# Introduction to Cosmology and Dark Matter

## Lecture III


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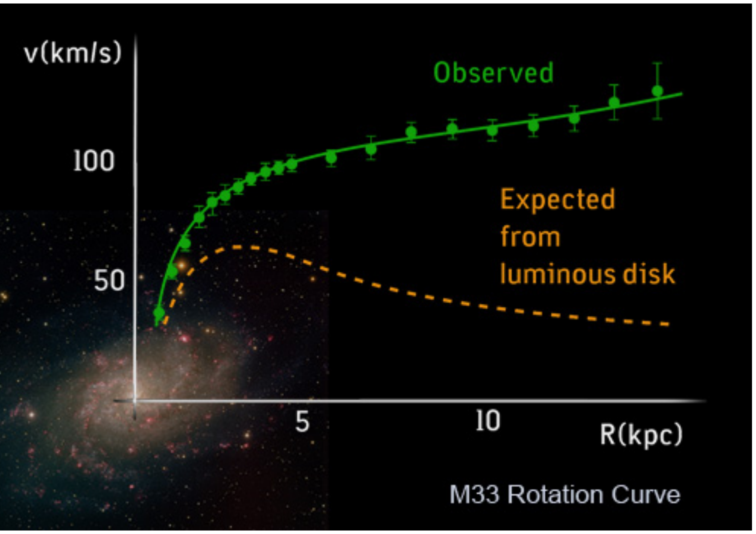
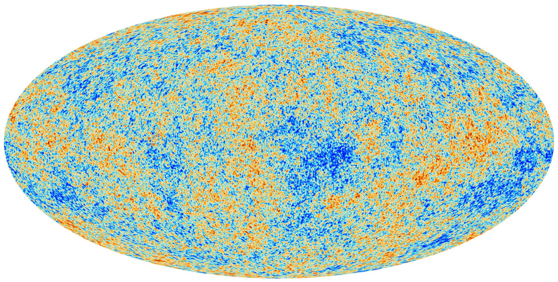
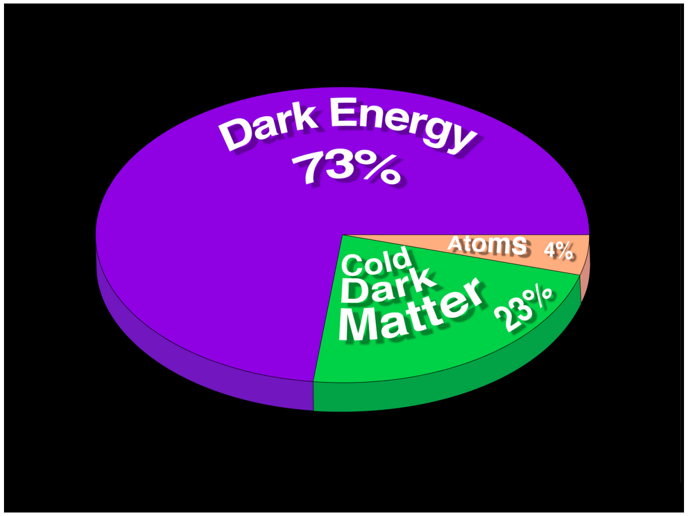





TRISEP Lectures, TRIUMF, June 25, 2025

# Short Review From Lecture II

This expressions are true, even if particles are non-relativistic today.

Today, $T_0 = 2.7\ K$ ; $1 K = 8.6 \times 10^{-5} eV$

$T_0 \sim 2.3 \cdot 10^{-5} eV$. This is smaller than the heavier neutrino masses.

Considering $\rho_{\nu,0} = n_{\nu,0}\, m_\nu \longrightarrow \Omega_{\nu,0} = \frac{\rho_{\nu,0}}{\rho_c}$

$$\rho_c = 0.5 \times 10^4 \frac{eV}{cm^3} \Rightarrow \Omega_\nu = 15.6 \times 10^{-2} \frac{g_\nu}{g_{*,s}(T_f)} \frac{m_\nu}{eV}$$

Neutrinos decoupled at $T \sim 1 MeV$

$$g_\nu = 2 \times \frac{3}{4} = 1.5$$

$$g_{*s}(T \sim 1 MeV) = 2 + \frac{7}{8} \cdot \underbrace{(2+2)}_{e^- + e^+} + \frac{7}{8} \times \overbrace{2}^{\nu + \bar{\nu}} \times \overbrace{3}^{N_g} = 10.75$$

$$\boxed{\Omega_\nu = \frac{m_\nu}{45\, eV}}$$

$2K < m_\nu < 1 MeV$
(Decoupled while being relativistic)

Since experimentally $m_\nu < 0.1\ eV$, each neutrino species contributes with less than $\sim 2$ per mille of the Universe $\rho$. Neutrinos are not the observed Dark Matter.

These relic neutrinos must be there and have never been observed !!

# Evolution of $g_*$ with T

When all particles of the SM were relativistic $T > 100\,GeV$

$$g_* = 2 \times 15 \times 3 + 2 \times 4 + 4 + 2 \times 8 = 118$$

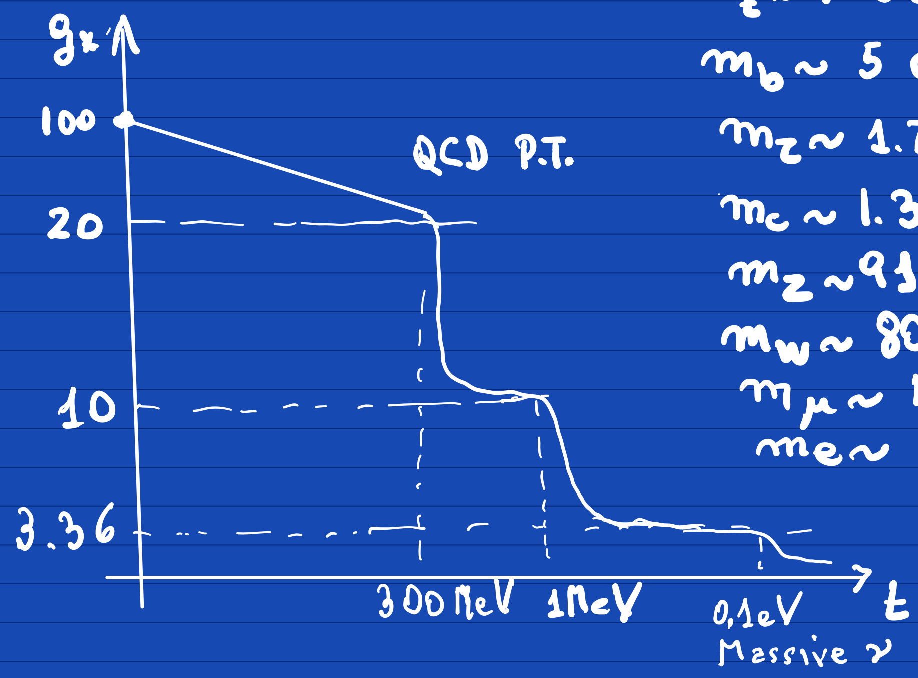
15 : $\nu_L + e_L + e_R + \underbrace{3}_{N_c} u_L + 3u_R + 3d_L + 3d_R$

2 number of d.o.f per helicity.

3 : Number of generations

2×4 : EW Gauge Bosons
4 : Higgs

2×8 : Gluons.




$m_t \sim 175\,GeV$
$m_b \sim 5\,GeV$
$m_\tau \sim 1.77\,GeV$
$m_c \sim 1.3\,GeV$
$m_Z \sim 91\,GeV$
$m_W \sim 80\,GeV$
$m_\mu \sim 105\,MeV$
$m_e \sim 511\,KeV$

# Cold Relics

The case of cold relics is a little bit more complicated, since the equilibrium density depends exponentially on T

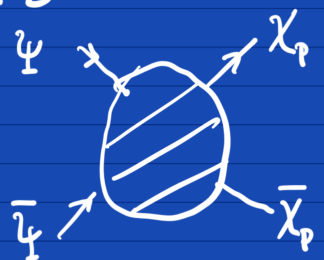
$$n_\psi^{Eq} = g_\psi \left(\frac{m_\psi T}{2\pi}\right)^{3/2} \exp\left[-m_\psi/T\right]$$

If the particle would suffer no interactions the number density would be such that

$$n a^3 = \text{constant} \Rightarrow a^3 \frac{dn}{dt} + 3a^2 n \frac{da}{dt} = 0$$

$$\frac{dn}{dt} + 3Hn = 0 \quad ; \quad H = \frac{\dot{a}}{a}$$

But the particle before decoupling is interacting with other particles in the plasma




Number of particles affected by annihilation and inverse annihilation processes.

The number density may be obtained by solving the so-called Boltzmann equation

Let me mention that a more precise treatment of the Boltzmann equation leads to the replacement of the factor 5 in $Y(T_f)$ by $3.79(n+1)$ and hence,

$$\Omega_\psi \sim \frac{1.07 \times 10^9 (n+1)\, x_f^{n+1}\, \mathrm{GeV}^{-1}}{\left(g_{*s}/g_*^{1/2}\right) M_{Pl} \cdot \sigma_0}$$

Also, the subleading coefficient in the determination of $x_f$ changes from $\left(\frac{1}{2} - n\right)$ to $\left(-\frac{1}{2} - n\right)$. The overall behaviour is unchanged.

As we said before, these cold relics ma be a candidate for the observed DARK MATTER. A simple exercise is to ask what would be the $\sigma_0$ that would lead to $\Omega_\psi \sim 0.25$ for $n=0$ and $x_f \sim 25$. One obtains $\sigma_0 \sim 10^{-8}\,\mathrm{GeV}^{-2}$ implying that a weak scale mass particle with weak scale cross sections leads to the proper DARK MATTER DENSITY.

End of Review

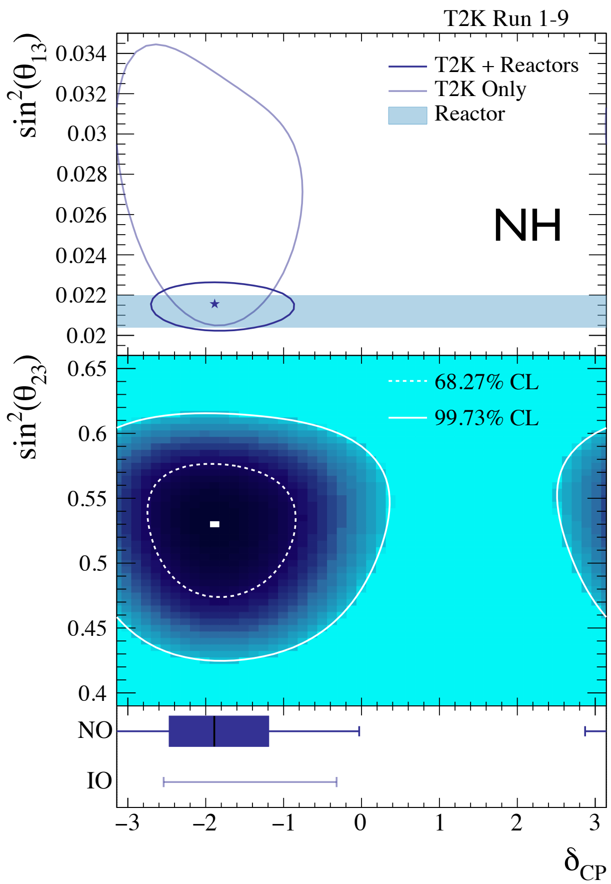
# Open Question VI
# Is CP violated in the neutrino sector ?

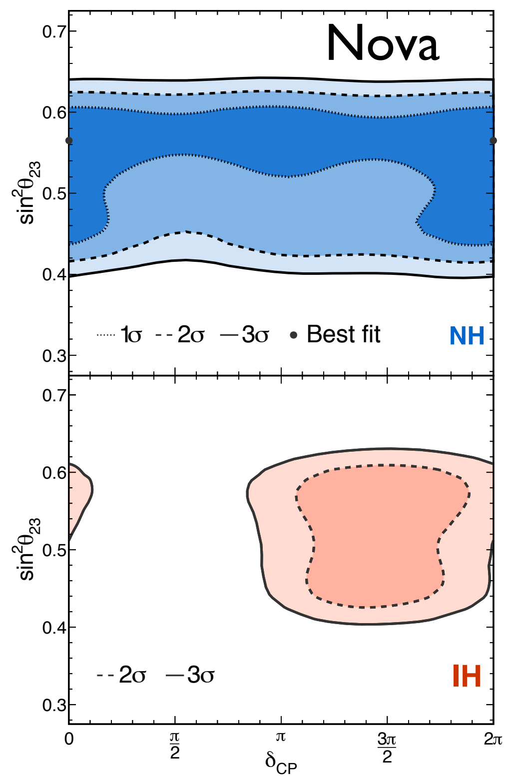
Best test : $\nu_\mu \to \nu_e$ oscillations.

$$P_{\mu e} = 4c_{13}^2 s_{13}^2 s_{23}^2 \sin^2 \Delta_{31} + 4c_{13}^2 c_{23}^2 s_{12}^2 c_{12}^2 \sin^2 \Delta_{21}$$
$$+ 8c_{13}^2 c_{12} c_{23} s_{12} s_{13} s_{23} \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} + \delta_{13}).$$

$$\Delta_{ij} = \frac{(m_i^2 - m_j^2)L}{4E}$$

## Hints of sizable CP-violation




C.W. rule

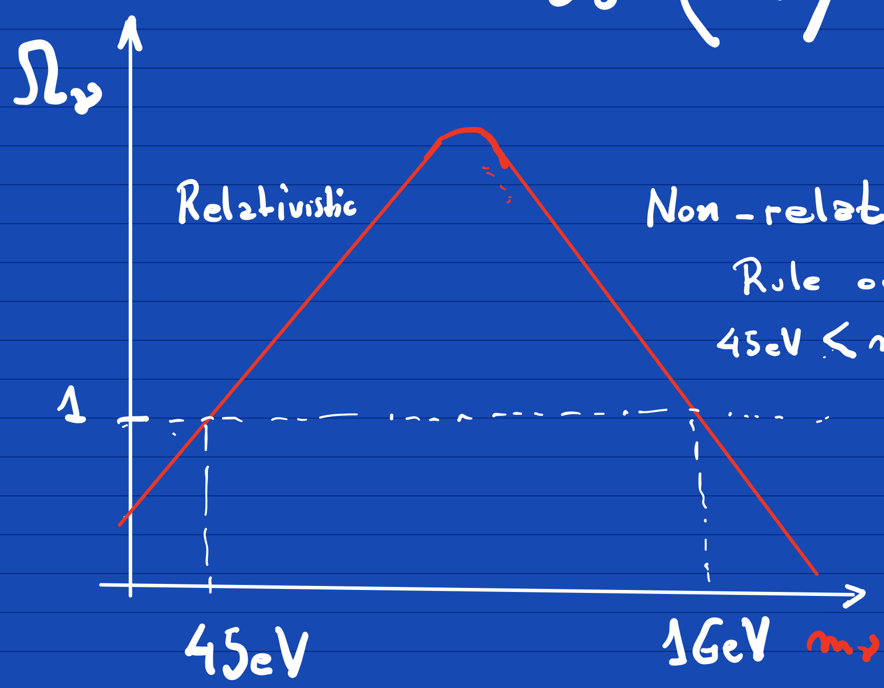
$\theta_{12} \sim 34^o$

$\theta_{23} \sim 45^o$

$\theta_{13} \sim 9^o$

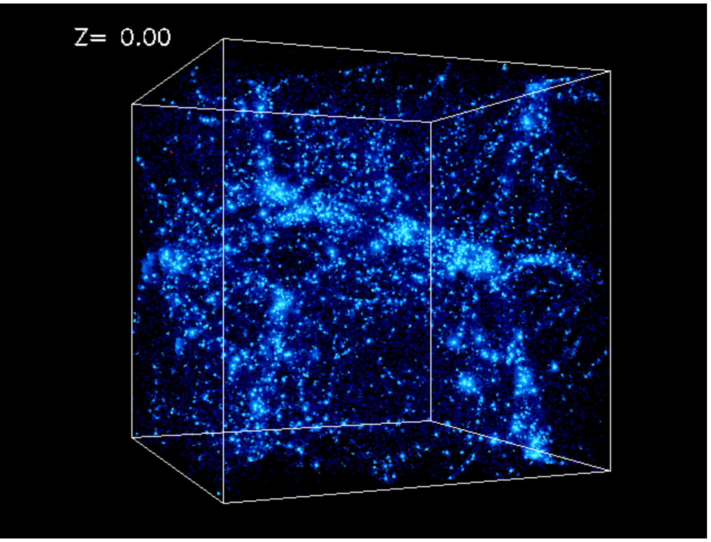
Let us emphasize that, for weakly interacting particles with $\langle \sigma |v| \rangle \sim \frac{G_F m^2}{4\pi}$, small masses imply smaller cross sections and if one computes $\Omega$ as before one obtains a value of $\Omega > 1$, what is of course unacceptable. Therefore, this particles must be heavier than about 1 GeV to be good DM candidates. This bound applies to "heavy neutrinos" or Higgsinos in supersymmetry.

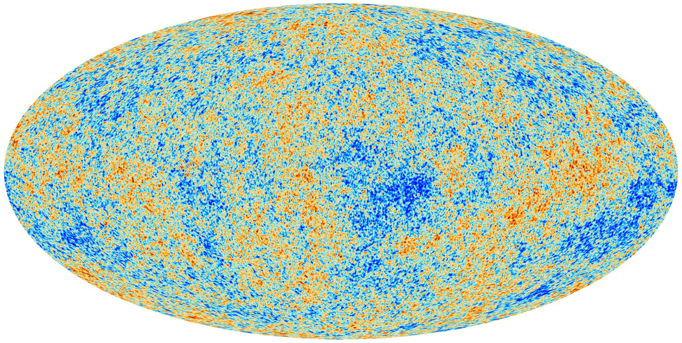
Just from cosmology $(\Omega_\nu)$

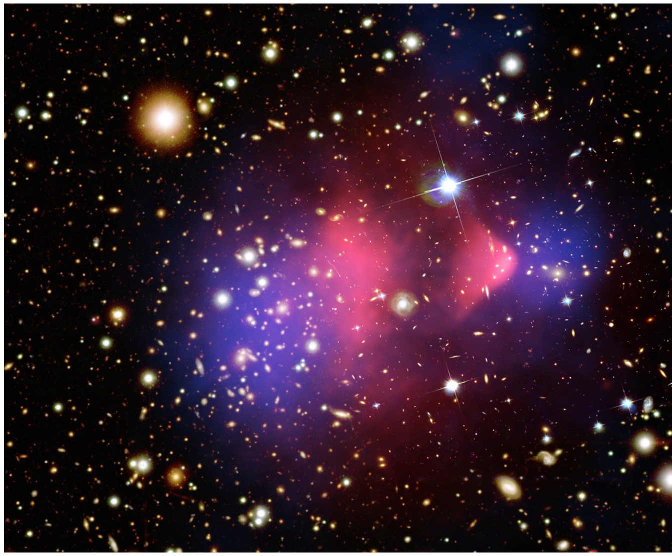


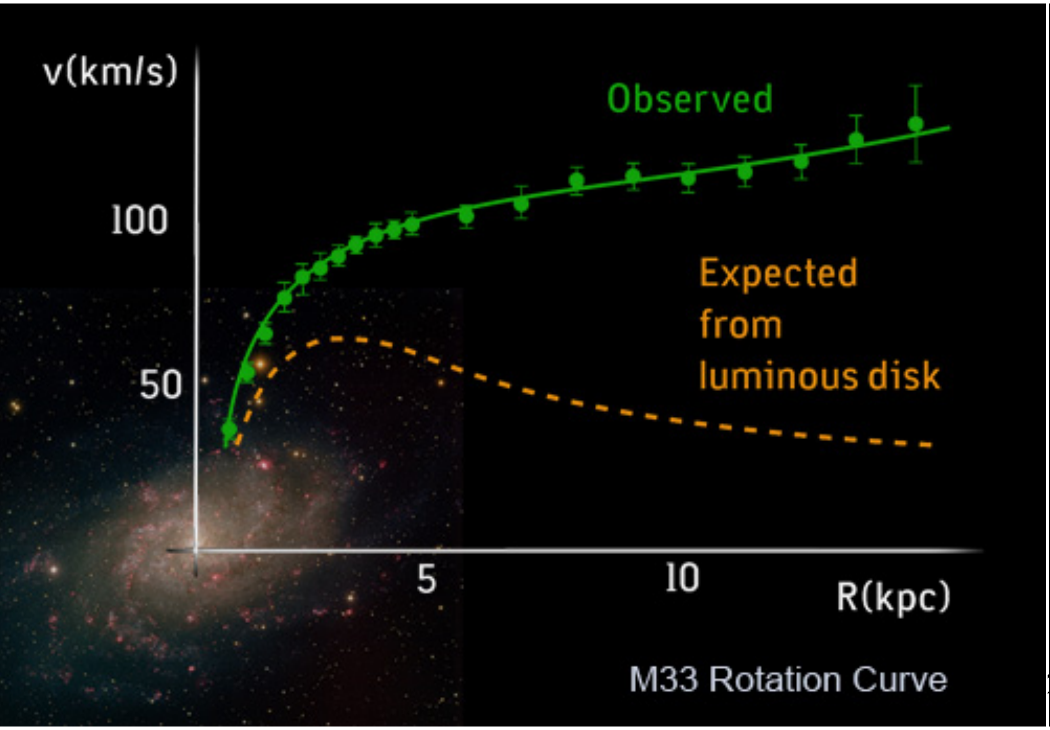
# What is the Dark Matter ?

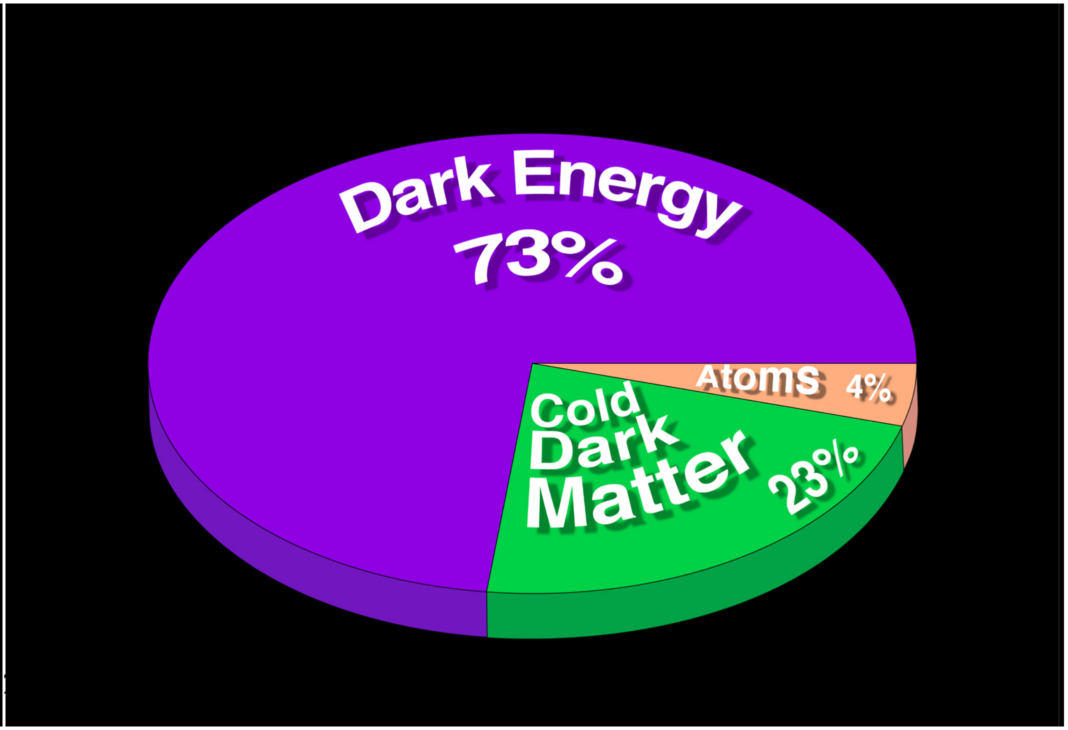
## Existence of Dark Matter Supported by overwhelming indirect evidence





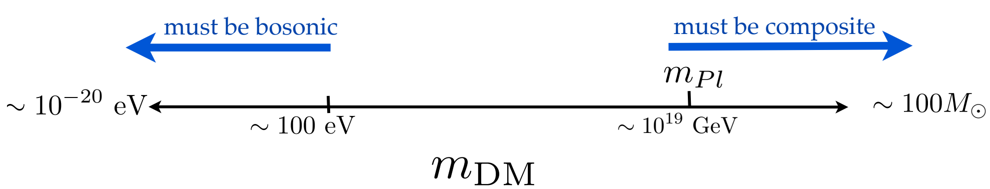


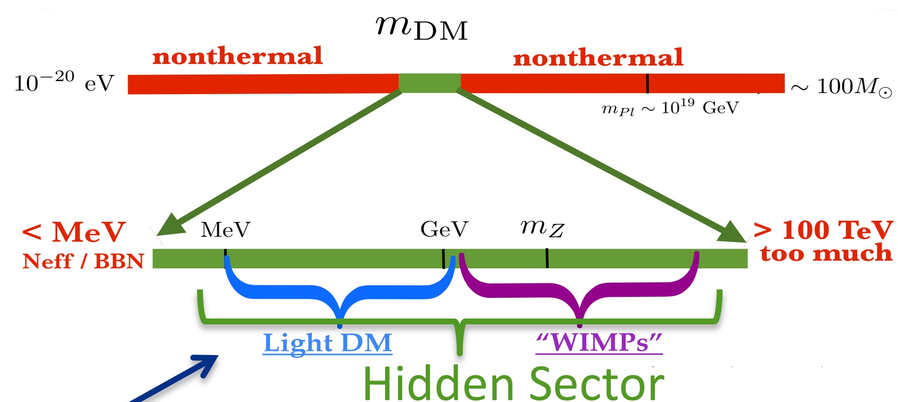
# What do we know about Dark Matter ?

**- very little -**

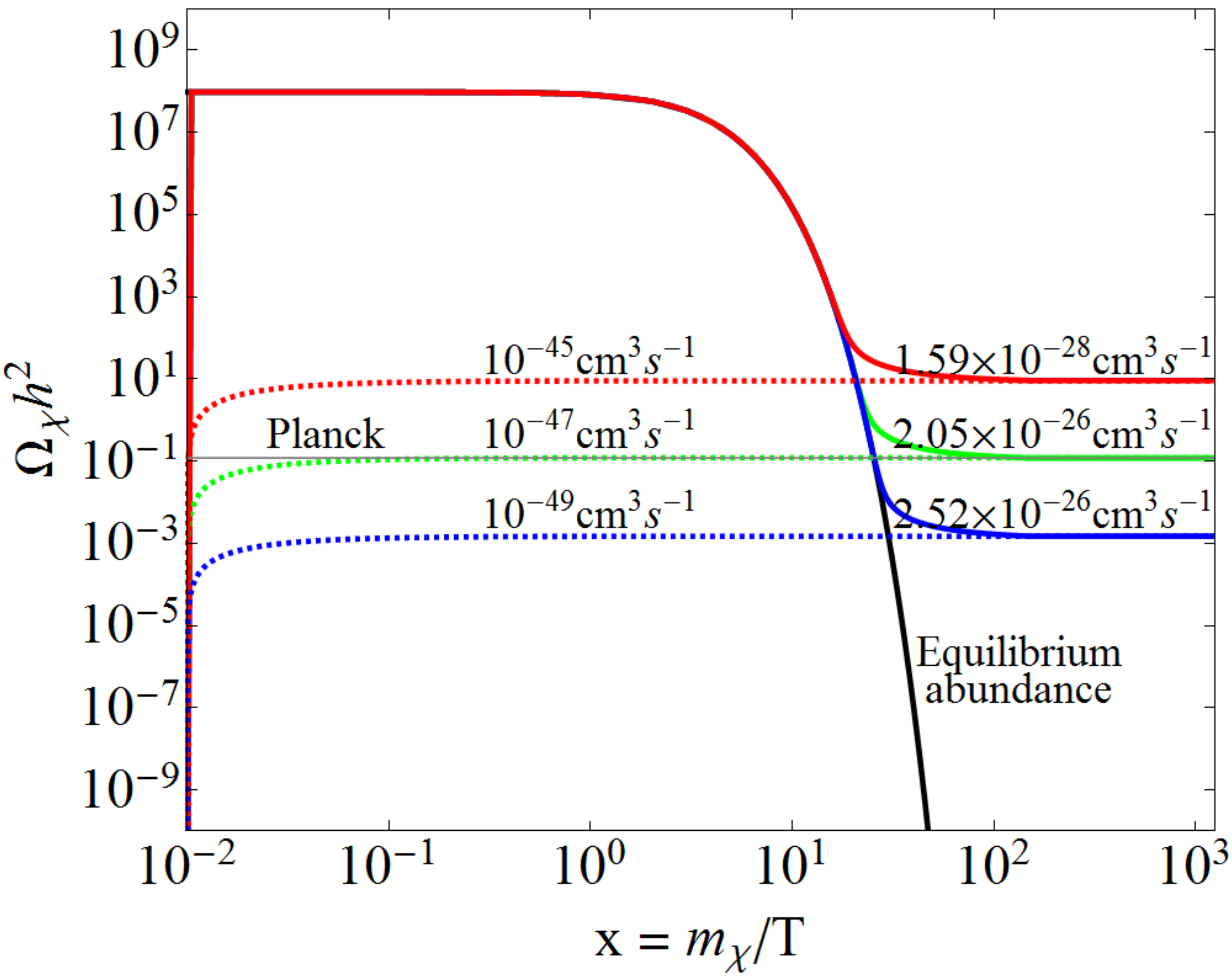
- Couples gravitationally
- It is the most abundant form of matter
- It can be part of a larger invisible/dark sector with new dark forces
- It must be made of something different that all the particles we know, it can be made of particles or compact objects, or better described as wavelike disturbances
- Its mass can be anything from as light as $10^{-22}$ eV to as heavy as primordial black holes of tens of solar masses




Folding in assumptions about early Universe cosmology can provide some guidance

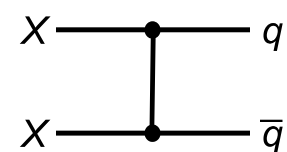


# Dark Matter as a Big Bang Relic




Kolb and Turner
The Early Universe

$$\Omega_X \propto \frac{1}{\langle \sigma v \rangle} \sim \frac{m_X^2}{g_X^4}$$




$m_X$ ~ 100 GeV, $g_X$ ~ 0.6 → $\Omega_X$ ~ 0.1

Weak scale size masses and couplings roughly consistent with ΩDM

WIMPS

# Preservation of R-Parity:

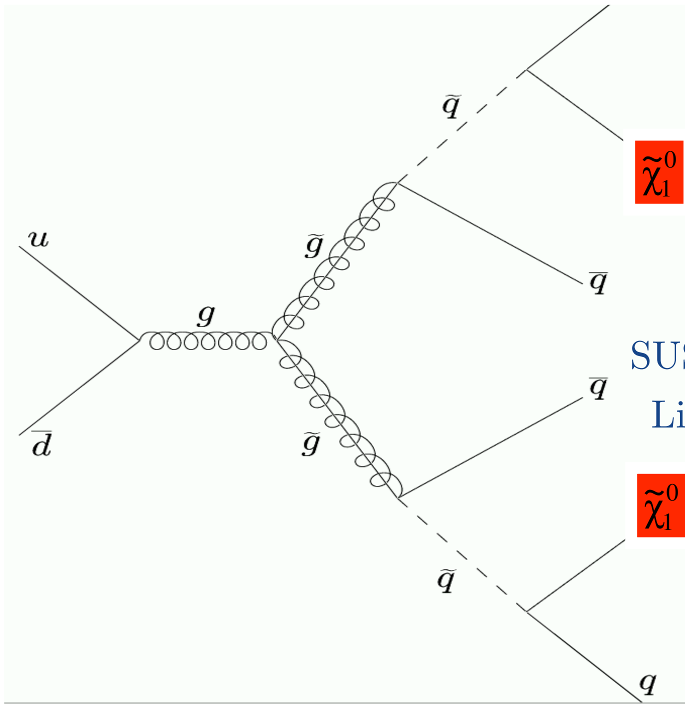
# Supersymmetry at colliders

## Gluino production and decay: Missing Energy Signature

*Supersymmetric Particles tend to be heavier if they carry color charges.*

*Particles with large Yukawas tend to be lighter.*

*Charge-less particles tend to be the lightest ones.*




SUSY Particles $R_P = -1$

SM Particles $R_P = 1$

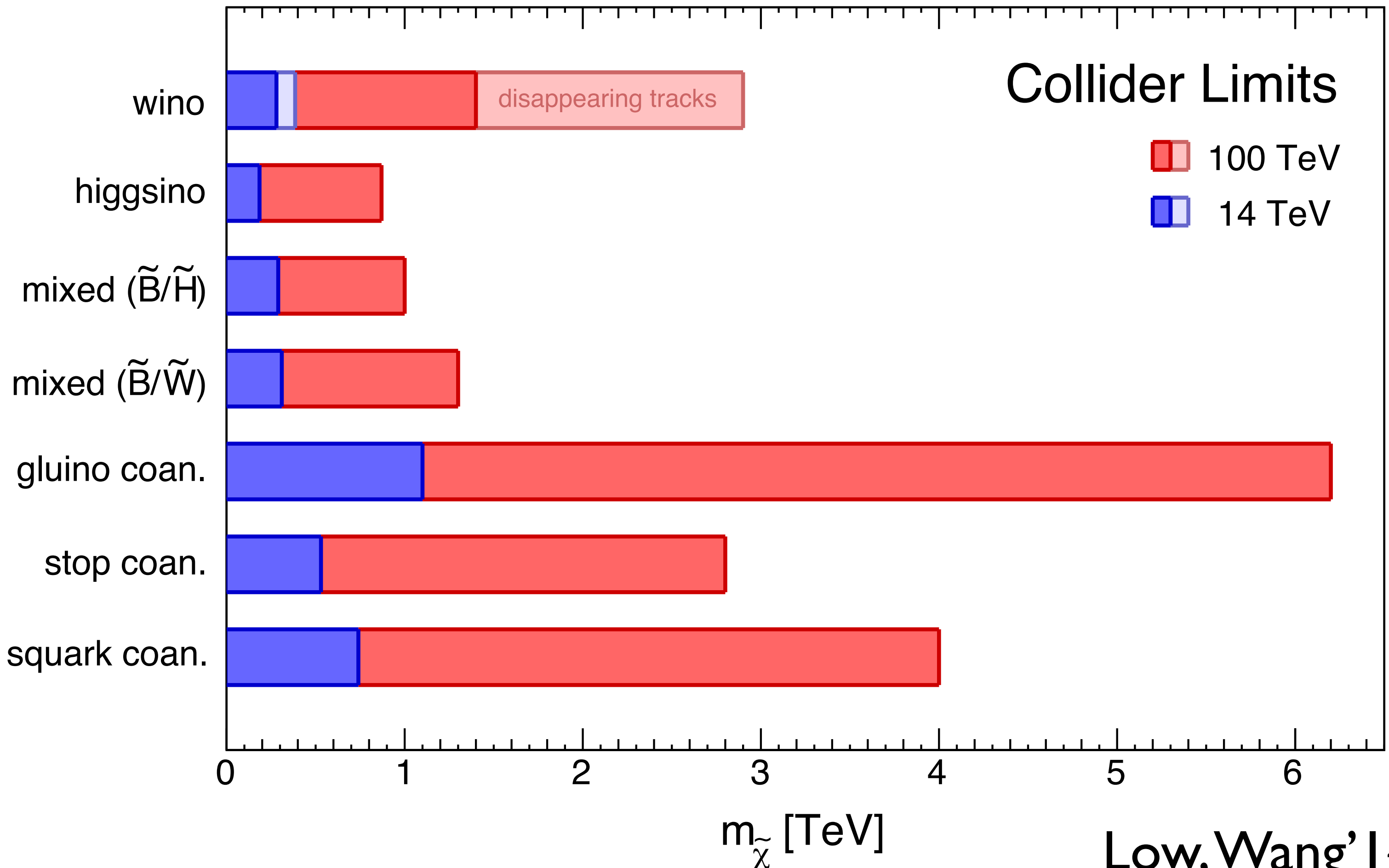
SUSY Particles produced in Pairs

Lightest SUSY Particle is Stable

➢ **Lightest supersymmetric particle = Excellent Cold dark matter candidate.**

# Dark Matter in SUSY Theories is a neutral partner of either the Higgs or Gauge Bosons

## Future Colliders : Direct Production Limits

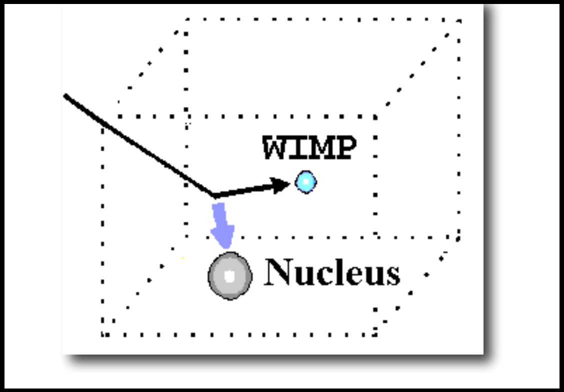


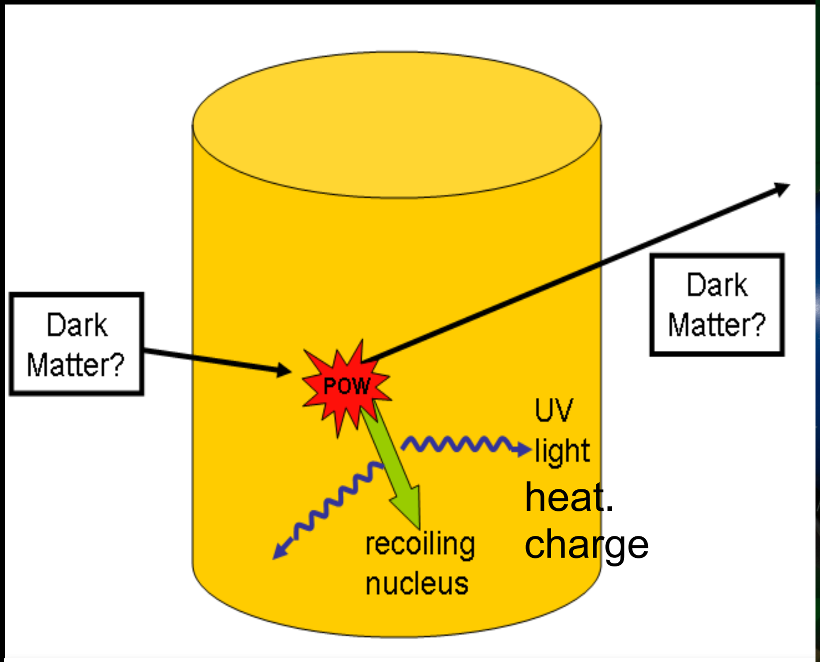

Low, Wang'14

100 TeV collider will probe most promising regions

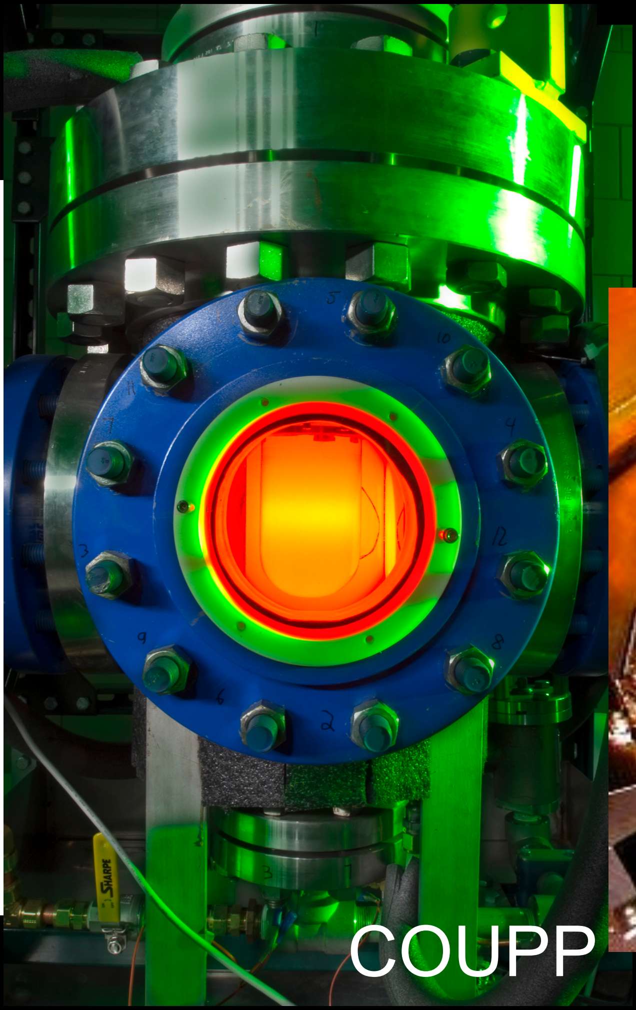
# Dark Matter Search in Direct Detection Experiments

It can *collide with a single nucleus in your detector*




Underground detector





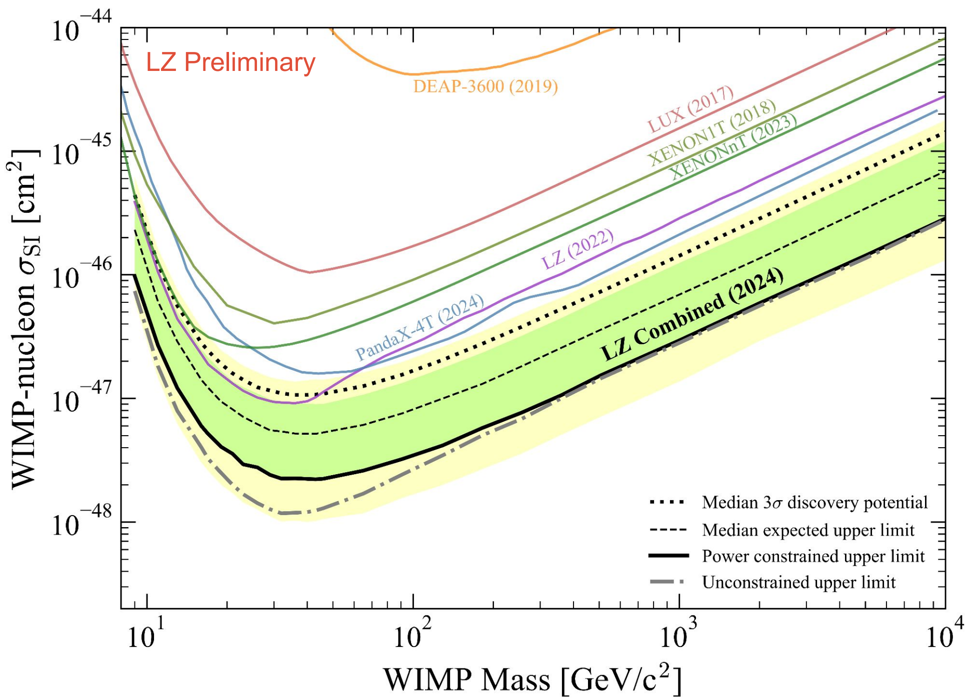
XENON, LUX

also GoGeNT
DAMIC
DarkSide

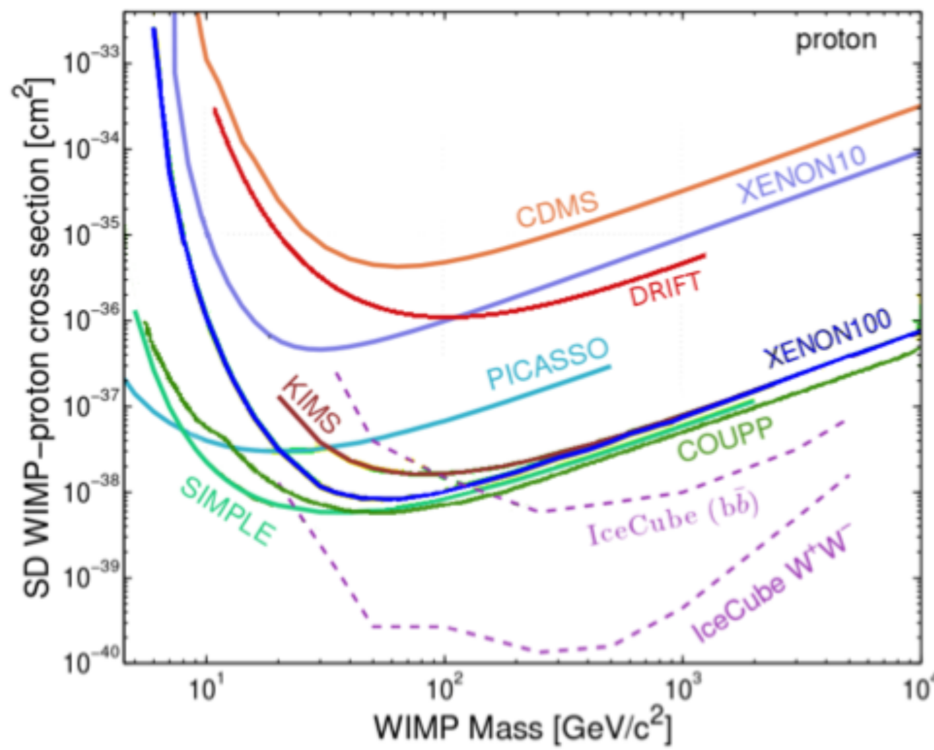
# Current Bounds from Direct Dark Matter Detection

Current Limits

$$1 \text{ pb} = 10^{-36} \text{ cm}^2, \qquad 1 \text{ zb} = 10^{-45} \text{ cm}^2$$




Spin Independent Interactions

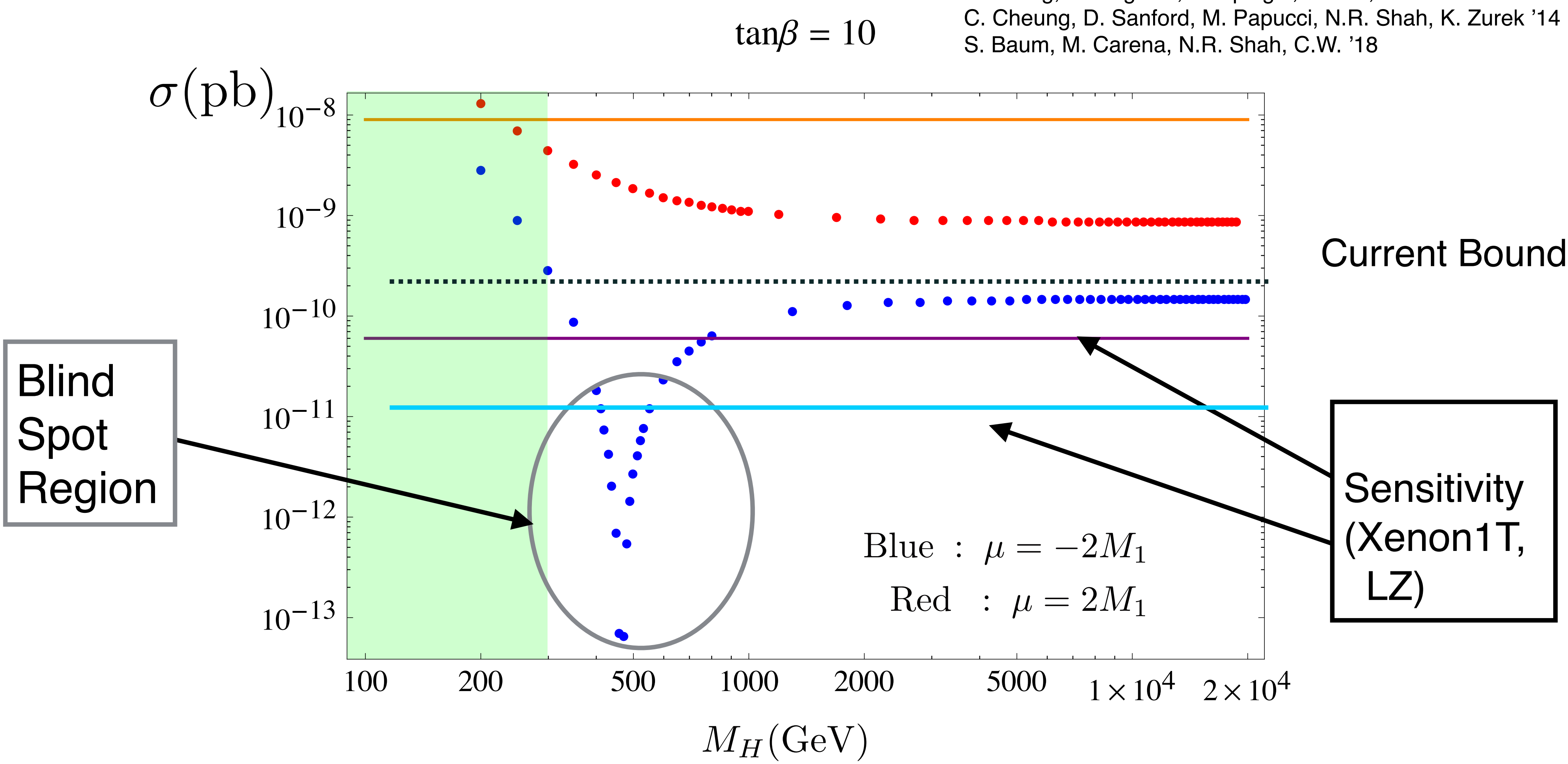



Spin Dependent Interactions

# Dependence of the cross section on the heavy Higgs mass

Blind Spots : $$2\left(m_{\chi^0} + \mu\sin 2\beta\right)\frac{1}{m_h^2} = -\mu\tan\beta\frac{1}{M_H^2}$$

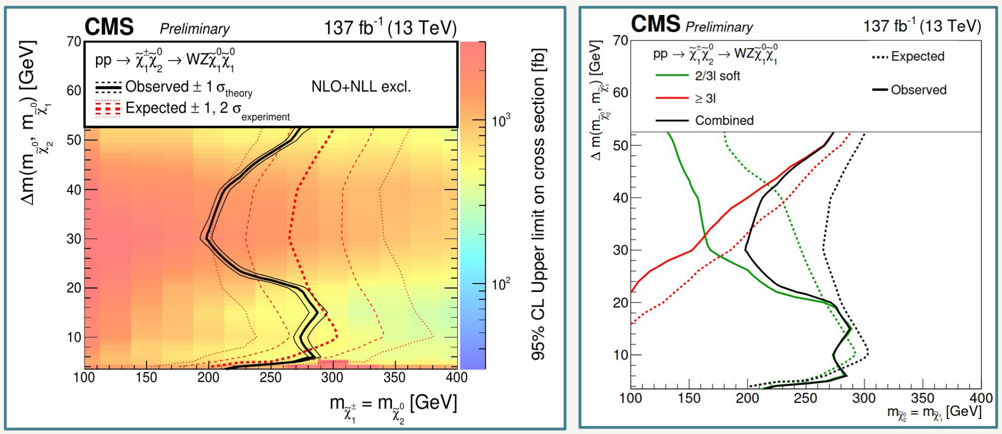
P. Huang, C.W.'14
P. Huang, R. Roglans, D. Spiegel, Y. Sun, C.W.'17
C. Cheung, D. Sanford, M. Papucci, N.R. Shah, K. Zurek '14
S. Baum, M. Carena, N.R. Shah, C.W. '18

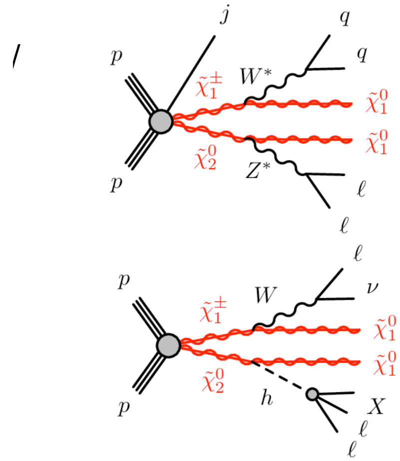


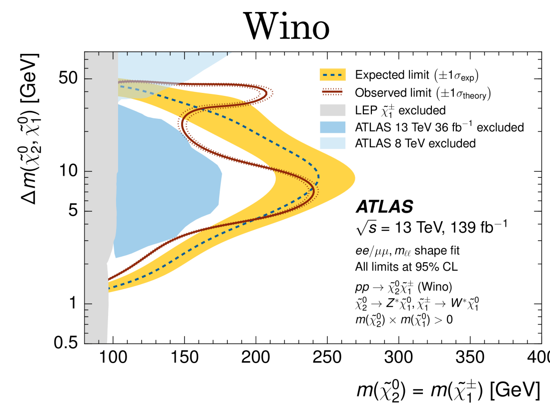

$m_{\chi^\pm} = |\mu|$, $m_\chi^0 = M_1$, $M_1 = 200$ GeV

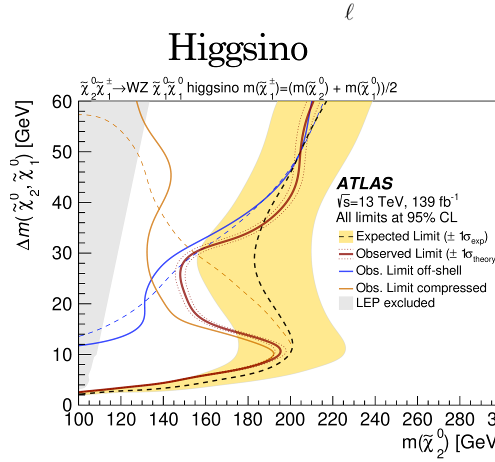
# There may be surprises, like in collider searches

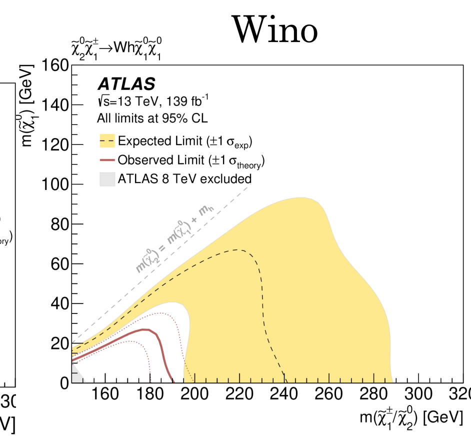
- The 2/3l soft and ≥3l analyses complement each other in the compressed region
  - Orthogonal lepton $p_T$ ranges but different selections (e.g. MET for 2/3l soft)
    → Challenging to be fully optimal in the crossover regime





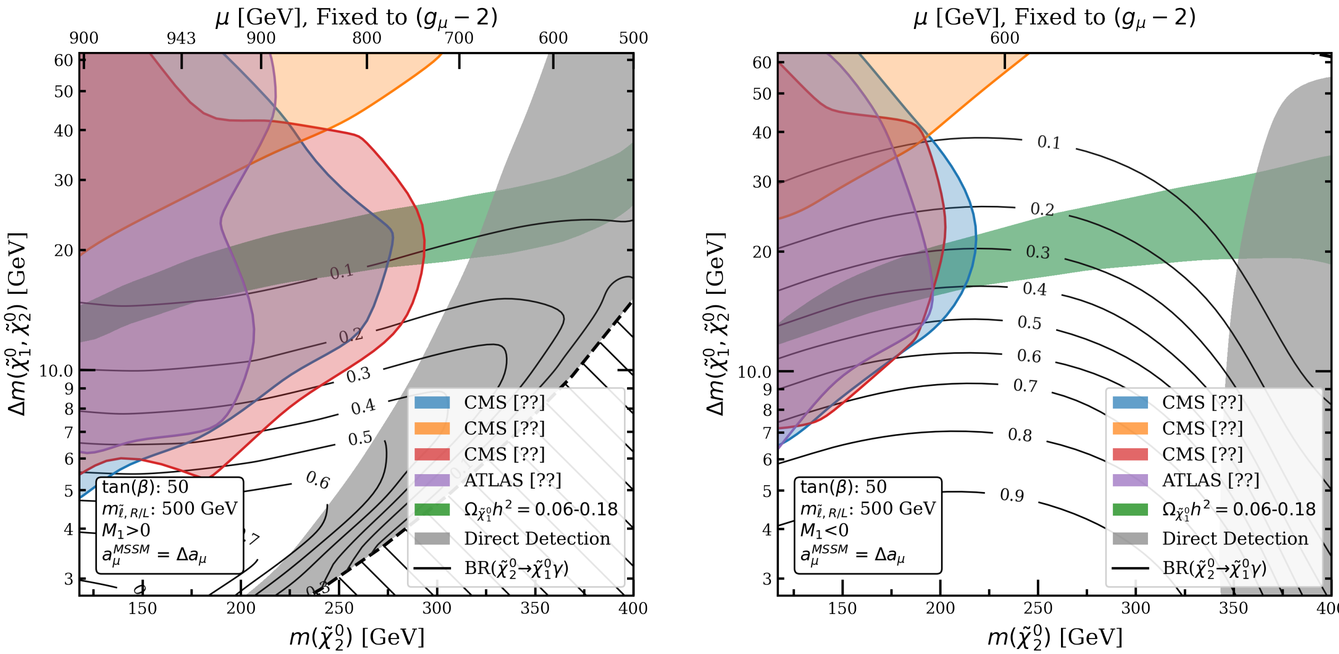



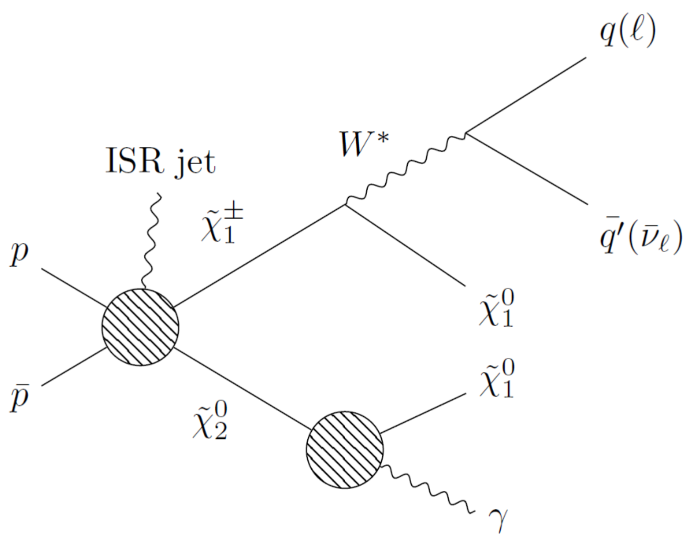

**Excesses in regions consistent with co-annihilating Dark Matter**

# Same region of Parameters

S. Baum, M. Carena, N. Shah, C. Wagner'21
D. Rocha, T. Ou, 2305.02354,
S. Roy, C.W., 2401.08917



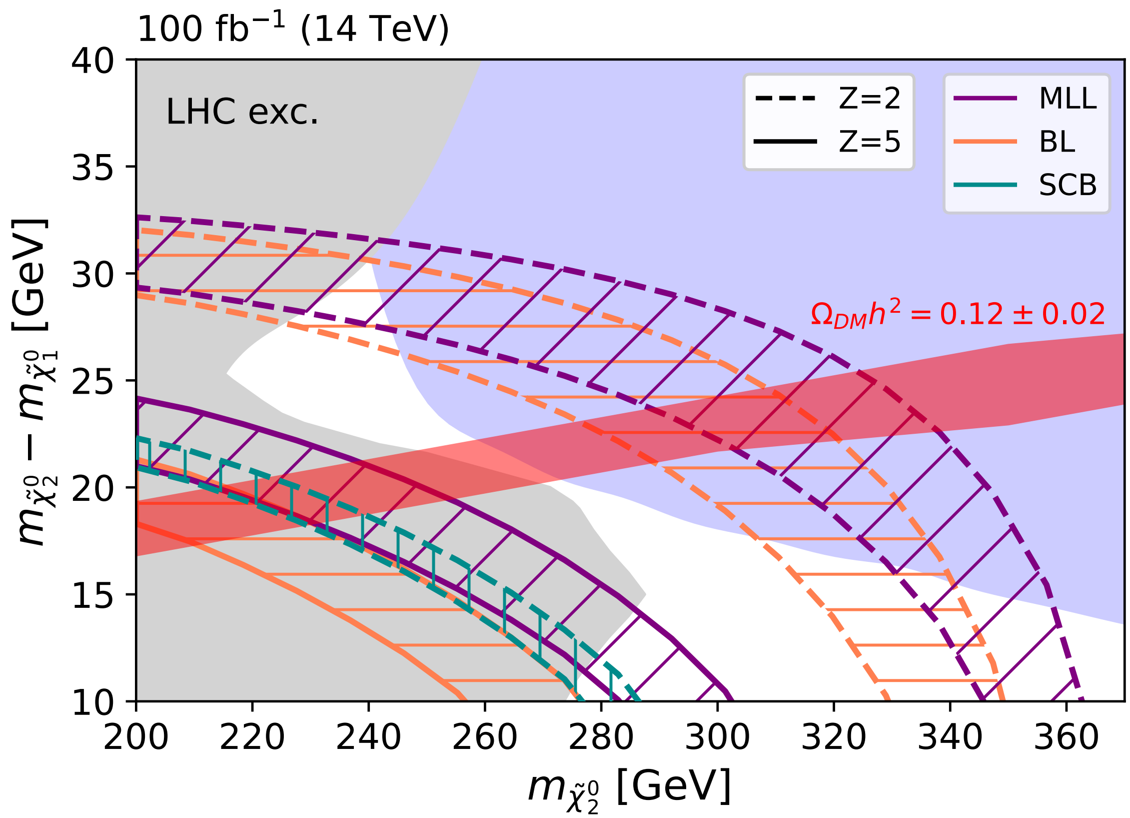

Large regions of parameter space that can be probed at the LHC for negative $M_1$




Enhanced radiative decays into photons provide a novel signature

# Updated Experimental Constraints and Searches for Photons in the Final State

Arganda, Carena, de los Rios, Perez, Rocha, Sanda Seoane and C.W. arXiv:2410.13799




Only a narrow band of allowed values if one takes the standard relic density prediction. To be tested at next LHC run

# Axion or Coherent Oscillations

Let's take a field with $\mathcal{L}$

$$\mathcal{L} \sim A^2 \left[ \frac{\dot{\varphi}^2}{2} - \frac{m_\varphi^2 \varphi^2}{2} \right]$$

where I assumed that the field is homogeneous with no $\vec{x}$ dep.

Taking the action in an expanding Universe to be

$$S = \int d^4x \; a^3(t) \, \mathcal{L}$$

The equation of motion

$$\ddot{\varphi} + 3H\dot{\varphi} + m_\varphi^2 \varphi = 0$$

At high T, the "friction" term H dominates and the solution $\varphi \sim \varphi_i =$ constant.

But as the Universe cools down, $H < m_a$

$$3H\dot{\varphi} + m_\varphi^2 \varphi \sim 3H^2 \frac{d\varphi}{d\ln(a)} + m_\varphi^2 \varphi$$

The field starts to oscillate.

$$\rho_\varphi \sim A^2 \left[\frac{\dot{\varphi}^2}{2} + m_\varphi^2 \frac{\varphi^2}{2}\right]$$

In average $\langle \dot{\varphi}^2 \rangle \sim m_\varphi^2 \langle \varphi^2 \rangle$

$$\rho_\varphi \sim A^2 \langle \dot{\varphi}^2 \rangle \sim \frac{A^2 m_\varphi^2 \varphi_i^2}{2}$$

Mutiplying the Eq. of motion by $\dot{\varphi}$

$$\dot{\varphi} \frac{d\dot{\varphi}}{dt} + 3H\dot{\varphi}^2 + \frac{m_\varphi^2}{2} \frac{d\varphi^2}{dt} = 0$$

$$\frac{d}{dt}\left(\frac{\dot{\varphi}^2}{2} + \frac{m_\varphi^2 \varphi^2}{2}\right) = -3H\dot{\varphi}^2 + \dot{m}_\varphi \langle \varphi^2 \rangle$$

↑ Admits $m_\varphi(T)$

In average then

$$\boxed{\rho_\varphi = \text{const.} \frac{m_\varphi(T)}{a^3}}$$

Behaves like cold DM once $m_\varphi(T) \to$ ct.

$$\rho \sim m\, n \sim m/a^3$$

Axions are an example where the mass varies with $T$ but settles at a constant value at $T < \Lambda_{QCD} \sim 300$ MeV

For the particular case of axions, the amplitude $A \sim f_{PQ}$, with
$f_{PQ}\, m_a \sim m_\pi f_\pi$

$$n_a = \frac{f_{PQ}^2\, m_a^2\, \varphi_i^2}{m_a\, 2}$$

and equating $m_a \sim H(T_f)$

$$\boxed{n_a \sim \frac{\varphi_i^2\, m_\pi^2 f_\pi^2}{m_a\, 2}}$$

Here everything is of order 100 MeV

$$\frac{m_a n_a}{s(T_f)} \sim \frac{10^{-4} \cdot \varphi_i^2}{2} \frac{\mathrm{GeV}^{+4}}{2\pi^2 g_* T_f^3/45}$$

$$\frac{n_a(T_0) m_a}{\rho_c} = \frac{10^{-4} \varphi_i^2\, \mathrm{GeV}^{+4}}{2\pi^2 g_* \frac{T_f^3}{45}} \cdot \frac{3000/\mathrm{cm}^3}{0.5 \times 10^{-5} \frac{\mathrm{GeV}}{\mathrm{cm}^3} \times 2}$$

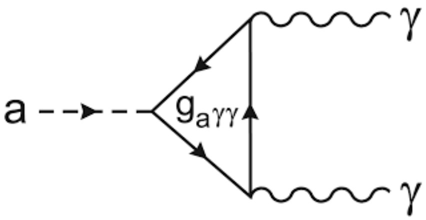
$$\boxed{1.66\, g_* \frac{T_f^2}{M_{Pl}} = m_a} \; ; \quad \boxed{\Omega_a = \frac{0.7 \times 10^5}{g_* T_f^3} \mathrm{GeV}^3 \varphi_i^2}$$

To get the proper relic density $\Omega_a \sim 0.1$

$T_f \sim 10^2\,\mathrm{GeV} \Rightarrow \boxed{m_a \sim 10^{-14}\,\mathrm{GeV} = 10^{-5}\,\mathrm{eV}}$

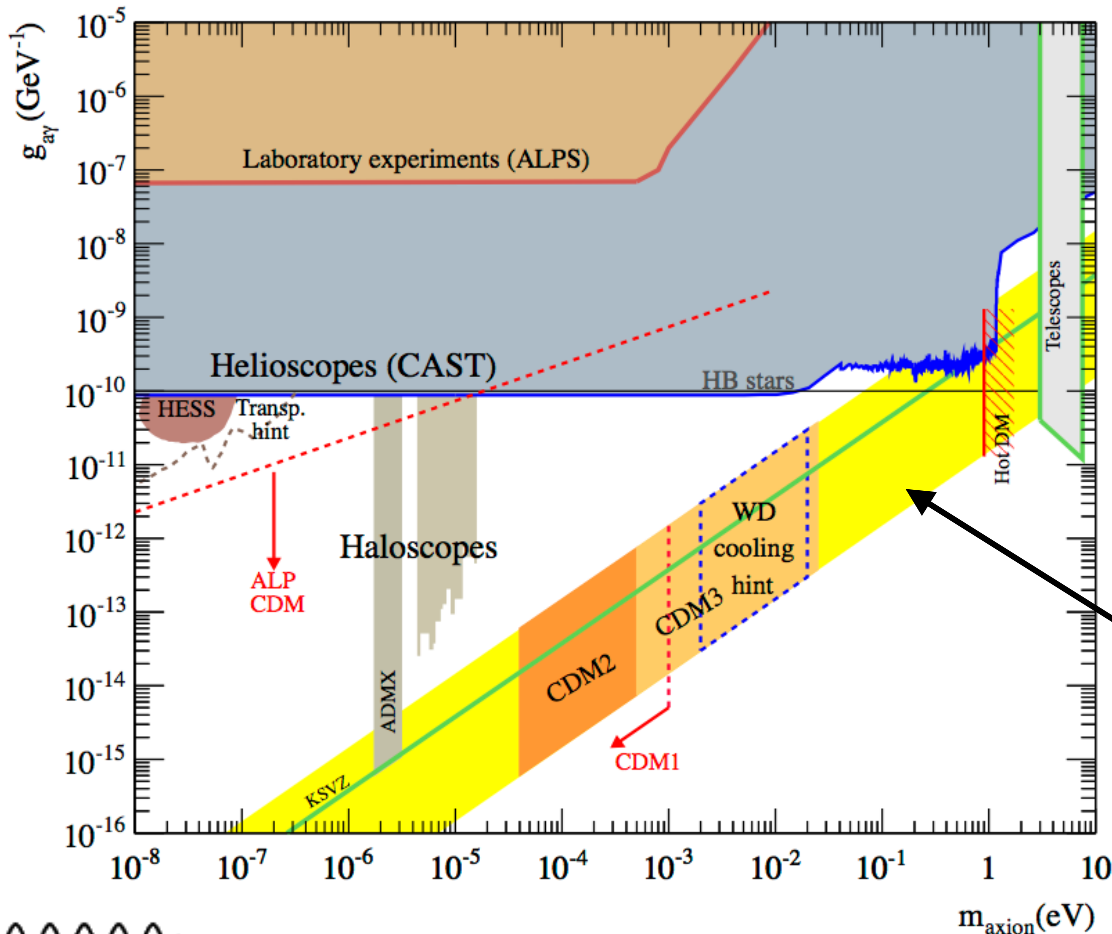
$(\varphi_i \sim \mathcal{O}(1))$

Standard Solution : Promote θ to be a field, a (axion), whose v.e.v is zero

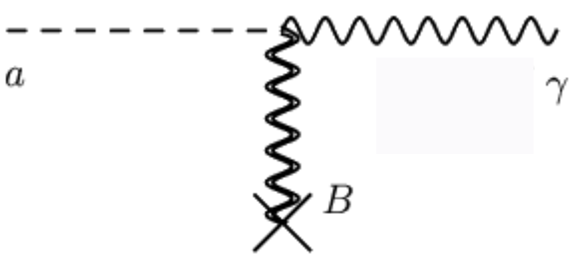



Axions : Solve the strong CP Problem
They are also a good CDM candidate

Axions produced in solar core (conversion to X Rays) : J. Collar




QCD Axion




Hallo Axions : Resonant Magnetic Cavity Searches

## Baryons.

An important relic of the Big Bang may be baryons. However, we don't see any anti-baryons.

Moreover, if we take the cross section mediated by pions,

$$\langle \sigma |v| \rangle \sim c\, m_\pi^{-2} \quad , \quad m_\pi \sim 135\,\text{MeV}$$

$$x_F \sim 42 \quad ; \quad T_f = 22\,\text{MeV}$$

$$Y = \frac{n_B}{s} = 7 \times 10^{-20} / c$$

This value is nine orders of magnitude lower than the value we estimated from observations.

A possible explanation is that a different baryon number and CP violating process must exist to generate a very small asymmetry, of order $10^{-10}$ between Baryons and anti-B. The remaining anti-B annihilated against the Baryons. As said above, the process is called Baryogenesis.

# Big Bang Nucleosynthesis

I will only present a simplified analysis

Let's start by presenting some light nuclei properties.

$$B_A = Z\, m_p + (A-Z)\, m_n - m_A$$

| Atom | $B_A$ | $B_A/A$ |
|---|---|---|
| $^1H$ | 13.6 eV | |
| $^2H \equiv D$ | 2.2 MeV | 1.1 MeV |
| $^3H$ | 6.9 MeV | 2.3 MeV |
| $^3He$ | 7.7 MeV | 2.6 MeV |
| $^4He$ | 28.3 MeV | 7.1 MeV |
| $^7Li$ | 37.9 MeV | 5.4 MeV |

$^4He$ maximizes the binding energy per baryon, and beyond $^1H$ is the preferred nuclei to form at $T \sim$ MeV.

Again, since $\frac{n_B}{n_\gamma} \ll 1$, the effective "Nucleosynthesis" T is much smaller than $B_A$, of order 70 KeV.

One peculiarity here is that the neutron, when it is outside a nucleus is unstable, with $\tau_n \sim 890\,s$.
Observe that this is of the order of the lifetime of the Universe

$$t_{BBN}(70\,KeV) \sim \frac{\tau_n}{3}$$

We should start by knowing how many neutrons per protons are there..
For that we should notice that the reaction $p + e \longleftrightarrow n + \nu$ is in chemical equilibrium until $\nu$ decouple, which happens at 0.8 MeV.

$$\frac{n_n}{n_p} \sim \exp\left[-\frac{Q}{T} + \frac{\mu_e - \mu_\nu}{T}\right] \sim \exp\left[-\frac{Q}{T}\right]$$

$$Q = m_n - m_p \sim 1.3\,MeV$$

$$\frac{n_n}{n_p} = \exp\left[-\frac{1.3}{0.8}\right] \sim \frac{1}{5} \quad \text{at } 0.8\,MeV$$

If the proton would be stable, essentially all neutrons will form $^4He$ and the $^4He$ mass fraction

$$Y = \frac{4 n_{He}}{n_p + n_n} = \frac{2 n_n}{n_p + n_n} = \frac{2/5}{6/5} = \frac{1}{3}$$

But there is a reduction in the number of neutrons,

$$\frac{n_n(70\,KeV)}{n_n(0.8\,MeV)} \sim \exp\left[-\frac{\Delta t}{\tau_n}\right] \sim \frac{3}{4}$$

Implying that $n_n/n_p\,(70\,KeV) \sim 3/20$

$$Y \sim \frac{6/20}{1 + \frac{3}{20}} = \frac{6}{23} \sim 0.25$$

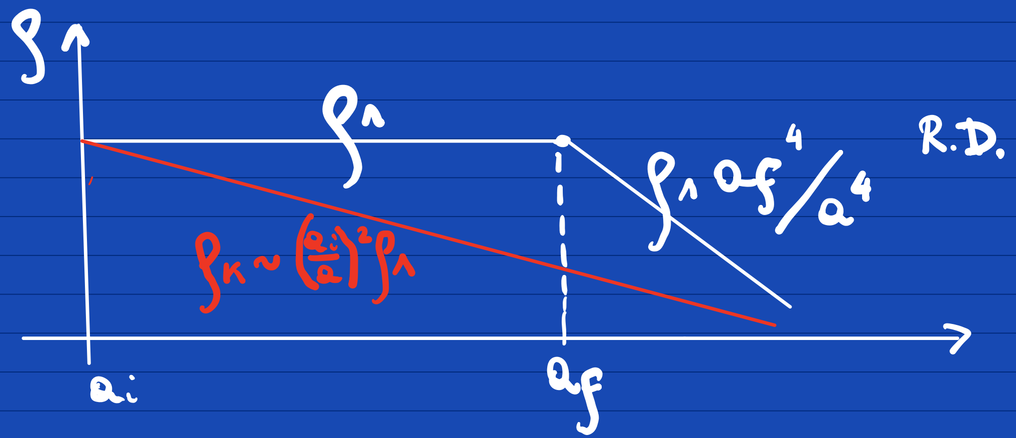
So, approximately one fourth of the baryons form $^4He$, while the rest remain as free protons $\longrightarrow$ $^1H$.

The 0.8 MeV and, more importantly, $\Delta t$ & 70 KeV value depend on the expansion of the Universe, and hence on $g_*$.

Hence, BBN allows to constrain $g_*$ !

# Quantitative requirements for inflation

Let's take for simplicity a Universe which was dominated by a CC from a time $t_i$ until $t_f$ and then it was radiation dominated until today




Here we assume that at the beginning the "curvature energy density" is similar to $\rho_\Lambda$.

$$H^2(a_i) = \frac{K}{a_i^2} + \Lambda = \frac{8\pi G}{3}\left(\rho_K + \rho_\Lambda\right)$$

$$H^2(a) = \frac{K}{a^2} + \Lambda = \frac{8\pi G}{3}\left(\rho_K \frac{a_i^2}{a^2} + \rho_\Lambda\right)$$

$a_i < a < a_f$

$$H^2(a) = \frac{8\pi G}{3}\left(\rho_K \frac{a_i^2}{a^2} + \rho_\Lambda \left(\frac{a_f}{a}\right)^4\right)$$

$a_f < a < a_0$

Also, $$\frac{a_i}{a_f} \sim \exp\left[-\Lambda^{1/2}(t_f - t_i)\right]$$

where we assumed $\rho_\Lambda$ dominates after some short period of time, what is OK since $\rho_K \sim a^{-2}$ and $a$ is growing exponentially.

Now, what we want to require is that once we go to R.D. the curvature remains small.

$$\rho_K \left(\frac{a_i}{a_0}\right)^2 \ll \rho_\Lambda \left(\frac{a_f}{a_0}\right)^4$$

If $\rho_K \sim \rho_\Lambda$

$$\left(\frac{a_i}{a_f}\right)^2 \ll \left(\frac{a_f}{a_0}\right)^2$$

$$\frac{a_f}{a_0} \sim \frac{T_0}{T_f} \qquad (a \cdot T)^3 \propto S = \text{const.}$$

Now, $T_0 \sim 2 \times 10^{-4}$ eV. Taking $T_f \sim \mathcal{O}(10^{16}\,\text{GeV})$

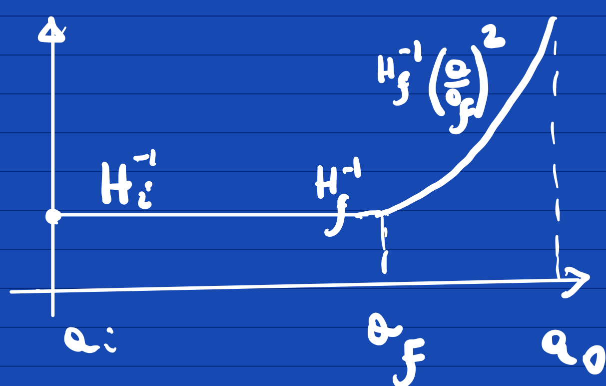
$$\frac{a_f}{a_0} \sim 10^{-29} \Rightarrow \frac{a_i}{a_f} \ll 10^{-29}$$

$$\Lambda^{1/2}(t_f - t_i) > 70$$ Seventy e-folds.

Inclusion of MD leads to similar bound.

Apart from the flatness problem, inflation addresses the so-called horizon problem, namely CMB coming from opposite directions look the same and these points were not in causal correlation with each other. The way to solve this is to assume that all points in the observable Universe proceed from a small, highly correlated region that was blown away by inflation

$$d_{H_0} \sim H_0^{-1} < H_i^{-1} \frac{a_0}{a_i}$$




$$H_0^{-1} = H_f^{-1} \cdot \left(\frac{a_0}{a_f}\right)^2 \ll H_f^{-1} \frac{a_0}{a_i} \Rightarrow \left(\frac{a_0}{a_f}\right) \ll \frac{a_f}{a_i}$$

Same condition as for flatness!!