Introduction to Cosmology and Dark Matter

Lecture II

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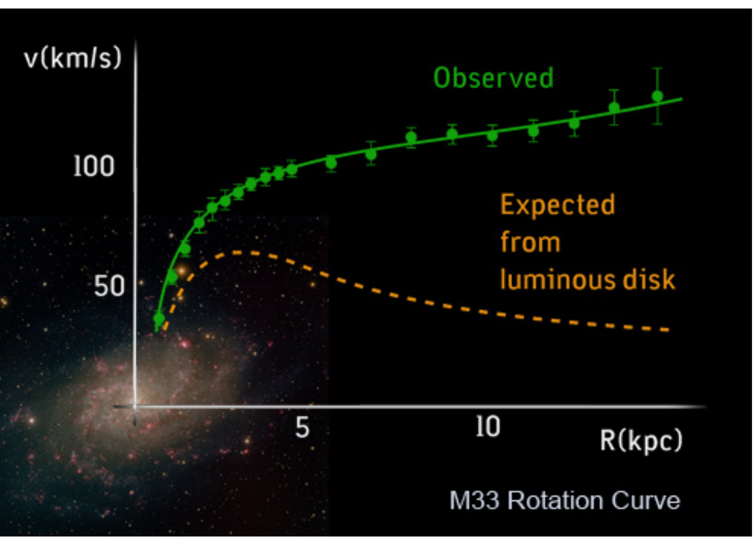
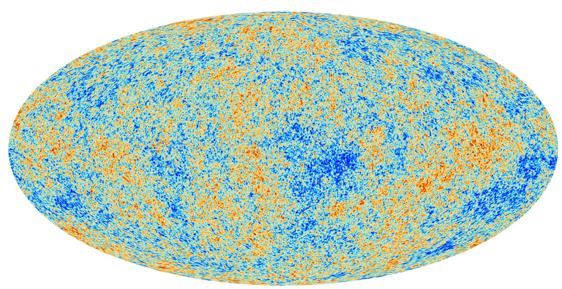
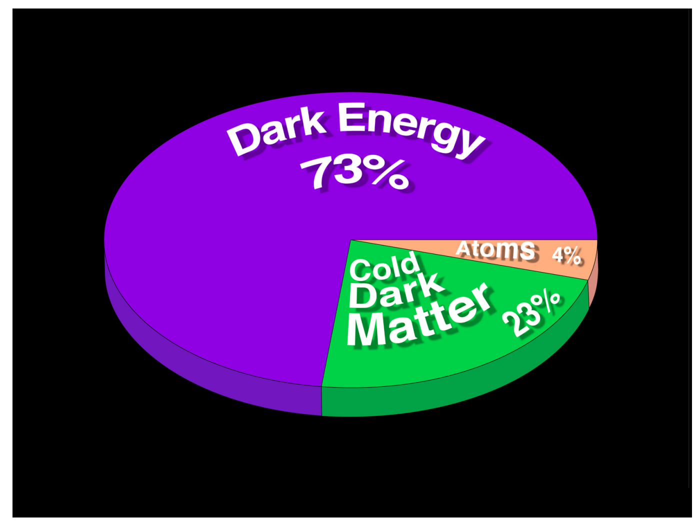






TRISEP Lectures, TRIUMF, June 24, 2025

FIFA Club World Cup · Today, 12:00 p.m.

Auckland City vs Boca Juniors

Group stage · Group C · Matchday 3 of 3

Short Review From Lecture I

What about a cosmological constant?

ρ_Λ : constant ; $P_\Lambda = -\rho_\Lambda$

We will define the equation of state parameter

$$\boxed{P = w\rho} \quad \text{, and if} \quad \boxed{\rho \propto a^{-n} \text{, } w = \frac{n}{3} - 1}$$

Friedmann Equation

I am not going to derive it, although it is a straightforward application of GR. Taking the "00" component of the Einstein's Equation

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho}$$

One can define the Hubble expansion parameter $H = \dfrac{\dot{a}}{a}$

$$\boxed{H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}}$$

Relativistic particles

$\rho = \rho_m + \rho_{rad} + \rho_\Lambda$

We shall define ρ_c as the energy density necessary to obtain a flat Universe $(K=0)$.

$$\Omega_X = \frac{\rho_X}{\rho_c} \quad ; \quad \rho_c \sim 10^{-5}\,\frac{\text{GeV}}{\text{cm}^3}\,.\,h^2 = \frac{H_0^2}{(8\pi G/3)}$$

$$H_0 = h \times 100\,\frac{\text{Km}}{\text{s Mpc}} \quad \left(\begin{array}{l}\text{Experimentally} \\ h \sim 0.7\end{array}\right)$$

Now, for a flat universe

$$\Omega_M + \Omega_{Rad} + \Omega_\Lambda = 1$$

Also, for a universe in which $\rho \propto a^{-n}$

$$\frac{\dot{a}}{a} \propto a^{-n/2} \Rightarrow \frac{da}{a^{(1-n/2)}} \propto dt$$

Matter domination $n=3$, $a(t) \propto t^{2/3}$

Radiation domination $n=4$, $a(t) \propto t^{1/2}$

Cosmological constant $n=0$, $a(t) \propto e^{Ht}$

Observe that for $n<2$ the temporal relation of $1-\Omega_X$ would be inverted.

In particular, for $n=0$

$$\boxed{\frac{1-\Omega_2}{1-\Omega_1} = \exp\left[-2H_\Lambda(t_2-t_1)\right]}$$

Hence, one could start with $1-\Omega_1$ of order one at $t_1 << t_2$, and it will be driven to 1 at t_2! We will assume that there was such an early time period where $H^2 \sim \Lambda$ was a constant, and we will call this period **INFLATION**, in which the scale factor grew exponentially, rendering the Universe flat.

Let me emphasize that, as suggested before, we seem to be entering a new "inflationary" period.

	Bosons	Fermions	Non-relativistic
n_i $\xi(3)\sim 1.2$	$\frac{\xi(3)}{\pi^2} g_i T^3$	$\frac{3}{4}\frac{\xi(3)}{\pi^2} g_i T^3$	$g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-\frac{m_i}{KT}}$
ρ_i	$\left(\frac{\pi^2}{30}\right) g_i T^4$	$\frac{7}{8}\left(\frac{\pi^2}{30}\right) g_i T^4$	$m_i n_i$
p_i	$\frac{1}{3}\rho_i$	$\frac{1}{3}\rho_i$	$n_i T \ll \rho_i$ $\rho \to 0$

$$\rho_R = \frac{\pi^2}{30} \cdot g_* T_\gamma^4$$

$$g_* = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T_\gamma}\right)^4 + \frac{7}{8}\sum_{\text{fermions}} \left(\frac{T_i}{T_\gamma}\right)^4$$

In radiation domination

$$H = \frac{\dot a}{a} = \frac{T^2}{M_{Pl}} g_*^{1/2} \left(\frac{8\pi^3}{90}\right)^{1/2} = 1.66\, g_*^{1/2} \frac{T^2}{M_{Pl}}$$

But $a = t^{1/2} \Rightarrow \frac{\dot a}{a} = \frac{1}{2t}$

$$t = 0.3\, g_*^{-1/2} \frac{M_{Pl}}{T^2} \sim \left(\frac{T}{\text{MeV}}\right)^{-2} \text{sec}$$

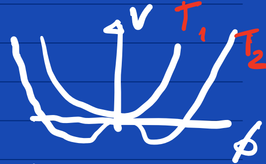
Radiation domination continues until $T \sim$ eV, $t \sim 39{,}000$ years.

Many interesting things happen in the evolution of the Universe

At very high energies, quark, leptons, Higgs and gauge bosons are in thermal equilibrium

$T \sim 100$ GeV $\longrightarrow$ Higgs acquire a vacuum expectation value (particle masses)

$T_1 > T_2$



$T \sim 300$ MeV $\longrightarrow$ QCD phase transition

quarks, gluons $\longrightarrow$ Baryons, Mesons

$T \sim$ MeV : neutrinos decouple. $[e, \gamma, p, n, \nu]$

$T \sim 0.1$ MeV : Nucleosynthesis

$T \sim$ eV : Recombination. Neutral atoms. Cosmic Microwave Background comes from photons cooling down from that era.

<u>Neutrino Decoupling</u>

$$n \langle \sigma \cdot v \rangle \sim H \sim \frac{T^2}{M_{Pl}}$$

$$\langle \sigma \cdot v \rangle \sim \frac{T^2}{M_W^4} \sim T^2 G_F^2$$

$$G_F^2 T^5 \sim \frac{T^2}{M_{Pl}} \implies T^3 = G_F^{-2} M_{Pl}^{-1} \sim 10^{-9}\,\text{GeV}^3$$

$$T \sim 10^{-3}\,\text{GeV} \equiv 1\,\text{MeV}$$

End of Review

Neutrino Temperature

Once neutrinos decouple, photons remain in equilibrium with electrons, which however soon become non-relativistic and hence annihilate into photons

$$g_* \left(m_e \lesssim T_\gamma \lesssim 1\,\mathrm{MeV} \right) = 2 + 4 \cdot \frac{7}{8} = \frac{11}{2}$$

$$T_\gamma \sim T_\nu$$

$$g_* \left(T_\gamma < m_e \right) = 2 \qquad \left(T_\gamma \neq T_\nu \right)$$

$$\frac{11}{2} \cdot T_\nu^3 = 2 \cdot T_\gamma^3 \qquad \text{entropy cons.}$$

At $T_\gamma < m_e$ $\qquad T_\nu \equiv \left(\frac{4}{11}\right)^{1/3} T_\gamma$

where $\qquad s = \frac{S}{V} = \frac{\rho + P}{T}$ (entropy density)

$$s = \frac{2\pi^2}{45} g_{*s} .$$

$$g_{*,0} = 2 + \underbrace{\frac{7}{8} \cdot 2 \cdot 3}_{\text{Neutrinos}} \left(\frac{4}{11}\right)^{4/3} \sim 3.36$$

$$g_{*s,0} = 2 + \frac{7}{8} \cdot 2 \cdot 3 \cdot \left(\frac{4}{11}\right) \sim 3.91$$

$$\left.\begin{aligned} s &= \frac{2\pi^2}{45} \cdot g_{*s,0}\, T^3 = 2970\ \mathrm{cm}^{-3} \\ n_\gamma &= 2\zeta(3)/\pi^2\ T^3 = 422\ \mathrm{cm}^{-3} \end{aligned}\right\} s \sim 7 n_\gamma$$

Evolution of g_* with T

When all particles of the SM were relativistic $T > 100\,\text{GeV}$

$$g_* = 2\times 15 \times 3 + 2\times 4 + 4 + 2\times 8 = 118$$

$15 : \nu_L + e_L + e_R + \overset{N_c}{3} u_L + 3u_R + 3d_L + 3d_R$

2 number of d.o.f per helicity.

3 : Number of generations

$\begin{cases} 2\times 4 : \text{EW Gauge Bosons} \\ 4 : \text{Higgs} \end{cases}$

2×8 : Gluons.

$m_t \sim 175\,\text{GeV}$

$m_b \sim 5\,\text{GeV}$

$m_\tau \sim 1.77\,\text{GeV}$

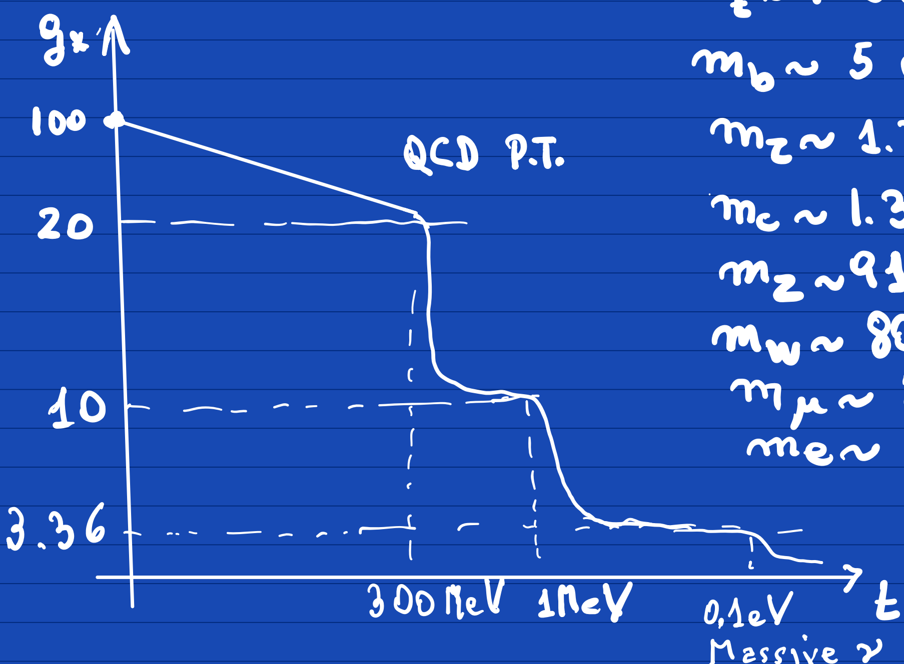
$m_c \sim 1.3\,\text{GeV}$

$m_Z \sim 91\,\text{GeV}$

$m_W \sim 80\,\text{GeV}$

$m_\mu \sim 105\,\text{MeV}$

$m_e \sim 511\,\text{KeV}$



Baryons

Recall that $\Omega_B = \frac{\rho_B}{\rho_c} = \frac{n_B \cdot \overbrace{1\,GeV}^{m_{n,p}}}{\rho_c}$

$$n_B = \rho_c \frac{\Omega_B}{GeV}$$

$$\frac{n_B}{n_\gamma} = \frac{\rho_c}{GeV} \cdot \frac{\Omega_B}{422\,cm^{-3}} = \frac{0.5\times 10^{-5}}{422} \cdot \Omega_B$$

$$\frac{n_B}{n_\gamma} = 1.4 \times 10^{-8}\,\Omega_B$$

$\Omega_B < 1$ and measurements $\rightarrow \Omega_B \sim 0.05$

$$\frac{n_B}{n_\gamma} \sim 6 \cdot 10^{-10} \quad ; \quad \frac{n_B}{s} = 8 \cdot 10^{-11}$$

Very few Baryons per photon!

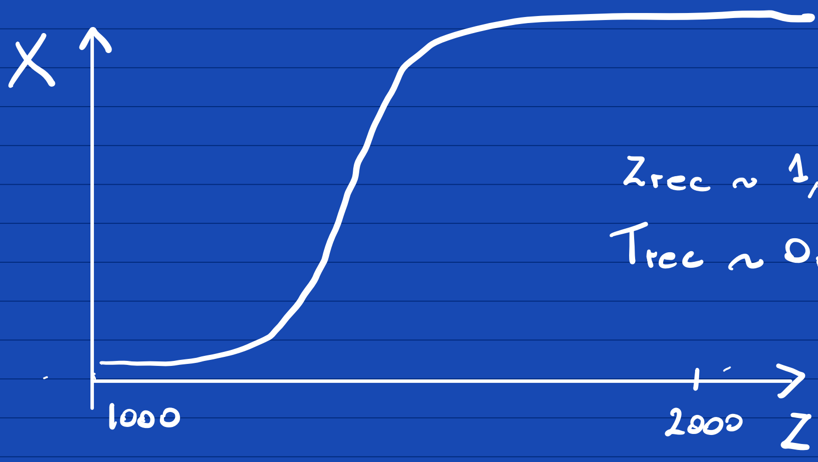
This has as an implication that when protons and electrons try to form atoms $T_{rec} < 13.6\,eV = B$

$$\mu_p + \mu_e = \mu_H \qquad n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_i - m_i}{T}\right)$$

$$B = m_p + m_e - m_H$$

$$\frac{n_H\, n_B}{n_p \cdot n_e} = \frac{g_H}{g_p \cdot g_e} (2\pi)^{3/2} \frac{n_B}{T^3} \left(\frac{T}{m_e}\right)^{3/2} \exp\left(\frac{B}{T}\right)$$

$$X = \frac{n_p}{n_B} \; ; \; n_p = n_{e^-} \Rightarrow \frac{1-X}{X^2} \sim \eta \left(\frac{T}{m_e}\right)^{3/2} \exp\left(\frac{B}{T}\right)$$



$$\frac{a}{a_0} = \frac{1}{1+z}$$

$z_{rec} \sim 1{,}100$

$T_{rec} \sim 0.3\ eV$

Assuming matter domination $a(t) \sim t^{2/3}$

$$t_0 = \frac{2}{3H_0} \quad ; \quad t_{rec} \sim \frac{2}{3H_0}\left(1+z_{rec}\right)^{-3/2}$$

$t_{rec} \sim 370{,}000$ yrs.

At this time, the Universe become neutral and photons suffer their last scattering. The CMB is an experimental evidence of this process.

Let me mention that the fact that we only see matter and not anti-matter is a big puzzle that shows that proton and neutron numbers are not just determined by going out of equilibrium by suppressed scattering. Some other, baryon and CP violating process determines their number, something called Baryogenesis.

Relics of the Big Bang

We define a relic as a particle which is either stable or has a lifetime much larger than the age of the Universe and hence its number is conserved since they decoupled from the plasma. DARK MATTER may be an example of such relics.

Particles may decouple when they are still relativistic, when $T_f \gg m$, where T_f is the "freeze out" decoupling T. An example are the neutrinos.

$$n_\psi = \frac{\zeta(3)}{\pi^2} g_\psi T^3 \begin{pmatrix} 1 \\ 3/4 \end{pmatrix} \begin{matrix} \text{Bosons} \\ \text{Fermions} \end{matrix}$$

Now, since then $n_\psi \cdot a^3$ is conserved,

$$s_f = \frac{2\pi^2}{45} g_{*s}(T_f)\, T_f^3 \qquad \left(s = \frac{\rho + P}{T} \right)$$

$$\frac{n_\psi}{s_f} = \frac{45\,\zeta(3)}{2\pi^4} \frac{g_\psi}{g_{*s}(T_f)}$$

For $s_0 = 2990\ \mathrm{cm}^{-3}$ $\Longrightarrow$ $\boxed{n_{\psi,0} \sim 830 \left(\frac{g_\psi}{g_{*s}(T_f)} \right) \mathrm{cm}^{-3}}$

This expressions are true, even if particles are non-relativistic today.

Today, $T_0 = 2.7\ K$; $1K = 8.6 \times 10^{-5} eV$

$T_0 \sim 2.3 \cdot 10^{-5} eV$. This is smaller than the heavier neutrino masses.

Considering $\rho_{\nu,0} = n_{\nu,0}\, m_\nu \longrightarrow \Omega_{\nu,0} = \frac{\rho_{\nu,0}}{\rho_c}$

$$\rho_c = 0.5 \times 10^4 \frac{eV}{cm^3} \Rightarrow \Omega_\nu = 15.6 \times 10^{-2} \frac{g_\nu}{g_{*,s}(T_f)} \frac{m_\nu}{eV}$$

Neutrinos decoupled at $T \sim 1 MeV$

$$g_\nu = 2 \times \frac{3}{4} = 1.5$$

$$g_{*s}(T \sim 1 MeV) = 2 + \frac{7}{8} \cdot \underbrace{(2+2)}_{e^- + e^+} + \frac{7}{8} \times \overbrace{2}^{\nu + \bar{\nu}} \times \overbrace{3}^{N_g} = 10.75$$

$$\boxed{\Omega_\nu = \frac{m_\nu}{45\, eV}}$$

$2K < m_\nu < 1 MeV$

(Decoupled while being relativistic)

Since experimentally $m_\nu < 0.1\ eV$, each neutrino species contributes with less than ~ 2 per mille of the Universe ρ. Neutrinos are not the observed Dark Matter.

These relic neutrinos must be there and have never been observed !!

Cold Relics

The case of cold relics is a little bit more complicated, since the equilibrium density depends exponentially on T

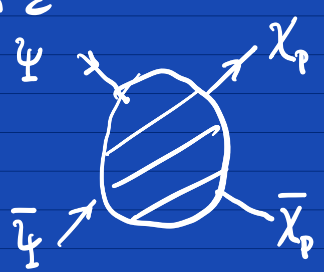
$$n_\psi^{Eq} = g_\psi \left(\frac{m_\psi T}{2\pi} \right)^{3/2} \exp\left[-m_\psi/T\right]$$

If the particle would suffer no interactions the number density would be such that

$$n a^3 = \text{constant} \Rightarrow a^3 \frac{dn}{dt} + 3a^2 n \frac{da}{dt} = 0$$

$$\frac{dn}{dt} + 3Hn = 0 \quad ; \quad H = \frac{\dot{a}}{a}$$

But the particle before decoupling is interacting with other particles in the plasma

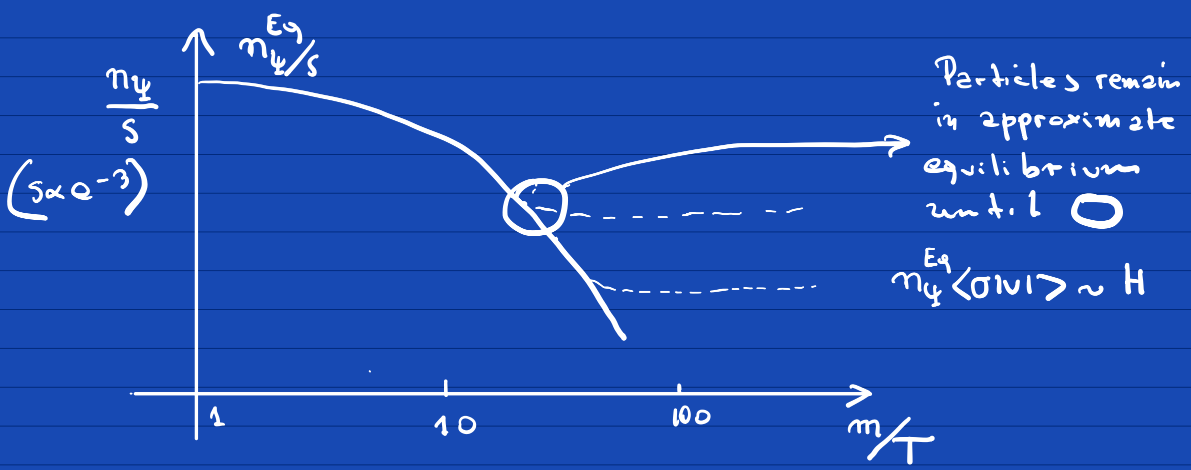


Number of particles affected by annihilation and inverse annihilation processes.

The number density may be obtained by solving the so-called Boltzmann equation

$$\frac{dn_\psi}{dt} + 3H\,n_\psi = -\left\langle \sigma_{\psi\bar\psi\to x\bar x}\cdot|v|\right\rangle\left(n_\psi^2 - \left(n_\psi^{Eq}\right)^2\right)$$

Here, we assume that the particle is not precisely in equilibrium, but is driven towards equilibrium through interactions with the plasma. Observe that if $H \ll n_\psi \langle \sigma_{\psi\bar\psi\to x\bar x}\cdot|v|\rangle$, particles will be driven to equilibrium. On the contrary, when $H \gg n_\psi\langle\sigma\cdot|v|\rangle$, the number density would just evolve, with $n_\psi a^3 = \text{const.}$



$$\frac{d(n_\psi/s)}{dt} = \frac{1}{s}\frac{dn_\psi}{dt} - \frac{1}{s^2}\,n_\psi\cdot\overbrace{\frac{ds}{dt}}^{-3Hs}$$

$$= -\frac{1}{s}\left\langle\sigma\cdot|v|\right\rangle\left(n_\psi^2 - \left(n_\psi^{Eq}\right)^2\right)$$

$$t = 0.3\,g_*^{-1/2}\,M_{PL}/T^2 = 0.3\,g_*^{-1/2}\,x^2\,M_{PL}/m^2;\quad x = \frac{m}{T}$$

$$dt = 0.3\,g_*^{-1/2}\,M_{PL}/m^2\cdot 2x\cdot dx = \frac{1}{H(m)}\,x\cdot dx$$

$$\boxed{\frac{dY}{dx} = -\frac{x\,s\,\langle\sigma\cdot|v|\rangle}{H(m)}\left(Y^2 - Y_{Eq}^2\right)}\qquad Y = \frac{n_\psi}{s}$$

Let us estimate $n_\psi(T_f)/s$; $x_f = \frac{m}{T_f}$

$$Y(T_f) = \frac{45}{2\pi^2 (2\pi)^{3/2}} \frac{g_\psi}{g_{*s}(T_f)} x_f^{3/2} e^{-x_f}$$

Value of $n_\psi(T_f)$ determined by the condition

$$n_\psi \langle \sigma |v| \rangle \sim H \sim 1.66\, g_*^{1/2} \frac{T_f^2}{M_{PL}}$$

Now, assume that

$$\langle \sigma |v| \rangle \propto \sigma_0\, x^{-n} \begin{cases} n=0 & \text{s-wave} \\ n=1 & \text{p-wave} \end{cases}$$

$$1.66\, g_*^{1/2}\, x_f^{-2}\, m_\psi^2 = \frac{\sigma_0 x_f^{-n}}{(2\pi)^{3/2}} \left[g_\psi\, m_\psi^3\, x_f^{-3/2}\, e^{-x_f} \right]$$

$$\boxed{e^{x_f} = 3.8\; 10^{-2} \left(\frac{g_\psi}{\sqrt{g_*}} \right) m_\psi M_{PL} \sigma_0\, x_f^{-n+1/2}}$$

It is clear from here that due to the appearance of M_{PL}, the solution is $x_f \gg 1$ in which case the exponential factor grows $\boxed{e^{x_f} = A\, x_f^{-n+1/2}}$, $A \gg 1$

$$\boxed{x_f \sim \ln(A) + \left(\tfrac{1}{2} - n\right) \ln\left[\underbrace{\ln(A)}_{\sim x_f} \right]}$$

From here I can determine x_f, which for a weak scale $\sigma_0 \sim \frac{G_F^2 m^2}{4\pi}$ gives values of

$$x_f \sim \ln\left[10^{-2} \times 10^{2} \times 10^{-7} \times 10^{19}\right] \sim 27$$

So, for masses of order 100 GeV, $T_f \sim$ few GeV

Coming back to $\frac{n_\psi}{s} \propto x_f^{3/2} e^{-x_f}$

$$x_f^{3/2} e^{-x_f} = \frac{1}{3.8 \times 10^{-2}} \frac{g_*^{1/2}}{g_\psi} \frac{x_f^{n+1}}{m_\psi M_{Pl} \sigma_0}$$

From here one can determine

$$\frac{n_\psi(T_f)}{s(T_f)} = \frac{n_{\psi,0}}{s_0}$$

$\rho_\psi = n_\psi . m_\psi$ becomes dependent on m_ψ only through x_f, and taking $\rho_c = 0.5 \times 10^{-5} \frac{\text{GeV}}{\text{cm}^3}$ we compute Ω_ψ

$$Y(T_f) \sim \frac{45}{2\pi^2 (2\pi)^{3/2}} \frac{g_\psi}{g_{*s}(T_f)} \cdot \frac{1}{3.8 \times 10^{-2}} \frac{g_*^{1/2}}{g_\psi} \cdot \frac{x_f^{n+1}}{m_\psi M_{Pl} \sigma_0}$$

$$\sim \frac{5 \, . \, x_f^{n+1}}{(g_{*,s}/g_*^{1/2})\, m_\psi m_{Pl} \sigma_0}$$

$$\Omega_\psi \sim \frac{1.3 \times 10^{9} \, x_f^{n+1} \, \text{GeV}^{-1}}{(g_{*s}/g_*^{1/2})\, M_{PL} \cdot \sigma_0}$$

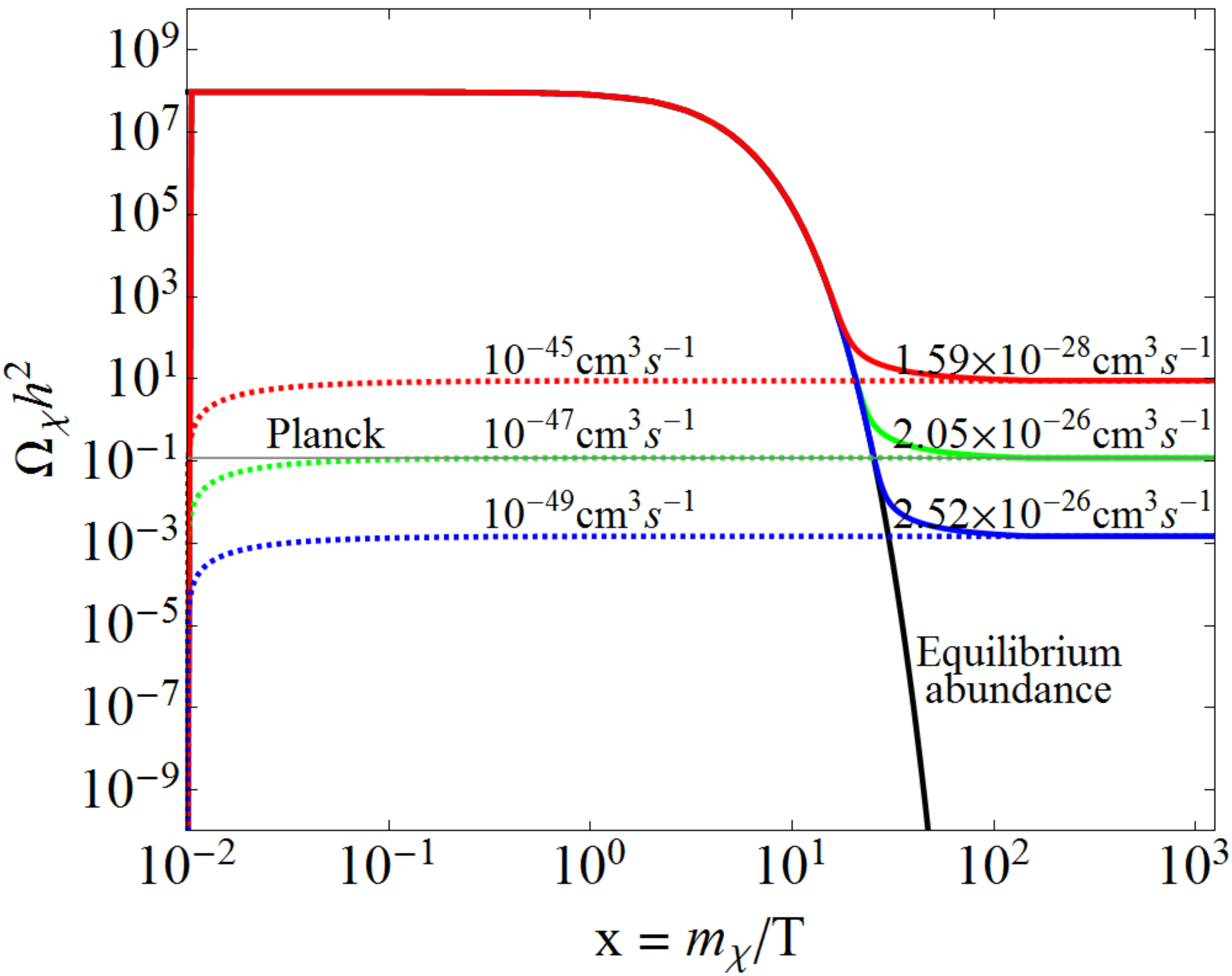
Let me mention that a more precise treatment of the Boltzmann equation leads to the replacement of the factor 5 in $Y(T_f)$ by $3.79(n+1)$ and hence,

$$\boxed{\Omega_\psi \sim \frac{1.07 \times 10^9 \, (n+1) \, x_f^{n+1} \, \mathrm{GeV}^{-1}}{\left(g_{*s}/g_*^{1/2}\right) \, M_{PL} \cdot \sigma_0}}$$

Also, the subleading coefficient in the determination of x_f changes from $\left(\frac{1}{2} - n\right)$ to $\left(-\frac{1}{2} - n\right)$. The overall behaviour is unchanged,

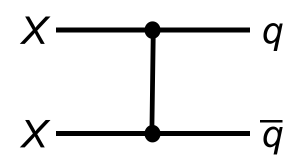
As we said before, these cold relics may be a candidate for the observed DARK MATTER. A simple exercise is to ask what would be the σ_0 that would lead to $\Omega_\psi \sim 0.25$ for $n=0$ and $x_f \sim 25$. One obtains $\sigma_0 \sim 10^{-8} \, \mathrm{GeV}^{-2}$ implying that a weak scale mass particle with weak scale cross sections leads to the proper DARK MATTER DENSITY.

Dark Matter as a Big Bang Relic



Kolb and Turner
The Early Universe

$$\Omega_X \propto \frac{1}{\langle \sigma v \rangle} \sim \frac{m_X^2}{g_X^4}$$

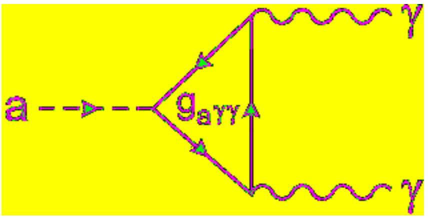


$m_X \sim 100$ GeV, $g_X \sim 0.6 \rightarrow \Omega_X \sim 0.1$

Weak scale size masses and couplings roughly consistent with ΩDM

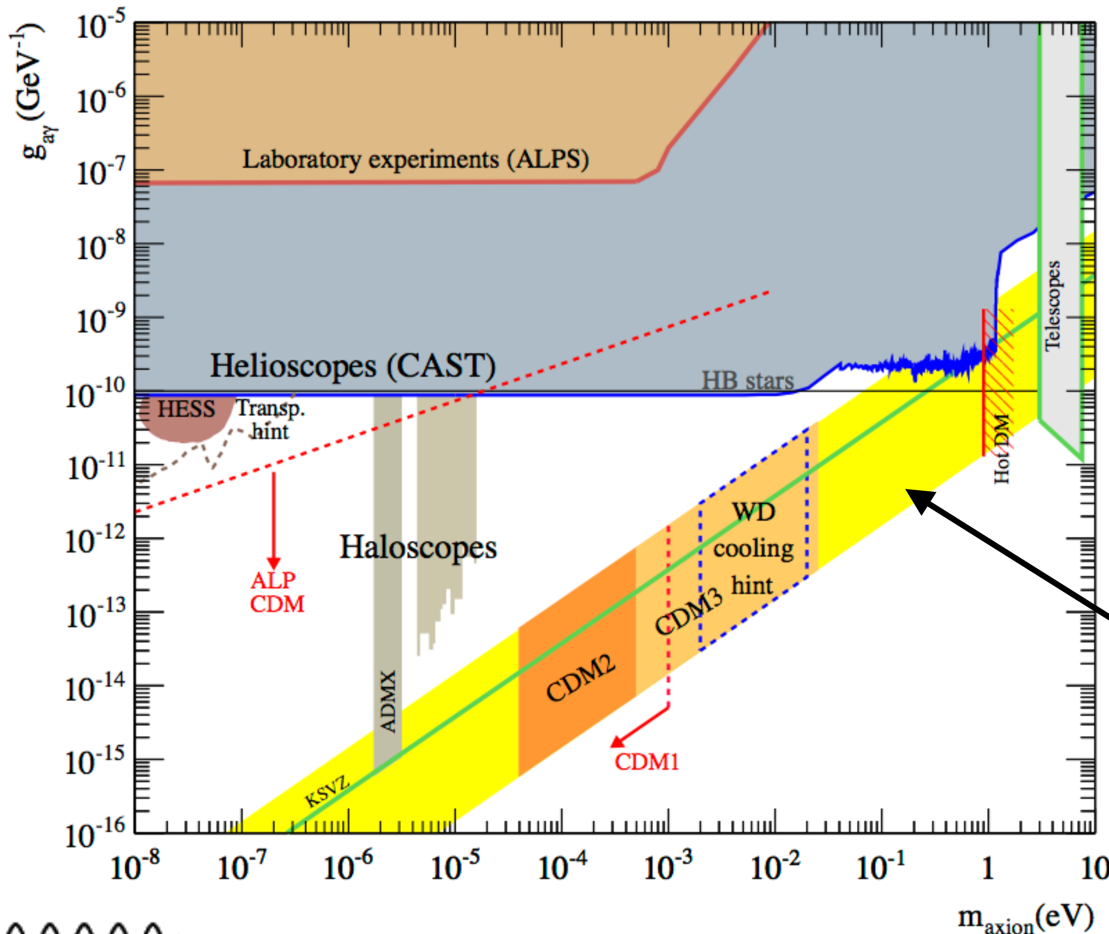
WIMPS

Standard Solution : Promote θ to be a field, a (axion), whose v.e.v is zero

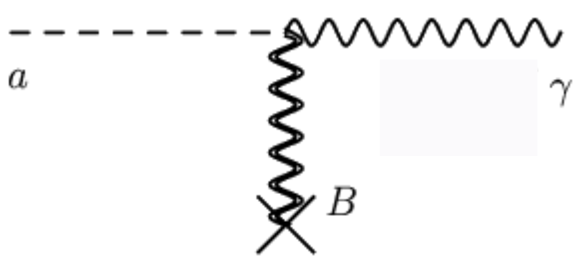


Axions : Solve the strong CP Problem
They are also a good CDM candidate

Axions produced in solar core (conversion to X Rays) : J. Collar



QCD Axion



Hallo Axions : Resonant Magnetic Cavity Searches