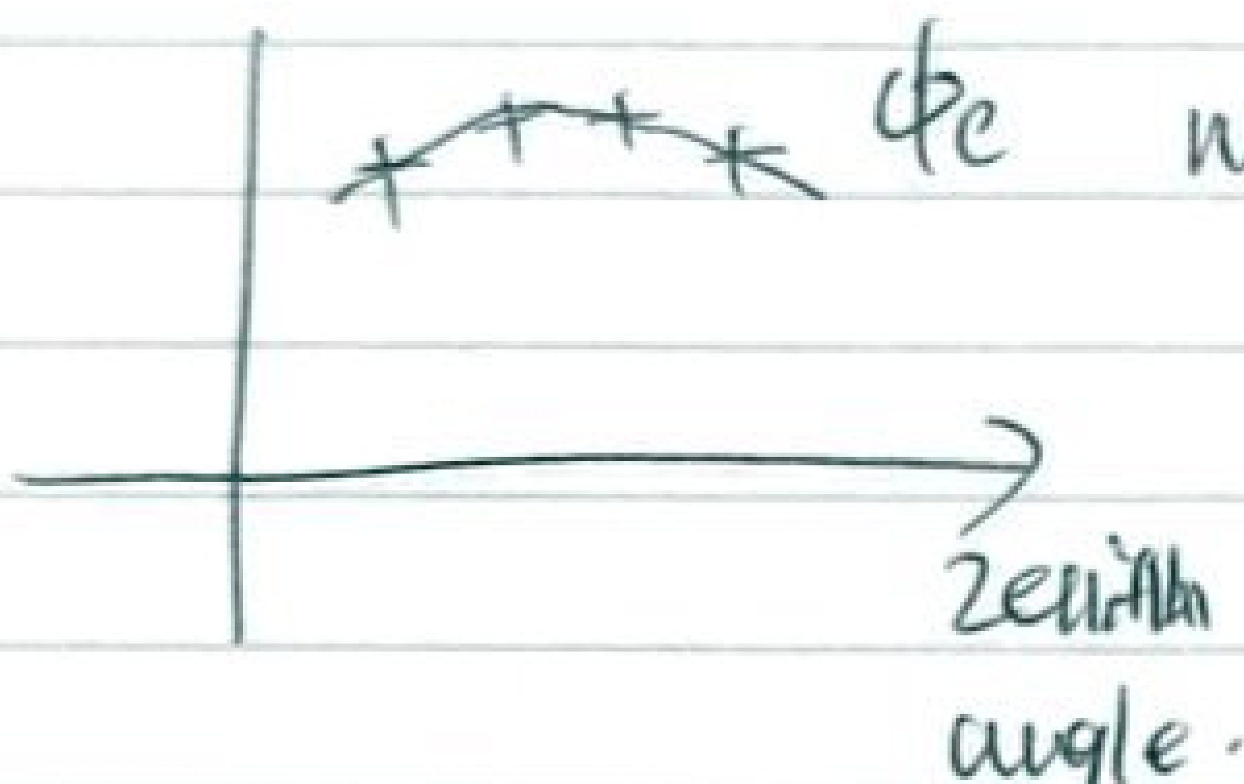
At production. $\phi_\mu : \phi_e \simeq 2:1$.

energy GeV +



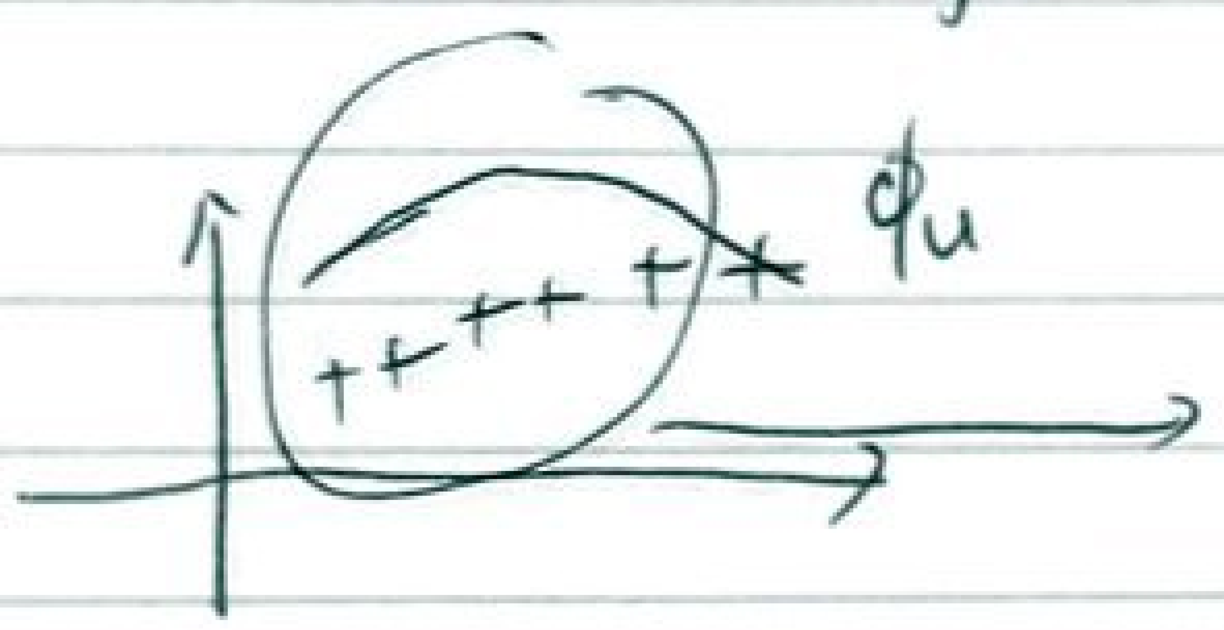
baseline vary from 10km – 10^4km.

signature:



no oscillation.

data match prediction.



where oscillation happens.

$\nu_\mu \to \nu_e$. so one measures the reduced flux of ν_μ

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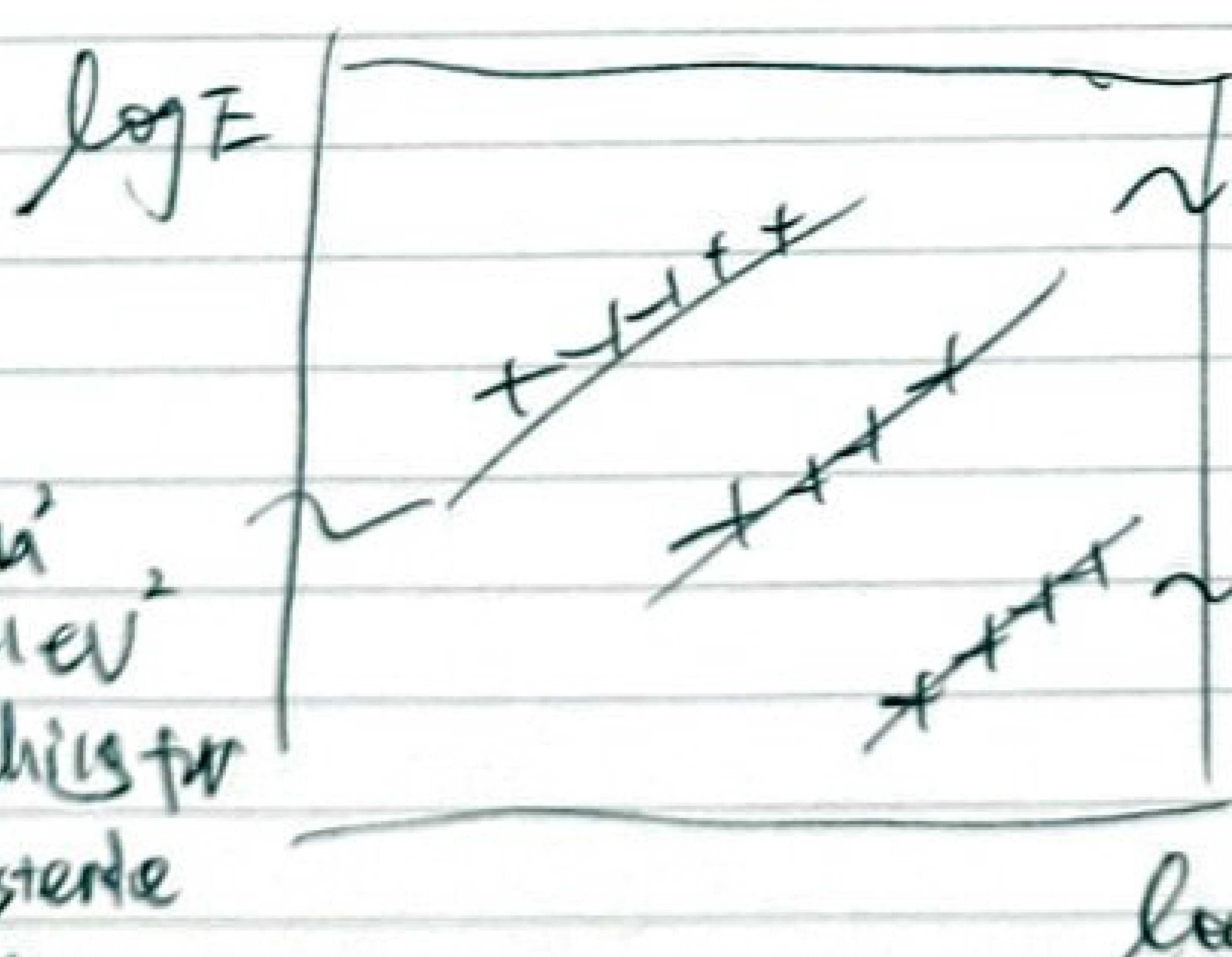
what are actually oscillating?



$\Delta m^2 \sim \Delta m^2_{32} \sim 2\times10^{-3}\,eV^2$ "atmos" sector.

a summary of exp.

$\Delta m^2 \sim \Delta m^2_{21} \sim 7\times10^{-5}\,eV^2$ "solar" sector

$\Delta m^2 \sim 1eV^2$ searching for $\sim 1eV$ sterile neutrino most experiments are pretty well approximated by two-level system oscillation, eg i.e. $P = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$

reason: $\Delta m^2_{32} \simeq \Delta m^2_{31} \gg \Delta m^2_{21}$

if $\frac{L}{E} \sim \frac{1}{\Delta m^2_{32}} \sim \frac{1}{\Delta m^2_{31}} \rightarrow$ 2level oscillation.

if $\frac{L}{E} \sim \frac{1}{\Delta m^2_{21}} \rightarrow$

← not participating.

$\therefore$ solar: θ_{12}, Δm^2_{21}, two level.
atmos: θ_{32}, Δm^2_{32}, two level.
} at leading order.
to build intuition

Last missing piece: matter effect.

In vacuum. we used this:

$$i\frac{d}{dL}|\nu_i\rangle = \frac{m_i^2}{2E}|\nu_i\rangle.$$

we can solve this in flavor basis instead.

$$i\frac{d}{dL}|\nu_\beta\rangle = U_{\beta i}\frac{m_i^2}{2E}U^*_{\alpha i}|\nu_\alpha\rangle$$

Or, in 2-flavor case:

$$i \frac{d}{dL} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta, & \sin 2\theta \\ \sin 2\theta, & \cos 2\theta \end{pmatrix} \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix}.$$

↑ solve this. you will get the usual $P = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right)$

Now, in the presence of (high-density) matter,
ν's dispersion relation will get modified.
similarly to γ traveling in medium.

consider the presence of electron. & only CC interaction.

$\mathcal{L} \supset -2\sqrt{2} G_F (\bar{\nu}_{eL} \gamma_\mu \nu_{eL})(\bar{e}_L \gamma^\mu e_L)$ integrating out W.

$$\langle \bar{e}_L \gamma_\mu e_L \rangle = \frac{N_e}{2} \delta_{\mu 0}.$$

$\Rightarrow$ EoM for neutrinos: $(i\not{\partial} - \sqrt{2} G_F N_e \gamma_0) |\nu_e\rangle = 0.$

$E = p + \underbrace{\sqrt{2} G_F N_e}_{\text{potential } V}.$

$\frac{\Delta m^2}{4E}\begin{pmatrix} -C_2 & S_2 \\ S_2 & C_2 \end{pmatrix}$ from above.

$$\Rightarrow H = H_{vacuum} + \begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix}$$

✓ with this H, you can solve the Schrödinger Equation,

assuming constant matter density.

define $\xi = \frac{2\sqrt{2} G_F N_e}{\Delta m^2 / E}$ $\quad P_{\alpha\beta} = \dfrac{\sin^2 2\theta \, \sin^2\left(\frac{\Delta m^2 L}{4E} \sqrt{1+\xi^2-2\xi\cos 2\theta}\right)}{1+\xi^2-2\xi\cos 2\theta}.$

↑ sensitive to the sign of Δm^2.

we have only clearly measured matter effect from the Sun, where matter density is NOT constant.

how solar ν actually oscillate is the so-called MSW effect.

But, matter effect $\to$ the sign of δm^2 holds.
so we know $\delta m_{21}^2 > 0$.

with the two level approximation, & matter effect for solar ν - you can get good intuition for all oscillation exp measure angles θ_{12}, θ_{13} & θ_{23}. and Δm_{21}^2, Δm_{32}^2.

Next gen exp DUNE, Hyper-K. aim to measure δ_{CP}, MH

both use $P_{\mu e}$ & $P_{\bar{\mu}\bar{e}}$ need 3-flavor & matter effect.

in constant matter density (here it's a good approximation for Earth surface).

$\Delta_{31} \equiv \Delta m_{31}^2 L/4E$.

(DUNE)

$$P_{\mu e} \simeq \sin^2\theta_{23} \sin^2 2\theta_{13} \Delta_{31}^2 \frac{\sin^2(\Delta_{31} - VL/2)}{(\Delta_{31} - VL/2)^2}$$

$$+ \sin 2\theta_{23} \sin 2\theta_{13} \sin 2\theta_{12} \frac{\sin(\Delta_{31} - VL/2)}{\Delta_{31} - VL/2} \Delta_{31}\Delta_{21} \frac{\sin(VL/2)}{VL/2}$$

$$\times \cos(\Delta_{31} + \delta_{cp})$$

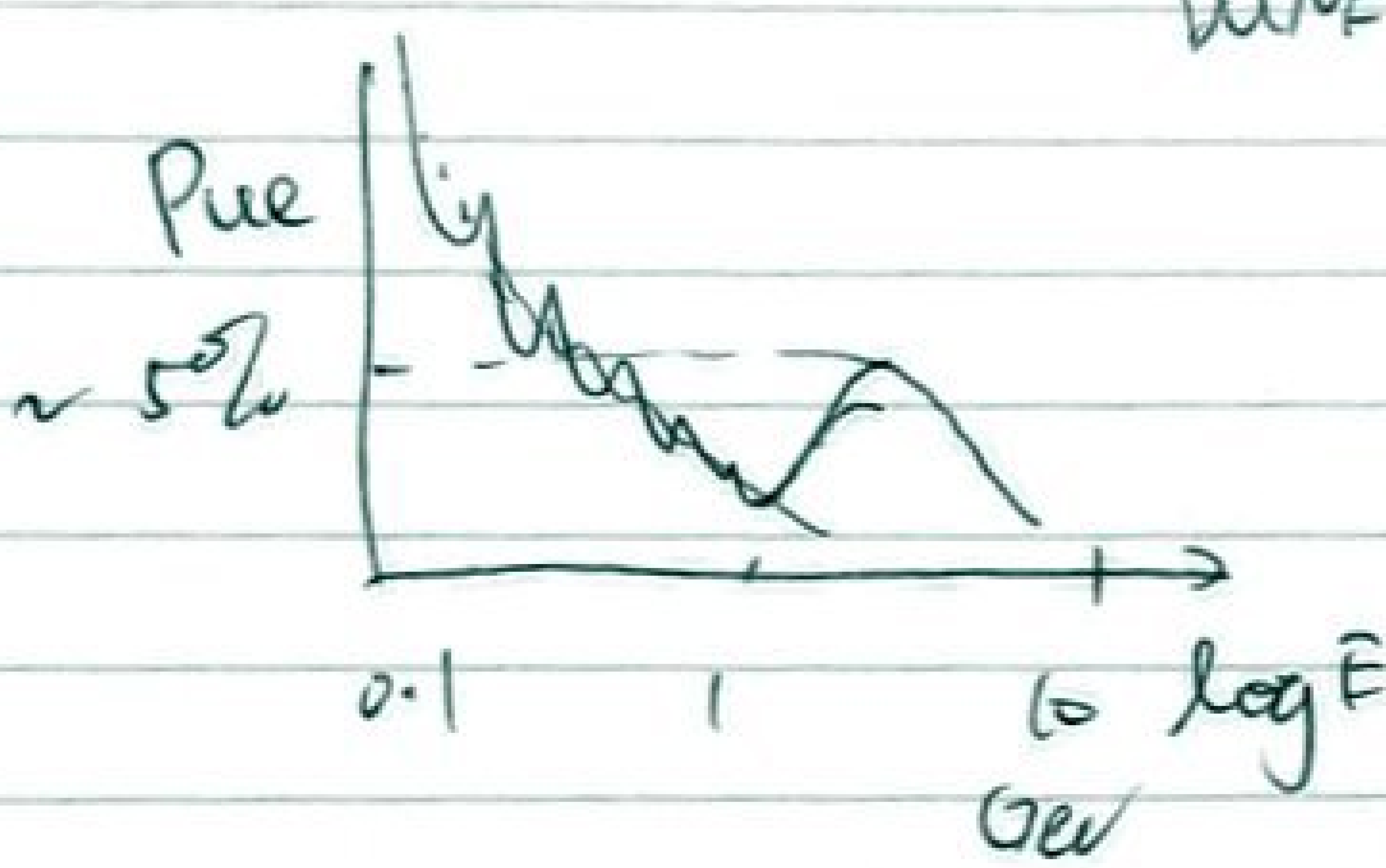
note: "long baseline" νL need to be not tiny.

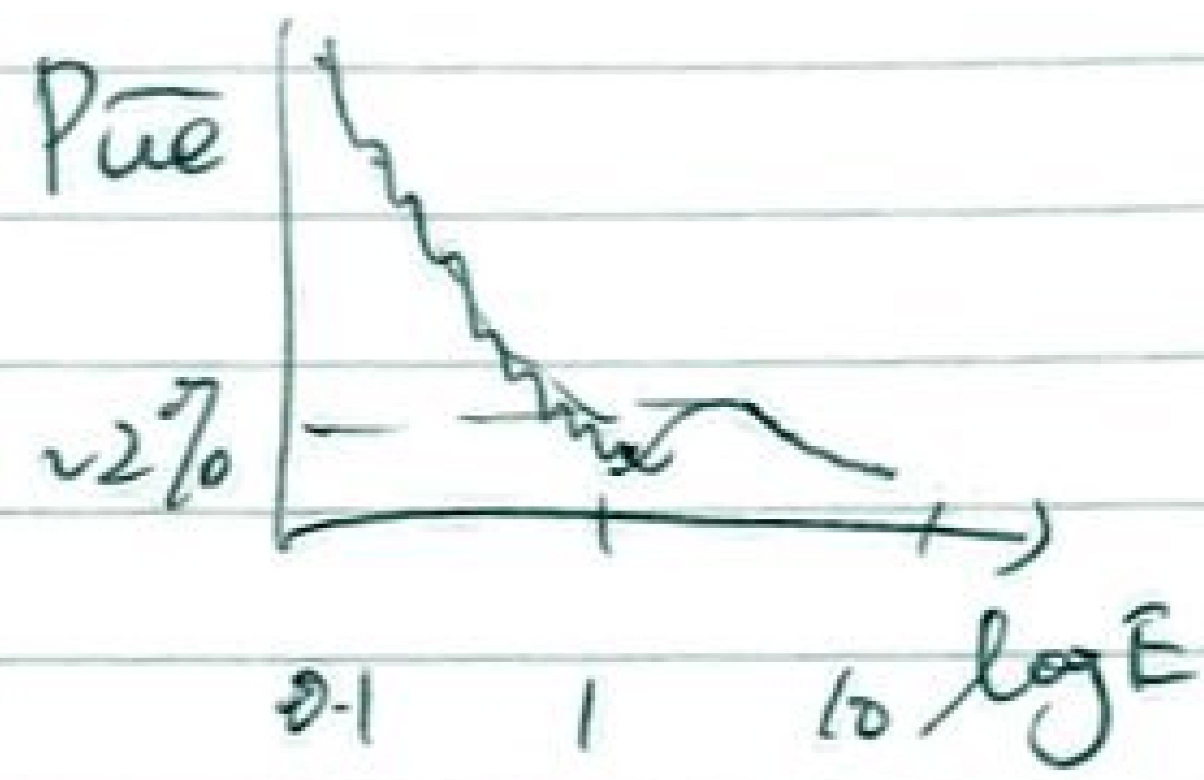
νL term together with Δ_{31} $\Rightarrow$ ~~mass~~ sign of Δ_{31}

δ_{cp} shows up in 2nd term.

$\Rightarrow$ MH. δ_{cp}. both measured in $P_{\mu e}$ & $P_{\bar{\mu}\bar{e}}$.

DUNE





DUNE, Hyper-K aim to measure δ_{cp} $\sim 10°$ which causes $\sim 5\%$ change in peak height.