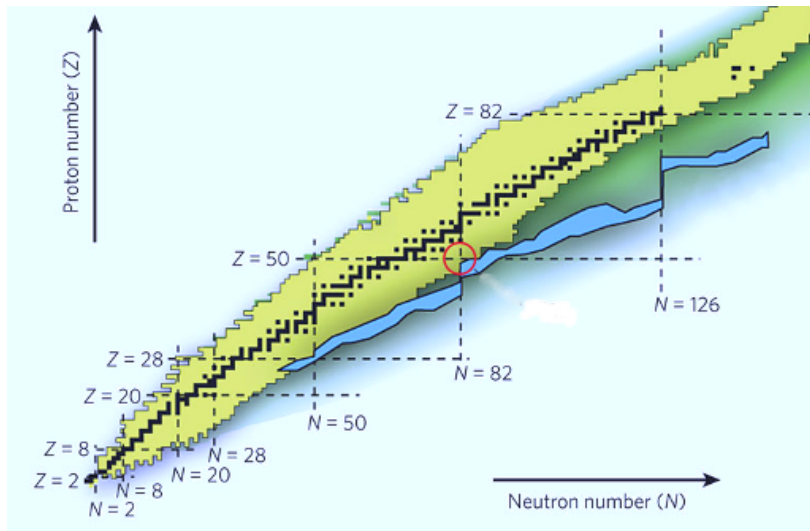


Applications of *ab initio* and phenomenological nuclear reaction theory

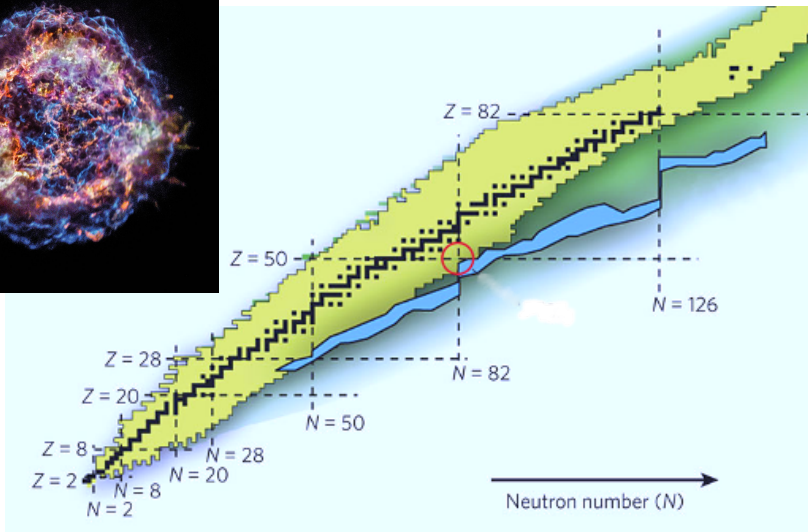
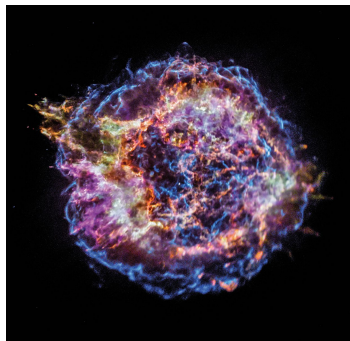
Mack C. Atkinson



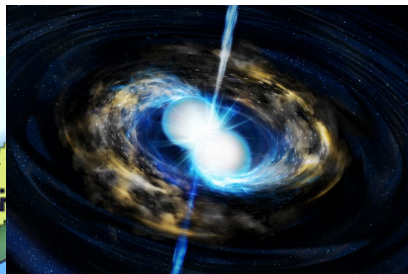
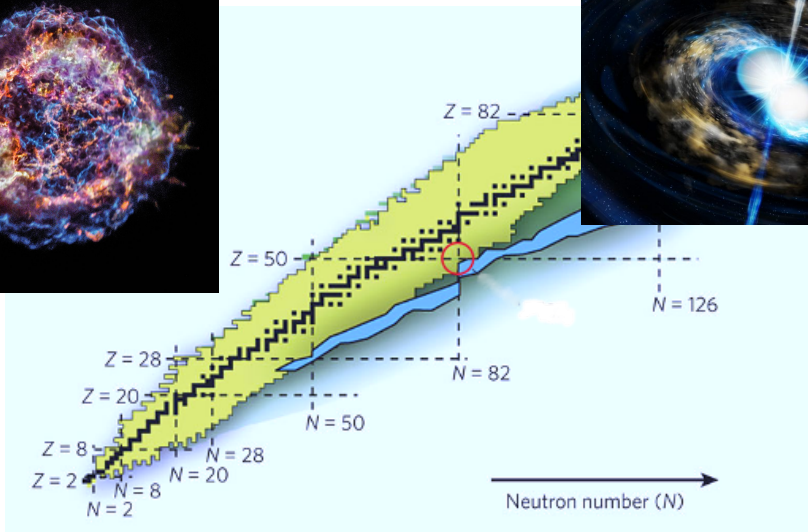
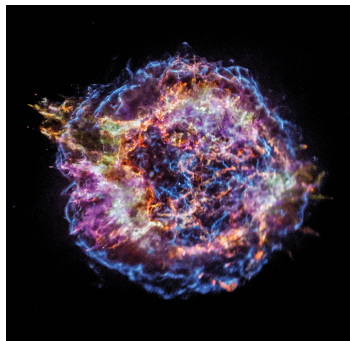
Nuclear physics is ubiquitous among many fields of science



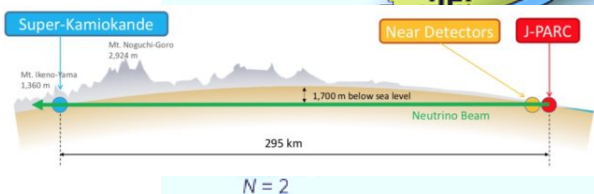
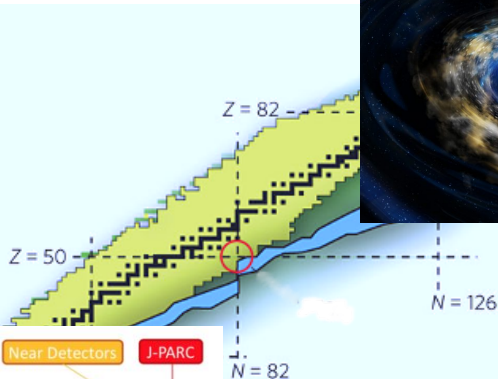
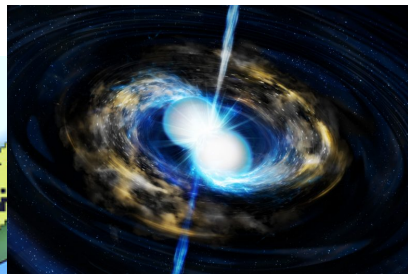
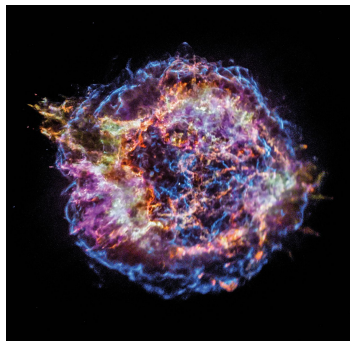
Nuclear physics is ubiquitous among many fields of science



Nuclear physics is ubiquitous among many fields of science

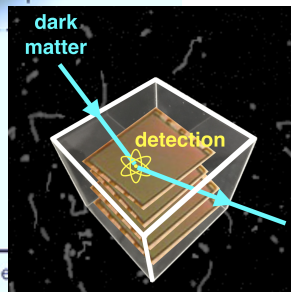
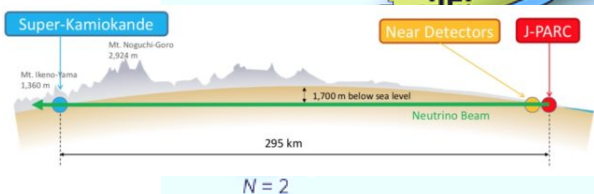
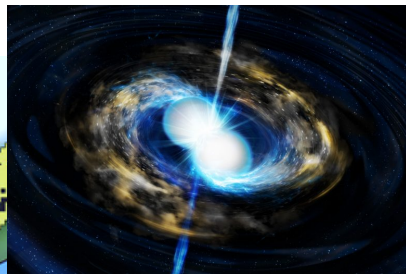
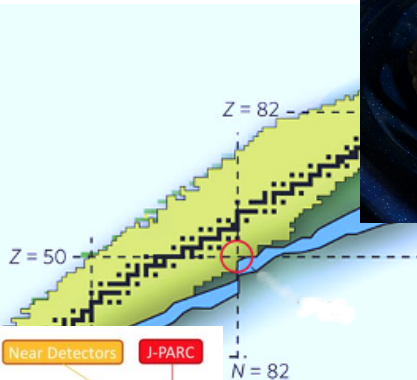
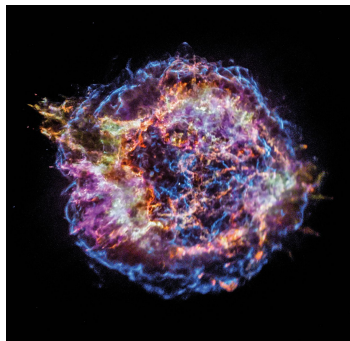


Nuclear physics is ubiquitous among many fields of science

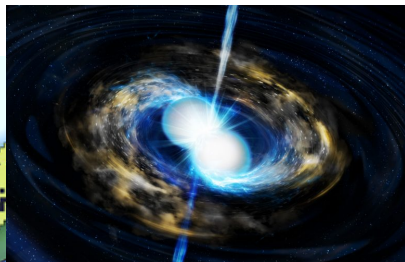
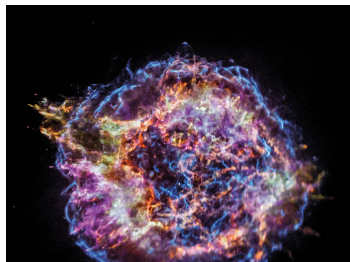


Neutron number (N)

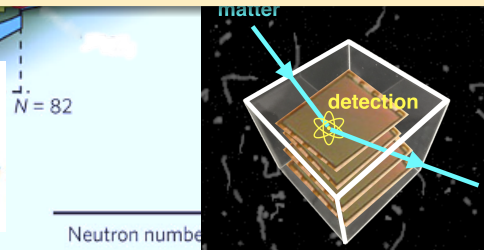
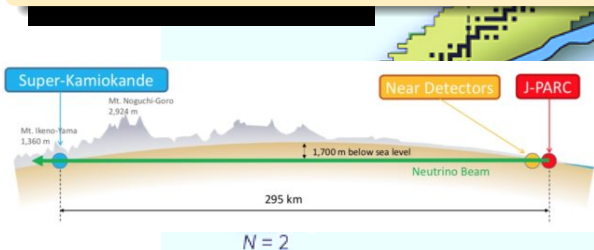
Nuclear physics is ubiquitous among many fields of science



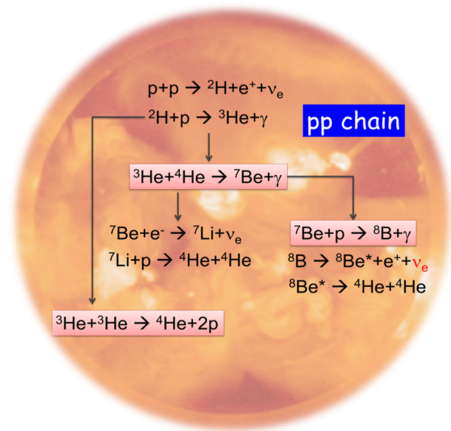
Nuclear physics is ubiquitous among many fields of science



Accurate nuclear reaction calculations are vital

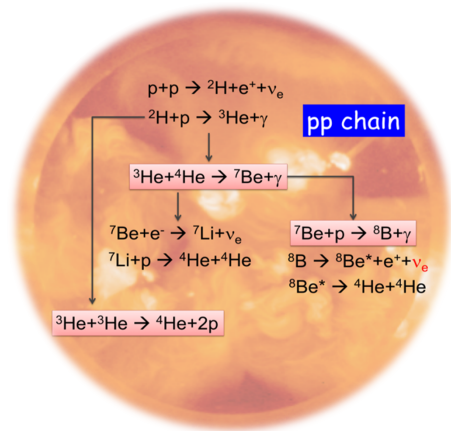


For example, fusion reactions like ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be}$ in our sun



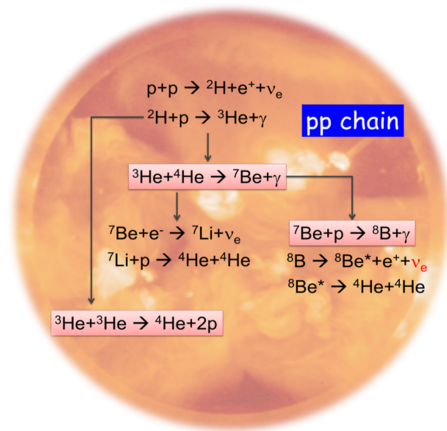
For example, fusion reactions like ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be}$ in our sun

- Neutrinos observed from the sun do not match solar model predictions



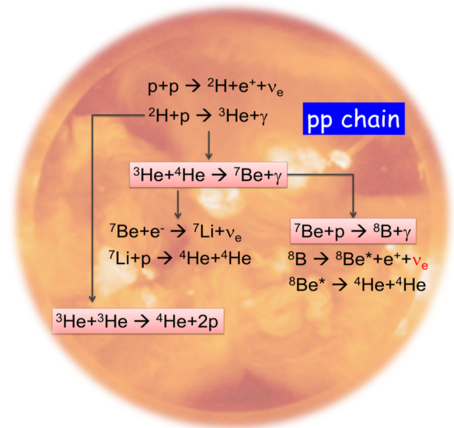
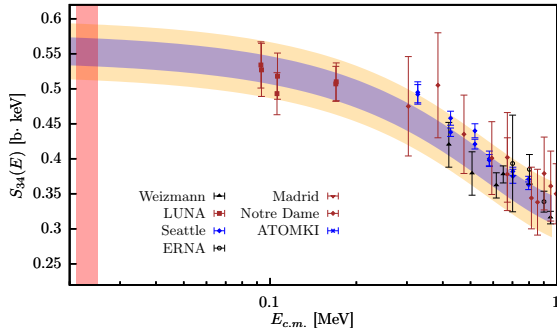
For example, fusion reactions like ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be}$ in our sun

- Neutrinos observed from the sun do not match solar model predictions
- Largest uncertainty comes from nuclear reaction rates



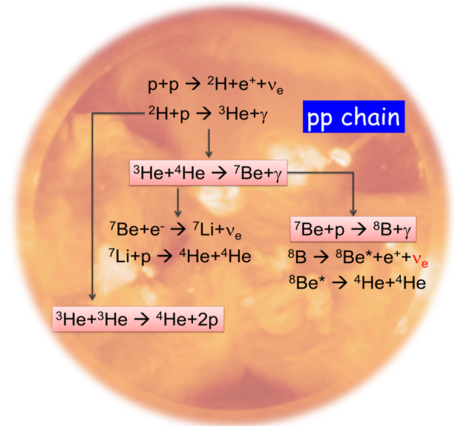
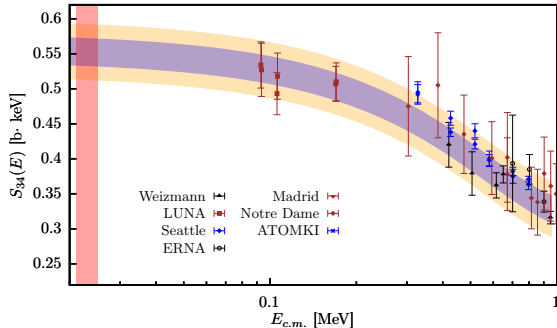
For example, fusion reactions like ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be}$ in our sun

- Neutrinos observed from the sun do not match solar model predictions
- Largest uncertainty comes from nuclear reaction rates



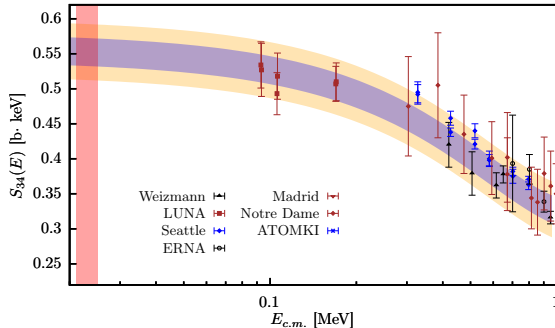
For example, fusion reactions like ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be}$ in our sun

- Neutrinos observed from the sun do not match solar model predictions
- Largest uncertainty comes from nuclear reaction rates
- Need nuclear theory to accurately predict $S_{34}(E)$



For example, fusion reactions like ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be}$ in our sun

- Neutrinos observed from the sun do not match solar model predictions
- Largest uncertainty comes from nuclear reaction rates
- Need nuclear theory to accurately predict $S_{34}(E)$
- How to calculate this?

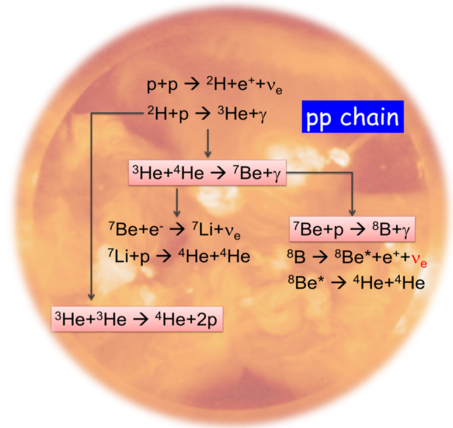


Adelberger *et al.*, Rev Mod Phys **83** 195 (2011)

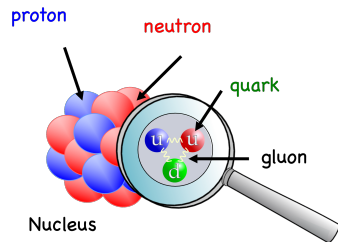
mackenzie.c.atkinson@gmail.com

Mack C. Atkinson

LLNL

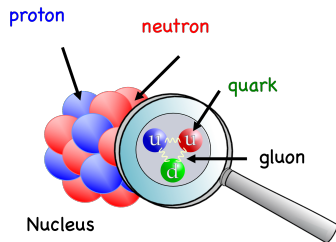


General picture of the nuclear many-body problem



General picture of the nuclear many-body problem

$$(T + V)\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

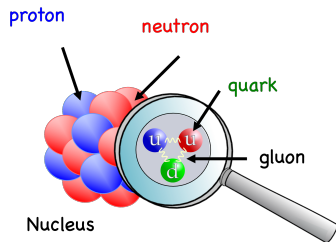


General picture of the nuclear many-body problem

$$(T + V)\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

$$V = \frac{1}{2} \sum_{k \neq l=1}^A V_{NN}(\mathbf{r}_k, \mathbf{r}_l)$$

No exact form of $V_{NN}(\mathbf{r}, \mathbf{r}')$

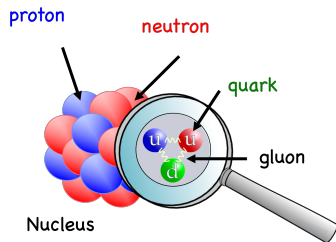
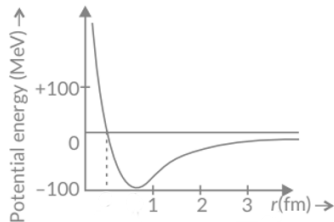


General picture of the nuclear many-body problem

$$(T + V)\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

$$V = \frac{1}{2} \sum_{k \neq l=1}^A V_{NN}(\mathbf{r}_k, \mathbf{r}_l)$$

No exact form of $V_{NN}(\mathbf{r}, \mathbf{r}')$



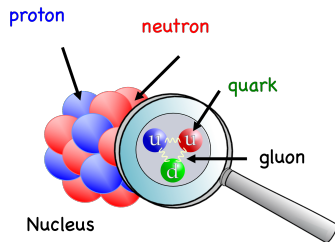
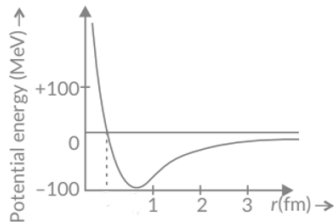
General picture of the nuclear many-body problem

$$(T + V)\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

$$V = \frac{1}{2} \sum_{k \neq l=1}^A V_{NN}(\mathbf{r}_k, \mathbf{r}_l)$$

V_{NN} is fit to NN scattering data

No exact form of $V_{NN}(\mathbf{r}, \mathbf{r}')$



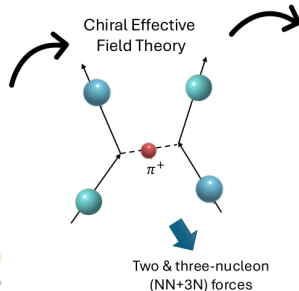
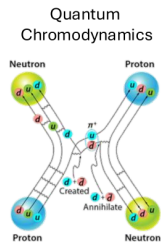
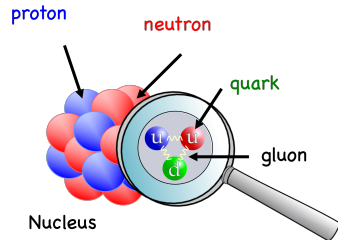
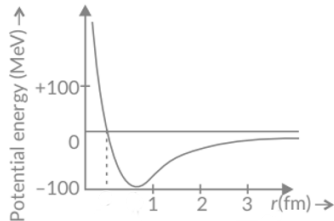
General picture of the nuclear many-body problem

$$(T + V)\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A) = E\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A)$$

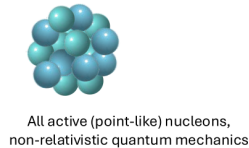
$$V = \frac{1}{2} \sum_{k \neq l=1}^A V_{NN}(\mathbf{r}_k, \mathbf{r}_l)$$

V_{NN} is fit to NN scattering data

No exact form of $V_{NN}(\mathbf{r}, \mathbf{r}')$



Many-body calculations



The *ab initio* method: No-core shell model with continuum (NCSMC)

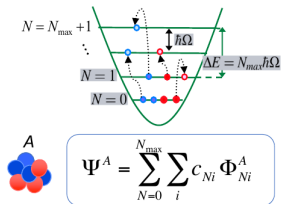
$$\left\langle \Psi_{bs} \left({}^7\text{Be} \right) \left| \hat{\mathcal{M}}_{\text{EM}} \right| \Psi_{sc} \left({}^3\text{He} + \alpha \right) \right\rangle$$

The *ab initio* method: No-core shell model with continuum (NCSMC)

$$\left\langle \Psi_{bs} \left({}^7\text{Be} \right) \left| \hat{\mathcal{M}}_{\text{EM}} \right| \Psi_{sc} \left({}^3\text{He} + \alpha \right) \right\rangle$$



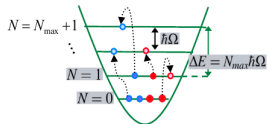
The *ab initio* method: No-core shell model with continuum (NCSMC)



$$\left\langle \Psi_{bs} \left({}^7\text{Be} \right) \left| \hat{\mathcal{M}}_{\text{EM}} \right| \Psi_{sc} \left({}^3\text{He} + \alpha \right) \right\rangle$$



The *ab initio* method: No-core shell model with continuum (NCSMC)



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

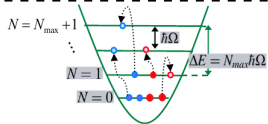
$$\hat{H} = \hat{T} + \hat{V}_{NN} + \hat{V}_{NNN}$$

$$\hat{H} |\Psi^A\rangle = E |\Psi^A\rangle$$

$$\left\langle \Psi_{bs} (^7\text{Be}) \left| \hat{\mathcal{M}}_{\text{EM}} \right| \Psi_{sc} (^3\text{He} + \alpha) \right\rangle$$



The *ab initio* method: No-core shell model with continuum (NCSMC)



$N = N_{\max} + 1$
 $N = 1$
 $N = 0$
 $\hbar\Omega$
 $\Delta E = N_{\max}\hbar\Omega$

A

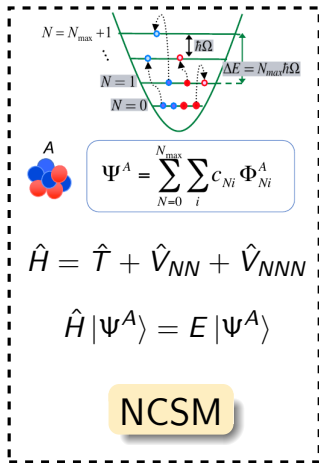
$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$
$$\hat{H} = \hat{T} + \hat{V}_{NN} + \hat{V}_{NNN}$$
$$\hat{H} |\Psi^A\rangle = E |\Psi^A\rangle$$

NCSM

$$\left\langle \Psi_{bs} \left({}^7\text{Be} \right) \left| \hat{\mathcal{M}}_{\text{EM}} \right| \Psi_{sc} \left({}^3\text{He} + \alpha \right) \right\rangle$$



The *ab initio* method: No-core shell model with continuum (NCSMC)

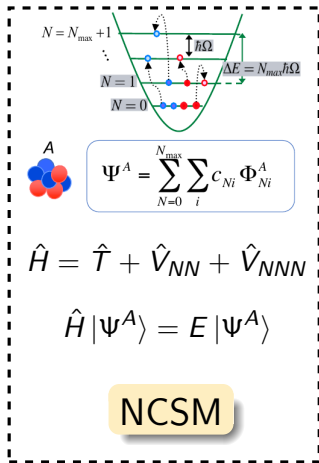


$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} (A) \\ \text{cluster} \end{array}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} (A-a) \\ \text{cluster} \end{array}, \nu \right\rangle$$

$$\left\langle \Psi_{bs} (^7\text{Be}) \left| \hat{\mathcal{M}}_{\text{EM}} \right| \Psi_{sc} (^3\text{He} + \alpha) \right\rangle$$



The *ab initio* method: No-core shell model with continuum (NCSMC)



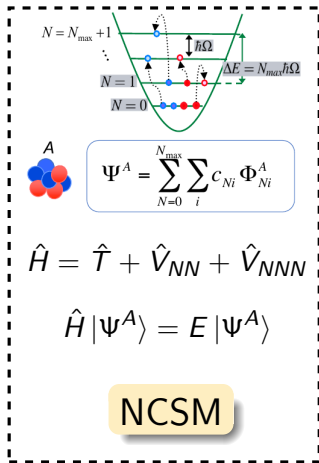
$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} [A] \\ \text{Be} \end{array}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} \text{He} + \alpha \\ (A-a) \end{array}, \nu \right\rangle$$

$\uparrow$
 $|^7\text{Be}\rangle$

$$\left\langle \Psi_{bs} (^7\text{Be}) \left| \hat{\mathcal{M}}_{\text{EM}} \right| \Psi_{sc} (^3\text{He} + \alpha) \right\rangle$$



The *ab initio* method: No-core shell model with continuum (NCSMC)



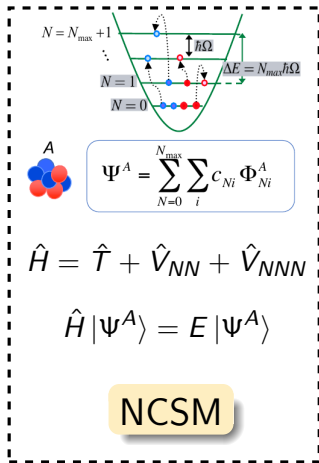
$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} [A] \\ \text{Be} \end{array}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} \text{He} \\ (a) \end{array}, \nu \right\rangle$$

$\uparrow$ $|^7\text{Be}\rangle$
 $\uparrow$ $|\alpha\rangle \otimes |^3\text{He}\rangle$

$$\left\langle \Psi_{bs} (^7\text{Be}) \left| \hat{\mathcal{M}}_{\text{EM}} \right| \Psi_{sc} (^3\text{He} + \alpha) \right\rangle$$



The *ab initio* method: No-core shell model with continuum (NCSMC)



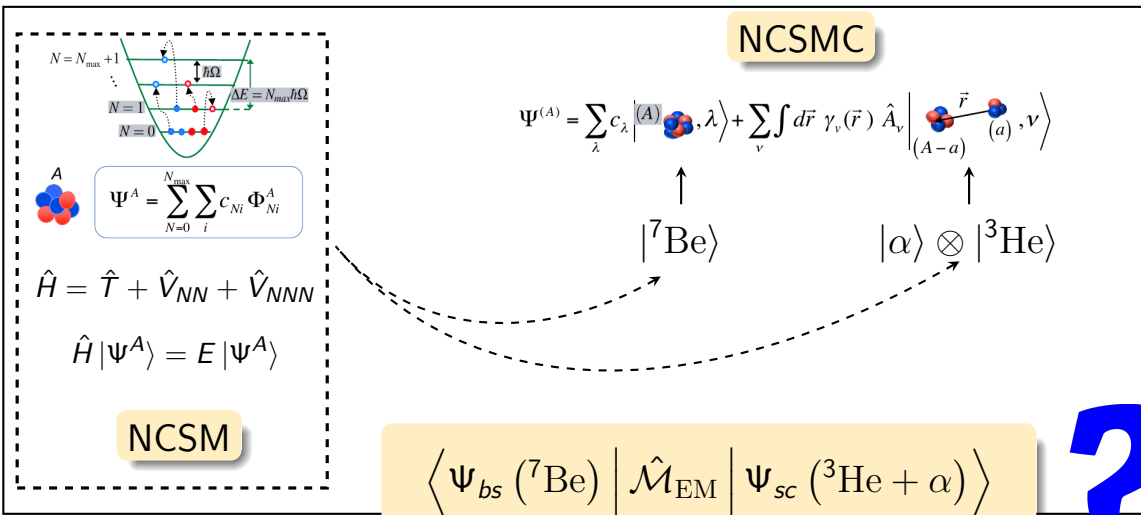
$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} [A] \\ \text{cluster} \end{array}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} \text{cluster} \\ (A-a) \end{array}, \nu \right\rangle$$

$\uparrow$ $|^7\text{Be}\rangle$
 $\uparrow$ $|\alpha\rangle \otimes |^3\text{He}\rangle$

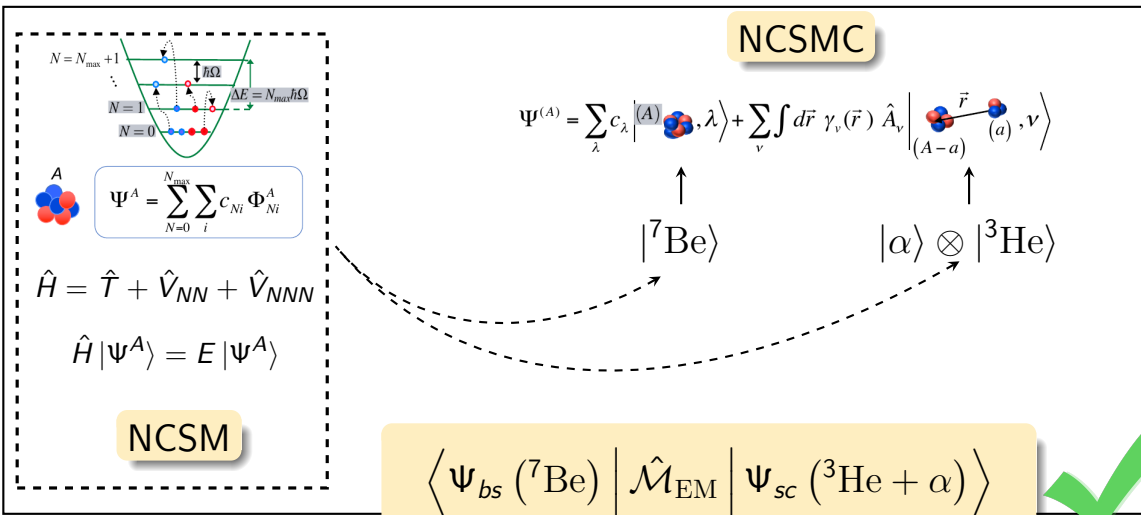
$$\left\langle \Psi_{bs} (^7\text{Be}) \left| \hat{\mathcal{M}}_{\text{EM}} \right| \Psi_{sc} (^3\text{He} + \alpha) \right\rangle$$



The *ab initio* method: No-core shell model with continuum (NCSMC)



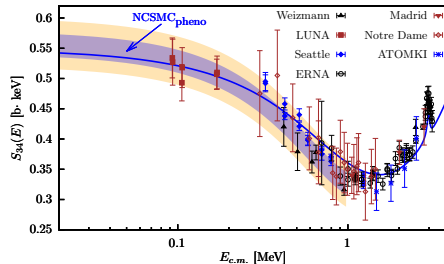
The *ab initio* method: No-core shell model with continuum (NCSMC)



NCSMC prediction consistent with higher-energy fusion data

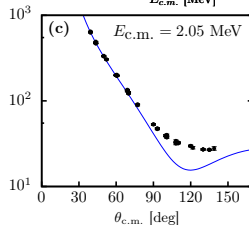
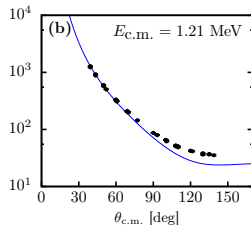
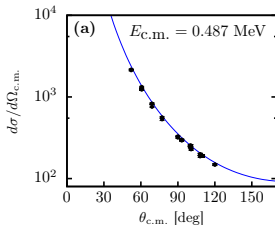
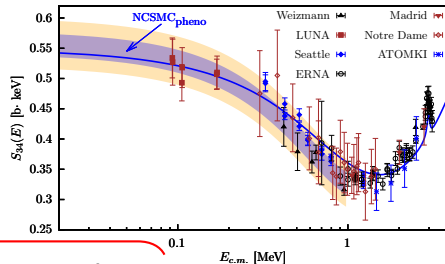


NCSMC prediction consistent with higher-energy fusion data



NCSMC prediction consistent with higher-energy fusion data

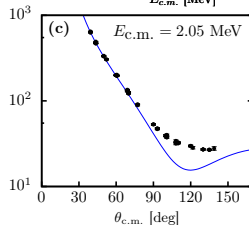
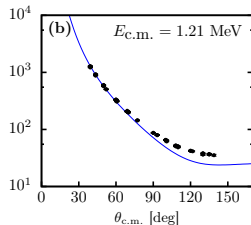
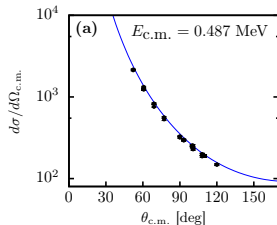
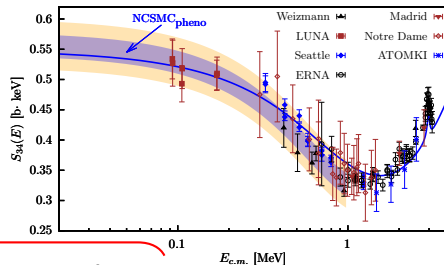
$$\langle \psi_{bs} (^7\text{Be}) | \hat{\mathcal{M}}_{\text{EM}} | \psi_{sc} (^3\text{He} + \alpha) \rangle$$



NCSMC prediction consistent with higher-energy fusion data

- Elastic-scattering and fusion cross sections should be consistent

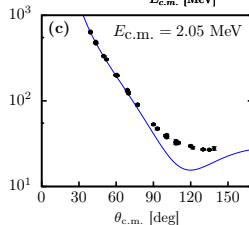
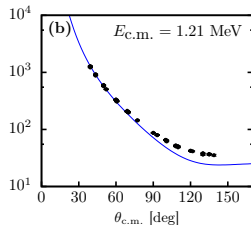
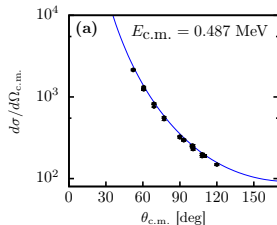
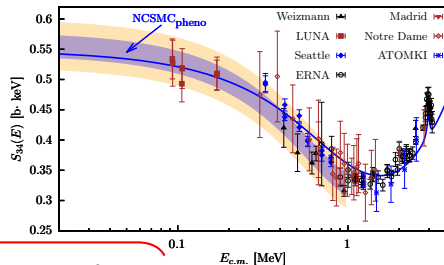
$$\langle \psi_{bs} (^7\text{Be}) | \hat{\mathcal{M}}_{\text{EM}} | \psi_{sc} (^3\text{He} + \alpha) \rangle$$



NCSMC prediction consistent with higher-energy fusion data

- Elastic-scattering and fusion cross sections should be consistent
- Missing ${}^6\text{Li}+p$?

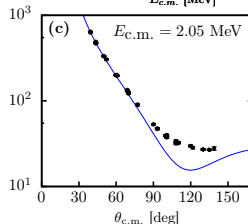
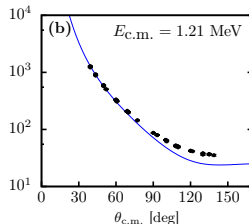
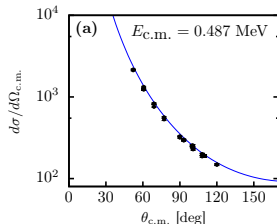
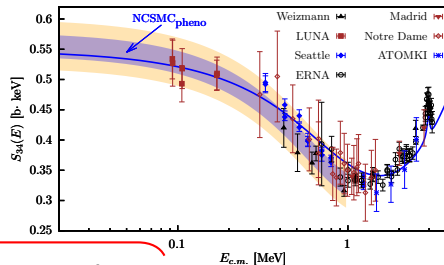
$$\langle \Psi_{bs} ({}^7\text{Be}) | \hat{\mathcal{M}}_{\text{EM}} | \Psi_{sc} ({}^3\text{He} + \alpha) \rangle$$



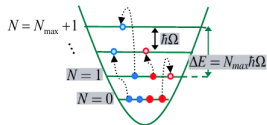
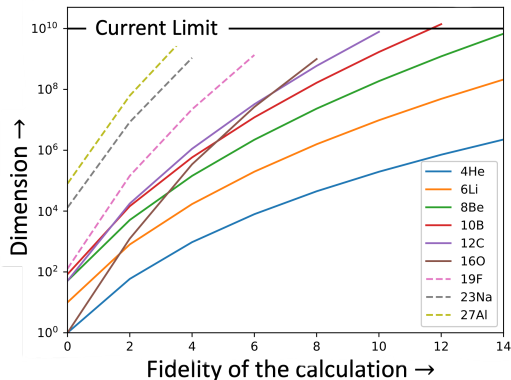
NCSMC prediction consistent with higher-energy fusion data

- Elastic-scattering and fusion cross sections should be consistent
- Missing ${}^6\text{Li}+p$?
- Could also point to unconstrained aspects of V_{NN}

$$\langle \psi_{bs} ({}^7\text{Be}) | \hat{\mathcal{M}}_{\text{EM}} | \psi_{sc} ({}^3\text{He} + \alpha) \rangle$$

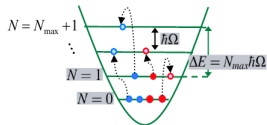
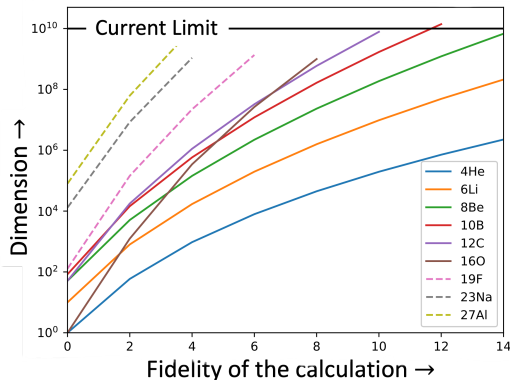


NCSMC method is intractable for larger nuclei – how to proceed?



$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

NCSMC method is intractable for larger nuclei – how to proceed?

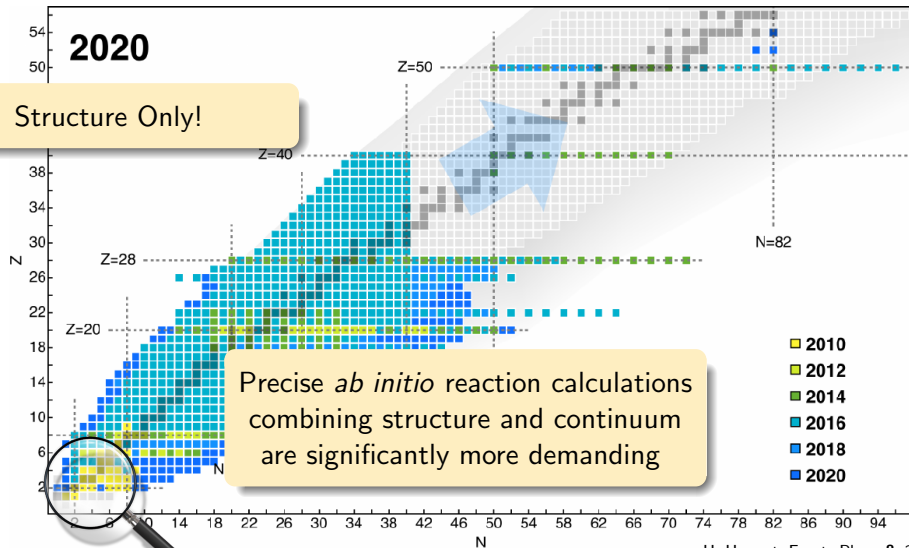


$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{nucleus} \end{matrix}, \lambda \right\rangle + \sum_v \int d\vec{r} \gamma_v(\vec{r}) \hat{A}_v \left| \begin{matrix} (A-a) \\ \text{nucleus} \end{matrix}, \vec{r}, v \right\rangle$$

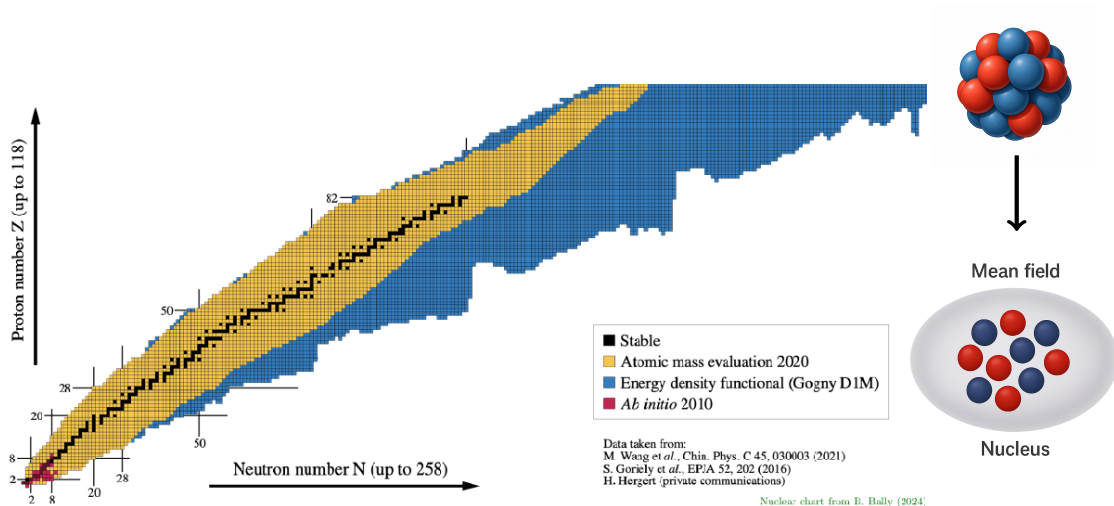
Scaling even worse for full NCSMC

NCSMC method is intractable for larger nuclei – how to proceed?



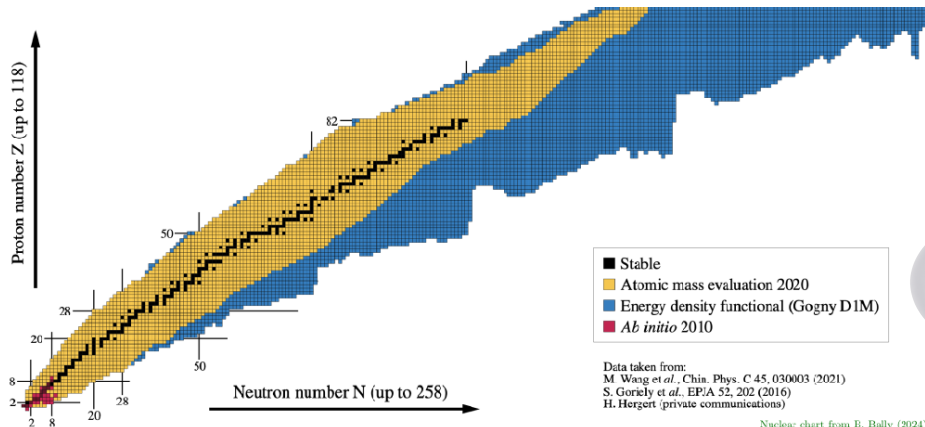
H. Hergert, Front. Phys. 8, 379 (2020)

Larger coverage with mean-field theory (Density Functional Theory)

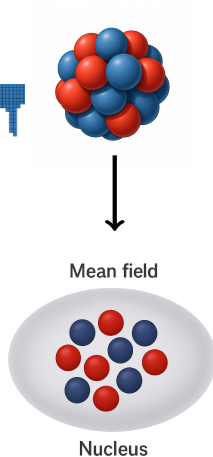


Larger coverage with mean-field theory (Density Functional Theory)

Still structure only!



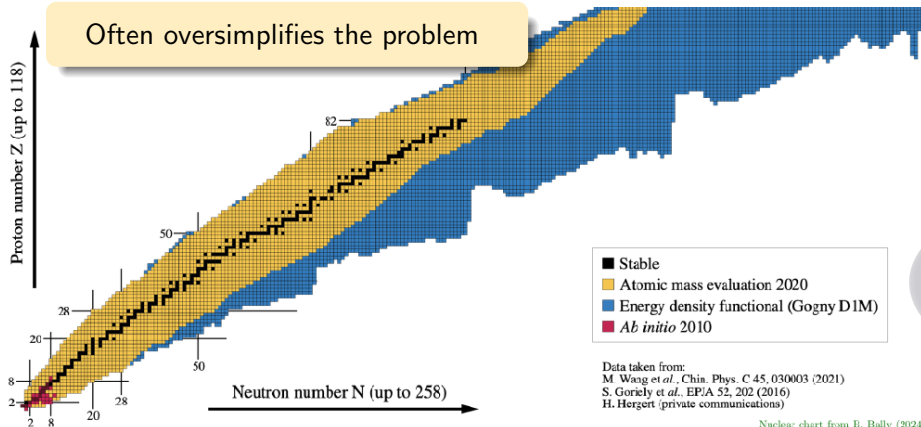
Nuclear chart from B. Bally (2024)



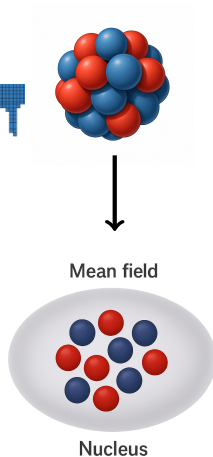
Larger coverage with mean-field theory (Density Functional Theory)

Still structure only!

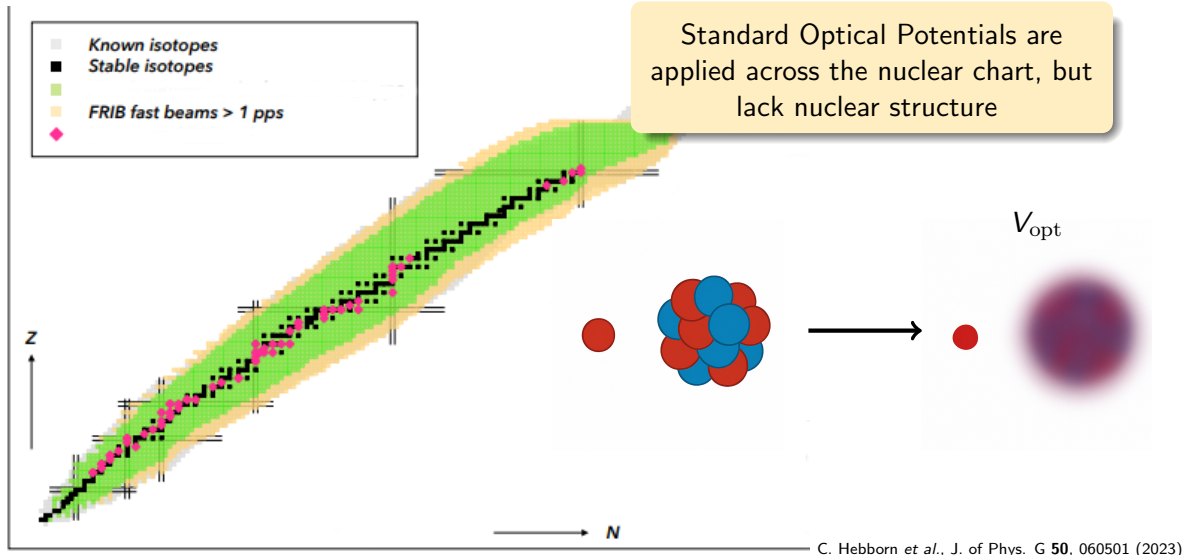
Often oversimplifies the problem



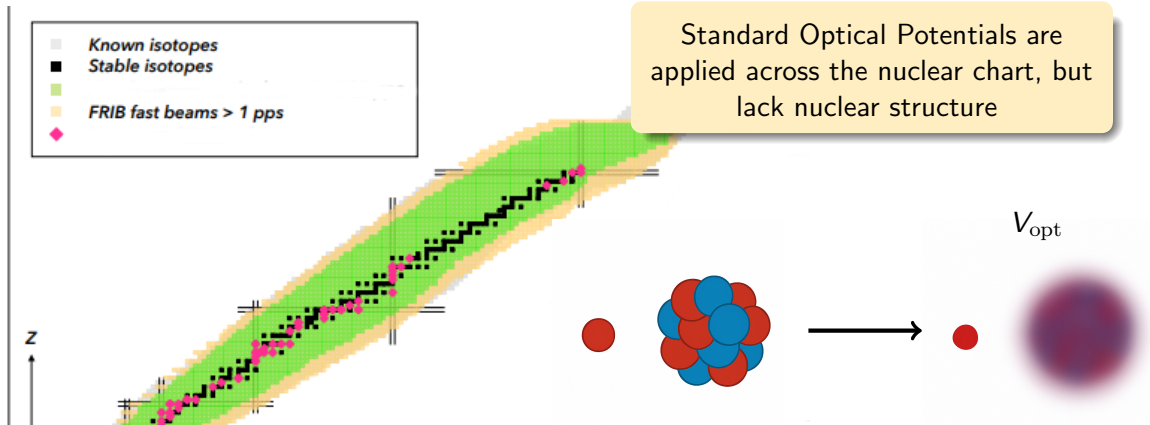
Nuclear chart from B. Bally (2024)



Scattering and structure are typically treated separately → not ideal!



Scattering and structure are typically treated separately → not ideal!

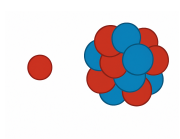


Dispersive Optical Model

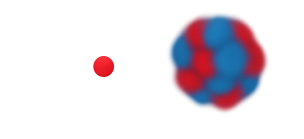
The DOM strikes a balance between computational cost and a consistent treatment of bound and continuum dynamics

Dispersive Optical Model (DOM): uniting structure and continuum

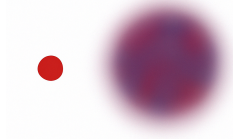
Exact



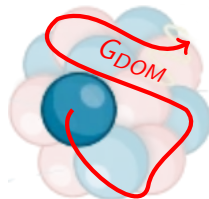
DOM



V_{opt}

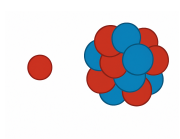


Dispersive Optical Model (DOM): uniting structure and continuum



G_{DOM} : Single-Particle Propagator

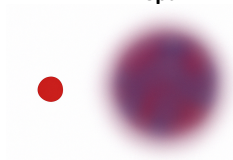
Exact



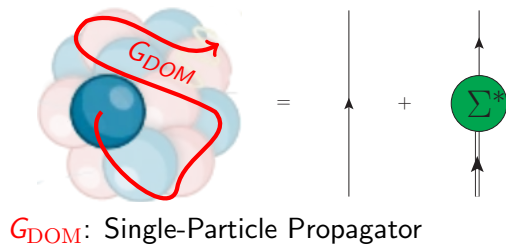
DOM



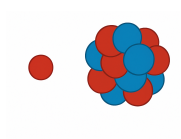
V_{opt}



Dispersive Optical Model (DOM): uniting structure and continuum



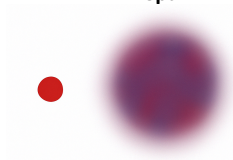
Exact



DOM

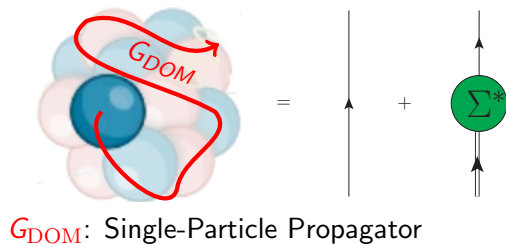


V_{opt}

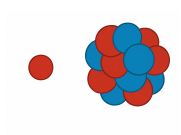


Dispersive Optical Model (DOM): uniting structure and continuum

- Self-energy: $\Sigma^*(r, r'; E) = V_{\text{opt}}(r, r'; E)$



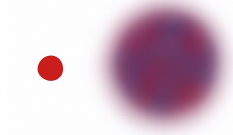
Exact



DOM



V_{opt}

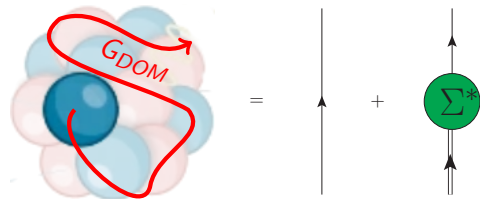


Dispersive Optical Model (DOM): uniting structure and continuum

- Self-energy: $\Sigma^*(r, r'; E) = V_{\text{opt}}(r, r'; E)$

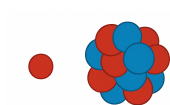
Dispersive Correction

$$\text{Re}\Sigma_{\ell j}(r, r'; E) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} dE' \frac{\text{Im}\Sigma_{\ell j}(r, r'; E')}{E - E'}$$



G_{DOM} : Single-Particle Propagator

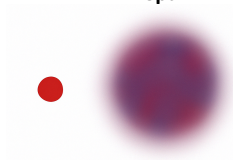
Exact



DOM



V_{opt}

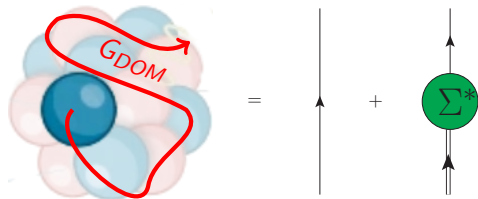


Dispersive Optical Model (DOM): uniting structure and continuum

- Self-energy: $\Sigma^*(r, r'; E) = V_{\text{opt}}(r, r'; E)$

Dispersive Correction

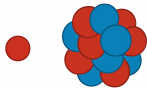
$$\text{Re}\Sigma_{\ell j}(r, r'; E) = \frac{1}{\pi} \mathcal{P} \int_{-\infty}^{\infty} dE' \frac{\text{Im}\Sigma_{\ell j}(r, r'; E')}{E - E'}$$



G_{DOM} : Single-Particle Propagator

- This constraint ensures bound and scattering quantities are simultaneously described

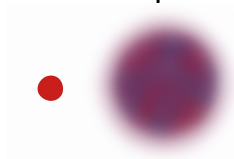
Exact



DOM



V_{opt}

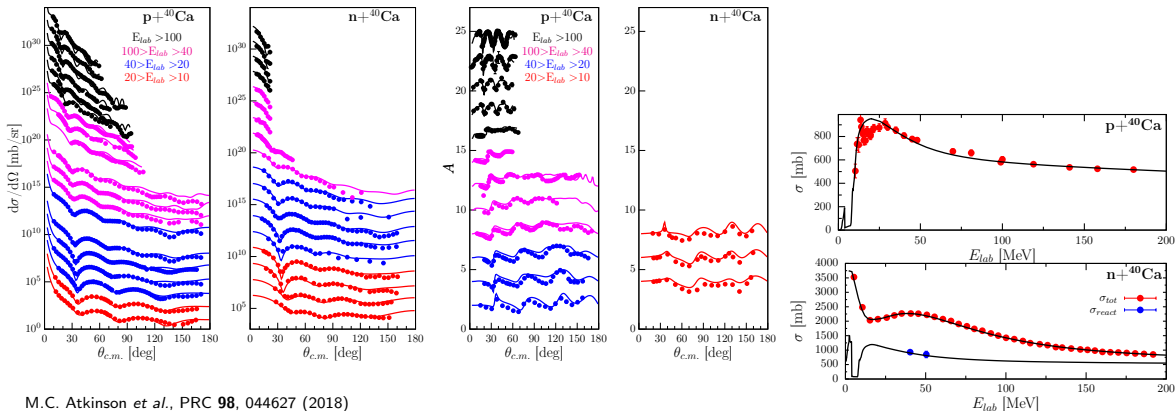


Constraining the self-energy of ^{40}Ca

- Parameters of self-energy varied to minimize χ^2

Constraining the self-energy of ^{40}Ca

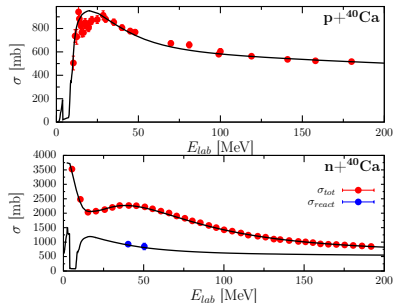
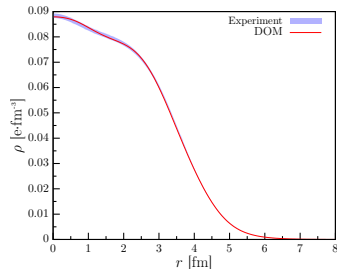
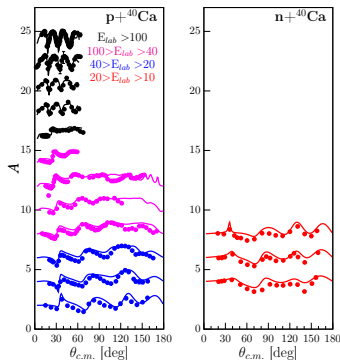
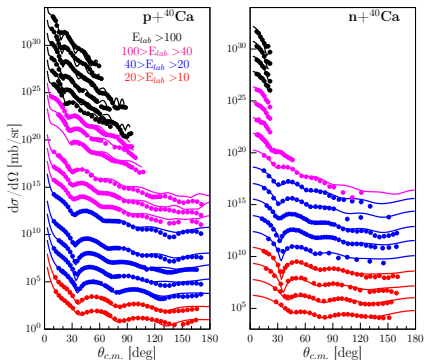
- Parameters of self-energy varied to minimize χ^2



M.C. Atkinson *et al.*, PRC **98**, 044627 (2018)

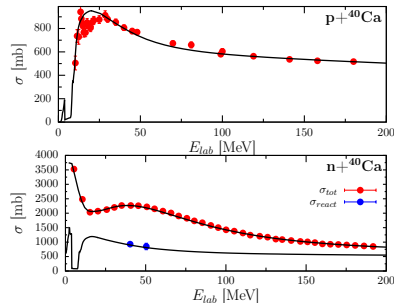
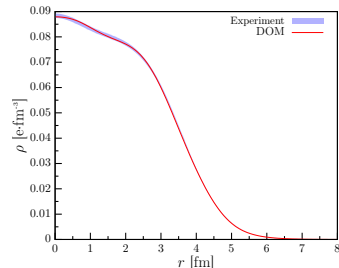
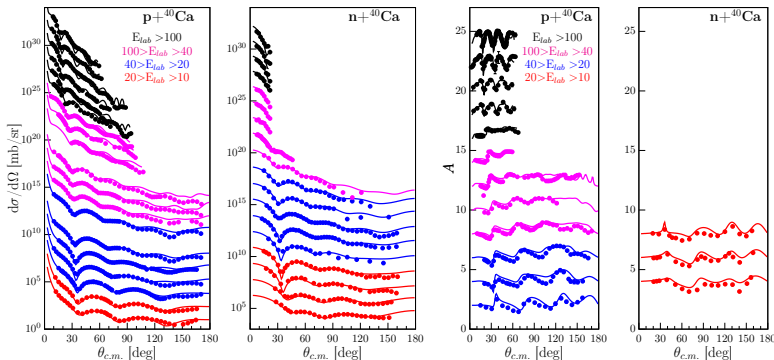
Constraining the self-energy of ^{40}Ca

- Parameters of self-energy varied to minimize χ^2



Constraining the self-energy of ^{40}Ca

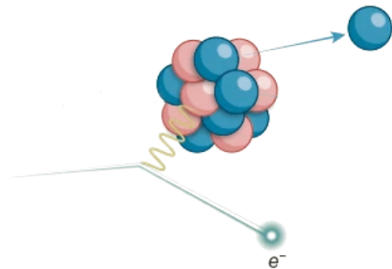
- Parameters of self-energy varied to minimize χ^2
- Reproducing the data ensures an accurate self-energy



M.C. Atkinson *et al.*, PRC **98**, 044627 (2018)

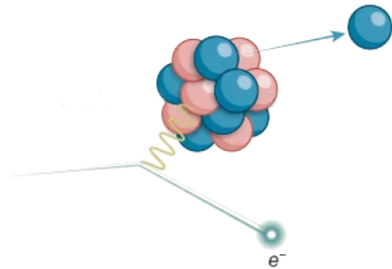
Electron scattering: the exclusive $^{40}\text{Ca}(e, e'p)^{39}\text{K}$ reaction

- Electron interaction means clean knockout reactions



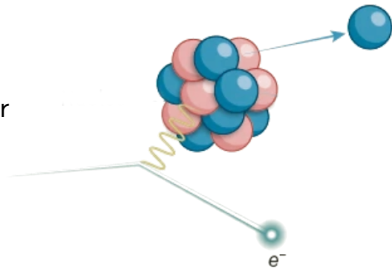
Electron scattering: the exclusive $^{40}\text{Ca}(e, e'p)^{39}\text{K}$ reaction

- Electron interaction means clean knockout reactions
- Cross section is closely tied to the boundstate wavefunction



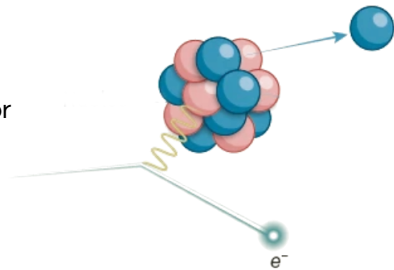
Electron scattering: the exclusive $^{40}\text{Ca}(e, e'p)^{39}\text{K}$ reaction

- Electron interaction means clean knockout reactions
- Cross section is closely tied to the boundstate wavefunction
- Evidence of physics beyond mean-field: Spectroscopic Factor

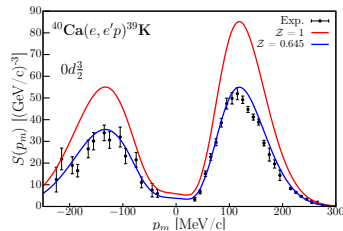


Electron scattering: the exclusive $^{40}\text{Ca}(e, e'p)^{39}\text{K}$ reaction

- Electron interaction means clean knockout reactions
- Cross section is closely tied to the boundstate wavefunction
- Evidence of physics beyond mean-field: Spectroscopic Factor

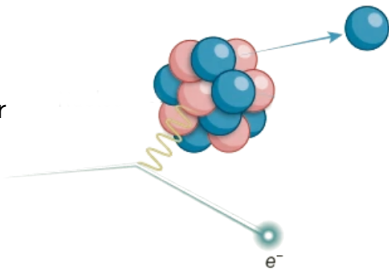


Old Mean-Field

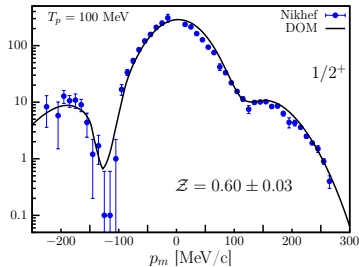
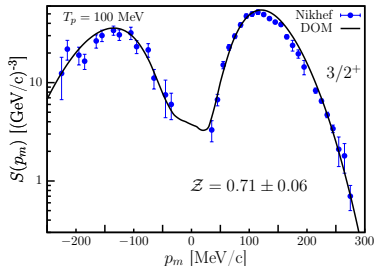


Electron scattering: the exclusive $^{40}\text{Ca}(e, e'p)^{39}\text{K}$ reaction

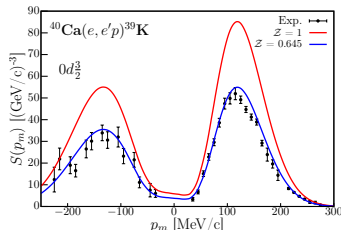
- Electron interaction means clean knockout reactions
- Cross section is closely tied to the boundstate wavefunction
- Evidence of physics beyond mean-field: Spectroscopic Factor
- DOM allows first consistent calculation



DOM Prediction



Old Mean-Field

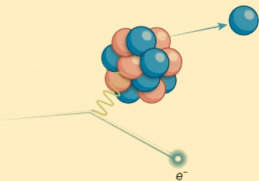


Now consider knockout with a hadronic probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$

- Proton knockout can be used in both stable and exotic nuclei

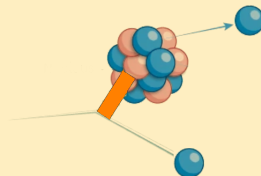
M.C. Atkinson *et al.*, *Front. Phys.* **12**, 1505982 (2025)

Electron probe: $^{40}\text{Ca}(e, e'p)^{39}\text{K}$



M.C. Atkinson *et al.*, *PRC* **105**, 014622 (2022)

Proton probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$

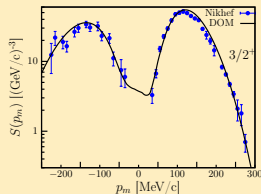


Now consider knockout with a hadronic probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$

- Proton knockout can be used in both stable and exotic nuclei

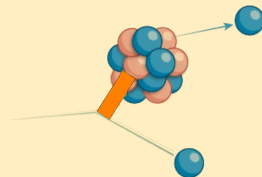
M.C. Atkinson *et al.*, Front. Phys. **12**, 1505982 (2025)

Electron probe: $^{40}\text{Ca}(e, e'p)^{39}\text{K}$



M.C. Atkinson *et al.*, PRC **105**, 014622 (2022)

Proton probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$

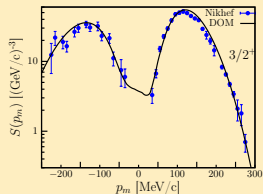


Now consider knockout with a hadronic probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$

- Proton knockout can be used in both stable and exotic nuclei

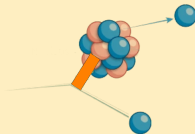
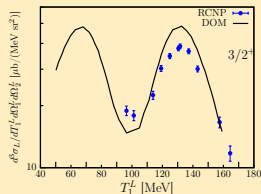
M.C. Atkinson *et al.*, Front. Phys. **12**, 1505982 (2025)

Electron probe: $^{40}\text{Ca}(e, e'p)^{39}\text{K}$



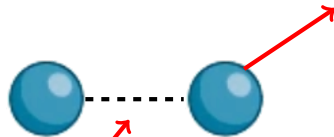
M.C. Atkinson *et al.*, PRC **105**, 014622 (2022)

Proton probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$



Now consider knockout with a hadronic probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$

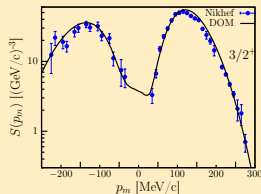
- Proton knockout can be used in both stable and exotic nuclei
- Discrepancy caused by using free V_{pp}



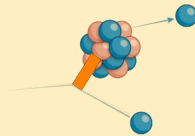
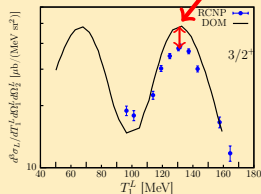
M.C. Atkinson *et al.*, Front. Phys. **12**, 1505982 (2025)

M.C. Atkinson *et al.*, PRC **105**, 014622 (2022)

Electron probe: $^{40}\text{Ca}(e, e'p)^{39}\text{K}$



Proton probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$



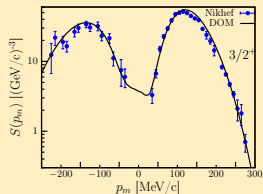
Now consider knockout with a hadronic probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$

- Proton knockout can be used in both stable and exotic nuclei
- Discrepancy caused by using free V_{pp}
- Dress V_{pp} to include effect of proton propagating through nucleus (propagator G_{DOM})

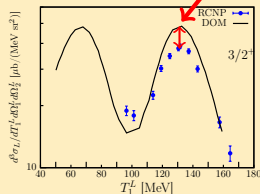
M.C. Atkinson *et al.*, Front. Phys. **12**, 1505982 (2025)

M.C. Atkinson *et al.*, PRC **105**, 014622 (2022)

Electron probe: $^{40}\text{Ca}(e, e'p)^{39}\text{K}$

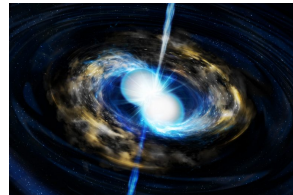
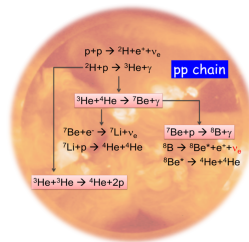
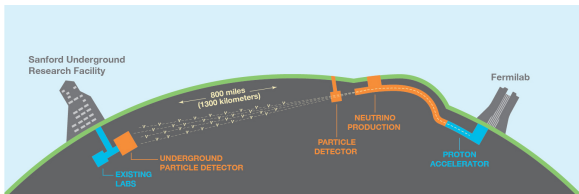
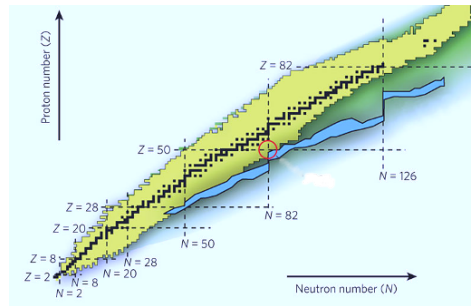


Proton probe: $^{40}\text{Ca}(p, 2p)^{39}\text{K}$



Outlook

- Use dressed V_{pp} in reactions throughout nuclear chart
- Synergizes with global DOM effort
 - Reactions with exotic nuclei (e.g. ^{52}Ca)
- Investigate e and ν scattering with ^{40}Ar for DUNE
- Work already begun to analyze JLab experiment on $^{40}\text{Ar}(e, e'p)^{39}\text{Cl}$
- Investigate impact of V_{NN} on Helium fusion



Thanks

- Willem Dickhoff
- Robert Charity
- Hossein Mahzoon
- Lee Sobotka
- Natalie Calleya



- Petr Navratil



- Sofia Quaglioni
- Kostas Kravvaris
- Cole Pruitt



- Louk Lapikás
- Henk Blok



- Kazuyuki Ogata



- Kazuki Yoshida

