Geometry and Energy in Effective Field Theory

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SMEFT meets ChEFT - Sep 29, 2025

Based on 2307.03187, 2504.18537 and 2410.21563, 2504.10617

Motivation

Story begins in practical pheno calculations for **SMEFT**

Idea: bottom-up EFT to systematically classify “all” BSM physics (knowledge of UV **not required!**)

Assumptions: new physics decoupled $\Rightarrow \Lambda \sim$ few TeV $\gg v$ and at the accessible scale only SM fields + symmetries

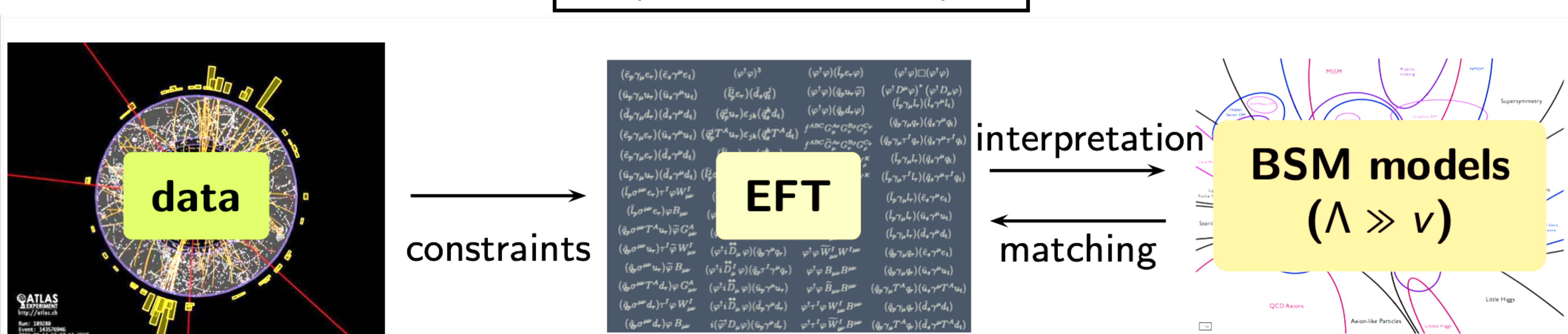
Violate B/L number

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{1}{\Lambda}\mathscr{L}_5 + \frac{1}{\Lambda^2}\mathscr{L}_6 + \frac{1}{\Lambda^3}\mathscr{L}_7 + \frac{1}{\Lambda^4}\mathscr{L}_8 + \ldots : \qquad \mathscr{L}_n = \sum_i C_i \mathscr{O}_i^{(d=n)}$$

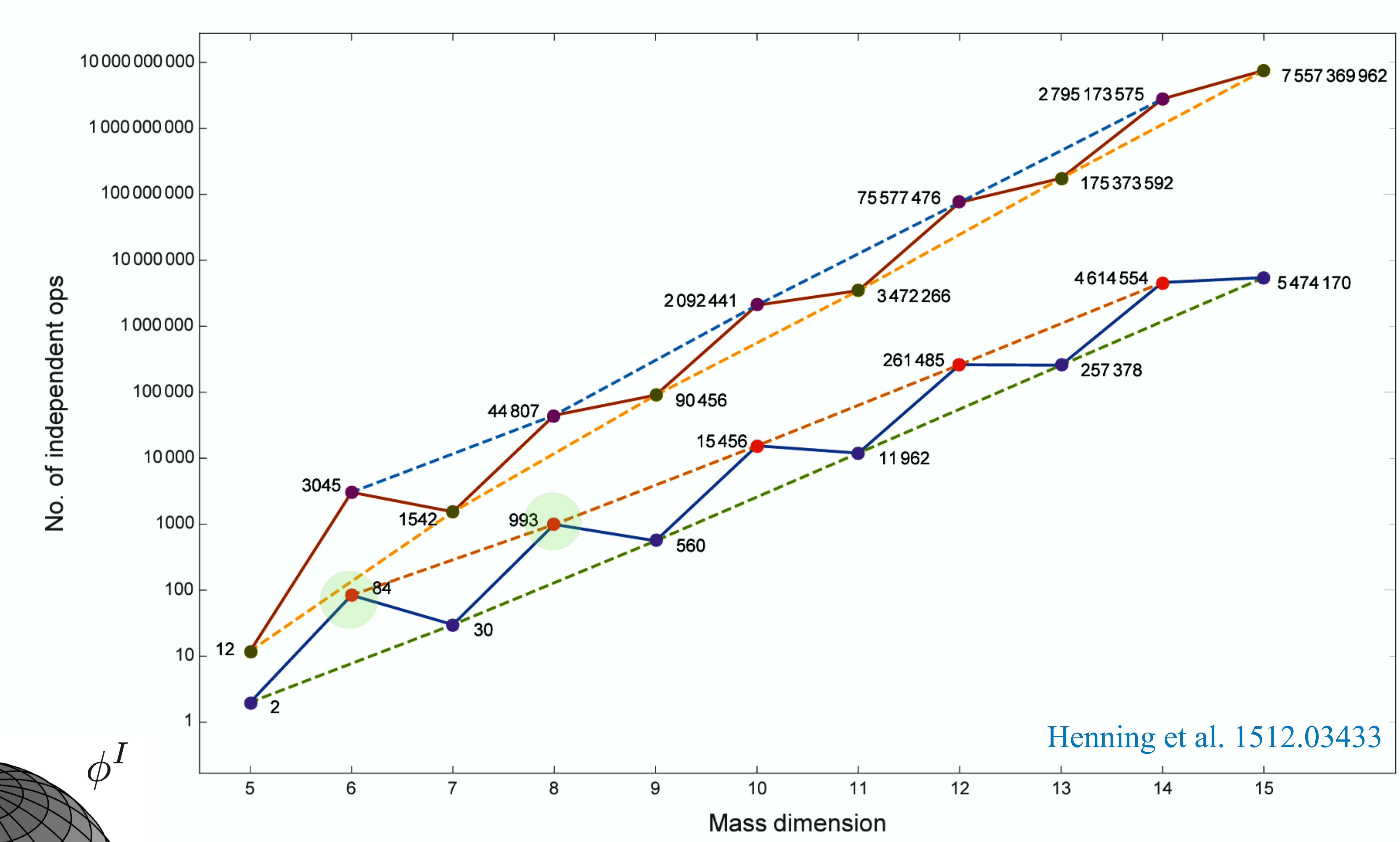
Majorana ν masses

“Leading” SM deviations (LEP constrained)

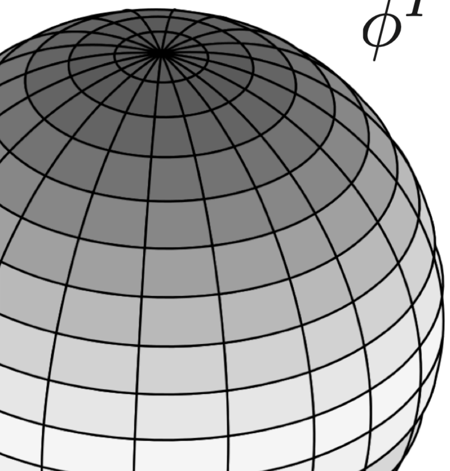
Current Frontier!



Operator growth



ϕ^I



Operator growth

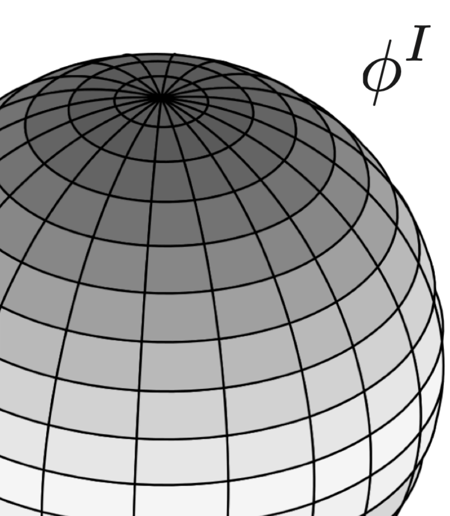
Some of the many operators in SMEFT Lagrangian…

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

Provocative *prompts*:



1) How can we calculate anything?

2) How can we discover anything?

1) Can we simplify higher order calculations?

General scalar field theory

NLSM: A scalar field theory can be written as

$$\mathscr{L} = \frac{1}{2} h_{IJ}(\boldsymbol{\phi})(\partial_\mu \phi)^I (\partial^\mu \phi)^J - V(\boldsymbol{\phi})$$

Riemannian metric in field-space is $h_{IJ}(\phi)$ wrt field multiplet ϕ^I

Expanding around flat-space ⇒ **higher-dim operators**

$$h_{IJ} = \delta_{IJ} + h_{IJ,K}\phi^K + h_{IJ,KL}\phi^K\phi^L + \ldots$$

Scalar EFT ↔ field theory on curved **scalar manifold**

Can include higher-derivative **metric-independent** operators E.g.

$$\lambda_{IJKL}(\phi)\partial_\mu\phi^I\partial^\mu\phi^J\partial_\nu\phi^K\partial^\nu\phi^L$$

Or try to include in the metric

Cheung et al 2202.06972, Cohen et al 2202.06965, 2410.21378, Craig and Yu-Tse Alan Lee 2307.15742, Cohen, Xu-Xiang Li, Zhang 2509.20449

Field re-definitions

Insight: S-matrix is **field re-definition invariant** $\leftrightarrow$ Lagrangian can change but not physical observables

Non-derivative **field re-definition** $\leftrightarrow$ **coord change** on scalar field-space manifold

$$\phi^I \to \varphi^I(\boldsymbol{\phi})$$

Then the field-space metric **transforms as a tensor**

$$g_{IJ}(\boldsymbol{\phi}) \to g'_{IJ}(\boldsymbol{\varphi}) = \left(\frac{\partial \phi^K}{\partial \varphi^I}\right)\left(\frac{\partial \phi^L}{\partial \varphi^J}\right) g_{KL}(\boldsymbol{\phi})$$

and the derivative of the scalar **transforms as a vector**

$$\partial_\mu \phi^I \to \partial_\mu \varphi^I = \left(\frac{\partial \varphi^I}{\partial \phi^J}\right) \partial_\mu \phi^J$$

$\Rightarrow$ Lagrangian density can be made **manifestly field redef invariant!**

Gains: amplitudes

Riemann curvature

$$R_{IJKL} = h_{IM}\left(\partial_K \Gamma^M_{LJ} + \Gamma^M_{KN}\Gamma^N_{LJ}\right) - (K \leftrightarrow L)$$

with **covariant derivative** ∇_I and **Christoffel symbol**

$$\Gamma^I_{JK} = \frac{1}{2} h^{IL}(h_{JL,K} + h_{LK,J} - h_{JK,L})$$

4-point Born amplitude $\phi_I \phi_J \to \phi_K \phi_L$ (massless fields)

$$\boxed{A^4_{IJKL} = R_{IJKL} s_{IK} + R_{IKJL} s_{IJ}\,,} \quad s_{ij} = (p_i + p_j)^2$$

Amplitudes depend on **geometric invariants** (new cross-check on top of gauge invariance!)

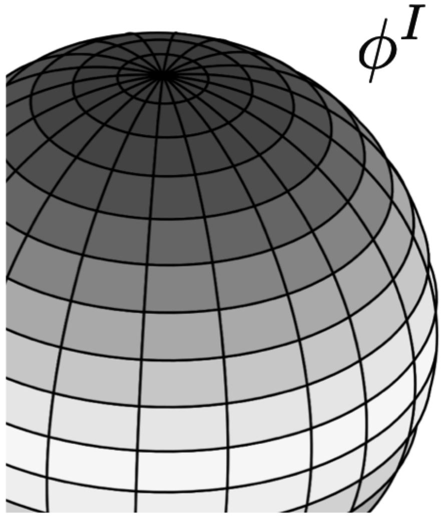
Bose symmetry $\leftrightarrow$ R_{IJKL} symmetries **Bianchi IDs**

$$R_{IJKL} + R_{IKLJ} + R_{ILJK} = 0 \qquad R_{IJMN;L} + R_{IJLM;N} + R_{IJNL;M} = 0$$

Geometry in the bosonic sector

S-matrix is **field re-definition invariant** $\leftrightarrow$ Lagrangian can change but **not** physical observables

Key insight: Field redefinitions are a **diffeomorphism** on scalar manifold



$$\phi^I \to \phi'^I(\phi)$$

$$\updownarrow$$

$$x^\mu \to x'^\mu(x)$$

$$\mathcal{L} = \frac{1}{2} \underline{h_{IJ}(\phi)} \partial_\mu \phi^I \partial^\mu \phi^J - V(\phi)$$

Target space metric
Resumming an infinite tower of operators

$$h_{IJ}(\phi) \to h'_{IJ}(\phi') = h_{AB}(\phi) \frac{\partial \phi^A}{\partial \phi'^I} \frac{\partial \phi^B}{\partial \phi'^J}$$

Diffeo-invariant quantities $\longleftrightarrow$ ***field-basis invariant quantities***

curvatures, covariant derivatives, … | *amplitudes, RG equations, …*

Lagrangian manifestly **field-basis invariant** $\rightarrow$ **applied** in amplitudes, soft theorems, double copy, hidden symmetries…

Alonso, Manohar, Martin, Trott, Jenkins; Cheung, Cohen, Craig, Sutherland, Zhang,…

Geometry in the fermionic sector

General Lagrangian (re-sums even more operators to **all orders**):

BA, Helset, Manohar, Pagès, Shen, JHEP 201 (2023)

$$\mathscr{L} = \frac{1}{2} h_{IJ}(\phi)(D_\mu \phi)^I (D^\mu \phi)^J - V(\phi) - \frac{1}{4} g_{AB}(\phi) F^A_{\mu\nu} F^{B\mu\nu}$$
$$+ \frac{1}{2} i k_{\bar{p}r}(\phi) \left(\bar{\psi}^{\bar{p}} \gamma^\mu \overleftrightarrow{D}_\mu \psi^r \right) + i \omega_{\bar{p}rI}(\phi)(D_\mu \phi)^I \bar{\psi}^{\bar{p}} \gamma^\mu \psi^r - \bar{\psi}^{\bar{p}} \mathscr{M}_{\bar{p}r}(\phi) \psi^r + \bar{\psi}^{\bar{p}} \sigma_{\mu\nu} \mathscr{T}^{\mu\nu}_{\bar{p}r}(\phi, F) \psi^r$$
$$+ C_{\bar{p}r\bar{s}t}(\phi) \left(\bar{\psi}^{\bar{p}} \gamma^\mu \psi^r \right) \left(\bar{\psi}^{\bar{s}} \gamma^\mu \psi^t \right)$$

Under fermion **field re-definition** $\psi^p \to R^p_{\ s}(\phi) \psi^s$

$$k_{\bar{p}r} \to \left[(R^\dagger)^{-1} k R^{-1} \right]_{\bar{p}r},$$

$$\omega_{\bar{p}rI} \to \left[(R^\dagger)^{-1} \omega_I R^{-1} \right]_{\bar{p}r} + \frac{1}{2} \left[(R^\dagger)^{-1} k (\partial_I R^{-1}) \right]_{\bar{p}r} - \frac{1}{2} \left[(\partial_I (R^\dagger)^{-1}) k R^{-1} \right]_{\bar{p}r}$$

$\Rightarrow k_{\bar{p}r}$ transforms as a **Hermitian** metric and $\omega_{\bar{p}rI}$ transforms as an **anti-Hermitian** connection

Unified field supermanifold

Promoting bosonic manifold to a Grassmanian **supermanifold**

BA, Helset, Manohar, Pagès, Shen, JHEP 201 (2023)
BA, Helset, Pagès, Shen 2504.18537

We can group the fields into a **supermultiplet** and **supermetric**

$$\Phi^a = \begin{pmatrix} \phi^I \\ A_\mu^A \\ \psi^p \\ \bar{\psi}^{\bar{p}} \end{pmatrix} \qquad {}_a\bar{g}_b(\Phi) = \begin{pmatrix} h_{IJ} + (\bar{\psi}\omega^-)_{s\{I}\, k^{s\bar{t}} (\omega^+\psi)_{\bar{t}J\}} & 0 & (\bar{\psi}\omega^-)_{rI} & (\omega^+\psi)_{\bar{r}I} \\ 0 & -g_{AB}\eta_{\mu_A\mu_B} & 0 & 0 \\ -(\bar{\psi}\omega^-)_{pJ} & 0 & 0 & k_{\bar{r}p} \\ -(\omega^+\psi)_{\bar{p}J} & 0 & -k_{\bar{p}r} & 0 \end{pmatrix}$$

Derived by requiring metric **transforms as tensor** under field redefinition

$${}_a\bar{g}_b \to \left(\frac{\delta\Phi^c}{\delta\Phi'^a}\right) {}_c\bar{g}_d \left(\frac{\delta\Phi^d}{\delta\Phi'^b}\right)$$

Note: Supermetric is supersymmetric as it's equal to its supertranspose but not in field content!

General amplitudes

The 4-point $\psi^p \phi^I \to \psi^{\bar{r}} \phi^J$ **massless** scattering amplitude

$$\mathcal{A}_{pI\bar{r}J} = (\bar{u}_{\bar{r}} \not{\phi}_I u_p) \bar{R}_{\bar{r}pJI}$$

The 5-point $\psi^p \phi^I \to \psi^{\bar{r}} \phi^J \phi^K$

$$\mathcal{A}_{pI\bar{r}JK} = (\bar{u}_{\bar{r}} \not{\phi}_J u_p) \bar{\nabla}_K \bar{R}_{\bar{r}pIJ} + (\bar{u}_{\bar{r}} \not{\phi}_K u_p) \bar{\nabla}_J \bar{R}_{\bar{r}pIK}$$

$$\bar{\nabla}_K \bar{R}_{\bar{r}pIJ} = \bar{R}_{\bar{r}pIJ,K} - \bar{\Gamma}^{\bar{s}}_{\bar{r}K} \bar{R}_{\bar{s}pIJ} - \bar{\Gamma}^{s}_{pK} \bar{R}_{\bar{r}sIJ} - \bar{\Gamma}^{L}_{IK} \bar{R}_{\bar{r}pLJ} - \bar{\Gamma}^{L}_{JK} \bar{R}_{\bar{r}pIL}$$

Turning on the scalar potential and fermion mass matrix

$$\begin{aligned} \mathcal{A}_{pI\bar{r}J} =& (\bar{u}_{\bar{r}} \not{\phi}_I u_p) \left(\bar{R}_{\bar{r}pJI} + k^{s\bar{t}} \left(\frac{\mathcal{M}_{\bar{r}s;I} \mathcal{M}_{\bar{t}p;J}}{s_{\bar{r}I}} - \frac{\mathcal{M}_{\bar{r}s;J} \mathcal{M}_{\bar{t}p;I}}{s_{pI}} \right) \right) \\ & - (\bar{u}_{\bar{r}} u_p) \left(\mathcal{M}_{\bar{r}p;IJ} - h^{LK} \frac{\mathcal{M}_{\bar{r}p;L} V_{;IJK}}{s_{IJ}} \right) , \end{aligned}$$

1-loop application: Renormalisation (bosonic e.g.)

One-loop RGE from 2nd variation of action   t'Hooft '74 and Alonso, Manohar et al '20

$$A^{B\mu_B} = \mathsf{A}^{B\mu_B} + \zeta^{B\mu_B} - \frac{1}{2}\widetilde{\Gamma}^{(B\mu_B)}_{jk}\eta^j\eta^k + \ldots$$

$$\phi^I = \Phi^I + \eta^I - \frac{1}{2}\widetilde{\Gamma}^I_{jk}\eta^j\eta^k + \ldots$$

$$\eta^i = \begin{pmatrix} \eta^I \\ \zeta^{A\mu_A} \end{pmatrix}$$

in **geodesic (RNC) coordinates**

(can also use Fermi NC to help ID field non-analyticities 2509.07101 — see Yu-Tse Lee's talk!)

$$\frac{\mathrm{d}^2\phi^i}{\mathrm{d}\lambda^2} + \Gamma^i_{jk}(\phi)\frac{\mathrm{d}\phi^j}{\mathrm{d}\lambda}\frac{\mathrm{d}\phi^k}{\mathrm{d}\lambda} = 0$$

➡

$$\phi^i = \phi^i_0 + \lambda\eta^i - \frac{1}{2}\lambda^2\,\Gamma^i_{jk}(\phi_0)\,\eta^j\eta^k + \ldots$$

$$\phi^i \to \phi^i + \eta^i - \frac{1}{2}\Gamma^i_{jk}\eta^j\eta^k + \mathcal{O}(\eta^3)$$

gives **covariant** result e.g. $\eta\eta$-variation

$$\delta_{\eta\eta}S = \frac{1}{2}\int d^4x\,\Big\{h_{IJ}\left(\widetilde{\mathscr{D}}_\mu\eta\right)^I\left(\widetilde{\mathscr{D}}_\mu\eta\right)^J + \Big[-\widetilde{R}_{IKJL}(D_\mu\phi)^K(D^\mu\phi)^L - (\nabla_I\nabla_J V)$$
$$- \frac{1}{4}\left(\nabla_I\nabla_J g_{AB} - \Gamma^C_{IA}g_{CB,J} - \Gamma^C_{IB}g_{AC,J}\right)F^{A\mu\nu}F^B_{\mu\nu} - h_{IK}h_{JL}g^{AB}t^K_A t^L_B\Big]\eta^I\eta^J\Big\}$$

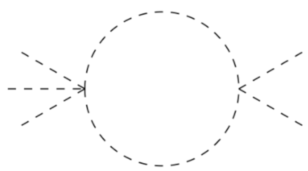
1-loop application: Renormalisation

The 2nd variation of action gives 1-loop RGE (super compact expressions!) t'Hooft '74, Alonso, Manohar et al '20

Pure bosonic loop pole:

Helset, Manohar, Jenkins 2212.03253

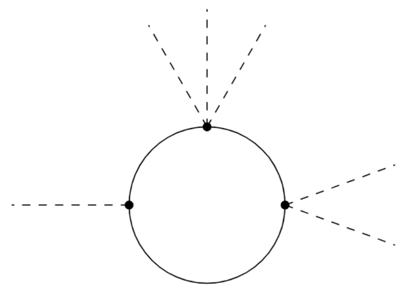
$$\Delta S = \frac{1}{32\pi^2\epsilon}\int \mathrm{d}^4x \left\{\frac{1}{12}\mathrm{Tr}\left[Y_{\mu\nu}Y^{\mu\nu}\right] + \frac{1}{2}\mathrm{Tr}\left[\mathcal{X}^2\right]\right\}$$



Pure fermionic loop pole:

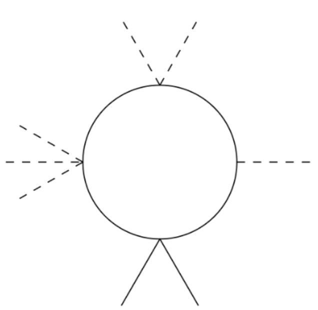
BA, Helset, Manohar, Pagès, Shen 2307.03187

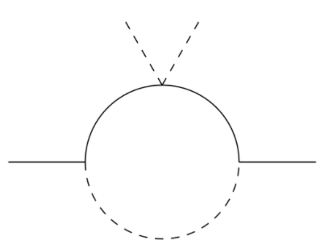
$$\begin{aligned}\Delta S = \frac{1}{32\pi^2\epsilon}\int \mathrm{d}^4x \Big\{&\frac{1}{3}\mathrm{Tr}\left[\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu}\right] + \mathrm{Tr}\left[(\mathcal{D}_\mu\mathcal{M})(\mathcal{D}^\mu\mathcal{M}) - (\mathcal{M}\mathcal{M})^2\right] \\ &- \frac{16}{3}\mathrm{Tr}[(\mathcal{D}_\mu\mathcal{T}^{\mu\alpha})(\mathcal{D}_\nu\mathcal{T}^{\nu\alpha}) - (\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^2] \\ &- 4i\mathrm{Tr}[\mathcal{Y}_{\mu\nu}(\mathcal{M}\mathcal{T}^{\mu\nu} + \mathcal{T}^{\mu\nu}\mathcal{M})] - 8\mathrm{Tr}(\mathcal{M}\mathcal{T}^{\mu\nu})^2\Big\}\end{aligned}$$



Mixed bosonic-fermionic pole:

BA, Helset, Pagès, Shen 2504.18537



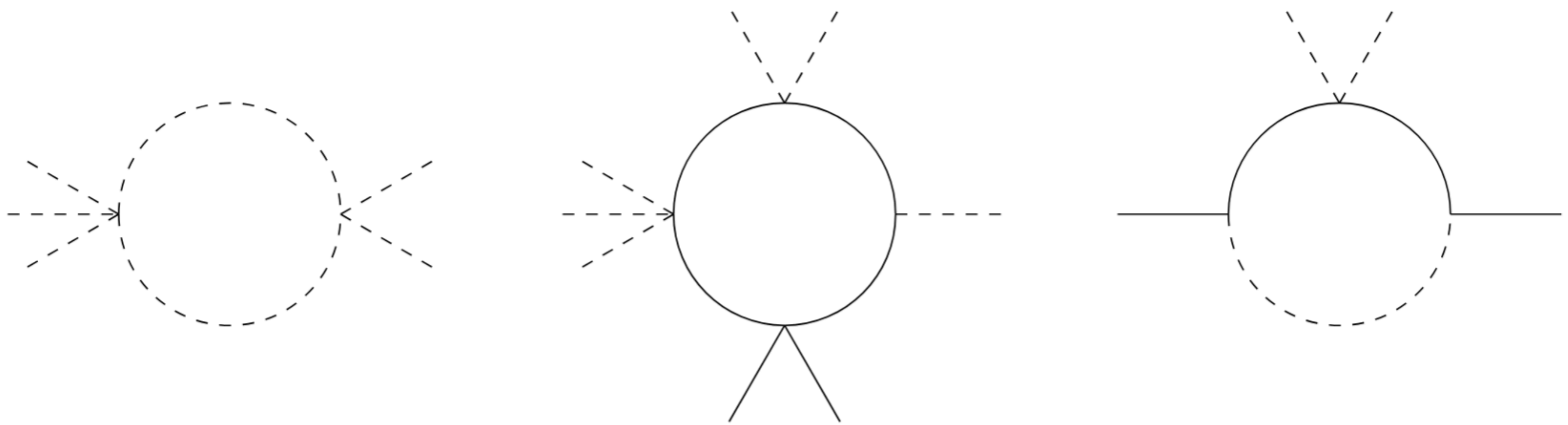


$$\begin{aligned}\Delta S^{(2)}_{\mathrm{mix}} = \frac{1}{32\pi^2\epsilon}\int \mathrm{d}^4x \Big\{&\mathrm{Tr}[\bar{N}(i\not{D} + 2\mathcal{M})N] - 2\mathrm{Tr}[i\bar{Q}NX + \mathrm{h.c.}] \\ &+ \mathrm{Tr}[i\bar{Q}(i\overleftarrow{\not{D}} + \mathcal{M} - \sigma_{\alpha\beta}\mathcal{T}^{\alpha\beta})(i\not{D} + 2\mathcal{M})N + \mathrm{h.c.}] \\ &+ \mathrm{Tr}[\bar{Q}(i\not{D} - \mathcal{M} + \sigma_{\alpha\beta}\mathcal{T}^{\alpha\beta})QX + \mathrm{h.c.}] \\ &- \mathrm{Tr}[\bar{Q}(i\overleftarrow{\not{D}} + \mathcal{M} - \sigma_{\alpha\beta}\mathcal{T}^{\alpha\beta})(i\not{D} + 2\mathcal{M})(i\not{D} - \mathcal{M} + \sigma_{\gamma\delta}\mathcal{T}^{\gamma\delta})Q]\end{aligned}$$

Renormalisation

with identified **covariant parts, e.g.**

$$
\begin{aligned}
[\mathcal{Y}_{\mu\nu}]^p{}_r &= [\mathcal{D}_\mu, \mathcal{D}_\nu]^p{}_r = \bar{R}^p{}_{rIJ}(D_\mu\phi)^I(D_\nu\phi)^J + \left(\bar{\nabla}_r t^p_A\right) F^A_{\mu\nu} ,\\
(\mathcal{D}_\mu \mathcal{M})^p{}_r &= k^{p\bar{t}}(\mathcal{D}_\mu \mathcal{M}_{\bar{t}r}) = k^{p\bar{t}}\left[D_\mu \mathcal{M}_{\bar{t}r} - \bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I \mathcal{M}_{\bar{s}r} - \bar{\Gamma}^{s}_{Ir}(D_\mu\phi)^I \mathcal{M}_{\bar{t}s}\right] ,\\
(\mathcal{M}\mathcal{M})^p{}_r &= k^{p\bar{t}}\mathcal{M}_{\bar{t}q}k^{q\bar{s}}\mathcal{M}_{\bar{s}r} ,\\
(\mathcal{D}_\mu \mathcal{T}^{\alpha\beta})^p{}_r &= k^{p\bar{t}}(\mathcal{D}_\mu \mathcal{T}^{\alpha\beta}_{\bar{t}r}) = k^{p\bar{t}}\left[D_\mu \mathcal{T}^{\alpha\beta}_{\bar{t}r} - \bar{\Gamma}^{\bar{s}}_{I\bar{t}}(D_\mu\phi)^I \mathcal{T}^{\alpha\beta}_{\bar{s}r} - \bar{\Gamma}^{s}_{Ir}(D_\mu\phi)^I \mathcal{T}^{\alpha\beta}_{\bar{t}s}\right] ,\\
(\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^p{}_r &= k^{p\bar{t}}\mathcal{T}^{\mu\nu}_{\bar{t}q}k^{q\bar{s}}\mathcal{T}^{\alpha\beta}_{\bar{s}r} .
\end{aligned}
$$



Mapping to the SMEFT

We can apply formalism to the SMEFT by **identification**

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^2 + i\phi^1 \\ \phi^4 - i\phi^3 \end{pmatrix} \qquad A^B_\mu = \begin{pmatrix} G^{\mathscr{A}}_\mu \\ W^a_\mu \\ B_\mu \end{pmatrix}$$

with **scalar metric**

$$\begin{aligned} h_{IJ} =& \delta_{IJ} \left[1 + \frac{1}{4} \left({}^8C^{(1)}_{H^6D^2} - {}^8C^{(2)}_{H^6D^2} \right) (\phi^K \phi^K)^2 \right] + \left(-2 \ {}^6C_{H^4\Box} \right) \phi^I \phi^J \\ &+ \frac{1}{2} \left[{}^6C_{H^4D^2} + {}^8C^{(2)}_{H^6D^2} (\phi^K \phi^K) \right] \mathcal{H}_{IJ}(\phi) \,, \end{aligned}$$

$$\mathcal{H}_{IJ}(\phi) = \phi_I \phi_J + \begin{bmatrix} \phi_2^2 & -\phi_1\phi_2 & -\phi_2\phi_4 & \phi_2\phi_3 \\ -\phi_1\phi_2 & \phi_1^2 & \phi_1\phi_4 & -\phi_1\phi_3 \\ -\phi_2\phi_4 & \phi_1\phi_4 & \phi_4^2 & -\phi_3\phi_4 \\ \phi_2\phi_3 & -\phi_1\phi_3 & -\phi_3\phi_4 & \phi_3^2 \end{bmatrix}$$

and **gauge metric**

$$g_{AB} = \begin{bmatrix} [g_{GG}]_{\mathscr{A}\mathscr{B}} & 0 & 0 \\ 0 & [g_{WW}]_{ab} & [g_{WB}]_a \\ 0 & [g_{BW}]_b & g_{BB} \end{bmatrix}$$

SMEFT

Again applying formalism to the SMEFT

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^2 + i\phi^1 \\ \phi^4 - i\phi^3 \end{pmatrix} \qquad A^B_\mu = \begin{pmatrix} G^{\mathscr{A}}_\mu \\ W^a_\mu \\ B_\mu \end{pmatrix} \qquad \psi^p = \begin{pmatrix} \ell^p_L \\ q^p_L \\ e^p_R \\ u^p_R \\ d^p_R \end{pmatrix}$$

and identifying e.g. for RH electrons in SMEFT

$$M_{\bar{p}r} \supset [Y_e]^\dagger_{\bar{p}r} H - {}^6C_{\underset{\bar{p}r}{leH^3}} H(H^\dagger H) - {}^8C_{\underset{\bar{p}r}{leH^5}} H(H^\dagger H)^2$$

$$T^{\mu\nu}_{\bar{p}r} \supset {}^6C_{\underset{\bar{p}r}{leBH}} H \frac{1}{2}\left(B^{\mu\nu} - i\tilde{B}^{\mu\nu}\right) + {}^8C_{\underset{\bar{p}r}{leBH^3}} H(H^\dagger H)\frac{1}{2}\left(B^{\mu\nu} - i\tilde{B}^{\mu\nu}\right)$$

$$\omega_{R,\bar{p}rI} \supset + i(\phi\gamma_4)_I \, {}^6Q^{(1)}_{\underset{\bar{p}r}{e^2H^2D}}$$

Plug and Play: SMEFT RGEs

Many new bosonic (e.g. on next slide) and fermionic RGEs calculated in SMEFT

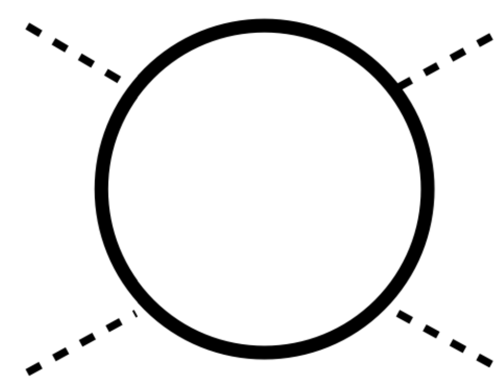
Fermionic loop corrections to bosonic operators:

$$
\begin{aligned}
{}^6\dot{C}_{H^4\Box} &= \frac{2}{3} g_1^2 \kappa_1 + 2 g_2^2 \kappa_2 - 2\kappa_9 - 6\kappa_{10} - 2\kappa_{11}, \\
{}^6\dot{C}_{H^4 D^2} &= \frac{8}{3} g_1^2 \kappa_1 - 8\kappa_9 + 4\kappa_{11} \,.
\end{aligned}
$$

BA, Helset, Manohar, Pagès, Shen, JHEP 201 (2023)

Geometry combines terms together

$$
\begin{aligned}
\kappa_1 &= \left[y_e \, {}^6C_{e^2 \underset{tt}{H^2} D} + 2 y_\ell \, {}^6C^{(1)}_{\ell^2 \underset{tt}{H^2} D} + N_c y_u \, {}^6C_{u^2 \underset{tt}{H^2} D} + N_c y_d \, {}^6C_{d^2 \underset{tt}{H^2} D} + 2 N_c y_q \, {}^6C^{(1)}_{q^2 \underset{tt}{H^2} D} \right] \\
\kappa_9 &= \mathrm{Tr} \left[-Y_e Y_e^{\dagger} {}^6C_{e^2 H^2 D} + Y_e^{\dagger} Y_e \, {}^6C^{(1)}_{\ell^2 H^2 D} - N_c Y_d Y_d^{\dagger} {}^6C_{d^2 H^2 D} + N_c Y_d^{\dagger} Y_d \, {}^6C^{(1)}_{q^2 H^2 D} \right. \\
&\qquad \left. + N_c Y_u Y_u^{\dagger} {}^6C_{u^2 H^2 D} - N_c Y_u^{\dagger} Y_u \, {}^6C^{(1)}_{q^2 H^2 D} \right] , \\
\kappa_{11} &= \mathrm{Tr} \left[-N_c Y_d Y_u^{\dagger} {}^6C_{udH^2D} - N_c Y_u Y_d^{\dagger} {}^6C^{\dagger}_{udH^2D} \right]
\end{aligned}
$$



Mixed fermion-bosonic loop contributions up to dimension 8 computed recently in BA, Helset, Pagès, Shen 2504.18537

Takeaway: This organisation let's us easily calculate higher dimensional loops

Bosonic fermion loop dimension 8 example

$$
\begin{aligned}
{}^{8}\dot{C}_{H^8} =& \lambda\left(-\frac{4}{3}g_1^2\,{}^{6}C_{H^4D^2}-\frac{8}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_1\\
&+\left(-8g_2^2\,{}^{6}C_{H^6}+\lambda\left(\frac{64}{3}g_2^2\,{}^{6}C_{H^4\square}-4g_2^2\,{}^{6}C_{H^4D^2}-\frac{16}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\right)\kappa_2\\
&+\left(6\,{}^{6}C_{H^6}-16\lambda\,{}^{6}C_{H^4\square}+2\lambda\,{}^{6}C_{H^4D^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})\\
&-\frac{4}{3}\lambda g_1^2\kappa_1^{(8)}-\frac{4}{3}\lambda g_2^2\kappa_2^{(8)}-\frac{4}{3}\lambda g_2^2\kappa_3-\frac{4}{3}\lambda g_2^2\kappa_4-\frac{8}{3}\lambda g_1^2\kappa_5+\frac{4}{3}\lambda g_2^2\kappa_5+\frac{1}{3}\lambda(g_1^2-g_2^2)\kappa_6\\
&+4\lambda\kappa_7^{(8)}-8\lambda\kappa_8+4\lambda\kappa_9^{(8)}+4\lambda\kappa_{10}^{(8)}+4\lambda\kappa_{12}-4\lambda\kappa_{13}-4\lambda\kappa_{14}-4\lambda\kappa_{15}-4\lambda\kappa_{16}\\
&-4\lambda\kappa_{17}-4\kappa_{21}^{(8)}+2\kappa_{22}-\frac{20}{3}\lambda g_1g_2\tau_2-\frac{8}{3}\lambda g_2^2\tau_3'+4\lambda g_2\tau_{18}+8\lambda g_1\tau_{20}+2\lambda g_2\tau_{26}\,.
\end{aligned}
$$

$$
\begin{aligned}
{}^{8}\dot{C}^{(1)}_{H^6D^2} =& \left(2g_1^2\,{}^{6}C_{H^4D^2}+\frac{16}{3}g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_1\\
&+\left(-\frac{32}{3}g_2^2\,{}^{6}C_{H^4\square}+\frac{2}{3}g_2^2\,{}^{6}C_{H^4D^2}+8g_1g_2\,{}^{6}C_{WBH^2}\right)\kappa_2\\
&+\left(8\,{}^{6}C_{H^4\square}+{}^{6}C_{H^4D^2}\right)(-\kappa_7+4\kappa_{10}+2\kappa_{11})\\
&+2g_1^2\kappa_1^{(8)}+\frac{10}{3}g_2^2\kappa_2^{(8)}+2g_2^2\kappa_3+\frac{8}{3}g_2^2\kappa_4+4g_1^2\kappa_5-\frac{10}{3}g_2^2\kappa_5-\frac{1}{2}g_1^2\kappa_6+g_2^2\kappa_6\\
&+2\kappa_8-6\kappa_9^{(8)}-10\kappa_{10}^{(8)}\underbrace{-2}_{+4}\kappa_{11}^{(8)}-6\kappa_{12}+6\kappa_{13}+6\kappa_{14}+10\kappa_{15}+6\kappa_{16}+10\kappa_{17}\\
&\underbrace{+2}_{-2}\kappa_{18}-\kappa_{19}+4\kappa_{20}+\frac{32}{3}g_1g_2\tau_2+\frac{20}{3}g_2^2\tau_3'-8g_2\tau_{18}-12g_1\tau_{20}-6g_2\tau_{26}
\end{aligned}
$$

What next?

Incorporating higher derivatives into metric via functional geometry and jet bundles Brivio et al. `23, ’25 Cohen et al. ’22, ’24, Craig et al. ’23, ’24,…

Incorporating invariance under gauge field redefinitions

BA, Xu-Xiang Li, Martin, Pagès, *In preparation*

RGEs for 4+ fermion operators and higher gauge field operators

Implementation of geoSMEFT organization in public codes e.g. MATCHETE

Geometry of odd-dimensional operators? Neutrino Physics!

2) Can we make the SMEFT more discovery-friendly?

Energy Expansion of SMEFT

Geometry responsible for **all-order operator resummation** ⇒ amazing calculation tool!

No information on which operators have **largest impact** in high energy processes for **HL-LHC** ⇒ **Energy-enhanced SMEFT**

Idea: provide a **general prescription** to make energy-enhancement **manifest at the Lagrangian level**

BA, Martin, JHEP 29 (2025)
BA, Martin, PRD 112 (2025)

New power counting

Recall naive **SMEFT expansion**

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{i,j} \frac{c_j^{(i)}}{\Lambda^i} \mathcal{O}_j^{(4+i)}$$

With **generic** amplitude expansion

$$|\mathcal{A}|^2 = |A_{\text{SM}}|^2 \left\{ 1 + \frac{2\,\text{Re}(A_{\text{SM}}^* A_6)}{\Lambda^2 |A_{\text{SM}}|^2} + \frac{1}{\Lambda^4}\left(\frac{|A_6|^2}{|A_{\text{SM}}|^2} + \frac{2\,\text{Re}(A_{\text{SM}}^* A_8)}{|A_{\text{SM}}|^2} \right) + \cdots \right\}$$

Then note that any on-shell n-leg vertex **scales as**

$$\mathcal{V}_k^{(n)} \sim \frac{E^{q_k} v^{p_k}}{\Lambda^D}$$

Make energy manifest in $\mathscr{L}$ by **new power counting**

$$(\Lambda, E, v) \sim \begin{cases} (\lambda^{-3}, \lambda^{-2}, \lambda^{-1}) & \text{if } \Lambda \gg E \gg v, \\ (\lambda^{-3}, \lambda^{-1}, \lambda^{-1}) & \text{if } \Lambda \gg E \sim v, \end{cases}$$

At the **Lagrangian level:**

$$\mathcal{L}_{\text{SMEFT}}^{(n)} = \sum_j g_{\text{SM}}^j \lambda_j^{(n)} \mathcal{O}_{\text{SM}}^j + \sum_{i,k} c_k^{(i)} \frac{\lambda_k^{(n)}}{\Lambda^i} \mathcal{O}_k^{(4+i)}$$

with **vertex-dependent** parameter

$$\lambda_j^{(n)} = \lambda^{3D - 2q_{\max} - p_{\min}}$$

BA, Martin, 2504.10617

λ-protocol

Given $\{n, O\}$ for O with $\{N_f, N_H, N_X, N_D\}$:

i) Mass dimension of O is $D = \frac{3}{2}N_f + N_H + 2N_X - 4$

ii) On-shell $\mathcal{V}_k^{(n)}$ vertex dimension: $d = 4 - n$

iii) Fewest vevs to pull out: $p_{\min} = \max[\mathrm{N_f} + \mathrm{N_X} - \mathrm{N_H}, 0]$

iv) Most powers of E you can keep: $q_{\max} = D + d - p_{\min}$

For all sub-leading terms: also attainable with extra protocol by careful replacement of $E \to v$

Note: for $n = 2,3$ **kinematics frozen** by on-shell conditions and momentum conservation → scaling fixed by lowest dim operator. Only $n \geq 4$ can form independent **Mandelstam invariants**

BA, Martin, 2504.10617

Enhancement Tables

We explicitly provide **maximally enhanced** operator tables up to $n = 6$ and dimension 10 e.g. 2- and 4-point:

Dimension-6 Operators		
Operator	$\lambda^{(2)}$	Vertices
H^6	λ^4	$h^2 : v^2$
H^4D^2	λ^4	$(\partial h)^2 : v^2$
H^2X^2	λ^4	$(\partial V)^2 : v^2$
$\psi^2 H^3$	λ^4	$\psi^2 : v^2$
Dimension-8 Operators		
Operator	$\lambda^{(2)}$	Vertices
H^8	λ^8	$h^2 : v^4$
H^6D^2	λ^8	$(\partial h)^2 : v^4$
H^4X^2	λ^8	$h^2 : v^4$
$\psi^2 H^5$	λ^8	$\psi^2 : v^4$
Dimension-10 Operators		
Operator	$\lambda^{(2)}$	Vertices
H^{10}	λ^{12}	$h^2 : v^6$
H^8D^2	λ^{12}	$(\partial h)^2 : v^6$
H^6X^2	λ^{12}	$h^2 : v^6$
$\psi^2 H^7$	λ^{12}	$\psi^2 : v^6$

Dimension-6 Operators		
Operator	$\lambda^{(4)}$	Vertices
H^4D^2	λ^2	$(\partial h)^2 h^2 : E^2,\ \ h^3 V : vE,\ \ h^2V^2 : v^2$
H^2X^2	λ^2	$h^2V^2 : E^2,\ \ hV^3 : vE,\ \ V^4 : v^2$
X^3	λ^2	$V^4 : E^2$
$\psi^2 HX$	λ^2	$\psi^2V^2 : vE,\ \ \psi^2\, h\, \partial V : E^2$
ψ^2H^2D	λ^2	$\psi^2\, V\, h : vE,\ \ \psi^2\, h\, \partial h : E^2$
ψ^4	λ^2	$\psi^4 : E^2$
Dimension-8 Operators		
Operator	$\lambda^{(4)}$	Vertices
X^4	λ^4	$V^4 : E^4$
H^4D^4	λ^4	$(\partial h)^4 : E^4,\ \ h^3V : E^3v,\ \ h^2V^2 : E^2v^2,\ \ hV^3 : E\,v^3$
$H^2X^2D^2$	λ^4	$(\partial h)^2V^2 : E^4,\ \ (\partial h)V^3 : E^3v,\ \ V^4 : E^2v^2$
$\psi^2H^2D^3$	λ^4	$\psi^2(\partial h)^2 : E^4,\ \ \psi^2(\partial h)V : E^3v,\ \ \psi^2V^2 : E^2\,v^2$
ψ^2X^2D	λ^4	$\psi^2(\partial^2 V)(\partial V) : E^4$
ψ^2HXD^2	λ^4	$\psi^2(\partial h)(\partial V) : E^4,\ \ \psi^2(\partial V)^2 : E^3v$
ψ^4D^2	λ^4	$\psi^2(\partial\psi)^2 : E^4$
Dimension-10 Operators		
Operator	$\lambda^{(4)}$	Vertices
H^4D^6	λ^6	$(\partial h)^4 : E^6,\ \ h^3V : E^5v,\ \ h^2V^2 : E^4v^2,\ \ hV^3 : E^3\,v^3,\ \ V^4 : E^4\,v^2$
X^4D^2	λ^6	$(\partial V)^2V^2 : E^6$
ψ^2HXD^4	λ^6	$\psi^2(\partial h)(\partial V) : E^6,\ \ \psi^2(\partial V)^2 : E^5v$
$\psi^2H^2D^5$	λ^6	$(\partial h)^2(\partial\psi)^2 : E^6,\ \ (\partial h)(\partial\psi)^2V : E^5v,\ \ (\partial\psi)^2V^2 : E^4v^2$
ψ^4D^4	λ^6	$(\partial\psi)^4 : E^6$

Non-trivial E-dependence for 4-point and higher!

All sub-leading contributions in λ can be **easily determined** with counting algorithm for any n and # of ψ, X, H, D in operator

BA, Martin, 2504.10617

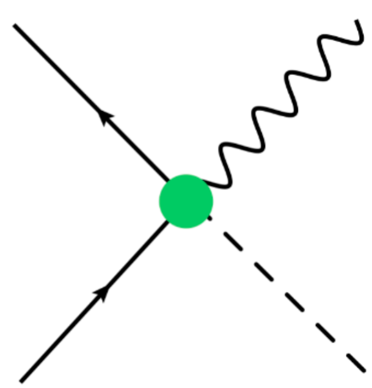
Assumptions and Examples

Assume all $c_j^{(i)}/\Lambda^i$ roughly equal $\leftrightarrow$ weakly coupled UV physics

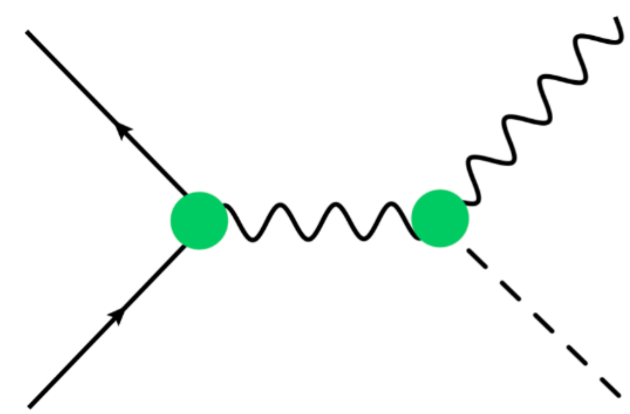
Further enhancements: tree-level UV origin, same chirality for flavour universal/ MFV assumptions

We explicitly do 4,5,6-point examples: $q\bar{q} \to VH$, $gg \to t\bar{t}H$, di-Higgs VBF

Checkout simplest $q\bar{q} \to VH$ (after simplifying)



$$\mathcal{A}^{(2)}_{qqVh} = \frac{\lambda^2}{\hat{\Lambda}^2} c_{\psi^2 HX} + \frac{\lambda^3}{\hat{\Lambda}^2} c_{\psi^2 H^2 D} + \frac{\lambda^4}{\hat{\Lambda}^4} c_{\psi^2 HXD^2} + \frac{\lambda^5}{\hat{\Lambda}^4} c_{\psi^2 H^2 D^3} + \frac{\lambda^5}{\hat{\Lambda}^4} c_{\psi^2 H^4 D} + \cdots .$$



$$\mathcal{A}^{(1)}_{qqVh} = \lambda\, g^{\mathrm{SM}}_{\bar{q}qV}\, g^{\mathrm{SM}}_{hVV} + \frac{\lambda^3}{\hat{\Lambda}^2} g^{\mathrm{SM}}_{\bar{q}qV}\, c_{H^2X^2} + \frac{\lambda^4}{\hat{\Lambda}^2} g^{\mathrm{SM}}_{hVV}\, c_{\psi^2 HX}$$

BA, Martin, 2504.10617

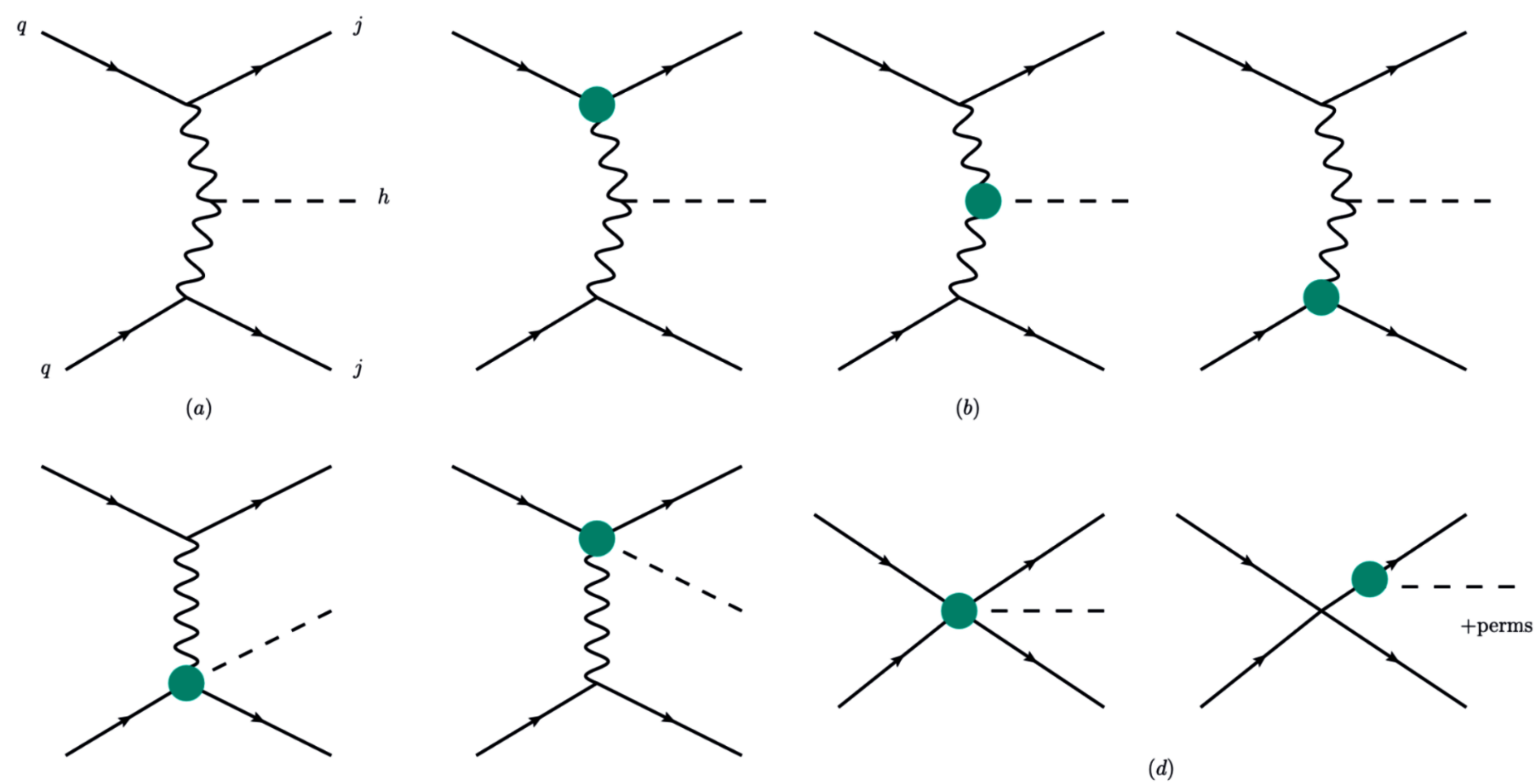
Example in detail: VBF Higgs production

Need process with **high E kinematics** $\leftrightarrow$ amplify effects of high-dim operators

Our aim: Determine which operators are E-enhanced and push to **unconstrained** $\mathcal{O}(1/\Lambda^4)$ [BA and Martin 2410.25163]

SMEFT corrs. to SM couplings
$g_{ffV,\text{SMEFT}} = g_{ffV,\text{SM}} + \mathcal{O}(v^2/\Lambda^2)$

New vertex structures on top of SM



E-enhanced topology starting at dimension 6 (c)

E-enhanced five particle vertices only present at dimension 8

Energy-enhanced geoSMEFT operators

In regime $E \gg v$ the terms in $\mathscr{A}_6$ and $\mathscr{A}_8$ that incorporate the highest powers of E carry the largest impact

$2 \to 3$ amplitudes have mass dimension -1 with e.g. **scaling**

$$\mathscr{A}_{\rm SM} \sim g_{\rm SM}^3 \frac{v}{E^2}, \quad \mathscr{A}_{Hq}, \mathscr{A}_{Hu,d} \sim g_{\rm SM}^2 \frac{c_6\, v}{\Lambda^2}, \quad \mathscr{A}_{q^2H^2XD}, \mathscr{A}_{q^2H^2D^3} \sim g_{\rm SM}^2 \frac{c_8\, v\, E^2}{\Lambda^4}, \quad \mathscr{A}_{q^4H^2} \sim \frac{c_8\, v\, E^2}{\Lambda^4}$$

The ratio of $D = 8$ interference piece to the $D = 6$

$$\frac{\mathscr{A}^*_{\rm SM}\mathscr{A}_8}{\mathscr{A}^*_{\rm SM}\mathscr{A}_6} \sim \left(\frac{c_8}{c_6}\right)\left(\frac{E^2}{\Lambda^2}\right)$$

For fixed $\Lambda \sim \mathrm{TeV}$ the Wilson coefficients for E-enhanced $D = 6$ operators such as $c_{Hq}^{(3)} \ll 1$ to be consistent with LHC/LEP [Ellis et al. '20]

Numerical analysis and resonant operators

Implemented LHC-like VBF selection cuts on $m_{j_1 j_2}$, $\Delta\eta_{j_1 j_2}$, $p_{T,H}$

[Araz et al '20]

Numerical analysis needed to confirm **EFT validity** up to $(D=8)^2$ terms; **minimum** $\Lambda \approx 1.2\,\mathrm{TeV}$

ID'd $D=8$ operators with **largest contributions** consistent with analysis: $c^{(3)}_{q^2H^2D^3}$ and $c^{(3)}_{q^2H^4}$

Operator $c^{(4)}_{q^2H^2D^3}$ is significant but causes **EFT breakdown** at $\Lambda = 1.2\,\mathrm{TeV}$ due to $\hat{s}^3$ scaling $\Rightarrow$ **exclude** since requires $\Lambda > 3\,\mathrm{TeV}$

Type	(480 GeV, 2.5)	SM Deviation (%)	(600 GeV, 3.0)	SM Deviation (%)
SM	0.1375(2)	-	0.1239(2)	-
$D=6$	$0.1357(7)^{+0.0089}_{-0.0090}$	[−7.9, +5.2]	$0.1219(6)^{+0.0077}_{-0.0063}$	[−6.8, +4.5]
$D=6+(6\times6)$	$0.1355(7)^{+0.0087}_{-0.0077}$	[−7.1, +4.9]	$0.1221(6)^{+0.0080}_{-0.0065}$	[−6.8, +4.9]

Type	(480 GeV, 2.5)	SM Deviation (%)	(600 GeV, 3.0)	SM Deviation (%)
SM	0.1375(2)	-	0.1239(2)	-
Coefficients at $D=8$				
$c^{(1)}_{q^4H^2}$	**0.1396(2)**	**+1.5**	**0.1261(2)**	**+1.8**
$c^{(2)}_{q^4H^2}$	0.1367(3)	0.6	0.1234(2)	-0.4
$c^{(3)}_{q^4H^2}$	**0.1512(3)**	**10.0**	**0.1359(2)**	**+9.7**
$c^{(1)}_{d^4H^2}$	0.1376(2)	+0.1	0.1240(2)	+0.1
$c^{(1)}_{u^4H^2}$	0.1380(3)	+0.4	0.1250(2)	+0.9
$c^{(1)}_{u^2d^2H^2}$	0.1374(3)	-0.1	0.1238(2)	-0.1
$c^{(1)}_{q^2d^2H^2}$	0.1377(3)	+0.1	0.1222(3)	-1.4
$c^{(2)}_{q^2d^2H^2}$	0.1370(3)	-0.4	0.1237(3)	-0.2
$c^{(1)}_{q^2u^2H^2}$	0.1372(2)	-0.2	0.1239(3)	0.0
$c^{(2)}_{q^2u^2H^2}$	0.1385(2)	+0.7	0.1252(3)	+1.0
$c^{(1)}_{q^2BH^2D}$	0.1374(3)	-0.1	0.1243(3)	+0.3
$c^{(3)}_{q^2BH^2D}$	0.1374(3)	0.0	0.1243(2)	+0.2
$c^{(1)}_{q^2WH^2D}$	0.1375(2)	+0.2	0.1241(2)	+0.2
$c^{(3)}_{q^2WH^2D}$	**0.1408(3)**	**+2.4**	**0.1270(2)**	**+2.5**
$c^{(5)}_{q^2WH^2D}$	0.1372(3)	-0.2	0.1240(3)	+0.1
$c^{(1)}_{u^2WH^2D}$	0.1381(2)	+0.4	0.1241(3)	+0.2
$c^{(1)}_{u^2BH^2D}$	0.1375(3)	0.0	0.1242(2)	+0.2
$c^{(1)}_{d^2WH^2D}$	0.1373(3)	-0.1	0.1239(2)	0.0
$c^{(1)}_{d^2BH^2D}$	0.1375(3)	0.0	0.1241(2)	+0.2
$c^{(1)}_{q^2H^2D^3}$	0.1376(3)	+0.1	0.1240(2)	+0.1
$c^{(2)}_{q^2H^2D^3}$	0.1372(3)	-0.2	0.1240(2)	+0.1
$c^{(3)}_{q^2H^2D^3}$	**0.1439(3)**	**+4.7**	**0.1299(2)**	**+4.8**
$c^{(4)}_{q^2H^2D^3}$ (*)	**0.1419(3)**	**+3.2**	**0.1280(3)**	**+3.3**
$c^{(1)}_{u^2H^2D^3}$	0.1380(3)	+0.4	0.1244(3)	+0.4
$c^{(1)}_{d^2H^2D^3}$	0.1371(2)	-0.3	0.1239(2)	0.0

(*) $(D=8)^2 > (D=8)\times\mathrm{SM}$

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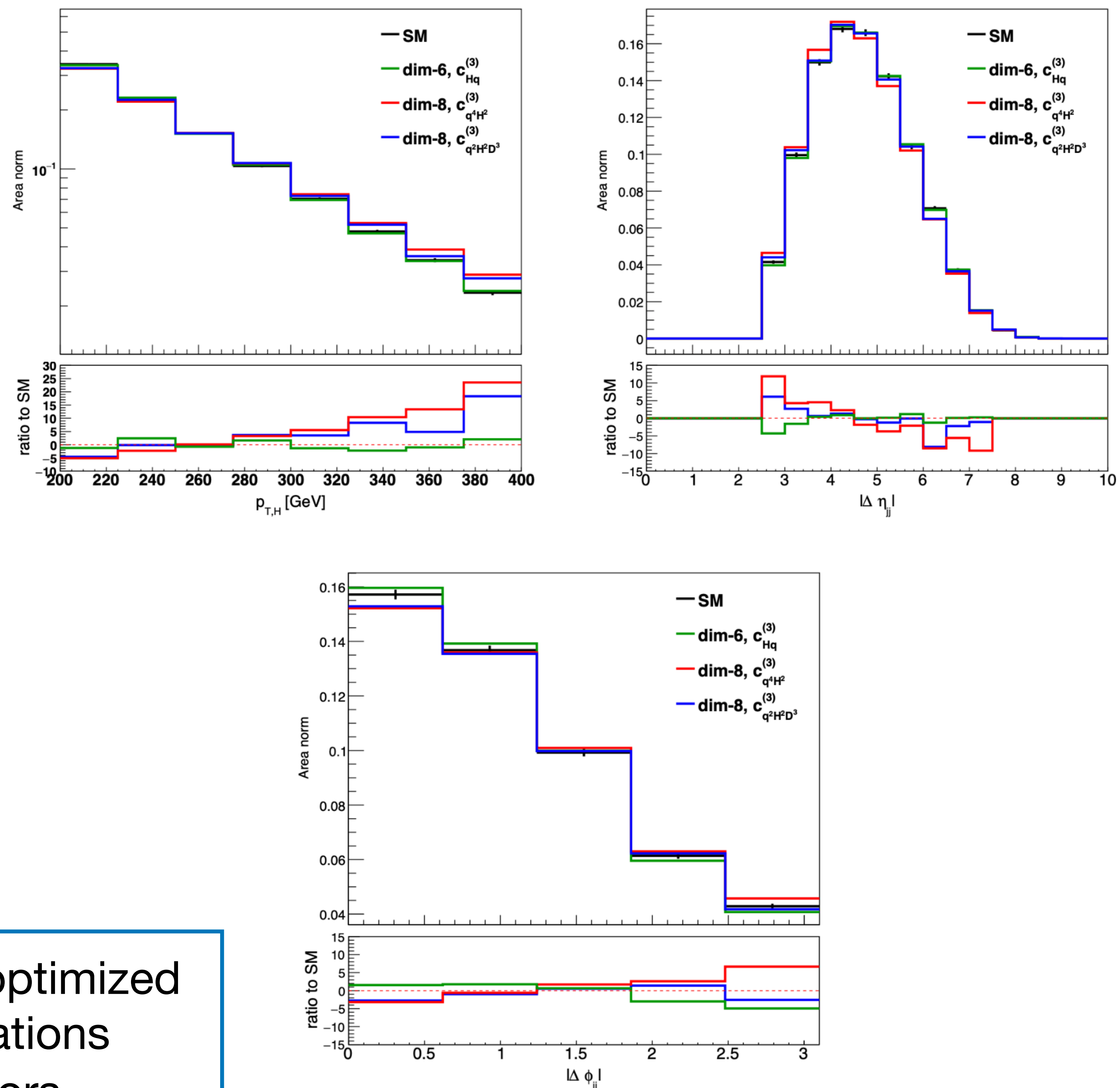
Observable distributions

$D = 8$ operators influence high p_T^H regions more than $D = 6$ operators

Small c_6 **LEP constrained** values largely suppress $D = 6$ impacts

Angular distributions **subtle differences** among SMEFT operator

Operators $c_{Hq}^{(3)}$ and $c_{q^2H^2}^{(3)}$ minimally affect angular distributions while $c_{q^2H^2D^3}^{(3)}$ causes noticeable shifts



Takeaway: Observables at high p_T^H, optimized kinematic cuts and observable correlations **needed to distinguish** $D = 8$ operators

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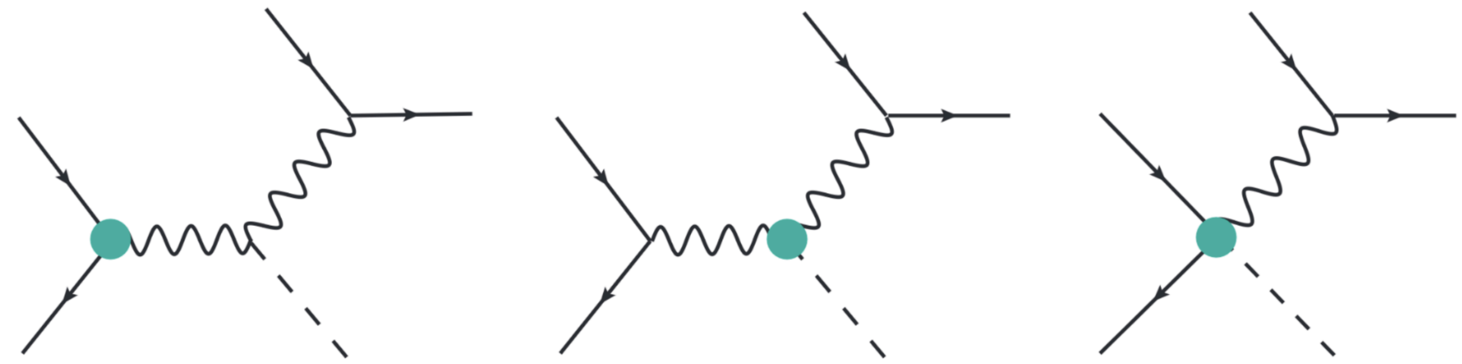
Crossed-process: Associated production $pp \to V(\bar{q}q)H$

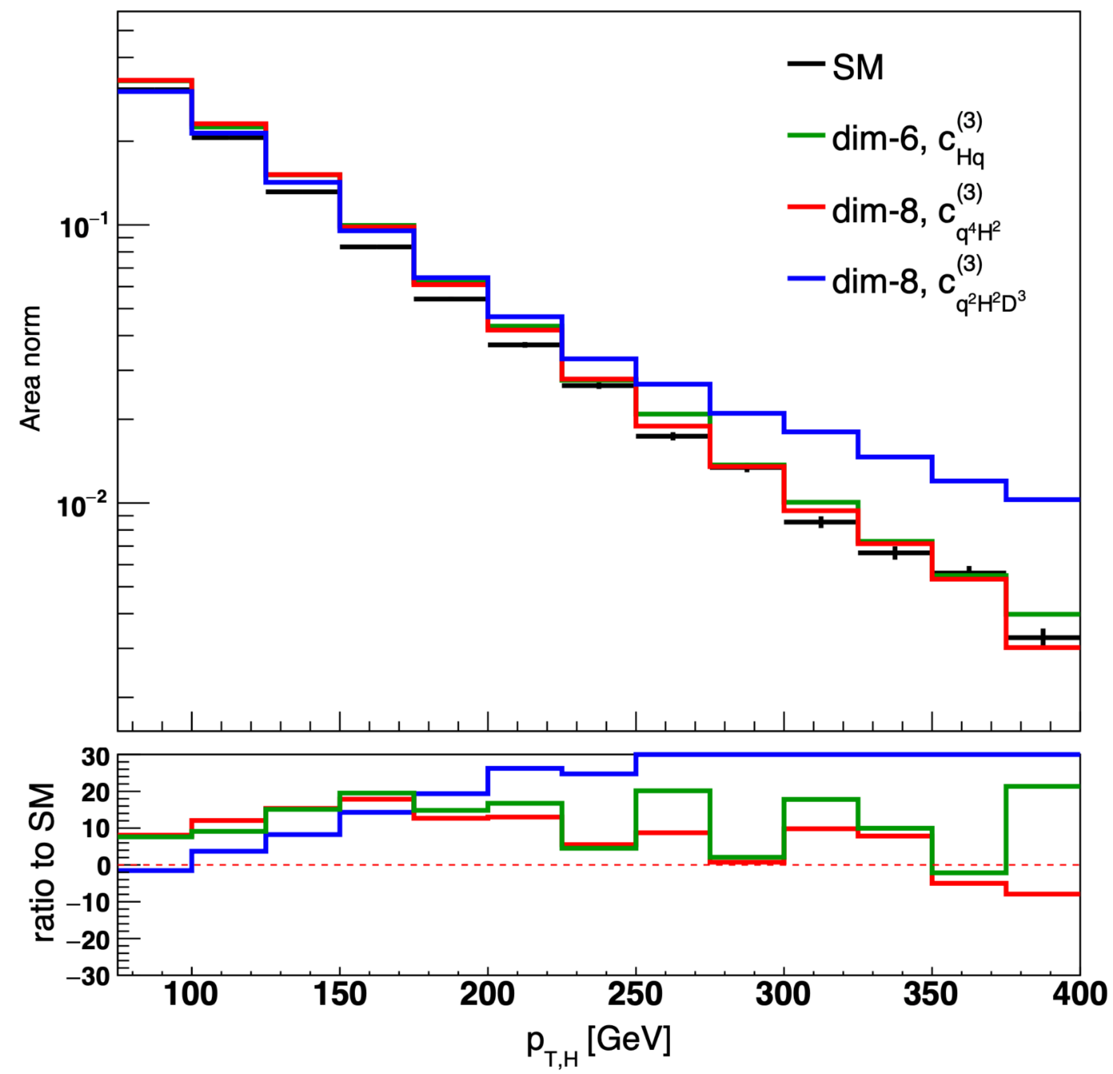
Crossing initial fermion transforms VBF topology to $pp \to V(\bar{q}q)H$



Simulated $pp \to Z(\bar{q}q)H$ with $75\,\mathrm{GeV} \le \mathrm{p_{T,Z}} \le 400\,\mathrm{GeV}$ and $70 \le m_{jj} \le 110\,\mathrm{GeV} \leftrightarrow$ STSX binning strategy [Corbett et al '23]

Operator $c^{(3)}_{q^2H^2D^3}$ significantly impacts $p_{T,H}$ **affecting both** VBF and VH production

Operator $c^{(3)}_{q^4H^2}$ **negligible** effect on VH production since analysis cuts break crossing symmetry
$\Rightarrow$ **deviations only in VBF**



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What next?

Application in SMEFT fits, PDFs, and uncertainty estimation

BA, Hobbs, Martin *In preparation*

BA, Martin, Shephard *In preparation*

More detailed analyses of other important HL-LHC & future collider processes

Systematically refining even further beyond kinematics

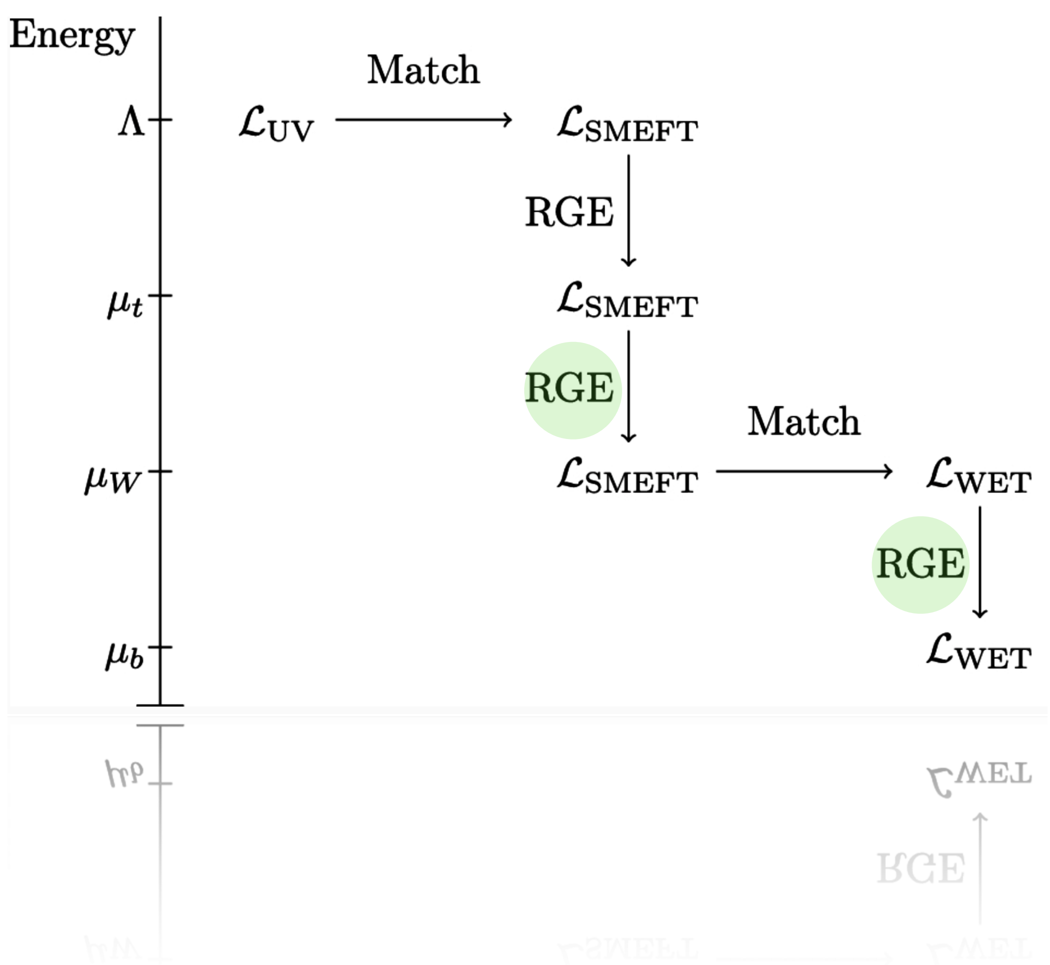
BA, Helset, Martin *In preparation*

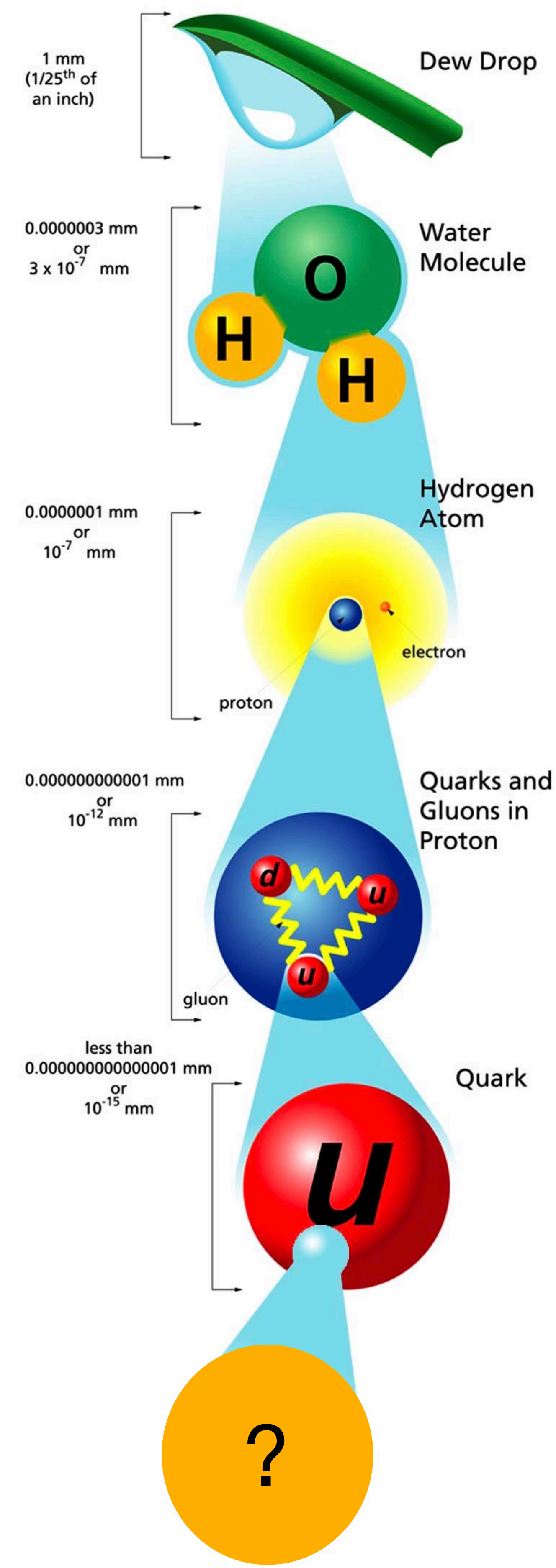
New formalisms, ideas welcome!

Back-up

Outline

- EFT structure in general
  - Field space geometry **for calculations**
  - Scattering amplitudes
  - Beyond scalars
  - 1-loop RGEs for SMEFT
  - Energy-enhanced expansion **for discovery**





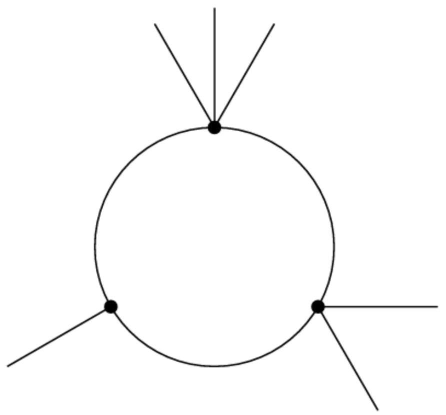
1-loop application: Renormalisation

The 2nd variation has the form [t'Hooft '74, Alonso, Manohar et al '20]

$$\delta_{\eta\eta} S = \frac{1}{2} \int \mathrm{d}^4 x \left\{ h_{IJ} (\mathscr{D}_\mu \eta)^I (\mathscr{D}_\mu \eta)^J + X_{IJ} \eta^I \eta^J \right\}$$

and 1-loop pole is given by

$$\Delta S = \frac{1}{32\pi^2 \epsilon} \int \mathrm{d}^4 x \left\{ \frac{1}{12} \mathrm{Tr}\left[Y_{\mu\nu} Y^{\mu\nu} \right] + \frac{1}{2} \mathrm{Tr}\left[\mathcal{X}^2 \right] \right\}$$



applied to **scalar-gauge theory**

$$\left[\widetilde{\mathscr{D}}_\mu, \widetilde{\mathscr{D}}_\nu \right]^i{}_j = \left[\widetilde{Y}_{\mu\nu} \right]^i{}_j = \widetilde{R}^i{}_{jkl} (D_\mu Z)^k (D_\nu Z)^l + \widetilde{\nabla}_j \widetilde{t}^i_C F^C_{\mu\nu} \qquad Z^i_\mu = \begin{bmatrix} (D_\mu \phi)^I \\ F^{A\,\mu_A}_\mu \end{bmatrix}$$

$$\widetilde{\mathscr{D}}_\mu \begin{bmatrix} \eta^I \\ \zeta^A_\lambda \end{bmatrix} = \partial_\mu \begin{bmatrix} \eta^I \\ \zeta^A_\lambda \end{bmatrix} + \begin{bmatrix} t^I_{C,J} A^C_\mu + \Gamma^I_{LJ} (D_\mu \phi)^L & -\Gamma^I_{CB} F^C_{\mu\sigma} \\ \Gamma^A_{CJ} F^C_{\mu\lambda} & -f^A_{CB} A^C_\mu \eta_{\lambda\sigma} + \Gamma^A_{LB} (D_\mu \phi)^L \eta_{\lambda\sigma} \end{bmatrix} \begin{bmatrix} \eta^J \\ \zeta^B_\sigma \end{bmatrix}$$

with parts read from each 2nd variation

$$\mathcal{X}^I{}_J = h^{IK} X_{KJ} \qquad \mathcal{X} = \begin{bmatrix} [\mathcal{X}_{\eta\eta}]^I{}_J & [\mathcal{X}_{\eta\zeta}]^I{}_{(B\mu_B)} \\ [\mathcal{X}_{\eta\zeta}]^{(A\mu_A)}{}_J & [\mathcal{X}_{\zeta\zeta}]^{(A\mu_A)}{}_{(B\mu_B)} \end{bmatrix}$$

Renormalisation

One-loop RGE from **2nd variation** of action $\psi^a \to \psi^a + \chi^a$

$$\delta_{\bar{\chi}\chi} S = \int \mathrm{d}^4 x \left\{ \frac{1}{2} i k_{\bar{p}r} \left(\bar{\chi}^{\bar{p}} \gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \chi^r \right) - \bar{\chi}^{\bar{p}} \mathcal{M}_{\bar{p}r} \chi^r + \bar{\chi}^{\bar{p}} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar{p}r} \chi^r \right\}$$

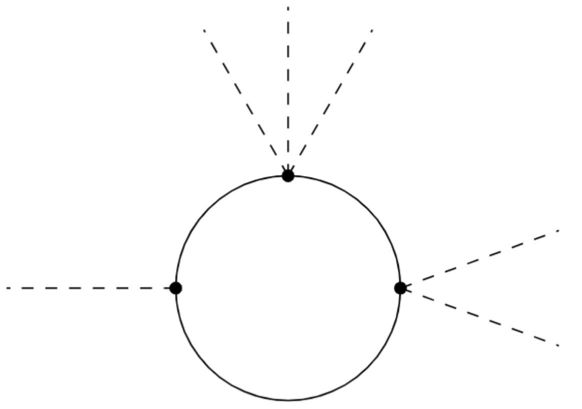
with **covariant derivative** $\mathscr{D}_\mu = \partial_\mu \mathbf{1} + \omega_\mu$ and fermion fluctuations $\chi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}$

The metric, mass and dipole terms

$$k = \begin{pmatrix} \kappa_L & 0 \\ 0 & \kappa_R \end{pmatrix}, \quad \mathcal{M} = \begin{pmatrix} 0 & M \\ M^\dagger & 0 \end{pmatrix} \quad \mathcal{T}^{\mu\nu} = \begin{pmatrix} 0 & T^{\mu\nu} \\ T^{\mu\nu\dagger} & 0 \end{pmatrix} \quad \omega_{\bar{p}rI} = \begin{pmatrix} \omega_{L,\bar{p}rI} & 0 \\ 0 & \omega_{R,\bar{p}rI} \end{pmatrix}$$

gives **covariant** result for $\chi\bar{\chi}$-variation

$$\begin{aligned} \Delta S = \frac{1}{32\pi^2 \epsilon} \int \mathrm{d}^4 x \Big\{ & \frac{1}{3} \mathrm{Tr}\left[\mathcal{Y}_{\mu\nu} \mathcal{Y}^{\mu\nu}\right] + \mathrm{Tr}\left[(\mathcal{D}_\mu \mathcal{M})(\mathcal{D}^\mu \mathcal{M}) - (\mathcal{M}\mathcal{M})^2\right] \\ & - \frac{16}{3} \mathrm{Tr}[(\mathcal{D}_\mu \mathcal{T}^{\mu\alpha})(\mathcal{D}_\nu \mathcal{T}^{\nu\alpha}) - (\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^2] \\ & - 4i \mathrm{Tr}[\mathcal{Y}_{\mu\nu}(\mathcal{M}\mathcal{T}^{\mu\nu} + \mathcal{T}^{\mu\nu}\mathcal{M})] - 8 \mathrm{Tr}(\mathcal{M}\mathcal{T}^{\mu\nu})^2 \Big\} \end{aligned}$$



SMEFT: bosons

We can apply general EFT to the SMEFT by **identification**

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi^2 + i\phi^1 \\ \phi^4 - i\phi^3\end{pmatrix} \qquad A^B_\mu = \begin{pmatrix} G^{\mathscr{A}}_\mu \\ W^a_\mu \\ B_\mu \end{pmatrix}$$

with **scalar metric**

$$\begin{aligned} h_{IJ} =& \delta_{IJ}\left[1 + \frac{1}{4}\left({}^{8}C^{(1)}_{H^6D^2} - {}^{8}C^{(2)}_{H^6D^2}\right)(\phi^K\phi^K)^2\right] + \left(-2 \;\; {}^{6}C_{H^4\Box}\right)\phi^I\phi^J \\ &+ \frac{1}{2}\left[{}^{6}C_{H^4D^2} + {}^{8}C^{(2)}_{H^6D^2}(\phi^K\phi^K)\right]\mathcal{H}_{IJ}(\phi)\,, \end{aligned}$$

$$\mathcal{H}_{IJ}(\phi) = \phi_I\phi_J + \begin{bmatrix} \phi_2^2 & -\phi_1\phi_2 & -\phi_2\phi_4 & \phi_2\phi_3 \\ -\phi_1\phi_2 & \phi_1^2 & \phi_1\phi_4 & -\phi_1\phi_3 \\ -\phi_2\phi_4 & \phi_1\phi_4 & \phi_4^2 & -\phi_3\phi_4 \\ \phi_2\phi_3 & -\phi_1\phi_3 & -\phi_3\phi_4 & \phi_3^2 \end{bmatrix}$$

and **gauge metric**

$$g_{AB} = \begin{bmatrix} [g_{GG}]_{\mathscr{A}\mathscr{B}} & 0 & 0 \\ 0 & [g_{WW}]_{ab} & [g_{WB}]_a \\ 0 & [g_{BW}]_b & g_{BB} \end{bmatrix}$$

SMEFT: fermions

Again applying formalism to the SMEFT

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^2 + i\phi^1 \\ \phi^4 - i\phi^3 \end{pmatrix} \qquad A^B_\mu = \begin{pmatrix} G^{\mathscr{A}}_\mu \\ W^a_\mu \\ B_\mu \end{pmatrix} \qquad \psi^p = \begin{pmatrix} \ell^p_L \\ q^p_L \\ e^p_R \\ u^p_R \\ d^p_R \end{pmatrix}$$

with SM Lagrangian

$$\mathcal{L} = -\frac{1}{4} F^A_{\mu\nu} F^{A\,\mu\nu} + (D_\mu H)^\dagger (D^\mu H) - \lambda \left(H^\dagger H - \frac{1}{2} v^2 \right)^2 + \delta_{\bar{p}r} i \bar{\psi}^{\bar{p}} \gamma^\mu D_\mu \psi^r - \bar{\psi}^{\bar{p}} \mathcal{M}_{\mathrm{SM},\bar{p}r} \psi^r$$

and identifying e.g. for RH electrons in SMEFT

$$M_{\bar{p}r} \supset [Y_e]^\dagger_{\bar{p}r} H - {}^6C_{\underset{\bar{p}r}{leH^3}} H (H^\dagger H) - {}^8C_{\underset{\bar{p}r}{leH^5}} H (H^\dagger H)^2$$

$$T^{\mu\nu}_{\bar{p}r} \supset {}^6C_{\underset{\bar{p}r}{leBH}} H \frac{1}{2} \left(B^{\mu\nu} - i \tilde{B}^{\mu\nu} \right) + {}^8C_{\underset{\bar{p}r}{leBH^3}} H (H^\dagger H) \frac{1}{2} \left(B^{\mu\nu} - i \tilde{B}^{\mu\nu} \right)$$

$$\omega_{R,\bar{p}rI} \supset + i (\phi \gamma_4)_I \, {}^6Q^{(1)}_{\underset{\bar{p}r}{e^2H^2D}}$$

Renormalisation

One-loop RGE from **2nd variation** of action $\psi^a \to \psi^a + \chi^a$

$$\delta_{\bar\chi\chi} S = \int \mathrm{d}^4x \left\{ \frac{1}{2} i k_{\bar p r} \left(\bar\chi^{\bar p} \gamma^\mu \overleftrightarrow{\mathcal{D}}_\mu \chi^r \right) - \bar\chi^{\bar p} \mathcal{M}_{\bar p r} \chi^r + \bar\chi^{\bar p} \sigma_{\mu\nu} \mathcal{T}^{\mu\nu}_{\bar p r} \chi^r \right\}$$

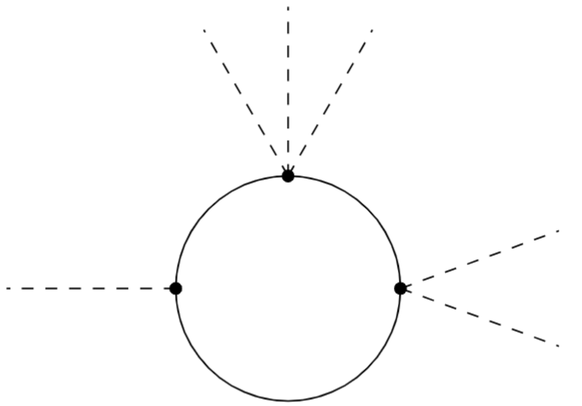
with **covariant derivative** $\mathscr{D}_\mu = \partial_\mu \mathbf{1} + \omega_\mu$ and fermion fluctuations $\chi = \begin{pmatrix} \chi_L \\ \chi_R \end{pmatrix}$

The metric, mass and dipole terms

$$k = \begin{pmatrix} \kappa_L & 0 \\ 0 & \kappa_R \end{pmatrix}, \qquad \mathcal{M} = \begin{pmatrix} 0 & M \\ M^\dagger & 0 \end{pmatrix} \qquad \mathcal{T}^{\mu\nu} = \begin{pmatrix} 0 & T^{\mu\nu} \\ T^{\mu\nu\dagger} & 0 \end{pmatrix} \qquad \omega_{\bar p r I} = \begin{pmatrix} \omega_{L,\bar p r I} & 0 \\ 0 & \omega_{R,\bar p r I} \end{pmatrix}$$

gives **covariant** result for $\chi\bar\chi$-variation

$$\begin{aligned} \Delta S = & \frac{1}{32\pi^2\epsilon} \int \mathrm{d}^4x \left\{ \frac{1}{3} \mathrm{Tr}\left[\mathcal{Y}_{\mu\nu}\mathcal{Y}^{\mu\nu}\right] + \mathrm{Tr}\left[(\mathcal{D}_\mu\mathcal{M})(\mathcal{D}^\mu\mathcal{M}) - (\mathcal{M}\mathcal{M})^2\right] \right. \\ & - \frac{16}{3}\mathrm{Tr}[(\mathcal{D}_\mu \mathcal{T}^{\mu\alpha})(\mathcal{D}_\nu \mathcal{T}^{\nu\alpha}) - (\mathcal{T}^{\mu\nu}\mathcal{T}^{\alpha\beta})^2] \\ & \left. - 4i\mathrm{Tr}[\mathcal{Y}_{\mu\nu}(\mathcal{M}\mathcal{T}^{\mu\nu} + \mathcal{T}^{\mu\nu}\mathcal{M})] - 8\mathrm{Tr}(\mathcal{M}\mathcal{T}^{\mu\nu})^2 \right\} \end{aligned}$$



More beyond geometry?

Recall: Higher-dim operators suppressed by $1/\Lambda$ so amp-squared SMEFT series

$$|\mathscr{A}|^2 = |A_{\rm SM}|^2 \left\{ 1 + \frac{2\text{Re}(A_{\rm SM}^* A_6)}{\Lambda^2 |A_{\rm SM}|^2} + \frac{1}{\Lambda^4}\left(\frac{|A_6|^2}{|A_{\rm SM}|^2} + \frac{2\text{Re}(A_{\rm SM}^* A_8)}{|A_{\rm SM}|^2} \right) + \cdots \right\}$$

Key Insight: Higher-dim operator effects can grow with $E \Rightarrow$ overcome naive suppression by powers of $1/\Lambda$ when $E \sim \Lambda$

Geometry $\leftrightarrow$ metric re-summation of higher-dimensional operators in $(\phi^2 \sim (HH^\dagger) \sim v^2)/\Lambda^2$ but **not** $E/\Lambda \Rightarrow$ **need more** for $E \gg v$

ID higher-dim **multi-particle operators** that grow with energy and have the most significant impact on high-energy processes

Energy-enhanced geoSMEFT operators

In regime $E \gg v$ the terms in $\mathscr{A}_6$ and $\mathscr{A}_8$ that incorporate the highest powers of E carry the largest impact

$2 \to 3$ amplitudes have mass dimension -1 with naive **scaling**

[BA, Martin, *In preparation*]

$$\mathscr{A}_{\text{SM}} \sim g_{\text{SM}}^3 \frac{v}{E^2}, \quad \mathscr{A}_{Hq}, \mathscr{A}_{Hu,d} \sim g_{\text{SM}}^2 \frac{c_6\, v}{\Lambda^2}, \quad \mathscr{A}_{q^2H^2XD}, \mathscr{A}_{q^2H^2D^3} \sim g_{\text{SM}}^2 \frac{c_8\, v\, E^2}{\Lambda^4}, \quad \mathscr{A}_{q^4H^2} \sim \frac{c_8\, v\, E^2}{\Lambda^4}$$

The ratio of $D=8$ interference piece to the $D=6$

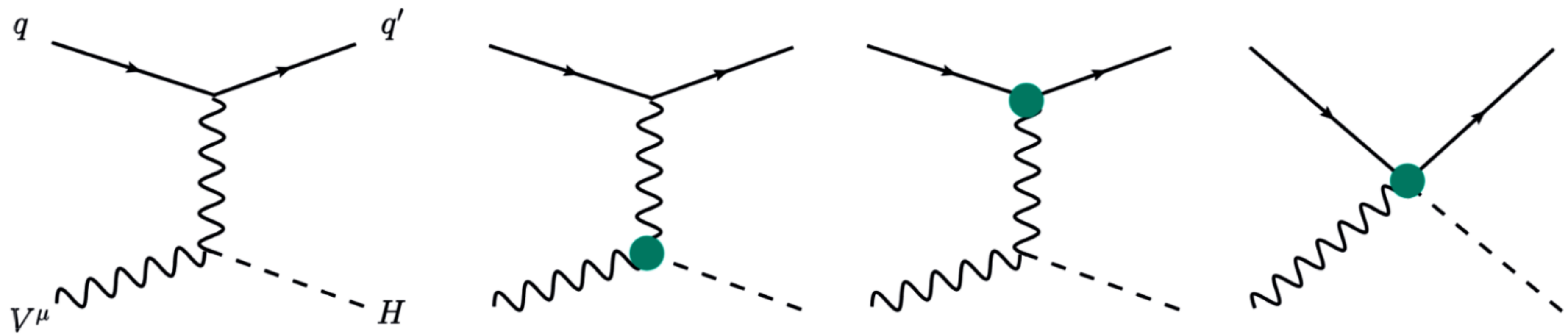
$$\frac{\mathscr{A}^*_{\text{SM}}\mathscr{A}_8}{\mathscr{A}^*_{\text{SM}}\mathscr{A}_6} \sim \left(\frac{c_8}{c_6}\right)\left(\frac{E^2}{\Lambda^2}\right)$$

For fixed $\Lambda \sim \text{TeV}$ the Wilson coefficients for E-enhanced $D=6$ operators such as $c_{Hq}^{(3)} \ll 1$ to be consistent with LEP

[Ellis et al. '20]

Analysis of energy-enhanced contributions to VBF

Consider $qV \to q'H$ as proxy for VBF to ID most enhanced SMEFT operators



High-E limit $\hat{t} \gg m_V$ with V_L effects grow the strongest with E once $qV \to q'H$

$$\mathscr{A}(qZ_{L,\mu} \to qH) \underset{\hat{t}\gg m^2_{Z,H}}{=} -i\langle \bar{q} | \gamma_\mu p^\mu_H | q] \frac{1}{\hat{t}} \left(g_{Zq_Lq_L} g^{(1)}_{HZZ} + g^{(1)}_{ZHq_Lq_L} \frac{\hat{t}}{\Lambda^2} + (g^{(2)}_{ZHq_Lq_L} - g^{(3)}_{ZHq_Lq_L}) \frac{\hat{t}^2}{2\Lambda^4} \right)$$

4-particle contact terms scale with higher powers of $\hat{t}$

$$\mathscr{A}(qW_{L,\mu} \to q'H) \underset{\hat{t}\gg m^2_{W,H}}{=} -i\langle \bar{q} | \gamma_\mu p^\mu_H | q] \frac{1}{\hat{t}} \left(g_{Wq_Lq'_L} g^{(1)}_{HWW} + g^{(1)}_{WHq_Lq'_L} \frac{\hat{t}}{\Lambda^2} + (g^{(2)}_{WHq_Lq'_L} - g^{(4)}_{WHq_Lq'_L}) \frac{\hat{t}^2}{2\Lambda^4} - g^{(3)}_{WHq_Lq'_L} \frac{\hat{t}\,(2\hat{s} + \hat{t})}{2\Lambda^4} \right)$$

New terms involving quark momenta $\propto \hat{s}\hat{t}$ and **dominate** when $\hat{s}$ is large but $\hat{t}$ remains small; other SMEFT contributions are **suppressed** in $\hat{t}$

Total cross-sections

Effective W approximation: treating incoming W as proton constituent in the $2 \to 3$ process ⇒ convolving the W-boson PDF with the $qV \to q'H$ in the limit $\hat{t} \to 0$

[Dawson '84]

Dominant $D = 6$ terms are suppressed at large $\hat{s}$ with W_T ⇒ Focus on W_L integrating scattering angle up to $\theta^*_{\max} = p_{T,W}/E_W$

$$\int^{\theta_{\max}} d\theta^* \, 2\,\mathrm{Re}(A_{\mathrm{SM}} A^{(6)})_{W_L} \sim \frac{v^2\,\hat{s}}{\Lambda^2\, m_W^2} \qquad \int^{\theta_{\max}} d\theta^* \, |A^{(6)}|^2_{W_L} \sim \frac{v^2\,\hat{s}}{\Lambda^4},$$

Dominant $D = 8$ **interference terms** from operators leads to **different scaling** for ~ $c^{(3)}_{q^2H^2D^3}, c^{(3)}_{q^2H^2WD}$ vs. $c^{(4)}_{q^2H^2D^3}$ ↔ operators with different Lorentz structures

$$\int^{\theta_{\max}} d\theta^* \, 2\,\mathrm{Re}(A_{\mathrm{SM}} A_3^{(8)})_{W_L} \sim \frac{v^2\,\hat{s}^2}{\Lambda^4\, m_W^2} \qquad \int^{\theta_{\max}} d\theta^* \, 2\,\mathrm{Re}(A_{\mathrm{SM}} A_{24}^{(8)})_{W_L} \sim \frac{v^2\,\hat{s}}{\Lambda^4}$$

Squared terms exhibit larger differences

$$\int^{\theta_{\max}} d\theta^* \, |A_3^{(8)}|^2_{W_L} \sim \frac{v^2\,\hat{s}^3}{\Lambda^8} \qquad \int^{\theta_{\max}} d\theta^* \, |A_{24}^{(8)}|^2_{W_L} \sim \frac{v^2\,\hat{s}\, m_W^4}{\Lambda^8}$$

[BA and Martin 2410.25163]

Effective W approximation

Additionally: The operator $c^{(3)}_{q^2H^2D^3}$ interferes with the SM for W_T

$$\int^{\theta_{\max}} d\theta^* \, 2\,\mathrm{Re}(A_{\rm SM}A^{(8)}_{24})_{W_T} \sim \frac{v^2\,\hat{s}}{\Lambda^4} \qquad \int^{\theta_{\max}} d\theta^* \, |A^{(8)}_{24}|^2_{W_T} \sim \frac{v^2\,\hat{s}\,m_W^4}{\Lambda^8}$$

But this weaker interference is **offset** by larger transverse W PDFs

[Dawson '84]

Determining whether T or L effects dominate requires **numerical analysis** beyond $2 \to 2$ approximations

New pure contact $D = 8$ vertices from q^4H^2 operators contribute in VBF with largest effect from $(LL)(LL)$ helicity structures

$$\mathscr{A}(u_L d_L \to u_L d_L H) \sim v c^{(3)}_{q^4H^2}\langle 34\rangle[12]$$

[BA and Martin 2410.25163]

Energy-enhanced contributions to VBF

Dimension 6

	Operator	relevant ψ
$Q_{H\psi}^{(1)}$	$i(\bar{\psi}_p\gamma^\nu\psi_r)H^\dagger \overleftrightarrow{D}_\mu H$	$\psi=\{q,u,d\}$
$Q_{H\psi}^{(3)}$	$i(\bar{\psi}\gamma^\nu\sigma^I\psi)\,H^\dagger \overleftrightarrow{D}_\mu\sigma_I H$	$\psi=\{q\}$

Remaining HVV and ffV vertices suppressed

Dimension 8

	Operator	relevant ψ
$Q_{\psi^2H^2D^3}^{(1)}$	$i(\bar{\psi}_p\gamma^\mu\psi_r)\left[(D_\nu H)^\dagger(D^2_{(\mu,\nu)}H)-(D^2_{(\mu,\nu)}H)^\dagger(D_\nu H)\right]$	$\psi=\{q,u,d\}$
$Q_{\psi^2H^2D^3}^{(2)}$	$i(\bar{\psi}_p\gamma^\mu\overleftrightarrow{D}_\nu\psi_r)\left[(D_\mu H)^\dagger(D_\nu H)+(D_\nu H)^\dagger(D_\mu H)\right]$	$\psi=\{q,u,d\}$
$Q_{\psi^2H^2D^3}^{(3)}$	$i(\bar{\psi}_p\gamma^\mu\sigma^I\psi_r)\left[(D_\nu H)^\dagger\tau^I(D^2_{(\mu,\nu)}H)-(D^2_{(\mu,\nu)}H)^\dagger\sigma^I(D_\nu H)\right]$	$\psi=\{q\}$
$Q_{\psi^2H^2D^3}^{(4)}$	$i(\bar{\psi}_p\gamma^\mu\sigma^I\overleftrightarrow{D}_\nu\psi_r)\left[(D_\mu H)^\dagger\tau^I(D_\nu H)+(D_\nu H)^\dagger\tau^I(D_\mu H)\right]$	$\psi=\{q\}$

	Operator
$Q_{q^4H^2}^{(1)}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_p\gamma_\mu q_r)(H^\dagger H)$
$Q_{q^4H^2}^{(2)}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_p\gamma_\mu\sigma^I q_r)(H^\dagger\sigma^I H)$
$Q_{q^4H^2}^{(3)}$	$(\bar{q}_p\gamma^\mu\sigma^I q_r)(\bar{q}_p\gamma_\mu\sigma^I q_r)(H^\dagger H)$
$Q_{u^4H^2}^{(1)}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{u}_p\gamma_\mu u_r)(H^\dagger H)$
$Q_{d^4H^2}^{(1)}$	$(\bar{d}_p\gamma^\mu d_r)(\bar{d}_p\gamma_\mu d_r)(H^\dagger H)$
$Q_{u^2d^2H^2}^{(1)}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{d}_p\gamma_\mu d_r)(H^\dagger H)$
$Q_{q^2u^2H^2}^{(1)}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{u}_p\gamma_\mu u_r)(H^\dagger H)$
$Q_{q^2u^2H^2}^{(2)}$	$(\bar{q}_p\gamma^\mu\sigma^I q_r)(\bar{u}_p\gamma_\mu u_r)(H^\dagger\sigma^I H)$
$Q_{q^2d^2H^2}^{(1)}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{d}_p\gamma_\mu d_r)(H^\dagger H)$
$Q_{q^2d^2H^2}^{(2)}$	$(\bar{q}_p\gamma^\mu\sigma^I q_r)(\bar{d}_p\gamma_\mu d_r)(H^\dagger\sigma^I H)$

	Operator	relevant ψ
$Q_{\psi^2BH^2D}^{(1)}$	$(\bar{\psi}_p\gamma^\nu\psi_r)D^\mu(H^\dagger H)B_{\mu\nu}$	$\psi=\{q,u,d\}$
$Q_{\psi^2BH^2D}^{(2)}$	$i(\bar{\psi}_p\gamma^\nu\psi_r)(H^\dagger\overleftrightarrow{D}^\mu H)B_{\mu\nu}$	$\psi=\{q,u,d\}$
$Q_{\psi^2BH^2D}^{(3)}$	$(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)D^\mu(H^\dagger\sigma^I H)B_{\mu\nu}$	$\psi=\{q\}$
$Q_{\psi^2BH^2D}^{(4)}$	$i(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)B_{\mu\nu}$	$\psi=\{q\}$
$Q_{\psi^2WH^2D}^{(1)}$	$(\bar{\psi}_p\gamma^\nu\psi_r)D^\mu(H^\dagger\sigma^I H)W^I_{\mu\nu}$	$\psi=\{q,u,d\}$
$Q_{\psi^2WH^2D}^{(2)}$	$i(\bar{\psi}_p\gamma^\nu\psi_r)(H^\dagger\overleftrightarrow{D}^{I\mu}H)W^I_{\mu\nu}$	$\psi=\{q,u,d\}$
$Q_{\psi^2WH^2D}^{(3)}$	$(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)D^\mu(H^\dagger H)W^I_{\mu\nu}$	$\psi=\{q\}$
$Q_{\psi^2WH^2D}^{(4)}$	$i(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)(H^\dagger\overleftrightarrow{D}^\mu H)W^I_{\mu\nu}$	$\psi=\{q\}$
$Q_{\psi^2WH^2D}^{(5)}$	$\epsilon_{IJK}(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)D^\mu(H^\dagger\sigma^J H)W^K_{\mu\nu}$	$\psi=\{q\}$
$Q_{\psi^2WH^2D}^{(6)}$	$i\epsilon_{IJK}(\bar{\psi}_p\gamma^\nu\sigma^I\psi_r)(H^\dagger\overleftrightarrow{D}^{J\mu}H)W^K_{\mu\nu}$	$\psi=\{q\}$

From 993 to 41 E-enhanced operators for VBF up to $D=8$

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