Introduction to Cosmology and Dark Matter

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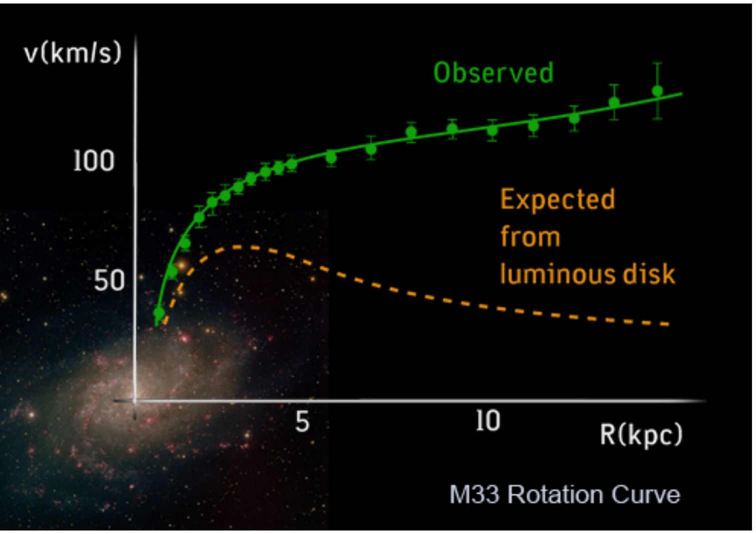

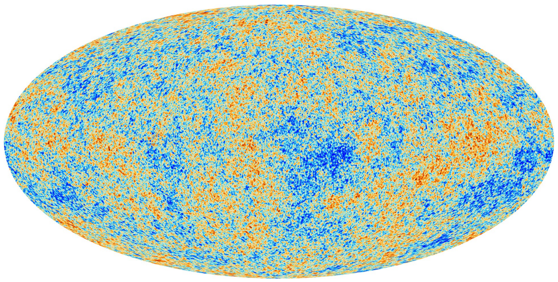

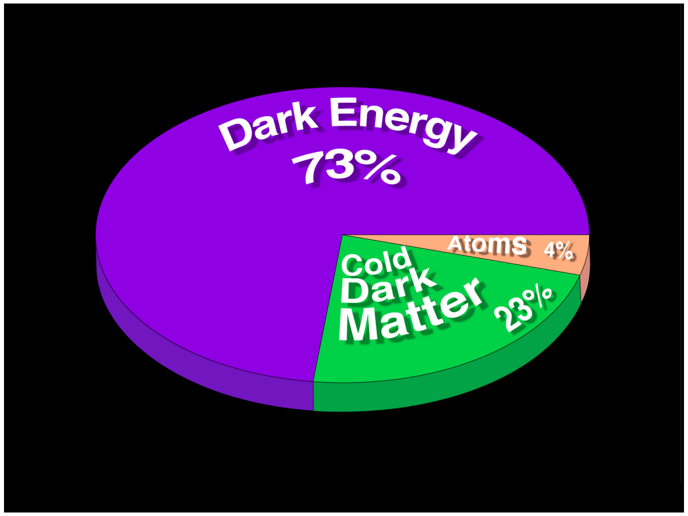

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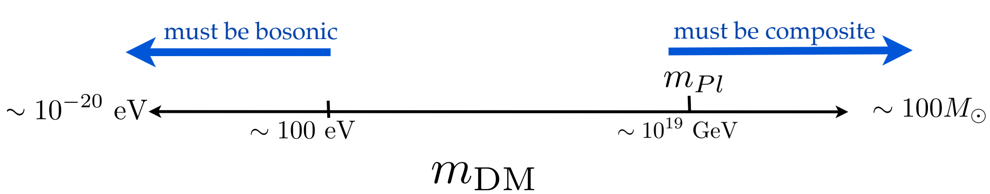
Cosmology : Studies the Origin and Evolution of the Universe

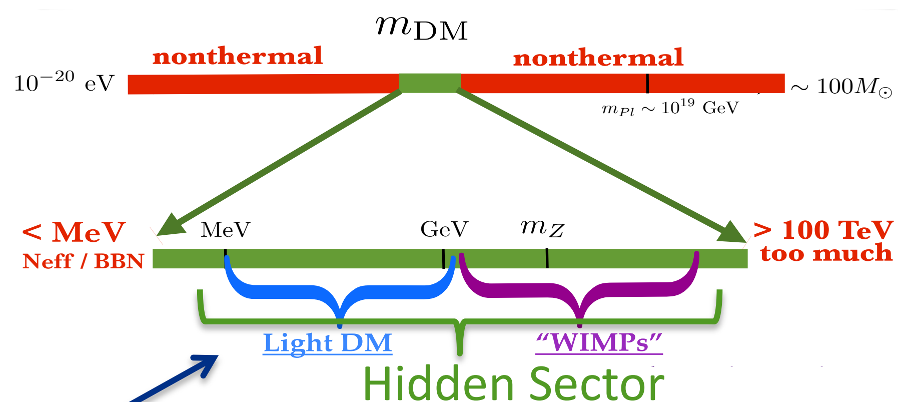
What do we know about Dark Matter ?

- very little -

- Couples gravitationally
- It is the most abundant form of matter
- It can be part of a larger invisible/dark sector with new dark forces
- It must be made of something different that all the particles we know, it can be made of particles or compact objects, or better described as wavelike disturbances
- Its mass can be anything from as light as 10^{-22} eV to as heavy as primordial black holes of tens of solar masses



Folding in assumptions about early Universe cosmology can provide some guidance



Cosmology and Dark Matter

The observable Universe is formed by planets, stars, galaxies and clusters of galaxies. The sun, for instance has a mass $M_\odot \sim 2 \times 10^{30}$ Kg. Nearest star is a few light years away. We will use the parsec (pc) units. 1 pc = 3.26 light years.

Our galaxy has about 10^{11} stars, consisting of a disk of radius 12.5 Kpc and width 0.3 Kpc. Typical separation of galaxies is 1Mpc. Galaxies can form clusters, but beyond 100 Mpc the Universe looks smooth. The size of the observable Universe is about 4×10^3 Mpc.

Cosmological Principle: At large scales, the Universe looks the same at each point. There is no preferred point. The Universe is homogeneous and isotropic!

Important observable properties is the fact that the Universe seems to be expanding. The further away galaxies seem to be the faster to depart from us. Another important observation is the cosmic Microwave background; a bath of photons with a black body spectrum characterized by a $T \sim 2.7K$. This is consistent with photons that were in thermal equilibrium with charged particles and have cooled down due to the Universe expansion. These photons are cosmological relics, and stopped interacting once neutral atoms formed.

To describe the Universe expansion, one can assume that the main long range effect is given by gravity, hence one should use general relativity. Due to the homogeneity and isotropy of the Universe, things are simple.

Metric:

$$ds^2 = c^2 dt^2 - a(t)^2 \left(dx^2 + dy^2 + dz^2\right)$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -a^2 & & \\ & & -a^2 & \\ & & & -a^2 \end{pmatrix}$$

t: Proper time for an observer at a given comoving coordinate (x, y, z). $a(t)$ is called the scale factor

This assumes that the Universe has no curvature. In spherical coordinates and assuming some curvature, $c=1$

$$ds^2 = dt^2 - a(t)^2 \left\{ \frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2 \right\}$$

where

$$\boxed{k = \begin{cases} <0 & \text{"open"} \quad \text{Hyperbolic} \\ 0 & \text{"flat"} \\ >0 & \text{"closed"} \end{cases}}$$

All measurements point towards a flat universe, $k=0$, at present. We will speculate about the reason later.

Now, we have to apply the Einstein Eqs.

$$R_{\mu\nu} - \frac{1}{2} R g^{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g^{\mu\nu}$$

$R_{\mu\nu}$: Four dimensional curvature tensor

$R = g^{\mu\nu} R_{\mu\nu}$; Λ: Cosmological Constant

$$T^{\mu\nu} = \mathrm{diag}(\rho, -P, -P, -P)$$

Perfect fluid energy momentum tensor

Before doing anything, we shall consider that the Universe expands adiabatically $\Rightarrow$ $dE = -P\,dV$

Lets consider the expansion of a section of the Universe of radius $r \cdot a(t)$, and derive the variation with a

$$d\left(\rho\, r^3 a(t)^3\right) = -P\, d\left(r^3 a(t)^3\right)$$

$$r^3 \frac{d\left(\rho a^3\right)}{d a^3} = \left(\rho + \frac{1}{3}\frac{d\rho}{d(\ln a)}\right) r^3$$

$$(P+\rho) = -\tfrac{1}{3}\, d\rho / d\ln a$$

$$\boxed{P = -\rho\left(1 + \frac{1}{3}\frac{d(\ln\rho)}{d(\ln a)}\right)}$$ Equation of state

For instance, for a fluid composed by non-relativistic particles,

$$\rho_m = n(a)\cdot m \propto \frac{n(a_i)\,a_i^3}{a^3}\, m$$

where $n(a)$ is the number density, and a_i is an initial value.

$$n(a_i)\,a_i^3 = n(a)\,a^3$$ Particle number conservation

Hence $a\dfrac{d(\ln\rho_m)}{da} = \dfrac{d\ln\rho_m}{d\ln a} = -3 \Rightarrow \boxed{P_m = 0}$

For photons, instead the energy density should consider the redshift since the wavelength increases with a

$$\frac{a(t)}{a_i} = \frac{\lambda}{\lambda_i} \Rightarrow E_\gamma \propto \frac{1}{\lambda_\gamma} \sim \frac{1}{a}\frac{a_i}{\lambda_i}$$

Hence $\rho \sim \dfrac{n_i}{\lambda_i}\cdot\dfrac{a_i^4}{a^4} \Rightarrow \boxed{P_\gamma = -\rho_\gamma\left(1 - \frac{4}{3}\right) = \frac{\rho_\gamma}{3}}$

What about a cosmological constant?

ρ_Λ : constant ; $P_\Lambda = -\rho_\Lambda$

We will define the equation of state parameter

$\boxed{P = w\rho}$, and if $\boxed{\rho \propto a^{-n}, \; w = \frac{n}{3} - 1}$

Friedmann Equation

I am not going to derive it, although it is a straightforward application of GR. Taking the "00" component of the Einstein's Equation

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho}$$

One can define the Hubble expansion parameter $H = \frac{\dot{a}}{a}$

$$\boxed{H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}}$$

Relativistic particles

$$\rho = \rho_m + \rho_{rad} + \rho_\Lambda$$

We shall define ρ_c as the energy density necessary to obtain a flat Universe $(K=0)$.

$$\Omega_X = \frac{\rho_X}{\rho_c} \quad ; \quad \rho_c \sim 10^{-5} \frac{\text{GeV}}{\text{cm}^3} \cdot h^2 = \frac{H_0^2}{(8\pi G/3)}$$

$$H_0 = h \times 100 \frac{\text{Km}}{\text{s Mpc}} \quad \left(\begin{array}{l}\text{Experimentally} \\ h \sim 0.7\end{array}\right)$$

Now, for a flat universe

$$\Omega_M + \Omega_{Rad} + \Omega_\Lambda = 1$$

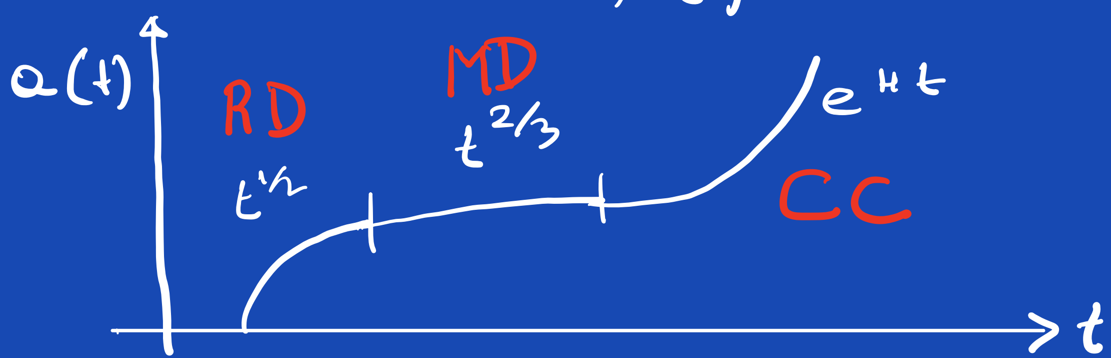
Also, for a universe in which $\rho \propto a^{-n}$

$$\frac{\dot{a}}{a} \propto a^{-n/2} \Rightarrow \frac{da}{a^{(1-n/2)}} \propto dt$$

Matter domination $n=3$, $a(t) \propto t^{2/3}$

Radiation domination $n=4$, $a(t) \propto t^{1/2}$

Cosmological constant $n=0$, $a(t) \propto e^{Ht}$



Flatness Problem

As we said before, today all evidence suggests that $k \sim 0$.

$$1 - \Omega_0 = -\frac{k}{H_0^2 a_0(t)^2} \qquad \left(\text{"0" means today}\right)$$

Now, if $\rho \propto a^{-n} \propto H^2$

$$\boxed{\frac{1-\Omega_i}{1-\Omega_0} = \left(\frac{a_0}{a_i}\right)^{2-n} = \left(\frac{t_0}{t_i}\right)^{\frac{2(n-2)}{n}} \quad n \neq 0}$$

MD $\qquad 1 - \Omega_0 = (1-\Omega_i)\left(\frac{t_0}{t_i}\right)^{2/3}$

RD $\qquad 1 - \Omega_0 = \left(1-\Omega_i\right)\left(\frac{t_0}{t_i}\right)^{1/2}$

Since $t_0 \gg t_i$, in order to get $1-\Omega_0 \sim 1$, we need Ω_i to be extremely close to 1. What ensures the flatness of the early Universe?

Observe that for $n<2$ the temporal relation of $1-\Omega_X$ would be inverted.

In particular, for $n=0$

$$\boxed{\frac{1-\Omega_2}{1-\Omega_1} = \exp\left[-2H_\Lambda(t_2-t_1)\right]}$$

Hence, one could start with $1-\Omega_1$ of order one at $t_1 << t_2$, and it will be driven to 1 at t_2! We will assume that there was such an early time period where $H^2 \sim \Lambda$ was a constant, and we will call this period **INFLATION**, in which the scale factor grew exponentially, rendering the Universe flat.

Let me emphasize that, as suggested before, we seem to be entering a new "inflationary" period.

Age of the Universe

For most of the Universe history it was either matter or Λ dominated

$$t = \int_0^{a_0(t)} \frac{da}{\dot{a}(t)} = \int_0^{a(t)} \frac{da}{\left(\frac{8\pi G}{3}\rho\right)^{1/2} a}$$

$$= \int_0^{a_0(t)} \frac{da}{\left(\frac{8\pi G}{3}\right)^{1/2} \left(\frac{\rho_{m,0}\, a_0^3}{a} + \rho_\Lambda a^2\right)^{1/2}}$$

Let's change variables to

$$x = \frac{a}{a_0} = \frac{1}{1+z} \; ; \text{ Redshift factor}$$

Currently, experimental evidence $\Omega_{m,0} \sim 0.3$; $\Omega_\Lambda = 0.7$

$$H_0^2 = \frac{8\pi G}{3}\left(\rho_{m,0} + \rho_\Lambda\right)$$

$$t_0 = \frac{1}{H_0}\int_0^1 \frac{dx \quad x^{1/2}}{\left[\Omega_{M,0} + \Omega_{\Lambda,0} x^3\right]^{1/2}} =$$

$$= \frac{1}{H_0}\,\frac{2}{3}\int_0^1 \frac{d\,x^{3/2}}{\left[\Omega_{M,0} + \Omega_{\Lambda,0} x^3\right]^{1/2}}$$

$$\boxed{t_0 = \frac{2}{3H_0}\,\Omega_{\Lambda,0}^{-1/2}\,\mathrm{arcsinh}\left(\frac{\Omega_{\Lambda,0}}{\Omega_{M,0}}\right)^{1/2}}$$

For $\Omega_{M,0} = 0.3$, $\Omega_{\Lambda,0} = 0.7 \Longrightarrow \boxed{t_0 = \frac{0.97}{H_0}}$

Observe that for $\Omega_{M,0} = 1$, $\Omega_{\Lambda,0} = 0$ we would get $t_0 = 2/3 \cdot 1/H_0$, so Ω_Λ increases the Universe lifetime.

Experimentally $H_0^{-1} \sim 13.5$ Gyr, which is of the order of the oldest stars age.

Equilibrium Thermodynamics

In the early Universe, particle interactions lead to a thermal equilibrium. Particles continue interacting until the rate

$\Gamma H^{-1} < 1$ Less than one interaction per age of the Universe.

$$n = \frac{g}{(2\pi)^3} \int f(\vec{p})\, d^3\vec{p}$$

$$\rho = \frac{g}{(2\pi)^3} \int E(\vec{p}) \cdot f(\vec{p})\, d^3\vec{p}$$

$$P = \frac{g}{(2\pi)^3} \int \frac{|\vec{p}|^2}{3E(\vec{p})} f(\vec{p})\, d^3\vec{p}$$

g = # spin states

$g_\gamma = 2 = g_{e^-} = g_{e^+}$

$g_\nu = g_{\bar{\nu}} = 1$

$g_W = 3$

$g_q = g_{\bar{q}} = 2$

N_c of them.

$$f(\vec{p}) = \left[\exp\left(\frac{E-\mu}{T}\right) \pm 1\right]^{-1}$$

$+$ Fermions

$-$ Bosons

	Bosons	Fermions	Non-relativistic
n_i $\xi(3) \sim 1.2$	$\frac{\xi(3)}{\pi^2} g_i T^3$	$\frac{3}{4} \frac{\xi(3)}{\pi^2} g_i T^3$	$g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-\frac{m_i}{KT}}$
ρ_i	$\left(\frac{\pi^2}{30}\right) g_i T^4$	$\frac{7}{8}\left(\frac{\pi^2}{30}\right) g_i T^4$	$m_i n_i$
P_i	$\frac{1}{3}\rho_i$	$\frac{1}{3}\rho_i$	$n_i T \ll \rho_i$ $\rho \to 0$

$$\rho_R = \frac{\pi^2}{30} \cdot g_* T_\gamma^4$$

$$g_* = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T_\gamma}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} \left(\frac{T_i}{T_\gamma}\right)^4$$

In radiation domination

$$H = \frac{\dot{a}}{a} = \frac{T^2}{M_{PL}} g_*^{1/2} \left(\frac{8\pi^3}{90}\right)^{1/2} = 1.66\, g_*^{1/2} \frac{T^2}{M_{PL}}$$

But $a = t^{1/2} \Rightarrow \frac{\dot{a}}{a} = \frac{1}{2t}$

$$t = 0.3\, g_*^{-1/2} \frac{M_{PL}}{T^2} \sim \left(\frac{T}{\text{MeV}}\right)^{-2} \text{sec}$$

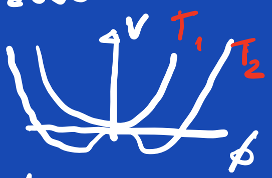
Radiation domination continues until $T \sim eV$, $t \sim 39{,}000$ years.

Many interesting things happen in the evolution of the Universe

At very high energies, quark, leptons, Higgs and gauge bosons are in thermal equilibrium

$T \sim 100$ GeV $\longrightarrow$ Higgs acquire a vacuum expectation value (particle masses)

$T_1 > T_2$



$T \sim 300$ MeV $\longrightarrow$ QCD phase transition

quarks, gluons $\longrightarrow$ Baryons, Mesons

$T \sim$ MeV : neutrinos decouple. $[e, \gamma, p, n, \nu]$

$T \sim 0.1$ MeV : Nucleosynthesis

$T \sim$ eV : Recombination. Neutral atoms.

Cosmic Microwave Background comes from photons cooling down from that era.

<u>Neutrino</u> <u>Decoupling</u>

$$n \langle \sigma \cdot v \rangle \sim H \sim \frac{T^2}{M_{Pl}}$$

$$\langle \sigma \cdot v \rangle \sim \frac{T^2}{M_W^4} \sim T^2 G_F^2$$

$$G_F^2 T^5 \sim \frac{T^2}{M_{Pl}} \Rightarrow T^3 = G_F^{-2} M_{Pl}^{-1} \sim 10^{-9} \text{GeV}^3$$

$$T \sim 10^{-3} \text{GeV} \equiv 1 \text{MeV}$$