The Electroweak Chiral Lagrangian (HEFT):

Implications for BSM Higgs Physics

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Standard Model EFT meets Chiral EFT , TRIUMF, Vancouver 29 Sep- 3 Oct 2025

Content of this talk is devoted to the purpose of this meeting

Standard Model EFT meets Chiral EFT :
Main purpose of this talk is not reviewing but generating discussions between SMEFT/HEFT communities

Content organized through four main issues:

- ★ Basics of the EChL
- ★ Introducing Loops: Renormalization in the HEFT. RGEs. Running coefficients
- ★ Matching HEFT to UV theories. Matching HEFT to SMEFT
- ★ Pheno implications for MultiHiggs production at colliders (focus on HH and HHH)

Basics of the EChL

The Electroweak Chiral Lagrangian: Before Higgs discovery (I)

Let's go back in time to the origin ……….around 35-40 years ago !

Central Point of Chiral Lagrangians: Non Linearity of Goldstone Bosons Dynamics

Seminal works of the EChL:

S. Weinberg, *Phenomenological Lagrangians, Physica* A96, 327 (1979)

T. Appelquist and C. Bernard, *Strongly Interacting Higgs Bosons*, PRD22, 200 (1980)

A.Longhitano, *Heavy Higgs Bosons in the Weinberg Salam Model*, PRD22, 1166 (1980)

M.Chanowitz and M.K.Gaillard, *The TeV Physics of Strongly Interacting W's and Z's*, NPB261, 379 (1985)

A.Dobado and M.J. Herrero, *Phenomenological Lagrangian Approach to the SBS of the SM*, PLB228,495(1989)

A.Dobado, D.Espriu and M.J.Herrero, *Chiral Lagrangians as a tool to probe the SSB of the SM at LEP*, PLB255,405(1991)

F. Feruglio , *The chiral approach to the electroweak interactions*, Int. J. Mod. Phys. A 08, 4937 (1993).

GBs self couplings given by a derivative expansion, like in low energy QCD. Parallelism EChL↔ChL.

The building of this Effective Field Theory followed the guidelines of ChPT (Gasser, Leuwyler , Annals Phys.158(1984)142)

The 3 GBs are those of Spontaneous Chiral Symmetry breaking $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$

GBs transform non-linearly ; U $(2 \times 2$ matrix) transforms linearly $U \to LUR^+$

EChL

Chidim2 $O(\partial^2)$ Chidim4 $O(\partial^4)$

$$\mathscr{L}^{EW}_{EChL} = \mathscr{L}^{EW}_2 + \mathscr{L}^{EW}_4 + .. \qquad U(\omega_a) = e^{i\omega_a \tau_a / v}$$

$$\mathscr{L}^{EW}_2 = \frac{v^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^+) \; ; v = 246\,\mathrm{GeV}$$

Perturb. expansion in $\left(\frac{p}{\Lambda_{EChL}}\right)^n$; $4\pi v \sim 3\,\mathrm{TeV}$ ← Scale of chiral loops

$\Lambda_{EChL} = \mathrm{Min}\left(4\pi v, M_{resonances}\right)$

$SU(2)_{L+R}$ is EW isospin (custodial symmetry $\to \rho \sim 1$)

ChL

$$\mathscr{L}^{QCD}_{ChL} = \mathscr{L}^{QCD}_2 + \mathscr{L}^{QCD}_4 + .. \qquad U(\pi_a) = e^{i\pi_a \tau_a / f_\pi}$$

$$\mathscr{L}^{QCD}_2 = \frac{f_\pi^2}{4} \mathrm{Tr}(\partial_\mu U \partial^\mu U^+) \; ; f_\pi = 0.094\,\mathrm{GeV}$$

Perturb. expansion in $\left(\frac{p}{\Lambda_{ChL}}\right)^n$; $4\pi f_\pi \sim 1.2\,\mathrm{GeV}$ ← Scale of chiral loops

$\Lambda_{ChL} = \mathrm{Min}\left(4\pi f_\pi, M_{resonances}\right)$

$SU(2)_{L+R}$ is isospin symmetry of pion physics

The Electroweak Chiral Lagrangian: Before Higgs discovery (II)

Central Physics Issue of Chiral Lagrangians:
Non Linearity of GBs Dynamics is not just a choice of representation…
It reflects the Strongly Interacting Character of the Underlying Fundamental Theory.

As in ChPT of QCD,   the multi-GBs interactions in the EChL all grow with energy

$$\mathscr{L}_2^{\mathrm{EW}} = \frac{v^2}{4}\mathrm{Tr}(\partial_\mu U \partial^\mu U^+)\ ;\ v = 246\,\mathrm{GeV} \qquad U(\omega_a) = e^{i\omega_a \tau_a / v}$$

Example (4 GB's interaction vertex at LO):

$$i\Gamma_{\omega^+\omega^-\omega^+\omega^-} = -\frac{i}{3v^2}\left(2p_+\cdot p'_+ + 2p_-\cdot p'_- - (p_+ + p'_+)\cdot(p_- + p'_-)\right) \sim \mathscr{O}\left(\frac{s}{v^2}\right)$$

In contrast to SM (GBs linear ints) $\sim -2i\frac{m_{\mathrm{H}}^2}{v^2} \sim \mathscr{O}(s^0)$

and to SMEFT (GBs linear ints)

Simmilarly for higher number of GBs: 4GBs, 6 GBs, etc…self interactions at LO all grow with energy as $\mathscr{O}(s)$; at NLO as $\mathscr{O}(s^2)$; at NNLO as $\mathscr{O}(s^3)$; etc …

The growing with energy of GBs self-interactions announce Strongly Interacting Dynamics in GB's scattering and the potential emergent resonances. As it occurs in low energy QCD with $\pi\pi$ scattering

→

Enhancements in multiple boson production at colliders in TeV region with respect to SM rates

Generic BSM signals within EChL

The Electroweak Chiral Lagrangian: Before Higgs discovery (III)

Introducing EW gauge bosons in the EChL with $SU(2)_L \times U(1)_Y$ EW gauge symmetry

Bosons Sector : A.Longhitano, *Heavy Higgs Bosons in the Weinberg Salam Model*, PRD22, 1166 (1980)

Fermions introduced later: F. Feruglio , The chiral approach to the electroweak interactions, Int. J. Mod. Phys. A 08, 4937 (1993).

Here we consider Bosonic EChL (CP)

$\mathscr{L}^{\rm Bosonic}_{\rm EChL} = \mathscr{L}_2 + \mathscr{L}_4 + ..$

Same GBs $\omega_i\,(i=1,2,3)$

$SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$ $\quad SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$

CS in pure GB sector

CS Breaking by $g' \neq 0$

$$\mathscr{L}_2 = \frac{v^2}{4}{\rm Tr}[D_\mu U^\dagger D^\mu U] - \frac{1}{2g^2}{\rm Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] - \frac{1}{2g'^2}{\rm Tr}[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}] + \mathcal{L}_{GF} + \mathcal{L}_{FP}$$

(R_ξ)

$\mathscr{L}_4^{\rm Longhitano}(a_i) = \sum \mathscr{L}_i$; $a_i =$ EChL coefficients (dimensionless) (i=0..13 if CP)

non-CS

$$\mathscr{L}_0 = a_0\,(m_Z^2 - m_W^2)\,{\rm Tr}\Big[T\mathscr{V}_\mu\Big]{\rm Tr}\Big[T\mathscr{V}^\mu\Big]$$

CS

$$\mathscr{L}_1 = a_1{\rm Tr}\Big[U\hat{B}_{\mu\nu}U^\dagger\hat{W}^{\mu\nu}\Big] \quad \mathscr{L}_2 = ia_2{\rm Tr}\Big[U\hat{B}_{\mu\nu}U^\dagger[\mathscr{V}^\mu,\mathscr{V}^\nu]\Big] \quad \mathscr{L}_3 = -ia_3{\rm Tr}\Big[\hat{W}_{\mu\nu}[\mathscr{V}^\mu,\mathscr{V}^\nu]\Big]$$

$$\mathscr{L}_4 = a_4{\rm Tr}\Big[\mathscr{V}_\mu\mathscr{V}_\nu\Big]{\rm Tr}\Big[\mathscr{V}^\mu\mathscr{V}^\nu\Big] \quad \mathscr{L}_5 = a_5{\rm Tr}\Big[\mathscr{V}_\mu\mathscr{V}^\mu\Big]{\rm Tr}\Big[\mathscr{V}_\nu\mathscr{V}^\nu\Big]$$

non-CS

$$\mathscr{L}_6 = a_6{\rm Tr}\Big[\mathscr{V}_\mu\mathscr{V}_\nu\Big]{\rm Tr}\Big[T\mathscr{V}^\mu\Big]{\rm Tr}\Big[T\mathscr{V}^\nu\Big] \quad \mathscr{L}_7 = a_7{\rm Tr}\Big[\mathscr{V}_\mu\mathscr{V}^\mu\Big]{\rm Tr}\Big[T\mathscr{V}^\nu\Big]{\rm Tr}\Big[T\mathscr{V}_\nu\Big]$$

$$\mathscr{L}_8 = -\frac{a_8}{4}{\rm Tr}\Big[T\hat{W}_{\mu\nu}\Big]{\rm Tr}\Big[T\hat{W}^{\mu\nu}\Big] \quad \mathscr{L}_9 = -i\frac{a_9}{2}{\rm Tr}\Big[T\hat{W}_{\mu\nu}\Big]{\rm Tr}\Big[T[\mathscr{V}^\mu,\mathscr{V}^\nu]\Big]$$

$$\mathscr{L}_{10} = a_{10}{\rm Tr}\Big[T\mathscr{V}^\mu\Big]{\rm Tr}\Big[T\mathscr{V}_\mu\Big]{\rm Tr}\Big[T\mathscr{V}^\nu\Big]{\rm Tr}\Big[T\mathscr{V}_\nu\Big]$$

Removable by e.o.m

$$\mathscr{L}_{11} = a_{11}{\rm Tr}\Big[\mathscr{D}_\mu\mathscr{V}^\mu\mathscr{D}_\nu\mathscr{V}^\nu\Big]$$

$$\mathscr{L}_{12} = a_{12}{\rm Tr}\Big[T\mathscr{D}_\mu\mathscr{D}_\nu\mathscr{V}^\nu\Big]{\rm Tr}\Big[T\mathscr{V}^\mu\Big] \quad \mathscr{L}_{13} = \frac{a_{13}}{2}{\rm Tr}\Big[T\mathscr{D}_\mu\mathscr{V}_\nu\Big]{\rm Tr}\Big[T\mathscr{D}^\mu\mathscr{V}^\nu\Big]$$

$L_1 = a_5$
$L_2 = a_4$
$L_9 = a_3 - a_2$
$L_{10} = a_1$

Gasser, Leutwyler Annals Phys 158(1984)142

a_4, a_5 the most relevant CS

$\mathscr{L}_{4,5} \sim \mathcal{O}(\partial^4)$

Light bosonic fields: GBs ω_i and gauge bosons, W^i_μ and B_μ (Higgs less)

$$U = \exp\left(i\frac{\omega_i\tau_i}{v}\right)$$

(other non-linear reps. in literature)

$U \to LUR^+$

$$\hat{W}_\mu = \tfrac{g}{2}W^i_\mu\tau^i \qquad \hat{B}_\mu = \tfrac{g'}{2}B_\mu\tau^3$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp iW^2_\mu)\,, \quad Z_\mu = c_W W^3_\mu - s_W B_\mu\,, \quad A_\mu = s_W W^3_\mu + c_W B_\mu$$

Building Blocks

$$D_\mu U = \partial_\mu U - i\hat{W}_\mu U + iU\hat{B}_\mu$$

$$\mathcal{V}_\mu = (D_\mu U)U^\dagger\,, \quad \mathcal{D}_\mu O = \partial_\mu O + i[\hat{W}_\mu, O] \quad T = U\tau_3 U^\dagger$$

$$\hat{W}_{\mu\nu} = \partial_\mu\hat{W}_\nu - \partial_\nu\hat{W}_\mu - i[\hat{W}_\mu,\hat{W}_\nu] \quad \hat{B}_{\mu\nu} = \partial_\mu\hat{B}_\nu - \partial_\nu\hat{B}_\mu$$

Chidim counting: masses and derivatives count equally (as in ChL-QCD)

$$\partial_\mu,\quad m_W,\quad m_Z,\quad gv,\quad g'v \sim \mathcal{O}(p)$$

$$D_\mu U\,,\ \mathcal{V}_\mu\,,\hat{W}_\mu,\ \hat{B}_\mu \sim \mathcal{O}(p)$$

$$\mathcal{D}_\mu\mathcal{V}_\nu\,,\ \hat{W}_{\mu\nu},\ \hat{B}_{\mu\nu} \sim \mathcal{O}(p^2)$$

The Electroweak Chiral Lagrangian: After Higgs discovery (I)

To describe BSM Physics we focus on the Bosonic EChL $\mathscr{L}_{\rm HEFT} = \mathscr{L}^{\rm Bosonic}_{\rm EChL}$

The Bosonic EChL is reformulated including explicitly a light Higgs particle in addition to GBs and EW gauge bosons

(Our) Assumptions

CP, Lorentz , Gauge $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$, Global $SU(2)_L \times SU(2)_R \to SU(2)_{L+R}$

CS restored for $g' \to 0$
$(M_Z \to M_W, c_W \to 1)$
also called isospin limit

H is a singlet under all symmetries It is introduced via Polynomials in powers of $\left(\frac{H}{v}\right)^n$, and their derivatives

GB's are in non-linear representation $U(\omega^a) = e^{\omega^a \tau^a / v}$ $U \to L U R^+$

EW gauge bosons introduced via the gauge principle

Fermions assumed as in the SM,
i.e. no new interactions with fermions

Building Blocks

$$D_\mu U = \partial_\mu U - i\hat{W}_\mu U + iU\hat{B}_\mu \qquad \mathcal{V}_\mu = (D_\mu U)U^\dagger \, , \; \mathcal{D}_\mu O = \partial_\mu O + i[\hat{W}_\mu, O]$$

$$\hat{W}_\mu = \frac{g}{2} W^i_\mu \tau^i \qquad \hat{B}_\mu = \frac{g'}{2} B_\mu \tau^3 \qquad H \qquad \partial_\mu H$$

$$\hat{W}_{\mu\nu} = \partial_\mu \hat{W}_\nu - \partial_\nu \hat{W}_\mu - i[\hat{W}_\mu, \hat{W}_\nu] \qquad \hat{B}_{\mu\nu} = \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu) \, , \quad Z_\mu = c_W W^3_\mu - s_W B_\mu \, , \quad A_\mu = s_W W^3_\mu + c_W B_\mu$$

Chiral dim counting

$$\partial_\mu , m_W, m_Z , m_H, \lambda v , g v , g' v \sim \mathcal{O}(p)$$

$$D_\mu U \, , \; \mathcal{V}_\mu \, , \hat{W}_\mu, \; \hat{B}_\mu \sim \mathcal{O}(p)$$

$$\mathcal{D}_\mu \mathcal{V}_\nu \, , \; \hat{W}_{\mu\nu}, \; \hat{B}_{\mu\nu} \sim \mathcal{O}(p^2)$$

chiral dim

$$\mathscr{L}_{\rm HEFT}(a_i) = \mathscr{L}_2 + \mathscr{L}_4 + ..$$

a_i dimensionless $\quad$ H singlet

In contrast to

$$\mathscr{L}_{\rm SMEFT}(c_i) = \mathscr{L}_{\rm SM} + \mathscr{L}_6 + \mathscr{L}_8 + \ldots$$

$$\mathscr{L}_d = \sum_i \frac{c_i}{\Lambda^{d-4}} \mathcal{O}^i_d$$

canonical dim d

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\phi^0) \end{pmatrix}$$

c_i dimensionless $\quad$ H in doublet Φ

The Electroweak Chiral Lagrangian: After Higgs discovery (II)

Bosonic HEFT

$$\mathscr{L}^{\rm Bosonic}_{\rm EChL} = \mathscr{L}_2 + \mathscr{L}_4 + ..$$

$$\mathscr{L}_2 = \frac{v^2}{4}\left(1 + 2a\frac{H}{v} + b\frac{H^2}{v^2} + c\frac{H^3}{v^3} + ..\right) {\rm Tr}[D_\mu U^\dagger D^\mu U] + \frac{1}{2}\partial_\mu H \partial^\mu H - V(H) + \mathscr{L}_{\rm GF} + \mathscr{L}_{\rm FP}$$

$$V(H) = \frac{1}{2}m_H^2 H^2 + \kappa_3 \lambda v H^3 + \kappa_4 \frac{\lambda}{4} H^4$$

$$\mathcal{L}_{\rm GF} = -F_+ F_- - \frac{1}{2}F_Z^2 - \frac{1}{2}F_A^2$$

$$\mathcal{L}_{\rm FP} = \sum_{i,j=+,-,Z,A} \bar{c}^i \frac{\delta F_i}{\delta \alpha_j} c^j ,$$

$$F_\pm = \frac{1}{\sqrt{\xi}}(\partial^\mu W^\pm_\mu - \xi m_W \pi^\pm),$$

$$F_Z = \frac{1}{\sqrt{\xi}}(\partial^\mu Z_\mu - \xi m_Z \pi^3), \qquad F_A = \frac{1}{\sqrt{\xi}}(\partial^\mu A_\mu)$$

Gauge fixing functions in R_ξ

$$\mathscr{L}_4(a_i) = \sum_i \mathscr{F}_i(H)\, \mathscr{L}_i^{\rm Longhitano} +$$

Polynomials in H

Additional invariant terms built with the basic blocks

Now including also H's and ∂H's

On the relevance of the a_i's.....

For a complete list of operators, see Brivio et al 1311.1823

For a review, see : Brivio, Trott 1706.08945

a_4 , a_5 the most relevant for VBS, $VV \to VV$

$$-a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H\, \partial^\nu H}{v^2}{\rm Tr}\Big[\mathcal{V}_\mu \mathcal{V}_\nu\Big] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H\, \partial_\mu H}{v^2}{\rm Tr}\Big[\mathcal{V}^\nu \mathcal{V}_\nu\Big] + a_{dddd}\frac{1}{v^4}\partial_\mu H \partial^\mu H\, \partial_\nu H \partial^\nu H$$

Relevant in $WW \to HH$ and $ZZ \to HH$

Relevant in $HH \to HH$

Other notations:

Delgado et al 1911.06844

Dobado, Espriu 1911.06844

Gasser , Leutwyler Annals Phys158(1984)142

$$\eta = e = a_{dd\mathcal{V}\mathcal{V}1}$$

$$\delta = d = a_{dd\mathcal{V}\mathcal{V}2}$$

$$\gamma = a_{dddd}$$

$$L_1 = a_5 , L_2 = a_4$$

Main message: there is a hierarchy in the HEFT based on energy arguments

$\mathscr{O}(\partial^2)$ terms in $\mathscr{L}_2$ give $\mathscr{A} \sim \mathscr{O}(s)$

$\mathscr{O}(\partial^4)$ terms in $\mathscr{L}_4$ give $\mathscr{A} \sim \mathscr{O}(s^2)$

Most relevant coeffs. at LO a, b, c

Most relevant coeffs. at NLO $a_5, a_4, \eta, \delta, \gamma, ...$

In contrast to SMEFT where :

to get $\mathscr{A} \sim \mathscr{O}(s^2)$ dim8 operators are needed

effects of $\mathscr{O}_8$ suppressed by $\left(\frac{1}{\Lambda^4}\right)$

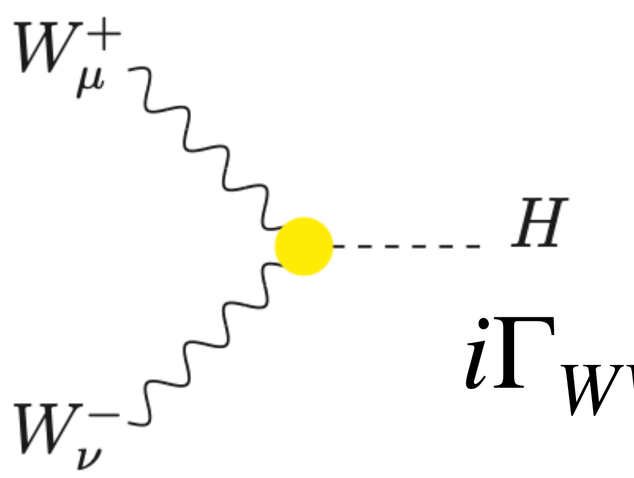
HH and HHH (EW) production with LO-HEFT: $a, b, c, \kappa_3, \kappa_4$

Easy connection of HEFT with the kappa formalism: **Choose Unitary gauge to see this connection (U=1)**

$$\mathcal{L}_{\rm HEFT}^{\rm LO} = \frac{v^2}{4}\left(1 + 2a\frac{H}{v} + b\frac{H^2}{v^2} + c\frac{H^3}{v^3}\right){\rm Tr}[D_\mu U^\dagger D^\mu U] + \frac{1}{2}\partial_\mu H \partial^\mu H - V(H) \; ; \quad m_H^2 = 2\lambda v^2 \; , \; m_W = gv/2 \; , \; m_Z = m_W/c_W$$

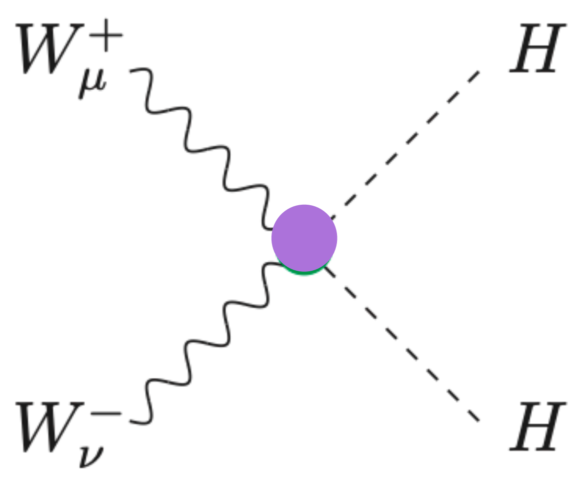
$$-\frac{1}{2g^2}{\rm Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}] - \frac{1}{2g'^2}{\rm Tr}[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}] + \mathcal{L}_{GF} + \mathcal{L}_{FP}, \quad V(H) = \frac{1}{2}m_H^2 H^2 + \kappa_3 \lambda v H^3 + \kappa_4 \frac{\lambda}{4} H^4$$

SM: $a = b = \kappa_3 = \kappa_4 = 1 \, , c = 0$ BSM: $a \neq 1 \, , b \neq 1 \, , \kappa_3 \neq 1 \, , \kappa_4 \neq 1 \, , c \neq 0$ (any of them provide BSM signals)



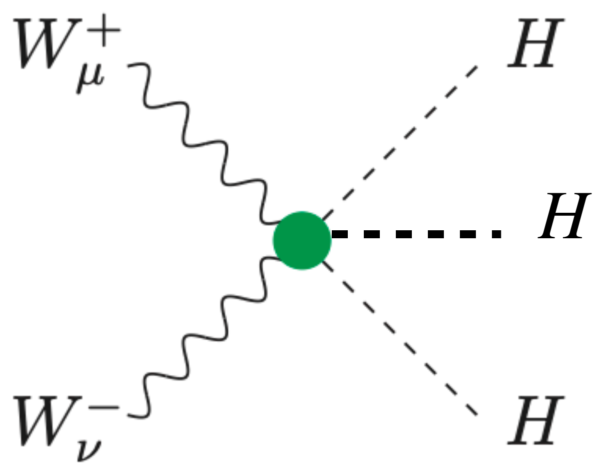
$a = \kappa_V$

$$i\Gamma_{WWH} = iagm_W g_{\mu\nu}$$



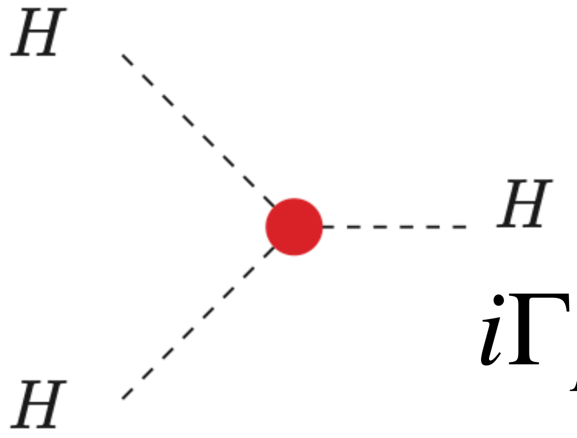
$b = \kappa_{2V}$

$$i\Gamma_{WWHH} = \frac{ibg^2}{2} g_{\mu\nu}$$



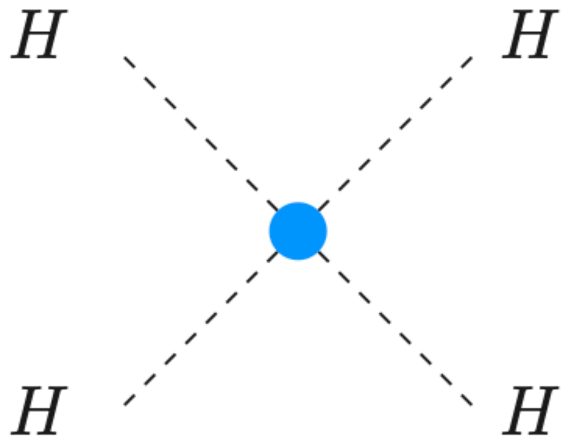
c

$$i\Gamma_{WWHHH} = \frac{i3cg^2}{2v} g_{\mu\nu}$$



$\kappa_3 = \kappa_\lambda$

$$i\Gamma_{HHH} = -i\kappa_3 6\lambda v$$



κ_4

$$i\Gamma_{HHHH} = -i\kappa_4 6\lambda$$

WW→HH gives access to a, b, κ_3

WW→HHH gives access also to c, κ_4

Constraints from ATLAS and CMS on kappa's

ATLAS (95%CL) : $\kappa_V \in [0.99, 1.11]$, $\kappa_{2V} \in [0.6, 1.5]$, $\kappa_\lambda \in [-1.2, 7.2]$,

Nature 607, 52 (2022)   PRL 133, 101801 (2024)

CMS (95%CL) : $\kappa_V \in [0.97, 1.09]$, $\kappa_{2V} \in [0.6, 1.4]$, $\kappa_\lambda \in [-1.2, 7.5]$,

Nature 607, 60 (2022)   PLB 861, 139210 (2025)

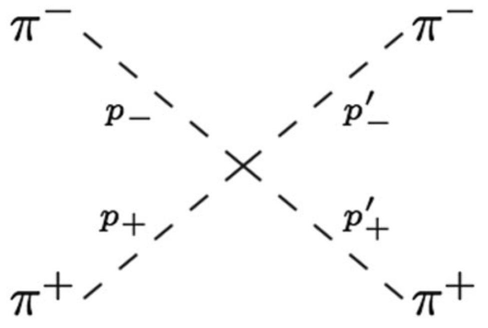
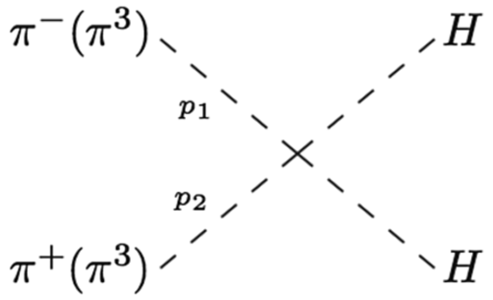
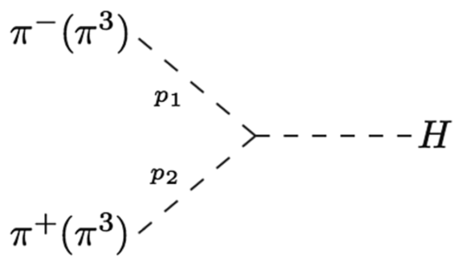
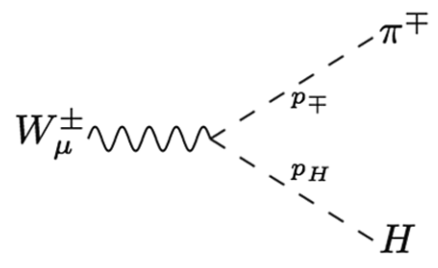
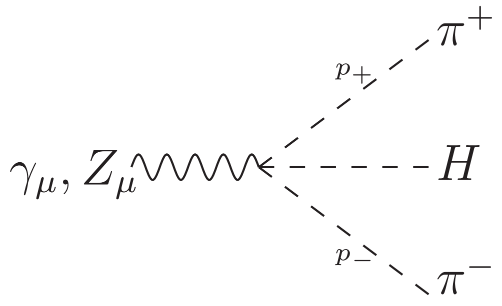
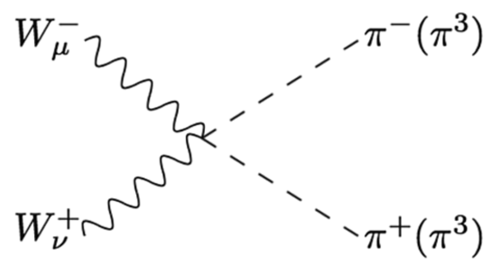
κ_4 practically unconstraint

c unconstraint

In HEFT with R_ξ gauge, diff interactions than in SM

LO-HEFT: different vertices than in SM (GBs non-linearity, H singlet)

Some examples: taken from M.Herrero, R. Morales, PRD104.075013(2021)

	LO-EChL $U(\pi^a) = e^{\pi^a \tau^a / v}$	**SM** $\Phi = \begin{pmatrix} i\pi^+ \\ \frac{v+H-i\pi_3}{\sqrt{2}} \end{pmatrix}$
	$-\frac{i}{3v^2}(2p + \cdot p'_+ + 2p_- \cdot p'_- - (p_+ + p'_+)\cdot(p_- + p'_-))$	$-2i\,\frac{m_H^2}{v^2}$
	$-\frac{2i}{v^2} b p_1 \cdot p_2$	$-i\,\frac{m_H^2}{v^2}$
	$-\frac{2i}{v} a p_1 \cdot p_2$	$-i\,\frac{m_H^2}{v}$
	$agp^\mu_\mp$	$\mp\frac{i}{2}g(p_\mp - p_0)^\mu$
	$2ia\frac{g}{v}\{s_W, \frac{c_W^2 - s_W^2}{2c_W}\}(p_- - p_+)^\mu$	0
	0	$\frac{i}{2}g^2 g^{\mu\nu}$
	H does NOT interact with ghosts !!	H interacts with ghosts

EChL and SM provide

Gauge Invariant amplitudes

$\mathscr{A}_{\text{unitary-gauge}} = \mathscr{A}_{R_\xi\text{-gauge}}$

The κ framework does not because GBs are not included

Gauge Invariance checked analytically in several processes: (Herrero,Morales)

$H \to \gamma\gamma\,,\gamma Z$ PRD102.075040(2020)

$WZ \to WZ$ PRD104.075013(2021)

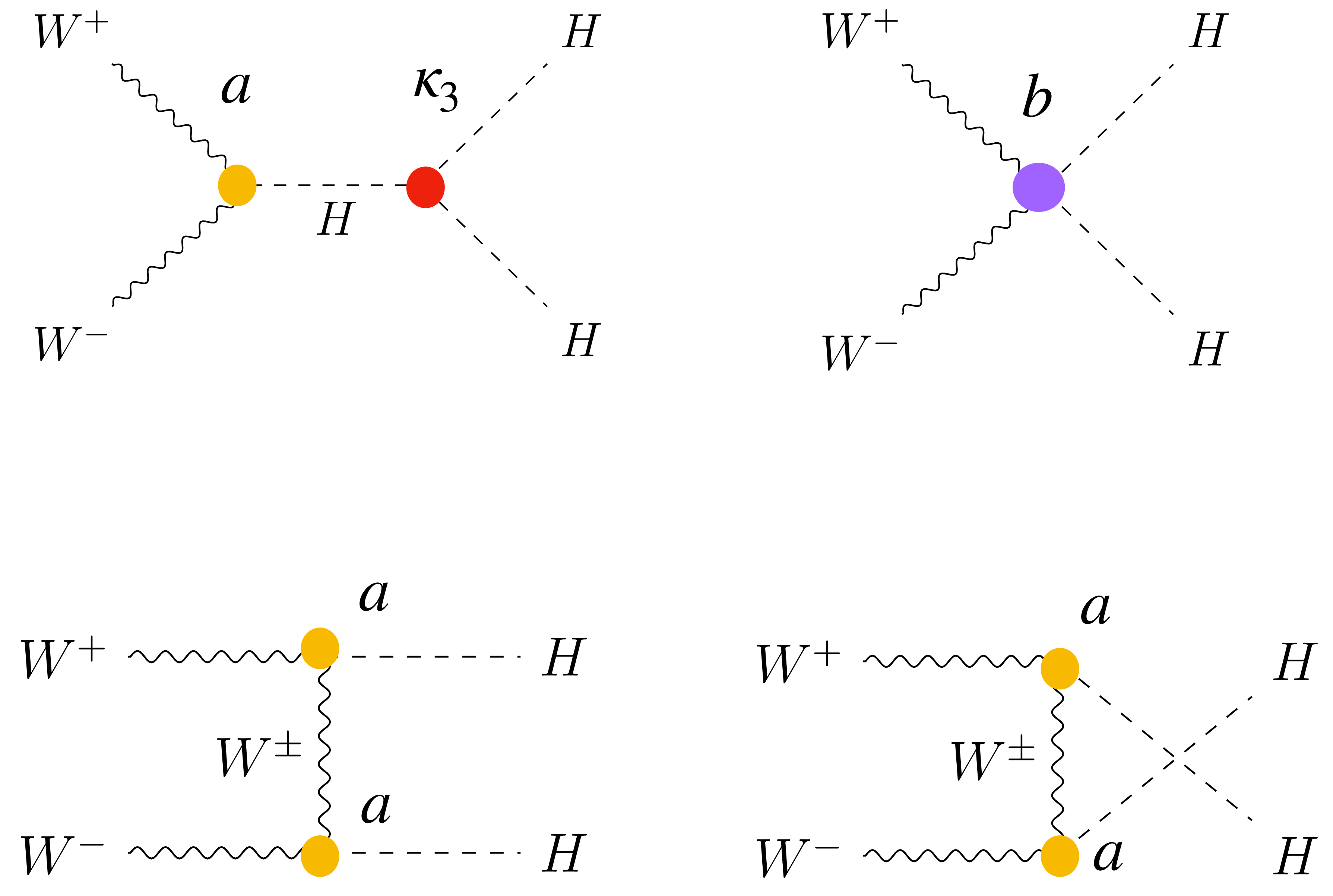
$WW \to HH$ PRD106.073008(2022)

$WW \to HHH$ Domenech et al 2506.21716

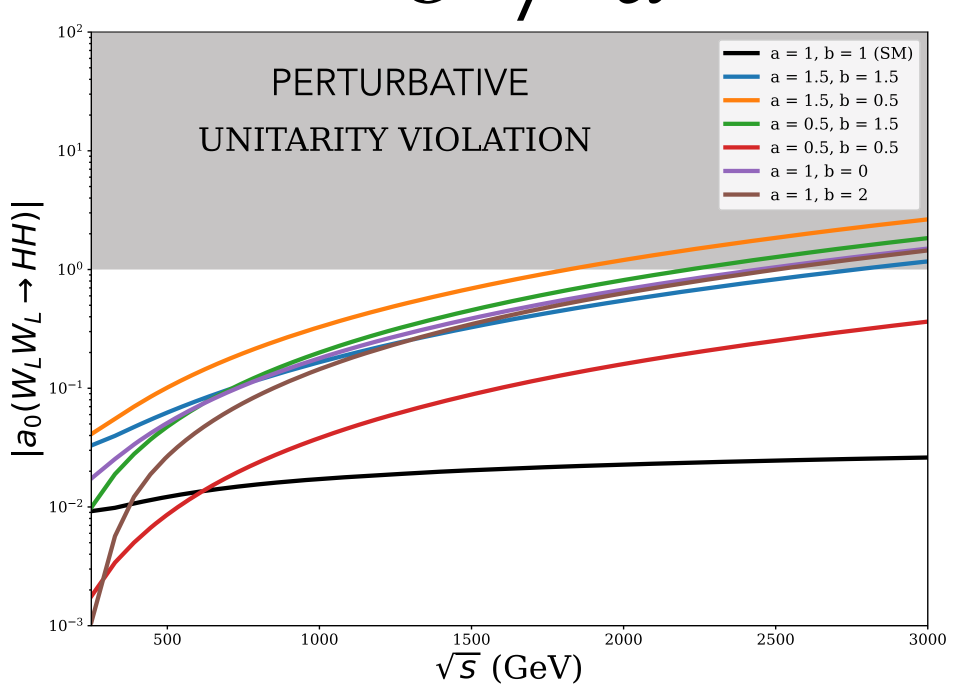
WW→HH gives access to a, b, κ_3 (LO-HEFT)

Important Issue 1): When looking for enhancements respect to SM not all the coefficients are equally relevant

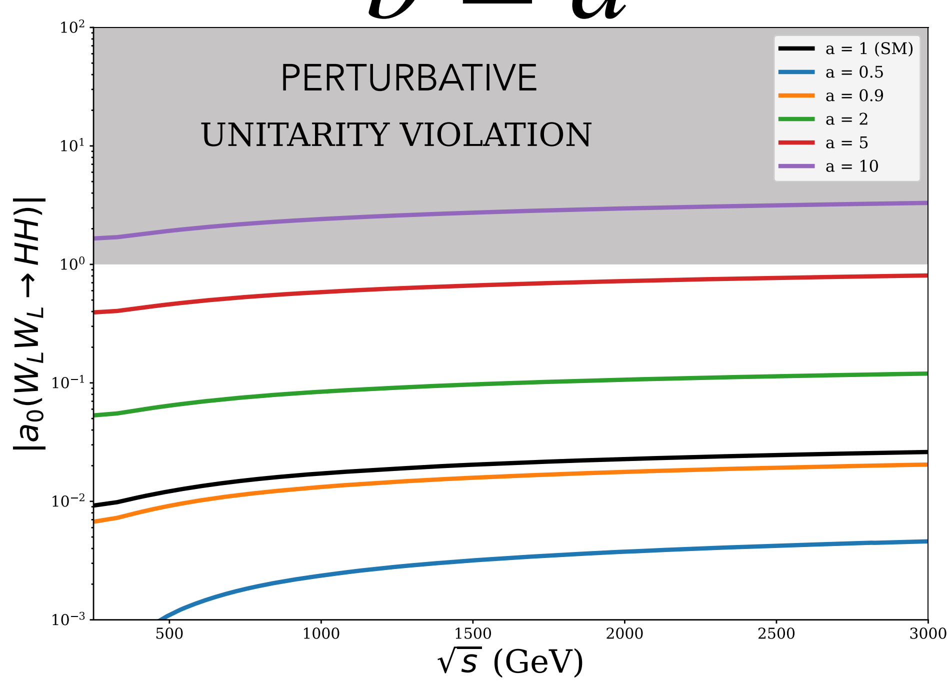
DIAGRAMS IN UNITARY GAUGE



$b \neq a^2$



$b = a^2$



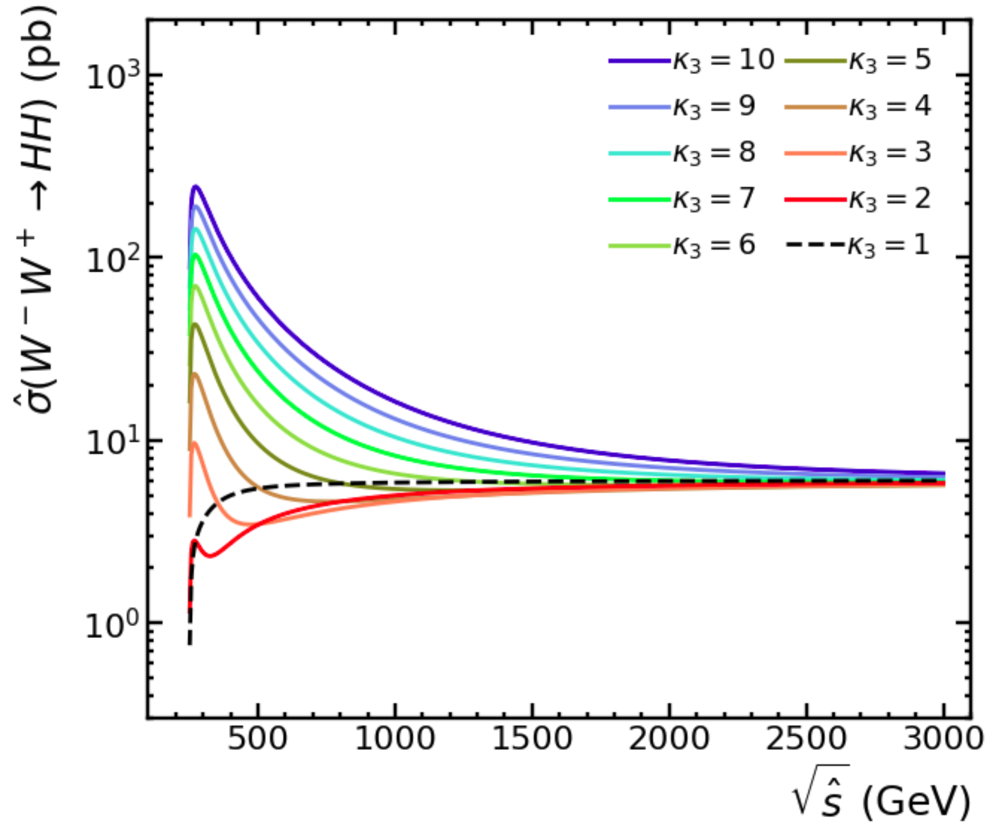
a, b

grow with energy

Unitarity Violation

Unless close to:

$b = a^2$



κ_3

Unitarity Preserved

Only relevant for low energy close to HH threshold

The role of $(a^2 - b)$ in $WW \to HH$ at LHC already studied in 2010 See Contino, et al 1002.1011

The most relevant are $a = \kappa_V$, $b = \kappa_{2V}$

Explained by behavior with energy

For $\sqrt{s} \gg m_W, m_H$

κ_3 effects are subleading

$$A^{\text{HEFT}}(W_L W_L \to HH) \simeq -\frac{1}{v^2}(a^2 - b)s$$

Important Issue 2): Unitary Violation if $a^2 - b \neq 0$

Important Issue 3): Gauge Invariance of the HEFT amplitude checked analitically $\mathscr{A}_{\text{unitary-gauge}} = \mathscr{A}_{R_\xi\text{-gauge}}$

See, for instance, 2208.05900, 2506.2171

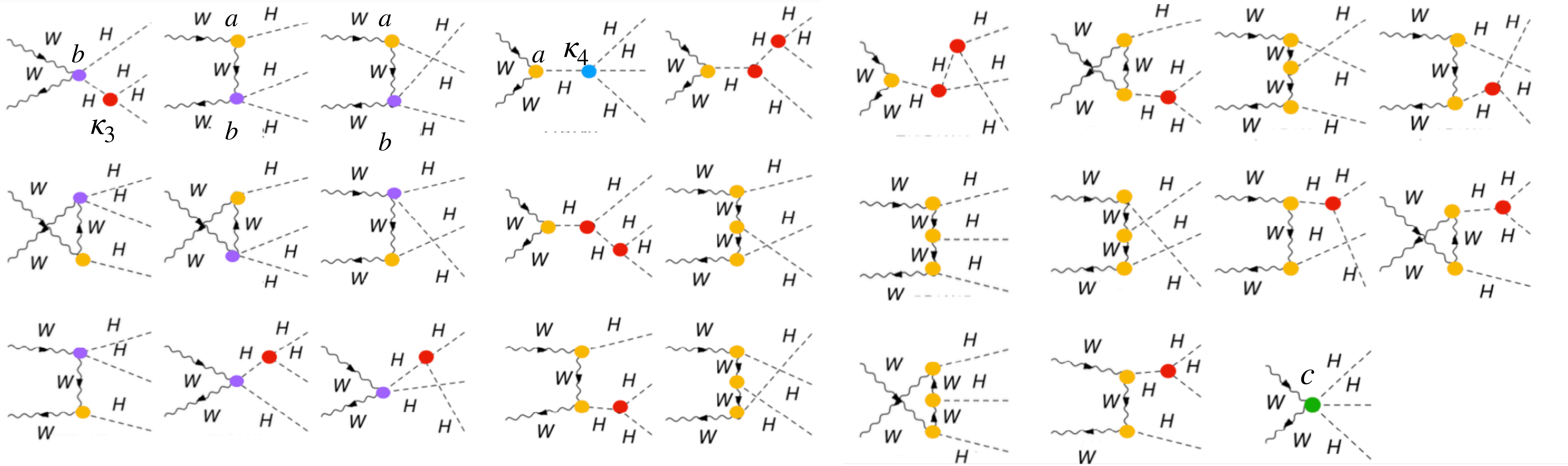
Kappa framework should not be be applied in R_ξ gauges
(for instance, in Feynman gauge which is the most frequently used in EW theory)

Similarly for ZZ→HH

WW → HHH gives access to $a, b, c, \kappa_3, \kappa_4$ (LO-HEFT)

DIAGRAMS IN UNITARY GAUGE

1) larger enhancements respect to SM from a, b, c than from κ_3, κ_4



The most relevant are

$a = \kappa_V$, $b = \kappa_{2V}$, c

Explained by behavior with energy

For $\sqrt{s} \gg m_W, m_H$

κ_3 and κ_4 effects are subleading

Only relevant close to HHH threshold

$$A^{\rm HEFT}(W_L W_L \to HHH) \simeq \frac{3}{v^3}\left(c + \frac{4}{3}a(a^2 - b)\right)s$$

Domenech et al 2506.2171 (full) Delgado el at 2311.04280 (scalar)

2): Unitary Violation if $c + \frac{4}{3}a(a^2 - b) \neq 0$

3) Gauge Invariance of the amplitude checked analytically $\mathscr{A}_{\rm unitary-gauge} = \mathscr{A}_{R_\xi - \rm gauge}$

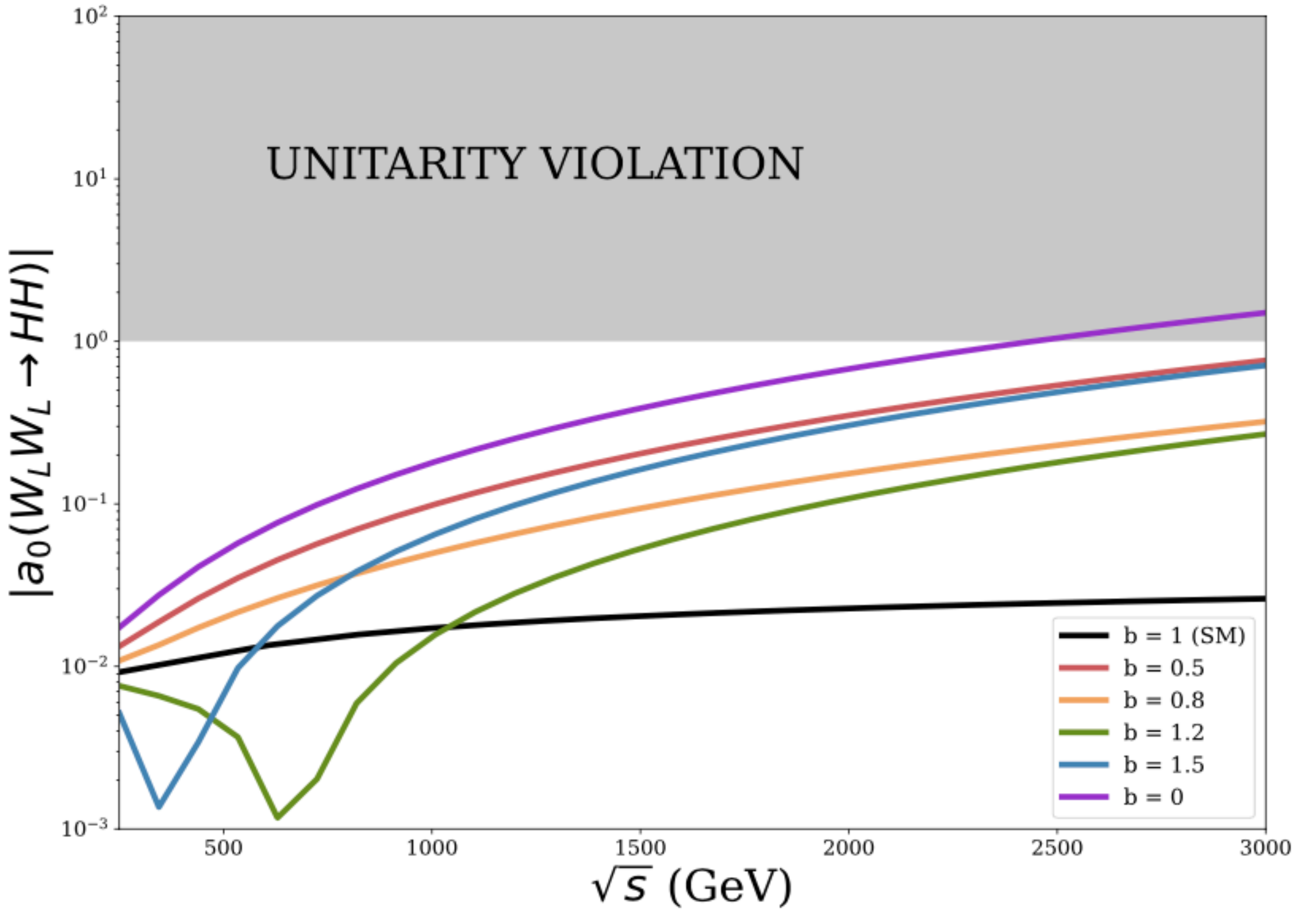
Domenech et al 2506.2171

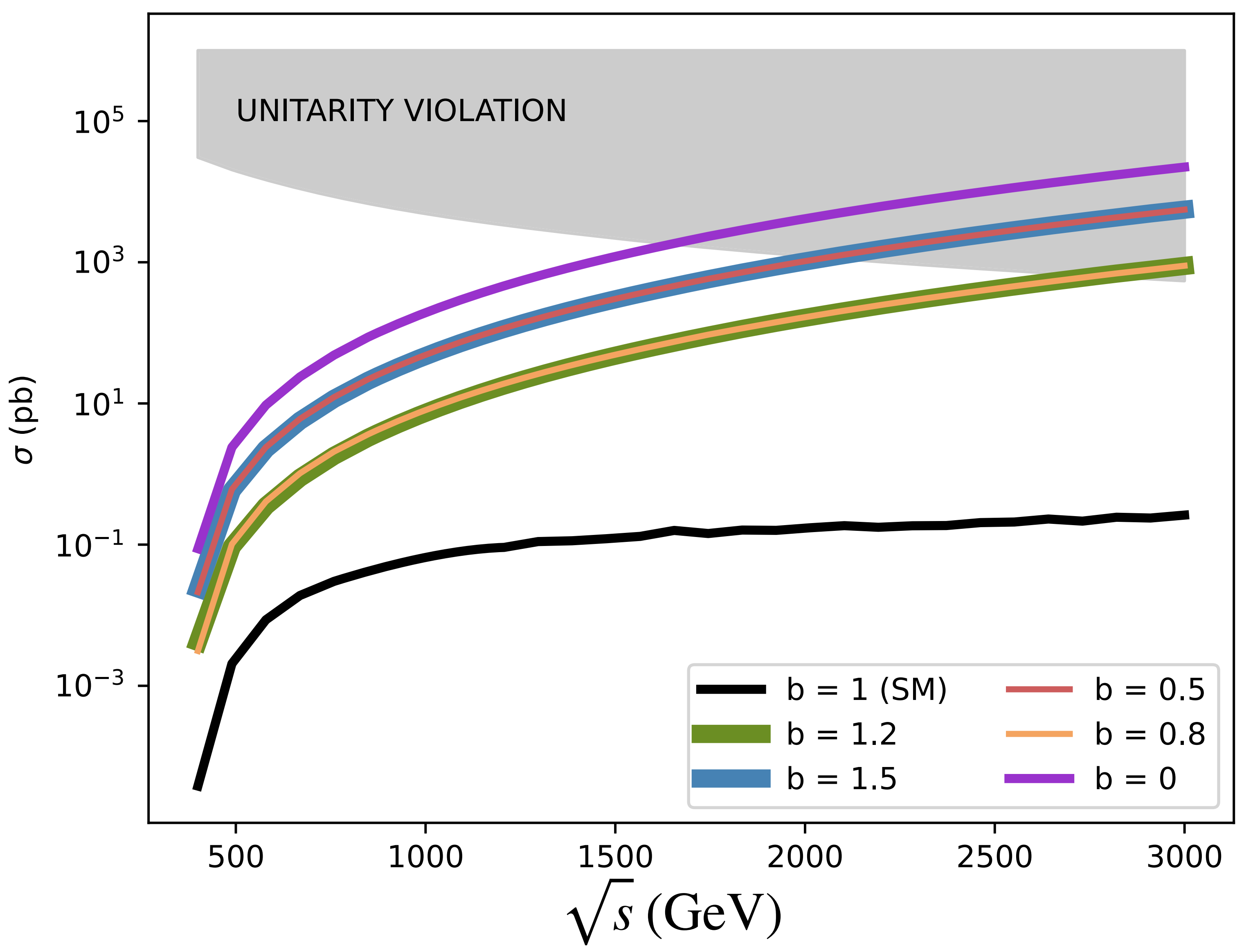
Similarly for ZZ→HHH

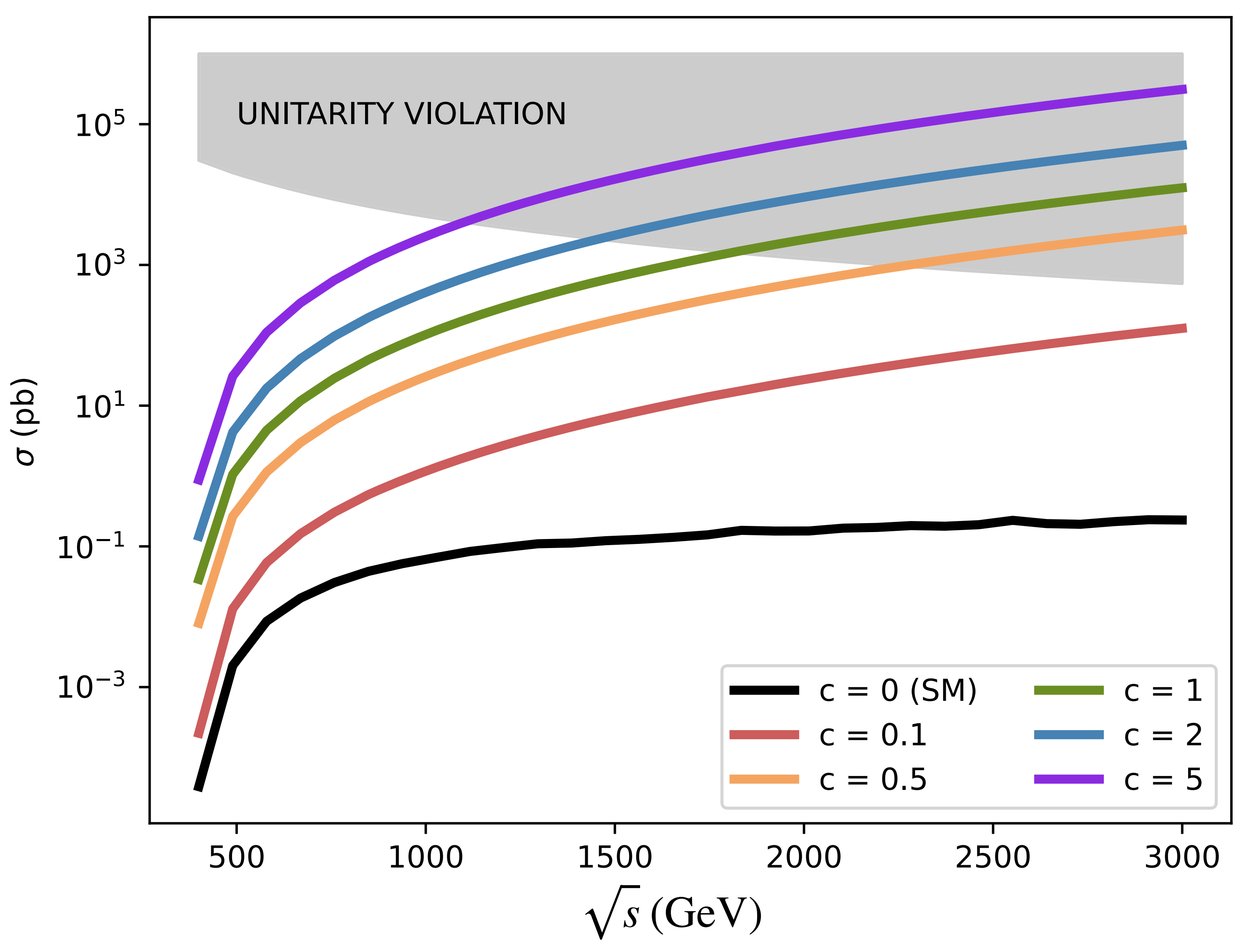
Unitarity in Multiple Higgs production with LO-HEFT

2407.20706, Phys. Rev. D 111, 055004 (2025), Anisha, Domenech, Englert, Herrero, Morales

$WW \to HH$ b coeff. $WW \to HHH$ b coeff. $WW \to HHH$ c coeff.







Restrictive constraints on c and b from unitarity

a already restricted from single H prod

When computing production rates at colliders, we will be restricted by unitarity

$$\epsilon_U = \frac{\text{rates preserving unitarity}}{\text{total rates}}$$

At LHC, we find important reduction factors

For HL-LHC events $6b2j$

$\epsilon_U \simeq 0.6$ **for** $b \in [0.5, 1.5]$

$\epsilon_U \simeq 0.7 - 0.5$ **for** $c \in [0.5, 1]$

UNITARITY CONDITIONS

$$|a_0(W_L W_L \to HH)(s)| \leq 1$$

$$\sigma(W_L W_L \to HHH)(s) \leq \frac{4\pi}{s}$$

NLO-HEFT relevant operators for HH

2208.05900, Phys. Rev. D 106(2022), 073008, Herrero, Morales (WW to HH)

$$\mathcal{V}_\mu = (D_\mu U)U^\dagger$$

$$\mathscr{L}_{\rm HEFT}^{\rm NLO+e.o.m} = \mathscr{L}^{\rm LO} - a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H\,\partial^\nu H}{v^2}{\rm Tr}\Big[\mathcal{V}_\mu\mathcal{V}_\nu\Big] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H\,\partial_\mu H}{v^2}{\rm Tr}\Big[\mathcal{V}^\nu\mathcal{V}_\nu\Big]$$
$$-\frac{m_{\rm H}^2}{4}\left(2a_{H\mathcal{V}\mathcal{V}}\frac{H}{v} + a_{HH\mathcal{V}\mathcal{V}}\frac{H^2}{v^2}\right){\rm Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big] + a_{Hdd}\frac{m_{\rm H}^2}{v^2}\frac{H}{v}\partial^\mu H\,\partial_\mu H$$
$$-\left(a_{HWW}\frac{H}{v} + a_{HHWW}\frac{H^2}{v^2}\right){\rm Tr}\Big[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Big] + i\left(a_{d2} + a_{Hd2}\frac{H}{v}\right)\frac{\partial^\nu H}{v}{\rm Tr}\Big[\hat{W}_{\mu\nu}\mathcal{V}^\mu\Big]$$

Full operators list given in the literature (see, for instance, Brivio et al 1311.1823) In different notation

summarized by: $a_{dd\mathcal{V}\mathcal{V}1} \leftrightarrow c_8$, $a_{dd\mathcal{V}\mathcal{V}2} \leftrightarrow c_{20}$, $a_{11} \leftrightarrow c_9$, $a_{HWW} \leftrightarrow a_W$, $a_{HHWW} \leftrightarrow b_W$, $a_{d2} \leftrightarrow c_5$, $a_{Hd2} \leftrightarrow a_5$, $a_{\Box\mathcal{V}\mathcal{V}} \leftrightarrow c_7$, $a_{H\Box\mathcal{V}\mathcal{V}} \leftrightarrow a_7$, $a_{d3} \leftrightarrow c_{10}$, $a_{Hd3} \leftrightarrow a_{10}$, $a_{\Box\Box} \leftrightarrow c_{\Box H}$, $a_{H\Box\Box} \leftrightarrow a_{\Box H}$, $a_{dd\Box} \leftrightarrow c_{\Delta H}$, $a_{H\mathcal{V}\mathcal{V}} \leftrightarrow a_C$ and $a_{HH\mathcal{V}\mathcal{V}} \leftrightarrow b_C$.

Reduction from 15 to 9 $a_i's$ NLO coefficients after the use of e.o.m

Out of these 9 $a_i's$

The most relevant are
$a_{ddVV1} \equiv \eta$, $a_{ddVV2} \equiv \delta$

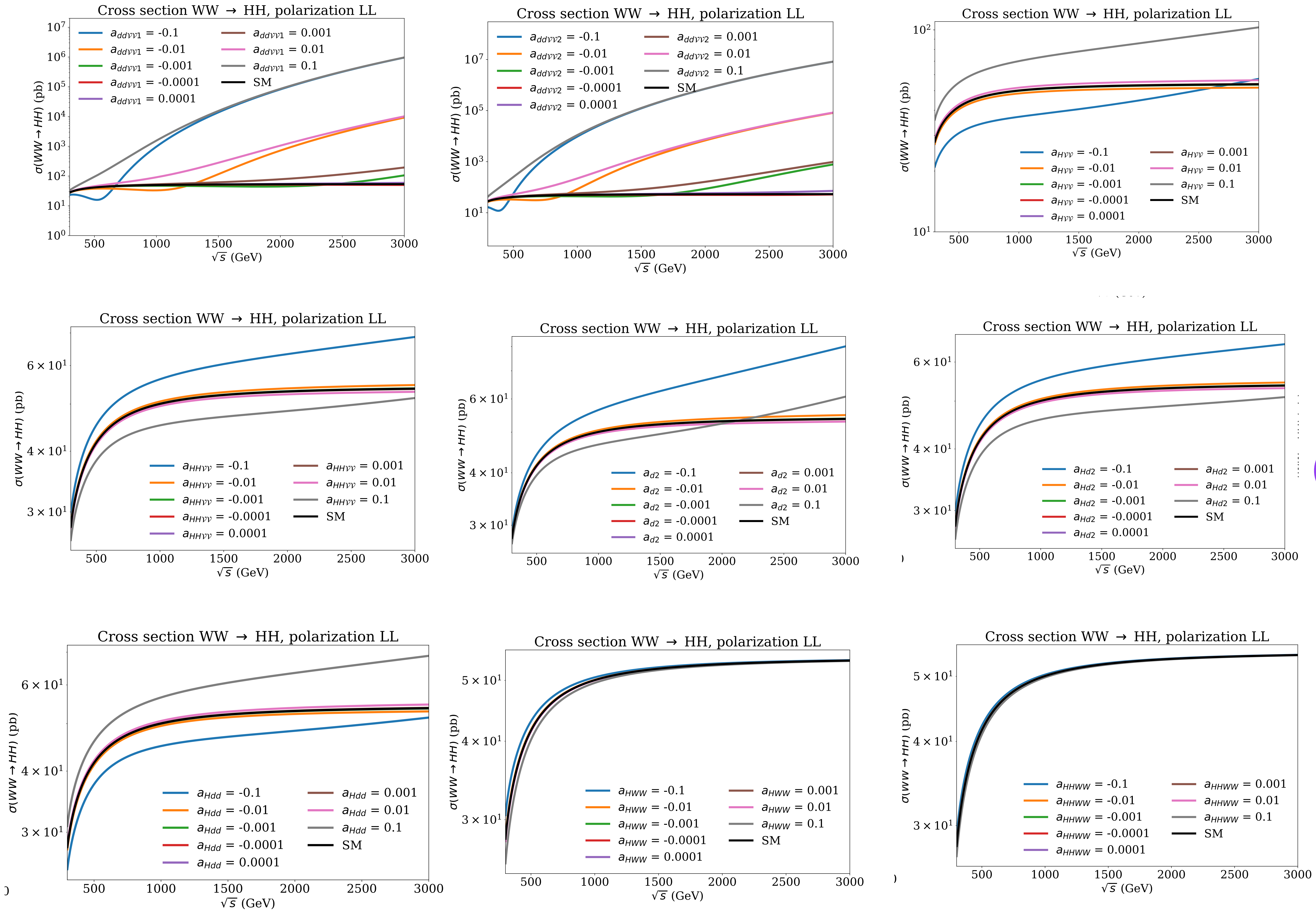
These operators contain 4 derivatives !!

Also checked numerically (see next)

Comparing the relevance of the various NLO $a_i's$ at $WW \to HH$

2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos

$a_{ddVV1} \equiv \eta$ $\qquad$ $a_{ddVV2} \equiv \delta$



For similar size of the $a_i's$ we find the largest xsections for

$a_{ddVV1} \equiv \eta \ , \ a_{ddVV2} \equiv \delta$

by several orders of magnitud !!

$$a_{ddVV1}(1/v^2)\,\partial^\mu H \partial^\nu H \mathrm{Tr}\left[(D_\mu U^+)(D_\nu U)\right] + a_{ddVV2}(1/v^2)\,\partial^\mu H \partial_\mu H \mathrm{Tr}\left[(D^\nu U^+)(D_\nu U)\right]$$

These are the NLO operators containing ∂^4

The most relevant are $a_{ddVV1} \equiv \eta \ , \ a_{ddVV2} \equiv \delta$

Introducing Loops: Bosonic HEFT Renormalization in R_ξ

Including loop corrections within bosonic-HEFT

(Herrero and Morales Series of works in R_ξ : 2005.03537, 2107.07890, 2208.05900)

★ Developed a practical program to include one-loop HEFT corrections by means of Green functions 1PIs

★ Easy to implement in physical scattering procceses

★ Based on computation of one-loop FDs (graphical/intuitive) easy to implement with usual tools FeynRules, FormCalc, Looptools etc..

★ Renormalization of the involved 1PI Green functions in generic R_ξ gauges, with generic off-shell legs (Running coeffs. is not enough. Complete Loop computation needed including finite contribs.)

★ Master equation to compute renormalized 1PI function within NLO HEFT (follow ChPT)

$$\hat{\Gamma}^{\rm NLO} = \Gamma^{\rm LO} + \Gamma^{a_i} + \Gamma^{\rm Loop} + \Gamma^{\rm CT}$$

$\hat{\Gamma}^{\rm NLO}$: Finite for all external (off-shell) momenta

Better not to use e.o.m, all operators needed

We use renorm. conditions: OS for W, Z,H…, MSbar for HEFT coefficients

$\Gamma^{\rm LO}$: From $\mathscr{L}_2$ FRs $a, b, \kappa_3, \kappa_4, \ldots$

Γ^{a_i}: From $\mathscr{L}_4$ FRs $a_i \to a_i + \delta a_i$ needed as new CTs to cancel new divergences from loops

$\Gamma^{\rm Loop}$: From loop diagrams computed with $\mathscr{L}_2$ FRs

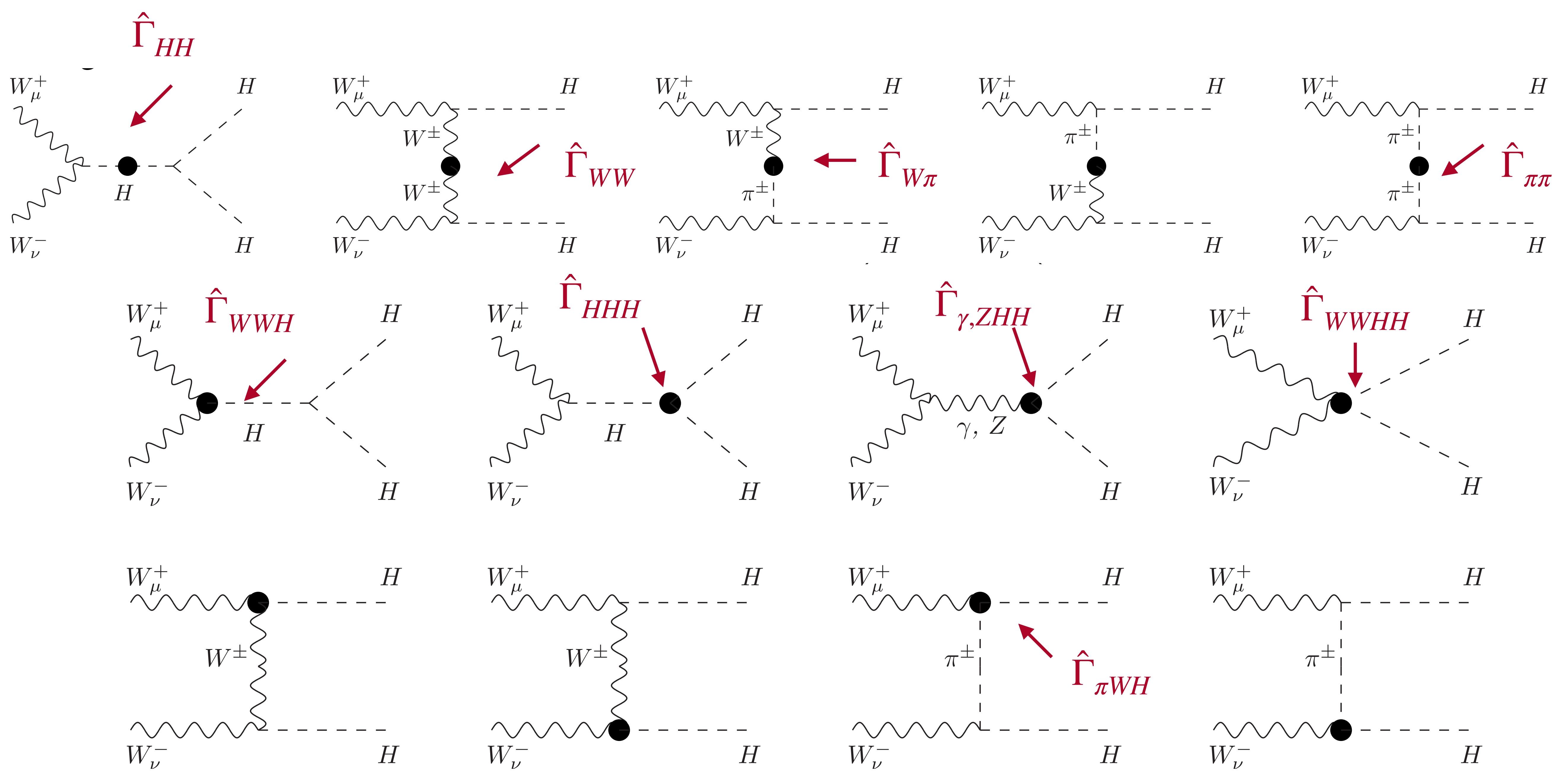
$\Gamma^{\rm CT}$: CTs generated from $\mathscr{L}_2$ $\delta Z_{W,Z,H..}\, \delta g, \delta g', \delta a, \delta b, \delta\kappa_3, \delta\kappa_4 \ldots$

Series of works in R_ξ: 2005.03537 (H decays to $\gamma\gamma$ and γZ), 2107.07890 (WZ to WZ), 2208.09334 (WW to HH) 2405.05385 (gg to HH, gg to HHH)

Example: 1-loop corrections in $WW \to HH$

M.J. Herrero and R.A Morales , PRD106,073008(2022) 2208.05900

Renormalized one-loop 1PIs $\hat{\Gamma}^{\rm NLO}_{\rm HEFT}$ in the R_ξ gauges = black balls contain all the bosonic loops

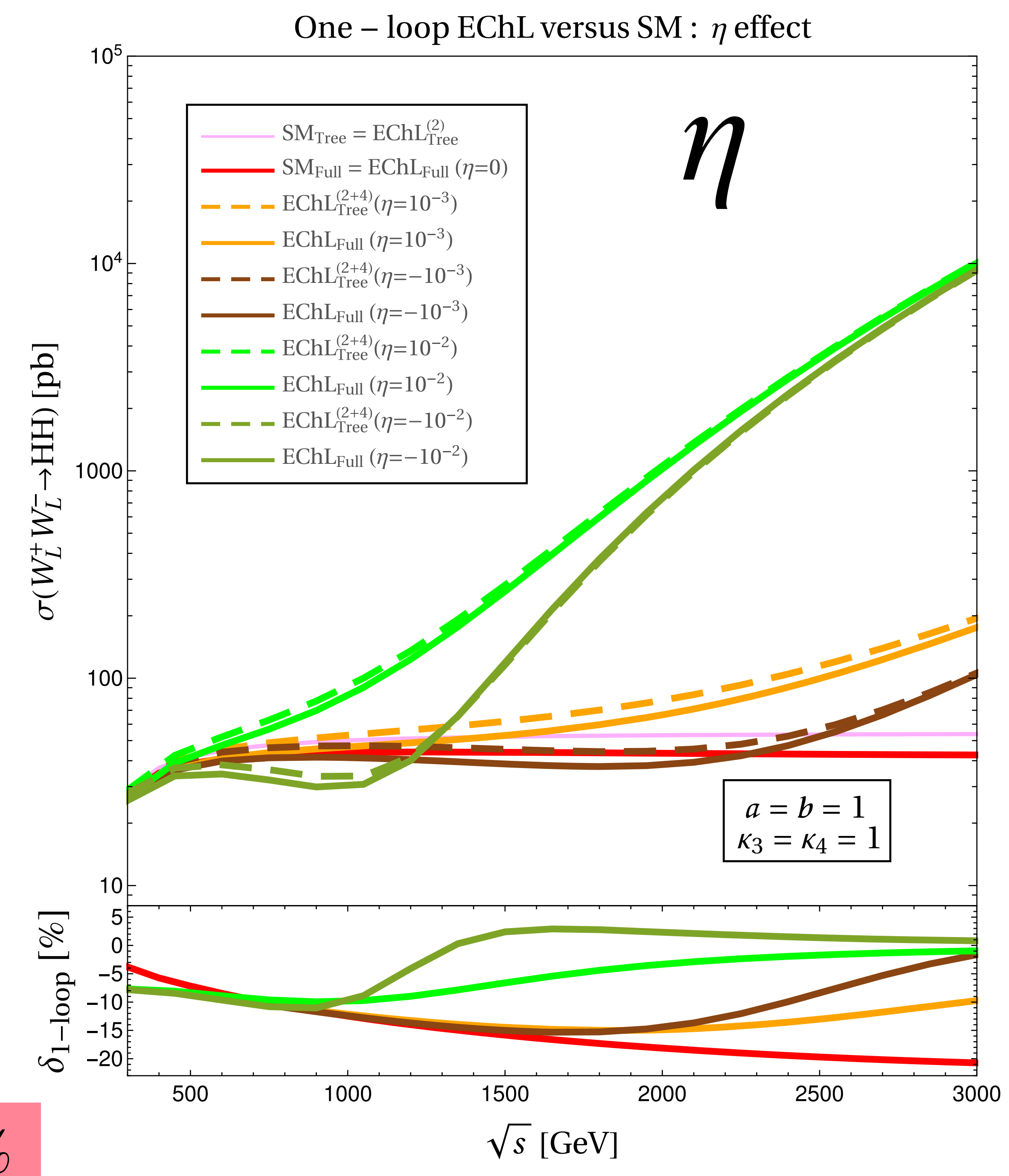


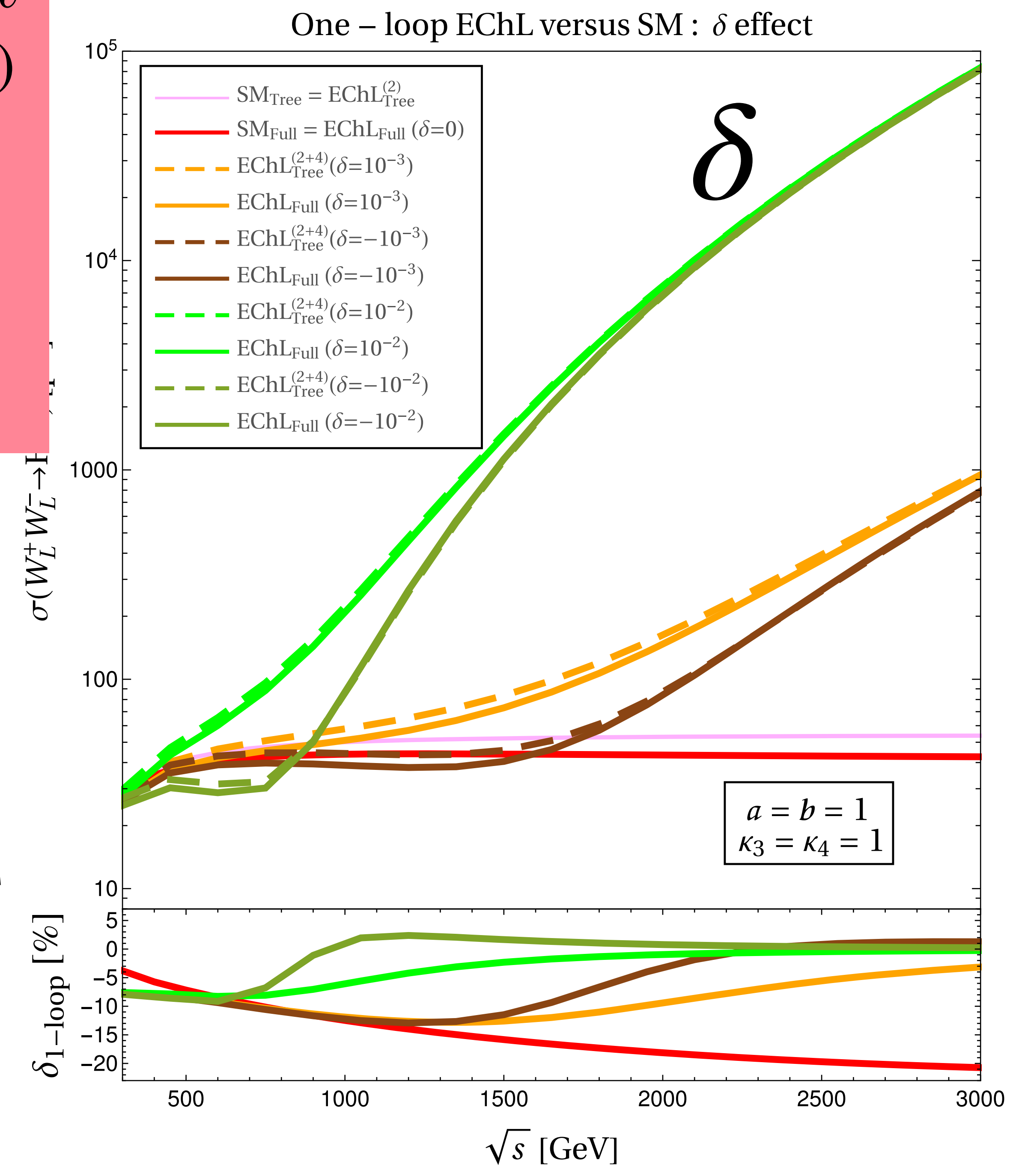
$a = b = \kappa_3 = 1$

In these plots

Size of $\delta_{1-\rm loop} \in (5, -15)\,\%$ comparable to SM $(-20\,\%)$

NLO/LO
different behaviour
with energy





We have computed all the loops and all the needed CTs

In particular, all the involved $\delta a_i's$

$$\delta_{1-\rm loop} = (\sigma_{\rm Full} - \sigma_{\rm Tree})/\sigma_{\rm Tree}$$

ξ independence checked in the full amplitude

Renormalization of HEFT coeffs and derived RGEs

Ex: $WW \to HH$ **All CTs derived computing loops**

M.J. Herrero and R.A Morales , PRD106,073008(2022) 2208.05900

$$\delta_\epsilon a = \frac{\Delta_\epsilon}{16\pi^2}\frac{3}{2v^2}\left((a^2-b)(a-\kappa_3)m_H^2 + a((1-3a^2+2b)m_W^2 + (1-a^2)m_Z^2)\right),$$

$$\delta_\epsilon b = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2v^2}\left((a^2-b)(8a^2-2b-12a\kappa_3+3\kappa_4)m_H^2 + 6a^2b(2m_W^2+m_Z^2) - 6b(m_W^2+m_Z^2) - 6b^2m_W^2\right),$$

$$\delta_\epsilon \kappa_3 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_H^2 v^2}\Big(\kappa_3(a^2-b+9\kappa_3^2-6\kappa_4)m_H^4 - 3(1-a^2)\kappa_3 m_H^2(m_W^2+m_Z^2) + 6(-2ab+2a^2\kappa_3+b\kappa_3)(2m_W^4+m_Z^4)\Big),$$

$$\delta_\epsilon a_{ddVV1} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a^4+a^2b+b^2}{3}, \qquad \delta_\epsilon a_{ddVV2} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{(a^2-b)(2a^2+b+6)}{12},$$

$$\delta_\epsilon a_{11} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a^2}{4}, \qquad \delta_\epsilon a_{H11} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{2}, \qquad \delta_\epsilon a_{HH11} = \frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4-5a^2b+b^2}{4},$$

$$\delta_\epsilon a_{HWW} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{12}, \qquad \delta_\epsilon a_{HHWW} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4-5a^2b+b^2}{24},$$

$$\delta_\epsilon a_{d2} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2-b)}{6}, \qquad \delta_\epsilon a_{Hd2} = \frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4-5a^2b+b^2}{6},$$

$$\delta_\epsilon a_{\Box VV} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{a(2+a^2)}{4}, \qquad \delta_\epsilon a_{H\Box VV} = \frac{\Delta_\epsilon}{16\pi^2}\frac{4a^4+a^2(4-3b)-2b}{4},$$

$$\delta_\epsilon a_{d3} = \frac{\Delta_\epsilon}{16\pi^2}\frac{a(a^2+b)}{2}, \qquad \delta_\epsilon a_{Hd3} = \frac{\Delta_\epsilon}{16\pi^2}\frac{-4a^4+a^2b+b^2}{2},$$

$$\delta_\epsilon a_{\Box\Box} = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3a^2}{4}, \qquad \delta_\epsilon a_{H\Box\Box} = \frac{\Delta_\epsilon}{16\pi^2}\frac{3(2a^2-b)}{2},$$

$$\delta_\epsilon a_{dd\Box} = \frac{\Delta_\epsilon}{16\pi^2}\frac{3(a^2-b)}{2}, \qquad \delta_\epsilon a_{Hdd} = 0, \qquad \delta_\epsilon a_{ddW}/2 = \delta_\epsilon a_{ddZ} = -\frac{\Delta_\epsilon}{16\pi^2}3(a^2-b),$$

$$\delta_\epsilon a_{HVV} = \delta_\epsilon a_{HHVV} = 0, \qquad \Delta_\epsilon = \frac{2}{\epsilon} - \gamma_E + \log(4\pi).$$

Combinations appearing in scattering amplitude :
(=use of e.o.m)

$$\delta_\epsilon \eta = \delta_\epsilon \tilde{a}_{ddVV1} = \delta_\epsilon(a_{ddVV1} - 4a^2a_{11} + 2aa_{d3}) = -\frac{\Delta_\epsilon}{16\pi^2}\frac{(a^2-b)^2}{3},$$

$$\delta_\epsilon \delta = \delta_\epsilon \tilde{a}_{ddVV2} = \delta_\epsilon\left(a_{ddVV2} + \frac{a}{2}a_{dd\Box}\right) = \frac{\Delta_\epsilon}{16\pi^2}\frac{(a^2-b)(7a^2-b-6)}{12},$$

$$\delta_\epsilon(a_{HVV} - 2a_{\Box VV} + 2aa_{\Box\Box}) = \frac{\Delta_\epsilon}{16\pi^2}a(1-a^2),$$

$$\delta_\epsilon(a_{HHVV} - 6\kappa_3 a_{\Box VV} - 4a_{H\Box VV} + 4ba_{\Box\Box} + 6\kappa_3 aa_{\Box\Box} + 4aa_{H\Box\Box}) = \frac{\Delta_\epsilon}{16\pi^2}(3\kappa_3 a(1-a^2) + 2b - 2a^2(2+3b) + 8a^4),$$

$$\delta_\epsilon(a_{Hdd} - a_{dd\Box}) = -\frac{\Delta_\epsilon}{16\pi^2}\frac{3a(a^2-b)}{2}. \qquad (4.15)$$

RGE easily derived for all these HEFT coefficients

$$a_i(\mu) = a_i(\mu') + \frac{1}{16\pi^2}\gamma_{a_i}\log\left(\frac{\mu^2}{\mu'^2}\right), \qquad \delta_\epsilon a_i = \frac{\Delta_\epsilon}{16\pi^2}\gamma_{a_i}$$

Interesting RGE invariants emerge for $a^2 = b$

$$\eta(\mu) = \eta(\mu') - \frac{1}{16\pi^2}\frac{1}{3}(a^2-b)^2\log\left(\frac{\mu^2}{\mu'^2}\right),$$

$$\delta(\mu) = \delta(\mu') + \frac{1}{16\pi^2}\frac{1}{12}(a^2-b)(7a^2-b-6)\log\left(\frac{\mu^2}{\mu'^2}\right)$$

Agreement of $\delta_\epsilon a_i' s$ with other computations in specific imits :
pure scalar (1311.5993,14091571)
isospin limit $m_W = m_Z$ (2109.02673)

On the size of Chiral Loops

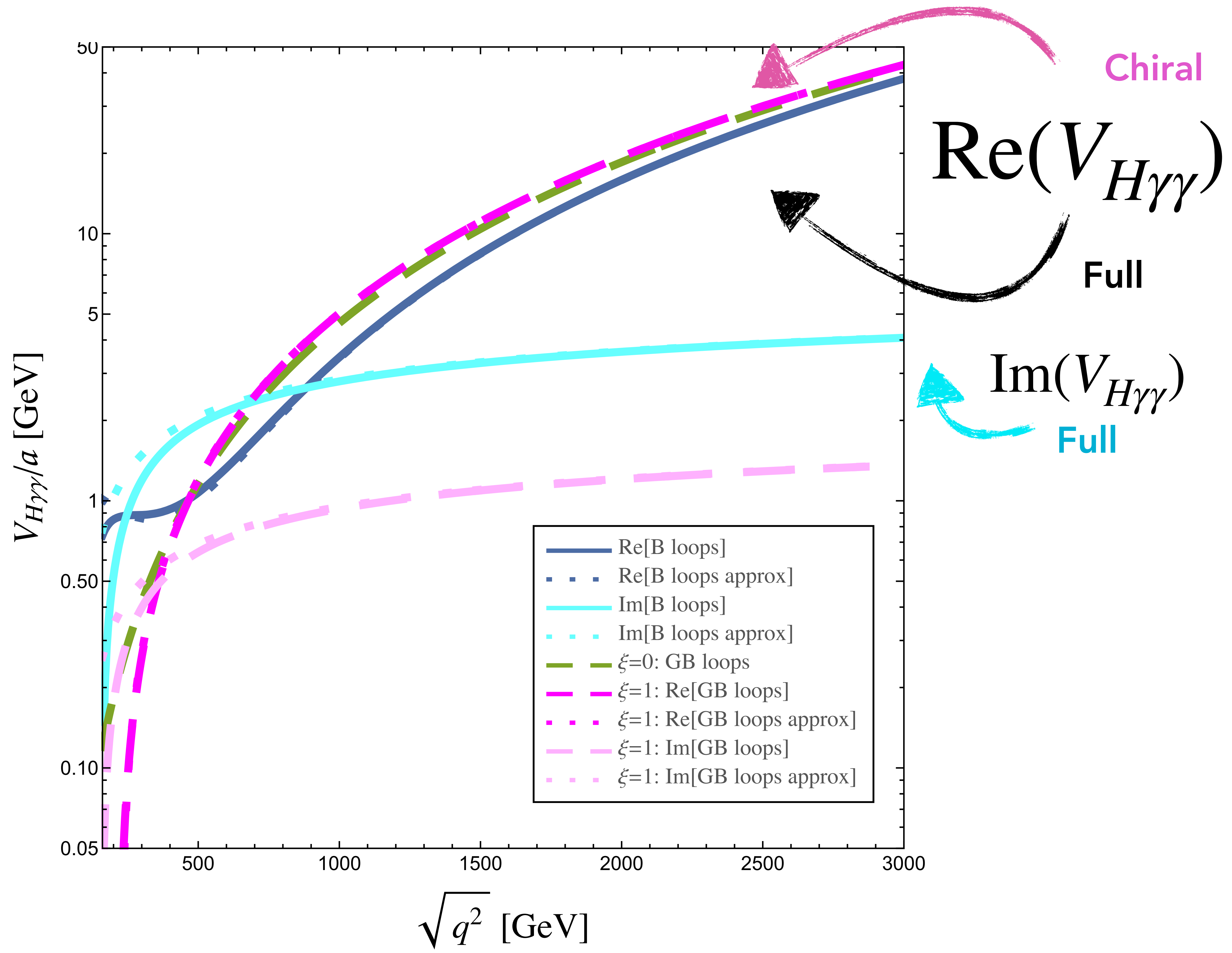
Chiral Loops in the Bosonic EChL means Scalar loops (Scalar= GBs and H)

Main issue is comparing size of pure scalar loops with other loops i.e pure gauge loops and mixed gauge-scalar loops

The size of the loop corrections can be large at large virtuality q !!!

Example 1: one-loop vertex functions $V_{H\gamma\gamma}$ and $V_{H\gamma Z}$ for off-shell $H(q^2 \neq m_H^2)$

Relevant for off-shell Higgs decays $H(q) \to \gamma\gamma$ and $H(q) \to \gamma Z$



M.J. Herrero and R.A Morales , PRD102,075040(2020) 2107.07890

$\mathrm{Re}(V_{H\gamma\gamma})$ grow with virtuality q^2 (Idem $\mathrm{Re}(V_{H\gamma Z})$)

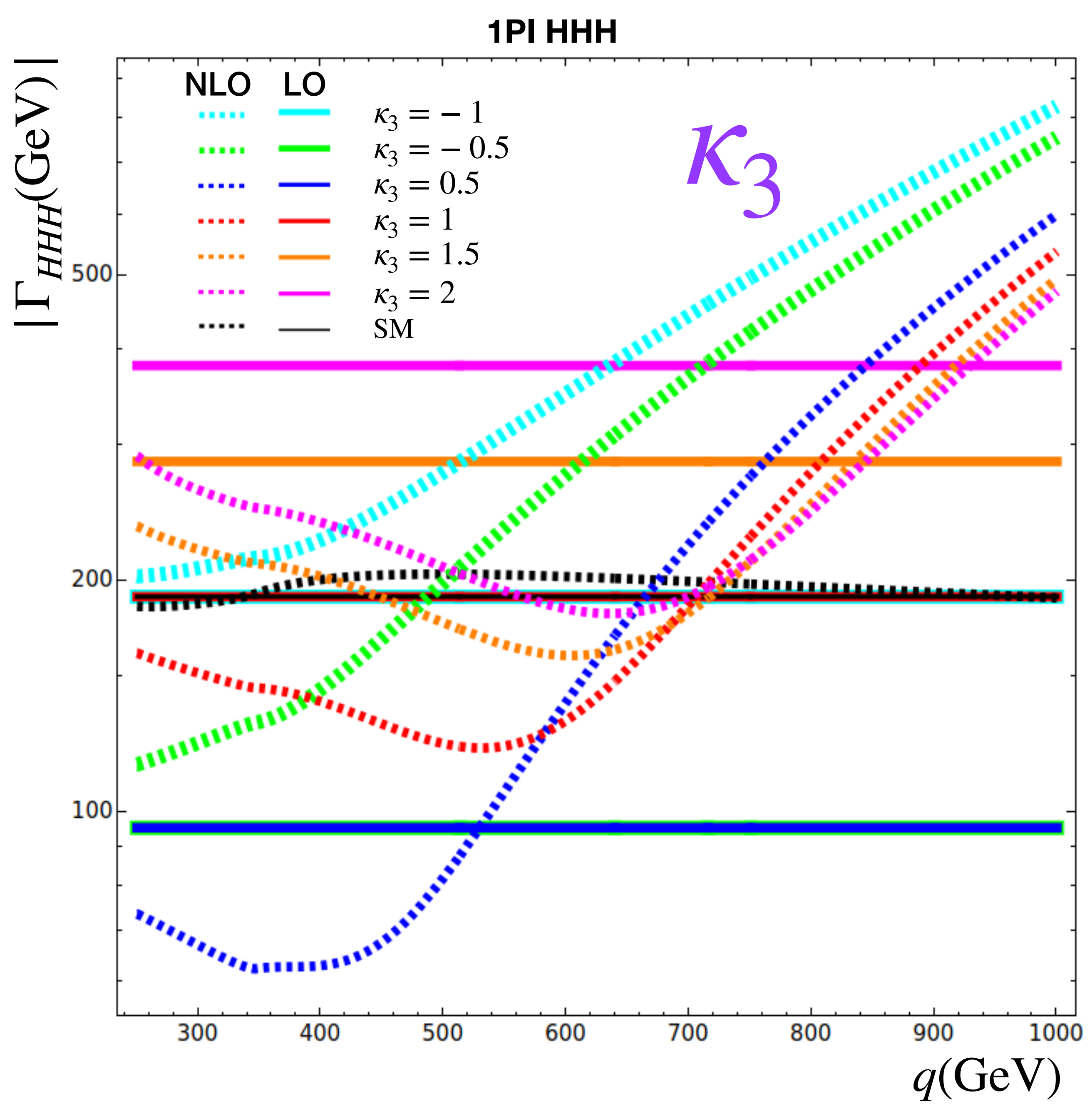
Chiral loops dominate at large q^2 if fermions assumed SM like

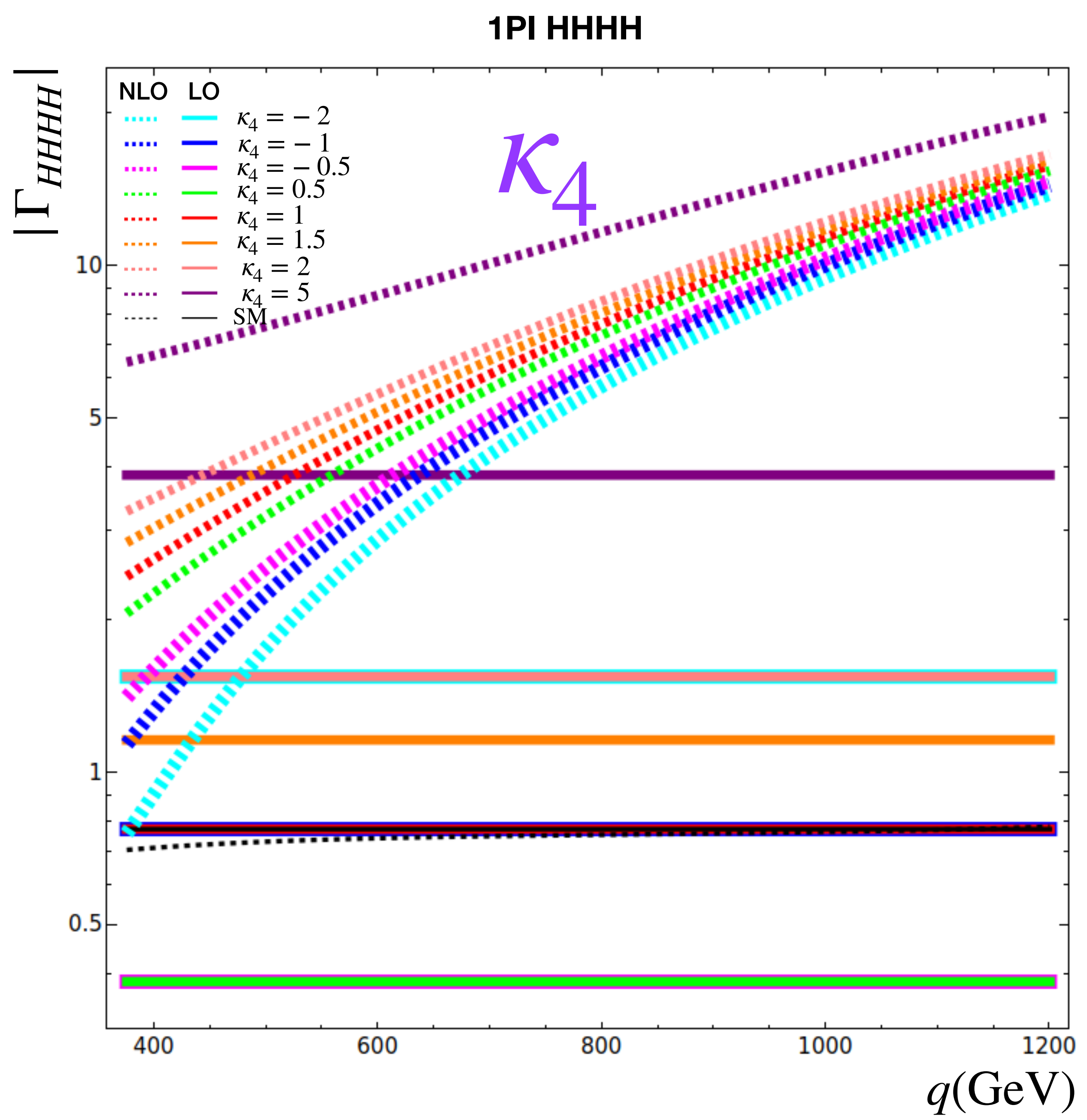
The full 1-loop corrections for
off-shell H
are ξ dependent

But the most relevant contributions
from chiral loops, i.e. $\mathcal{O}(q^2)$, are ξ independent

Example 2: Γ_{HHH} and Γ_{HHHH}

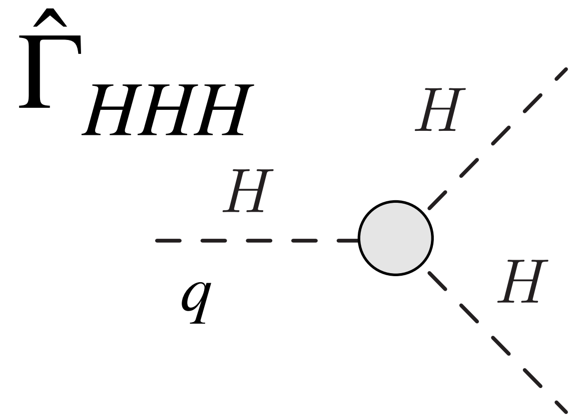
One-loop corrections in 1PIs: the case of Γ_{HHH} and Γ_{HHHH}

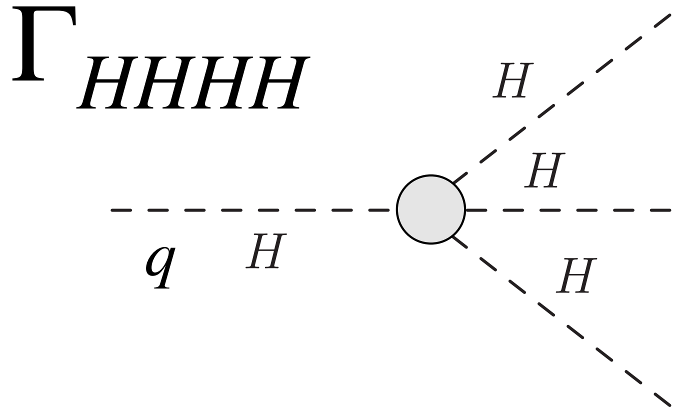




(details in 2208.05900 and 2405.05385)

In general, departures respect to the SM grow with offshellness q





Loops of bosons dominate if fermions assumed SM like

All CTs fixed
$\delta\kappa_3, \delta\kappa_4, \delta a_i's$ .. (see paper)

$$\delta_\epsilon\kappa_3 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_{\mathrm{H}}^2 v^2}\left(\kappa_3(a^2 - b + 9\kappa_3^2 - 6\kappa_4)m_{\mathrm{H}}^4 - 3(1-a^2)\kappa_3 m_{\mathrm{H}}^2(m_{\mathrm{W}}^2 + m_{\mathrm{Z}}^2) + 6(-2ab + 2a^2\kappa_3 + b\kappa_3)(2m_{\mathrm{W}}^4 + m_{\mathrm{Z}}^4)\right),$$

$$\delta_\epsilon\kappa_4 = -\frac{\Delta_\epsilon}{16\pi^2}\frac{1}{2m_{\mathrm{H}}^2 v^2}\left(\kappa_4(2a^2 - 2b + 9\kappa_3^2 - 6\kappa_4)m_{\mathrm{H}}^4 - 6(1-a^2)\kappa_4 m_{\mathrm{H}}^2(m_{\mathrm{W}}^2 + m_{\mathrm{Z}}^2) + 6(-2b^2 + 2a^2\kappa_4 + b\kappa_4)(2m_{\mathrm{W}}^4 + m_{\mathrm{Z}}^4)\right),$$

The size of the corrections can be large at large virtuality q

$\|\Gamma^{NLO}_{HHH}\|/\|\Gamma^{LO}_{HHH}\|$			$\|\Gamma^{NLO}_{HHHH}\|/\|\Gamma^{LO}_{HHHH}\|$		
κ_3	$q = 251$ GeV	$q = 1000$ GeV	κ_4	$q = 376$ GeV	$q = 1000$ GeV
-1	1.1	4.4	-2	0.49	6.2
-0.5	1.2	7.9	-1	1.5	13
0.5	0.77	6.3	-0.5	3.7	27
1	0.84	2.8	0.5	5.4	29
1.5	0.82	1.7	1	3.2	15
2	0.76	1.3	1.5	2.5	10
			2	2.1	7.9
			5	1.7	4.0
SM	0.97	1.0	SM	0.91	0.99

Non-linearity
H-singlet ⇒
growing with energy
of interactions within HEFT

↘

Larger corrections in HEFT than in SM

RGEs derived for κ_3 and κ_4 within HEFT

$$\kappa_i(\mu) = \kappa_i(\mu') + \frac{1}{16\pi^2}\gamma_{\kappa_i}\log\left(\frac{\mu^2}{\mu'^2}\right) \;;\; \delta_\epsilon\kappa_i = \frac{\Delta_\epsilon}{16\pi^2}\gamma_{\kappa_i}$$

Introducing Loops Improves Perturbative Unitarity

$W_L Z_L \to W_L Z_L$ **NLO-HEFT including loops** M. Herrero, R. Morales, PRD104.075013(2021)

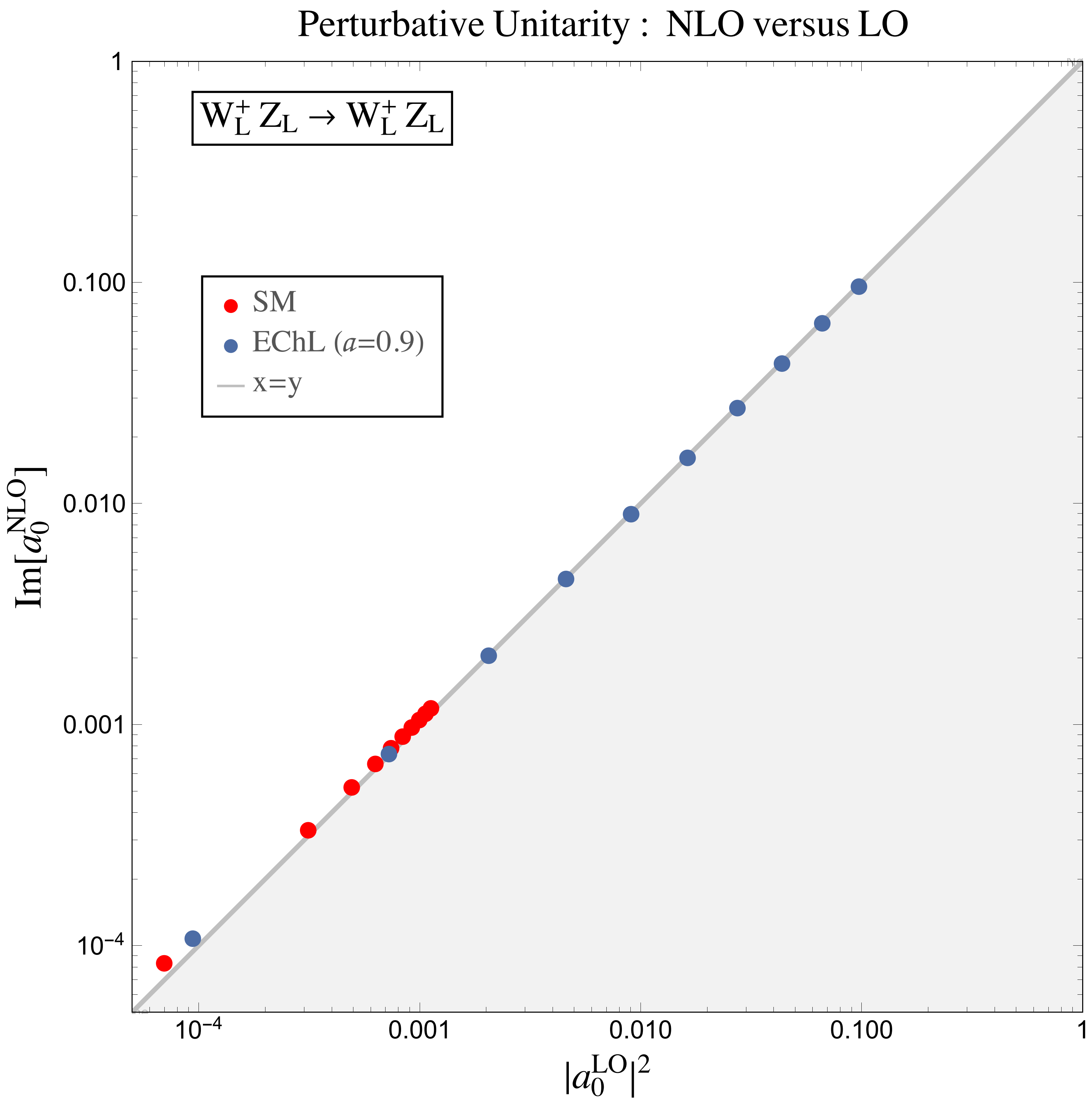


FIG. 8. Check of perturbative unitarity in the one-loop predictions for the lowest partial wave a_0 in both the EChL with $a = 0.9$ and the SM. The 10 blue (red) points are the EChL (SM) predictions for the ten chosen energies, $\sqrt{s}[\mathrm{GeV}] = 300$, 600, 900, 1200, 1500, 1800, 2100, 2400, 2700, 3000, from left to right along the diagonal line in this plot.

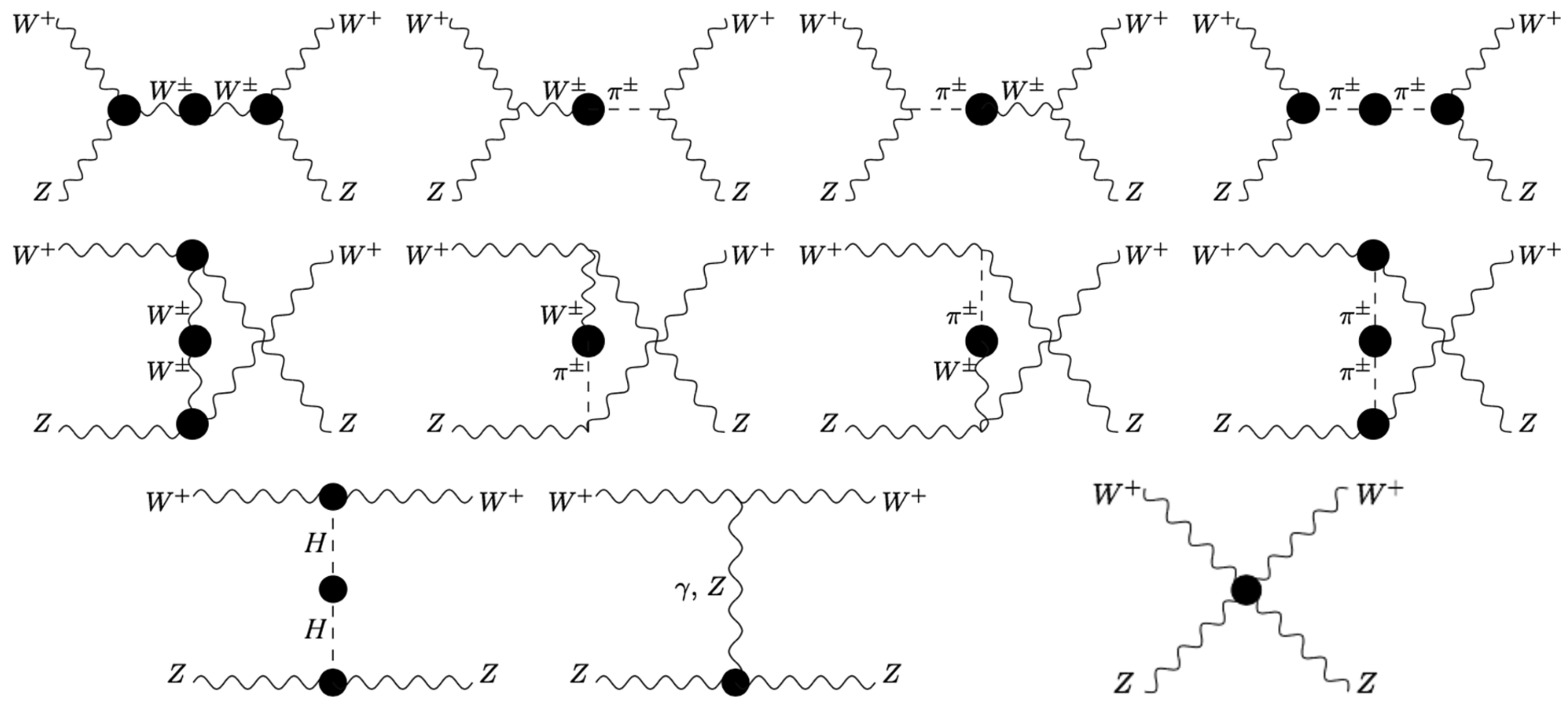


FIG. 1. Full 1PI functions (black balls) contributing to the full one-loop amplitude $\mathcal{A}(WZ \to WZ)$.

Black Balls include all bosonic loops: Scalar, Gauge and mixed Scalar-Gauge

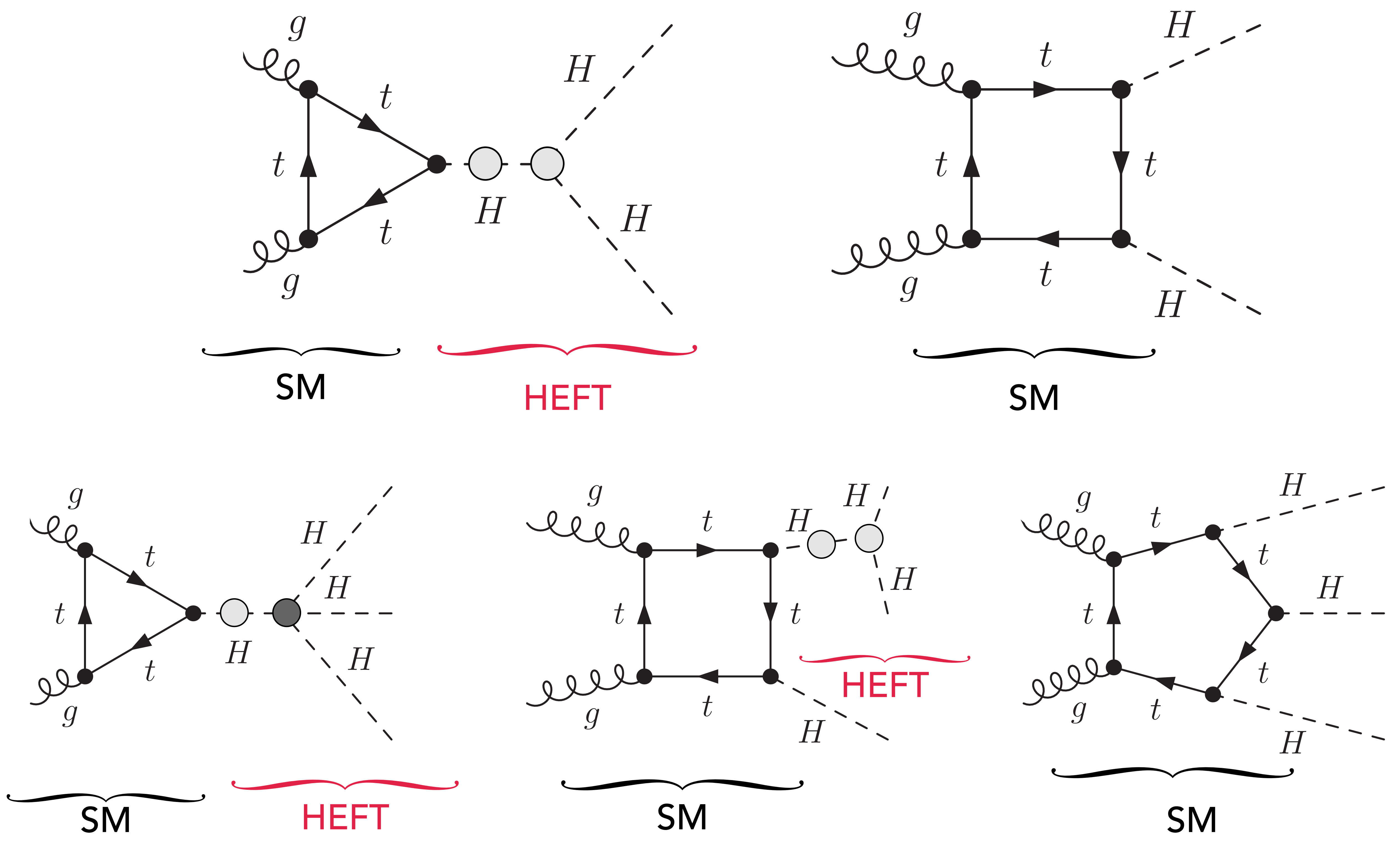
Loops generate an imaginary part such that fulfill the Perturbative Unitarity condition (as in ChPT)

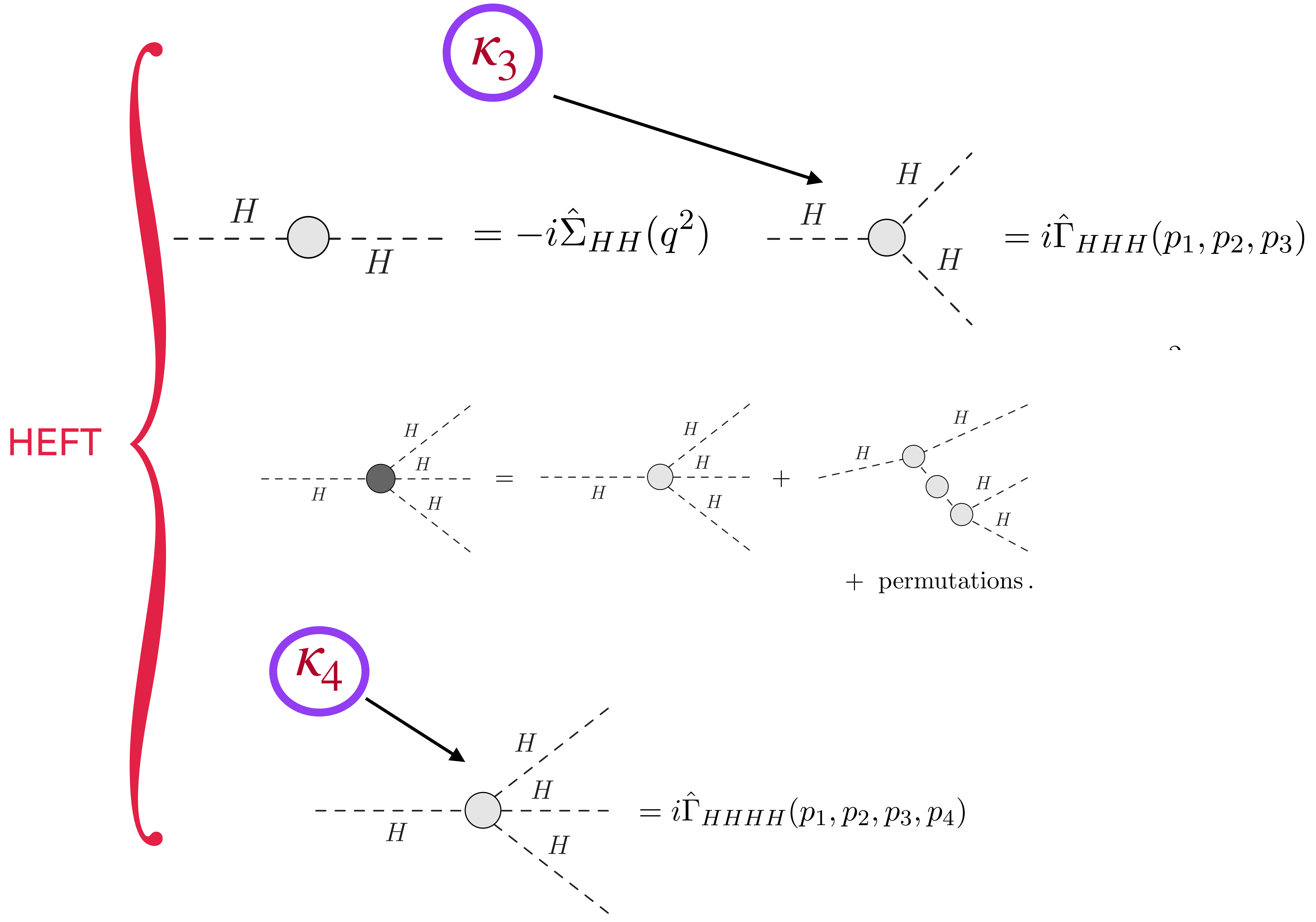
$$\mathrm{Im}[a_0^{\mathrm{NLO}}] \simeq |a_0^{\mathrm{LO}}|^2$$

Introducing EW loops in $gg \to HH$ and in $gg \to HHH$

Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , 2405.05385 (numerical estimates with VBFNLO)

Renormalized one-loop 1PIs $\hat{\Gamma}^{\rm NLO}_{\rm HEFT}$ computed in Feynman 'tHooft gauge = shaded balls contain the EW-NLO loops within HEFT





The loops in HHH and HHHH vertices and the non-trivial off-shell momenta dependencies produce relevant changes respect to LO

Renormalization of κ_3, κ_4 and of new $a_i's$ involved (see paper)

Most important message:
(EW) loop corrections using NLO-HEFT change
the sensitivity to κ_3 and κ_4 in HH and HHH production at LHC (see paper)

Matching HEFTs to UV theories. Matching HEFT to SMEFT

Matching amplitudes (HEFT versus UV)

We do matching at amplitude level (more useful to compare with data).
In contrast to other approaches: matching Lagrangians, matching Effective Actions …etc

Matching amplitudes requires:

$$\mathscr{A}^{\mathrm{HEFT}} = \mathscr{A}^{\mathrm{UV}}(\mathrm{m_{heavy}} \gg \mathrm{m_{light}})$$

- Setting the HEFT order (LO, NLO,..)
- Setting the n-loop order $\mathcal{O}(\hbar^n)$, same in both sides
- Setting the input parameters, in both sides
- Setting the proper large mass expansion in the UV theory

2307.15693, Phys.Rev.D 108 (2023)9, 095013, Arco, Domenech, Herrero, Morales

Matching HEFT and 2HDM Amplitudes

Matching several amplitudes:

Order	Process		Type
LO	$h \to WW^* \to Wf\bar{f}'$		tree
LO	$h \to ZZ^* \to Zf\bar{f}$		tree
LO	$W^+W^- \to hh$		tree
LO	$ZZ \to hh$		tree
LO	$hh \to hh$		tree
NLO	$h \to \gamma\gamma$	R_ξ	1-loop
NLO	$h \to \gamma Z$	R_ξ	1-loop

Choose input parameters:

(HEFT) $m_h, m_W, m_Z, a_i's$

(2HDM)
- m_h, m_W, m_Z, m_{12} (light)
- $m_H, m_A, m_{H^\pm}$ (heavy)
- $\tan\beta,\ \cos(\beta-\alpha)$ (free)

Proper large mass expansion is in $\left(\frac{m_{\rm light}}{m_{\rm heavy}}\right)^n$. Other parameters are derived $(\lambda_{h_ih_jh_k}, \ldots)$

Solution to the matching equations: HEFT versus 2HDM

2307.15693, Phys.Rev.D 108 (2023)9, 095013, Arco, Domenech, Herrero, Morales

Solving the matching equations implies identifying all Lorentz invariant structures involved and extracting the corresponding HEFT coeffs

$$(a = 1 - \Delta a\,,\, b = 1 - \Delta b\,,\, \kappa_3 = 1 - \Delta\kappa_3\,,\, \kappa_4 = 1 - \Delta\kappa_4)$$

$$\begin{aligned}
\Delta a|_{\rm 2HDM} &= 1 - s_{\beta-\alpha}\,,\\
\Delta b|_{\rm 2HDM} &= -c^2_{\beta-\alpha}(1 - 2c^2_{\beta-\alpha} + 2c_{\beta-\alpha}s_{\beta-\alpha}\cot 2\beta)\,,\\
\Delta\kappa_3|_{\rm 2HDM} &= 1 - s_{\beta-\alpha}(1+2c^2_{\beta-\alpha}) - c^2_{\beta-\alpha}\left(-2s_{\beta-\alpha}\frac{m^2_{12}}{m^2_h s_\beta c_\beta} + 2c_{\beta-\alpha}\cot 2\beta\left(1 - \frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right)\right)\,,\\
\Delta\kappa_4|_{\rm 2HDM} &= -\frac{c^2_{\beta-\alpha}}{3}\left(-7 + 64c^2_{\beta-\alpha} - 76c^4_{\beta-\alpha} + 12\left(1 - 6c^2_{\beta-\alpha} + 6c^4_{\beta-\alpha}\right)\frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right.\\
&\quad +4c_{\beta-\alpha}s_{\beta-\alpha}\cot 2\beta\left(-13 + 38c^2_{\beta-\alpha} - 3(-5+12c_{\beta-\alpha})\frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right)\\
&\quad \left.+4c^2_{\beta-\alpha}\cot^2 2\beta\left(3c^2_{\beta-\alpha} - 16s^2_{\beta-\alpha} + 3(-1+6s^2_{\beta-\alpha})\frac{m^2_{12}}{m^2_h s_\beta c_\beta}\right)\right)\,,\\
a_{h\gamma\gamma}|_{\rm 2HDM} &= -\frac{s_{\beta-\alpha}}{48\pi^2}\,,\\
a_{h\gamma Z}|_{\rm 2HDM} &= -\frac{(2c^2_{\rm w} - 1)s_{\beta-\alpha}}{96c^2_{\rm w}\pi^2}\,.
\end{aligned}$$

Posterior computations within HEFT are in agreement with ours: 2311.16897 (PC-3), 2312.13885

These contributions are $\mathcal{O}\left(\frac{m_{\rm light}}{m_{\rm heavy}}\right)^0$

Leading terms in the large $m_{\rm heavy}$ expansion

Summarize the Non-Decoupling effects of the heavy Higgs bosons at low energies

They are valid for arbitrary $t_\beta\,,\, c_{\beta-\alpha}$

when $c_{\beta-\alpha} \ll 1$ is required (quasi-alignement)

These non-decoupling effects from the heavy bosons are not obtained in the SMEFT where all effects are decoupling $\left(\frac{m_{\rm light}}{m_{\rm heavy}}\right)^n,\ n \geq 2$

Interesting correlations found

$$(\Delta b + 2\Delta a)_{\rm 2HDM} = 0$$

$$(\Delta\kappa_4 + 2\Delta\kappa_3)_{\rm 2HDM} = 0$$

Relating HEFT and SMEFT

Different approaches in the literature:

Comparing Lagrangian Functions (with field redefinitions),
Comparing Effective Actions (with the path integral),
Comparing Green Functions (with diagrammatic methods)..

We choose comparing scattering amplitudes

for several MultiHiggs processes (with external physical bosons)

$$WW \to H \quad ZZ \to H \quad WW \to HH \quad ZZ \to HH \quad WW \to HHH \quad ZZ \to HHH \quad HH \to HH$$

And solve jointly
(for the 7 channels)
'The Matching Equation'
In terms of relations among coefficients

$$A_{\rm HEFT}(a_i) = A_{\rm SMEFT}(c_j)$$

The correspondence
between both theories
is across different orders!

$$a_i \leftrightarrow c_j$$

Domenech et al 2506.2171

For truncated HEFT (up to chidim4) and
truncated SMEFT (up to dim6). Both tree level.

We set input parameters
In the on-shell scheme, i.e,
m_H, m_W, m_Z, G_F (or v)

This is a consequence
of the different counting

Matching HEFT/SMEFT amplitudes in (EW) HH and HHH with full bosonic EFT (scalar and gauge)

D. Domenech, M.Herrero, R.Morales, A. Salas-Bernardez, 2506.2171

A proper matching HEFT/SMEFT of full bosonic sector must include both scalar and gauge (L and T) particle content

Relevant HEFT operators and coefficients $\mathscr{L}_{\rm HEFT} = \mathscr{L}_2 + \mathscr{L}_4$

$$\frac{v^2}{4}\left(1+2a\frac{H}{v}+b\frac{H^2}{v^2}+c\frac{H^3}{v^3}\right)\mathrm{Tr}[D_\mu U^\dagger D^\mu U]+\frac{1}{2}\partial_\mu H\partial^\mu H$$

$$-\frac{1}{2g^2}\mathrm{Tr}[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}]-\frac{1}{2g'^2}\mathrm{Tr}[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}]+\mathcal{L}_{GF}+\mathcal{L}_{FP},$$

$$-\left(\frac{1}{2}m_H^2H^2+\kappa_3\lambda v H^3+\kappa_4\frac{\lambda}{4}H^4\right)$$

$$-\left(a_{HWW}\frac{H}{v}+a_{HHWW}\frac{H^2}{v^2}\right)\mathrm{Tr}\left[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\right]$$

$$-\left(a_{HBB}\frac{H}{v}+a_{HHBB}\frac{H^2}{v^2}\right)\mathrm{Tr}\left[\hat{B}_{\mu\nu}\hat{B}^{\mu\nu}\right]$$

$$+\left(a_{H1}\frac{H}{v}+a_{HH1}\frac{H^2}{v^2}\right)\mathrm{Tr}\left[U\hat{B}_{\mu\nu}U^\dagger\hat{W}^{\mu\nu}\right]$$

$$+\left(a_{H0}\frac{H}{v}+a_{HH0}\frac{H^2}{v^2}+a_{HHH0}\frac{H^3}{v^3}\right)(m_Z^2-m_W^2)\,\mathrm{Tr}\left[U\tau^3U^\dagger\mathscr{V}_\mu\right]\mathrm{Tr}\left[U\tau^3U^\dagger\mathscr{V}^\mu\right]$$

$$D_\mu U = \partial_\mu U - i\hat{W}_\mu U + iU\hat{B}_\mu \qquad U = \exp\left(i\frac{\omega_i\tau_i}{v}\right) \qquad \mathcal{V}_\mu = (D_\mu U)U^\dagger$$

Relevant SMEFT operators and coefficients $\mathscr{L}^{\rm dim\,6}_{\rm SMEFT}$

$$(D_\mu\Phi)^\dagger(D^\mu\Phi)+m^2\Phi^\dagger\Phi-\frac{1}{4}\lambda(\Phi^\dagger\Phi)^2-\frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}+\mathcal{L}^{\rm SM}_{GF}+\mathcal{L}^{\rm SM}_{FP}$$

$$\frac{c_\Phi}{\Lambda^2}(\Phi^\dagger\Phi)^3+\frac{c_{\Phi\Box}}{\Lambda^2}(\Phi^\dagger\Phi)\Box(\Phi^\dagger\Phi)+\frac{c_{\Phi D}}{\Lambda^2}(\Phi^\dagger D^\mu\Phi)^*(\Phi^\dagger D_\mu\Phi)$$

$$+\frac{c_{\Phi W}}{\Lambda^2}(\Phi^\dagger\Phi)W^I_{\mu\nu}W^{I\,\mu\nu}+\frac{c_{\Phi B}}{\Lambda^2}(\Phi^\dagger\Phi)B_{\mu\nu}B^{\mu\nu}+\frac{c_{\Phi WB}}{\Lambda^2}(\Phi^\dagger\tau^I\Phi)W^I_{\mu\nu}B^{\mu\nu}$$

$$\Phi = \begin{pmatrix}\phi^+\\ \frac{1}{\sqrt{2}}(v+H+i\phi^0)\end{pmatrix} \qquad D_\mu = \partial_\mu + ig'B_\mu Y + igW^I_\mu T^I$$

14 HEFT coeffs (6 SMEFT coeffs) are involved in the matching

$$A_{\rm HEFT}(a_i) = A_{\rm SMEFT}(c_j)$$

We identify all Lorentz invariant structures in these amplitudes and solve the system of equations considering the previous set of multiHiggs processes

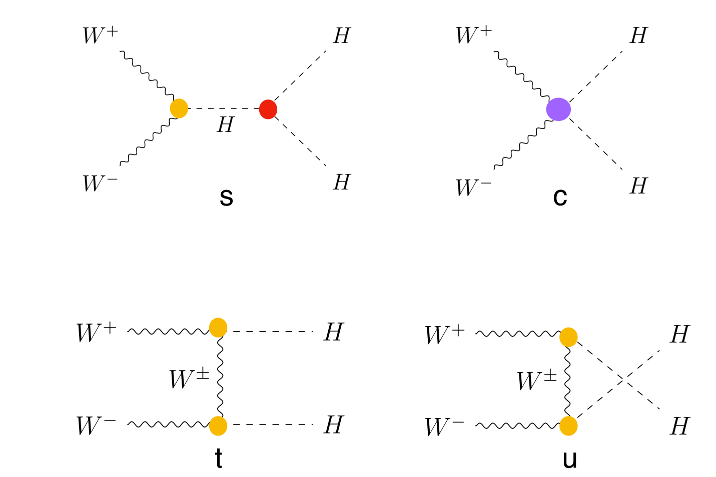
The solution is getting a_i in terms of c_j

Solutions to matching HEFT/SMEFT in HH and HHH with full bosonic EFT (scalar and gauge)

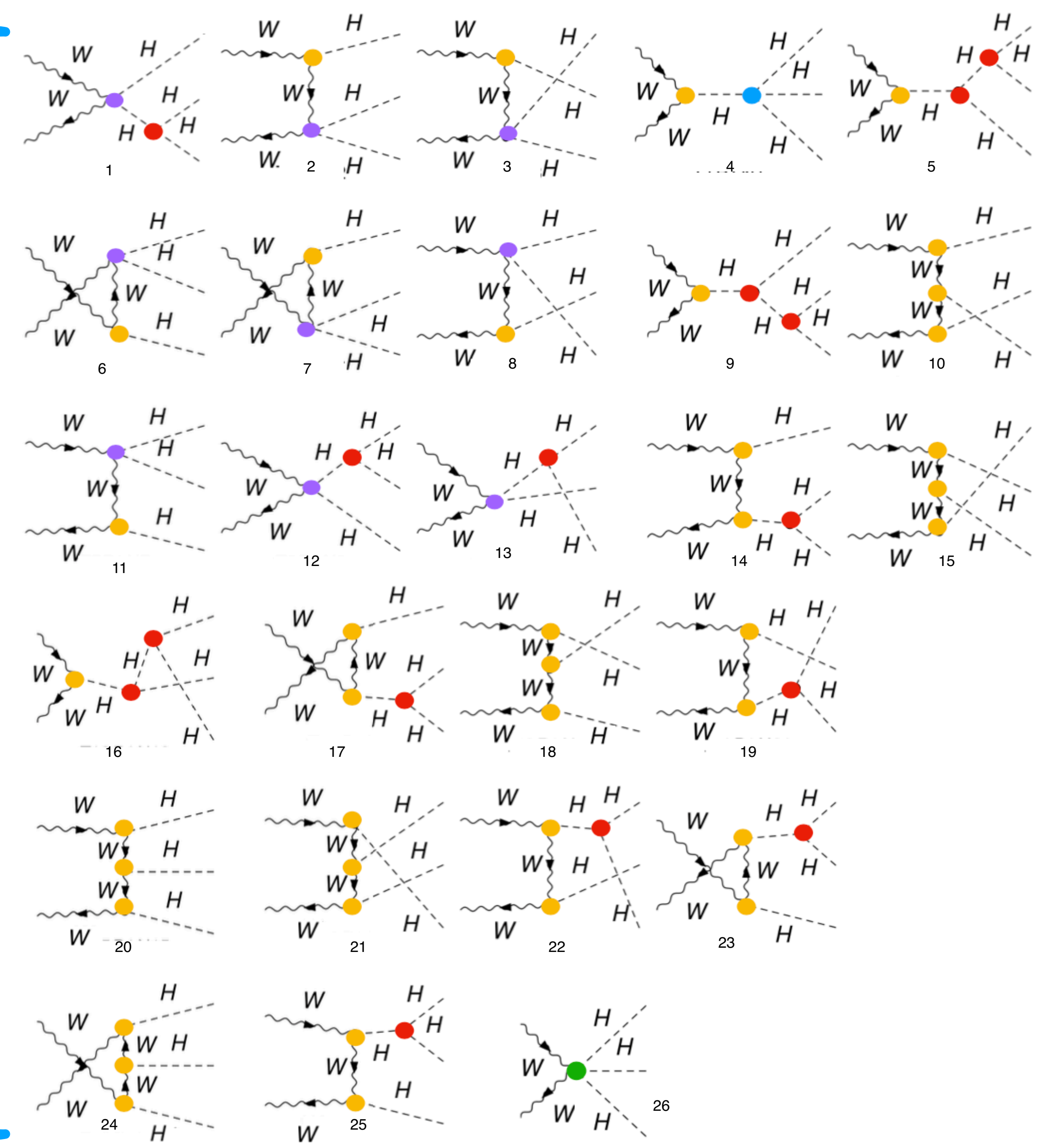
Domenech et al
2506.2171

We match amplitudes in the unitary gauge

HH



HHH



$\mathscr{A}_{\text{U-gauge}} = \mathscr{A}_{R_\xi\text{-gauge}}$

checked in all cases
both in HEFT and SMEFT

Relations HEFT/SMEFT coeffs occur through different orders

$$\Delta a = \frac{v^2}{\Lambda^2}\left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D}\right)$$

$$\Delta b = \frac{4v^2}{\Lambda^2}\left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D}\right)$$

$$c = -\frac{8v^2}{3\Lambda^2}\left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D}\right)$$

$$\Delta\kappa_3 = \frac{2v^4}{m_H^2\Lambda^2}c_\Phi + \frac{3v^2}{\Lambda^2}\left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D}\right)$$

$$\Delta\kappa_4 = \frac{12v^4}{m_H^2\Lambda^2}c_\Phi + \frac{50v^2}{3\Lambda^2}\left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D}\right)$$

$$a_{HWW} = -\frac{v^4}{2m_W^2\Lambda^2}c_{\Phi W}$$

$$a_{HHWW} = -\frac{v^4}{4m_W^2\Lambda^2}c_{\Phi W}$$

$$s_W^2 a_{HBB} = -\frac{v^4}{2m_Z^2\Lambda^2}c_{\Phi B}$$

$$s_W^2 a_{HHBB} = -\frac{v^4}{4m_Z^2\Lambda^2}c_{\Phi B}$$

$$s_W^2 a_{H0} = -\frac{v^4}{8m_Z^2\Lambda^2}c_{\Phi D}$$

$$s_W^2 a_{HH0} = -\frac{5v^4}{16m_Z^2\Lambda^2}c_{\Phi D}$$

$$s_W c_W a_{H1} = -\frac{v^4}{2m_Z^2\Lambda^2}c_{\Phi WB}$$

$$s_W c_W a_{HH1} = -\frac{v^4}{4m_Z^2\Lambda^2}c_{\Phi WB}$$

$$s_W^2 a_{HHH0} = -\frac{v^4}{4m_Z^2\Lambda^2}c_{\Phi D}$$

$a = 1 - \Delta a\,, b = 1 - \Delta b\,, \kappa_i = 1 - \Delta\kappa_i$

Uncorrelated coeffs in HEFT (H is a singlet)

$a, b, c, \kappa_3, \kappa_4, a_i's$ independent

Relations found in SMEFT (H is in a doublet)

$$\left(c + \frac{4}{3}a(a^2 - b)\right)_{\text{SMEFT}} = 0$$

$$\Delta b = 4\Delta a$$

$$c = -\frac{8}{3}\Delta a = -\frac{2}{3}\Delta b$$

$$\Delta\kappa_4 = 6\Delta\kappa_3 - \frac{4}{3}\Delta a$$

OK with pure scalar
2311.04280

$$2a_{HHWW} = a_{HWW}$$

$$2a_{HHBB} = a_{HBB}$$

$$2a_{HH0} = 5a_{H0}\ , a_{HHH0} = 2a_{H0}$$

$$2a_{HH1} = a_{H1}$$

NEW

Matching other HEFT coeffs requires dim 8 SMEFT

$$\eta \quad a_{ddVV1} = \frac{v^4}{4\Lambda^4}\left[c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)}\right]$$

$$\delta \quad a_{ddVV2} = \frac{v^4}{4\Lambda^4}c_{\phi^4}^{(3)}$$

Simmilarly to $a_{4,5}$ in $WZ \to WZ$

$$a_4 = \frac{v^4}{16\Lambda^4}c_{\phi^4}^{(2)}$$

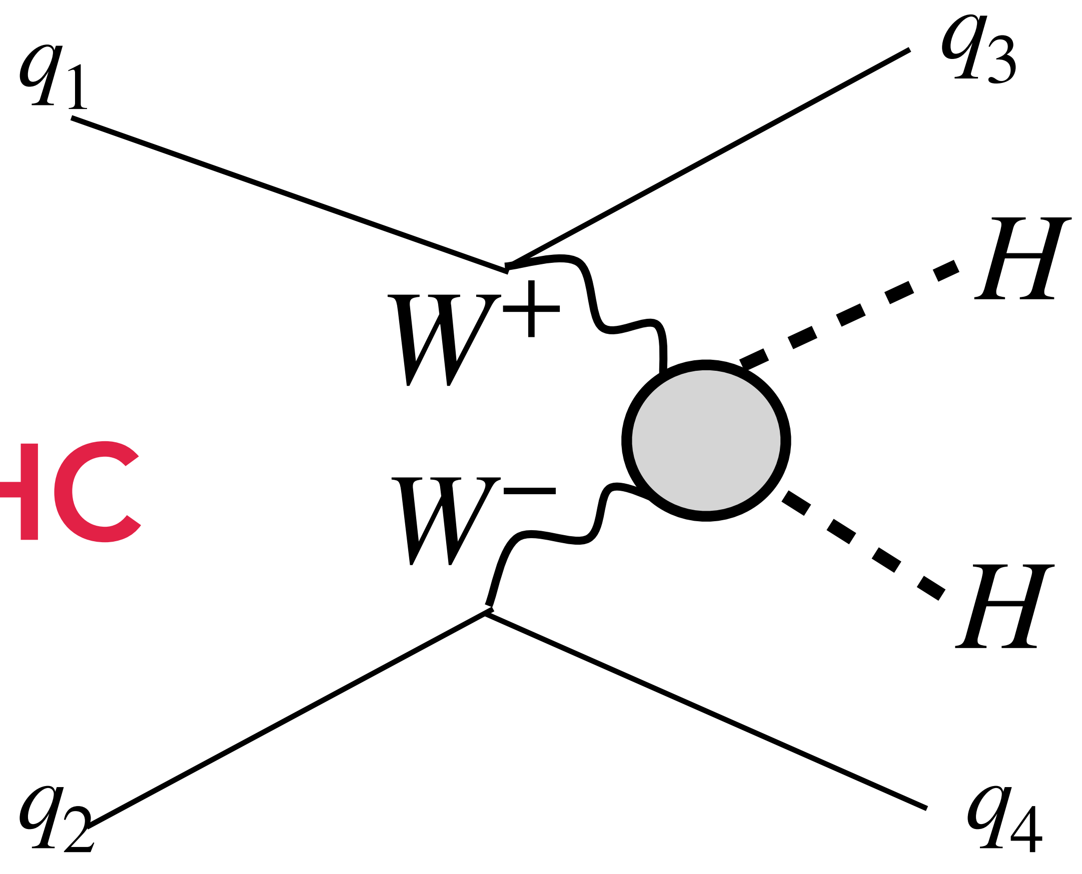
$$a_5 = \frac{v^4}{16\Lambda^4}c_{\phi^4}^{(3)}$$

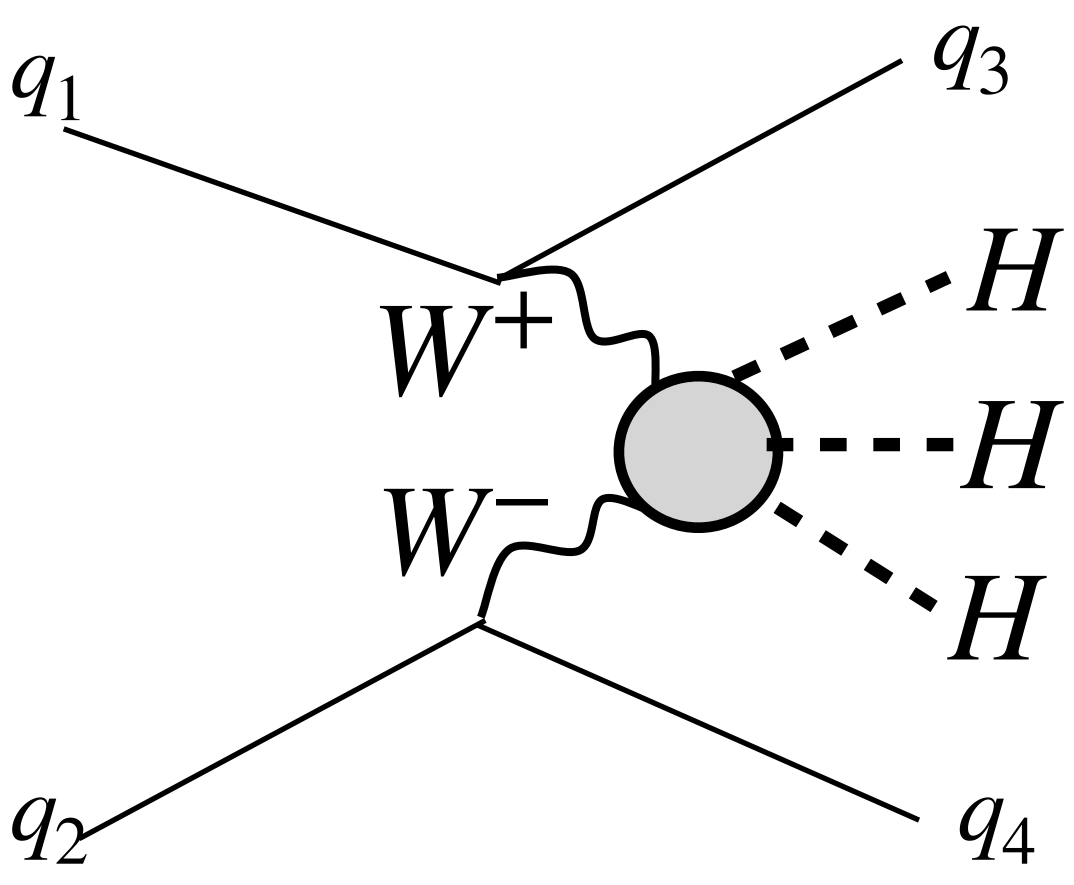
Pheno implications for MultiHiggs production at colliders

Most relevant phenomenology of the bosonic HEFT :

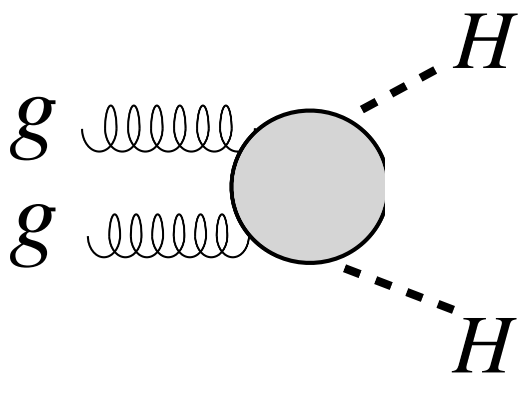
Golden Channels for BSM signals with EChL (HEFT): MultiBoson production

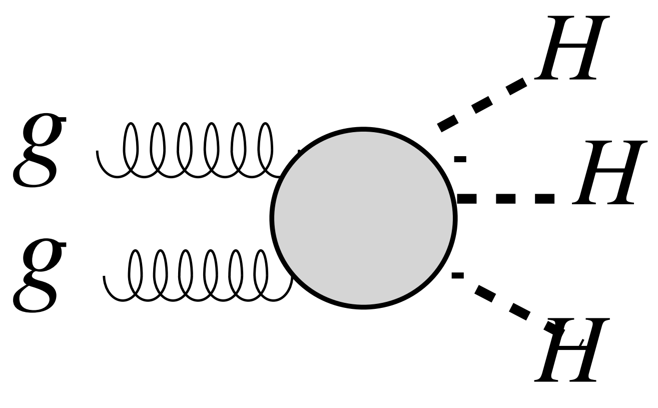
LHC





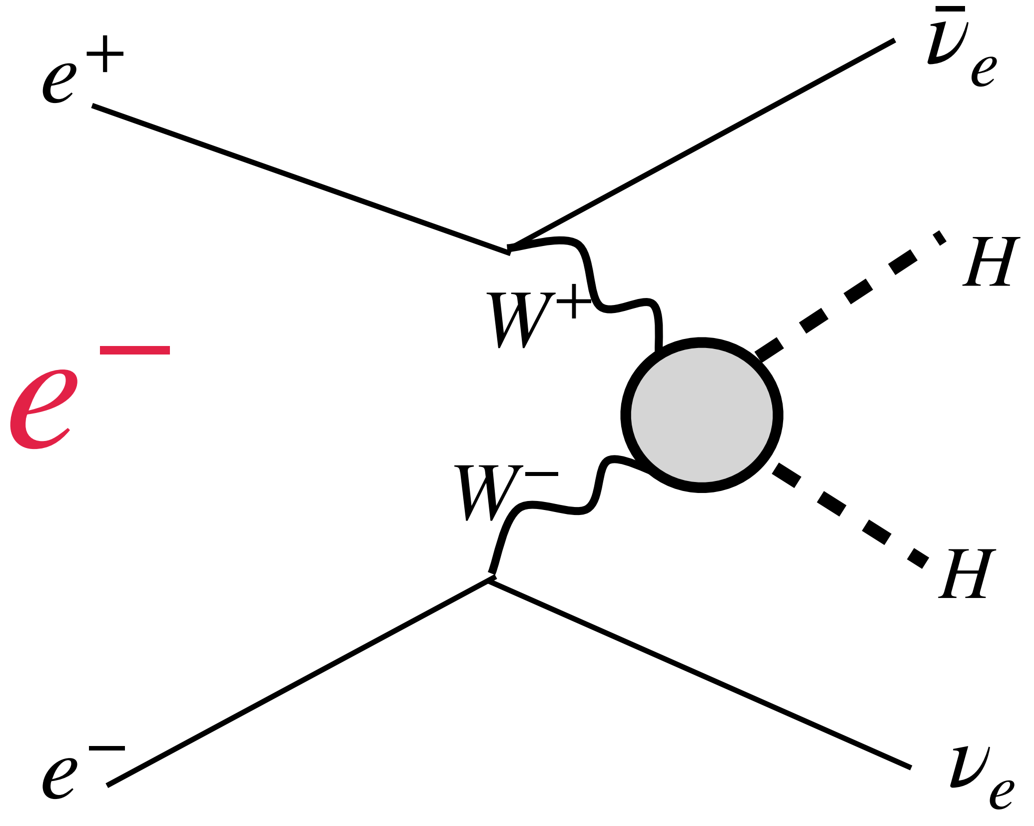
AND

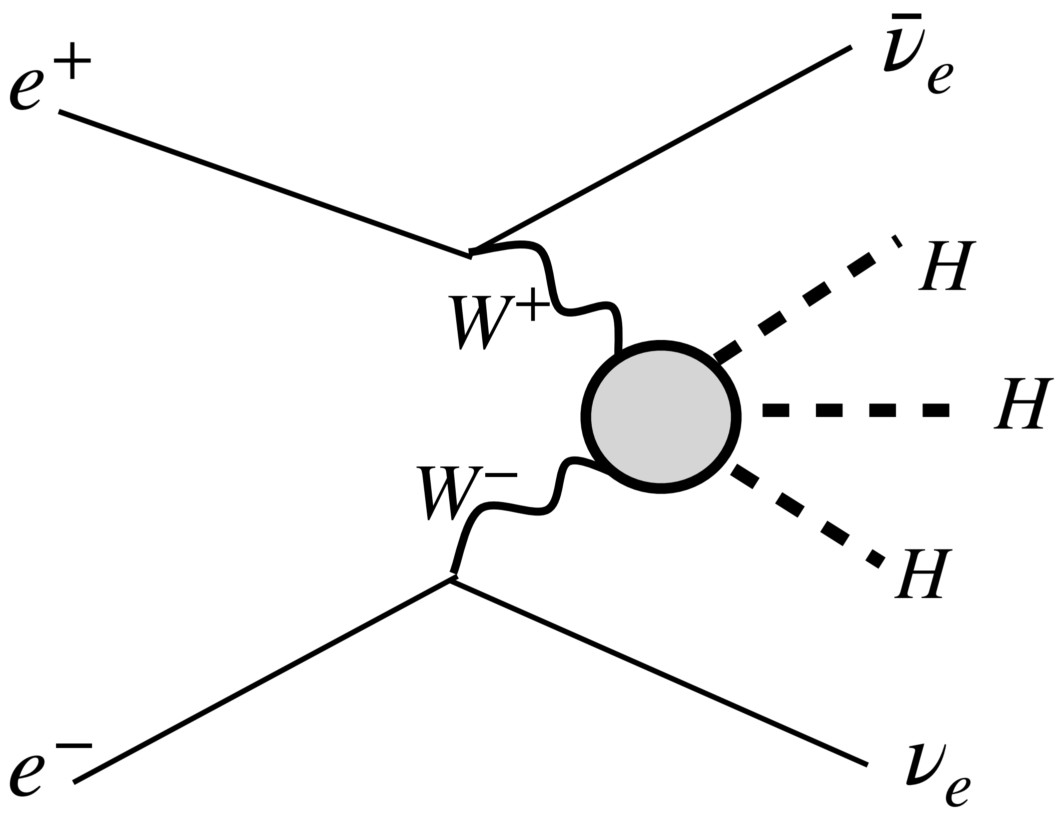




Do we learn anything comparing
HH and HHH production?

e^+e^-





Main goal is

Looking for **Enhacements** in multiple boson prod.
(gauge and Higgs): reminiscences of possible
underlying strong ints
(as in ChPT with multi π's)

*Different goal than constraining
BSM from global fits*

Main pheno implications of solutions found to the HEFT/SMEFT Matching

1) The relations found imply suppressed rates in the SMEFT compared to HEFT in multiple Higgs production from VBF

(Domenech et al 2506.2171) (see also Delgado et al 2311.04280 and talk by Cillero)

At $\mathcal{O}(\mathrm{TeV})$ energies (where the masses can be neglected):

$$A^{\mathrm{HEFT}}(W_LW_L \to HH) \simeq -\frac{1}{v^2}(a^2-b)s$$

$$A^{\mathrm{SMEFT}}(W_LW_L \to HH) \simeq \frac{s}{v^2}\Big(-2\frac{v^2}{\Lambda^2}(-c_{\Phi\Box}+\frac{1}{4}c_{\Phi D})\Big)$$

$$A^{\mathrm{HEFT}}(W_LW_L \to HHH) \simeq \frac{3}{v^3}\Big(c+\frac{4}{3}a(a^2-b)\Big)s$$

$$A^{\mathrm{SMEFT}}(W_LW_L \to HHH) \simeq \frac{s}{v^3}(0)+\mathcal{O}(s^0)$$

LO-HEFT amplitudes grow with s

HH If $(a^2-b)\neq 0$

HHH If $\left(c+\frac{4}{3}a(a^2-b)\right)\neq 0$

HEFT

Enhanced rates with respect to SM

Dim 6 -SMEFT amplitude suppressed by $(1/\Lambda^2)$

HH $\sim (s/\Lambda^2)$

HHH $\sim 0\times(s/\Lambda^2)$

(to get $\neq 0$ need to go to dim 8)
(then, amplitude even more suppressed)
$\sim (1/\Lambda^4)$

SMEFT

No significant enhanced rates with respect to SM

2) Constraints to HEFT coeffs can be derived from constraints to SMEFT coeffs (for instance, using global fits)

$\Delta a, \Delta b, c$ from constraints to $c_{\Phi\Box}, c_{\Phi D}$

$\Delta\kappa_3, \Delta\kappa_4$ from constraints to $c_{\Phi\Box}, c_{\Phi D}, c_\Phi$

a_{HWW}, a_{HHWW} from constraints to $c_{\Phi W}$

a_{HBB}, a_{HHBB} from constraints to $c_{\Phi B}$

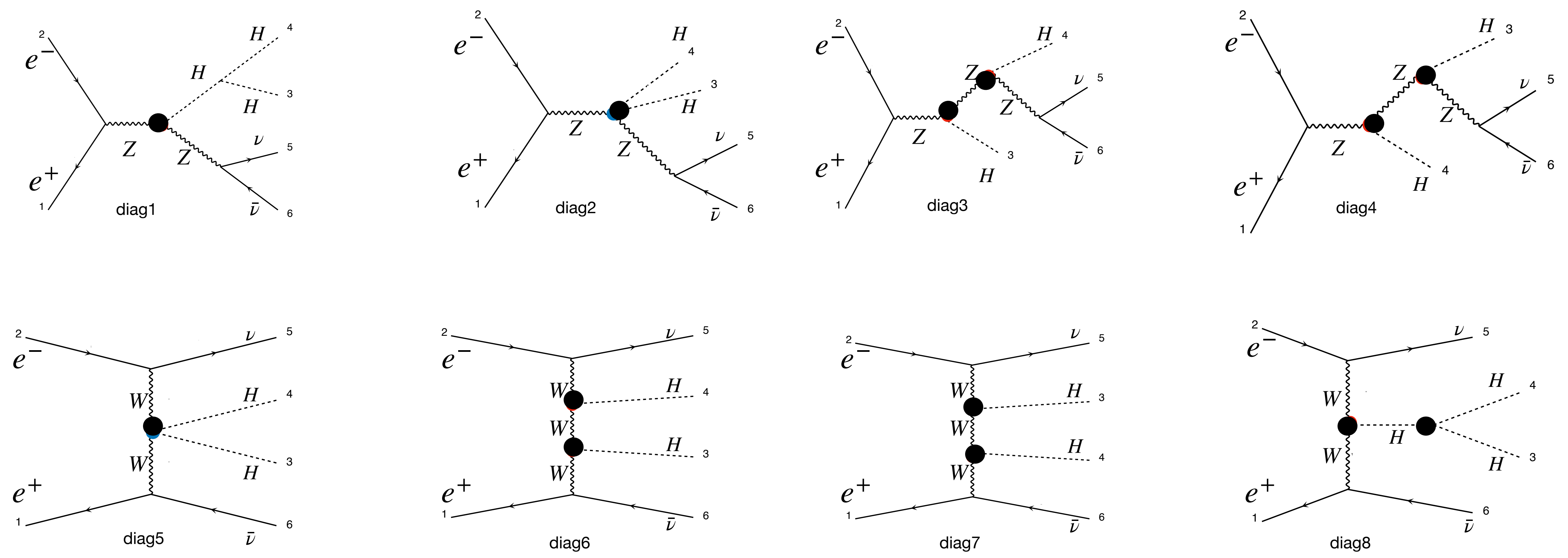
a_{H0}, a_{HH0} from constraints to $c_{\Phi D}$

a_{H1}, a_{HH1} from constraints to $c_{\Phi WB}$

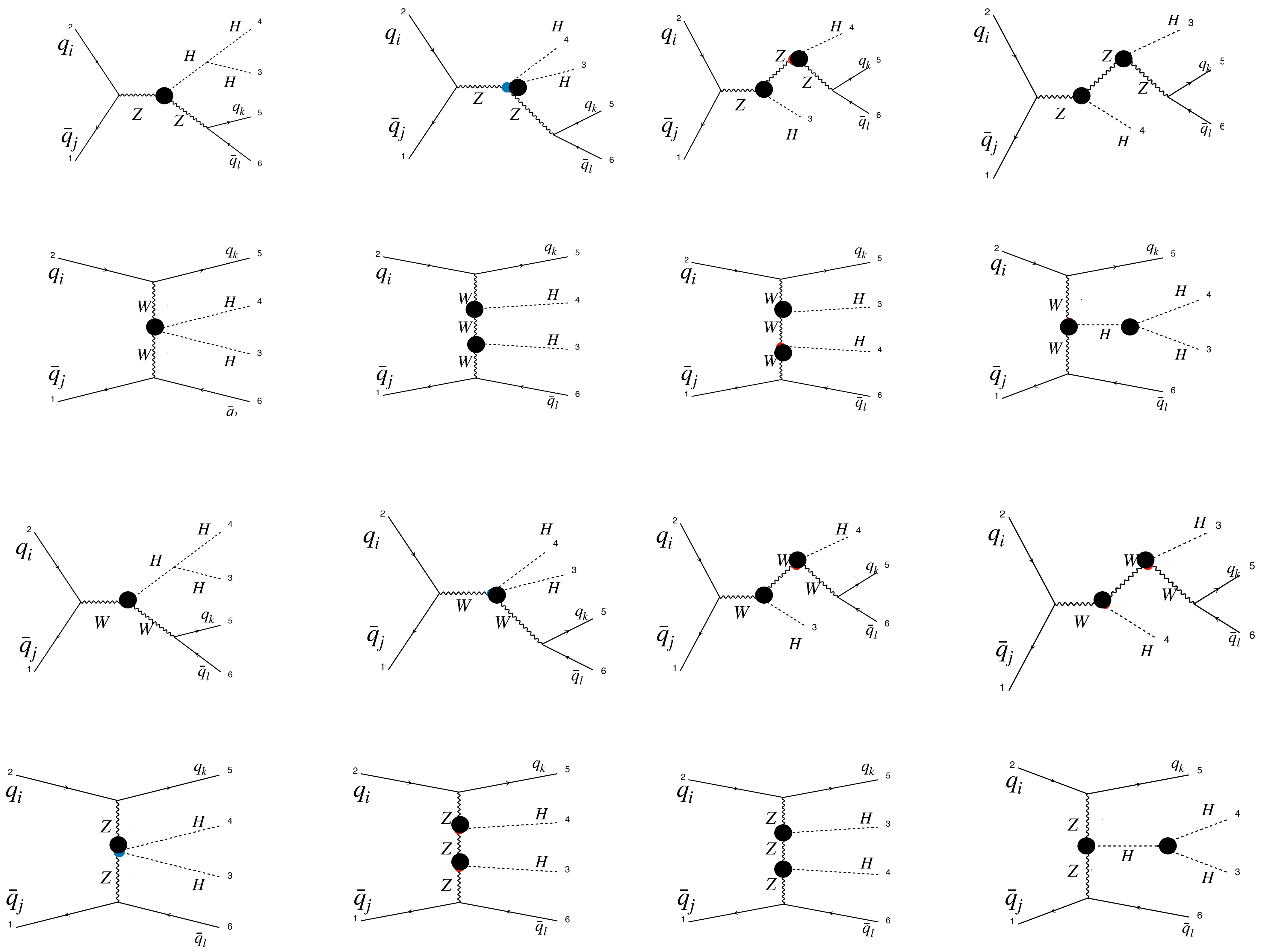
Predictions with HEFT at colliders: All diagrams must be included !!

Our Bosonic-HEFT model file is implemented in MadGraph5. It allows gauge choice.
For instance: HH requires including VBF and H(H) strahlung type diagrams

e^+e^- $\quad e^+e^- \to HH\nu\bar{\nu}$



LHC $\quad q_1\bar{q}_2 \to HHq_3\bar{q}_4$ (+ diags for $\bar{q}\bar{q}$ and for qq)



In e^+e^-explore $HH\not{E}_T$ events. We require $\not{E}_T > 20\,\mathrm{GeV}$

In pp explore $HHjj$ events
We require VBF cuts

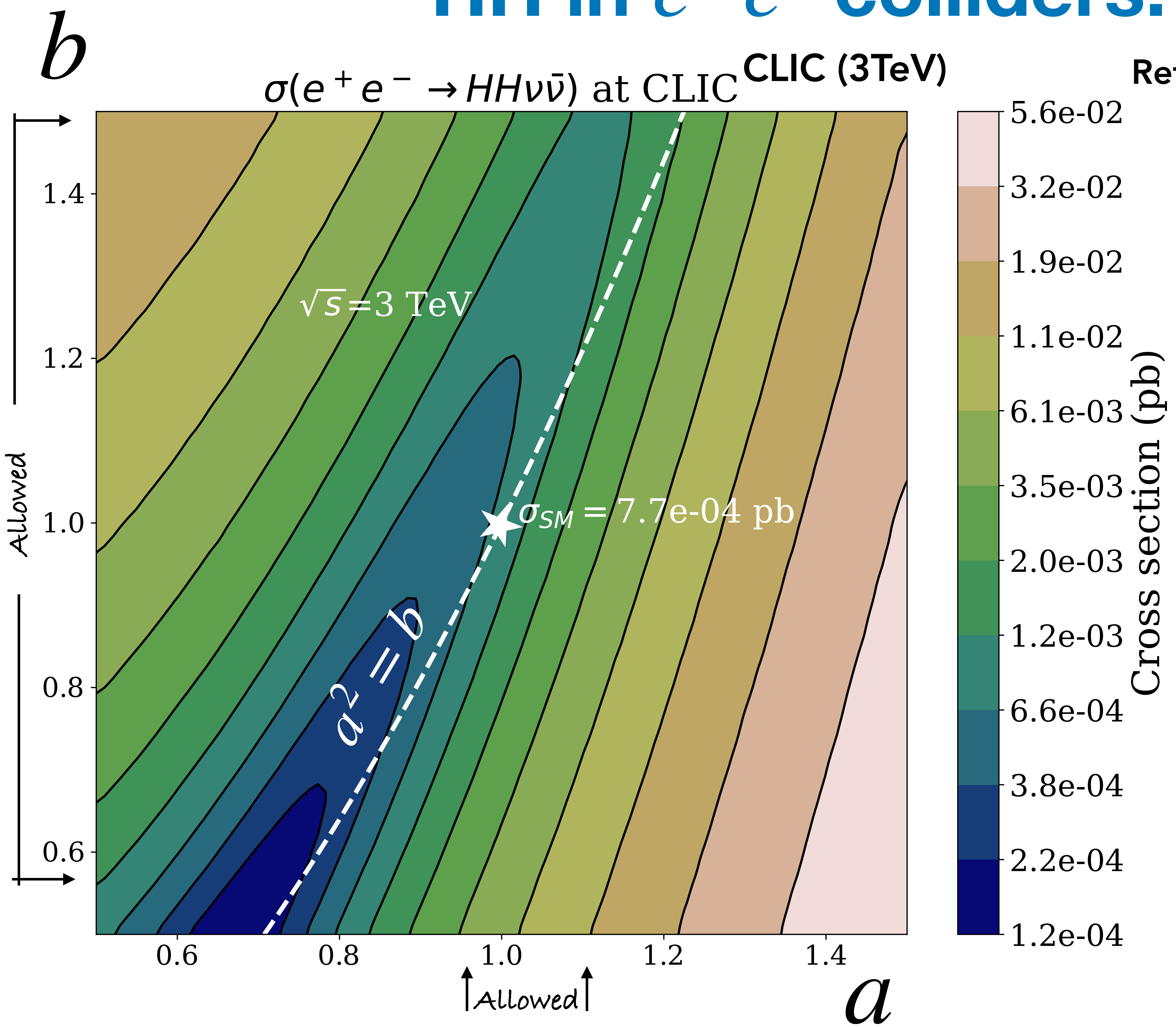
$$p_{T,j} \geq 20 \text{ GeV}, \qquad m_{jj} \geq 500 \text{ GeV},$$
$$2 < |\eta_j| < 5, \qquad \eta_{j_1} \times \eta_{j_2} < 0,$$

Two forward jets in different hemispheres at moderate transverse momentum with large invariant mass

BSM signals means deviations in σ and in $d\sigma's$ respect the SM rates. We also explore correlations.

HH in e^+e^-colliders: the role of $(a^2 - b)$

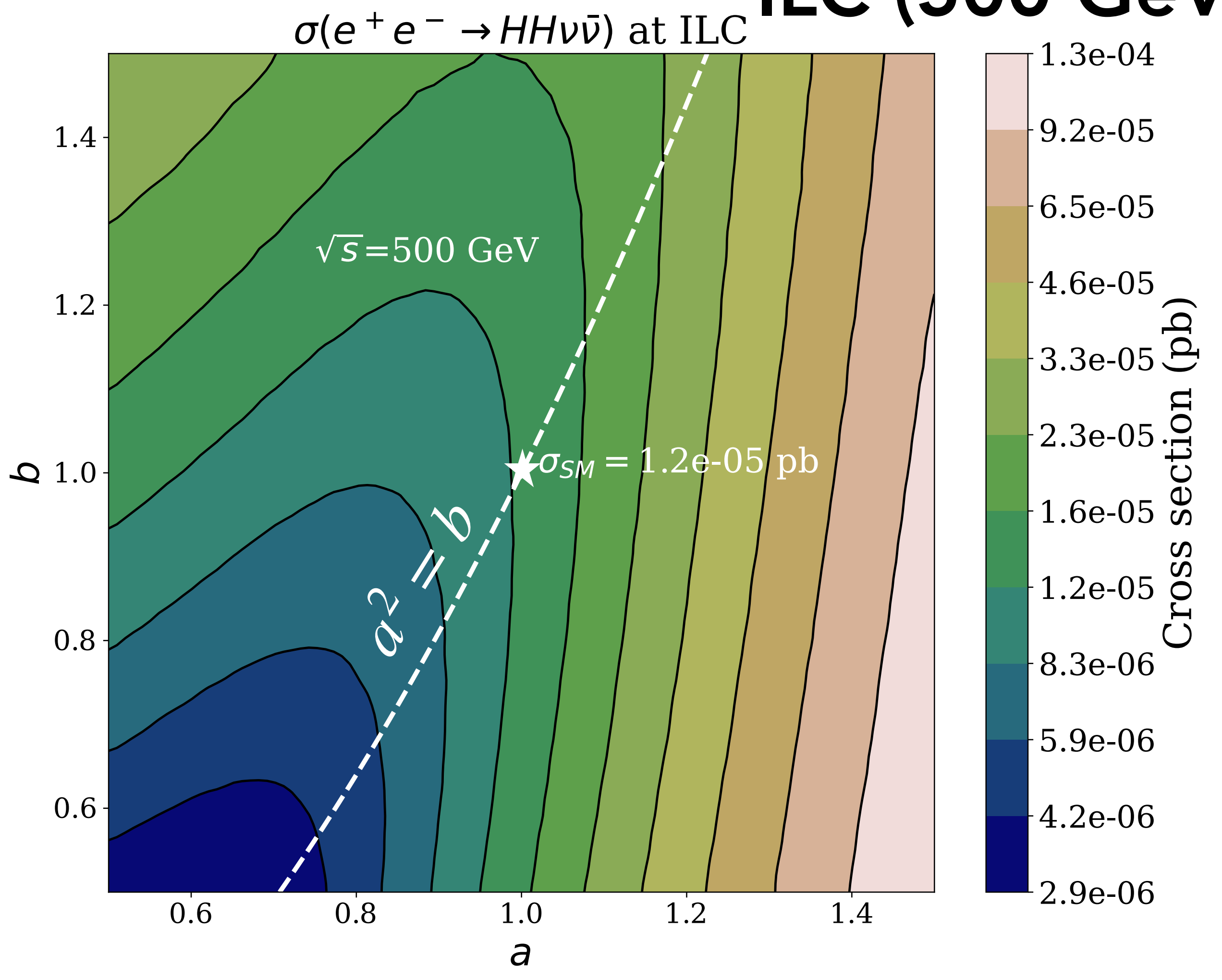
CLIC (3TeV)



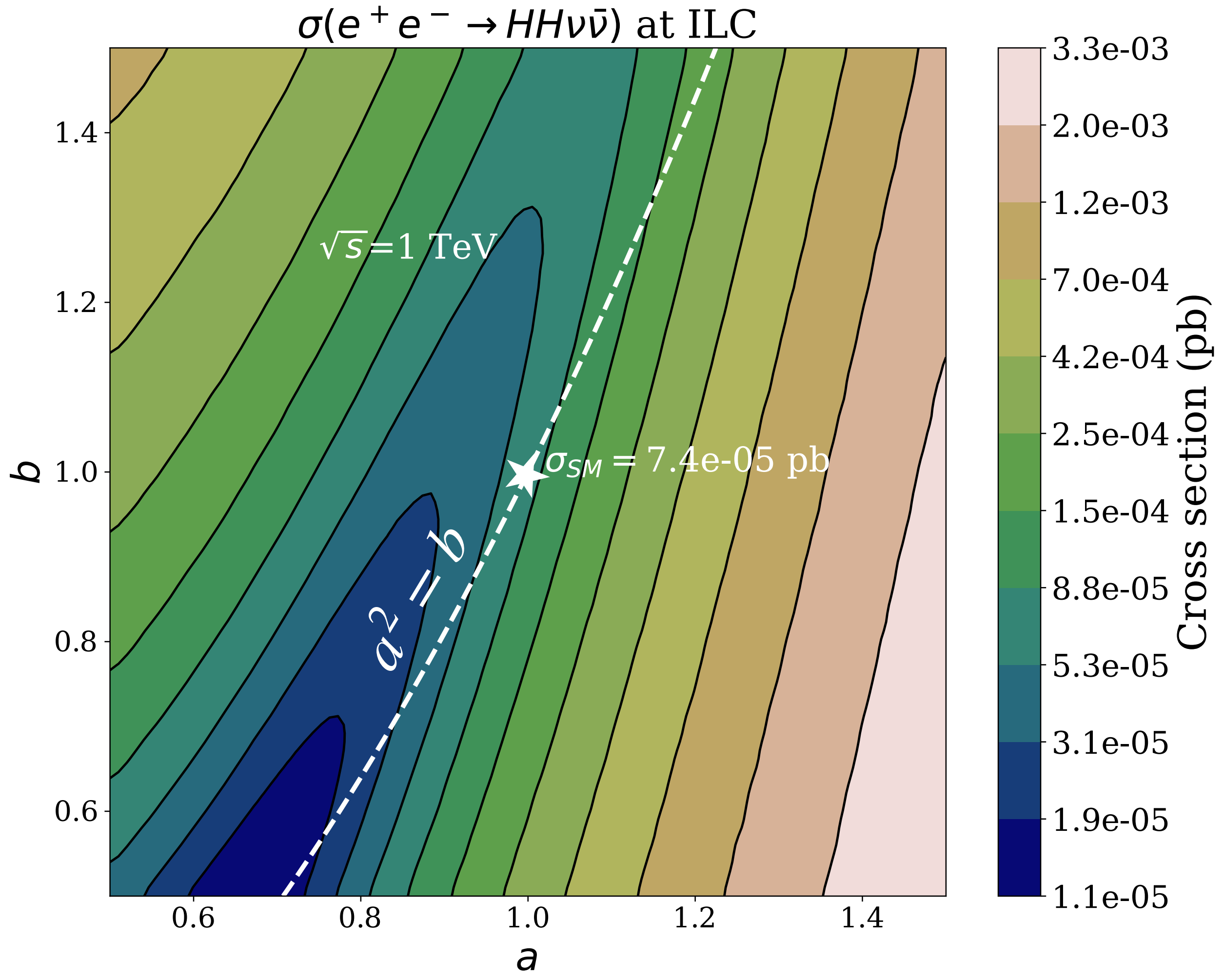
White line is for $b - a^2 = 0$, close to contours with lowest σ

Reference SM: $\sigma_{\rm SM}(e^+e^- \to HH\nu\bar{\nu}) = 7.7 \times 10^{-4}\,\text{pb}$ **at CLIC (3 TeV)**

ILC (500 GeV)



ILC (1TeV)



CLIC has the greatest σ as expected

Large cross section enhancements when $(b - a^2) \neq 0$

$b = 1 - \Delta b; a = 1 - \Delta a$

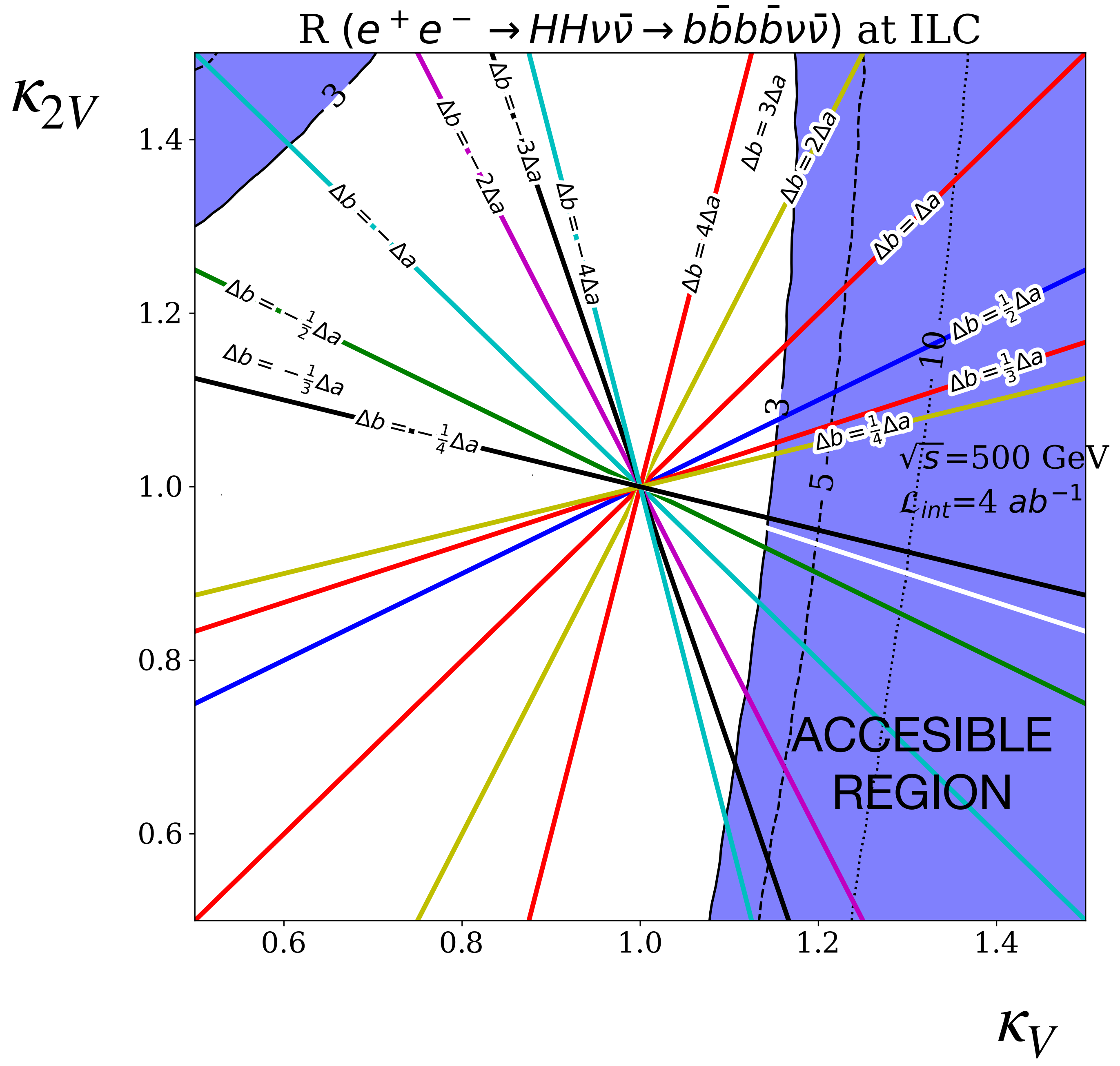
($\Delta b \neq 2\Delta a$)

2312.03877, EPJC 84 (2024)5, 503 ; 2011.13195, Davila, Domenech, Herrero, Morales

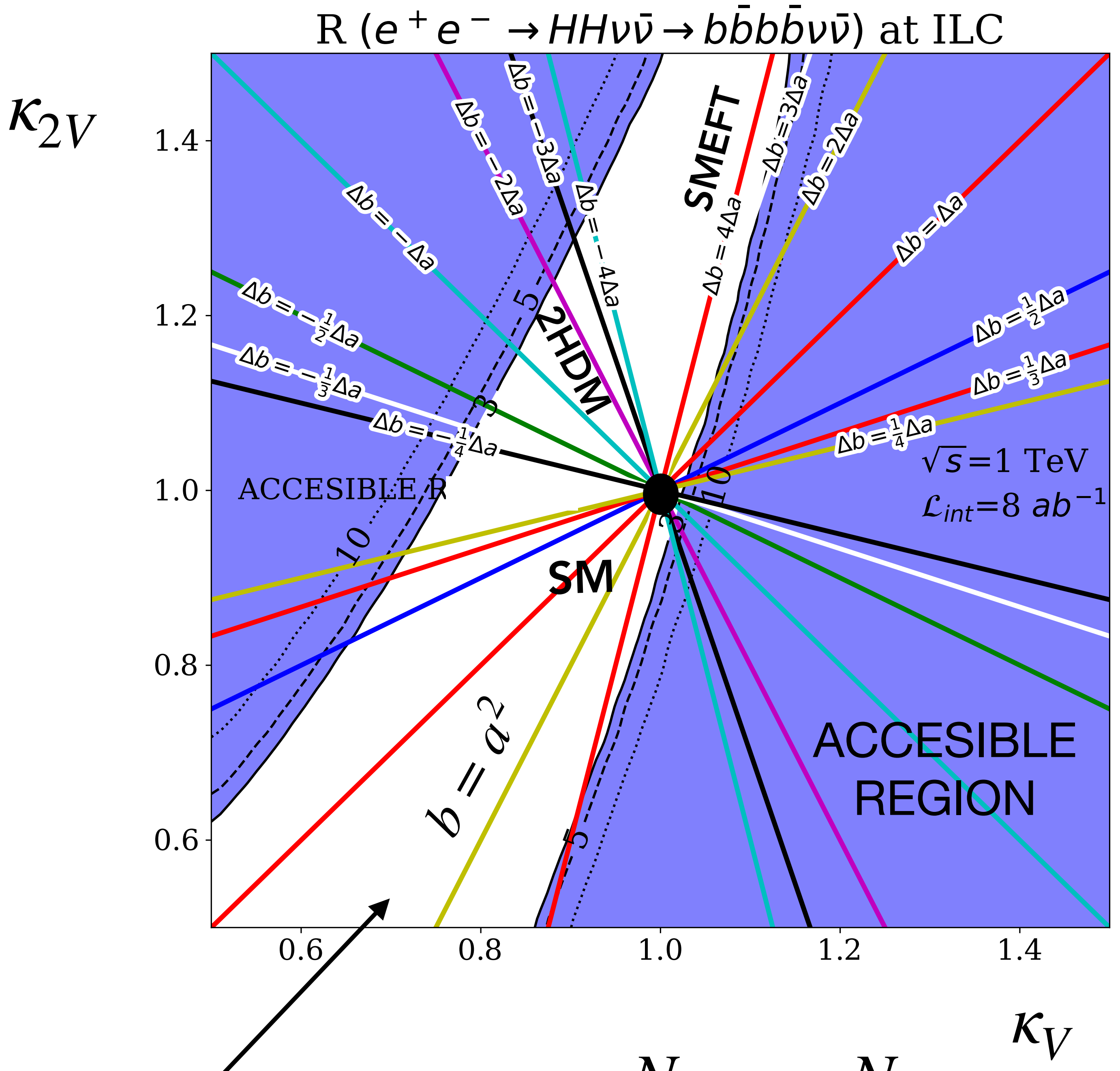
HH: Sensitivity to $(a, b) = (\kappa_V, \kappa_{2V})$ and correlations in e^+e^-

$e^+e^- \to HH\nu\bar{\nu} \to 4b + E_T^{miss}$ (Parton level simulations, no background, no detector sim.)

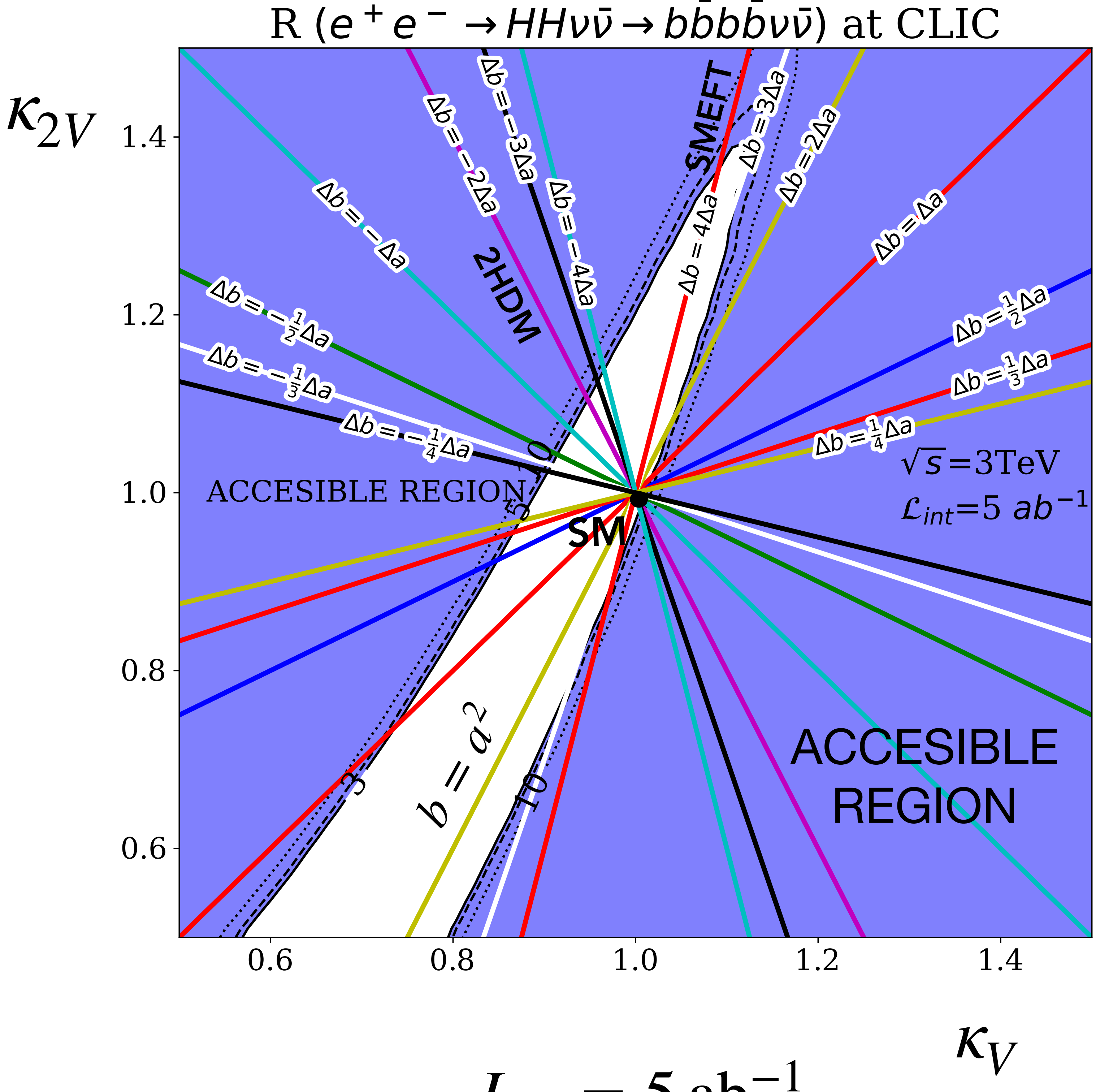
ILC 500 GeV



ILC 1 TeV



CLIC 3 TeV (is the best option)



Accessible Regions (in purple) defined as $R = \dfrac{N_{\rm BSM} - N_{\rm SM}}{\sqrt{N_{\rm SM}}} > 3$

Correlations $b \neq a^2$ defined by lines $\Delta b = C\Delta a$; $b = 1 - \Delta b$; $a = 1 - \Delta a$

Moving for instance in yellow line $b = a^2$ (equiv. to $\Delta b = 2\,\Delta a$) poorly accessible scenario (i.e. inside white region)

$L_{\rm int} = 5\ {\rm ab}^{-1}$

N=EVENTS with 4b+ETmiss

4 b-tagged jets $\epsilon_b = 0.8$

- ✓ $p_T^j > 20$ GeV
- ✓ $\Delta R_{jj} > 0.4$
- ✓ $|\eta^j| < 2$
- ✓ $E_T^{\rm mis} > 20$ GeV

Interesting cases: $\Delta b|_{\rm 2HDM} = -2\Delta a|_{\rm 2HDM}$ (magenta line)

$\Delta b|_{\rm SMEFT} = 4\Delta a|_{\rm SMEFT}$ (red line)

2312.03877 Davila, Domenech, Herrero, Morales, EPJC 84 (2024)5, 503 ; 2011.13195 2307.15693 2208.05452

(Parton level simulations, no background, no detector sim.)

Accessibility to NLO-HEFT $(\eta, \delta)=(a_{ddVV1}, a_{ddV2})$ at e^+e^-

2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos

Minimal detection cuts

$p_T^b > 20$ GeV $|\eta^b| < 2$

$\Delta R_{bb} > 0.4$ $\not{E}_T > 20$ GeV

b-tagging efficiency of 80%

Signal with greater statistics: $e^+e^- \to HH\nu\bar{\nu} \to b\bar{b}b\bar{b}\nu\bar{\nu}$

4b jets + missing ET

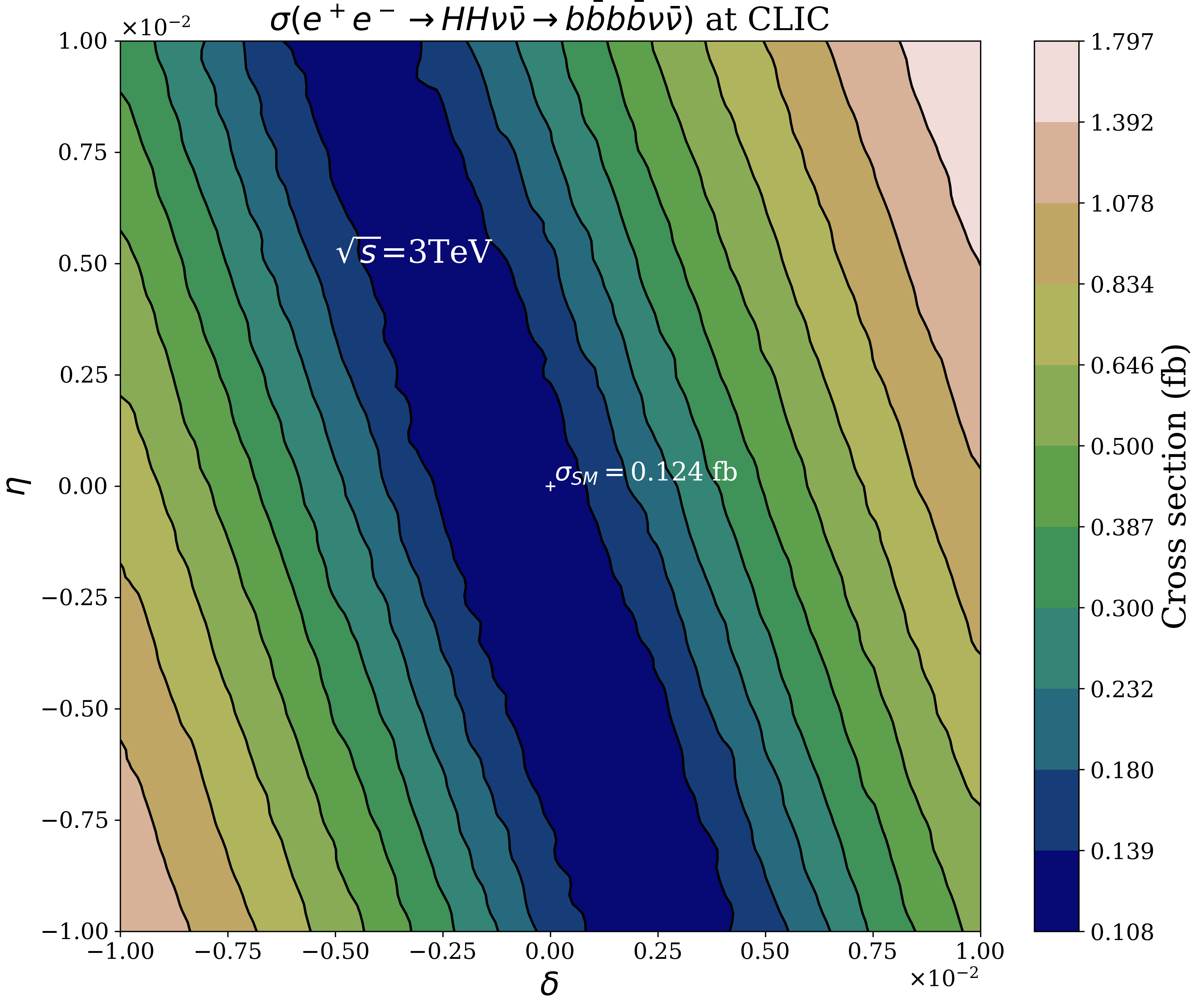
$$R = \frac{N_{BSM} - N_{SM}}{\sqrt{N_{SM}}}$$

Greater accessibility in CLIC (3TeV)

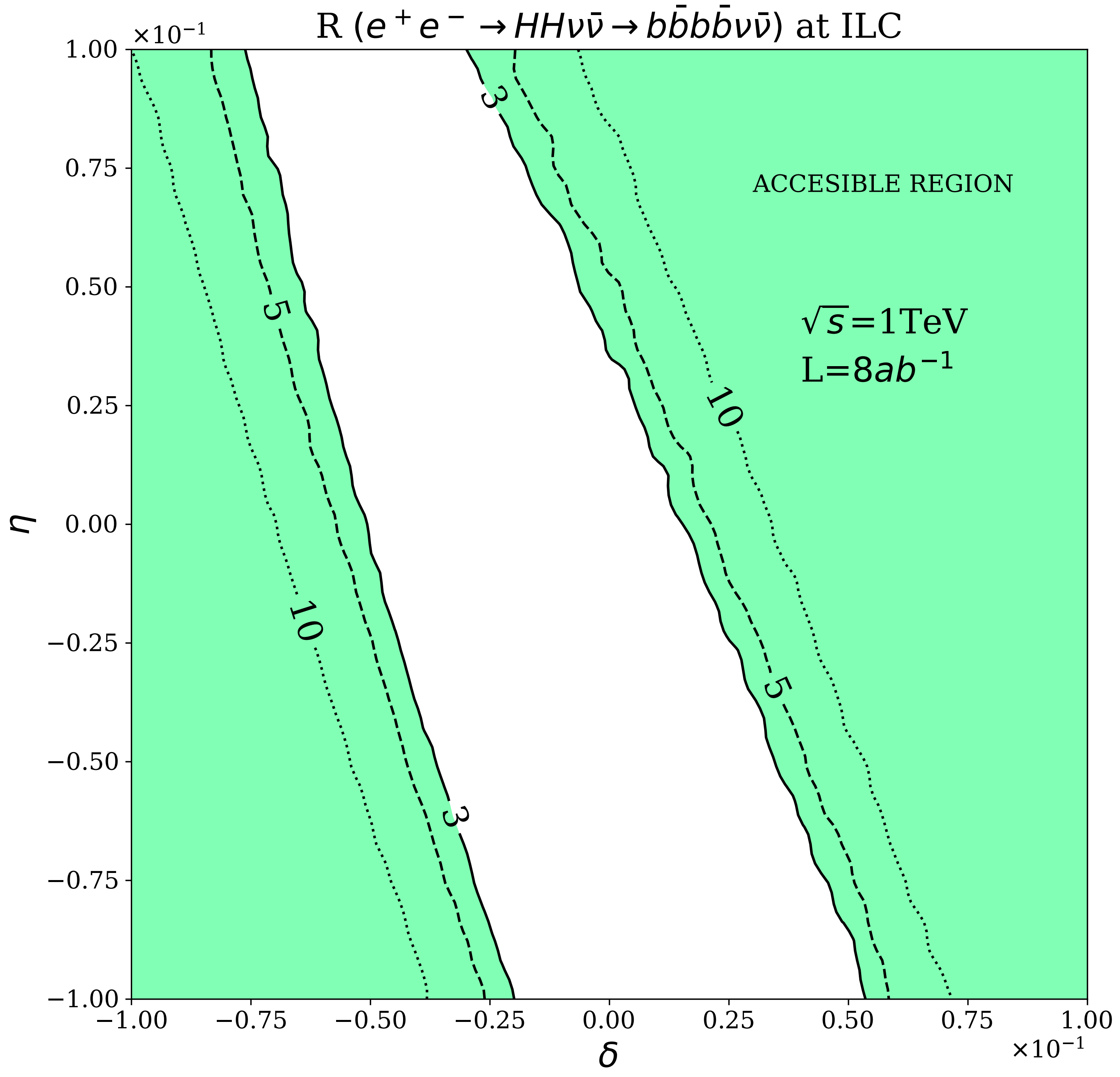
Expected reach $\eta, \delta \sim \mathcal{O}(10^{-3})$

Accesible region: R > 3

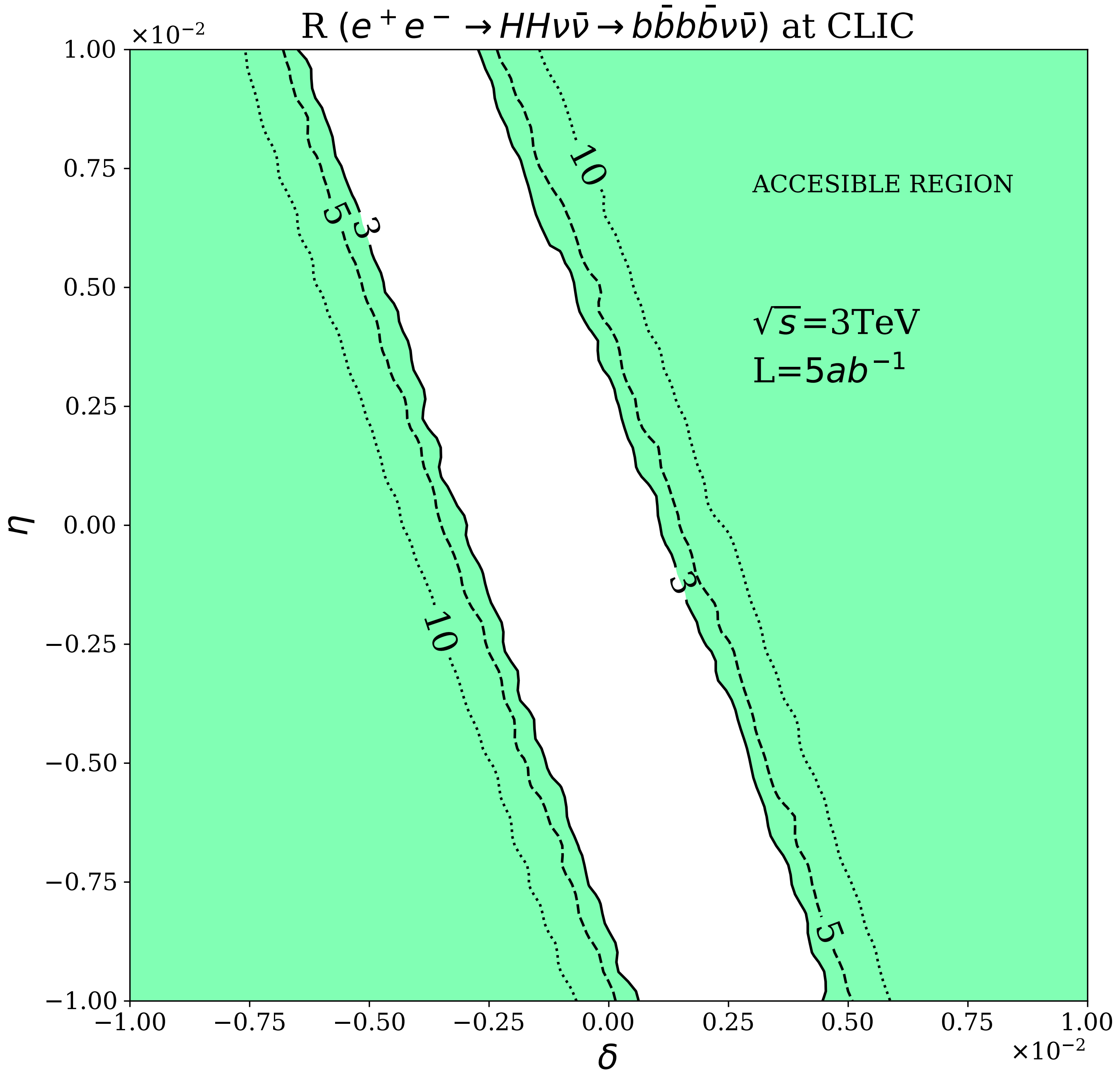
CLIC 3 TeV (is the best option)



ILC



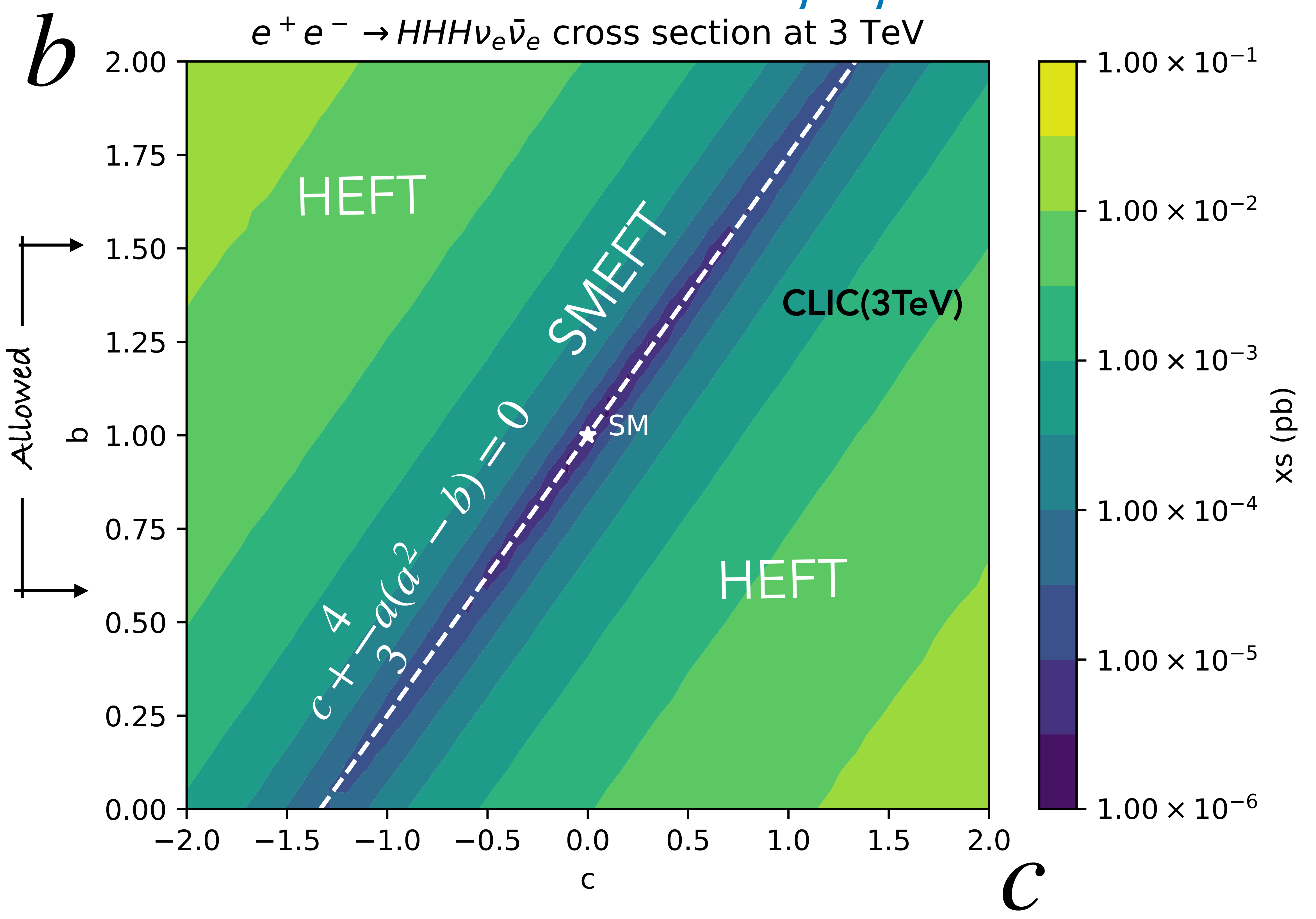
CLIC



Access to b and c in HHH production at lepton colliders

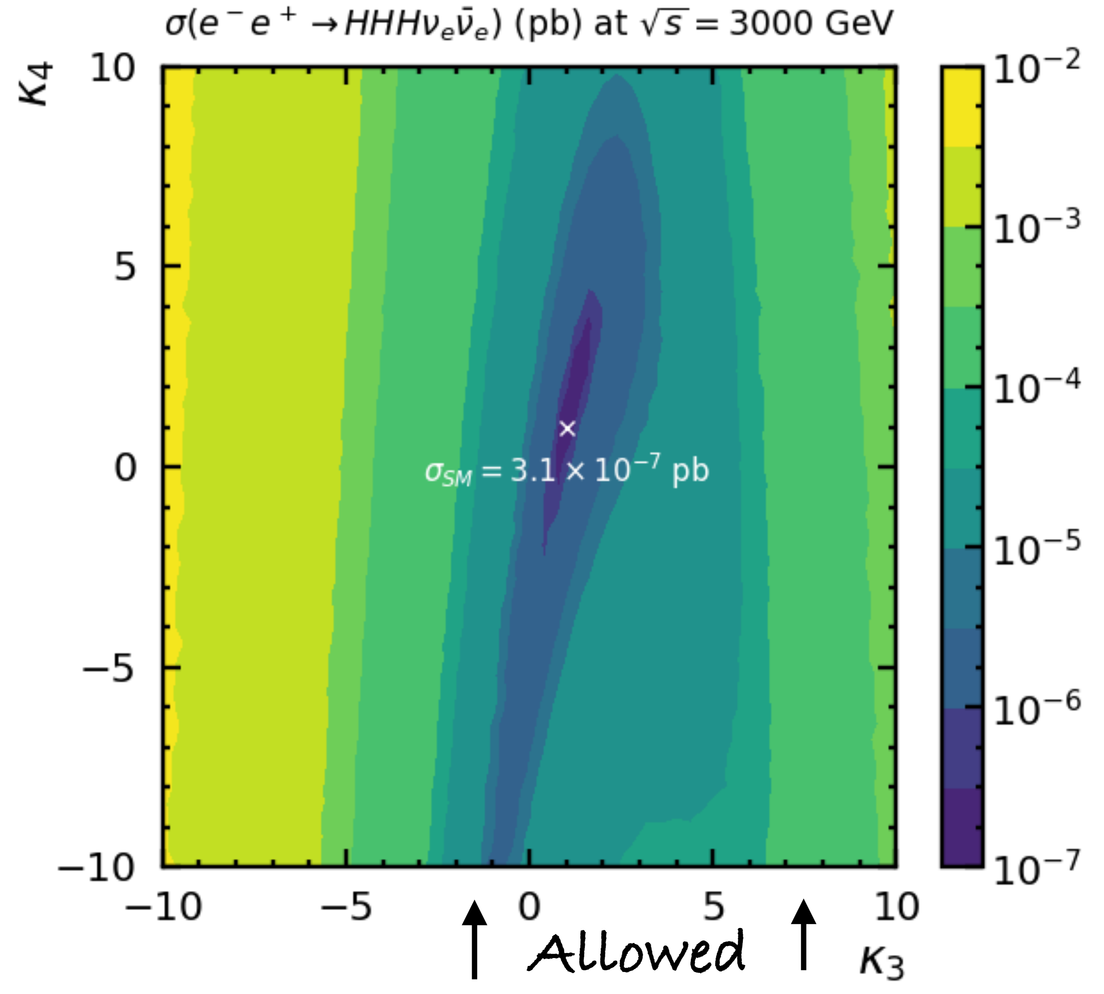
CLIC e^+e^- 3 TeV and $\mu^+\mu^-$3 TeV are the best options

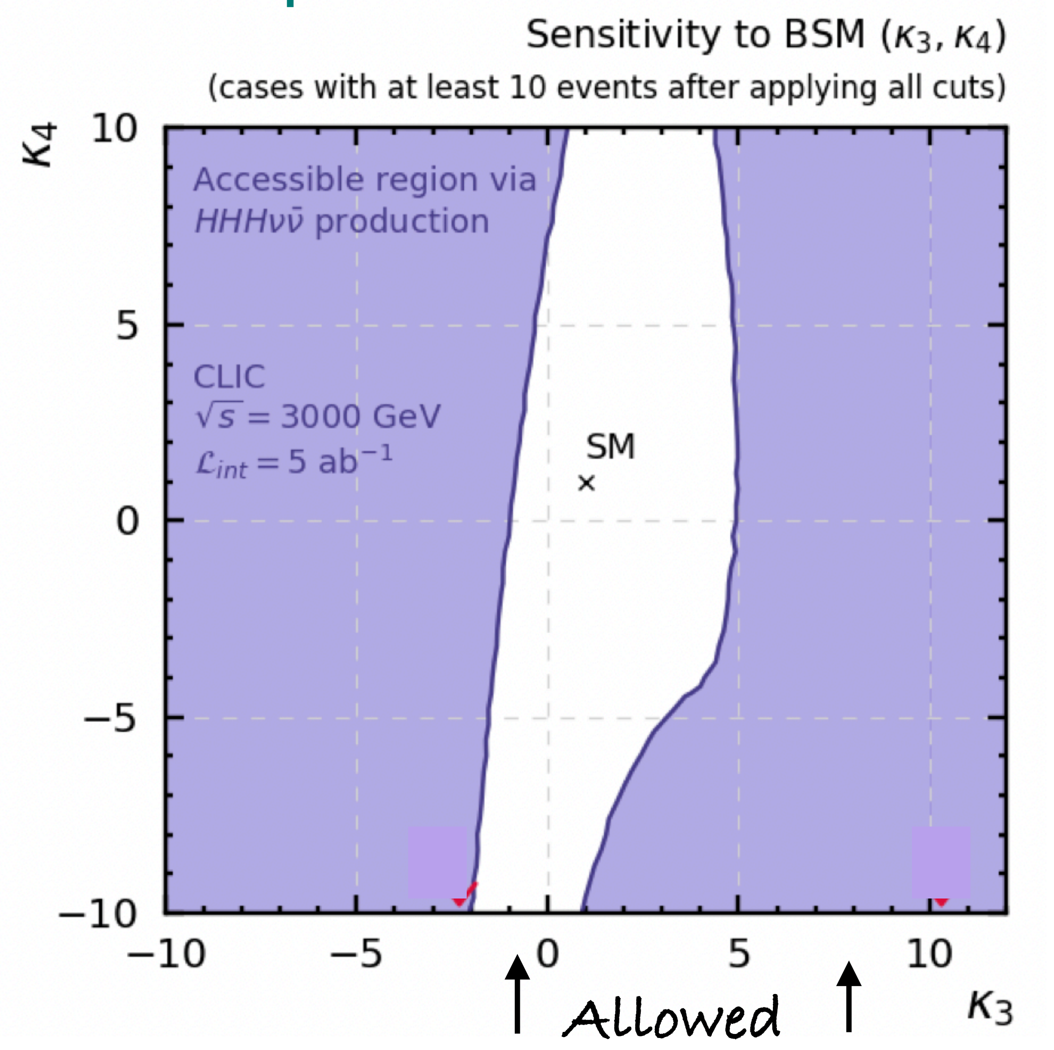
2407.20706, Phys. Rev. D 111, 055004 (2025),
Anisha, Domenech, Englert, Herrero, Morales



Reference SM: $\sigma_{\rm SM}(e^+e^- \to HHH\nu\bar{\nu}) = 3\times10^{-7}$ pb

sensitivity to κ_3 and κ_4 is poor





2011.13195, EPJC 81 (2021)3, 260,
González-López, Herrero, Martínez-Suárez

sensitivity in the HEFT to b and c is large

$\sigma(\Delta\kappa_4 = -0.25) = \mathcal{O}(10^{-7})$pb $\quad \sigma(\Delta\kappa_3 = -0.25) = \mathcal{O}(10^{-6})$pb

Large enhacements in $\sigma(e^+e^- \to HHH\nu\bar{\nu})_{\rm HEFT}$ compared to SM if $c+\frac{4}{3}a(a^2-b)\neq 0$

Small $\Delta b, c$ produces larger enhancements respect to SM rates, compared to $\Delta\kappa_{3,4}$

Explained by: $A(W_LW_L \to HHH) \simeq \frac{3s}{v^3}\left[c+\frac{4}{3}a\left(a^2-b\right)\right] + \mathcal{O}(s^0)$

$\sigma(\Delta b = \pm 0.25) \simeq 2\times10^{-4}$pb $\quad \sigma(c = \pm 0.25) \simeq 2\times10^{-4}$pb

$\Rightarrow N_U^{6b} = 51$ (~ 0 if SM)

(EW) HHjj and HHHjj production at LHC ($\sqrt{s} = 14\,\text{TeV}$)

Sensitivity to a, b, c

Anisha, Domenech, Englert, Herrero, Morales, PRD111, 055004 (2025),

All numerical computations are done with VBFNLO and MG5

Include ALL diagrams: VBF, Z MultiHiggs-strahlung ..

Sensitivity to a, b, c is maily via Vector Boson Fusion topology

VBF cuts

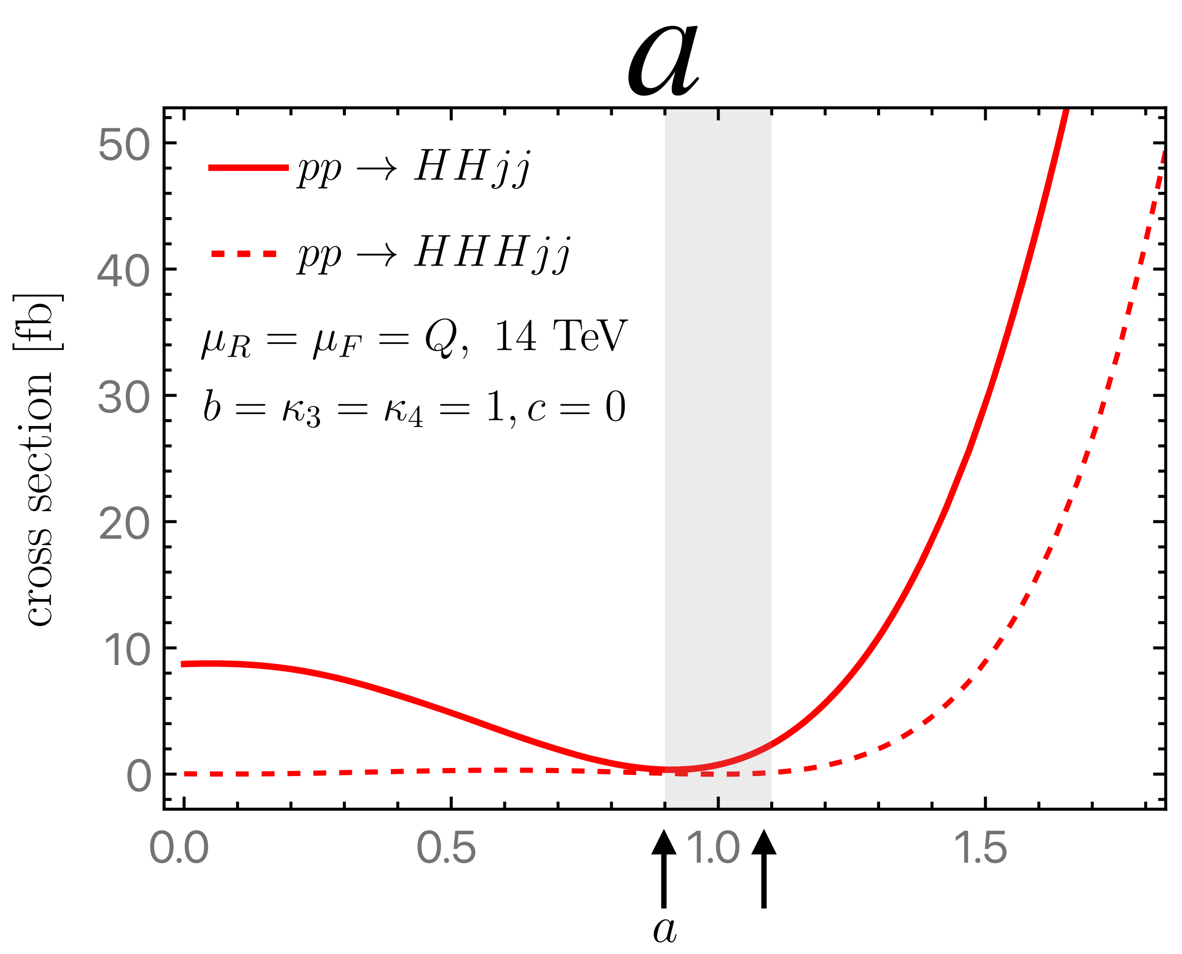
$$p_{T,j} \geq 20\ \text{GeV}, \quad m_{jj} \geq 500\ \text{GeV}, \quad 2 < |\eta_j| < 5, \quad \eta_{j_1} \times \eta_{j_2} < 0,$$

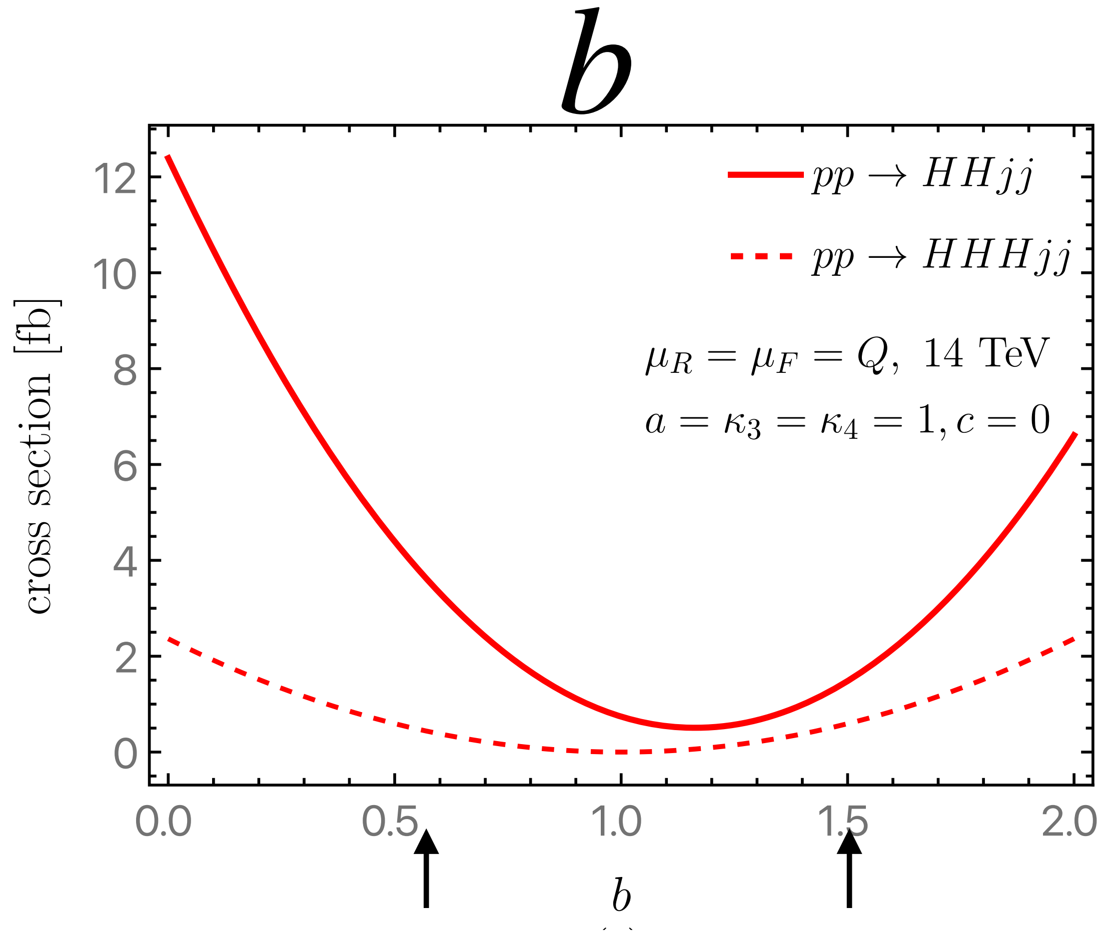
Two forward jets in different hemispheres at moderate transverse mom with large invariant mass

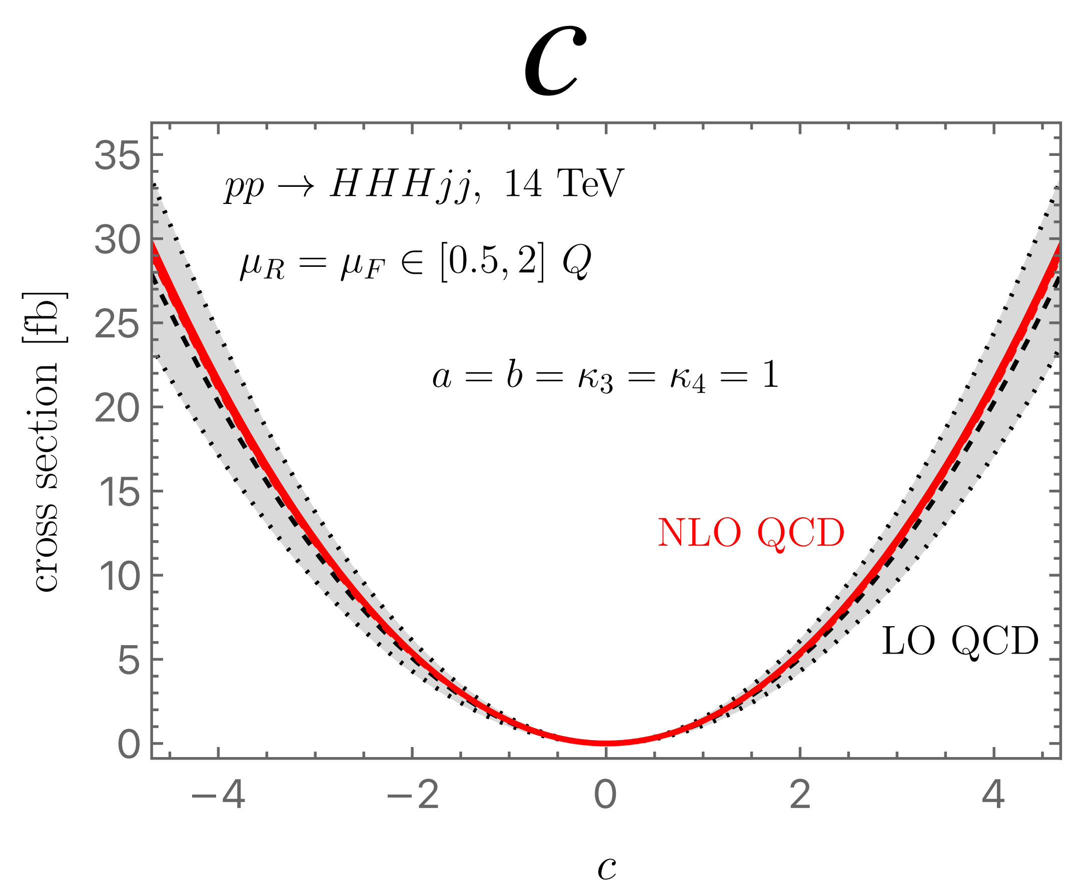
Reference SM

$\sigma_{SM}(pp \to HHjj) = 8 \times 10^{-4}\,\text{pb}$

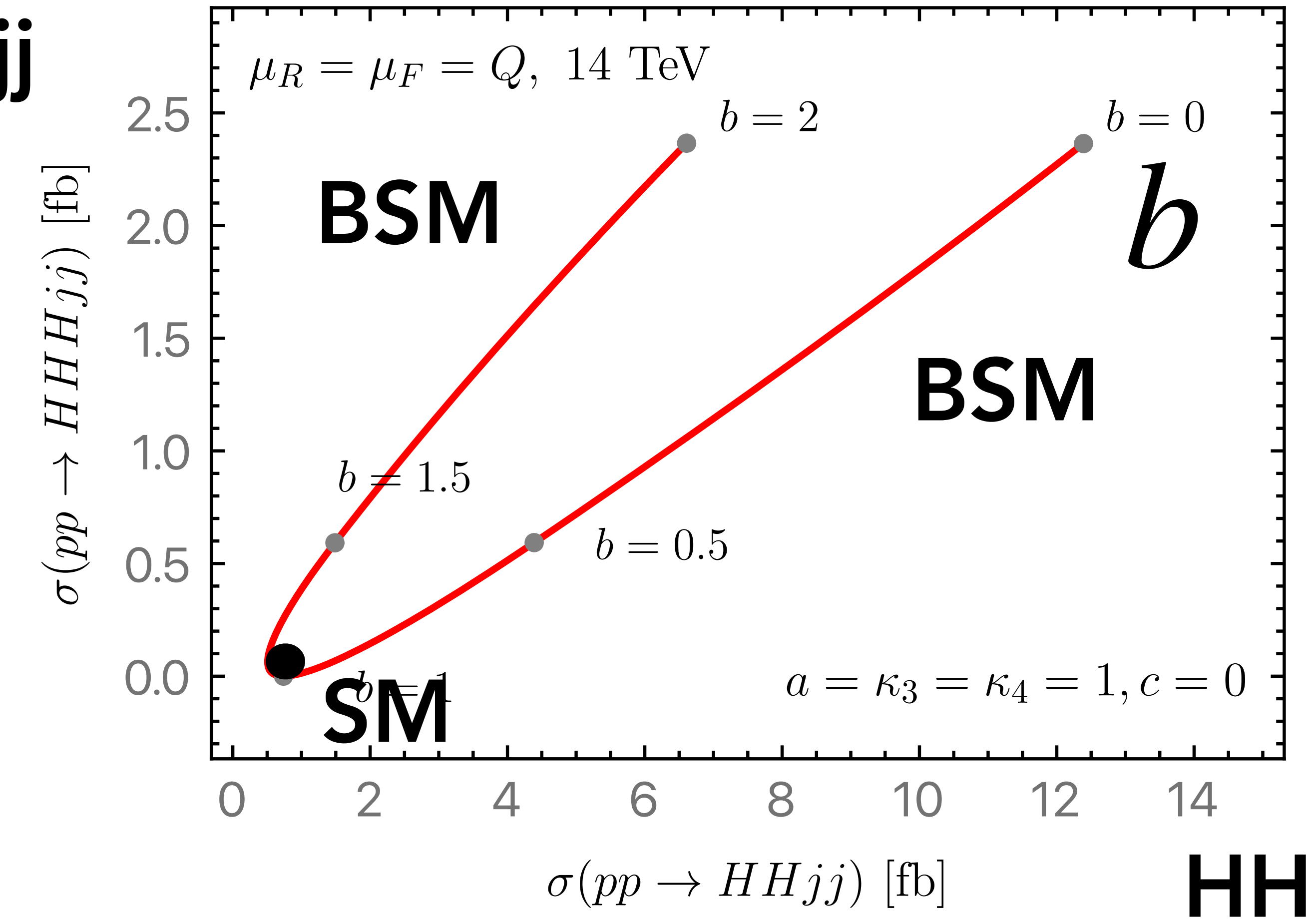
$\sigma_{SM}(pp \to HHHjj) = 2 \times 10^{-7}\,\text{pb}$







HHHjj

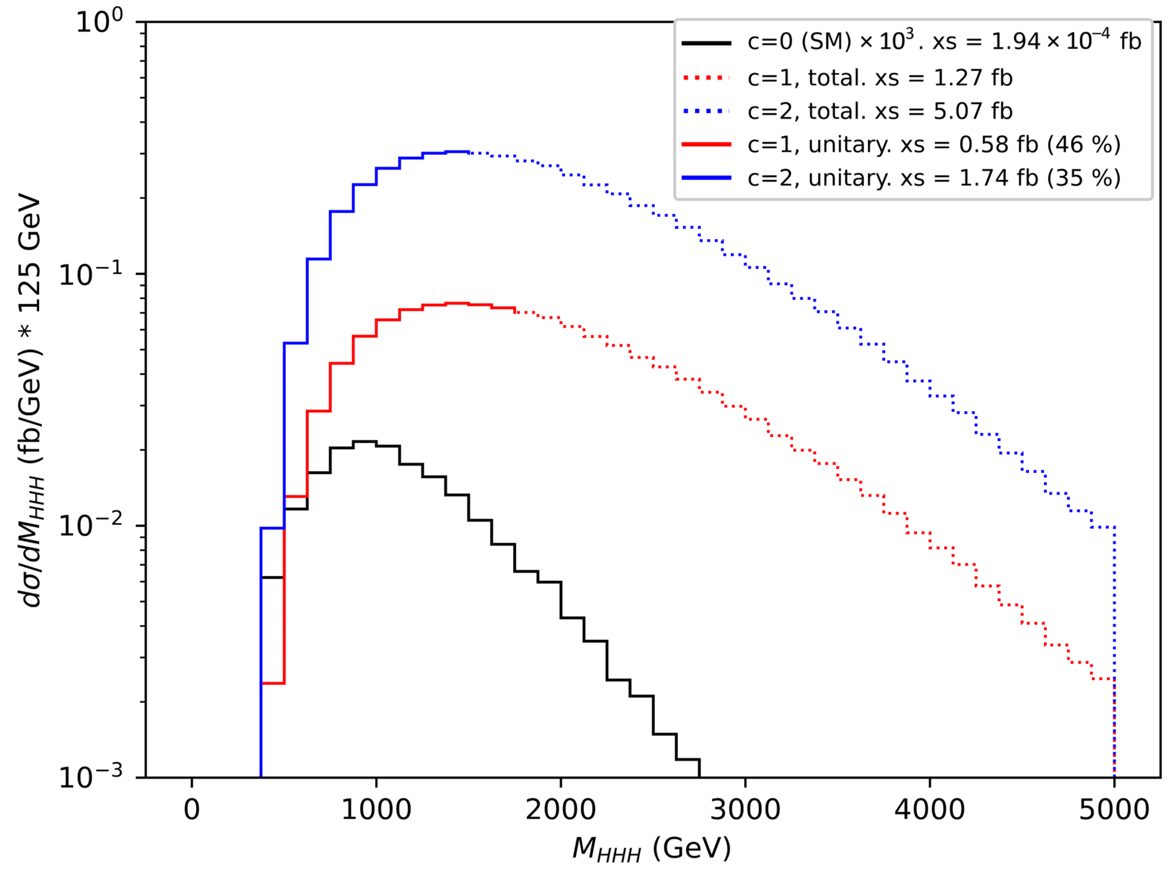


HHjj

We see $HHHjj \leftrightarrow HHjj$ correlations mainly from b

Big BSM enhancements HHHjj/HHjj with respect to SM

Larger if b>1 than in b<1



HHHjj

Big enhancements in invariant mass M_{HHH} distribution from $c \neq 0$

(Remind that c=0 in SM)

Large reduction factors To get unitarity preserved

$$\epsilon_U = \frac{\text{rates preserving unitarity}}{\text{total rates}}$$

HHHjj in pp colliders   Sensitivity to b, c

b	c	HL-LHC (14 TeV, 3 ab^{-1}) σ (fb)	ϵ_U	N_U^{6b}	FCC-hh (100 TeV, 30 ab^{-1}) σ (fb)	ϵ_U	N_U^{6b}
1.0	0.0	1.94×10^{-4}	1.00	0	1.27×10^{-2}	1.00	19
1.0	0.1	1.28×10^{-2}	9.96×10^{-1}	2	1.25×10^{1}	1.57×10^{-1}	3005
1.0	0.5	3.17×10^{-1}	6.84×10^{-1}	33	3.12×10^{2}	4.91×10^{-2}	23,511
1.0	1.0	1.27	4.60×10^{-1}	89	1.25×10^{3}	3.02×10^{-2}	57,887
1.0	2.0	5.07	3.50×10^{-1}	272	4.99×10^{3}	1.18×10^{-2}	90,194
1.0	5.0	3.17×10^{1}	1.69×10^{-1}	821	3.12×10^{4}	5.68×10^{-3}	271,802
1.5	0.0	5.56×10^{-1}	6.36×10^{-1}	54	5.50×10^{2}	4.11×10^{-2}	34,653
1.5	0.1	4.01×10^{-1}	6.35×10^{-1}	39	3.97×10^{2}	4.12×10^{-2}	25,060
1.5	0.5	3.53×10^{-2}	9.33×10^{-1}	5	3.41×10^{1}	1.14×10^{-1}	5959
1.5	1.0	1.48×10^{-1}	7.77×10^{-1}	18	1.42×10^{2}	6.91×10^{-2}	15,074
1.5	2.0	2.27	4.75×10^{-1}	166	2.23×10^{3}	2.49×10^{-2}	85,013
1.5	5.0	2.39×10^{1}	1.70×10^{-1}	623	2.35×10^{4}	5.83×10^{-3}	209,782
1.2	0.0	8.90×10^{-2}	8.37×10^{-1}	11	8.79×10^{1}	8.05×10^{-2}	10,867
1.2	0.1	3.46×10^{-2}	9.36×10^{-1}	5	3.42×10^{1}	1.14×10^{-1}	5982
1.2	0.5	7.06×10^{-2}	8.96×10^{-1}	10	6.88×10^{1}	1.01×10^{-1}	10,684
1.2	1.0	6.86×10^{-1}	5.84×10^{-1}	61	6.73×10^{2}	3.69×10^{-2}	38,136
1.2	2.0	3.82	4.14×10^{-1}	242	3.75×10^{3}	1.96×10^{-2}	112,846
1.2	5.0	2.84×10^{1}	1.69×10^{-1}	737	2.80×10^{4}	5.91×10^{-3}	253,718
0.8	0.0	8.95×10^{-2}	8.38×10^{-1}	12	8.80×10^{1}	8.02×10^{-2}	10,824
0.8	0.1	1.69×10^{-1}	7.69×10^{-1}	20	1.67×10^{2}	6.37×10^{-2}	16,284
0.8	0.5	7.41×10^{-1}	5.85×10^{-1}	67	7.31×10^{2}	3.65×10^{-2}	40,929
0.8	1.0	2.03	4.71×10^{-1}	147	2.00×10^{3}	2.48×10^{-2}	76,125
0.8	2.0	6.50	2.88×10^{-1}	287	6.40×10^{3}	1.18×10^{-2}	116,163
0.8	5.0	3.51×10^{1}	1.16×10^{-1}	624	3.46×10^{4}	3.71×10^{-3}	196,885
0.5	0.0	5.58×10^{-1}	6.37×10^{-1}	54	5.50×10^{2}	4.16×10^{-2}	35,126
0.5	0.1	7.38×10^{-1}	5.85×10^{-1}	66	7.28×10^{2}	3.89×10^{-2}	43,399
0.5	0.5	1.71	4.71×10^{-1}	124	1.69×10^{3}	2.47×10^{-2}	64,102
0.5	1.0	3.50	4.17×10^{-1}	224	3.45×10^{3}	1.96×10^{-2}	103,723
0.5	2.0	8.98	2.25×10^{-1}	310	8.85×10^{3}	8.44×10^{-3}	114,532
0.5	5.0	4.06×10^{1}	1.15×10^{-1}	720	4.00×10^{4}	3.84×10^{-3}	235,680
0.0	0.0	2.23	4.70×10^{-1}	161	2.20×10^{3}	2.40×10^{-2}	81140
0.0	0.1	2.58	4.10×10^{-1}	162	2.54×10^{3}	1.86×10^{-2}	72,706
0.0	0.5	4.22	2.86×10^{-1}	185	4.17×10^{3}	1.13×10^{-2}	72,099
0.0	1.0	6.85	2.85×10^{-1}	300	6.76×10^{3}	1.12×10^{-2}	116,597
0.0	2.0	1.40×10^{1}	2.25×10^{-1}	484	1.38×10^{4}	8.29×10^{-3}	175,548
0.0	5.0	5.07×10^{1}	1.15×10^{-1}	892	4.99×10^{4}	3.84×10^{-3}	293,866

SM → first row ($b = 1.0$, $c = 0.0$)

BSM from b, c → all remaining rows

Reference SM $\sigma_{SM}(pp \to HHHjj) = 2 \times 10^{-7}$ pb

N_U^{6b} = Number of events with 6b (+2 light jets with VBF topology)

Even after applying important reduction factors ϵ_U to preserve unitarity
We still get notable enhaments respect to to SM in events with 6b (+2 light jets) due to BSM with $b \neq 1$ or/and $c \neq 0$

In both studied cases

HL-LHC (14 TeV, 3 ab-1)

FCC-hh (100 TeV, 30 ab-1)

$a = 1$
Is set here

Sensitivity to b and c is large

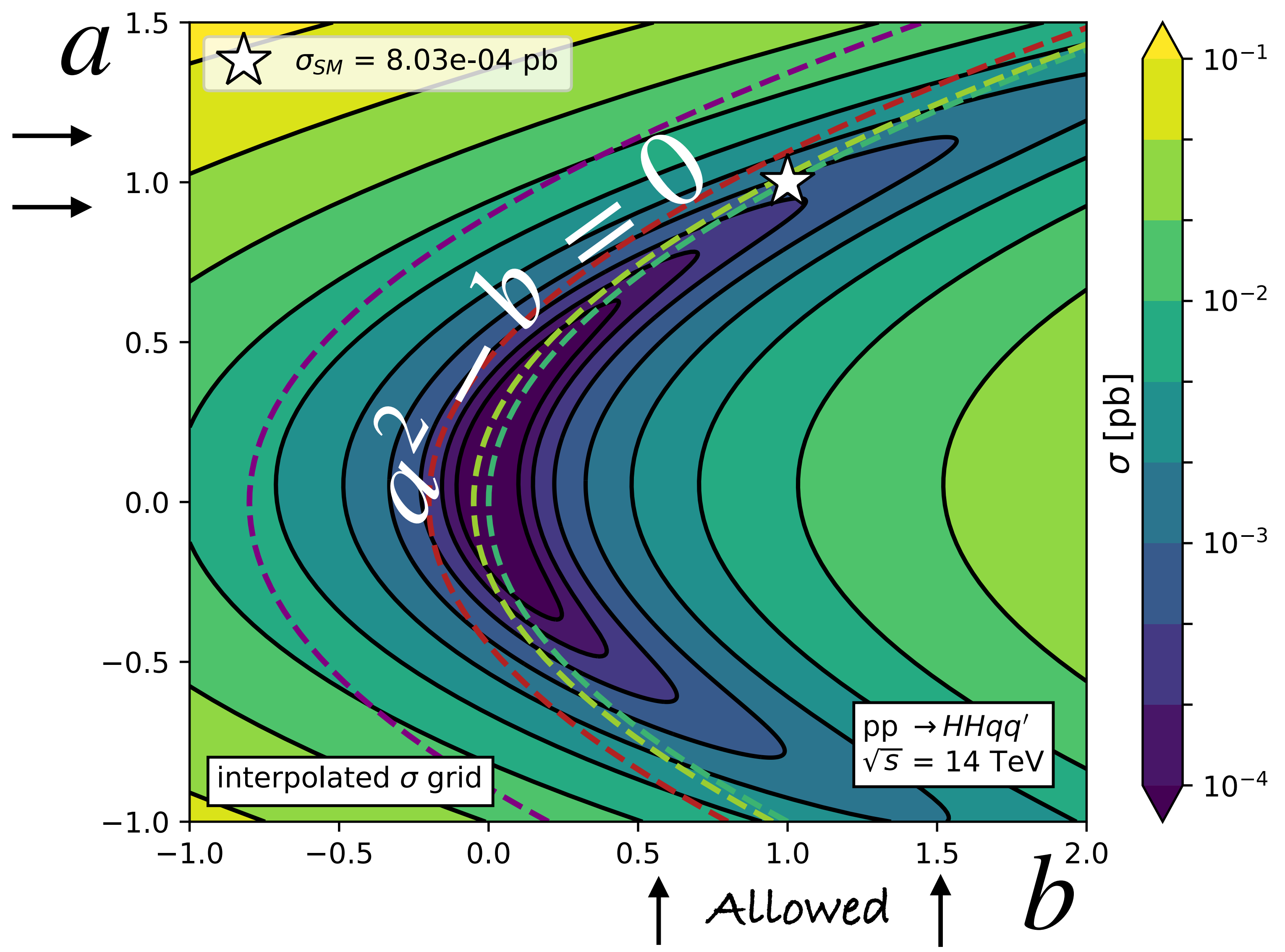
Examples at HL-LHC

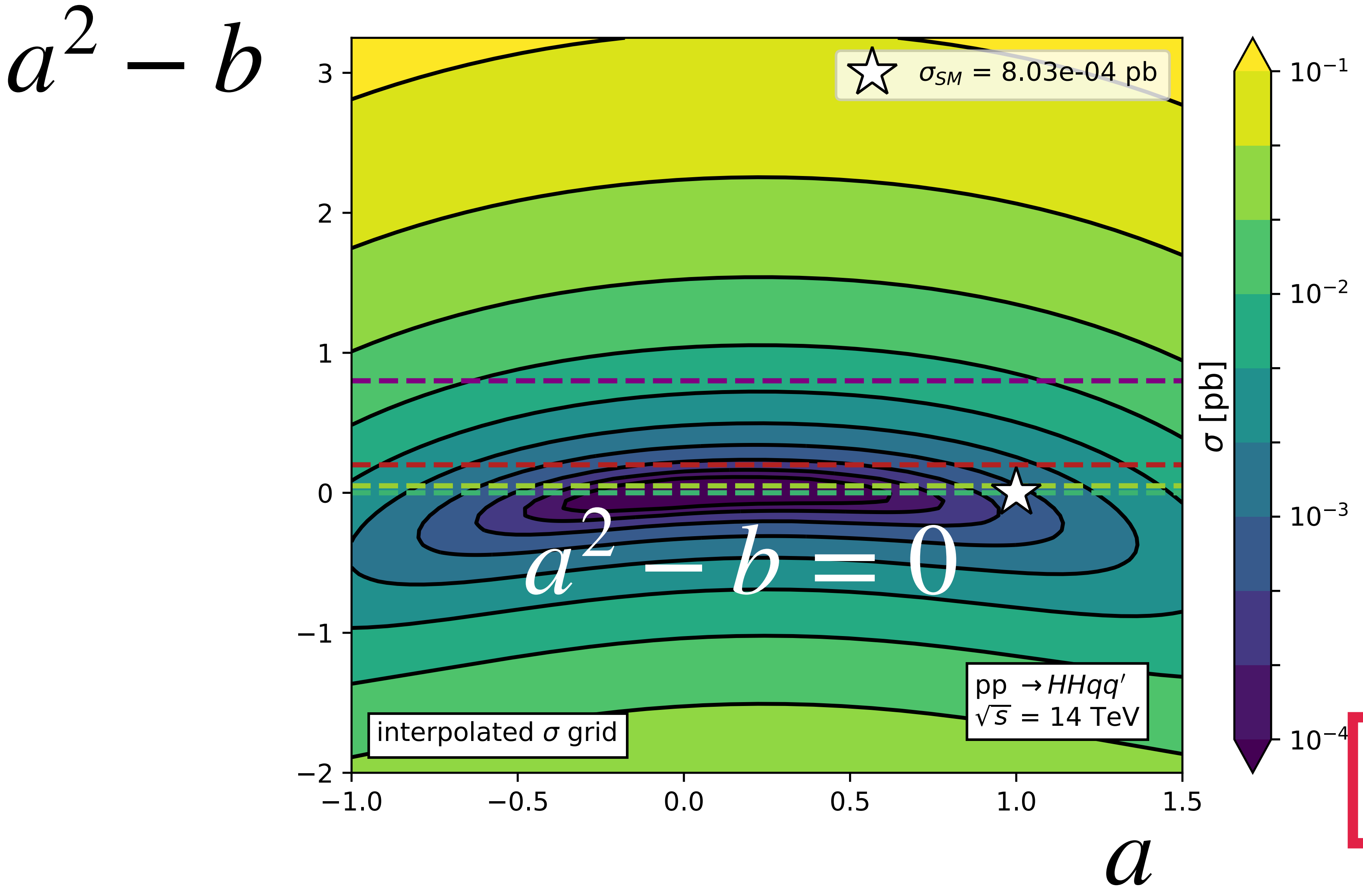
- $\Delta b = \pm 0.5$ **(allowed)** $N_U^{6b} = 54$
- $c = \pm 0.5$ **(allowed)** $N_U^{6b} = 33$

(~ 0 if SM)

Sensitivity to the combination $(a^2 - b)$ in HHjj at HL-LHC $(\sqrt{s} = 14\,\text{TeV})$

(D.Domenech, G. Garcia-Mir, M. Herrero, 2507.20988)





Cross-section Contour lines $\sigma_{\text{BSM}}(pp \to HHjj)$ **at LHC**

Compared to $\sigma_{\text{SM}}(pp \to HHjj) = 8 \times 10^{-4}\text{pb}$

Studied events are:

$\gamma\gamma b\bar{b}jj$

generated by MG5 +PYTHIA+DELPHES

The most sensitive parameter is

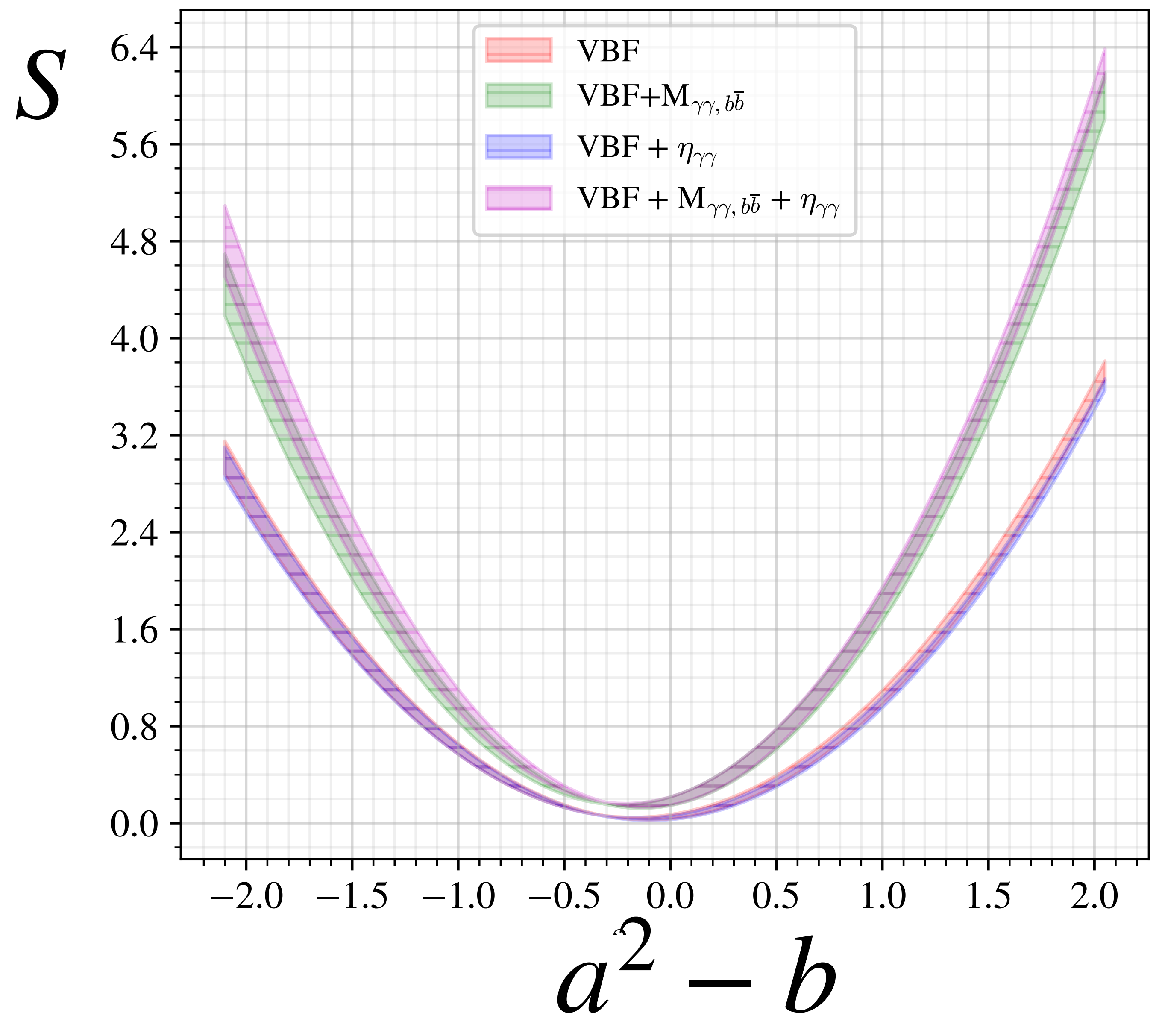
$(a^2 - b)$

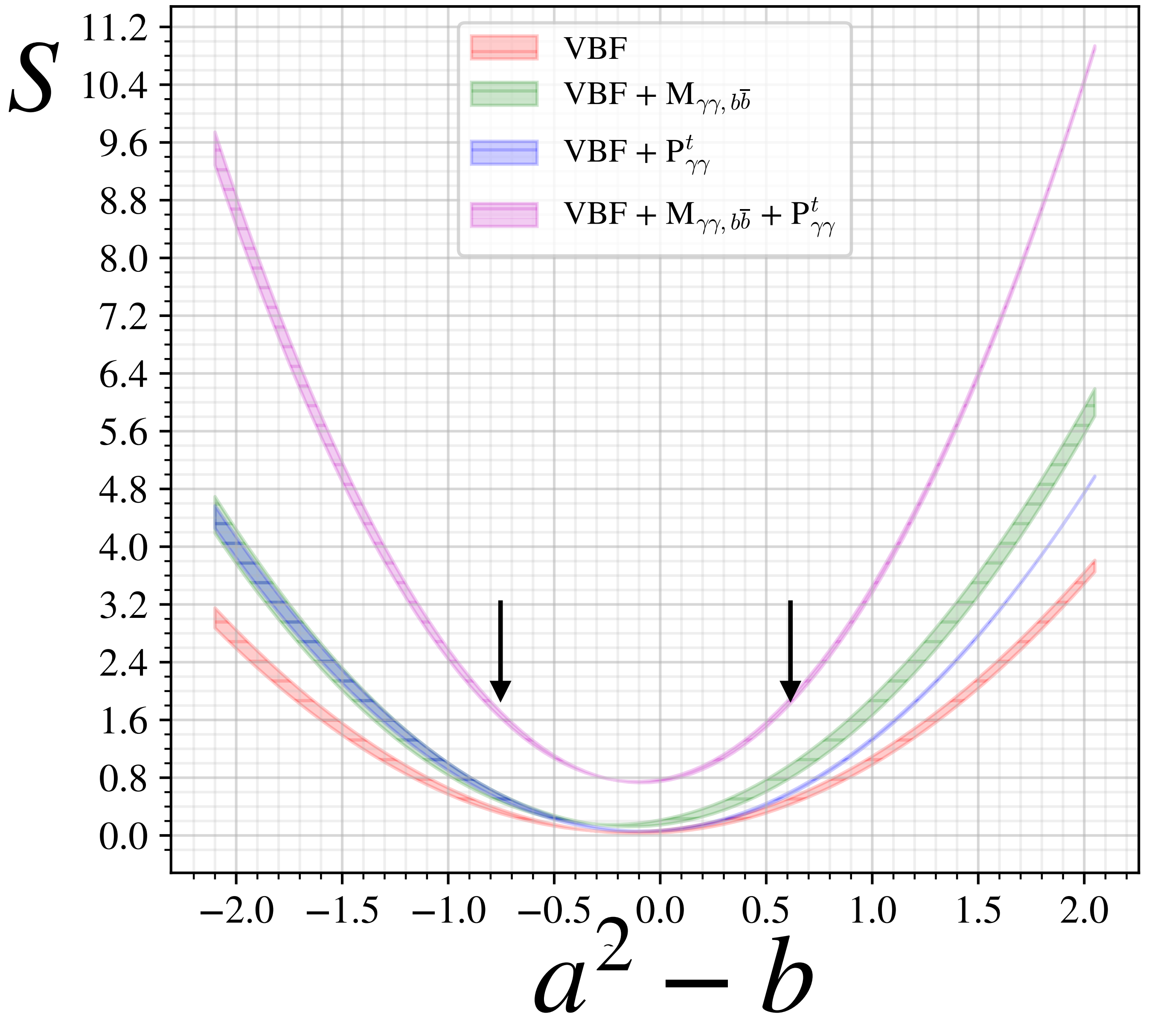
rather than a and/or b separately

(VBF cuts applied to jj)

$120 < M_{\gamma\gamma} < 130$ GeV

$50 < M_{b\bar{b}} < 150$ GeV





$$S = \sqrt{-2\left((N_S + N_B)\log\left(\frac{N_B}{N_S + N_B}\right) + N_S\right)}$$

Sensitivity S can be large with cuts requiring high transversally of $(\gamma\gamma)$

$P^T_{\gamma\gamma} > 200\,\text{GeV}$

$|\eta_{\gamma\gamma}| < 2$

$S \simeq 2$ If $-0.6 < (a^2 - b) < 0.6$

Large effects in $gg \to HH$, HHH from a_i coefficients (II)

Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , PRD110,095016(2024), 2405.05385

LHC $(\sqrt{s} = 13\,\text{TeV})$

$\mathcal{O}_{\Box\Box}$	$a_{\Box\Box} \frac{\Box H \Box H}{v^2}$	$\mathcal{O}_{H\Box\Box}$	$a_{H\Box\Box} \left(\frac{H}{v}\right) \frac{\Box H \Box H}{v^2}$
$\mathcal{O}_{Hdd}$	$a_{Hdd} \frac{m_H^2}{v^2} \left(\frac{H}{v}\right) \partial^\mu H \partial_\mu H$	$\mathcal{O}_{HHdd}$	$a_{HHdd} \frac{m_H^2}{v^2} \left(\frac{H^2}{v^2}\right) \partial^\mu H \partial_\mu H$
$\mathcal{O}_{ddW}$	$a_{ddW} \frac{m_W^2}{v^2} \left(\frac{H}{v}\right) \partial^\mu H \partial_\mu H$	$\mathcal{O}_{HddW}$	$a_{HddW} \frac{m_W^2}{v^2} \left(\frac{H^2}{v^2}\right) \partial^\mu H \partial_\mu H$
$\mathcal{O}_{ddZ}$	$a_{ddZ} \frac{m_Z^2}{v^2} \left(\frac{H}{v}\right) \partial^\mu H \partial_\mu H$	$\mathcal{O}_{HddZ}$	$a_{HddZ} \frac{m_Z^2}{v^2} \left(\frac{H^2}{v^2}\right) \partial^\mu H \partial_\mu H$
$\mathcal{O}_{dd\Box}$	$a_{dd\Box} \frac{1}{v^3} \partial^\mu H \,\partial_\mu H \,\Box H$	$\mathcal{O}_{Hdd\Box}$	$a_{Hdd\Box} \frac{1}{v^3} \left(\frac{H}{v}\right) \partial^\mu H \,\partial_\mu H \,\Box H$
$\mathcal{O}_{HH\Box\Box}$	$a_{HH\Box\Box} \left(\frac{H^2}{v^2}\right) \frac{\Box H \Box H}{v^2}$	$\mathcal{O}_{dddd}$	$a_{dddd} \frac{1}{v^4} \partial^\mu H \,\partial_\mu H \,\partial^\nu H \,\partial_\nu H$

Reference SM (from gluon gluon fusion)

$\sigma_{\rm SM}(pp \to HH) = 1.8 \times 10^{-2}\,\text{pb}$ $\quad \sigma_{\rm SM}(pp \to HHH) = 4 \times 10^{-5}\,\text{pb}$

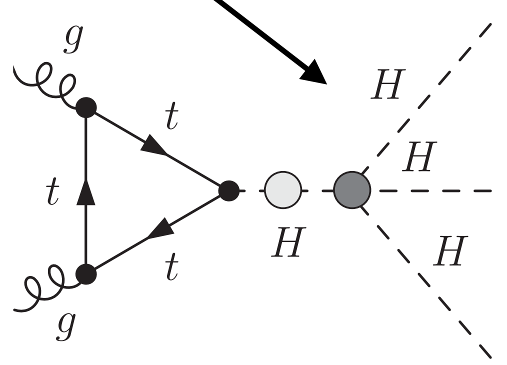
The largest BSM effects in HHH are from operators with higher number of derivatives, like $a_{dddd} \equiv \gamma, \ldots$ etc

Γ_{HHHH}

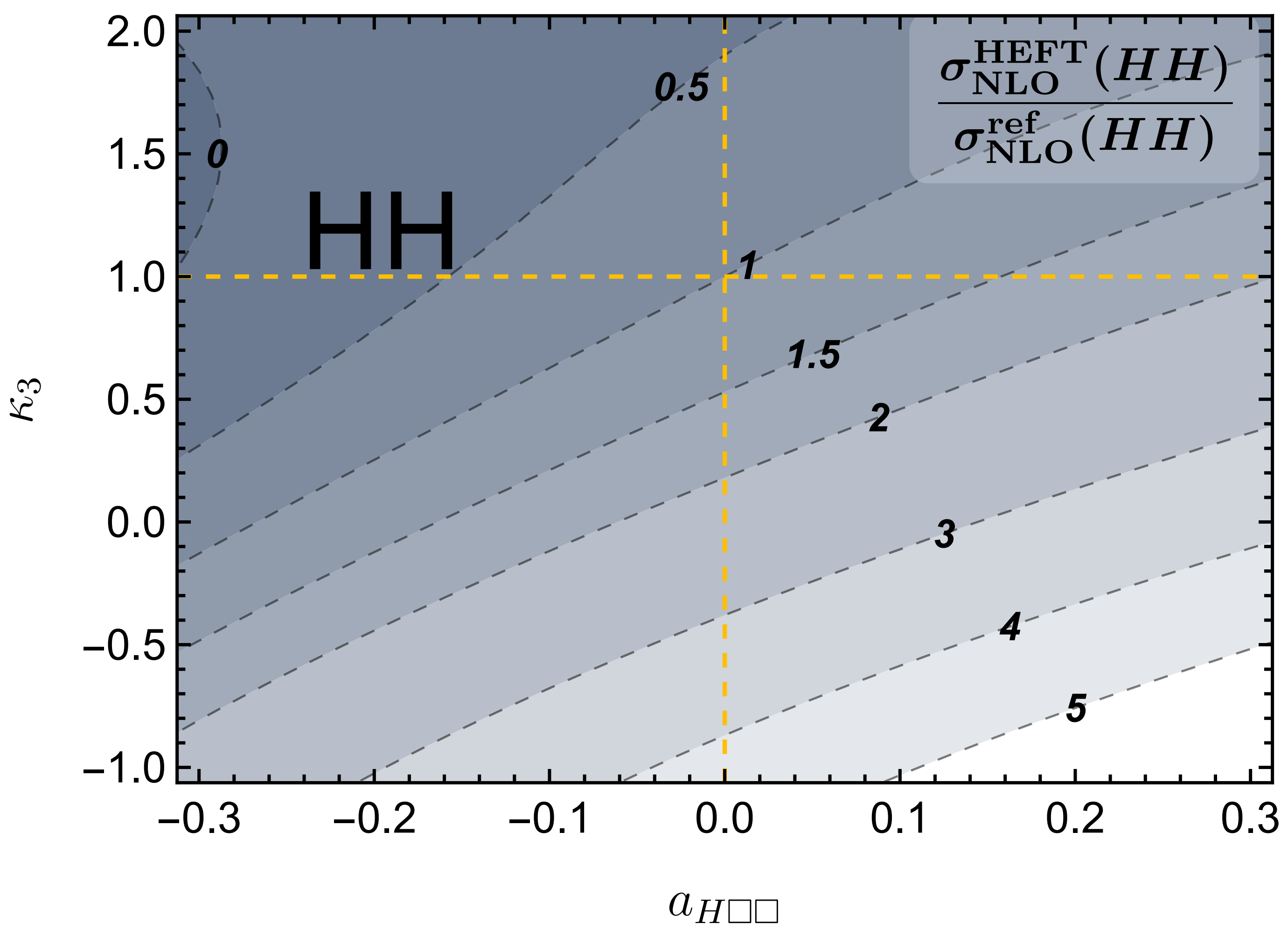
For instance, for $a_{dddd} = -0.040$ and $\kappa_3 = 1$

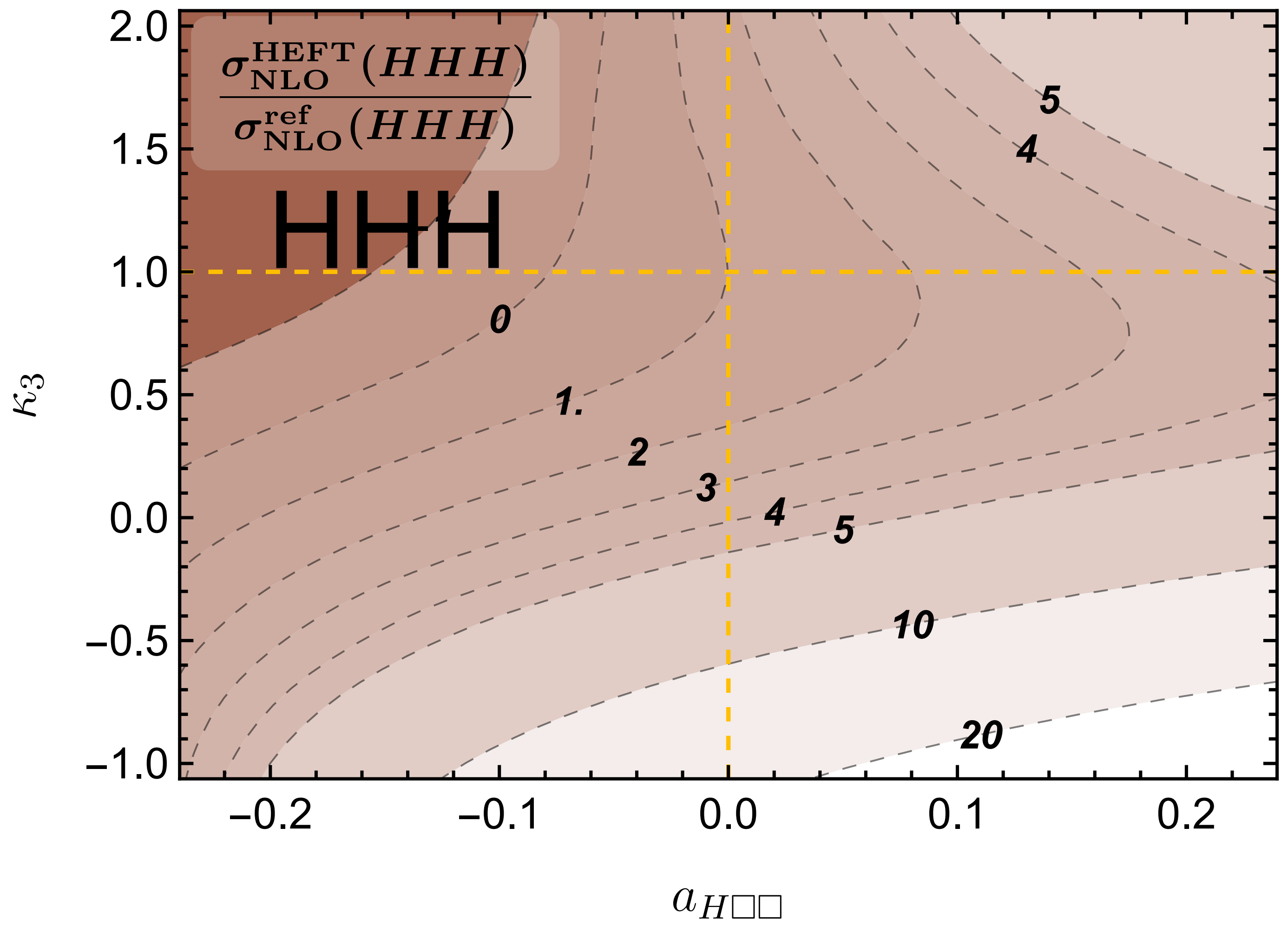
$\sigma^{\rm HEFT}(HHH) \sim 2\,\sigma^{\rm SM}(HHH)$ (100%) !!!

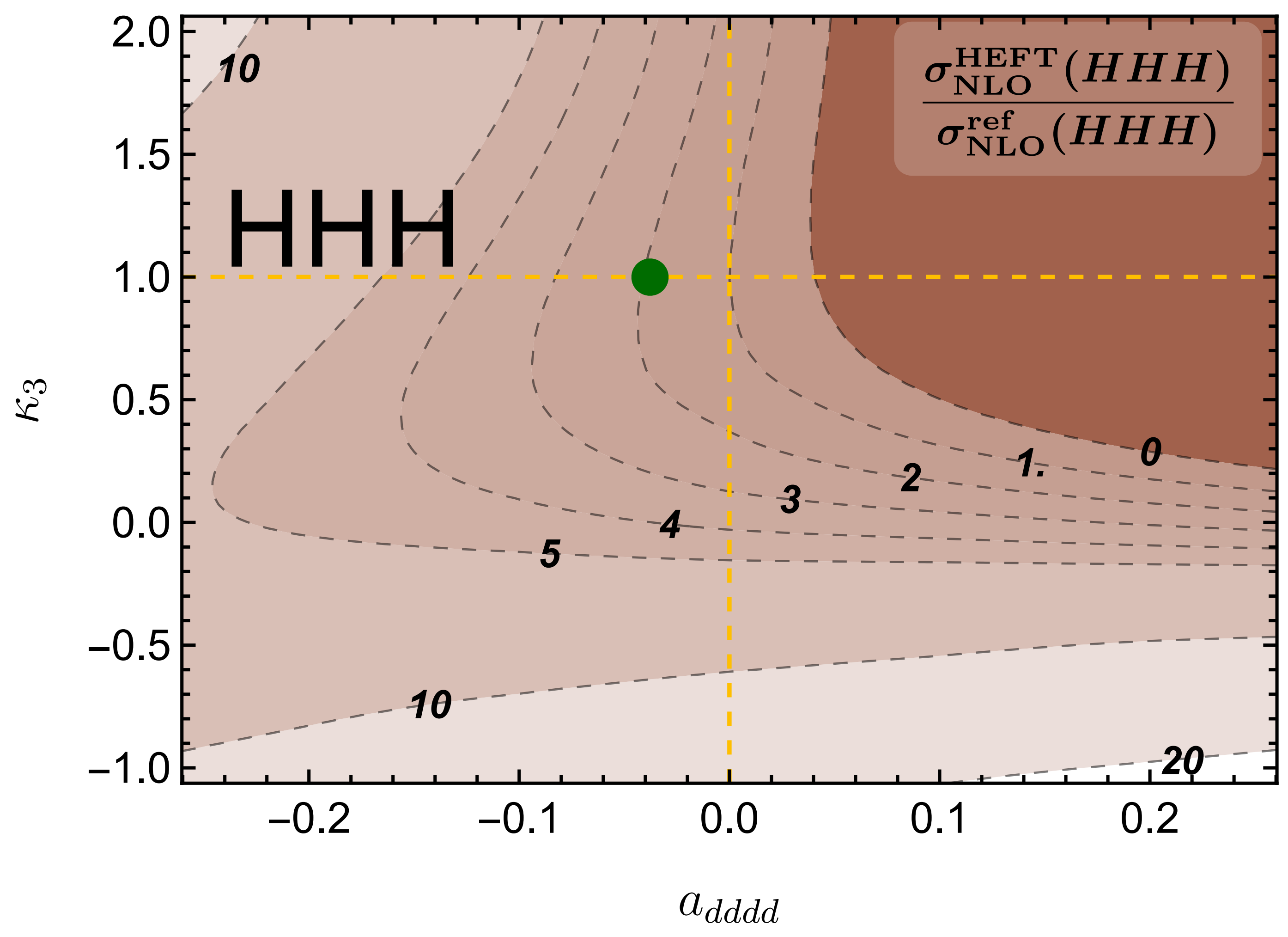
(green point ● in plot bellow)



Plots for other $a_i's$ in 2405.05385







Summary /Conclusions

Our main goal is looking for BSM signals at colliders

The EChL (HEFT) provides the proper tool to compute BSM rates both at tree and one-loop level.

Specially devoted to describe low energy behavior of non-decoupling (strongly interacting) UV scenarios. (i.e. below resonances)

Gauge invariant, Renormalizable in R_ξ, Improved Perturbative Unitarity

HL-LHC (14 TeV) and CLIC (3TeV): best access to HEFT coeffs.

We have studied the most relevant ones for Multiple Higgs production with focus in HH and HHH

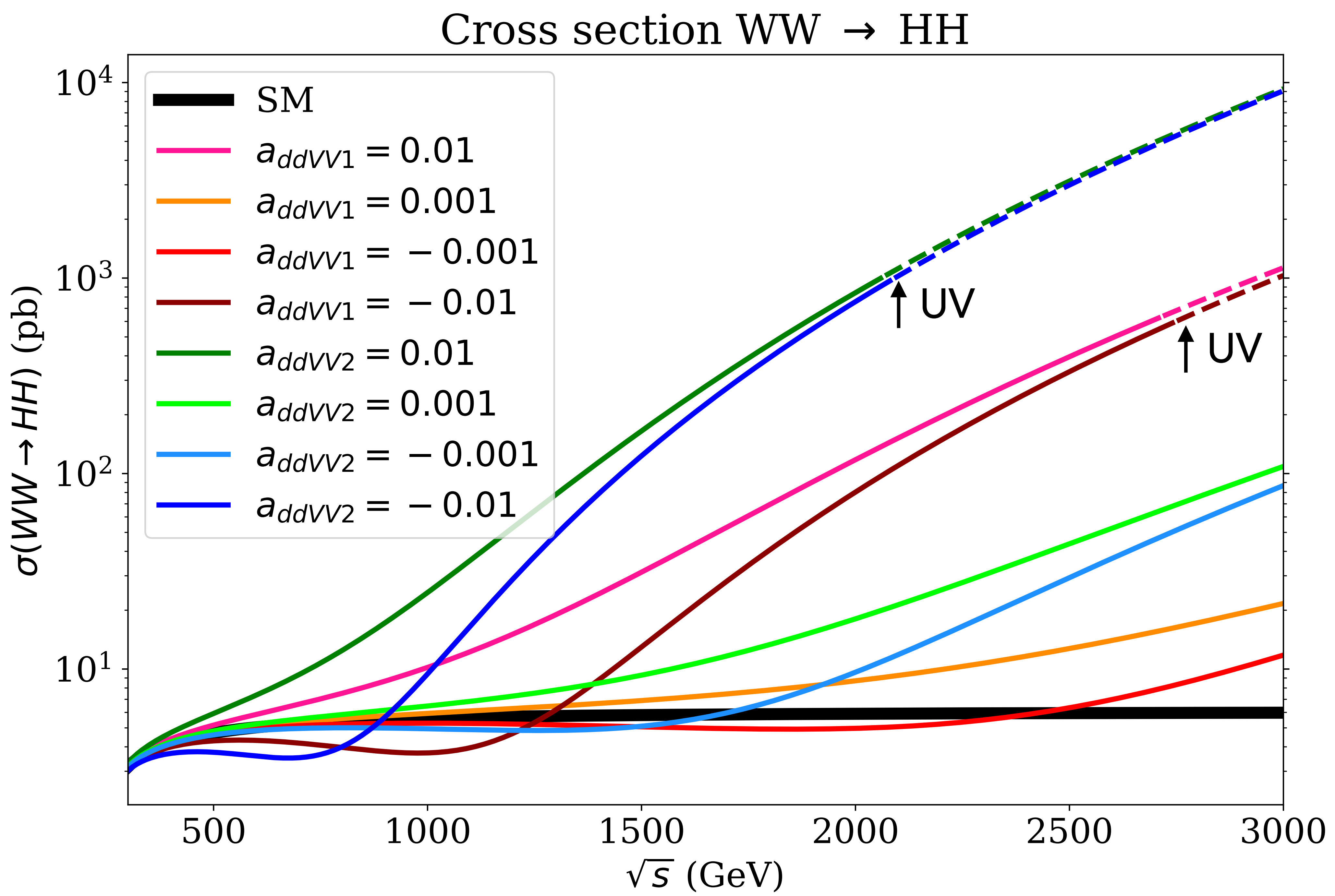
Studying specific combinations, like $(a^2 - b)$ in HH, $c + \frac{4}{3}a(a^2 - b)$ in HHH will help to find BSM signals (i.e. enhancements with respect to SM)

Back up slides

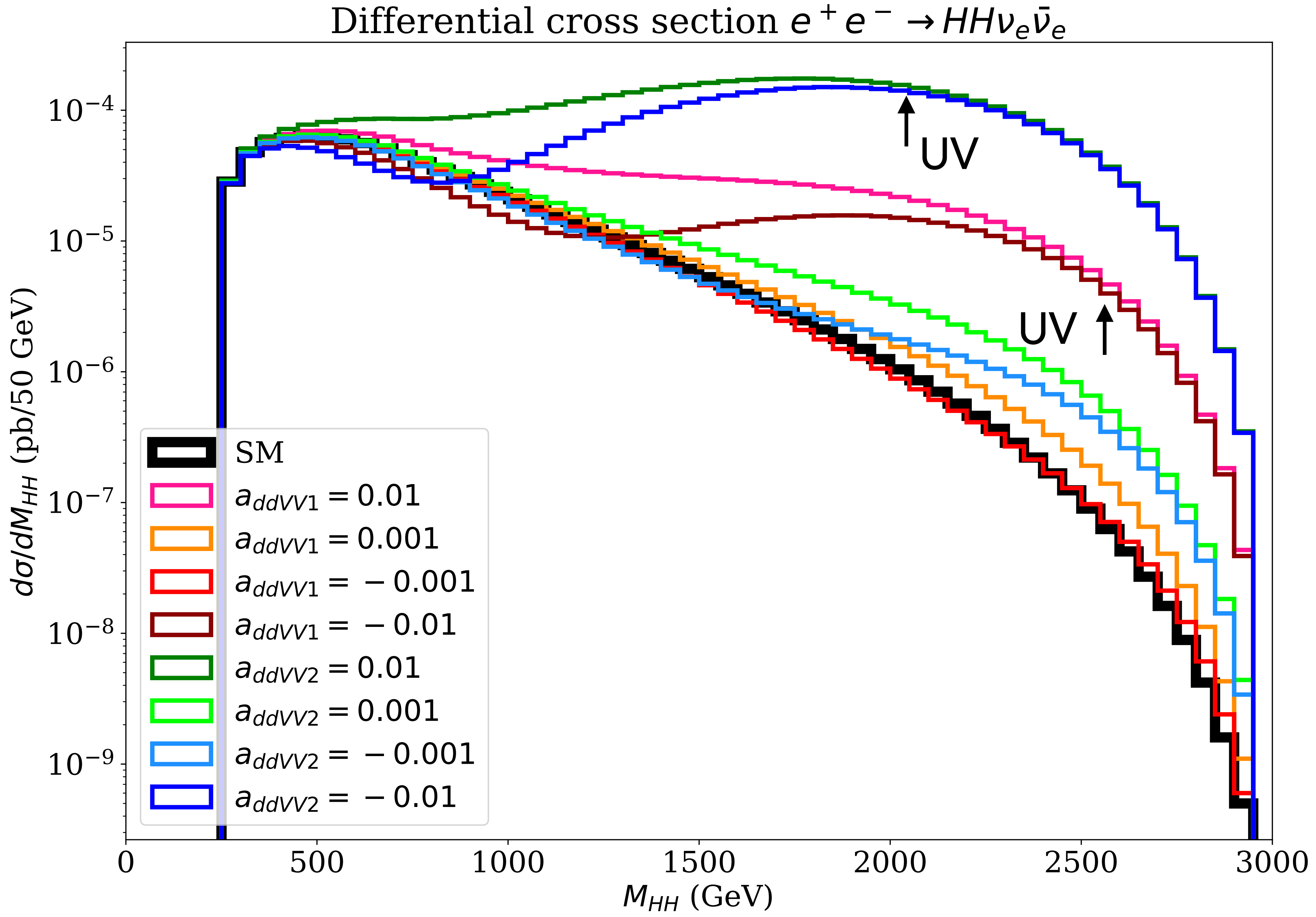
HH with NLO-HEFT: $\eta = a_{ddVV1}$; $\delta = a_{ddVV2}$ (Unitarity)

2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos

WW subprocess



Example: VBF from e^+e^- (CLIC 3 TeV)



↑UV= to the right of this point prediction enters in the Unitarity Violating region

enhancement in $WW \to HH$ at large $\sqrt{s}$ $\Rightarrow$ enhancement in $e^+e^- \to HH\bar{\nu}_e\nu_e$ at large invariant mass M_{HH}

Coefficient values ($\sim 10^{-2}$) for η and δ run into (perturbative) unitarity violation $\Rightarrow$ consequences for colliders $\Rightarrow$ Either restrict values or cut maximum m_{HH}

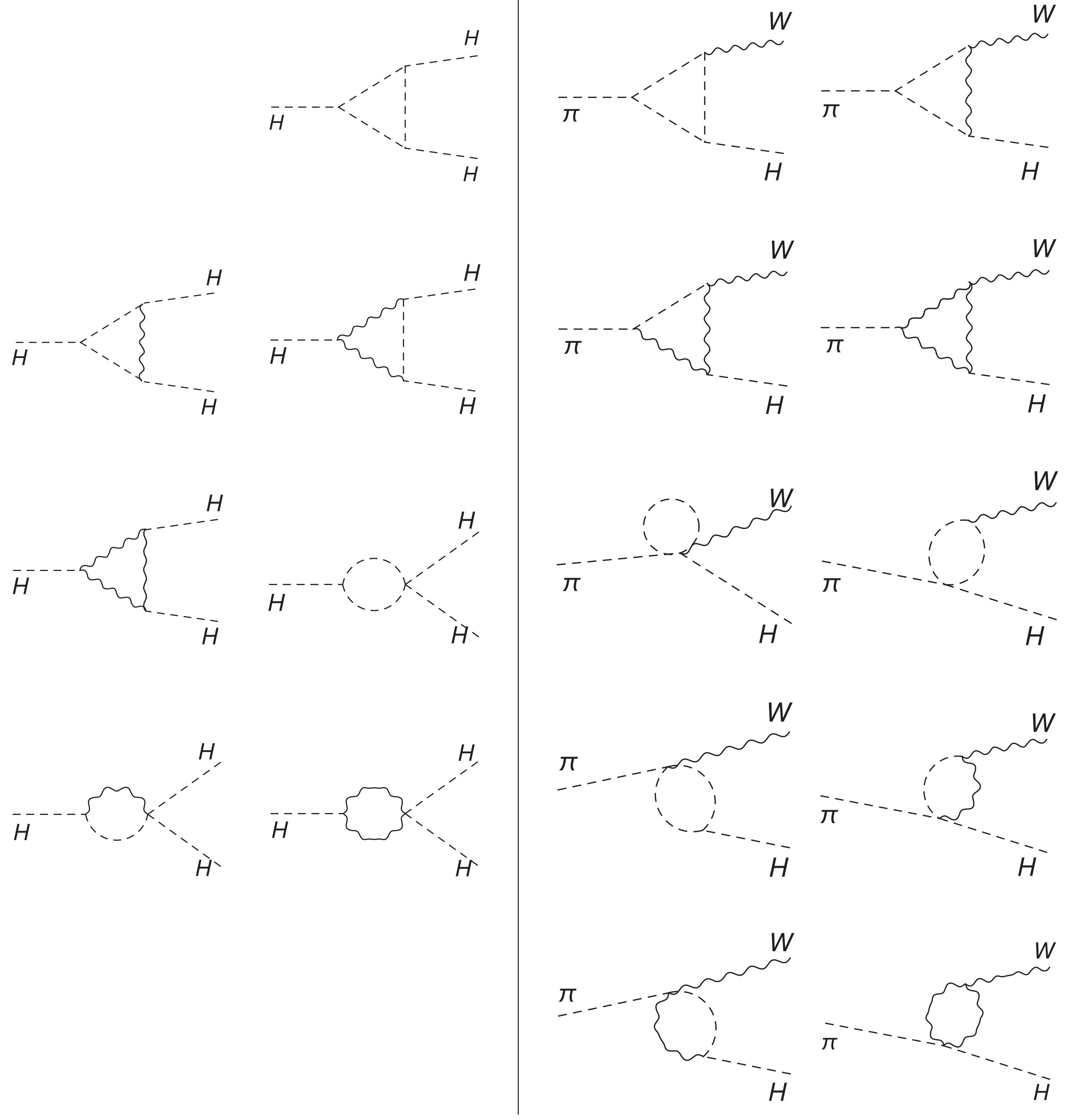
Some authors study emergent resonances in $WW \to HH$ that appear when imposing unitarity with the Inverse Amplitude Method (IAM)
See, for instance, review by Dobado and Espriu. Prog. Part. Nucl. Phys. 115, 103813 (2020).

Loop diagramas involved in WW -> HH

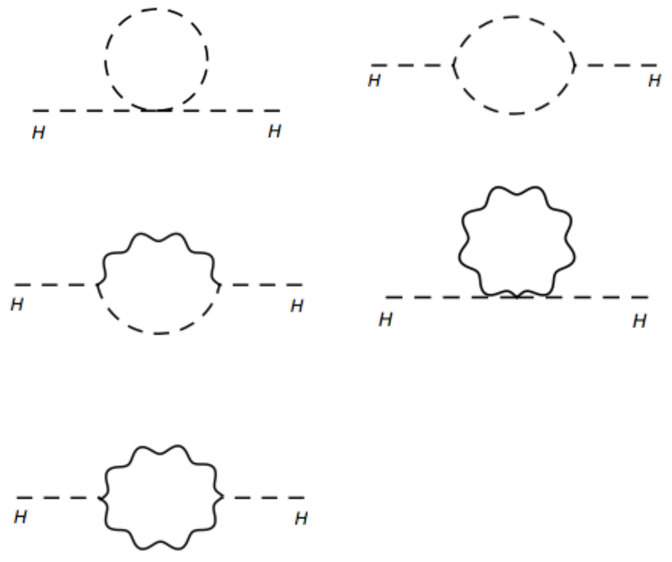
All bosonic loops in R_ξ: scalar (GB,H), gauge, and mixed scalar-gauge

3-point functions

Γ_{HHH} $\Gamma_{\pi WH}$



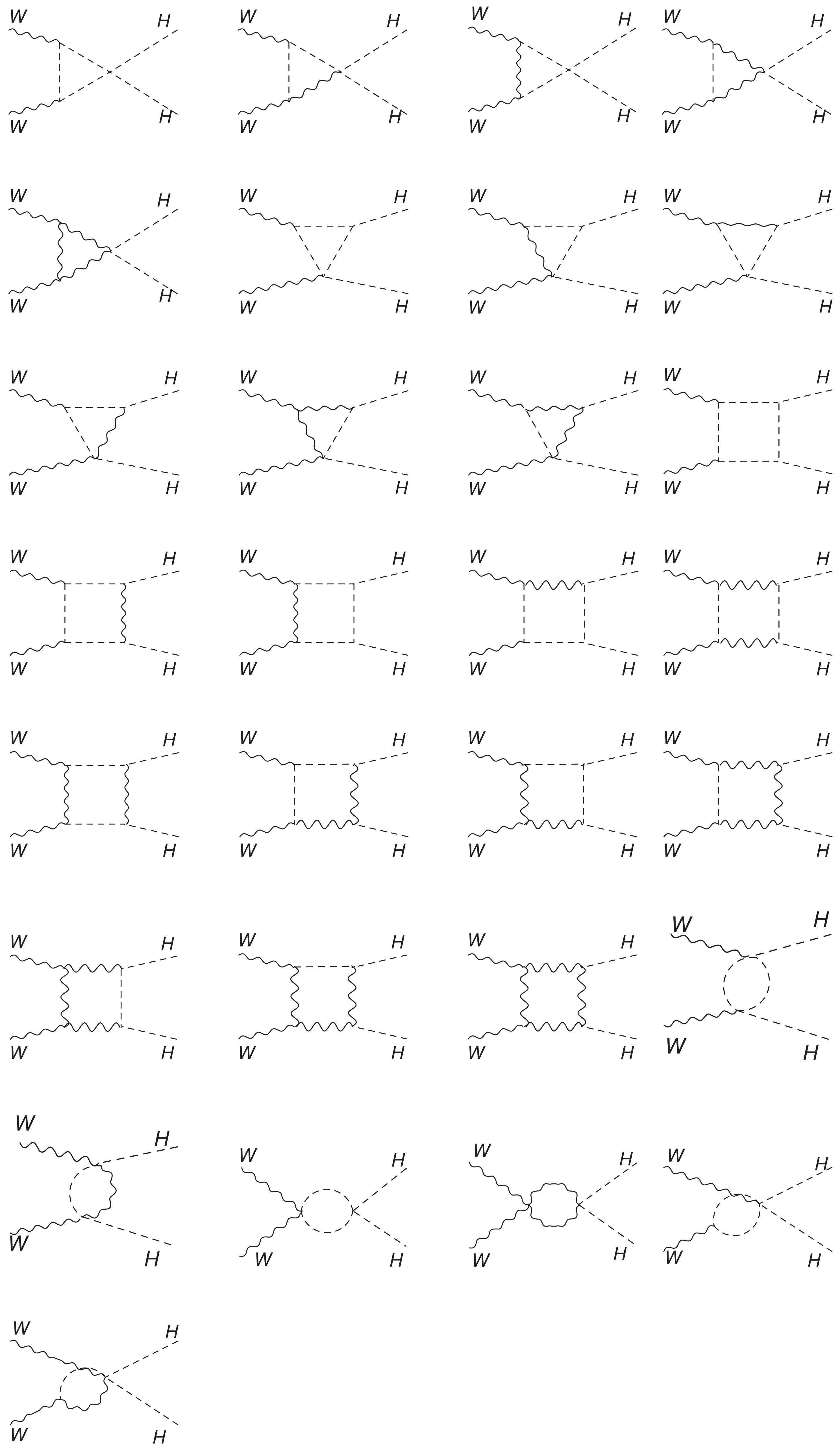
+ 2-point functions



+ ....

4-point functions

Γ_{WWHH}



NLO-HEFT relevant operators for HH

2208.05900, Phys. Rev. D 106(2022), 073008, Herrero, Morales (WW to HH)

$$\mathscr{L}_{\rm HEFT}^{\rm NLO} = \mathscr{L}^{\rm LO} - a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H\,\partial^\nu H}{v^2}{\rm Tr}\Big[\mathcal{V}_\mu\mathcal{V}_\nu\Big] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H\,\partial_\mu H}{v^2}{\rm Tr}\Big[\mathcal{V}^\nu\mathcal{V}_\nu\Big]$$
$$-\frac{m_{\rm H}^2}{4}\left(2a_{H\mathcal{V}\mathcal{V}}\frac{H}{v}+a_{HH\mathcal{V}\mathcal{V}}\frac{H^2}{v^2}\right){\rm Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big]$$
$$-\left(a_{HWW}\frac{H}{v}+a_{HHWW}\frac{H^2}{v^2}\right){\rm Tr}\Big[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Big]+i\left(a_{d2}+a_{Hd2}\frac{H}{v}\right)\frac{\partial^\nu H}{v}{\rm Tr}\Big[\hat{W}_{\mu\nu}\mathcal{V}^\mu\Big]$$
$$+\left(a_{\Box\mathcal{V}\mathcal{V}}+a_{H\Box\mathcal{V}\mathcal{V}}\frac{H}{v}\right)\frac{\Box H}{v}{\rm Tr}\Big[\mathcal{V}_\mu\mathcal{V}^\mu\Big]+a_{d3}\frac{\partial^\nu H}{v}{\rm Tr}\Big[\mathcal{V}_\nu\mathcal{D}_\mu\mathcal{V}^\mu\Big]$$
$$+\left(a_{\Box\Box}+a_{H\Box\Box}\frac{H}{v}\right)\frac{\Box H\,\Box H}{v^2}+a_{dd\Box}\frac{\partial^\mu H\,\partial_\mu H\,\Box H}{v^3}+a_{Hdd}\frac{m_{\rm H}^2}{v^2}\frac{H}{v}\partial^\mu H\,\partial_\mu H$$

$$\mathcal{V}_\mu = (D_\mu U)U^\dagger$$

e.o.m

$$\Box H = -m_h^2 H - \tfrac{3}{2}\kappa_3 m_h^2\tfrac{H^2}{v} - \tfrac{a}{2}v{\rm Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big] - \tfrac{b}{2}H{\rm Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big]$$
$${\rm Tr}\Big[\tau^j\mathcal{D}_\mu\mathcal{V}^\mu\Big] = -{\rm Tr}\Big[\tau^j\mathcal{V}^\mu\Big]\tfrac{2a}{v}\partial_\mu H$$

Full operators list given in the literature (see, for instance, Brivio et al 1311.1823)

summarized by: $a_{dd\mathcal{V}\mathcal{V}1}\leftrightarrow c_8$, $a_{dd\mathcal{V}\mathcal{V}2}\leftrightarrow c_{20}$, $a_{11}\leftrightarrow c_9$, $a_{HWW}\leftrightarrow a_W$, $a_{HHWW}\leftrightarrow b_W$, $a_{d2}\leftrightarrow c_5$, $a_{Hd2}\leftrightarrow a_5$, $a_{\Box\mathcal{V}\mathcal{V}}\leftrightarrow c_7$, $a_{H\Box\mathcal{V}\mathcal{V}}\leftrightarrow a_7$, $a_{d3}\leftrightarrow c_{10}$, $a_{Hd3}\leftrightarrow a_{10}$, $a_{\Box\Box}\leftrightarrow c_{\Box H}$, $a_{H\Box\Box}\leftrightarrow a_{\Box H}$, $a_{dd\Box}\leftrightarrow c_{\Delta H}$, $a_{H\mathcal{V}\mathcal{V}}\leftrightarrow a_C$ and $a_{HH\mathcal{V}\mathcal{V}}\leftrightarrow b_C$.

$$\mathscr{L}_{\rm HEFT}^{\rm NLO+e.o.m} = \mathscr{L}^{\rm LO} - a_{dd\mathcal{V}\mathcal{V}1}\frac{\partial^\mu H\,\partial^\nu H}{v^2}{\rm Tr}\Big[\mathcal{V}_\mu\mathcal{V}_\nu\Big] - a_{dd\mathcal{V}\mathcal{V}2}\frac{\partial^\mu H\,\partial_\mu H}{v^2}{\rm Tr}\Big[\mathcal{V}^\nu\mathcal{V}_\nu\Big]$$
$$-\frac{m_{\rm H}^2}{4}\left(2a_{H\mathcal{V}\mathcal{V}}\frac{H}{v}+a_{HH\mathcal{V}\mathcal{V}}\frac{H^2}{v^2}\right){\rm Tr}\Big[\mathcal{V}^\mu\mathcal{V}_\mu\Big]+a_{Hdd}\frac{m_{\rm H}^2}{v^2}\frac{H}{v}\partial^\mu H\,\partial_\mu H$$
$$-\left(a_{HWW}\frac{H}{v}+a_{HHWW}\frac{H^2}{v^2}\right){\rm Tr}\Big[\hat{W}_{\mu\nu}\hat{W}^{\mu\nu}\Big]+i\left(a_{d2}+a_{Hd2}\frac{H}{v}\right)\frac{\partial^\nu H}{v}{\rm Tr}\Big[\hat{W}_{\mu\nu}\mathcal{V}^\mu\Big]$$

The most relevant are
$a_{ddVV1}\equiv\eta$, $a_{ddVV2}\equiv\delta$

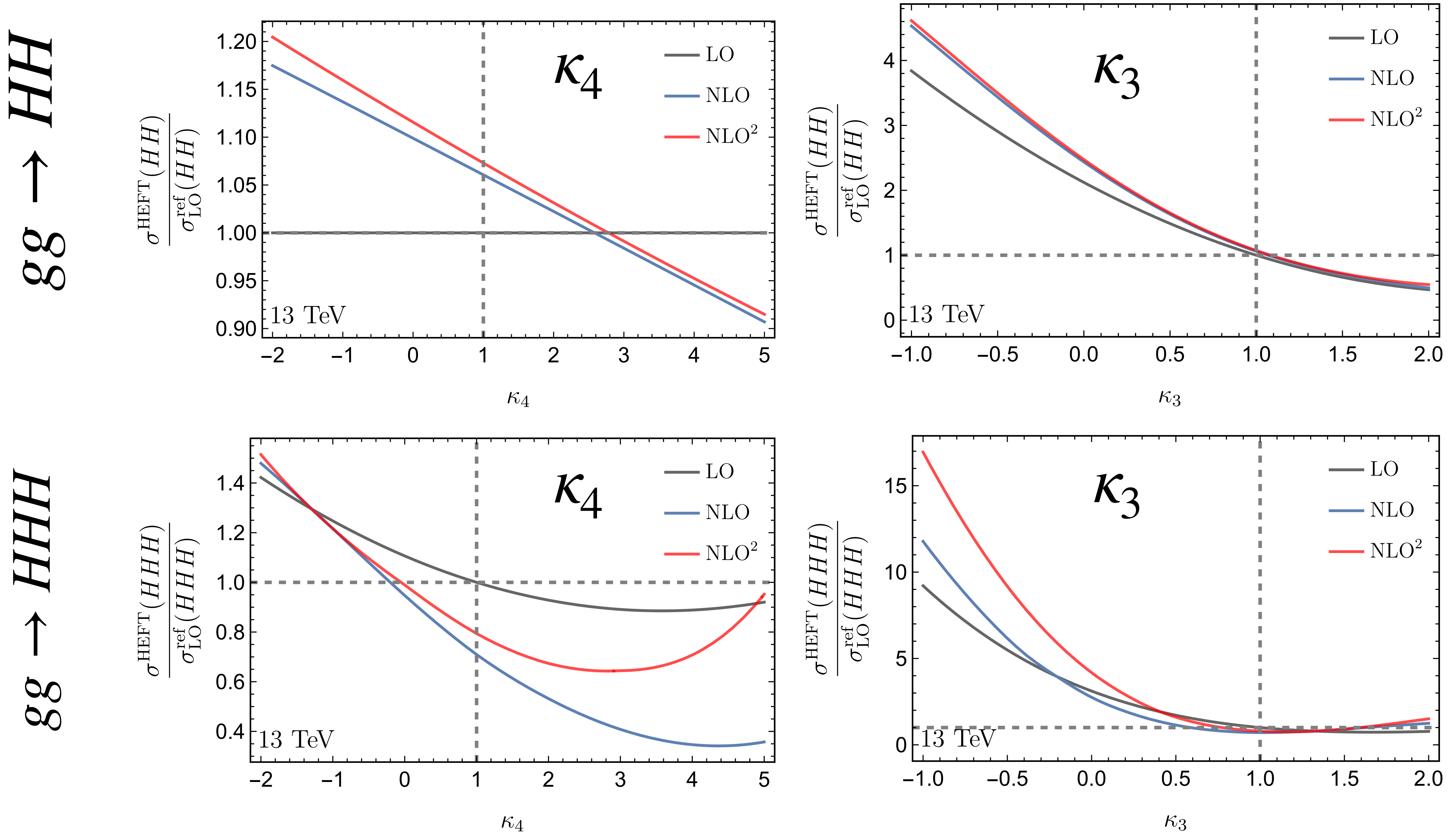
Reduction from 15 to 9 $a_i's$ NLO coefficients entering into $WW\to HH$

These operators contain 4 derivatives !!

Size of the EW loops in $gg \to HH$ and in $gg \to HHH$

Corrections at LHC (13 TeV) cross sections

Anisha, D.Domenech, C. Englert, M.J. Herrero, R.A.Morales , 2405.05385



$$\sigma^{\rm SM}_{\rm LO}(HH) = \sigma^{\rm ref}_{\rm LO}(HH) = 17.40\,{\rm fb}\,;\, \sigma^{\rm SM}_{\rm LO}(HHH) = \sigma^{\rm ref}_{\rm LO}(HHH) = 0.041\,{\rm fb}$$

All simulations done with BVFNLO

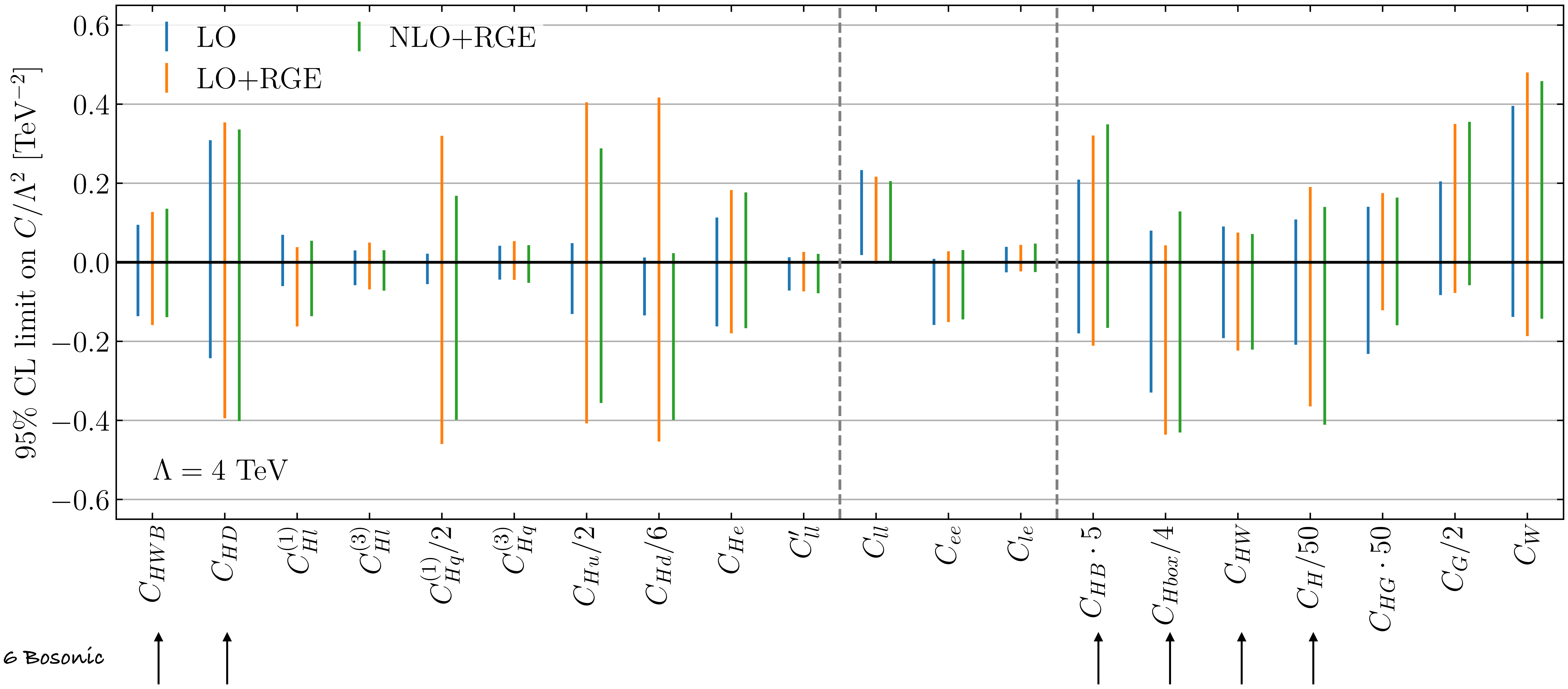
Most important message:
(EW) loop corrections within NLO-HEFT change
the sensitivity to κ_3 and κ_4 in HH and HHH production at LHC

The most relevant change is in κ_3
For κ_3<0, we find relevant
enhancements in the NLO/LO prediction
$\sigma(HH)$ of $\sim 10\,\%$
and in
$\sigma(HHH)$ of $\sim 30\,\%$ ($\sim 80\,\%$ if NLO^2)

Also large changes in κ_4
For κ_4>0, we find relevant
reductions in the NLO/LO prediction
$\sigma(HHH)$ of $\sim 50\,\%$

Example of Global Fits in the SMEFT (dim6)

R.Bartocci , A. Biekotter , T. Hurth, 2412.09674, JHEP 05 (2025) 203



Example of Global Fits in the SMEFT (dim6) (cont)

Using our relations found from HEFT/SMEFT matching……

The previous constraints on $\frac{c_i^{\rm SMEFT}}{\Lambda^2}({\rm TeV}^{-2})$ (for $\Lambda = 4\,{\rm TeV}$) from Global Fits, imply constraints on the HEFT coeffs….

For instance, for LO-HEFT coeffs. we get:

$$|\Delta a|_{\rm SMEFT} < 0.1 \qquad |\Delta b|_{\rm SMEFT} < 0.4 \qquad |c|_{\rm SMEFT} < 0.3$$

$|\Delta\kappa_3|_{\rm SMEFT} < 9.5$ $\qquad |\Delta\kappa_4|_{\rm SMEFT}$ practically unconstrained

And , for NLO-HEFT coeffs. we get:

$$|a_{H0}|_{\rm SMEFT} < 0.1 \qquad |a_{HH0}|_{\rm SMEFT} < 0.24 \qquad |a_{HWW}|_{\rm SMEFT} < 0.058 \qquad |a_{HHWW}|_{\rm SMEFT} < 0.029$$

$$|a_{HBB}|_{\rm SMEFT} < 0.058 \qquad |a_{HHBB}|_{\rm SMEFT} < 0.029 \qquad |a_{H1}|_{\rm SMEFT} < 0.053 \qquad |a_{HH1}|_{\rm SMEFT} < 0.026$$

$$|a_{HHH0}|_{\rm SMEFT} < 0.2$$

Comparing the size of coeffs HEFT/ SMEFT at $WW \to HH$

In particular $\mathcal{O}(\partial^4)$: $a_{ddVV1} \equiv \eta$, $a_{ddVV2} \equiv \delta$ HEFTchdim4 versus $c_\phi^{(1,2,3)}$ SMEFTdim8

2208.05452, Phys. Rev. D 106 (2022) 115027, Domenech, Herrero, Morales, Ramos

HEFT

chdim4

$$\mathcal{L}_4 = a_{ddVV1} \frac{\partial^\mu H \, \partial^\nu H}{v^2} \mathrm{Tr}\left[D_\mu U^\dagger D_\nu U\right] + a_{ddVV2} \frac{\partial^\mu H \, \partial_\mu H}{v^2} \mathrm{Tr}\left[D^\nu U^\dagger D_\nu U\right] + \ldots$$

SMEFT

dim8

$$\mathscr{L}_8 = \frac{c_{\phi^4}^{(1)}}{\Lambda^4}(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi) + \frac{c_{\phi^4}^{(2)}}{\Lambda^4}(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi) + \frac{c_{\phi^4}^{(3)}}{\Lambda^4}(D_\mu\phi^\dagger D_\mu\phi)(D^\nu\phi^\dagger D^\nu\phi) + \ldots$$ (in Warsaw Basis)

$$a_{ddVV1} = \frac{v^4}{4\Lambda^4}\left[c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)}\right]$$

$$a_{ddVV2} = \frac{v^4}{4\Lambda^4} c_{\phi^4}^{(3)}$$

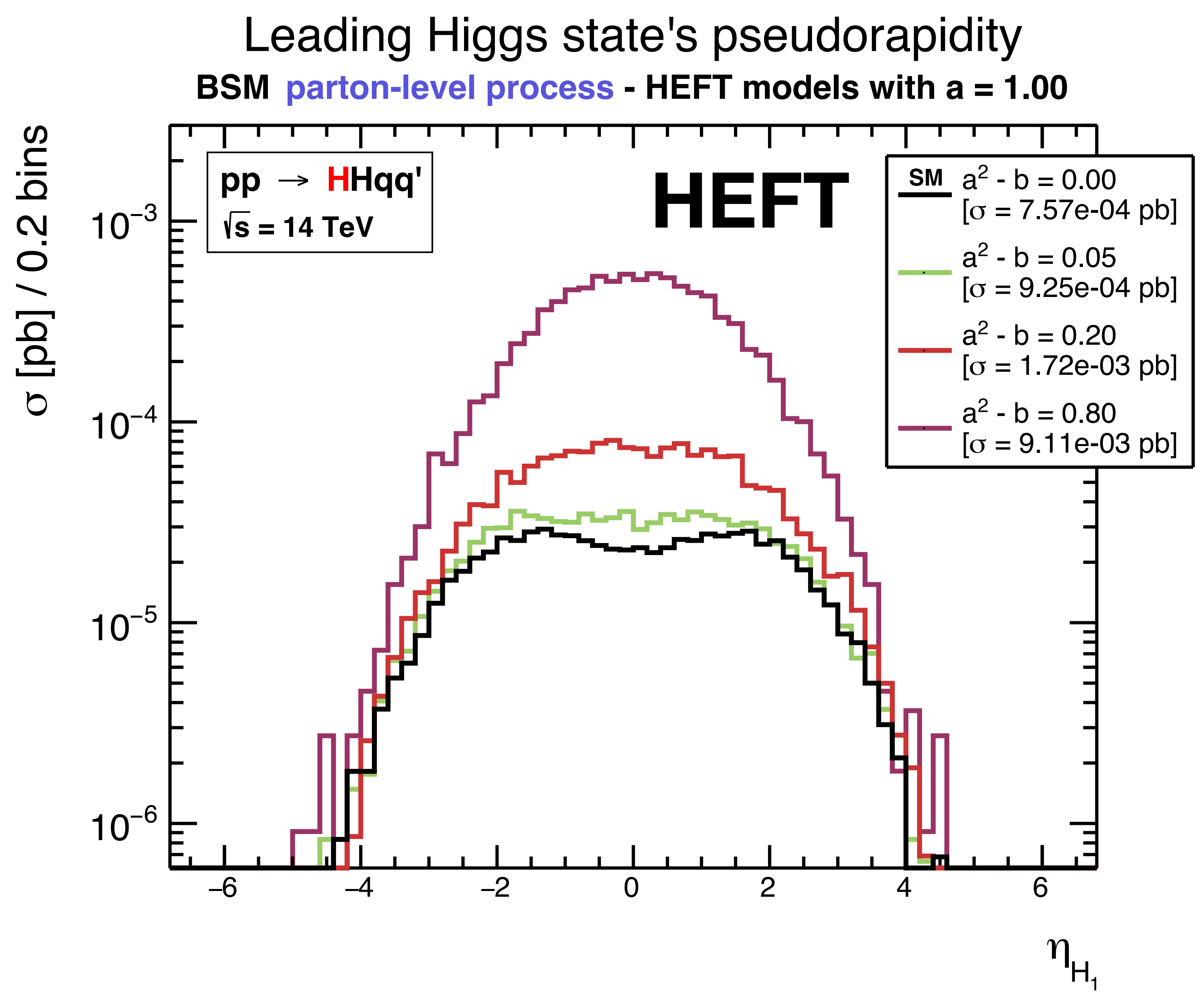
Matching : HEFT (SMEFT)	$\Lambda = 1$ TeV	$\Lambda = 2$ TeV	$\Lambda = 3$ TeV
	± 0.01 (± 11)	± 0.01 (± 175)	± 0.01 (± 885)
a_{ddVV1} $\left(c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)}\right)$	± 0.001 (± 1.1)	± 0.001 (± 17.5)	± 0.001 (± 88.5)
a_{ddVV2} $\left(c_{\phi^4}^{(3)}\right)$	± 0.0001 (± 0.11)	± 0.0001 (± 1.75)	± 0.0001 (± 8.85)

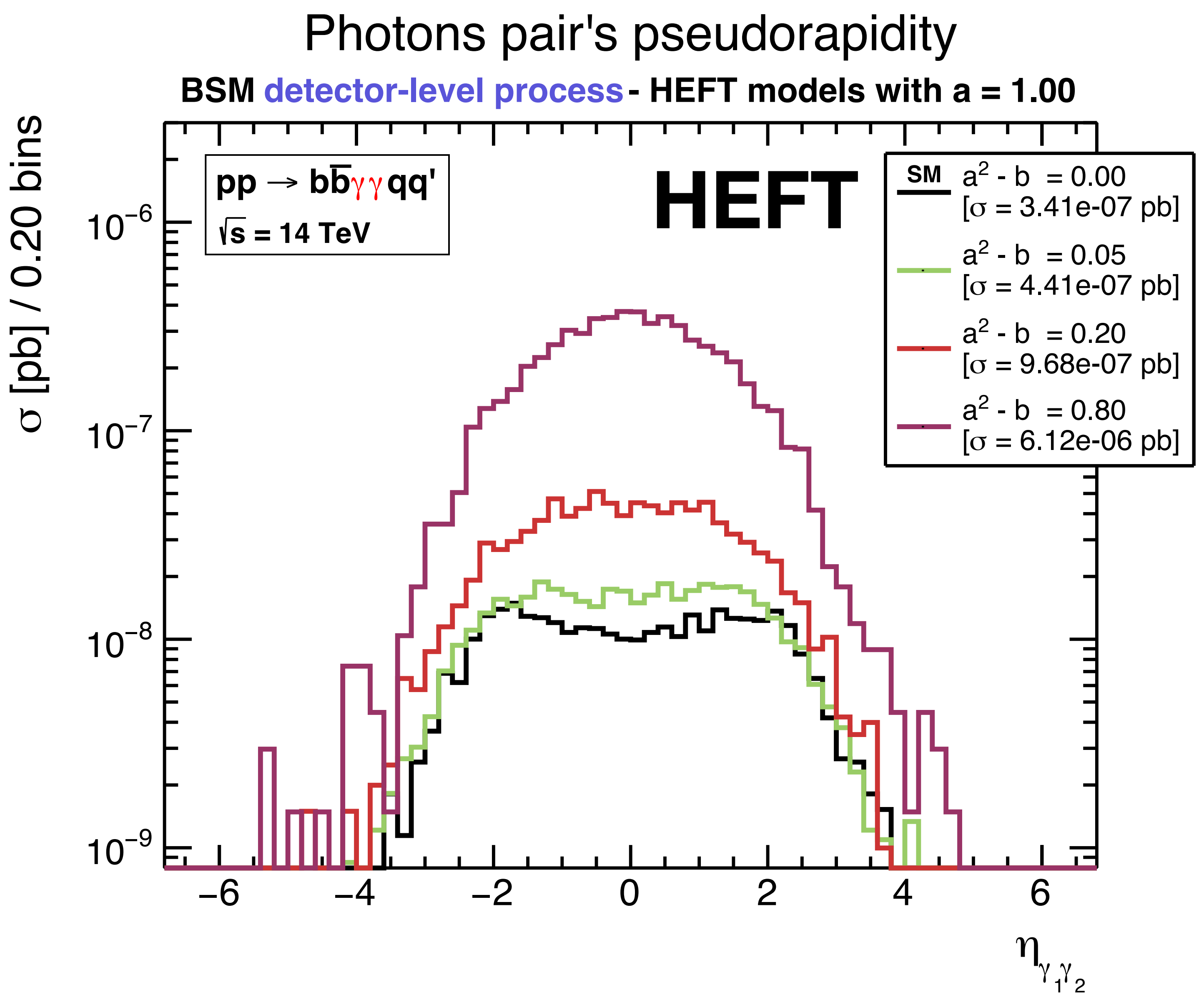
In SMEFT (dim 8) the predictions for $\sigma(WW \to HH)$ also grow with energy but produce smaller enhacements respect to SM (compared to HEFT) due to $1/\Lambda^4$ suppression

Several notations for the SMEFT coeffs in the Warsaw Basis (1008.4884, Gradkowski et al)

$$c_{\phi^4}^{(1)} = c_{\phi^4 D^4}^{(1)} = f_{S,2} \, , c_{\phi^4}^{(2)} = c_{\phi^4 D^4}^{(2)} = f_{S,0} \, , c_{\phi^4}^{(3)} = c_{\phi^4 D^4}^{(3)} = f_{S,1} \; \ldots\ldots\ldots\ldots$$

On the transversality of the final H's (and their decays) for $(a^2 - b) \neq 0$

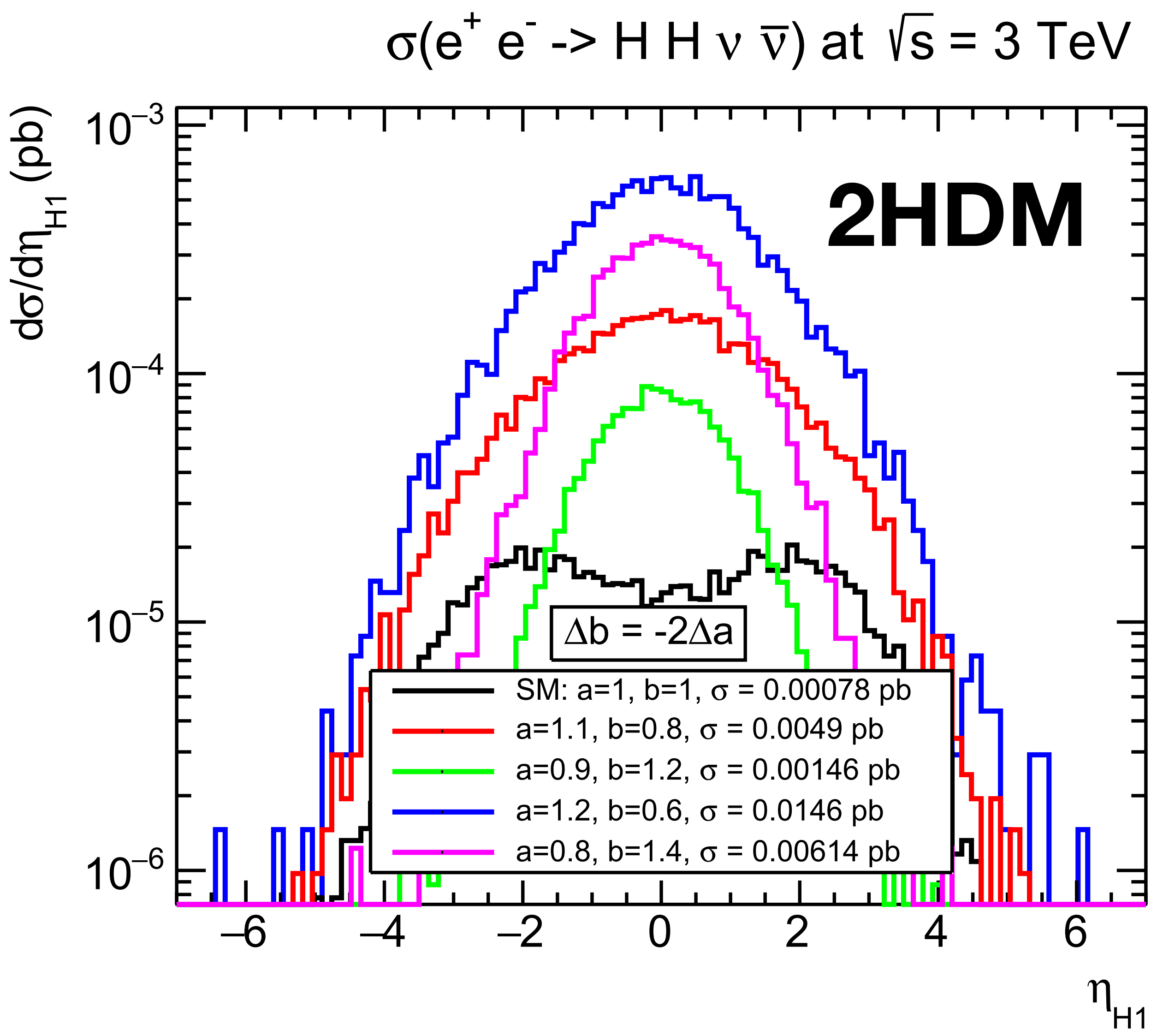


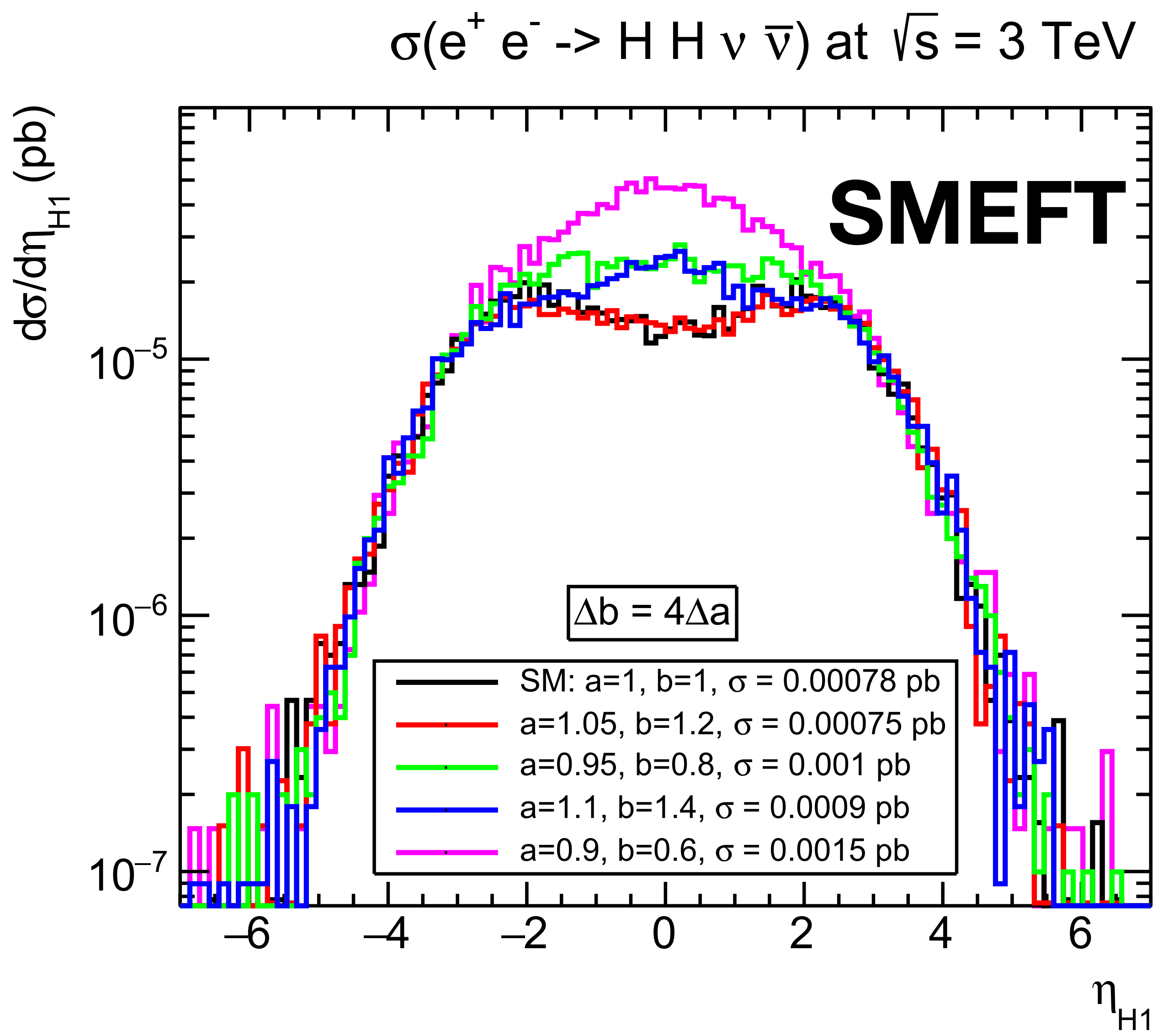


LHC (14 TeV)

$$pp \to HHjj \to b\bar{b}\gamma\gamma jj$$

2507.20988





CLIC (3 TeV)

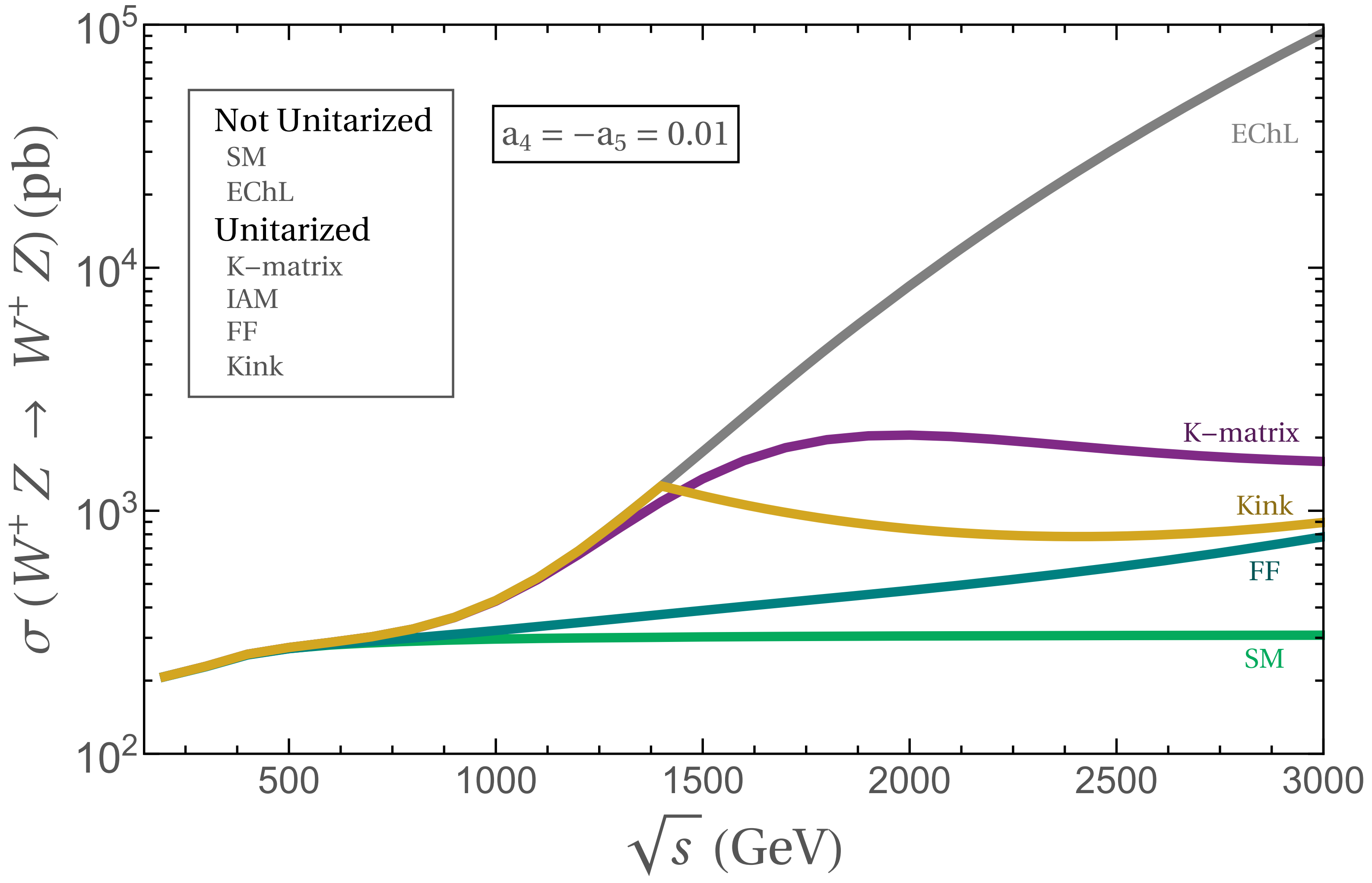
$$e^+e^- \to HH\nu\bar{\nu}$$

2312.03877

The constraints derived from data on the HEFT coefficients depend on the way chosen to solve the unitarity problem

C. Garcia-Garcia, M.Herrero, R.Morales, PRD100, 096003 (2019)

$WZ \to WZ$ a_4, a_5



$pp \to WZjj$ LHC (14 TeV)